NOISE IN HIGH DYNAMIC RANGE IMAGING

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ABSTRACT

High dynamic range (HDR) imaging is more and more widely used to increase the limited dynamic range of digital cameras and, in turn, to cover the dynamic range of the acquired scene. This image acquisition process can be subdivided into two steps. The first step is the *measurement* or *estimation* of the mostly non-linear camera transfer function (CTF). This is followed by the second step, the combination of a set of differently exposed images of the same scene into one HDR image after linearization with the inverse CTF. Each of the individual images in such an exposure set contains noise from the image acquisition process. Consequently, the calculated HDR image will as well contain noise, which fortunately is reduced by the weighted average of the images from the exposure set. We analyze the achieved gain in SNR for different weighting functions proposed in the literature and compare these with a plain average. Although these functions are based on reasonable intuitions, we show that the highest $\mathrm{SNR}_{\mathrm{gain}}$ is achieved with the plain average.

Index Terms— high dynamic range imaging, noise, noise reduction

1. INTRODUCTION

Natural scenes often cover a high dynamic range (HDR), which in general can not be acquired with a digital camera in a single image. For example, consider the situation of taking a photograph that contains the inside of a room and some exterior scene visible through a window. In most cases, the exterior scene will be bright due to the direct illumination by the sun, while the interior illumination is far darker. Hence, one will acquire an image, which shows properly exposed details in the room and a saturated window, in which all details of the exterior scene are lost. Alternatively, the exterior scene is properly exposed, but all details of the room are lost in the underexposed (dark) areas of that image.

To overcome this limitation, several methods have been developed that combine two or more differently exposed images into one image of greater dynamic extent, i.e., a HDR image. To this end, the mostly nonlinear camera transfer function (CTF) f has to be *measured* or *estimated*. Chart-based measurements [1] and radiometric measurements [2] have been applied to measure the CTF. Several methods have been proposed to estimate the CTF from a set of differently exposed images of the same scene. These are model-based estimates by fitting, e.g., a gamma curve [3], a polynomial [4], parametric functions [5, 6, 7], or a constrained piecewise linear model [8, 9], to the CTF. Alternatively, a smoothness constraint [10] has been applied to estimate the CTF. The influence of noise on the recovery of f has been briefly discussed in [4, 5].

Once the CTF f is known, i.e., measured or estimated, the differently exposed images can be aligned in range and combined into one HDR image. In general, the single images do partially overlap in the range domain, and will be combined by a weighted average into the resulting HDR image. This weighted average reduces noise in the resulting HDR image as compared to the noise in each single low dynamic range input image. Therefore, we will experimentally investigate this implicit noise reduction behavior of HDR imaging.

Hence, in this paper we will analyze the overall noise reduction behavior of HDR imaging. To this end, we first describe the imaging chain and the different noise sources in Section 2. Next, we summarize different weighting functions in Section 3, which are used to combine the images of an exposure set into a HDR image. Then we explain our experiments in Section 4, present the results in Section 5, and finally conclude in Section 6.

2. THE IMAGING CHAIN

Radiance from light sources or reflected by objects passes through the camera optics, resulting in an irradiance $E(x, y, \lambda)$ incident on the imaging sensor. This irradiance is filtered by the spectral curves of N_τ color filters $\tau(\lambda)$, e.g., τ_R, τ_G, and τ_B in case of the R-,G-, and B-filters of the Bayer pattern, in front of the sensor and by the spectral sensitivity $R(\lambda)$ of the sensor itself. The quantum efficiency $\eta(\lambda)$ of the sensor can, without loss of generality, be incorporated into the spectral sensitivity of the sensor. Integrated over the area A of the sensor element this results in an amount of radiant power $\phi_{m,n}$ incident on the sensor element at pixel position (m, n)

$$\phi_{m,n} = \iint_{x=m,y=n}^{m+\Delta x,n+\Delta y} E^*(x, y)\, dA \qquad (1)$$

with

$$E^*(x,y) = \int_{\lambda=0}^{\infty} E(x,y,\lambda)\tau(\lambda)R(\lambda)\,d\lambda \tag{2}$$

yielding the radiant energy $Q_{m,n} = t \cdot \phi_{m,n}$ collected in the sensor element during the exposure time t. In the presence of noise n_Q, e.g., thermal noise, read out noise, or quantum noise, this radiant energy is altered. Next, the nonlinear behavior of the imaging system, e.g., the nonlinear sensor sensitivity near the noise floor and the full well capacity, or nonlinearities in the electronics, change the sensor response. These nonlinearities are reflected in the camera transfer function f. The resulting output I further includes additional noise n_f, e.g., quantization noise. Hence, in a set of exposures Q_j with exposure times t_j ; $j = 0 \,..\, N_j - 1$ the sensor output intensity I_j for exposure time t_j is given by

$$I_{j,m,n} = f(Q_{j,m,n} + n_Q) + n_f \,;\, Q_{j,m,n} = t_j \phi_{m,n} \tag{3}$$

Without fixed pattern noise (FPN), which is accounted for by flat field correction, the major noise influences n_Q can be modelled as Gaussian noise [11]. We can calculate an estimate $\hat{\phi}_{m,n}$ of $\phi_{m,n}$, which will exhibit the full dynamic range. To this end, the partial estimates $\hat{\phi}_{j,m,n}$ from each individual low dynamic range image, given by

$$\hat{\phi}_{j,m,n} = \frac{1}{t_j} f^{-1}(I_{j,m,n}) \tag{4}$$

are combined by

$$\hat{\phi}_{m,n} = \frac{\sum w(\hat{\phi}_{j,m,n}, I_{j,m,n})\hat{\phi}_{j,m,n}}{\sum w(\hat{\phi}_{j,m,n}, I_{j,m,n})} \tag{5}$$

with w being a weighting function that represents the reliability of the individual estimates $\hat{\phi}_{j,m,n}$.

3. THE WEIGHTING FUNCTION

Different weighting functions have been proposed in the literature, obviously having influence on the noise reduction through the weighted average in the calculation of a HDR image. Therefore, we will now briefly outline three different weighting functions that have been proposed in the literature using the previously established notation. As a fourth alternative we suggest a straightforward averaging with constant weights.

The first weighting function is based on the observation that the output of the sensor is most reliable in areas of highest slope of the CTF and least reliable in areas where the slope of the CTF is lowest. Hence, the weighting function w_{Mann} has been chosen to be the derivative of f

$$w_{\text{Mann}}(\hat{Q}_j) = \frac{d}{d\hat{Q}} f(\hat{Q}) \tag{6}$$

by Mann et al. [3, 5] and consequently emphasizes ranges of $\hat{Q}$ with strongest contrast transfer.

Alternatively, a triangular function w_{Debevec}, given by

$$w_{\text{Debevec}}(I_j) = \begin{cases} I - I_{\min} & ; I \leq \frac{1}{2}(I_{\min} + I_{\max}) \\ I_{\max} - I & ; I > \frac{1}{2}(I_{\min} + I_{\max}) \end{cases} \tag{7}$$

has been proposed by Debevec et al. [10]. This function weights pixel values near the black or white level as unreliable and gray values centered between $I_{\min}$ and $I_{\max}$ are considered most reliable.

Mitsunaga et al. [4] introduced a weighting function $w_{\text{Mitsunaga}}$, which is given by the estimated amplitude signal to noise ratio $\text{SNR}_{\text{amplitude}}$

$$\text{SNR}_{\text{amplitude}} = \frac{\hat{Q}}{\sigma_Q} = \frac{f^{-1}(I_j)}{\sigma_I \frac{d}{dI} f^{-1}(I_j)} \tag{8}$$

with σ_Q being the standard deviation of the noise n_Q, σ_I being the standard deviation of the noise in the image, which is used to estimate the standard deviation σ_Q by local linearization. If σ_I is independent of the measurement value I_j the weighting function is given by

$$w_{\text{Mitsunaga}}(I_j) = \frac{f^{-1}(I_j)}{\frac{d}{dI} f^{-1}(I_j)} \tag{9}$$

Applying the rule for the derivative of the inverse this can be rewritten as

$$w_{\text{Mitsunaga}}(Q_j) = \hat{Q}_j \frac{d}{d\hat{Q}} f(\hat{Q}) \tag{10}$$

which is the weighting function w_{Mann} multiplied by a linear function.

A fourth alternative w_{Mean} is the uniform weight of pixels by one, i.e., the average

$$w_{\text{Mean}} = 1 \tag{11}$$

which rates all estimates $\hat{Q}_j$ as equally certain.

Common to all of these weighting functions is that they are set to 0 for pixels, which are either underexposed or saturated. That is, the weighted average is calculated only over those pixels that fall between the shoulder regions of the CTF.

4. EXPERIMENTAL SETUP

We generated a synthetic HDR test image with a linear decrease in ϕ over the x-direction of the image. This image exhibits a uniform histogram. Next, we calculated exposure sets with $N_j = 8$ low dynamic range images from this HDR image using four different camera transfer functions. The first three are given by

$$f_\gamma(Q) = \alpha + \beta Q^\gamma \tag{12}$$

Table 1. This table gives experimental results of the mean SNR_{gain} [dB] over 100 experiment runs for each combination of input SNR (different amount of n_Q), CTF, quantization, and weighting function. The standard deviation of all entries ranges from $\sigma_{\text{min}} = 0.0399$ to $\sigma_{\text{max}} = 0.0495$ with mean $\bar{\sigma} = 0.0458$. Note that the SNR_{gain} is highest in almost all cases for the weighting function w_{Mean}. For $\gamma = 1.0$, w_{Mean} and w_{Mann} indeed are the same.

SNR [dB]	weighting function	f_γ						f_{arctan}	
		$\gamma = 0.5$ $\alpha = -0.348, \beta = 2.46$		$\gamma = 1.0$ $\alpha = -0.071, \beta = 3.57$		$\gamma = 2.0$ $\alpha = -0.004, \beta = 11.16$		$m = 0.02$ $M = 0.3$	
		without quant.	with quant.	without quant.	with quant.	without quant.	with quant.	without quant.	with quant.
15	w_{Mann}	4.73	4.71	5.10	5.06	3.97	3.87	5.03	5.00
	w_{Debevec}	4.13	4.13	3.79	3.79	2.73	2.73	3.90	3.90
	$w_{\text{Mitsunaga}}$	4.75	4.74	3.97	3.96	2.58	2.57	4.36	4.34
	w_{Mean}	5.10	5.08	5.10	5.06	5.10	4.80	5.11	5.08
20	w_{Mann}	4.97	4.95	5.41	5.35	4.05	3.89	5.34	5.28
	w_{Debevec}	4.27	4.26	3.87	3.86	2.61	2.61	4.00	4.00
	$w_{\text{Mitsunaga}}$	4.97	4.95	4.05	4.03	2.43	2.41	4.51	4.49
	w_{Mean}	5.41	5.38	5.41	5.35	5.41	4.79	5.41	5.37
30	w_{Mann}	5.14	5.03	5.63	5.40	4.04	3.65	5.54	5.36
	w_{Debevec}	4.32	4.29	3.87	3.83	2.44	2.42	4.00	3.98
	$w_{\text{Mitsunaga}}$	5.11	5.06	4.04	4.00	2.20	2.16	4.58	4.53
	w_{Mean}	5.63	5.54	5.63	5.40	5.63	2.71	5.63	5.47

with $\gamma = 0.5$, $\gamma = 2.0$, and $\gamma = 1.0$ and parameters α and β as given in Table 1. Note that for $\gamma = 1.0$ this is a linear camera transfer function. A fourth camera transfer function is given by

$$f_{\text{arctan}}(Q) = \frac{4}{\pi}\text{arctan}\left(\frac{Q-m}{M-m}\right) \tag{13}$$

The parameters are given in Table 1. From each of these exposure sets we reconstructed the HDR image using each of the weighting functions.

To investigate the influence of the sensor noise n_Q, we added noise to the HDR image before calculation of each low dynamic range image, resulting in exposure sets with different SNR and independent Gaussian noise in each of the single exposures. We then calculated the mean SNR and mean SNR improvement, i.e., the SNR_{gain} in the recovered HDR images for each of these experiment setups over 100 runs (see Table 1).

To analyze the influence of the quantization alone, we calculated the SNR in the recovered HDR images for each combination of CTF and weighting function, using $n_Q = 0$ and 8 bit quantization for the low dynamic range images. This results in an upper bound for the SNR of the recovered HDR image (see Table 2). We then repeated the experiments now with both noise n_Q at different SNR and with 8 bit quantization for each of the low dynamic range images, 100 times and once again calculated each mean SNR_{gain} (see Table 1).

5. RESULTS

Table 1 gives numerical results obtained from these experiments for different input SNR, all combinations of weighting function and CTF, and, with and without quantization step for the low dynamic range images. One would expect the SNR_{gain} to be $\text{SNR}_{\text{gain}} = 0\,\text{dB}$ in case of no overlap of the images in the range domain, i.e., disjoint parts of the scene input dynamic range are observed by always only one low dynamic range image with a particular exposure setting. Furthermore, one would expect the SNR_{gain} to be smaller than $\text{SNR}_{\text{gain}} = 10\log_{10} k$ in case of k images having the same exposure, i.e., a simple average of k images without increase of the dynamic range. In our experimental setup, with eight exposures in an exposure set, at average each individual pixel is observed in $k = 4$ images. Hence, the expected SNR_{gain} is $\text{SNR}_{\text{gain}} = 10\log_{10} 4 \approx 6\,\text{dB}$. This is a good match for the experimental results given in the table. Remarkably, in almost all cases the SNR_{gain} obtained from the weighted average is highest with the weighting function w_{Mean}.

However, due to the quantization noise inherent in each individual low dynamic range image an upper bound of the overall obtainable SNR is given in Table 2. In case the input noise drops and, hence, the SNR in the input HDR image increases, the SNR is limited by the upper bound given in Table 2. Even the weighting will not improve the SNR beyond these values, because the quantization noise remains as the major noise source in the HDR image.

Table 2. This table shows the SNR [dB] in the recovered HDR image. The input HDR image contains is given in floating point precission and contains practically no noise, but each low dynamic range image is quantized with 8 bit, introducing a quantization noise n_f in each of the images of the exposure set. These SNR are the best SNR to be achieved with this HDR imaging system.

weighting function	f_γ $\gamma = 0.5$	f_γ $\gamma = 1.0$	f_γ $\gamma = 2.0$	f_{arctan}
w_{Mann}	52.26	49.73	45.48	51.12
w_{Debevec}	55.42	55.81	54.00	55.81
$w_{\text{Mitsunaga}}$	56.54	56.21	54.27	56.54
w_{Mean}	54.70	49.73	35.97	51.94

To verify this upper bound, we have increased the input SNR, i.e., we reduced the noise in the input HDR image, for the different combinations of weighting function and CTF towards the values given in Table 2. As expected the SNR_{gain} dropped to $\text{SNR}_{\text{gain}} = 0\,\text{dB}$, if the SNR approaches this upper bound.

6. CONCLUSIONS

We have experimentally investigated the noise reduction behavior of different weighting functions proposed in the literature for the calculation of HDR images. The weighting functions published so far are based on reasonable intuitions, e.g., strongest contrast transfer or SNR dependent weighting. Our results experimentally show that indeed a straightforward average outperforms these weighting functions. At first, one might be surprised by this result. However, HDR images are almost always calculated from only a few exposures of an exposure set from which even fewer images overlap in the range domain. That is, the weighted average is calculated on very few (mostly two to four) images. If the weighting function even further reduces the influence of some of these images to a few percent, this can be compared to an average of fewer images than available. The plain average not only improves the SNR_{gain}, but even further decreases the computational cost of HDR imaging since no weighting function has to be evaluated, although the latter is less important.

The CTF is assumed to be known exactly in these experiments. In practical applications, however, the CTF contains errors as well. This should be included in the analysis in future work.

So far this is an experimental investigation of the influence of the weighting function on the SNR in the recovered HDR image. Note, that the plain average is the maximum likelihood estimate of the mean of the noisy signal. Thus an analytical investigation of the noise, probably in context of the grayvalue distribution of the scene, should be carried out as future work.

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