Optimisation & Operations Research Haide College, Spring Semester

Assignment 1 (5%) Due: 30 March 2025, 23:59

10 marks

1. *Translate the following scenario into a linear program.*

   A maths lecturer is visiting OUC.

   It is currently 9:40am and they have a class that starts at 10:10am. They would like a coffee before class, so will go from their office to the coffee shop and then to the classroom. The route they will take is shown in the picture below.

   For the first part of their journey they will walk slowly (shown in blue). For the second part of their journey they will walk fast (shown in purple). For the last part of their journey they will run (shown in red).

   The length of each part of this journey shown in the picture an indication only, the lecturer would like to figure out how long each part of the route should be.

   The total length of the journey is 1 km. The lecturer would like to use less than 250 kJ in energy to make this journey. Speed and energy required for slow walking, fast walking and running are shown in the table below.

	Slow walk	Fast walk	Run
Speed (km/hr)	3	6	12
Energy (kJ/hr)	734	1304	3586

   Similar to the route shown in the picture they intend to walk (slow or fast) for most of the journey. The distance walked at a fast pace should be **no more than** the distance walked slowly. The distance run should be **no more than half** the distance walked fast.

   It takes 15 minutes to order and drink a coffee (during this time the lecturer stops at the coffee shop and neither walks or runs).

   How far should the lecturer walk slowly, walk fast and run in order to get to class as quickly as possible?

a) State the variables (including units), the objective and the constraints.

Solution:

- Variables

$$\begin{aligned} x_1 &= \text{distance slow walk (km)} \\ x_2 &= \text{distance fast walk (km)} \\ x_3 &= \text{distance run (km)} \end{aligned}$$

- Objective: minimise $z = \frac{1}{3}x_1 + \frac{1}{6}x_2 + \frac{1}{12}x_3$
- Constraints: such that

$$\begin{aligned} \frac{734}{3}x_1 + 13046x_2 + \frac{3586}{12}x_3 \leq 250 \text{ (energy constraint)} \\ -x_1 + x_2 \leq 0 \text{ (distance walked slow vs fast)} \\ -x_2 + 2x_3 \leq 0 \text{(distance walks fast vs run)} \\ x_1 + x_2 + x_3 = 1(totaldistance) \\ x_1, x_2, x3 \geq 0 \text{ (non-negativity)} \end{aligned}$$

b) **MATLAB Grader.** Enter your linear program from part (a), and solve it using `linprog`.

Solution:

```
% Assignment 1, Q1(b)
% Mike Chen, March 2025

% Use linprog to solve the "lecturer gets to class on time" problem

% Enter the LP as a minimisation problem (minimise time).

f = [1/3; 1/6; 1/12];

A = [734/3 1304/6 3586/12;
-1 1 0;
0 -1 2];

b = [250; 0; 0];

Aeq = [1 1 1];
beq = 1;

lb = zeros(1,3);
ub = [];

% Call linprog:
[x,fval] = linprog(f,A,b,Aeq,beq,lb,ub)
```

```
% energy used
energy = A(1,:)*x

% minutes early
early_time = (0.5-0.25-fval)*60
```

c) Using the output from `linprog`, state the optimal solution. This should include the value of the objective and the values of the variables. Make sure you include units for all quantities.

Solution: The optimal solution is $x_1 = 0.4$, $x_2 = 0.4$, $x_3 0.2$, with $z = 0.2167$.
To minimise travel time ("get to class as quickly as possibole") length of the different parts should be

- slow walking: 400 m
- fast walking: 400 m
- running: 200 m

Total time is 13 minutes (or 28 minutes if you include the coffee break).

d) How much energy does the lecturer use to make this journey?

Solution: Substituting the optimal solution into the left-hand side of the energy constraint gives:

$$\text{energy} = \frac{734}{3} \times 0.4 + 13046 \times 0.4 + \frac{3586}{12} \times 0.2 \approx 244.6\, kJ.$$

e) Does the lecturer get to the classroom on time? How many minutes is the lecturer early (or late!) to class?

Solution: The lecturer had 30 minutes to make it to class. They spent 15 minutes having a coffee and 13 minutes walking/running. This means they would take 28 minutes to get to the classroom, arriving at 10:08am - 2 minutes early!

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10 marks

2. Consider the following linear program

$$\max \quad z = 2x + 2y,$$

subject to

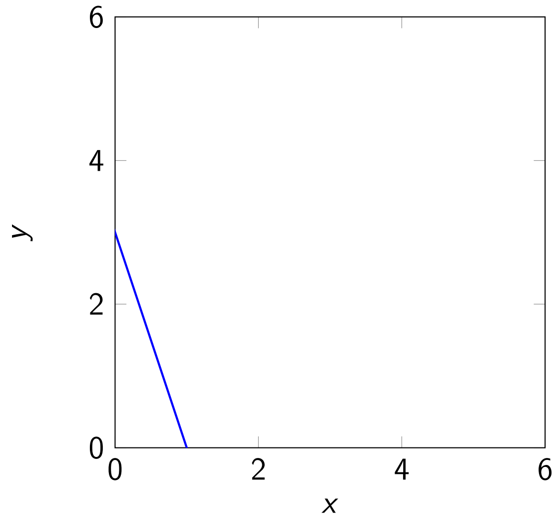
$$\begin{aligned} -3x - y &\le -3 \\ -2x + y &\le 3 \\ y &\le 5 \\ 3x + y &\le 14 \\ 2x - y &\le 6 \end{aligned}$$

with $x, y \ge 0$.

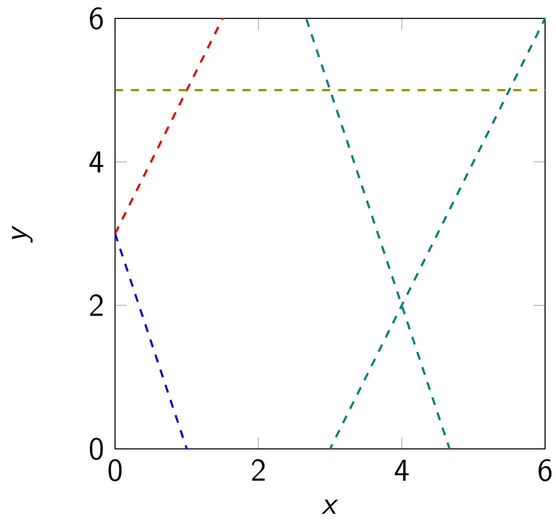
a) Draw the feasible region. Do this either by hand or with MATLAB (or similar). Your drawing should include:

- the edges of the feasible region
- the vertices of the feasible region

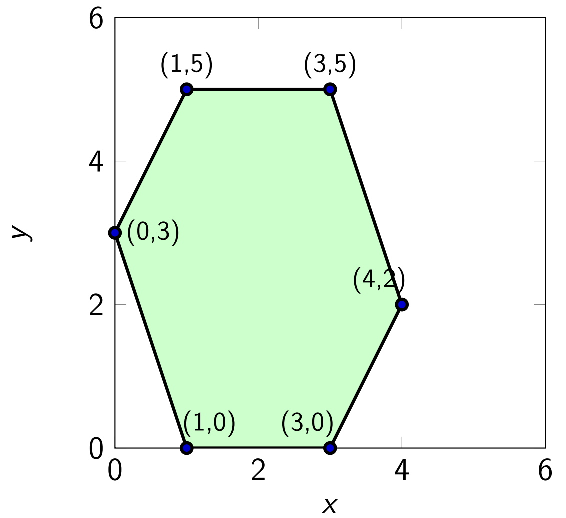
Hint: start by drawing one line for each constraint to show the edges of the feasible region. To get you started, here is a line representing the first constraint:



Solution: Here are the lines for the five constraints:



This gives a feasible region that looks like this:



b) Consider the region from part (a). State the feasible vertices. How many are there?

Solution: There are 6 feasible vertices: (1,0), (0,3), (1,5), (3,5), (4,2) and (3,0).

c) Calculate the objective at each feasible vertex. What is the optimal solution?

Solution:
The values of z at the feasible vertices are

vertex	$z = 2x + 2y$
(1,0)	2
(0,3)	6
(1,5)	12
(3,5)	16
(4,2)	12
(3,0)	6

So z is maximised at $(3, 5)$ with value $z = 16$.

d) You should find that **two pairs** of vertices have the same value of the objective function. Explain what is happening here with reference to both the objective function and the shape of feasible region.

Finally, state **with some explanation** a different objective function of the form $z = ax+by$ with the following properties:

- there are two pairs of vertices with the same value of the objective
- there is a unique maximum to the linear program
- the coefficients of the objective are not equal, that is $a \neq b$.

Solution: The values at (0,3) & (3,0), with $z = 6$, and (1,5) & (4,2), with $z = 12$, are the two pairs of points. In both cases the value of the objective is the same because the points lie on the same contour line of the the objective function.
From inspecting the above plots, notice we can satisfy these properties in two different

ways:

- **Option 1.** Find an objective function where (0,3) & (3,5) lie on the same contour line, which will mean that (1,0) & (4,2) also share a contour line.

  Such an objective function will be of the form $z = c(2x - 3y)$, taking a maximum value at either (1,5) or (3,0) depending on the value of c.

  Some examples:

  - $z = 4x - 6y$, with max of 12 at (3,0)
  - $z = -6x + 9y$, with a max of 39 at (1,5)

- **Option 1.** Find an objective function where (1,0) & (1,5) lie on the same contour line, which will mean that (3,0) & (3,5) also share a contour line.

  Such an objective function will be of the form $z = cx$, taking a maximum value at either (0,3) or (4,2) depending on the value of c.

  Some examples:

  - $z = x$, with max of 4 at (4,2)
  - $z = -3x$, with a max of 0 at (0,3)

e) **MATLAB Grader.** Check your answer using `linprog`.

Solution:

```
%% Assignment 1, Q2(f)
% *Mike Chen, Feb 2025*
%
% Check your answer to the LP in Question 2.

%% Write the LP as a minimisation problem
f = [1;-5];
A = [-3 -1; -1 1; 1 3; 1 0; 1 -4];
b = [-3;3;17;5;1];

Aeq = [];
beq = [];

lb = zeros(1,2);
ub = [];

% Solve in linprog
[x,fval] = linprog(f,A,b,Aeq,beq,lb,ub)
```