THE UNIVERSITY of ADELAIDE

Optimsation & Operations Research

Haide College, Spring Semester

Tutorial 1

These questions are on Topic 1: Optimisation & linear programming

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1. **Translation:** A farmer has three crops he can grow: wheat, rice and cotton. Each results in revenue (per acre) of \$100, \$300, \$200. However, the rice and cotton must be irrigated taking 110 kilolitres and 100 kilolitres of water per acre per day, respectively, and the farmer has only 3000 kilolitres of water available per day for the whole farm. All must be fertilised, with respective costs of \$40, \$30 and \$20 per acre.

   Interrogate the problem and formulate an optimisation problem to tell the farmer how much of each crop to grow on his/her 50 acre farm to maximise his/her profits.

   Hints: remember to look for three things:

   1. the variables (the things you can control);
   2. the objective (the thing you want to maximise or minimise); and
   3. the constraints (there are 2 main constraints here, but don't forget non-negativity).

   Tabulate the data, and then construct the optimisation in standard form.

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2. **Translation:** Consider the following *portfolio management* problem. A bank has \$1 million to invest in variety of bonds offered by the government and other agencies. Assume that there are 4 bonds considered here, with quantities related to each denoted with $i = 1, 2, 3, 4$.

   Each bond has a *rated quality*, q_i, an *after-tax yield*, y_i and a *years to maturity*, m_i (how long the investment is committed). The portfolio manager must try to maximise the return on investment, but must also meet other criteria:

   1. the average quality of the portfolio of bonds cannot be worse than 1.5 (note that for quality, a low number corresponds to high-quality)
   2. the average years to maturity of the portfolio of bonds should not exceed 4 years.

a) What are the variables? *Hint: define variables* x_1, x_2, x_3 *and* x_4.

b) What is the objective?

c) Write a series of linear constraints. *Hint: there should be three.*

d) What are the bounds on the variables?

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3. **Interpretation:** Imagine that we took a problem expressed in the form

$$\begin{aligned} \max \quad & z = \mathbf{c}^T \mathbf{x} \\ \text{subject to} \quad & A'\mathbf{x} \leq \mathbf{b} \\ & \mathbf{x} \geq \mathbf{0} \end{aligned}$$

   and when we converted it into standard inequality form, we arrived at a Tableau for the equalities, on which we performed a series of row operations to obtain

$$\begin{array}{|ccccccc|}
1 & 1 & 0 & 1 & 0 & 0 & 5 \\
0 & 3 & 1 & 0 & -1 & 0 & 7 \\
0 & 3 & 0 & 1 & -1 & 1 & 4
\end{array}$$

a) Write down a solution to this problem with three basic, and three non-basic variables (Hint: it should be possible to do so immediately).

b) Interpret this solution in the light of the optimisation problem specified in terms on inequalities above.

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4. **Calculations:** Translate the following problem into standard equality form.

$$\begin{array}{lrcrcrcr}
\min & z & = & 2x_1 & - & 2x_2 & + & 3x_3 \\
\text{subject to} & & & & & & & \\
& & & -x_1 & + & 2x_2 & + & x_3 & \leq & 4 \\
& & & 2x_1 & - & x_2 & + & 2x_3 & \geq & -2
\end{array}$$

with $x_1 \geq 0$, $x_2 \leq 0$, and x_3 free.

Hints: some tricks you will need:

1. You need to convert it into a maximisation problem.
2. You need to convert a $\geq$ constraint into a $\leq$
3. You need to swap a non-positive variable with a non-negative one.
4. You need to replace a free variable with two non-negative variables.
5. You need to add slack variables to convert the constraints into equalities.
