THE UNIVERSITY of ADELAIDE

Optimisation & Operations Research

Haide College, Spring Semester

Assignment 2 (5%)

Due: 13 April, 23:59

Instructions:

- Answer the below questions and submit via Cloudcampus as a single PDF file.
- It is strongly recommended that you type your assignment (for example with LaTeX/Overleaf, as a MATLAB Live Script). A handwritten/scanned submission is acceptable provided it is neat and legible.
- Show all your working. Marks will be deducted if your answers do not include sufficient explanation, are illegible or are otherwise difficult to understand.
- In submitting this assignment you agree to have abided by the principles of academic integrity. You may discuss your assignment with other students but the written submission must be your own work and reflect your own understanding of the material.
- You must submit your assignment by the due date. Late assignments will incur a 50% penalty up to 48 hours after the due date, and a 100% penalty after that. You may request an extension, but please do this at least 24 hours before the due date.

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7 marks

1. Consider the following linear program.

$$\begin{aligned} \max \quad & z = 4x_1 + 3x_2 + 2x_3 + x_4, \\ \text{subject to} \quad & 10x_1 + 7x_2 - x_3 + x_4 \leq 18, \\ & 3x_2 + 8x_3 - 6x_4 \leq 16, \\ & 15x_1 - 20x_2 - 40x_3 - 50x_4 \leq 27, \end{aligned}$$

with x_1, x_2, x_3 and $x_4 \geq 0$.

6 a) Solve the above linear program using Simplex.
At each step explain why a pivot has been chosen, clearly stating the mandatory and (if applicable) discretionary rules which have been applied.
When you reach the final step, clearly explain why the algorithm has stopped.
State your optimal solution (values of the variables and objective).

1 b) **MATLAB Grader.** Check your answer with MATLAB Grader.

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6 marks

2. Consider the following primal linear program (P),

$$\begin{array}{llrcrcrcr} (P) & \max z = & 3x_1 & - & x_2 & + & x_3 & & \\ & \text{subject to} & 2x_1 & - & 5x_2 & + & 2x_3 & \leq & 8 \\ & & 5x_1 & + & 4x_2 & - & x_3 & \leq & 14 \\ & & \multicolumn{7}{l}{x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0} \end{array}$$

2 a) State the dual problem (D) of (P). As usual, your should include the objective and all constraints.

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1 b) Find the complementary slackness relations for the above primal problem (P).

3 c) You are given that (P) has the optimal solution

$$x_1^* = 0, x_2^* = 12, x_3^* = 34.$$

Use this optimal solution and the complementary slackness relations from Question 2(b) to find the optimal solution of the dual (D).
Note: no marks will given for a Simplex Method solution.

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7 marks

3. Consider the following linear problem

$$\begin{aligned} \max \quad z = 4x_1 + 2x_2 + x_3, \\ \text{subject to} \quad x_1 \leq 5, \\ 4x_1 + x_2 \leq 25, \\ 8x_1 + 4x_2 + x_3 \leq 125, \end{aligned}$$

with $x_1, x_2, x_3 \geq 0$. This is an example of Klee-Minty problems, which is a class of linear programs often used to test the performance of algorithms.

2 a) Solve the above problem using Simplex with Bland's rules for column/row choice. In addition to the usual mandatory rules, these rules are:

- column choice: if more than one $-c_j < 0$ choose the option with smallest j.
- row choice: if more than one row has equal smallest b_i/a_{ij}, choose the row with smallest i.

In your submission, rather than writing out each tableau, record the following information about each step:

- The choice of pivot location (i, j)
- The value of the objective
- The value of the variables (x_1, x_2, x_3)
- The non-basic variables.

A table or list would be an appropriate way to present this information.

2 b) Now solve the problem again, but this time with different discretionary rules. In addition to the usual mandatory rules apply the following

- column choice: if more than one $-c_j < 0$ choose the minimum value of $-c_j$ (the most strongly negative). Where two values are equal, choose the option with the smallest j.
- row choice: if more than one row has equal smallest b_i/a_{ij}, choose the row with largest a_{ij}. Where two values are equal, choose the option with the smallest i.

In your submission, include the details of each step as in Question 3(a) presented as a table or list.

2 c) Visualise the trajectories of the solutions from Questions 3(a) and 3(b). To do this make a 3D plot of the (x_1, x_2, x_3) co-ordinates of each step of the Simplex.
Hint: One way to make such a plot is with MATLAB's `plot3` function (see function documentation for examples). You may find it helpful to indicate the feasible region as part of your plot(s).

1 d) Briefly discuss how Simplex proceeds with the different discretionary choice rules from Question 3(a) and 3(b). Refer to your plots from Question 3(c) as part of your answer.
