Optimsation & Operations Research

Haide College, Spring Semester

Tutorial 2

These questions are on Topic 2: Simplex Algorithm

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1. **Interpretation:**

   Imagine we start with a LP

$$\begin{array}{rcrcrcrcl} \max\ z & = & 6x_1 & + & 14x_2 & + & 13x_3 & & \\ \text{subject to} & & & & & & & & \\ & & \frac{1}{2}x_1 & + & 2x_2 & + & x_3 & \leq & 24 \\ & & x_1 & + & 2x_2 & + & 4x_3 & \leq & 60 \\ & & & & & & x_i & \geq & 0 \end{array}$$

   which we put into standard equality form (adding slack variables), and then into the tableau

x_1	x_2	x_3	x_4	x_5	z	b	basic variable
1/2	2	1	1	0	0	24	x_4
1	2	4	0	1	0	60	x_5
-6	-14	-13	0	0	1	0	

   We perform Simplex, and end up with the Tableau

x_1	x_2	x_3	x_4	x_5	z	b	basic variable
1	6	0	4	-1	0	36	x_1
0	-1	1	-1	1/2	0	6	x_3
0	9	0	11	1/2	1	294	

   The optimal solution is therefore $\mathbf{x}^* = (36, 0, 6)$, with $z^* = 294$

a) How close to equality are the original constraints at this solution?

b) Intepret that "closeness" in the light of the value of the slack variables.

c) If we were to increase one of the constraint values, say $60 \to 61$, we could increase one of the slack variables – which one and by how much?

d) Use the final row of the tableau to estimate the potential affect of this on the value of z^*

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2. Consider the following Linear Program

$$\begin{array}{lrcrcrcrcl} (P) & \max z & = & -x_1 & + & x_2 & + & 2x_3 & - & 12 \\ & \text{s.t.} & & -2x_1 & + & 2x_2 & + & x_3 & \leq & 6 \\ & & & 3x_1 & + & x_2 & - & x_3 & \leq & 9 \\ & & & \multicolumn{7}{c}{x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0.} \end{array}$$

   Find the optimal solution to this linear program using the Simplex algorithm.

   Explain each step along the way. Make sure to identify pivot locations, and how they were chosen, and include values of slack variables in your solution.

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3. *Translate the following scenario into a linear program.*

A food stall in the OUC canteen is creating a recipe for a new dish.

It is to be a delicious beef noodle soup made up of the following ingredients:

- Noodles
- Beef
- Vegetables
- Broth

The nutritional and other information of the four ingredients is given in the below table.

	Noodles	**Beef**	**Vegetables**	**Broth**
Energy (per 100g serve)	560 kJ	1046 kJ	54 kJ	13 kJ
Protein (per 100g serve)	4.1 g	35 g	2.5g	1 g
Vitamin C (per 100g)	0 mg	0 mg	19.7 mg	0 mg
Volume (per 100g)	156.2 mL	104.6 mL	94.8 mL	100 mL
Cost (per 100g)	¥0.8	¥2.6	¥2.1	¥0.3

The soup as a whole should meet the following nutritional requirements:

- Energy: must have at least 2000 kJ
- Protein: must have at least 30 g of protein
- Vitamin C: must have at least 30 mg

The soup should exactly fill a standard 1000 mL bowl. It should have no more than 400 mL of broth to leave plenty of room for the other ingredients.

To maximise profit, the noodle soup should be produced as cheaply as possible.

a) State the variables (including units), the objective and the constraints.

b) Solve your linear program from part (a) using `linprog`.

c) Using the output from `linprog`, state the optimal solution. This should include the value of the objective and the values of the variables. Make sure you include units for all quantities.

d) State the nutritional information for this noodle soup. This should include energy (kJ), protein (g) and vitamin C (mg).

e) Advise the food stall on a price for the noodle soup and state the resulting profit per bowl. (**Hint:** There is no "right" answer here, but you should explain your reasoning.)

Bonus questions

1. **Calculations:** Consider the LP with the Simplex Tableau:

x_1	x_2	x_3	x_4	x_5	x_6	z	b
0	1	-2	0	3	1	0	3
1	2	4	0	1	0	0	4
4	-2	1	1	1	0	0	2
-3	-1	-5	0	0	7	1	10

a) Explain why each of the following positions would not be a suitable choice for the next pivot position, if the Simplex Method were to be applied to the above tableau.

   i. Row 1, Column 1
   ii. Row 1, Column 3
   iii. Row 3, Column 4
   iv. Row 2, Column 1
   v. Row 2, Column 8
   vi. Row 4, Column 3

b) Nominate two distinct entries that *could* be selected as suitable pivot positions for the Simplex Method.

c) What happens to the value of the objective function if you pivot in Column 5?
