Math 143 Midterm 1

1/27/15

Mike Robertson Section: Name:________________________

This test is scored out of 200 points. No notes or calculators allowed. Point values of problems are indicated after the problem number.

1. (25) Both the integral given and the series given can be thought of as "infinite sums". Compare and contrast these sums, but focus primarily on what the series is. You're to use this exercise to show off that you know how a series is defined and how a value is assigned to it.

$$\int_0^1 f(x)\,dx$$

$$\sum_{n=1}^{\infty} a_n$$

2. (25) Use basic principles of series to find the value that $\sum_{n=2}^{\infty} 3\left(\frac{4}{5}\right)^n$ converges to. You may not rely entirely on the formula that the book found using these basic principles.

3. (50) Show that the given series converge or diverge by clearly stating the test used and hypotheses of these tests. You need not justify the most obvious of claims, but should justify any questionable ones.

   (a) $\sum \frac{3^n}{(2n+1)!}$

   (b) $\sum \frac{(-1)^n}{\sqrt{2n+1}}$

   (c) $\sum \frac{n^2+1}{n^4}$

4. (25) Determine the interval on which the following power series converges.

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n2^n}(x-3)^n$$

5. (25) Use the method of Taylor/Maclaurin to find the first three non-zero terms in the power series for $f(x) = \sqrt{3x^2+x}$ centered at $x=1$. Do not find this by manipulation of a known power series.

6. (50) Manipulate known power series for $e^x, \sin x$, $\cos x$ and $(1+x)^k$ to find the following as power series. Recall that $\frac{d}{dx}\sin^{-1}x = \frac{1}{\sqrt{1-x^2}}$.

   (a) $\frac{\sin 5x}{x}$

   (b) $\int \sin^{-1} x\ dx$

   (c) $x^2 e^{x^3}$