THE UNIVERSITY of ADELAIDE

Optimsation & Operations Research

Haide College, Spring Semester

Tutorial 5

These questions are on Topic 4: Integer programming problems, and Topic 5: More sophisticated algorithms

---

1. **Calculations:** Perform Branch and Bound on the following problem:

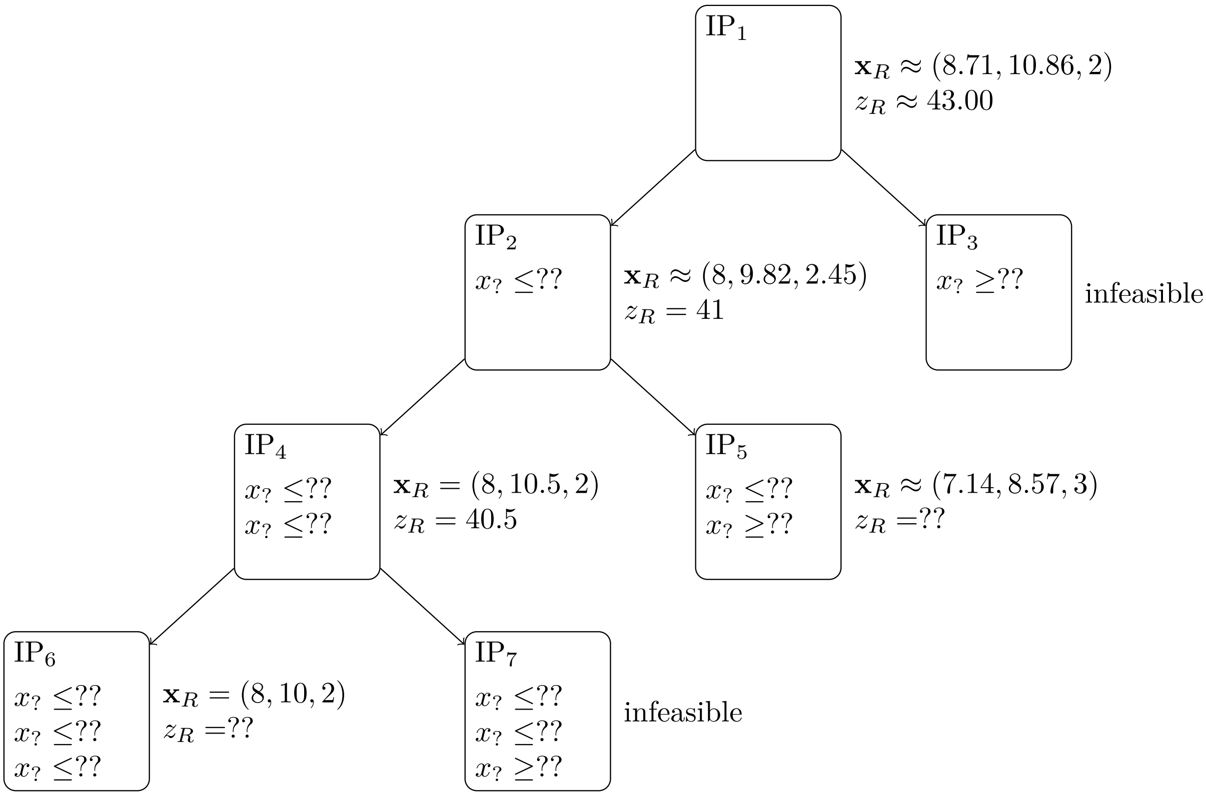
$$
\begin{array}{lrcl}
\text{maximise} & 2x_1 - x_2 & & \\
\text{subject to} & x_1 + x_2 & \leq & 7 \\
 & 2x_1 - 3x_2 & \leq & 12 \\
 & x_1 \geq 0, x_2 \geq 0 & & \\
 & x_1, x_2 \text{ integer} & &
\end{array}
$$

---

2. The following ILP (Integer Linear Program)

$$
\begin{array}{rcrcrcrcl}
\max z & = & 3x_1 & + & x_2 & + & 3x_3 & & \\
 & & -2x_1 & + & 4x_2 & - & 4x_3 & \leq & 18 \\
 & & 5x_1 & - & 3x_2 & + & x_3 & \leq & 13 \\
 & & -x_1 & + & 2x_2 & + & 3x_3 & \leq & 19
\end{array}
$$

with the $x_i \geq 0$ and integer, has been solved using Branch and Bound. The steps in the solution tree are shown in the figure below.



Note that the subproblems in the figure only show additional constraints added to the subproblem in addition to the original constraints of the ILP. Note also that the figure does not show all solutions to relaxed versions of the IPs (you will have to do some working yourself).

a) Write out a completed version of the tree diagram above by filling in :
   - the constraints for all nodes
   - the value of the objective z_R for the relaxed version of IP_5 and IP_6.

b) Explain why the tree is fathomed at IP_3, IP_5, IP_6 and IP_7.

c) Use the figure (and your working) to derive the optimal solution to the ILP.

Bonus questions

1. **Translation:** *Ambulance depot location.* The ambulance service needs to locate a number of depots around the city so that it can get to any potential accident within 15 minutes.

   - There are M possible locations where we could place a depot.
   - Each location i can reach a subset S_i of the population within the required time.

   In the ambulance depot problem, the subsets might be described by regions represented say by circles around the hub, but in general we simply use sets, which makes the problem formulation more general.

   Each location also has a cost $c_i > 0$. We need choose which potential depot sites to use to:

   - minimise the cost of the overall system, and
   - ensure that every person is covered by at least one depot.

   Formulate the problem of minimising the cost of the ambulance depots as an Integer Linear Program.
