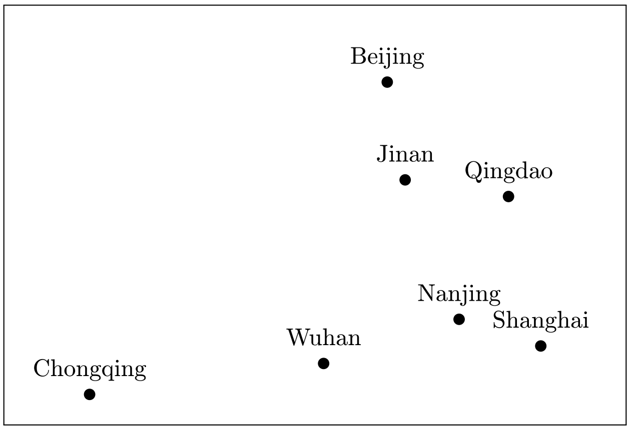
Optimsation & Operations Research

Haide College, Spring Semester

Tutorial 4

These questions are on Topic 4: Integer programming problems.

Consider the seven Chinese cities shown in the map below.



1. A tourist would like the visit all of the cities shown above exactly once, starting and finishing in Qingdao. They would like to do this in the shortest distance possible.

   The distances between each pair of cities in kilometres are given in the table below.

	Qingdao	**Shanghai**	**Beijing**	**Wuhan**	**Jinan**	**Chongqing**	**Nanjing**
Qingdao	0	560	550	840	310	1480	480
Shanghai	560	0	1070	690	740	1440	270
Beijing	550	1070	0	1050	360	1460	900
Wuhan	840	690	1050	0	720	750	460
Jinan	310	740	360	720	0	1250	540
Chongqing	1480	1440	1460	750	1250	0	1200
Nanjing	480	270	900	460	540	1200	0

   What route around the above cities should the tourist take?

a) This is an example of a type of problem covered in this course. State the name of this type of problem and describe a greedy heuristic to find a solution.

b) Apply the greedy heuristic to find a route around the seven cities **starting and ending in Qingdao**. State the order of your route and the total distance travelled.

c) Is the route you found with the greedy heuristic the shortest route possible? Suggest what could be improved about the route you found.

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2. A student at Haide College enjoys travelling to other cities by train. They would like to travel as many kilometres as possible, but not spend more than ¥2400.

   The cost of a return train ticket from Qingdao to the other six cities is:

From Qingdao to ...	**Shanghai**	**Beijing**	**Wuhan**	**Jinan**	**Chongqing**	**Nanjing**
Return ticket (¥)	600	700	1000	300	1700	600

The distance from Qingdao to each city is given in the table for part (a).

Which cities should the student visit?

a) This is an example of a type of problem covered in this course. State the name of this type of problem and describe a greedy heuristic to find a solution.

b) Apply the greedy heuristic to find which cities the student should visit. State the total number of kilometres travelled and the total cost of the tickets.

c) The greedy solution to this problem is not the optimal solution. With reference to your answer to 2(a), are there any aspects of the greedy solution that are particularly non-optimal? In view of this, can you suggest a better solution?

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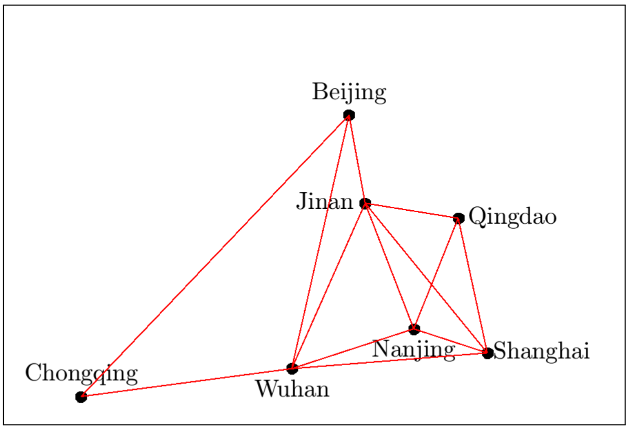
3. Another student from Haide College is a fan of high-speed rail. Assume that our 7 cities are connected in a high-speed rail network as shown below (this is a simplified version of the real network). Also assume that the distacnce by rail is the same as that given in the table in Question 1.



Starting in Qingdao, what is the shortest path to visit each city using this high-speed rail network?

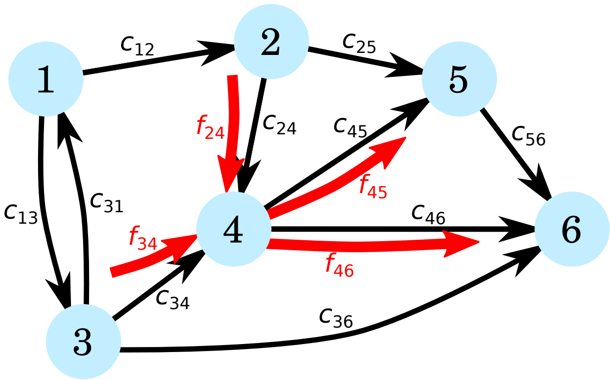
a) This is an example of a type of problem covered in this course. State the name of this type of problem and which (greedy) algorithm can be used to find a solution.

b) Apply the greedy heuristic to find a routes around the seven cities **starting in Qingdao**. State your solution both in the usual notation of the algorithm, as well as writing out the shortest routes from Qingdao to each of the other cities.

Hint: in applying this algorithm, it might be easier to denote each city with a number (Qingdao = 1, Shanghai = 2 and so on).

Bonus questions

1. **Translation:** A *flow network* is a directed graph $G = (V, E)$ where each edge (u, v) has a capacity $c_{u,v}$, and each edge could receive a flow $f_{u,v}$ of some commodity (*e.g.*, electricity or traffic). See the figure below for an example.



A flow network is constrained in various ways:

- Flows are non-negative.
- The flow on an edge may not exceed the capacity. For example, in the figure, we must choose $f_{24} \leq c_{24}$. In particular, if no edge exists (*e.g.*, between nodes 1 and 4), this flow must be zero.
- Flows are conserved, *i.e.*, everything that flows into a node must also flow out, except at a source or sink, For instance, in the figure

$$f_{34} + f_{24} = f_{45} + f_{46}.$$

- The total flow in at the source must be balanced by the flow out at the sink.

The *maximum flow* problem is the problem: given source node s and destination sink t, what is the maximum flow that can pass across the network between these two (any flow that enters at the source must exit at the sink).

a) Formulate this problem as a LP in general terms, *i.e.*, write the objective and constraints as a set of general equations or inequalities (you don't need to translate into standard form yet).

b) Take the particular problem above (assuming all possible flows, not just those illustrated), and write it in a form suitable for input to MATLAB, *i.e.*, work out the matrices A and A_{eq} and vectors $\mathbf{b}$, $\mathbf{b}_{eq}$ and $\mathbf{c}$, assuming that node 1 is a source (*i.e.*, it can supply an arbitrary amount of flow) and 6 is a sink (it can absorb an arbitrary amount). You should have 1 variable for each edge, and 1 constraint for each node (conservation) and for each edge (capacity).

   (NB: don't solve – there are better approaches than just solving directly as a LP, *e.g.*, the Ford-Fulkerson algorithm).

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2. **Calculations:** Consider the primal LP

$$
\begin{array}{llrl}
\text{(P)} & \max \quad z & = 3x_1 - x_2 + 7x_3 & \\
& \text{subject to} & -x_1 + x_2 + 2x_3 & \leq 8 \\
& & x_1, x_2, x_3 & \geq 0
\end{array}
$$

a) Write down the dual.

b) Show by inspection that the dual is infeasible.

c) What can you conclude about the solution to (P)?

d) Now describe the relationship between primal and dual again if we restricted the variables to be integers.