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Metamerism and shade sorting

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Abstract: Metameric objects look alike in colour under one light source, and different under other light sources. The measures or indices of metamerism are of two types, special and general. Various aspects of metamerism and metameric indices are discussed in this chapter.

Textile and garment manufacturers require that all the pieces making up a product should match in colour. Grouping rolls of cloth by shade is increasingly important as apparel designers have to use coloured pieces very carefully and the consumers are becoming more quality-conscious. The clustering method of shade sorting has been covered in this chapter.

Key words: metameric indices, reflectance spectra, spectral decomposition, shade sorting, sequencing and tapering, adaptive clustering.

5.1 Introduction

If two objects have identical tristimulus value under a particular illuminant, it is not necessary that they will have identical tristimulus values under other illuminants too. Two objects, therefore, may look alike under one light source, but they may look different under other light sources. This property of a pair of objects is known as metamerism. A survey (ISCC, 1988) showed that the concept of metamerism is not clear to industrial colourists. Metamers are rare in nature, but they are more common in synthetic colourant mixtures.

Metamerism is a problem for the colourist. In most colouration processes, a standard colour of unknown composition is matched by using mixture of different colourants. A colourist may be happy with his matched sample under a particular light, but when the user observes the standard and sample under a different light, he may be surprised to see distinct colour differences between them. Moreover, a composite material may be composed of different components coloured separately or collected from different sources. Disastrous mismatches at different portions of the system may occur with change of light source. A dress designer may require that all portions of a garment say, button, belt, different pieces of textile materials should match under all possible light sources. This is a very difficult task (if not impossible), because the same colourants should be used for all the materials to get non-metameric or universal match. This, however, does not necessarily mean that if we use different colourants for standard and product, they will always give metameric match.

Metamerism, i.e. colour match failure with change of illuminating light source, adds a new dimension to the colour control systems. Different aspects of metamerism have been studied and discussed by many researchers (Roy Choudhury and Chatterjee, 1992). Computer colourant formulations normally predict low metameric recipes. However, the reliability of the present measures of metamerism, on which these predictions are based, may be questioned (Roy Choudhury and Chatterjee, 1996) as different newer aspects of colour appearance have been revealed recently.

A range of metamers are used for evaluating daylight simulators corresponding to D55, D65 and D75. For each type of illuminant, a set of eight metamers was provided as part of the CIE method, five in the visible range (400–700 nm) and three in the ultraviolet (300–400 nm). An updated version was published in 1999, which included a new set of metamers for evaluating the simulator corresponding to D50. Colour differences (CIELAB or CIELUV) were calculated for each metamer under both the CIE illuminant and the test simulator using the CIE 1964 supplementary colorimetric observer. The CIE special metamerism index, which is the average colour difference of the metamers, was defined for both visible and ultraviolet ranges (denoted as MI_{vis} and MI_{uv}, respectively). The daylight simulator is rated from A to E, based on metamerism index value of <0.25, 0.25–0.5, 0.5–1.0, 1.0–2.0, >2.0 CIELAB colour difference units, respectively (Xu *et al.*, 2003).

5.2 Defining metamerism

A number of definitions have been proposed for metamerism, but the most acceptable one is by Judd and Wyszecki (1963):
Metameric objects are objects that, when illuminated by a given reference illuminant, reflect stimuli of different spectral power distributions (SPD) that produce the same colour under the same viewing conditions.

It is generally found that if the reflectance values do not intersect at all, then the match mainly differs in lightness; if they intersect once, then the chromaticities generally differ in the red-green region; and if they intersect twice, the chromaticities differ in the green-purple direction (Moridian, 1986). Our colour vision is trichromatic, therefore only three aspects of a reflected light are recorded by the visual system (Moradian and Rigg, 1987). Since colour is three dimensional, the metamers must intersect, at least, at three wavelengths so that they match under, at least, one illuminant. When the reflectance curves intersect at three or more widely spaced wavelengths in the visible region of the spectrum, it may be possible that the samples match in lightness as well as chromaticity under a particular illuminant. In spite of difference in spectral reflectance (R_λ) they may have the same tristimulus values under the said reference illuminant because these are integrated values, and unequal values at a particular wavelength may be compensated by unequal values at other wavelengths. However, the compensation fails when the illuminant is changed, resulting in some colour difference between the objects. When two coloured specimens match under a reference illuminant, say D65 (fluorescent lamp having colour temperature 6500 K, approximately similar to daylight) and mismatches under a test illuminant A (tungsten filament lamp), the match is termed metameric to illuminant A.

5.3 Types of metamerism

Five types of metamerism reported (Wyszecki, 1985) are:

1. Illuminant metamerism
2. Observer metamerism
3. Geometric metamerism
4. Field-size metamerism
5. Instrumental metamerism.

5.3.1 Illuminant metamerism

So far the metamerism due to change in illuminant has been discussed, because this is the most important type of metamerism and has received widest attention. Unless specifically mentioned, metamerism indicates illuminant metamerism only.

5.3.2 Observer metamerism

It is common in industrial practice to seek to control production with a tolerance on the basis of illuminant metamerism, disregarding observer variations. Any money on such control will be spent largely in vain when the metameric pairs are exposed to the eye of random purchasers or observers (Hemmendinger and Bottiger, 1978). Observer metamerism denotes that a pair is judged matched by an observer and mismatched by another. An index of metamerism is expected to assess these failures of colour matches between different observers. The phenomenon is important, not only for determining acceptability of perceived colour differences in metameric pairs, but also for use as a reference in the assessment of illuminant metamerism.

The extent to which the colour matching properties of normal trichromats differ from one observer to another is not readily specified. For large-field (10°) colour matching, Wyszecki and Stiles (1982) proposed a set of colour matching functions obtained for 20 observers, of which the first 18 were used in colour matching investigation of Stiles and Burch (1959). The selection was made of observers with greatest reliability and experience in trichromatic matching, and is a good representation of colour normal observers covering a wide range of age from about 20 to 60 years old. CIE TC 1–07 committee on observer metamerism submitted its report in favour of the above 20 colour matching functions.

Strocka (1978) proposed an index of observer metamerism. For a particular metameric pair, the indices of metamerism, M_i may be calculated in terms of CIELAB colour difference units for each of the 20 individual observers by replacing CIE standard observer functions while calculating tristimulus values. The indices may further be calculated for 'k' standard metameric pairs. The mean index for the 20 observers is a meaningful measure of observer metamerism (Equation [5.1]).

$$\text{Observer metameric index, } \overline{M_k} = \sum_{i=1}^{20} \frac{M_{ik}}{20} \qquad [5.1]$$

However, the calculation with 20 observer functions is too extensive to be suitable in practice. Strocka also pointed out that more than two observers deviating in a different way from a standard observer are not necessary to define a meaningful measure of observer metamerism.

It is convenient, therefore, to replace the CIE 1964 standard observer with a single test observer. Allen (1969) proposed a test observer, called a standard deviate observer, which was derived from a statistical analysis of the variances and the covariances of the above 20 observers. A new, more effective standard deviate observer has also been proposed by Nayatani *et al.* (1983, 1985) by using an analytical approach different from that used by Allen and then optimizing the new standard deviate observer using 22 metameric pairs. The optimized deviate observer gave observer metamerism indices with almost complete correlation to those averaged for the 20 observers. A simple observer metamerism index can be evaluated as the size of the colour difference between a metameric pair caused by substituting a standard deviate observer for the reference observer (CIE 1964 standard observer).

Dichroic ladders (Wardman *et al.*, 1996) are very effective tools for testing defective colour vision. They can also be used to detect differences between observers with normal colour vision. A dichroic ladder comprises a series of coloured samples, each differing in hue slightly from the next. The samples are prepared using a mixture of dyes which are colour-constant. The colour appearances of the samples, therefore, change very little when the light source is changed. Metal complex (1:1) dyes on wool are suitable for the purpose. A separate sample called floater is also prepared by dyeing with a mixture of other dyes (mostly levelling acid dyes), which are less colour-constant. The observer is asked to compare the colour of the floater with the colour of the samples in the ladder, placed side-by-side in a row, and to select the sample in the ladder that is the closest visual match to the floater. The experiment can be repeated under several light sources as the floater changes its hue, while samples of the ladder remain almost unchanged with the change of light source. By comparing the matches made by the individuals under various light sources, the colour vision of the individuals can be compared. The use of dichroic ladders also permits an assessment to be made of the accuracy of the computation of the CIE tristimulus values which specify colour appearance instrumentally. In a study (Wardman *et al.*, 1996), a good agreement was observed between visually and instrumentally selected matching ladder samples under simulated daylight (illuminant D65). Under tungsten light (illuminant A) and a departmental store lamp (illuminant TL84), most of the observers selected samples from the ladder as a match to the floater that were redder and greener for the two illuminants respectively than the instrumentally determined match. The error may be due to inappropriate weights for the two illuminants used while computing the CIE tristimulus values.

5.3.3 Geometric metamerism

Geometric metamerism arises when two object colours, that match under one set of conditions, no longer match when the geometries of illumination and viewing are changed. This may be due to differences of gloss and texture of the two members of the pair. However, the phenomenon has not yet been studied in detail.

5.3.4 Field-size metamerism

Field-size metamerism occurs when the match fails due to change of the size of visual field or distance of viewing. It is a special case of observer metamerism. Different sets of colour matching functions for the same observer are used for different sizes of visual field – 1931 and 1964 CIE standard observers are for the fields of 1–4° and above 4° angular subtense respectively. The same observer may detect metamerism by observing Maxwell spot, when the visual field of angular subtense greater than 4° is used for viewing. Maxwell spot is not generally noticeable, but occasionally it may be a striking feature and may complicate visual colour matching. The object may appear non-uniform in colour with ill-defined boundary of the Maxwell spot and a diameter of about 4°. As the point of reference is shifted from place to place, the Maxwell spot moves with it. The colour difference between the two areas is a measure of field-size metamerism. Most of the difference between the two sets of colour matching functions can be attributed to yellow macular pigment which covers the fovea and the surrounding areas. The index is calculated by assessing colour difference of a coloured object using 2° and 10° colour matching functions.

5.3.5 Instrumental metamerism

Instrumental metamerism arises when two objects match one set of instrumental conditions, but fail in another case. This mainly happens with different tristimulus colorimeters and with spectrophotometers with different geometries. Plates XXIV, XXV, XXVI (see colour section between pages 146 and 147) shows a metameric fabric set under three illuminants: (a) daylight fluorescent lamp (D65); (b) departmental store lamp (TL84); and (c) tungsten lamp (illuminant A). The figure clearly shows that the colour of the three samples changes differently under three light sources.

5.4 Numerical methods of generating metamers

Metamers are encountered in all colour reproduction processes. But for scientific studies, metamers generated theoretically by numerical methods are preferred, as these are not unduly constrained by the particular colouration process. Several numerical methods of generating metamers have been reported in the literature (Wyszecki, 1958; Takahama and Nayatani, 1972; Schmitt, 1976; Ohta, 1982)) such as:

1. Metameric black method
2. Linear combination method
3. Least square method
4. Monte Carlo method
5. Frequency limited method
6. Ohta's method
7. Matrix R method.

5.4.1 Metameric black method

The above method was developed by Wyszecki (1958); reflectance curves are generated by setting the tristimulus values each equal to zero – hence the name, metameric black. The spectral reflectance function of metameric blacks $\rho_b(\lambda)$ under a given illuminant $S(\lambda)$ fulfils the following conditions (Equation [5.2]):

$$\begin{aligned} \sum_{\lambda} \rho_{\lambda}(\lambda) S(\lambda) \bar{x}(\lambda) \Delta\lambda = 0 \\ \sum_{\lambda} \rho_{\lambda}(\lambda) S(\lambda) \bar{y}(\lambda) \Delta\lambda = 0 \\ \sum_{\lambda} \rho_{\lambda}(\lambda) S(\lambda) \bar{z}(\lambda) \Delta\lambda = 0 \end{aligned} \qquad [5.2]$$

There are an infinite number of metameric blacks that fulfil the above equations.

Metameric blacks are not physically realizable as the reflectance values are negative at some wavelengths, but they can be added to a given spectral function $\rho_0(\lambda)$ to generate a large number of spectral reflectance functions having positive values at all wavelengths (Equation [5.3]):

$$\rho(\lambda) = \rho_0(\lambda) + \rho_b(\lambda) \qquad [5.3]$$

The new function will be useful only if $0 \leq \rho(\lambda) \leq 1.0$. This can be achieved by multiplying $\rho_b(\lambda)$ by an appropriate scaling factor before adding it to $\rho_0(\lambda)$.

Wyszecki (1958) derived 27 linearly independent spectral reflectance functions under illuminant C for CIE 1931 colour matching functions. Each reflectance function has all but four zero reflectance values. Out of the four, three non-zero values are at the fixed wavelengths of 450, 520 and 620 nm, and the fourth, having a value of 1.0, varies in position along the spectrum. A more elegant set of spectral reflectance functions of metameric black was further derived by orthogonalizing the functions. Each pair of functions fulfils the following condition (Equation [5.4]):

$$\sum_{\lambda} \rho_b^i(\lambda) \rho_b^j = 0 \qquad [5.4]$$

with $i, j = 1, 2, 3, \ldots, 27$ and $i \neq j$.

The metameric black method was adopted for the study of colour-rendering properties of light sources and for comparison of colour matching functions.

5.4.2 Linear combination method

In this method, three different sets of spectral reflectance curves, either generated artificially or determined from spectrophotometric measurements of real objects, are linearly combined to give identical tristimulus values under one illuminant and unequal values under a different illuminant (Richter, 1958; Wyszecki and Stiles, 1982). The method can be extended to generate metameric stimuli with respect to more than one illuminant and/or observer.

A spectral reflectance function, $\rho_i(\lambda)$ can be derived by linear combination of three functions, ρ^1, ρ^2 and ρ^3, as follows (Equation [5.5]):

$$a_{i1}\rho_i^1(\lambda)+a_{i2}\rho_i^2(\lambda)+a_{i3}\rho_i^3(\lambda)=\rho_i(\lambda) \tag{5.5}$$

By finding unique solution of three simultaneous linear equations for the tristimulus values X, Y, Z, one can calculate the factors a_{i1}, a_{i2}, a_{i3}.

5.4.3 Least squares method

Takahama and Nayatani (1972) proposed a method in which a number of real reflectance functions are modified by least squares technique to give the same tristimulus values under one illuminant. The method consists of changing each given $\rho_\alpha(\lambda)$ function of the initial collection of spectral reflectance functions to a new function $\rho_\alpha^*(\lambda)$ that meets the conditions (Equation [5.6]):

$$\begin{aligned}
&\sum_\lambda \rho_\alpha^*(\lambda)S(\lambda)\bar{x}(\lambda)\Delta\lambda = X_\alpha^* = X\\
&\sum_\lambda \rho_\alpha^*(\lambda)S(\lambda)\bar{y}(\lambda)\Delta\lambda = Y_\alpha^* = Y\\
&\sum_\lambda \rho_\alpha^*(\lambda)S(\lambda)\bar{z}(\lambda)\Delta\lambda = Z_\alpha^* = Z\\
&\sum_\lambda \left[\rho_\alpha^*(\lambda)-\rho_\alpha(\lambda)\right]^2 = \text{minimum}
\end{aligned} \tag{5.6}$$

The computation procedure is simpler than those used in earlier studies. Smooth spectral reflectance functions of object colours can be produced, which resemble those found in paints and dye products.

5.4.4 Monte Carlo method

In this method (Wyszecki and Stiles, 1982) random numbers are used to generate a large number of metamers having identical tristimulus values under one illuminant by computer very quickly. Random numbers $\rho^j(\lambda_i)$ are assigned to a fixed set of wavelengths λ_i (i = 1–M) that satisfy the following condition (Equation [5.7]):

$$0 \le \rho^j(\lambda_i) \le 1.0 \tag{5.7}$$

5.4.5 Frequency limited method

The earlier methods generally give step functions instead of smooth curves. Smooth curves can be generated by this method – the reflectance at a particular wavelength being linked with that at neighbouring wavelengths (Stiles *et al.*, 1977). A basic collection of spectral reflectance functions ρ (λ) adopted is defined as follows (Equation [5.8]):

$$\rho(\lambda)=\frac{1}{2}+\frac{1}{2}\phi(\lambda)$$
$$\text{where } \phi(\lambda)=\sum_{i=-\infty}^{+\infty}\frac{\psi_i}{2}\frac{\sin^2\left[\pi(\lambda-\lambda_0)\omega-\frac{i\pi}{2}\right]}{\left[\pi(\lambda-\lambda_0)\omega-\frac{i\pi}{2}\right]^2} \quad [5.8]$$
$$-1\le\psi_i\le 1\left(i=0,\ \pm 1,\ \pm 2,\text{etc.}\right)$$

where ω is a constant representing the limiting frequency and λ_0 is a zero-adjustment factor. ∞ may be replaced with N when it is a sufficiently large number. When the limiting frequency of the spectral reflectance function is set at ω = 1/50, the spectral reflectance function achieves typically four oscillations within the visible spectrum, making them resemble real spectral reflectance curves.

5.4.6 Ohta's method

Ohta (1982) proposed a simplified method for formulating pseudo-object-colour reflectance function based on the dependency of reflectance on neighbouring wavelengths. It is reasonable to assume that the value of reflectance at a particular wavelength ρ_i is not independent of that of ρ_{i-1} and ρ_{i+1}, but instead is closely dependent on both of them. A simple way for formulating the dependency is to use the value $\bar{\delta}_i$ defined by Equation [5.9]:

$$\bar{\delta}_i=\frac{\rho_{i-1}+\rho_{i+1}}{2}-\rho_i \quad [5.9]$$

with i from 2 to n – 1. If we set an upper limit Δ such that $\bar{\delta}_i \le \Delta$, the equation can be rewritten as Equation [5.10]:

$$\rho_{i+1}=2\rho_i-\rho_{i-1}+2r_{i-1}\Delta\ (i=2, 3, \ldots, n-1) \quad [5.10]$$

where $r_{i\text{-}1}$ is a uniformly distributed number satisfying the condition $-1 \leq r_{i\text{-}1} \leq 1$. The upper limit Δ for an actual object may well be lower than 0.03 for 10 nm wavelength intervals.

5.4.7 Matrix R method

Worthey (1988) derived metameric blacks by a linear algebra method using Cohen's Matrix R (Cohen and Kappauf, 1982; Cohen and Kappauf, 1985). In a similar way, Matrix R operations have been extended (Berns *et al.*, 1989) to create metameric stimuli that theoretically remain metameric with respect to an arbitrary number of sequential illuminants called multiple metamers.

The statistical significance of the artificial reflectances has been questioned by Thornton (1977). He proposed that the real reflectance spectra are much smoother due to surface scattering and rarely have more than three maxima over the visible range (Brill, 1987).

5.5 Metamerism and object-colour solid

The CIE Y, x, y values of all possible object colours under a particular illuminant can be plotted in three-dimensional space. MacAdam (1935) mathematically calculated the limits of such object colours and by connecting the Y, x, y values of these limiting objects a torpedo-shaped three-dimensional body called 'Object-Colour Solid' can be formed (Hunter, 1975). The upper end of the solid is tapered, indicating that fewer colours are available at high lightness, especially in the blue range. The points on the surface of the object-colour solid have maximum possible excitation purity, chroma or saturation. Each of these points represents one optimal object-colour stimulus, i.e. a unique spectral reflectance curve having no metamer. However, a number of metamers exist for points within the solid, as shown by the study made by Stiles and Wyszecki (1962). The maximum number of metamers occurs in the centre of the solid. The most important application of a set of metamers generated with respect to a given illuminant and observer is to determine the magnitude of the colour mismatches that will occur when the illuminant and/or the observer is changed. Such data are useful for studying colour-rendering properties of light sources and for comparison of different sets of colour matching functions. When the illuminant is changed from reference to test illuminant, every single point in the tristimulus space will expand into a cloud of points, the boundaries of which are called the colour mismatch gamut, or the theoretical limits of metamerism. Their size, shape and location in the tristimulus space will depend on the SPD of the new illuminant. Three methods, namely metameric black, Monte Carlo and deterministic methods, are based on statistical techniques and therefore provide 95% boundary colour mismatch ellipsoids. In the metameric black method, an initial $\rho_i(\lambda)$ is required for each point in space. There is a great deal of freedom in choosing the spectral reflectance function, but its effect on the boundary of chromaticity mismatches is not fully understood. Multivariate statistical technique is used for calculation of a boundary in *x, y, Y* space in the Monte Carlo method. In deterministic method developed by Stiles and Wyszecki (1962), the actual production of spectral reflectance functions is not required. Instead, a basic collection of spectral reflectance is assumed, in which each member function is thought of as a set of reflectance values ρ_i, where $0 \le \rho_i \le 1.0$. They assumed that for every wavelength interval *i*, all reflectance values ρ_i are between zero and unity and that the variation of ρ_i of the *i*-th interval is independent of the variation in reflectance values in all other wavelength intervals. The assumptions are similar to those generally made in Monte Carlo technique. The basic collection is controlled by its frequency function $F(\rho_1,\ldots,\rho_M)$, which is split into the product of M functions, each depending on one reflectance value only.

$$
\begin{aligned}
&F(\rho_1,\ldots,\rho_{\mathrm{M}}) = F_1(\rho_1)\ldots F_M(\rho_M) \\
&\int_{\rho_i}^{1} F_i(\rho_i) = 1 \quad (i = 1 \text{ to } M)
\end{aligned}
\qquad [5.11]
$$

The next step is to determine the joint frequency function of the tristimulus values – six tristimulus values for each ρ_i function with respect to two given illuminants. Then, all tristimulus values of those object-colour stimuli are considered that, governed by the given collection of ρ_i functions, are metameric with respect to reference functions.

The 95% boundary ellipsoids computed by the different methods are, in general very similar in shape, orientation and volume. The differences are mainly caused by differences in the basic collections of spectral reflectance functions. None of the above statistical methods is capable of deducting the optimal boundary within which all of the original metamers must lie.

Theoretical reflectance curves generated by linear programming methods (Ohta and Wyszecki, 1975; Schmitt, 1976) can give optimal chromaticity mismatch boundaries. The optimal boundary is not an ellipsoid, and its shape and volume depend on the reference tristimulus values of the metamers characterized by the ρ_i functions. It embraces a considerably larger domain in the (*x, y, Y*) space than the 95% boundary ellipsoids determined by the statistical methods mentioned above.

Kuehni (1978) found that with real colourants (dyes), the position and shape of the boundary ellipsoids are similar, but the sizes are only one-third to two-thirds of the theoretical limits. It is reported (Berns *et al.*, 1988) that there is no statistical correlation, either between the illuminants or between their chromaticity mismatch gamuts. As a reference illuminant, D65 is probably the best choice, as it gives the smallest average mismatch gamut among different illuminants.

5.6 Wavelengths of intersections

For an object pair to be metameric, the reflectance curves of its members must be non-identical and must intersect at multiple wavelengths throughout the visible range. Stiles and Wyszecki (1968) verified on a theoretical basis that two stimuli, to be metameric, must cross at least three times. Kuehni (1978) noticed that for coloured textile samples, more four- or six-point crossovers can be observed than three-point crossings, due to their smooth and structureless reflectance curves as a result of scattering at or near the surface of the textile material. He also found that the locations of crossovers are highly randomized, with a distinct zone between 480 and 500 nm.

There has been large scale debate on the locations of spectral intersections of the members of metameric pairs. It is widely reported that there should be a minimum of three intersections – one at the short end, one at the middle and one at the long end of the visible spectrum. Many studies also show that the three crossover locations tend to occur in certain defined regions. These findings lead to hypotheses concerning fundamental aspects of the visual system. More specifically, we expect them to fall approximately at the wavelengths of the peaks of the three colour matching functions of the given observer. Thornton (1973) proposed that the wavelengths of the intersections are 448 ± 4, 537 ± 3 and 612 ± 8 nm. Thornton (1986) analysed the so-called Bayer set of metamers, a set of textile samples coloured with man-made colourants, and obtained average crossover wavelengths of 458, 541 and 611 nm. According to him, spectral intersection occurs at three wavelengths which are sensitivity peaks of the human visual system. He named these wavelengths as the 'prime wavelengths' and demonstrated their importance in light source design. Spectral crossing has important consequences for the design of illuminants. Thornton (1974) observed that if metameric reflectances cross at three specific wavelengths, then a three narrow band lamp emitting principally at these wavelengths should render the specimens identical in appearance.

Kuehni and Berns (1994) criticized that any such relationship is purely coincidental. Robertson (1994) commented that the location of intersections depends on the nature of metameric SPD – Thornton's method of deriving such SPD is simple, but arbitrary. Worthey (1994) said that the Thornton's proposal is valid only for a certain set of spectrally smooth reflectances.

Ohta and Wyszecki (1977) claimed that the variation of the intersecting wavelengths may be much greater. With numerical optimization of the reflectance differences in four zones of the wavelengths caused by three points of intersection and two end-points of the spectra, Ohta (1987) observed that the gamut of three wavelengths of intersections are confined within:

$$430 \le \lambda_1 \le 480, 500 \le \lambda_2 \le 580, 550 \le \lambda_3 \le 640 \text{ nm} \tag{5.12}$$

For strongly metameric colours, Ohta proposed convergence to $\lambda_1 = 450, \lambda_2 = 540$ and $\lambda_3 = 610$, which are close to those proposed by Thornton. According to him, Thornton's proposal is, therefore, applicable for strongly metameric colours.

Criticizing the conclusions made by Ohta, Kuehni (1988) stated that the intersections depend on the wavelength interval of the spectral data. Ohta's calculations were based on 10 nm intervals. Ohta's index of metamerism and the location of intersection change when the calculations are made at 5 or 1 nm.

Berns and Kuehni (1990) reported that crossover locations depend on the spectral properties of the metameric stimuli. For object colours, crossover can occur in nearly any region of the visible spectrum depending on the colourant absorption bandwidths. For a given set of colour matching functions, the only determining factor for crossover wavelengths is the said bandwidth. If absorption properties or SPDs are not allowed to constrain the metamers significantly, and when the degree of metamerism is very high, the crossovers tend to converge on the barycentric wavelengths – at 451, 540 and 597 nm for CIE 1931 2° observer and at 449, 534 and 595 nm for CIE 1964 10° observer.

Kuehni and Berns (1994) further claimed that as the absorption band width increased from 60 to 180 nm, the wavelengths of intersections (λ_1, λ_2 and λ_3) vary between the following ranges (Equation [5.13]):

$$438 \le \lambda_1 \le 460, 522 \le \lambda_2 \le 542, 599 \le \lambda_3 \le 628 \text{ nm} \tag{5.13}$$

Robertson (1994) commented that metamerism occurs when the signals generated in three retinal processes are equal for two different incident SPDs. This equality arises from the interactions of broad SPDs with broad sensitivity functions. The location of the intersections of metameric SPDs is determined by the nature of the SPDs and of the sensitivity functions. There is no fundamental significance to the location of intersections.

Roy Choudhury and Chatterjee (1996) studied the occurrence of metamerism during the textile dyeing process. Dyeing was carried out on thick opaque cotton tubular knitted fabrics with 160 commercially available dyes of different dye classes, namely direct, vat, azoic and reactive dyes in the depth range of 0.1–5% of the weight of material (g/L in case of azoic) following manufacturers' recommendations about application methods and depth restrictions. It is true that the self-shades (i.e. application of a single dye) are rarely metameric. However, self-shades provide a preliminary idea of variation of illuminant-dependent colour consistency among the dyes, which was utilized for preparing metameric pairs by shade-adjustments or using dyes of varying colour consistency. Many of the commercial dyes also contain toning agents and by-product components that may create metamerism. Selecting/preparing of metameric pairs was done by:

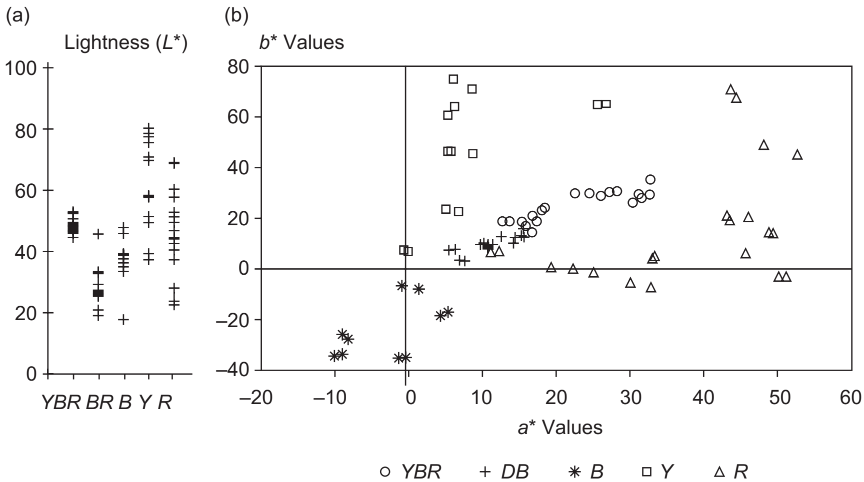
- Pair-wise comparison of the samples of under D65 lamp for closeness of shade.
- Selection of the pairs close in colour under D65, but showing distinct colour differences under tungsten and/or TL84 lamp.
- Shade-adjustment of the above pairs so that the members of the new pairs are further close in colour under D65 lamp. It was difficult to match shades belonging to two different dye classes, because each dye class had its own special tone/brightness and as such, often very close matching was not possible. With self-shades, only eight metameric pairs were obtained. One or both members of the remaining pairs were compound (mixture) shades.
- The selection of the modified pair as metameric, if the colour differences under tungsten and/or TL84 lamp persists.

Further to increase the reliability of assessments of the extent of metamerism, the metameric pairs were, subsequently, sorted into a few groups according to the shades or colours in terms of common colour names, such as dark brown. The five shade groups and the number of metameric pairs belonging to each group are as follows:

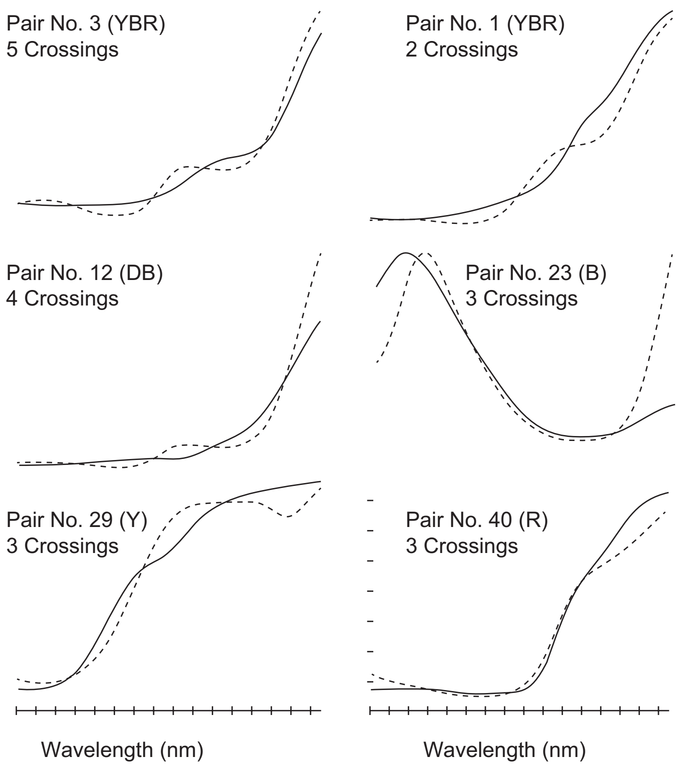
1. Yellowish and Reddish Browns (YBR) – 11 pairs
2. Dark Browns (DB) – 8 pairs
3. Blues (B) – 5 pairs
4. Yellows (Y) – 7 pairs
5. Reds and oranges (R) – 12 pairs.

Figure 5.1 shows lightness (L*) and a^*-b^* values of the members of 43 metameric pairs belonging to different shade groups. Lightness values approximately varied between 20 and 60 for shade groups YBR, DB and B and between 20 and 80 for shade groups *Y* and *R*. a^*-b^* values of majority of the samples lie between yellow and red axes.

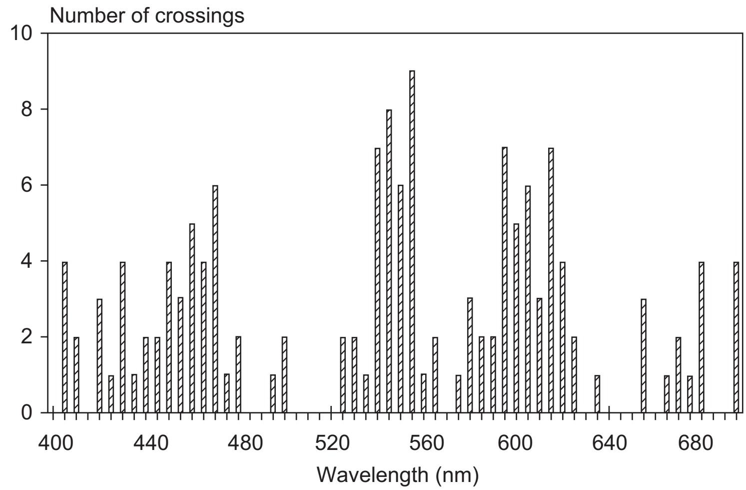
The reflectance curves of some metameric pairs from 400 to 710 nm are shown in Fig. 5.2. The reflectance curves of the respective members of the metameric pairs intersected at 2, 3, 4 or 5 wavelengths.



5.1 (a) Lightness (L*) and (b) a*-b* values of the members of 43 metameric pairs (shade-group-wise).



5.2 Reflectance curves of metameric pairs.



5.3 Frequency distribution of spectral crossings of the metameric pairs at 5 nm interval.

Generally for metamerism, the spectra of the pairs must cross at least at three wavelengths. However, some pairs having only two spectral crossings showed strong metamerism. The member spectra of such pairs are either very close to each other or just touching at another wavelength. Therefore, the pairs may have actually three crossings, but they are showing only two crossings, probably either due to error in spectrophotometric measurements or to the parameric nature (Section 5.9.1) of the pairs.

The frequency distribution of the 145 spectral crossings at 5 nm interval of the 43 metameric pairs is shown in Fig. 5.3. The spectral crossings were widely spread and occurred at most of the wavelength ranges. The frequencies of the intersections at different wavelength ranges varied from 1 to 9.

There were three prominent zones of spectral crossings, namely 450–470, 540–555 and 595–620 nm. These three zones are well within the three ranges proposed by Ohta (1987) – 430–480, 500–580 and 550–640 nm. The ranges for the three zones of spectral crossings were shorter probably because there was less number of samples under study. Ohta also proposed that, for strong metamerism, the pairs should have three crossings at 450, 540 and 610 nm. In the present study, three metameric pairs had spectral crossings close to the above three wavelengths.

5.7 Control of metamerism

To control metamerism in any colouration process, accurate measurement of metamerism is necessary. In manual shade matching, known colourants are used in different quantities and the shade is checked for colour difference against standard under a particular light source. When the prepared shade is within tolerable colour difference against standard, the sample is called a match to the standard. If we need a universal match, the pair is then to be checked under three or four light sources. If the colour difference under any light source is not acceptable, the shade may have to be discarded due to its metameric nature. With the invention of computer colour matching, the problem has been simplified to a large extent. In this case the computer with the help of mathematical calculations predicts recipes using multiple dyes. For universal match, the reflectance curves of the standard and that of the predicted recipe should be same (or the difference should be negligible). However, computerized spectral matching is very time consuming, tedious and difficult to attain. That is why most of the colour matching software relies on tristimulus match, i.e. the iteration process continues till the differences of the respective tristimulus values of the reference and predicted under a particular illuminant (i.e. theoretically defined light source) are within tolerance range. The computer programming cannot work on actual spectral data of light source as its SPD is variable. For each light source, there is an idealized illuminant whose SPD is defined, but it may not exist physically. The tolerance range may also be set on the basis of some colour difference formula, or in terms of some metameric index calculated from the tristimulus values.

When a spectrophotometric match is not feasible, a number of recommendations have been suggested by Longley (1976) for controlling metamerism during visual colour matching, based on practical experiences on colour matching. Referring to the locations of hues of colours to be mixed in Munsell hue circle, and considering perfect matching under daylight, he derived a number of rules as follows:

Rule 1. The closer or further apart in hues the yellow, orange and red chromatic colours to be mixed, the greener or redder will be the match, respectively under tungsten lamp.

Rule 2. When two brown combinations match in daylight and tungsten, the one with the most greenish yellow pigment will appear redder in cool white fluorescent lamp.

If the location of mixing colours moves in a clockwise direction, the new match will be greener under fluorescent lamp, and for counter-clockwise move, the match will be redder.

Rule 3. When yellow is mixed with any given green or blue, the greener the yellow, the greyer will be the mixed colour in tungsten. Conversely, the redder the yellow, the cleaner (i.e. brighter) and greener will be in tungsten. When mixed with any given yellow, the redder the shade of the blue, the greyer will be the mixed colour in tungsten. The greener the shade of blue or green, the cleaner will be the colour in tungsten.

Rule 4. When the hue difference between yellow and blue/green mixing colourants is increased, keeping daylight and tungsten match intact, the match becomes clearer in fluorescent lamp. Lowering the hue difference makes the colour greyer.

Such visual guides are approximate and, like every other rule, have exceptions.

5.8 Visual measurement of metamerism

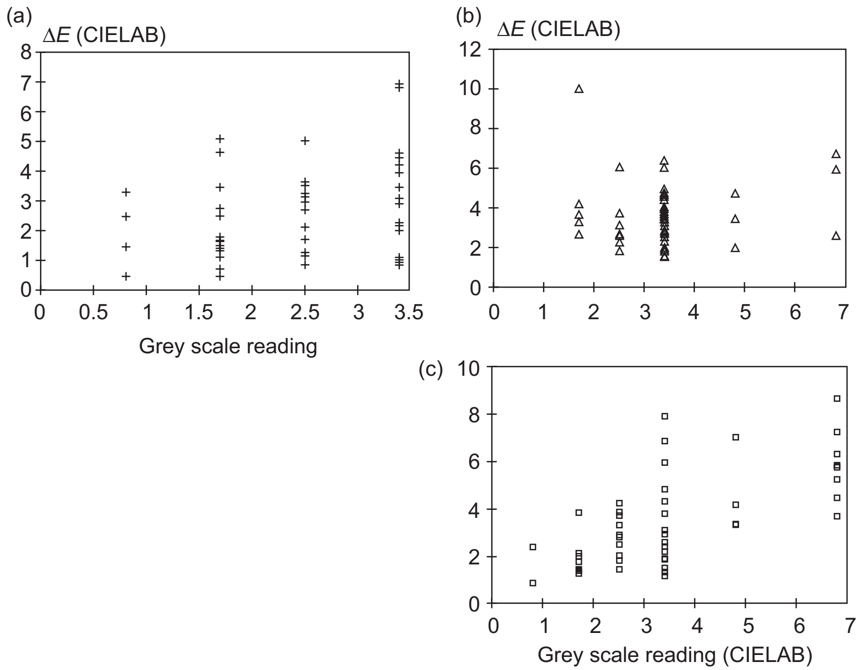
After observing that a pair of specimens is a visual match under a light source, the specimens are viewed by the same colour normal observer under one or more additional sources having distinctly different SPDs. If the first source is daylight, the second source may be an incandescent lamp. The larger the difference in the SPDs of the sources, the easier it is to detect a small degree of metamerism. A visual estimate of colour difference can be made with the help of grey scale. The grey scale consists of reference Munsell neutral grey chips paired with similar but progressively lighter Munsell grey chips. The relation between the grey scale rating and colour difference in CIELAB units is shown in Table 3.1. The metameric pair is placed in juxtaposition with the edge of the grey scale and compared with various grey pairs to determine which one most nearly matches the pair. The observation is preferably made under standardized viewing conditions inside a viewing chamber (ASTM, 1987).

Figure 5.4 shows a comparison of instrumental colour differences (CIELAB) against visual colour differences in terms of CIELAB values converted from a grey scale reading for 43 metameric pairs studied by Roy Choudhury and Chatterjee (1996). It can be seen that the grey scale method provides readings in terms of a few grading (e.g. 2, 2–3, 3, 3–4, 4, 4–5) and further interpolation is difficult and erroneous, particularly for metameric pairs where significant hue-change has occurred with change of illuminant. The method is, therefore, unable to provide fine scaling of colour differences, as in case of instrumental measures.

Both the D&H and Glenn colour rules are flat rectangular instruments, approximately 7 × 38 cm and 2.0 cm thick. They have two coloured slides, over which there is a mask with a small opening. Under controlled lighting conditions, observers are asked to move the slides back and forth until the two full samples visible through the opening look like the closest (metameric) match. One slide contains samples identified with letters. The other slide contains samples identified with numbers. After the observer has selected a colour match, the number and letter identifying the two swatches can be read. The alphanumeric matching pair obtained is strongly dependent on both the nature of the illuminant and on the state of the observer's colour vision. Variation in observer judgement of metameric colour matches is on

Table 5.1 Spearman's rank coefficients between visual metameric ranking and different measures of metamerism under three pairs of illuminants (Roy Choudhury and Chatterjee, 1996)

SHADE	MI	MI	MI	MI	MI	MI	MI	MI	MI	MI	MI	
GR.	BMAN	N+Y	RD	C	CS	CA	CR	CM	LABD	B+B	MC AVG.	
Reference illuminant D65 and test illuminant A												
YBR	0.20	0.25–0.55	0.40	0.30	0.13	0.15	0.13	0.18	0.18	0.17	0.14	
DB -0.48	0.74–0.31	0.71	0.93	0.48	0.76	0.00	0.52	0.57	0.50	0.40		
B	0.30	0.20 0.20	0.30	0.80	0.50	0.10	0.50	0.90	0.70	0.80	0.48	
Y -0.07–0.43	0.11	0.07–0.75	0.00–0.14	0.00–0.04–0.54	0.39–0.13							
R	0.32	0.07	0.68	0.37	0.80–0.37	0.80–0.37	0.30	0.22–0.50	0.21			
AVG.	0.05	0.17	0.03	0.37	0.42	0.15	0.33	0.05	0.37	0.23	0.27	0.21
Reference illuminant D65 and test illuminant TL84												
YBR	0.14	0.14–0.25	0.89	0.61	0.29	0.82	0.29	0.71	0.64	0.86	0.47	
DB	0.50–0.29–0.05	0.40	0.19	0.24	0.40	0.12	0.33	0.40	0.62	0.26		
B	0.60	0.40–0.40	0.30–0.50	0.60	0.10	0.60	0.70	0.90	0.90	0.38		
Y -0.04–0.04	0.18	0.25–0.14	0.29–0.14	0.29	0.50	0.54	0.93	0.24				
R	0.87–0.12	0.78–0.15	0.13–0.05	0.08–0.23	0.25	0.42	0.13	0.19				
AVG.	0.41	0.02	0.05	0.34	0.06	0.27	0.25	0.21	0.50	0.58	0.69	0.27
Reference illuminant TL84 and test illuminant A												
YBR -0.03–0.23–0.12–0.10–0.70	0.17–0.58	0.17 0.08–0.07–0.23–0.15										
DB	0.69	0.05	0.52	0.38	0.74	0.55–0.57	0.43	0.60	0.86	0.52	0.43	
B	0.50	0.60	0.60	0.50	0.20	0.10	0.50	0.10	0.10	0.90	0.40	0.41
Y	0.39–0.50	0.46	0.25–0.07–0.11	0.32	0.14	0.04	0.14	0.50	0.14			
R	0.82	0.38	0.83	0.47	0.58	0.32	0.30	0.32	0.77	0.67–0.53	0.45	
AVG.	0.47	0.06	0.46	0.30	0.15	0.20	0.53	0.23	0.32	0.50	0.13	0.22
G AVG.	0.31	0.08	0.18	0.34	0.21	0.21	0.37	0.17	0.40	0.44	0.36	0.23



5.4 Instrumental versus visual colour differences of 43 metameric pairs under three different illuminants : (a) Illuminant D65, (b) Illuminant A and (c) Illuminant TL84..

the same order of magnitude as the variation due to illuminant; i.e. illuminant and observer metamerism are about equally important. The obsolete Glenn Colourule has been revived, brought up to date and more carefully specified, using a durable polyester substrate dyed with well characterized disperse dyes. This revised colour rule is a simple tool for the study of the colour vision of normal and abnormal populations, and is based on the metamerism of subtractive mixtures. It can be used to study both observer metamerism and illuminant metamerism (Aspland and Shanbhag, 2006).

5.9 Metamerism indices

Visual methods mostly provide qualitative assessment of metamerism. A more reliable metamerism index (MI) is a measure of metamerism on the basis of spectrophotometric data. Ideally it should represent the extent of mismatch under various illuminants. The indices can be broadly classified into two groups (Hunter, 1975) namely:

1. Special index of metamerism
2. General index of metamerism.

5.9.1 CIE special index of metamerism

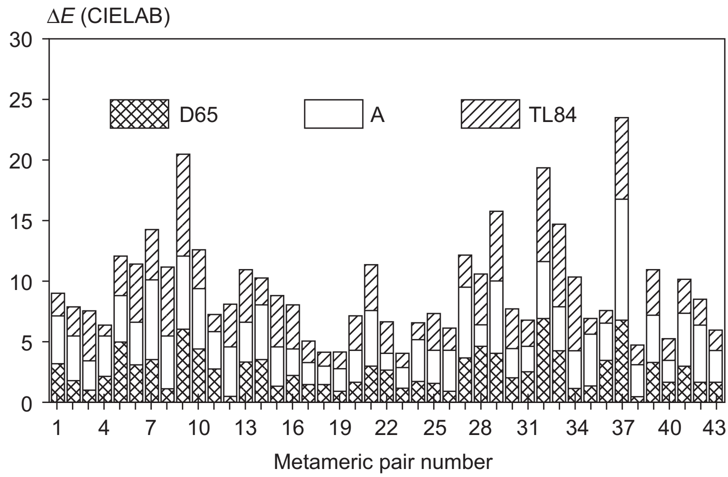
When two object colours (instrumentally) match under a standard illuminant and mismatch under a test illuminant, the special index of metamerism is the colour difference under test illuminant in terms of some standard colour difference equation such as CIELAB. The index was first proposed by Judd and Wyszecki (1963). The International commission on illumination (CIE) has recommended the CIELAB equation for object colours and the CIELUV equation for illuminating colours. Recently developed formulae such as CMC (l:c) may also be used.

Fifty-five metameric sample pairs were prepared using computer-predicted recipes from six different colour centres using cotton knit fabric. The colour difference of each sample pair was measured spectrophotometrically and was assessed visually by a panel of observers against a grey scale under three illuminants: reference illuminant D65, test illuminant A and TL84. In general, there was a positive agreement between observers' assessments, although there was some variation due to the spread of ages. The results of illuminant-specific special indices, CMC(2:1), were better than others, which included CIELAB, CMC(1:1), CIE94(1:1:1) and CIE94(2:1:1). In general, the performance of these five special indices was acceptable (Chow *et al.*, 1999).

The most important condition for evaluating the CIE special index of metamerism is the perfect matching of the specimens under, at least, one illuminant (reference illuminant). The exact match means both should have same tristimulus values. We cannot rely on visual matching because of the difference in colour vision characteristics between observers. However, identical tristimulus values exist only in the computer – it is very difficult to prepare a specimen with exactly same tristimulus as standard. In real situations, colour differences exist between the specimens even under the reference illuminant.

In the study by Roy Choudhury and Chatterjee (1996), the colour differences in ΔE (CIELAB) units of the 43 metameric pairs under three illuminants D65, A and TL84 varied from 0.47–6.94, 1.42–9.89 and 0.86–8.52, respectively, as shown in Fig. 5.5. The number of pairs having acceptable colour difference (i.e. less than 1 CIELAB unit) under the illuminant D65 was only five, and if we extend the limit to 1.5 CIELAB units, the number increases to 12. The hue-differences, ΔH^* (CIELAB) (not shown in the figure) were significantly high under illuminant A and much higher than those under D65. The hue-differences under TL84 were also higher than those under D65 and somewhat less than those under illuminant A for many pairs.

Some proposals (Robertson, 1983; Fairman, 1991) were made to rename the cases where colour differences exist under reference illuminant as paramerism, para-chromism, pseudo-metamerism, near-metamerism or



5.5 Colour differences between members of 43 metameric pairs.

approximate-metamerism, but none have been accepted universally. CIE (1971) advised to take suitable account of this difference – but no specific suggestion has been made. Many computer programs use additive corrections, while Broackes (1970) preferred multiplicative correction. Rodrigues and Besnoy (1980) proposed to compare the flare of the specimens (change of colours with quick change of illuminant) to compare the colour constancies of the individual samples. This is not a direct measure of metamerism, but a simple comparison of colour constancy properties of the standard and the specimen.

Some of the instrument manufacturers, such as M/s Hunterlab (1985), use another type of special index as follows (Equation [5.14]):

$$\text{Metameric index, MI (LABD)} \\ = [(\Delta L_{n1} - \Delta L_{n2})^2 + (\Delta a^*_{n1} - \Delta a_{n2})^2 + (\Delta b^*_{n1} - \Delta b^*_{n2})^2]^{1/2} \qquad [5.14]$$

where Δ indicates difference between standard and sample, and subscripts $n1$ and $n2$ indicate first and second illuminant. The L, a, b values may be of Hunter or CIELAB colour scale. This type of index does not distinguish between test and reference illuminants, but considers only the illuminant pairs.

Li and Berns (2007) compared different methods of parameric correction for the evaluation of metamerism. Three techniques have been used to correct this residual colour difference: an additive correction in $L^*a^*b^*$, a multiplicative correction in XYZ (recommended by the CIE) and parameric decomposition where the batch's spectrum is adjusted. The Matrix R technique had the worst spectral accuracy under the reference conditions, while principal component analysis (PCA) and independent component analysis (ICA) had similar and reasonable accuracies. Peyvandi and Amirshahi (2011) suggested a reliable approach for parameric correction using parameric decomposition based on PCA.

5.9.2 General indices of metamerism

The special index of metamerism is illuminant-specific. It needs one reference and at least one test illuminant. Test illuminants can be varied, and a number of indexes may be calculated or measured for a pair of specimens using same reference illuminant under which the specimens must match. To make the index illuminant independent and to specify the degree of metamerism of a pair of specimens by a single number, the general index of metamerism has been recommended. It is based on spectral difference between standard and trial, which is inevitable for metamers. Accordingly, Bridgeman and Hudson (1969) calculated the following index (Equation [5.15]):

$$\text{MI (BMAN)} = [\Sigma\, (R_{1\lambda} - R_{2\lambda})^2]^{1/2} \qquad [5.15]$$

where 1 and 2 denote the two members of the pair.

But the above equation does not take into account the variation of eyes' sensitivity to different spectral lights. Our eyes are more sensitive around 550 nm than at 400 or 700 nm. This variation in the eye's sensitivity at different wavelengths can be included by weighting the spectral differences with spectral sensitivity curves $(\bar{x}_\lambda, \bar{y}_\lambda, \bar{z}_\lambda)$ of our eyes (cones) – commonly known as colour matching functions. Such an index proposed by Nimeroff and Yurow (1965) is as follows (Equation [5.16]):

$$\text{MI (N + Y)} = \left[\sum\nolimits_\lambda \{\bar{x}_\lambda (\Delta R_\lambda)\}^2 + \sum\nolimits_\lambda \{\bar{y}_\lambda (\Delta R_\lambda)\}^2 + \sum\nolimits_\lambda \{\bar{z}_\lambda (\Delta R_\lambda)\}^2\right]^{1/2} \qquad [5.16]$$

Another formula, proposed by Chow *et al.* (1999), is based on weighting the spectral differences with the spectral luminous efficiency function. The index is calculated by means of Equation [5.17]:

$$\text{MI}(\text{P}) = \left[\sum_{\lambda=400}^{700} (V_\lambda \Delta R_\lambda)^2\right]^{1/2} \qquad [5.17]$$

The performance of the Nimersoff–Yurow index and the proposed metameric index by Chow were improved by weighting the spectral differences with the colour matching function and spectral luminous efficiency function, respectively, both functions compensating for the eye's sensitivity at different parts of the visible spectrum. In general, these indices failed to show any promising correlation with visual assessment (Chow *et al.*, 1999).

Moradian and Rigg (1987) suggested the use of the lightness index of Wyszecki ($L_\lambda = 25\,(R_\lambda)^{1/3} - 17$) instead of reflectance and the index becomes (Equation [5.18]):

$$\mathrm{MI(M+R)} = \left[\sum\nolimits_\lambda \left\{\bar{x}_\lambda(\Delta L_\lambda)\right\}^2 + \sum\nolimits_\lambda \left\{\bar{y}_\lambda(\Delta L_\lambda)\right\}^2 + \sum\nolimits_\lambda \left\{\bar{z}_\lambda(\Delta L_\lambda)\right\}^2\right]^{1/2} \qquad [5.18]$$

Khodadadi *et al.* (2007) calculated general MI using a power spectrum by Fourier transformation of spectral reflectance differences. This new approach is claimed to be a useful method to investigate the metamerism phenomenon.

5.10 Illuminant metamerism potentiality (IMP)

The colour position of a sample in the CIELAB colour space generally alters to another position when the combination of illuminant–observer changes. The change of colour position is referred to as colorimetric shift. Both the magnitude and direction of the colorimetric shift caused by change in the SPD of the illuminant depend on the spectral behaviour of each individual sample. From the theoretical point of view, a metameric pair of samples exhibits the metameric effect when the magnitudes and/or the directions of the colorimetric shifts of individual samples in the pair are different.

Peyvandi *et al.* (2012) attempted to develop a quantitative measure for objectively describing the metameric essence of a metameric pair. This measure is referred to as IMP – its magnitude is independent of changes in illumination (test illuminants). The properties and reliability of the IMP and analytical upper bound for the attainable metameric colour differences (MI) are illustrated by a numerical experiment, in which the correlation between IMP and the maximum of CIE94 colour difference of the pairs under various different test illuminants was examined.

The CIELAB IMP of a metameric pair can be defined as the highest possible rate of variation in the colour difference of the pair across all possible directions of small variation in SPD of the reference illuminant IMP of a metameric pair is a proportionality factor. It is not expressed in colour difference units, but nevertheless it can still be effectively used for mutual comparison (of the degrees) of metamerism for different metameric pairs.

It is expected that IMP should describe the upper limit of metamerism potential of a metameric pair under all kinds of test illuminant. So, numerical and practical experiments were carried out to study the reliability of IMP. The light sources A and 12 fluorescent lamps were selected as the test illuminants. Illuminant D65 and the 1964 standard observer were considered as the reference viewing condition. High positive correlations between IMP and the metamerism indices under four selected representative illuminants were found. It could be concluded that, also in this practical test, the estimated IMP reasonably explained the magnitude of metamerism under all illuminants under consideration. IMP acts as a single number quantity – it is a proportionality factor that successfully assists in describing the upper limit of colour differences of a metameric pair under a variety of light sources. On the basis of this research, IMP can reasonably provide a test-illuminant-independent estimate for evaluating the maximal magnitude of metamerism under different test light sources.

5.11 Spectral decomposition by Matrix R

Matrix R operations are based on the popular Wyszecki hypothesis (1953) that the colour processing mechanism preserves only part of the colour stimulus, the fundamental, evoking a single colour sensation and ejects the reminder, the residual, with no effect on the colour sensation. Matrix R is technically a projection operator using a generalized inverse. Matrix R operations emulate the colour processing system, extracting the fundamental while rejecting the residual. It is a square symmetrical matrix, i.e. each row is identical with the equal ranked column. Cohen and Kappauf (1982, 1985) derived Matrix R from a set of colour matching functions, Matrix A (Equation [5.19]),

$$R = A\,(A'\,A)^{-1}A' \qquad [5.19]$$

where A' is the transpose of Matrix A.

Theoretically any set of colour matching functions can be used. Any reflectance spectra can be decomposed into two components called fundamental stimulus and metameric black or residual stimulus using Matrix R. If the Matrix R is multiplied by the reflectance spectrum N (n by 1 matrix), the fundamental stimulus N^* ($N^* = RN$) is obtained. Again, residual stimulus B is given by subtracting fundamental stimulus from reflectance spectra, i.e. $B = N - N^*$. While calculating, scaled magnitudes (i.e. fractional quantities with respect to maximum value) are used.

5.11.1 Necessity of spectral decomposition

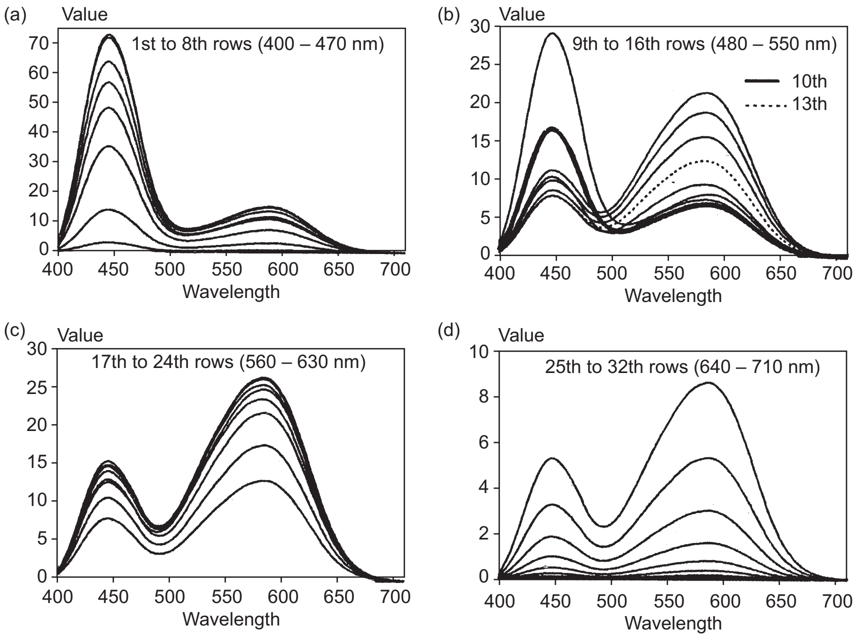
The most important criterion for evaluating CIE special index of metamerism is the perfect matching of the specimens under, at least, one illuminant (reference illuminant). However, in most of the cases, some colour differences exist between the specimens, even under the reference illuminant.

To have equal tristimulus values under reference illuminant, Fairman (1987) suggested spectral correction of one of the specimen using Cohen-Kappauf decomposition technique. Fundamental stimuli have the same tristimulus values as undecomposed spectra and thereby its special metamer. The fundamental alone is the cause of the colour sensation and the residual has no effect whatever on the evoked colour sensation as it has zero tristimulus values. When two metamers do not match under the reference illuminant, both may have difference in the fundamental stimuli and in the residual stimuli (i.e. metameric blacks). For perfect match under the reference illuminant, both should have same fundamental stimuli and, for metamerism, their residual stimuli should be different.

For matching under reference light, both the standard and trial should be decomposed, and the residual stimuli of the trial are to be added to the fundamental stimulus of the standard to get modified spectra of the trial having tristimulus values as those of standard under reference illuminant. Now the colour difference under any test illuminant can be calculated from the unmodified spectrum of the standard and modified spectra of the trial.

5.11.2 Computation of Matrix R

The Matrix R (32 × 32 matrix) was derived from the 1964 colour matching functions A ($\bar{x}_\lambda, \bar{y}_\lambda, \bar{z}_\lambda$, 32 × 3 matrix) using Equation [5.19]. The 32 rows (in groups of eight) of the Matrix R are plotted in Figs 5.6a–d. Each row or column represents every 10th nm wavelength between 400 and 710 nm. All the rows have two maxima – one at 450 nm (first peak), another at 590 nm (second peak). From the first row till 6th, values increase at each wavelength. At sixth row (top-most curve in Fig. 5.6a), the value reaches to a maximum of 74.3 at 450 nm and 15.7 at 590 nm. Then the values decrease for the subsequent rows and reach lowest values of 11.92 at 450 nm for 13th row and 6.84 at 590 nm for 10th row (Fig. 5.6b). The values again increase and reach maximum values of 15.7 and 27.01 for successive peaks at the 20th row (top-most curve in Fig. 5.6c). The values then decrease again for successive rows at all wavelengths and reach near unity at the 32nd row (lowest curve in Fig. 5.6d). As the matrix is symmetric, column values also change in a similar way. The colour matching functions used for deriving Matrix R were not weighted with SPD of any illuminant. Therefore, the fundamentals and the residuals obtained using the Matrix R may be considered to be equivalent to those under equi-energy spectrum.



5.6 Matrix R based on 1964 colour matching functions.

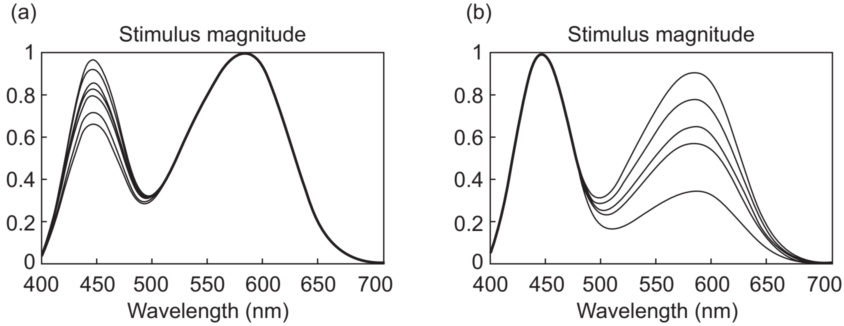
5.11.3 Spectral decomposition

The reflectance curves of the 43 metameric pairs were decomposed (Roy Choudhury and Chatterjee, 1996) into the fundamental stimuli by multiplying them with Matrix R, and the residual stimuli were derived by subtracting the fundamental from the original spectra. Spectral decomposition was carried out for the 43 metameric pairs. For most of the metameric pairs, fundamental stimuli differences were negligible. However, the residual differences were moderate to high.

Thirteen types of fundamental stimuli were observed among the metameric pairs under study, shown in Fig. 5.7. Eight types are shown in Fig. 5.7a and five other types in Fig. 5.7b. These include metameric pairs belonging to all shade groups, namely YBR, DB, B, Y and R.

5.11.4 Residual difference

As no improvement in the evaluation of special indices of metamerism was possible by spectral decomposition, due to near-identical fundamental stimuli of the respective members of most of the metameric pairs, efforts were made to propose improved general indices of metamerism on the basis of residual difference.



5.7 Thirteen types of fundamental stimuli obtained (a) eight types and (b) other five types.

The respective samples of each metameric pair under study differ only in residuals, and very little in fundamentals. Therefore, a general index of metamerism, MI (RD) was proposed on the basis on the square-root of the sum of the squares of the residual difference, similar to the Bridgeman's index (Bridgeman and Hudson, 1969) based on reflectance differences.

5.12 Colour constancy and metamerism

Robertson (1983) stated that colour constancy and metamerism are related. Metamerism may result from the failure of the colour constancy of one of the members of a pair of matched samples. Therefore, the degree of metamerism may probably be measured as a ratio or difference in colour constancy, though no work has yet been reported. For traditional colour order systems, the uniformity of the spacing of colours is disturbed when the light source is changed (Brill and Hemmendinger, 1985–86). The concept of colour constancy, i.e. names of object colours as assigned in natural daylight and the related colour order system, have been recently questioned by Brill *et al.* (1985–86). They showed that by changing the illuminant the whole hue circle may be reversed especially with an illuminant having a discontinuous SPD. If three reflectances $r1(\lambda)$, $r2(\lambda)$ and $r3(\lambda)$ are such that the chromaticity locus $[r2(\lambda), r3(\lambda)]/r1(\lambda)$ is convex and well-ordered in wavelength, then the objects having such reflectances will be immune to a chromaticity ordering reversal under change of illuminant (Brill *et al.*, 1985–86). An atlas will be consistent if the reflectance triads have the property stated above. The proof has been made by decomposition of tristimulus invariant Cohen's Matrix R into simpler tristimulus volume ratios (Brill, 1985).

The present measures of metamerism demand perfect matching under, at least, a single illuminant, in the absence of which the measures are unreliable or unpredictable. Moreover when metamerism exists between the members of a pair, or is created purposefully to generate 'Flare' effects, the reliability of present measures should be studied more intensively. When the illuminant is changed or intensity is varied, our eyes' sensitivity changes to some degree to compensate for the change of illuminance. Moreover, the object colours in complex viewing situations try to retain their colour relations intact in changing illuminating conditions. While the difference between the former phenomenon, i.e. chromatic adaptation and the latter, i.e. colour constancy may be debated, they are definitely correlated with the phenomenon of metamerism and may give further insight to its study.

While measuring the special index of metamerism, we consider the colour change in test illuminant. Here the final or resultant colour difference is measured. However, due to chromatic adaptation, the colour of the object reverses more or less to its original colour when the illuminant is changed. While measuring metamerism we do not consider this reversal process, called adaptive colour shift. When the illuminant is changed quickly there may be little scope for chromatic adaptation to occur, as it is a time dependent process. Therefore, the colour difference will be greater (McLaren, 1983). However, when the eye is fully adapted, colour difference is reduced.

Berns and Billmeyer (1983) proposed the following index of metamerism on the basis of chromatic adaptation transforms (Equation [5.20]):

$$\mathrm{MI\ (B+B)} = \left[\left(L^{/}_{\mathrm{STD}} - L^{/C}_{\mathrm{SAM}}\right)^2 + \left(a^{*/}_{\mathrm{STD}} - a^{*/C}_{\mathrm{SAM}}\right)^2 + \left(b^{*/}_{\mathrm{STD}} - b^{*/C}_{\mathrm{SAM}}\right)^2\right]^{1/2} \quad [5.20]$$

'*C*' stands for correction for difference in tristimulus values under reference illuminant. Correction is made in tristimulus values as follows (Equation [5.21]):

$$\begin{aligned} X^{/C}_{\mathrm{SAM}} &= X^{/}_{\mathrm{SAM}}\left(\frac{X^{D}_{\mathrm{STD}}}{X^{D}_{\mathrm{SAM}}}\right) \\ Y^{/C}_{\mathrm{SAM}} &= Y^{/}_{\mathrm{SAM}}\left(\frac{Y^{D}_{\mathrm{STD}}}{Y^{D}_{\mathrm{SAM}}}\right) \\ Z^{/C}_{\mathrm{SAM}} &= Z^{/}_{\mathrm{SAM}}\left(\frac{Z^{D}_{\mathrm{STD}}}{Z^{D}_{\mathrm{SAM}}}\right) \end{aligned} \quad [5.21]$$

/ stands for chromatic transformation from test to reference illuminant, and '*D*' stands for D65 (reference) illuminant.

Metamerism may also be due to failure of colour constancy of one of the objects resulting in mismatch under changed illuminant. So, probably some relation may be found between index of metamerism and colour constancy indices of the members of the metameric pair.

5.13 Performance of metameric indices

For the comparison of performances of the indices of metamerism (Roy Choudhury and Chatterjee, 1996), 43 metameric pairs were visually ranked in terms of the extent of visual metamerism. The ranking of the metameric pairs was made in order of increasing change in colour with change of the illuminant.

All the metameric pairs of a particular shade group were placed together side-by-side inside a colour matching cabinet under the reference (daylight) light source and then under the test light source. The pairs were checked visually for the colour differences (in fact, difference of colour differences) with the change of light source. Sufficient time was given to allow chromatic adaptation to act. The metameric pairs were ranked as per the extent of change of the visual colour difference with change of light source. Lowest rank (number) was allotted for the pair showing minimum change of colour difference with the change of light source and highest rank (number) was allotted for the metameric pair showing largest change of colour difference. The process was repeated two or three times for accurate ranking.

The following 11 established and proposed metameric indices were studied for correlation with visual metameric ranking:

1. Bridgeman – MI (BMAN)
2. Nimeroff and Yurrow – MI (N+Y)
3. Spectral Decomposition Residual differences – MI (RD)
4. Colour difference under test illuminant in CIELAB unit – MI (C)
5. Subtraction of colour differences under test and reference illuminants – MI (CS)
6. Addition of colour differences under test and reference illuminants – MI (CA)
7. Ratio of colour differences under test and reference illuminants – MI (CR)
8. Multiplication of colour differences under test and reference illuminants – MI (CM)
9. MI (LABD) based on L^*, a^*, b^* differences
10. MI (B+B) considering chromatic adaptation.
11. Difference of the colour constancy indices, CCI (MC) (Chapter 6) between the two members of the metameric pairs.

Table 5.1 shows Spearman's rank correlation coefficients between 11 metameric indices and shade-group-wise visual metameric ranking under three pairs of illuminants for five shade groups YBR, DB, B, Y and R. The correlations, in general, were moderate to poor. Some causes that may have diminished the correlations are:

1. Small number of observers (only four).
2. Difference in spectral energy distribution of light sources used in visual booth and those of illuminants used instrumental measurements.
3. The shade variations among the members of a particular shade group.
4. Unavoidable inherent errors in ranking method in the absence of external or reference tool like Grey Scale.

However, in some cases, good correlations were observed, especially under illuminant pair D65/TL84. This is probably due to the inherent similarity of these two illuminants. For a number of cases, the coefficients were negative, indicating inverse ranking. The number of such cases of negative ranking was more in the case of general indices.

All three general metameric indices showed poor correlations. Index weighted with colour matching functions, MI (N+Y) showed poorer correlations compared to the unweighted index, MI (BMAN). As the metameric pairs had identical fundamentals, the residual differences are expected to represent the degree of metamerism in a better way. But improvement in correlation was marginal in the case of MI (RD) as compared to MI (BMAN). On the contrary, it showed more instances of negative ranking.

Colour differences under test illuminant, MI (C), showed moderate correlations with visual ranking for some shade groups under D65/A and D65/TL84.

Addition (CA) and multiplication (CM) of colour differences under test and reference illuminants did not show any improvement in correlations with visual ranking. However, subtraction (CS) and ratio (CR) of these two colour differences showed some promising results. However, in five cases for CS and four cases for CR, negative ranking or inverse correlations were observed mostly for Y shade group.

Both the indices, MI (LABD) (based on ΔL^*, Δa^*, Δb^* differences under two illuminants) and MI (B+B), (based on chromatic adapted and corrected for tristimulus differences under reference illuminant), showed good correlations for some shade groups under each of the three illuminant pairs. MI (B+B) indices showed the best grand average coefficient, followed by MI (LABD). The rank coefficients for these two indices were also very similar. Metameric indices based on colour constancy indices (MC) showed some good correlations for most of the shade groups, irrespective of illuminant pair, but in a few cases the relations were inverse (negative ranking).

5.14 Instrumental shade sorting

Even after strict control of process variables during colouration for certain products, such as textiles and ceramics, it is impossible or uneconomical to limit colour variations among successive batches within certain tolerance. A collection of units or batches may be considered satisfactory commercial matches, but the collection may include units which would be unsatisfactory if they were placed adjacently. In such cases, more critical colour tolerances are required, and groups of acceptable materials need to be divided into subgroups whose overall colour differences are smaller. Such finer subdivision of coloured lots is known as shade sorting. If a large number of pieces have been dyed to match the same standard, but many are too far away to be commercially acceptable, a cheaper alternative to batch correction is shade sorting.

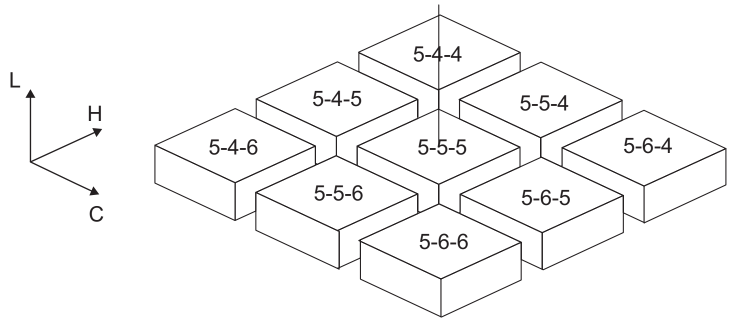
Shade sorting methods are being mostly used by the garment industry. Even though they source the material that has been 'passed' by an instrumental method of 'fail-pass' as per the criteria set by them, still they find that some colour differences become visually perceivable on stitching of the garment from the 'passed' lots of the same shade (Gulrajani, 2010).

In the majority of colour-shading applications, most of the sorting is still done visually. Manual shade sorting requires considerable experience of the kinds of variation in colour appearance that may be encountered for the particular type of goods. The sorting process will be easier if control samples are available. Visual observation should be under standardized conditions, as for certain goods colour appearance varies considerably with the angle of viewing and illumination and the uniformity of illumination.

Instrumental measurements are used to referee visual judgements, decide doubtful cases and maintain stability of shade ranges (Hunter, 1975). Instrumental sorting of samples is an objective process and provides a reproducible numerical record of the process. However, short and long term reproducibility and precision of the colorimetric values are the most important factors to decide whether instrumental shade sorting is worthwhile.

It is generally accepted that, for a large population of units acceptably matched to a standard, the colours of the units fall within the smooth, continuous surface boundary of an ellipsoid. The shape of the ellipsoid is such that all colours lying on the surface boundary are equally acceptable. By appropriate scaling of the dimensions of the axes, ellipsoids can be transformed into spheres. The procedures are independent of the colour difference formulae and the type of colour space used. In the simplest form of instrumental or numerical shade sorting, the volume of colour space containing acceptable coloured units, i.e. the acceptability ellipsoid is superimposed by a three-dimensional rectilinear grid. The first widely used system employed a rectangular solid whose faces were parallel to L, a^*, b^* planes, but in 1977 a more efficient solid was used whose faces were aligned parallel to L^*, C^* and H^*. Each block thus formed encompassed samples close in colour and was numbered by three digits, representing unit lengths of the three dimensions such as L^* (lightness), C^* (chroma) and H^* (hue) of CIELCH 1976 colour space in relation to the respective lengths of the ellipsoid. The method is known as 555 shade sorting (Simon, 1984). The advantage of the system is its numerical simplicity, and the colour relationship between the blocks can be inferred from their ascribed numerical values. In this system 'standard' shade is assigned the number '5' for all the three colour axes, i.e. L^*, a^* and b^* or L^*, C^*, h. Therefore, the 'standard' shade is termed as '555' and located at the centre of the 555 box. It is also known as the sort code of the sample. Other boxes have sort codes between 111 and 999. The dimensions of the three axis of the boxes are set as per the preset tolerance limits for the ΔL^*, Δa^*, Δb^* or ΔL^*, ΔC^* and ΔH^*. However, a sample may have a negative value of ΔL^*, or Δa^* or Δb^* or ΔC^* or ΔH^* that is at the periphery of the box, and another sample with plus value of ΔL^*, or Δa^* or Δb^* or ΔC^* or ΔH^* at the other end of the box, so the tolerance limit in the box becomes twice the pre-defined tolerance limit (Aspland and Howard, 1997).

A guideline for setting-up tolerance limits for individual colours in terms of ΔL^*, Δa^*, Δb^* is given by Li and others (1998). At different lightness levels, tolerance limit of Δa^* and Δb^* values are set between 0.4 and 0.8 – for yellows and violet, Δb^* tolerances are as high as 1.2. These specifications have been worked out on the basis of the experience of colourists handling the shade sorting process and provide the specification for all three dimensions for the different regions of the colour space. However, one may use one's own shade sorting criteria for adjusting the tolerance limit, according to his or her experience. These specifications are applicable to the Cartesian (L^*, a^*, b^*) coordinate. These are not entirely suitable for either the polar (L^*, C^*, h) coordinate or CMC micro-space concept, because both of these systems consider colour in terms of lightness, chroma and hue rather than lightness, redness-greenness and yellowness-blueness. Figure 5.8 shows 555 shade sorting blocks in the two dimensions (hue and chroma). Blocks with variations in lightness will remain above and below this set. Blocks starting with 6 (e.g. 655, 645, 666) remain in the upper block and those starting with 4 (e.g. 455, 445, 444) will remain below this set. The block 664 will contain colours that are lighter (higher L^*), more saturated or brighter (higher C^*) and little difference in hue (different H^*) than the colours in block 555. The primary disadvantage of the 555 shade sorting method is that borderline colours occupying the corners are 73% farther from the centre of the cube than are the centres of the sides. The problem is highlighted if the 555 block dimensions have not been selectively weighted to correspond to the ratios of the lengths of the semi-axes of the ellipsoids. The advantages of 555



5.8 A two-dimensional (hue and chroma) set of blocks in 555 shade sorting diagram. Lightness blocks are below and above this set.

shade sorting are its arithmetical simplicity and a well-defined relationship between the shade sorting blocks and the standard. The 555 shade sorting method is also referred to as the fixed-grid method, that relies on the closely packed array of boxes or blocks as discussed above. The dimensions of the three axis of the box depend on the preset tolerance resulting into rectangular or cubic blocks. The block sorting systems, does not optimize the sorting process to minimize the number of groups produced. They rely on allocation of samples into one of the blocks in the rigidly structured array around the standard. Block sorting systems may be satisfactory in a situation in which it is desirable to locate future production into previously established groups (Wardman *et al.*, 1992).

Another drawback of the fixed grid is that of the closely matching samples falling on the periphery of the adjacent boxes, resulting in more groups of samples adding to increased inventory, storage and handling problems.

The most efficient shade sorting solid would appear to be a sphere as it provides maximum volume for a given diameter. But there will be a significant space between adjacent spheres and batches falling in these spaces will not be sorted. McLaren (1983) proposed that the ideal sorting solid is truncated octahedron. However, as computation of six square and eight hexagonal faces is difficult, the next best solid proposed was a rhombic dodecahedron resulting from generating on each cube face a square pyramid whose peak lies at the centre of the next close packed cube. For the same diameter, the cube has only 36.8% of the volume of the sphere, while the above polygon has a volume of 47.7%. The newer sorting will result in 23% fewer groups, thereby increasing the sorting efficiency.

Another sorting system, called clemson color clustering (CCC) (Aspland *et al.*, 1987), determines the minimum number of spheres of specified size which are required to enclose all the sample points. Each group of dyed sample is called a cluster that is located in a sphere in colour space. The main difference between the CCC method and the 555 method is that in the 555 method the colour acceptability space (i.e. tolerance limit) is defined in a specified manner and the dyed samples falling within the acceptability space are 'put' into the boxes (cubes or polyhedra), while in the CCC method a minimum number of spheres (clusters) of specified size are created to house all the dyed samples. The diameter of the sphere (cluster) is the function of the maximum colour difference between all pairs of dyed samples in that cluster. This method does not take into consideration the concept of colour acceptability space while creating the cluster. In this technique, starting with one point (single sample) cluster, the other clusters are recursively merged to the 'parent' cluster to create a large cluster until the termination criterion is reached. The final number of clusters produced by this method determines the number and size of the shade sorting groups.

CCC shade sorting system is a 'dynamic' system. In the dynamic system the supplied dyed rolls (lots) or those in the stock grouped together initially may change when a new shipment is added to inventory. This may change the number of groups and also position of the initially grouped rolls from one cluster to another.

A limitation of the CCC clustering method is that the sorting is carried out without reference to a standard. This means that the groups produced are not coded according to their position in the colour space relative to the standard. Thus, the colour difference between the samples within a group and the standard coloured sample cannot be assessed.

A method to overcome this limitation of the CCC method has been proposed by Wardman *et al.* (1992). These investigators have suggested that initially a 'primary' cluster be created around the standard sample having samples with less than half of the value of the set tolerance limit of acceptability. In this way, all of the samples in the primary cluster will be an acceptable match to each other. The normal clustering method is then applied to the remaining samples that lie outside the primary cluster. This method has been termed the 'Scotsort method' (Gulrajani, 2010).

K-means (MacQueen, 1967) is one of the simplest unsupervised learning algorithms that solves clustering problem. The procedure follows a simple and easy way to classify a given data set through a certain number of clusters (assume *k* clusters) fixed *a priori*.

Shade sequencing is a traditional process of manually arranging lots of dyed pieces so that the colour difference between the adjacent pieces is not visually perceptible before stitching a garment. The shades can be sequenced by using a visual tapering method i.e. arranging lighter to darker or vice versa, which is a one-dimensional solution to the three-dimensional problem, since colours are described in terms of three parameters, namely lightness, chroma and hue. It fits into the argument that, notwithstanding the sophistication of instruments or the complexity of the mathematical formulae applied, there is always, at some stage of the process, someone who must look at the colour and decide whether to accept or reject it.

The problem of visual sequencing becomes complex when slight variation of hue is encountered along with the shade depth variation. However, when variation occurs primarily in L^* or C^* (linear variation) the tapering and sequencing becomes easy with good agreement between sorters. Similarly, when both L^* and C^* vary, tapering can be done smoothly, more so than the variation of one colour parameter.

Clustering of the tapered samples can be carried out by a pattern recognition technique known as the nearest neighbour technique. However, this technique cannot be applied when garment parts are cut at different times and then stitched, or when multiple products must match each other.

A further improvement in the combined sequencing and cluster technique has resulted in the development of the adaptive clustering technique. The adaptive clustering technique developed by SheLyn Inc. for shade sorting combines clustering, sequencing and historical analysis. Initially, ellipsoidal clusters are created based on user-defined ΔE_{cmc} tolerance. The data of the pieces that do not fall into any cluster are maintained, and when the new pieces arrive their data are compared with the left-out pieces. If found compatible, all these pieces are then used to create a new cluster; this newly formed cluster may be slightly shifted towards the centre of gravity of the cluster. The process is terminated when sufficient number of pieces has been added to the cluster. The pieces within each cluster are also sequenced. Thus, these clusters can 'adopt' based on evolving history. SheLyn Inc. has incorporated all these features in their Color iMatch Industrial software. This software is being used by GretagMacbeth (Laidlaw, 2008).

5.15 Conclusion

General metameric indices, Bridgeman's and Nimeroff and Yurow's and other indexes based on residual differences failed to show any promising correlation and showed inverse correlations in a large number of cases. The illuminant-dependent special indices are, therefore, more acceptable.

Colour differences under test illuminant, normally taken as a measure of the degree of metamerism, did not show high correlation, probably because the majority of the metameric pairs under study had colour differences under reference illuminants.

Mathematical transformations of colour differences under test and reference illuminants are unable to correlate with visual ranking in a better way. Subtraction and ratio of CIELAB colour difference values under test and reference illuminants showed good correlations in many cases. However, these measures resulted in more cases of negative ranking or inverse correlations.

Indices based on ΔL^*, Δa^*, Δb^* differences under two illuminants, MI (LABD) showed better performance. Another advantage of this type of indices is that they consider illuminant pairs only, and do not distinguish between test and reference illuminant. Berns and Billmeyer index (B+B) takes into account the inequalities of the two colour spaces (reference and test) and incorporates necessary corrections for differences in tristimulus values under reference illuminant. The correlations improved further and in fact showed best performance among the measures under study. The number of cases having negative ranking were also less for the above two indices.

Metameric indices based on colour constancy indices showed promising results, but also showed very low correlations for a number of metameric pairs and more cases of negative ranking.

The present work showed that indices based on ΔL^*, Δa^*, Δb^* differences under test and reference illuminant with or without modification for colour spaces as in case of MI (B+B) and MI (LABD) indices respectively performed best among the existing indices. Subtraction of colour differences under these two illuminants also showed good correlation in many cases. Further improvement may be made by incorporating chromatic adaptation in a better way. The difference of colour constancy indices of the members of the metameric pairs showed some correlations with the degree of metamerism, and this may lead to newer indices of metamerism in future.

Shade sorting of the acceptable fabric lots is necessitated because the fabric swatches from these dyed lots when placed together for stitching show visual colour difference. Even though one can sort coloured samples visually, it is not in practice due to poor observer-to-observer correlation and poor repeatability. Instrumental shade sorting is preferable, and considered more reliable. The first instrumental shade sorting method was evolved by Simon in 1961 and during over last 40 years many sorting methods have been developed that include 555 shade sorting, Clemson Colour Clustering, K-means Clustering and Adaptive Clustering.

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