学号：____ 姓名：____ 专业年级：____ 授课教师：____ 考场教室号：____ 座号：____

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中国海洋大学全日制本科课程期末考试试卷

_2023__ 年 Spring__ 季学期 考试科目：_Optimisation & Operations Research

学院：_海德学院_

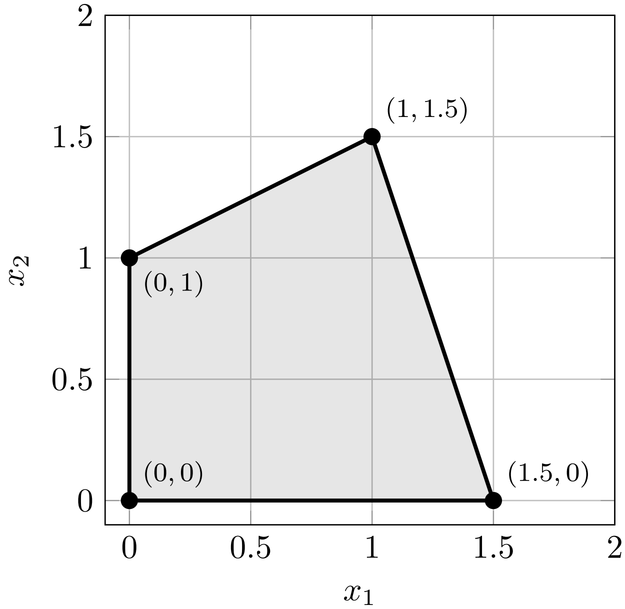
试卷类型：_SAMPLE EXAM__卷 命题人：_Dr Mike CHEN_ 审核人：_____

考试说明：本课程为闭卷考试，共__8_页，可携带计算器。

Question	1	2	3	4	5	Total
Marks	32	14	12	18	24	100

Question 1 [32 marks total]

(a) $\boxed{\text{6 marks}}$ Write the inequalities that describe the shaded feasible region specified by the thick lines. The (x_1, x_2) coordinates of each vertex are indicated.



Note that each line specifies one constraint. Ensure you include all constraints.

(b) $\boxed{\text{4 marks}}$ Fill in the missing parts of the Simplex Tableau corresponding to the following equations and optimisation objective:

$$\begin{array}{lrcrcrcr}
\max & z & = & -2x_1 & - & x_2 & - & 2 \\
\text{subject to} & & & & & & & \\
 & & - & x_1 & - & x_2 & \leq & 10 \\
 & & & 2x_1 & - & 2x_2 & \geq & 1
\end{array}$$

for non-negative variables x_1 and x_2.

			z	b
-1	-1		0	
			0	
			1	

Include the completed tableau on your answer sheet.

(c) $\boxed{\text{2 marks}}$ Explain the next step of the Simplex (phase II) algorithm for the following Tableau:

x_1	x_2	x_3	x_4	z	b
3	3	1	0	0	3
0	-1	0	1	0	2
-2	1	0	0	1	1

(d) $\boxed{\text{2 marks}}$ Explain the next step of the Simplex (phase II) algorithm.

x_1	x_2	x_3	x_4	x_5	x_6	x_7	z	b
0	$5/2$	1	$-1/2$	1	$-1/2$	0	0	$3/2$
1	$-1/2$	0	1	$-1/2$	$1/2$	0	0	$1/2$
0	$7/2$	0	0	$-1/2$	$1/2$	1	0	$7/2$
0	$11/2$	0	1	$3/2$	$-1/2$	0	1	$13/2$

(e) $\boxed{\text{2 marks}}$ Explain the next step of the Simplex (phase II) algorithm.

x_1	x_2	x_3	x_4	x_5	x_6	x_7	z	b
0	4	4	1	0	0	0	0	3
0	4	-1	0	1	0	0	0	5
1	$2/3$	1	0	0	$1/3$	0	0	$4/3$
0	$-4/3$	-1	0	0	$-2/3$	1	0	$7/3$
0	$7/3$	1	0	0	$2/3$	0	1	$2/3$

(f) $\boxed{\text{2 marks}}$ Explain the next step of the Simplex (phase II) algorithm.

x_1	x_2	x_3	x_4	z	b
2	0	1	0	0	1
3	-1	0	1	0	1
0	-1	0	0	1	-1

(g) $\boxed{\text{4 marks}}$ Is the following Tableau in feasible canonical form? **Explain your answer.**

x_1	x_2	x_3	x_4	x_5	x_6	x_7	z	b
1	2	1	$1/2$	$1/2$	0	0	0	2
2	-1	0	2	-1	1	0	0	1
-1	4	0	-1	0	0	1	0	3
1	5	0	2	1	0	0	1	-7

(h) $\boxed{\text{2 marks}}$ Complete the empty boxes for the next step of the Simplex algorithm.

x_1	x_2	x_3	s_4	s_5	z	b
2	0	1/2	1	−1/2	0	5/2
2	1	1/2	0	1/2	0	1/2
5	0	−1	0	1	1	3

x_1	x_2	x_3	s_4	s_5	z	b
		0			0	
		1			0	
		0			1	

Include the completed tableau on your answer sheet.

(i) $\boxed{\text{2 marks}}$ Complete the empty boxes for the next step of the Simplex algorithm.

x_1	x_2	x_3	x_4	x_5	x_6	z	b
4	4	2	1	1	0	0	3
0	3	−1	1	0	1	0	2
−1	−1	0	1	0	0	1	3

			1/4	1/4	0	0	3/4
			1	0	1	0	2
			5/4	1/4	0	1	

Include the completed tableau on your answer sheet.

(j) $\boxed{\text{2 marks}}$ Given the following Simplex Tableau, which variables are basic, and which are non-basic?

x_1	x_2	x_3	x_4	x_5	z	b
0	−1/4	1	1/4	0	0	1/4
4	0	0	0	1	0	6
1	−1/2	0	1/2	0	1	3/2

(k) $\boxed{\text{4 marks}}$ Given the following Simplex Tableau, what is the optimal solution to the problem?

x_1	x_2	x_3	x_4	x_5	x_6	z	b
0	1	1	1/4	0	0	0	1/4
1	1	0	1/4	1	0	0	5/4
4	2	0	0	0	1	0	2
0	2	0	1/2	0	0	1	−1/2

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Question 2 [14 marks total]

Translate the following problem into a Linear Program. Make sure you define all terms. Put the program into **standard inequality form**.

Do not solve the problem.

A manufacturer produces drinks. It can produce four types of drink:

- Orange
- Diet Orange
- Raspberry
- Diet Raspberry

The revenue for each type of drink is fixed at \$1 per liter, but the cost for each is different:

- the Diet versions cost \$0.05 per litre more than the regular version,
- Raspberry flavour costs \$0.04 per litre more than Orange.

Additionally,

- At least 20% of production must be devoted to each type of drink.
- The company can bottle at most 10,000 litres per week.

If the regular Orange drink costs $\$M$ per litre to make, and presuming that each type of drink makes at least a small profit, determine how the company should maximise profit.

(a) $\boxed{\text{2 marks}}$ What are the variables of the problem?

(b) $\boxed{\text{6 marks}}$ What is the objective?

(c) $\boxed{\text{4 marks}}$ What are the constraints of this problem?

(d) $\boxed{\text{2 marks}}$ State the complete problem written in standard inequality form.

Question 3 [12 marks total]

(a) $\boxed{\text{4 marks}}$ Write down the dual (D) of the following primal (P) linear program.

$$
\begin{array}{llrcrcrcr}
(P) & \max z & = & x_1 & + & 2x_2 & + & x_3 & + \ 3 \\
 & & & 2x_1 & + & x_2 & & & \leq \ 6 \\
 & & - & x_1 & + & x_2 & + & 2x_3 & = \ 4 \\
 & & & \multicolumn{6}{l}{x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0.}
\end{array}
$$

(b) $\boxed{\text{4 marks}}$ Write down the Complementary Slackness Relations (CSRs) for (P) and (D).

(c) $\boxed{\text{4 marks}}$ The solution to the primal is $x_1^* = 2/3$, $x_2^* = 14/3$ and $x_3^* = x_4^* = 0$, with $z^* = 13$. Calculate the solution to the Dual (D), verifying all of the CSRs in the process.

Question 4 [18 marks total]

Consider the following knapsack problem.

A hiker can choose from the following items when packing a knapsack:

Item	1 chocolate	2 raisins	3 camera	4 jumper	5 drink
w_i (kg)	0.5	0.4	0.8	1.6	0.6
v_i (value)	2.75	2.5	1	5	3.0

The hiker cannot carry more than 2.5 kg all together.

Use the **greedy heuristic** covered in the course to choose the number of each item to pack in order to maximise the total value of the goods packed, without violating the mass constraint.

Question 5 [24 marks total]

Solve the following optimisation problem using Branch and Bound.

Maximise

$$z = 5x_1 + 8x_2,$$

such that

$$4x_1 + 10x_2 \leq 11,$$

where $\mathbf{x}$ are non-negative and integer.

Illustrate your solution tree, and ensure you describe relaxed solutions to each along with the reasons to branch or bound at a particular point. You may find sketching the problem useful to understand the space.

End of examination questions.