THE UNIVERSITY of ADELAIDE

Optimisation & Operations Research

Haide College, Spring Semester

Practical 2 (1%) See website for practical and due dates

Work through the below instructions in MATLAB and MATLAB Grader as indicated. Submit to MATLAB Grader by the due date.

Functions to automate Simplex

Matt Roughan, Mike Chen

In this practical you will write some MATLAB functions to perform some parts of the Simplex Algorithm. You will then use these to solve some linear programming problems.
Assume the problem is given in the form:

$$\begin{aligned} \max \quad & z = \mathbf{c}^T\mathbf{x} + z_0 \\ \text{subject to} \quad & A\mathbf{x} \leq \mathbf{b} \\ & \mathbf{x} \geq \mathbf{0} \end{aligned}$$

where $\mathbf{b} \geq \mathbf{0}$, so that we can convert directly to standard equality form using slack variables, and the resulting Tableau will be in feasible canonical form, so we can skip Simplex Phase I.
Note. For this practical MATLAB Grader is for Questions 1 & 3 only.

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1. Write the a function `M = construct(A, b, c, z0)` to construct the Phase II Tableau as a matrix `M`. Input vectors should be column vectors.

   You may like to develop this by using the problem from Tutorial 2, Question 2 as a prototype. Recall that this problem is:

$$\begin{array}{lrrcrcrcl} (P) & \max z & = & -x_1 & + & x_2 & + & 2x_3 & - & 12 \\ & \text{s.t.} & & -2x_1 & + & 2x_2 & + & x_3 & \leq & 6 \\ & & & 3x_1 & + & x_2 & - & x_3 & \leq & 9 \\ & & & x_1 \geq 0, & & x_2 \geq 0, & & x_3 \geq 0. & & \end{array}$$

   Get started by writing out the constraint matrix and various vectors for this problem, then construct the corresponding tableau.

   MATLAB hints:

   - In MATLAB the command for an $n \times n$ identity matrix is `eye(n)`
   - matrices can be joined in a similar way to vectors, for instance `[eye(3);eye(3)]` or `[eye(3) eye(3)]`

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2. Write a function `[i,j] = choose(M)`, which takes a tableau M as input, and returns natural numbers i and j, which give the *entering* pivot row and column (the next pivot location).

   When you can't find a viable leaving variable, your code needs to indicate this by returning i and j values $i = j = \infty$. (**Hint:** floating point numbers can have the value `Inf`).

   Your function should apply the mandatory rules from the course notes on Simplex, and some choice of discretionary choice rules (for example Bland's rules or similar).

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3. Write a function called `Mout = pivot(Min,i,j)`. It should take 3 input arguments: an array M_{in}, representing a tableau and two natural numbers i and j which are the row and column of the pivot, respectively. It should return the array M_{out} after pivoting M_{in} at the point (i, j).

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4. Now write a script to solve the manufacturing problem from the course notes ("Our first problem") using your Simplex method. State the optimal solution (number of chairs and so on).
