THE UNIVERSITY of ADELAIDE

Optimisation & Operations Research

Haide College, Spring Semester

Assignment 4 (5%)

Due: 18 May, 23:59

Instructions:

- Answer the below questions and submit via Cloudcampus as a single PDF file.
- It is strongly recommended that you type your assignment (for example with LaTeX/Overleaf, as a MATLAB Live Script). A handwritten/scanned submission is acceptable provided it is neat and legible.
- Show all your working. Marks will be deducted if your answers do not include sufficient explanation, are illegible or are otherwise difficult to understand.
- In submitting this assignment you agree to have abided by the principles of academic integrity. You may discuss your assignment with other students but the written submission must be your own work and reflect your own understanding of the material.
- You must submit your assignment by the due date. Late assignments will incur a 50% penalty up to 48 hours after the due date, and a 100% penalty after that. You may request an extension, but please do this at least 24 hours before the due date.

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10 marks

1. Solve the following problem using Branch and Bound.

$$\begin{aligned} \max \quad & z = 3x_1 + 7x_2 + 5x_3, \\ \text{subject to} \quad & 2x_1 + 5x_2 + 4x_3 \leq 20, \\ & 2x_1 + 8x_2 + 5x_3 \leq 13, \\ & 3x_1 + 3x_2 + 10x_3 \leq 14, \end{aligned}$$

where $x_1, x_2, x_3 \geq 0$ and integer.

Perform the branch and bound procedure as described in the course notes, and illustrate this procedure with a tree diagram. Please refer to the above ILP as "IP1" and format your solution as per the parts below to facilitate marking. Note these parts are interconnected (you need to fathom leaf nodes to draw the tree diagram, for example).

5 a) Draw a tree diagram using the branching strategy described in the course notes.
Clearly indicate the extra constraints due to branching at each new leaf node on you diagram. Include the optimal solution (variables and objective) of each relaxed problems either in your tree diagram or in a separate table.

1 b) Include documented MATLAB code to solve each relaxed LP.
Note that since you must show all branches of the tree fathomed completely, you cannot solve the whole problem using MATLAB's `intlinprog`. It may be a good idea to check the solution that way.

3 c) For each *fathomed* nodes of the tree clearly explain why the node is fathomed. Present this information as a list of fathomed nodes, along with the reason they are fathomed.

1 d) State the optimal solution to the ILP.

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10 marks

2. Recall the following knapsack problem from Assignment 3.

Consider the following linear program representing a knapsack problem for six items with total value z and maximum volume 35.

$$\begin{array}{lrcrcrcrcrcrcl} \text{max.} & z & = & 19x_1 & + & 22x_2 & + & 30x_3 & + & 37x_4 & + & 11x_5 & + & 42x_6, \\ \text{s.t.} & & & 7x_1 & + & 6x_2 & + & 11x_3 & + & 13x_4 & + & 4x_5 & + & 13x_6 & \leq & 35, \end{array}$$

with $x_i = 0, 1$, for $i = 1, \ldots, 6$.

1 a) Relax the integral constraints (to give the constraint that all the x_i are $0 \leq x_i \leq 1$) and solve the above problem using the problem based approach in MATLAB. Upload your code to MATLAB Grader for checking, you do not need to include it in your PDF submission.

8 b) Find the optimal solution to the problem using the branch and bound as described in the course notes.

Since the tree for this problem is a large, so you should draw this diagram to show only the structure of the tree and the labels of the problems.

The detail of the relaxed problems and branching should then be provided in a table like the one below. Make sure you include reasons for fathoming.

It is **not** required that you submit any MATLAB code for this question, you just need to summarise results in your table.

Problem	Parent	New constraint	z_R	$\mathbf{x}_R$	Branched/fathomed	Reason for fathoming
1	-	-	...	...	...	...
2	1	...	...	...	...	...
3	1	...	...	...	...	...

Hint: To solve the relaxed problems in this question it is strongly recommended that you use the "problem-based" approach in MATLAB (see Practical 3 for details).

1 c) Finally, briefly comment on how effective branch and bound was for this problem. For example, would a brute force search have been more efficient?

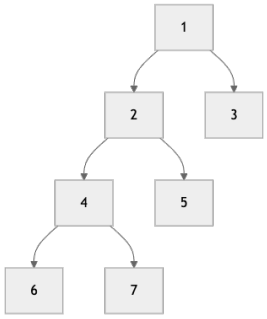
Example solution

Here is an example of the style formatting for Question 2.

Recall in Tutorial 5, Question 2 we performed Branch and Bound on the following problem:

$$\begin{array}{rcrcrcrcl} \max z & = & 3x_1 & + & x_2 & + & 3x_3 & & \\ & & -2x_1 & + & 4x_2 & - & 4x_3 & \leq & 18 \\ & & 5x_1 & - & 3x_2 & + & x_3 & \leq & 13 \\ & & -x_1 & + & 2x_2 & + & 3x_3 & \leq & 19 \end{array}$$

Here is a minimal tree diagram that would be sufficient for your assignment submission.



The table summarising the information from the relaxed problems, solution procedure and branching/fathoming would be:

Problem	Parent	New constraint	z_R	$\mathbf{x}_R$	Branched/fathomed	Reason for fathoming
1	-	-	43	(8.71,10.85,2)	branched	-
2	1	$x_1 \leq 8$	41.18	(8,9.82,2.45)	branched	-
3	1	$x_1 \geq 9$	-	-	fathomed	infeasible
4	2	$x_3 \leq 2$	40.5	(8,10.5,2)	branched	-
5	2	$x_3 \geq 3$	39	(7.14,8.57,3)	fathomed	$z_r < z_{ip}$ (IP6)
6	4	$x_2 \leq 10$	40	(8,10,2)	fathomed	integer feasible
7	4	$x_2 \geq 11$	-	-	fathomed	infeasible

See provided Live Script `Tute5_example.mlx` on Cloudcampus for an example of how to solve the above relaxed problems with a problem based workflow. The main feature here is that we store information about the solution procedure in a table. The optimsation problems are stored too, which makes is easier to add an extra constraint each time there is a new branched problem.