Control of an AUV (Autonomous Underwater Vehicle) under wave disturbances

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Introduction

The goal of this project is to design an underwater vehicle (a submarine in our case) and to study different **controller** strategies for **stabilizing** the submarine in presence of external disturbances, such as waves.

With underwater vehicle we define “a small vehicle that is capable of propelling itself beneath the water surface as well as on the water’s surface”.

Submarine Hull

The design of the classic submarine is called “teardrop-hull”, this type of structure has several advantages in terms of performance in the submerged phase due to the fact that it reduces hydrodynamic drag when the vehicle is submerged and it is structurally efficient against high external pressure, on the other hand it decreases the sea-keeping abilities when the vehicle is on the surface (also increasing the drag).

The modern small submarines, as the older ones, have a single hull while the huge submarines (as the ones dedicated to a military usage) requires a structure with multiple hulls, basically two, which are the pressure hull and the light hull: these two hulls are structured differently for different usages.

There is also a difference in the structure of the submarine: in fact if we look deeply at the history, after the Second World War most of the country affiliated or connected with Soviet Union, and obviously the Soviet Union itself, used submarines with a structure composed from double or multiple hull while the military alliance composed from the U.S.A and its allies normally used submarines with a single hull and a light hull only in specific parts of the submarine (as in the bow and the stern).

Light Hull

In a submarine vessel with a double-hull structure the light (outer or casing) hull is the external hull which forms the shape of submarine.

This external part defines the performance, in terms of hydrodynamic, of the submarine due to the fact that it affects the amount of power required to drive the vehicle through the water but it doesn’t have to withstand a pressure difference.

The light hull could be used as perfect place to put on equipment that could cause excessive stress if it is put in the inner hull (pressure hull). Obviously the light hull is very less heavy than the pressure hull and it helps also in terms of integrity of the vessel: in fact if the external hull takes some damage, the internal one will be untouched.

Pressure Hull

The principal aim of the strong (or pressure or internal) hull is to withstand the outside pressure in order to maintain the normal atmospheric pressure inside.

This hull is normally built with thick high-strength steel (the last type is the AHSS, or advanced high strength steel) with a complex internal structure and interconnection with the light hull. Normally the pressure hull is composed from a stiffening structure and it is divided by watertight bulkheads into many compartments.

The light hull and the pressure one are divided by a gap, often used to place some equipment which can tolerate the high external pressure (normally air and water tanks). The connection between these two hulls forms a three-dimensional structure (supported from steel elements and bars) which gives buckling stability and strength.

Pressure Hull

The circular cross section is the desired shape for the pressure hull, due to the fact that any other shape would be weakier: in fact there is no material that could resist to the compressive stress.

In the project design for a pressure hull, an engineer has to take as acceptable a safety factor of 1.5-2 and also the degree of precision has to be as high as possible in the construction of the pressure hull due to the fact that even a deviation from cross-sectional roundness in the order of 25 mm could decrease of a factor of 30% the hydrostatic load capacity.

One of the most important factors to rate the capacity of a submarine is the depth rating, which is represented obviously from the strength of the pressure hull: in fact the outside water pressure increases with depth. Approximately for each 10 meters of depth, the total number of atmospheres increases by one (so the pressure on the hull at, for example, 1000 meters is around 10100 kPa).

Ballast tanks

A ballast tank is a structure added during the construction of aquatic equipment to provide an adjustable point of equilibrium: commonly used in ship construction, as well as floating wind turbines, oil platforms, and submarines, a ballast tank can be filled with water to add or remove weight: this weight makes it possible for a sailing vessel or aquatic platform to maintain a specific level in the water despite varying weather conditions.

The concept of ballast is not new and has been followed since ancient times. In the earlier times, the sea-going vessels used solid ballast such as sandbags, rocks, iron blocks, etc. which were loaded/unloaded once the cargo loading or discharge operation was finished. This method helped to a certain extent to maintain the stability of the ship and its seaworthiness.

Ballast tanks

In ship construction, a ballast tank is usually centered at the lowest point of the hull. Additional ballast tanks may be installed in the fore and aft positions or on the starboard and port sides of the hull. By positioning multiple ballast tanks around the ship, it can withstand the high winds and waves that are commonly encountered in the ocean environment.

Ships used to transport goods incorporate ballast tanks to account for the weight of their payloads. An empty freighter ship might easily be capsized by high seas: by adding water to a ballast tank, the ship's center of gravity can be lowered, making it more stable. On the other hand, a heavily laden freighter could be swamped by high waves crashing over the bow. By pumping water out of the ballast tanks, the ship sits higher in the water and thus reduces its risk of being swamped.

Submarines use ballast tanks to provide their submerging and surfacing movements. By filling the ballast tanks with water, the craft becomes heavier and sinks below the surface of the water. When it is time to resurface, the ballast tanks are drained of water and the submarine becomes buoyant, rising to the surface.

Ballast tanks

Ballast tanks are continuously exposed to corrosive environment.
The application of coatings to water ballast tanks is the primary means of corrosion protection for ships and is recognized as one of the most important factors affecting integrity, maintenance cost and service life. Coatings serve mainly to minimize the corrosion rate, thereby potentially delaying the utilization of the built-in corrosion margins included in a vessel's structural scantlings.

As water is easily and abundantly available, it is used for providing the required stability and trim to the ship. This water is known as ballast water and process of taking ballast water into the ship is known as “ballasting”. The tanks on ships wherein the ballast water is filled are so known as ballast tanks.

The function of a ballast tank is controlled by a Kingston valve and a vent, which work together in a system of checks and balances. When the vent and valve are both opened, water flows into the tank, providing ballast. When only the Kingston valve is opened, the air pressure of the closed vent keeps the water out. This bit of physics is used by submarine pilots in times of war. Faster submergence times are possible by traveling in open water with the Kingston valve opened and the vents closed.

Sensors

Introduction

An underwater vehicle (a submarine in our case) is “a small vehicle that is capable of propelling itself beneath the water surface as well as on the water’s surface”.

Most large submarines consist of a cylindrical body with hemispherical (or conical) ends and a vertical structure, usually located amidships, that houses communications and sensing devices as well as periscopes. In modern submarines, this structure is the "sail" in American usage and "fin" in European usage (the tower-like structure found on the dorsal (topside) surface of submarines).

The technological process in the marine engineering has contributed to its success beyond all expectations and it is in particular due to the development of weapon systems (such as torpedoes and missiles) and sensors (especially the sonar) even more efficient.

Introduction

A sensor is a device that detects and responds to some type of input from the physical environment (like physical stimulus such as heat, light, sound, pressure, magnetism, or a particular motion) and transmits a resulting impulse converting it, for example, into data (as for measurement or operating a control) that can be interpreted by either a human or a machine.

Most sensors are electronic (the data is converted into electronic data), but some are more simple, such as a glass thermometer, which presents visual data. People use sensors to measure temperature, gauge distance, detect smoke, regulate pressure and a myriad of other uses.

Sonar

Sonar, short for **SO**und, **NA**vigation and **R**anging, is a method or device for detecting and locating objects, especially underwater, by means of sound waves sent out to be reflected by the objects.

It is very helpful for exploring and mapping the ocean because sound waves travel farther in the water than do radar and light waves.

The term sonar is also used for the equipment used to generate and receive the sound: the acoustic frequencies used in sonar systems vary from very low (infrasonic) to extremely high (ultrasonic).

The first recorded use of this technique was by Leonardo da Vinci in 1490, who used a tube inserted into the water to detect vessels by ear: moreover it was developed during World War I to counter the growing threat of submarine warfare with an operational passive sonar system in use by 1918.

Modern active sonar systems use an acoustic transducer to generate a sound wave which is reflected from target objects

Sonar

Two types of technology share the name “sonar”:

- **active** sonar transducers emit an acoustic signal or pulse of sound into the water: if an object is in the path of the sound pulse, the sound bounces off the object and returns an “echo” to the sonar transducer. If the transducer is equipped with the ability to receive signals, it measures the strength of the signal and, by determining the time between the emission of the sound pulse and its reception, the transducer can determine the range and orientation of the object;

- **passive** sonar systems are used primarily to detect noise from marine objects (such as submarines or ships) and marine animals like whales. Unlike active sonar, passive sonar does not emit its own signal, which is an advantage for military vessels that do not want to be found or for scientific missions that concentrate on quietly “listening” to the ocean. Rather, it only detects sound waves coming towards it.
  Passive sonar cannot measure the range of an object unless it is used in conjunction with other passive listening devices. Multiple passive sonar devices may allow for triangulation of a sound source.

Bourdon gauge

A bourdon tube pressure gauge is a mechanical pressure measuring instrument that reads the pressure without requiring any electrical power. It is generally used for the measurement of pressure from 0.6 to 7000 bar (8 to 10000 psi).

It is the most common type of pressure gauge and is used by industries for its high accuracy and precision, especially for high-pressure applications. Bourdon tube pressure gauges are suitable for liquid or gaseous media for vacuum, low and high-pressure applications.

Compared to other types of pressure gauge, the bourdon tube pressure gauge advantages are:

- relatively low cost;
- compact design;
- safety in the measurement of high-pressure ranges;
- measurement accuracy;
- use with heavy vibration application and dynamic pressure load;

Bourdon gauge

There are different types of Bourdon tubes:

- **c-type**: the most commonly found tube used for low-pressure applications (it can withstand pressures of up to 10,000 kilopascals). It consists of a tube bent into a coil or an arc: as the pressure in the tube increases, the coil unwinds and a pointer connected to the end of the tube can be attached to a lever and a pointer calibrated to indicate pressure;

- **spiral type**: this type of Bourdon tube is spiral in shape and suitable for pressures of up to 28,000 kilopascals;

- **helical type**: this type of Bourdon tube is helical in shape and suitable for pressures of up to 500,000 kilopascals.

While c-type Bourdon tubes are inexpensive to manufacture and easier to find, their pressure sensitivity is limited. Spiral and helical tubes are therefore used for high-pressure applications or those where more precision is required;

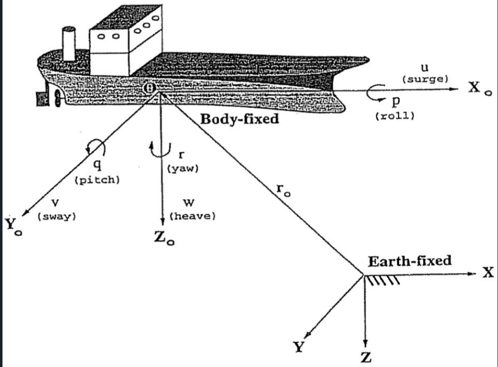
Motion Equation

Motion equation (1)

The equations of motion are usually represented using generalized position, velocity and forces defined by the state vectors:

$$\boldsymbol{\eta} := [x, y, z, \phi, \theta, \psi]^T$$
$$\mathbf{v} := [u, v, w, p, q, r]^T$$
$$\boldsymbol{\tau} := [X, Y, Z, K, M, N]^T$$

where η is the generalized position expressed in the North-East-Down (NED) reference frame $\{n\}$. When analyzing the motion of marine vehicles in 6-DOF it is convenient to define two coordinate frames as in figure:



Motion equation (2)

The moving coordinate frame $X_0Y_0Z_0$ is conveniently fixed to the vehicle and is called *the body-fixed reference frame* {*o*}.

The origin *O* of the body-fixed frame is usually chosen to coincide with the center of gravity when it is in the principal plane of symmetry or at any other convenient point.

For marine vehicles the body axes X_0,Y_0 and Z_0 coincide with the *principal axes of inertia* and are usually defined as:

X_0 - longitudinal axis (from aft to fore)
Y_0 - transversal axis (to starboard)
Z_0 - normal axis (directed downward)

A body-fixed reference frame {*o*} with the previous axes is rotating about the NED reference {*n*} frame with angular velocity $\boldsymbol{\omega} = [p, q, r]^T$.

Motion equation (3)

The generalized velocities $\dot{\eta}$ and **v** satisfy the following kinematics transformation:

$$\dot{\eta} = J(\eta)\mathbf{v}$$

$$J(\eta) = \begin{bmatrix} \mathbf{R}(\boldsymbol{\Theta}) & \mathbf{0}_{3\times3} \\ \mathbf{0}_{3\times3} & \mathbf{T}(\boldsymbol{\Theta}) \end{bmatrix}$$

where $\boldsymbol{\Theta} = [\phi, \theta, \psi]^T$ is the *Euler angles* and

$$\mathbf{R}(\boldsymbol{\Theta}) = \begin{bmatrix} c\psi c\theta & -s\psi c\phi + c\psi s\theta s\phi & s\psi s\phi + c\psi c\phi s\theta \\ s\psi c\theta & c\psi c\phi + s\phi s\theta s\psi & -c\psi s\phi + s\theta s\psi c\phi \\ -s\theta & c\theta s\phi & c\theta c\phi \end{bmatrix}$$

The matrix **R** is the *rotation matrix* and it is recognized as the Euler angle rotation matrix $\mathbf{R} \in SO(3)$ satisfying $\mathbf{R}\mathbf{R}^T = \mathbf{R}^T\mathbf{R} = \mathbf{I}$ and $\det(\mathbf{R}) = 1$, which implies that **R** is orthogonal.
Consequently, the inverse rotation matrix is given by $\mathbf{R}^{-1} = \mathbf{R}^T$.

Notation

The generalized velocity vector **v** and forces **τ** are both expressed in {*o*} and the 6-DOF states are defined according to the SNAME notation (1950):

- **surge** position x, linear velocity u, force X;
- **sway** position y, linear velocity v, force Y;
- **heave** position z, linear velocity w, force Z;
- **roll** angle ϕ, angular velocity p, moment K;
- **pitch** angle θ, angular velocity q, moment M;
- **yaw** angle ψ, angular velocity r, moment N;

So it is now possible to define a mathematical model of a marine craft, usually represented by a set of ordinary differential equations (ODEs) describing the motions in 6-DOF previously defined above.

Mathematical model (1)

Starting from translational motion equation:

$$m(\dot{v}_0 + \omega \times v_0 + \dot{\omega} \times r_G + \omega \times (\omega \times r_G) = f_0$$

and rotational motion equation:

$$I_0\dot{\omega} + \omega \times (I_0\omega) + mr_G \times (\dot{v} + \omega \times v_0) = m_0$$

and writing them in component form according to SNAME (1950) notation, that is:

$f_0 = \tau_1 = [X, Y, Z]^T$	external forces
$m_0 = \tau_2 = [K, M, N]^T$	moment of external forces about O
$v_0 = v_1 = [u, v, \omega]^T$	linear velocity of $X_0Y_0Z_0$
$\omega = v_2 = [p, q, r]^T$	angular velocity of $X_0Y_0Z_0$
$r_G = [x_G, y_G, z_G]^T$	center of gravity

Mathematical model (2)

It is possible to apply the previous notation to the first equations yields:

$$
\begin{aligned}
m\left[\dot{v} - wp + ur - y_G(r^2 + p^2) + z_G(qr - \dot{p}) + x_G(qp + \dot{r})\right] &= Y \\
m\left[\dot{w} - uq + vp - z_G(p^2 + q^2) + x_G(rp - \dot{q}) + y_G(rq + \dot{p})\right] &= Z \\
I_x\dot{p} + (I_z - I_y)qr - (\dot{r} + pq)I_{xz} + (r^2 - q^2)I_{yz} + (pr - \dot{q})I_{xy} & \\
+m\left[y_G(\dot{w} - uq + vp) - z_G(\dot{v} - wp + ur)\right] &= K \\
I_y\dot{q} + (I_x - I_z)rp - (\dot{p} + qr)I_{xy} + (p^2 - r^2)I_{zx} + (qp - \dot{r})I_{yz} & \\
+m\left[z_G(\dot{u} - vr + wq) - x_G(\dot{w} - uq + vp)\right] &= M \\
I_z\dot{r} + (I_y - I_x)pq - (\dot{q} + rp)I_{yz} + (q^2 - p^2)I_{xy} + (rq - \dot{p})I_{zx} & \\
+m\left[x_G(\dot{v} - wp + ur) - y_G(\dot{u} - vr + wq)\right] &= N
\end{aligned}
$$

The first three equations represent the translational motion while the three last equations represent the rotational motion: in summary the general 6-DOF rigid-body equations of motion.

Improved Mathematical Model

Introduction

The previous mathematical model was structured on the simplification that the control inputs of the 6-DOF system were all available on the 3 axis and on the 3 orientation angle.

The model didn’t take into account the presence of a single propeller and the control surfaces, that actually generate these forces in order to control the submarine. As well as the important role of the available sensors needed for the motion.

After the introduction of the physical structure of the submarine and its behaviour under pressure, now we focused on the role of the control surfaces and the propeller on the mathematical model, showing how a single actuator influences the different state variables and how and under which conditions the system can be properly decoupled.

Also we provided even a generalized model of an electric motor providing the relation between the control input of the motor and the propeller output force.

Improved submarine mathematical model

The starting point are the equations of motion for a rigid body with 6DOF. The equations are derived from the second law of dynamics:

$$\Sigma F = m\mathbf{a_G}$$

- a_G is the acceleration of the centre of mass;
- r_G is the position of the center of mass;
- w is the angular velocity;
- V is the linear velocity;

The acceleration is expressed as:

$$\mathbf{a_G} = \frac{\partial V}{\partial t} + \omega \times \mathbf{V} + \dot{\omega} \times \mathbf{r_G} + \omega \times \omega \times \mathbf{r_G}$$

Applying this equation in the first, the equations of a rigid body are:

Improved submarine mathematical model

Force equations

$$m[\dot{u} - vr + wq - x_G(q^2 + r^2) + y_G(pq - \dot{r}) + z_G(pr + \dot{q})]$$
$$= \sum X_{ext}$$

$$m[\dot{v} - wp + ur - y_G(r^2 + p^2) + z_G(qr - \dot{p}) + x_G(qp + \dot{r})]$$
$$= \sum Y_{ext}$$

$$m[\dot{w} - uq + vp - z_G(p^2 + q^2) + x_G(rp - \dot{q}) + y_G(rq + \dot{p})]$$
$$= \sum Z_{ext}$$

Moments equations

$$I_{xx}\dot{p} + (I_{zz} - I_{yy})qr - m[y_G(\dot{w} - uq + vp) - z_G(\dot{v} - wp + ur)]$$
$$= \sum K_{ext}$$

$$I_{yy}\dot{q} + (I_{xx} - I_{zz})rp - m[z_G(\dot{u} - vr + wq) - x_G(\dot{w} - uq + vp)]$$
$$= \sum M_{ext}$$

$$I_{zz}\dot{r} + (I_{yy} - I_{xx})pq - m[x_G(\dot{v} - wp + ur) - y_G(\dot{u} - vr + wq)]$$
$$= \sum N_{ext}$$

Forces and moments

It is important to focus our attention on the forces expressed in the right term of the 6DOF equations of motion. They are related to the coriolis effect, dumping effect of water, added mass inertial and, obviously, propulsion of the propellers.

The forces and moments are:

- hydrostatic forces;
- added mass inertia forces;
- hydrodynamic forces and moments;
- control surfaces forces and moments;
- propeller forces and moments;

Hydrostatic forces

The hydrostatic forces are the weight (W) and buoyancy (B), they acts respectively on the center of gravity and on the center of buoyancy. These forces, reported as acting on the center of mass are:

$$
\begin{aligned}
X_{HS} &= -(W - B)\sin\theta \\
Y_{HS} &= (W - B)\cos\theta\sin\phi \\
Z_{HS} &= (W - B)\cos\theta\cos\phi \\
K_{HS} &= -y_G W\cos\theta\cos\phi - z_G W\cos\theta\sin\phi \\
M_{HS} &= -z_G W\sin\theta - x_G W\cos\theta\cos\phi \\
N_{HS} &= -y_G W\cos\theta\sin\phi - z_G W\sin\theta
\end{aligned}
$$

Added mass inertia forces

The water which accelerates with the submarine creates and additional inertia, measured as added mass. This forces and moments are expressed as:

$$X_A = X_{\dot{u}}\dot{u} + X_{wq}wq + X_{qq}q^2 + X_{vr}vr + X_{rr}r^2$$
$$Y_A = Y_{\dot{v}}\dot{v} + Y_{\dot{r}}\dot{r} + Y_{ur_a}ur + Y_{wp}wp + Y_{pq}pq$$
$$Z_A = Z_{\dot{w}}\dot{w} + Z_{\dot{q}}\dot{q} + Z_{uq_a}uq + Z_{vp}vp + Z_{rp}rp$$
$$K_A = K_{\dot{p}}\dot{p}$$
$$M_A = M_{\dot{w}}\dot{w} + M_{\dot{q}}\dot{q} + M_{uw_a}uw + M_{vp}vp + M_{rp}rp + M_{uq_a}uq$$
$$N_A = N_{\dot{v}}\dot{v} + N_{\dot{r}}\dot{r} + N_{uv_a}uv + N_{wp}wp + M_{pq}pq + N_{ur_a}ur$$

The parameters can be obtained by estimations, empirical relationships and by the other terms already derived.

Effect of Drag - Hydrodynamic forces

The drag force in the front of submarine is:

$$D = \frac{1}{2}\rho C_D A_f V^2$$

Where C_D is related to the angle of attack with:

- $C_D = a\alpha^2 + b\alpha + c$, with α the angle of attack;
- A_f is the frontal area of the submarine;
- V is the fluid velocity.

The drag forces in all the directions are:

$$\begin{aligned} X_d = \quad & X_{u|u|}u\,|u| + X_{uv}uv + X_{uw}uw + \\ & X_{v|v|}v\,|v| + X_{w|w|}w\,|w| \\ Y_d = \quad & Y_{uv_d}uv + Y_{v|v|}v\,|v| \\ Z_d = \quad & Z_{uw_d}uw + Z_{w|w|}w\,|w| \end{aligned}$$

Where:

$$X_{u|u|} = -\frac{1}{2}(\rho A_f)\,c \qquad X_{uw} = X_{uv} = -\left(\frac{1}{2}\rho A_f\right) b$$

$$X_{w|w|_d} = X_{v|v|} = -\left(\frac{1}{2}\rho A_f\right)\left(a + \frac{c}{2}\right) \qquad -Y_{v|v|} = Z_{w|w|} = -\left(\frac{1}{2}\rho A_f\right) b$$

$$Z_{uw_d} = -Y_{uv_d} = -\left(\frac{1}{2}\rho A_f\right) c$$

Hull lift

The lift *L* acts on the center of pressure, it is perpendicular to the flow as the submarine moves in water. The lift causes the following moments and forces.

Lift forces and moments are represented by this relation:

$$L = \frac{1}{2}\rho C_L A_f V^2 \qquad C_{L_\alpha} = \frac{\partial C_L}{\partial \alpha}$$
$$M = \frac{1}{2}\rho C_M A_f V^2 \qquad C_{M_\alpha} = \frac{\partial C_M}{\partial \alpha}$$

The resulting forces and moments are:

$$Z_l = -\frac{1}{2}\rho A_f C_{L_\alpha}(u^2 + w^2)\alpha cos\alpha$$
$$M_l = \frac{1}{2}\rho A_f C_{M_\alpha}(u^2 + w^2)\alpha$$

$$Y_l = \frac{1}{2}\rho A_f C_{L_\beta}(u^2 + v^2)\beta cos\beta$$
$$N_l = \frac{1}{2}\rho A_f C_{M_\beta}(u^2 + v^2)\beta$$

Control surfaces

Submarine fins lift

$$L_{fin} = \frac{1}{2}\rho C_{L_{\delta f}} S_{fin} \delta_e v_e^2$$
$$M_{fin} = x_{fin} L_{fin}$$

Stern planes

$$\delta_e = \delta_s - \frac{w - x_{fin} q}{u}$$

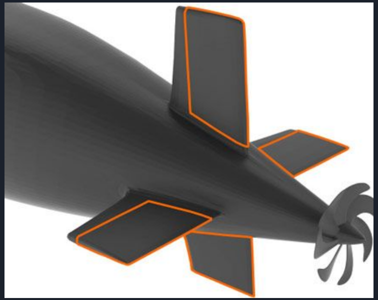
Vertical rudders

$$\delta_e = \delta_r + \frac{v + x_{fin} r}{u}$$

v_e = u and x_{fin} is the axial position of the fin post in body referenced coordinates

The rudders and the sterns planes generate a combined motion both on one axes and on one euler angle. The stern planes control the Pitch angle and generate a force on the Z-axis. While the rudders generates a force on the Y-axis and moment on Yaw angle

$$Y_r = \frac{1}{2}\rho C_{L\delta f} S_{fin}\left[u^2\delta_r + uv + x_{fin} ur\right]$$
$$Z_s = -\frac{1}{2}\rho C_{L\delta f} S_{fin}\left[u^2\delta_s - uw + x_{fin} uq\right]$$
$$M_s = -\frac{1}{2}\rho C_{L\delta f} S_{fin} x_{fin}\left[u^2\delta_s - uw + x_{fin} uq\right]$$
$$N_r = -\frac{1}{2}\rho C_{L\delta f} S_{fin} x_{fin}\left[u^2\delta_r + uv + x_{fin} ur\right]$$



Total result Results

$$\sum X_{ext} = X_{HS} + X_{u|u|}u|u| + X_{\dot{u}}\dot{u} + X_{uv}uv + X_{uw}uw + X_{v|v|}v|v| + X_{vr}vr + X_{w|w|}w|w| + X_{wq}wq + X_{qq}qq + X_{rr}rr + \underline{X_{prop}}$$

$$\sum Y_{ext} = Y_{HS} + \boxed{Y_{uu\delta_r}u^2\left(\delta_{r_{top}} + \delta_{r_{bottom}}\right)} + Y_{ur}ur + Y_{uv}uv + Y_{v|v|}v|v| + Y_{\dot{v}}\dot{v} + Y_{wp}wp + Y_{pq}pq + Y_{\dot{r}}\dot{r}$$

$$\sum Z_{ext} = Z_{HS} + \boxed{Z_{uu\delta_s}u^2\left(\delta_{s_{right}} + \delta_{s_{left}}\right)} + Z_{uw}uw + Z_{uq}uq + Z_{vp}vp + Z_{w|w|}w|w| + Z_{\dot{w}}\dot{w} + Z_{\dot{q}}\dot{q} + Z_{rp}rp$$

$$\sum K_{ext} = K_{HS} + K_{\dot{p}}\dot{p} + \boxed{K_{uu\delta_r}\left(\underline{-\delta_{r_{top}} + \delta_{r_{bottom}}}\right) + K_{uu\delta_s}\left(\underline{-\delta_{s_{right}} + \delta_{s_{left}}}\right)} + \underline{K_{prop}}$$

$$\sum M_{ext} = M_{HS} + \boxed{M_{uu\delta_s}u^2\delta_s} + M_{uw}uw + M_{uq}uq + M_{vp}vp + M_{\dot{w}}\dot{w} + M_{\dot{q}}\dot{q} + M_{rp}rp$$

$$\sum N_{ext} = N_{HS} + \boxed{N_{uu\delta_r}u^2\delta_r} + N_{ur}ur + N_{uv}uv + N_{\dot{v}}\dot{v} + N_{wp}wp + N_{pq}pq + N_{\dot{r}}\dot{r}$$

Propeller forces and moments

Roll moment is given by difference of angle between stern planes and rudders

Input of control surfaces

Actuators and DC Motor Dynamics

Bilinear Thruster Model

The relationship between the velocity vector of the vehicle and the vector of control variables expresses the function of the thruster force and moment:

$$\boldsymbol{\tau} = b(\nu, n)$$

The first order approximation of the thrust $\boldsymbol{T}$ and the torque $\boldsymbol{Q}$ for a single-screw propeller is:

$$T = \varrho D^4 K_T(J_0)|n|n$$

- n: propeller revolution;
- D: propeller diameter;
- ϱ: water density;
- V_a: speed of the water going into the propeller;
- $J_0 = V_a/(nD)$: advance number;
- K_T: thrust coefficient;

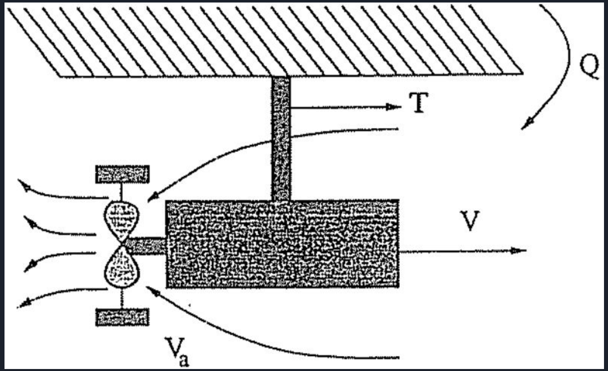
Bilinear Thruster Model

Starting from the point that we can rewrite $\mathbf{K}_T$ with the following approximation (it shows linear behavior in J_0):

$$K_T = \alpha_1 + \alpha_2 * (V_a/nD)$$

so the thruster force and the thruster torquer could be written as:

$$T(n, V_a) = T_{|n|n}\,|n|n + T_{|n|Va}\,|n|V_a$$
$$Q(n, V_a) = Q_{|n|n}\,|n|n + Q_{|n|Va}\,|n|V_a$$



Bilinear Thruster Model

In normal conditions V_a is connected with the speed of the vehicle as:

$$V_a = (1-w)V$$

normally w (called the wake fraction number) is normally equal to a number in the range of 0.1-0.4. So we can rewrite the equation of the thruster force as:

$$\tau = b_1 |n|n - b_2 |n|\nu$$

where:

- $b_1 = T_{|n|n} > 0$;
- $b_2 = -T_{|n|Va} (1-w) > 0$;

Obviously we can expand the system to a multivariable one, with the vector u composed and defined as $u_i = |n_i|n_i$ ($i = 1 \dots p$):

$$\tau = B_1 u - B_2(u)\nu$$

Affine Thruster Model

The bilinear model presented before could be approximated to an affine (linear with its input) model in the following form:

$$\boldsymbol{\tau} = \boldsymbol{B}\boldsymbol{u}$$

Letting *B* equal to B_1 and:

$$B_2(u)\nu \approx 0$$

That simplification implies that the propeller force in the *i-th* DOF developed by the *j-th* propeller can be described by:

$$\tau_i = B_{ij}u_j; \qquad B_{ij} = T_{|n|n}$$

The system we have studied is in the affine form due to the fact that the theory and the applications on the non-affine control systems is limited.

It is important to notice that the control variable **u** is expressed as:

$$u_i = |n_i|n_i$$

DC Motor Dynamics

Mostly of the thruster systems are composed from DC motors designed especially for operations in underwater environments. The dynamic model could be expressed in that way:

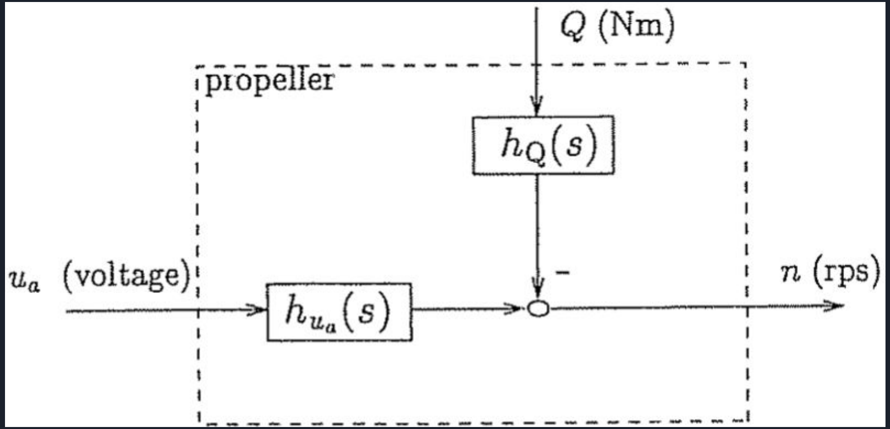
$$
\begin{aligned}
L_a \frac{di_a}{dt} &= -R_a\, i_a - 2\pi K_M\, n + u_a \\
2\pi J_m \frac{dn}{dt} &= K_M\, i_a - Q(n, V_a)
\end{aligned}
$$

- $\mathbf{L_a}$: armature inductance;
- $\mathbf{R_a}$: armature resistance;
- $\mathbf{u_a}$: armature voltage;
- K_M: motor torque constant;
- J_m: moment of inertia of the motor and the thruster;
- n: velocity of the motor in revolutions per second;
- $Q(n, V_a)$: load from the propeller;

$$Q(n, V_a) = Q_{|n|n}|n|n + Q_{|n|V_a}|n|V_a$$

DC Motor Dynamics

In the following block scheme, the electric dynamical model of a DC Motor is described:



The control input is given from the voltage while the output control variable is denoted with the number of round per second of the motor: the disturbances given from Q is applied.

DC Motor Dynamics

Obviously the DC Motor has physical constraints and limits, represented by hard non-linearities like:

- actuator saturation;
- Coulomb friction (kinetic friction, not dependent of the velocity and in opposite direction);
- dead-zones which represent frictional losses (they are torques);
- hysteresis, which represents the lack of complete reversibility due to the fact that with normal levels of field applied, when there is a reduction to zero of this field, some residual magnetism will remain: so you have to apply some current in the opposite direction and that leads to an energy loss (magnetize-demagnetize cycle). So there is an inefficiency of the DC motors because hysteresis is constant and so are the losses.

We can neglect these effects and applying the Laplace's transformation we have:

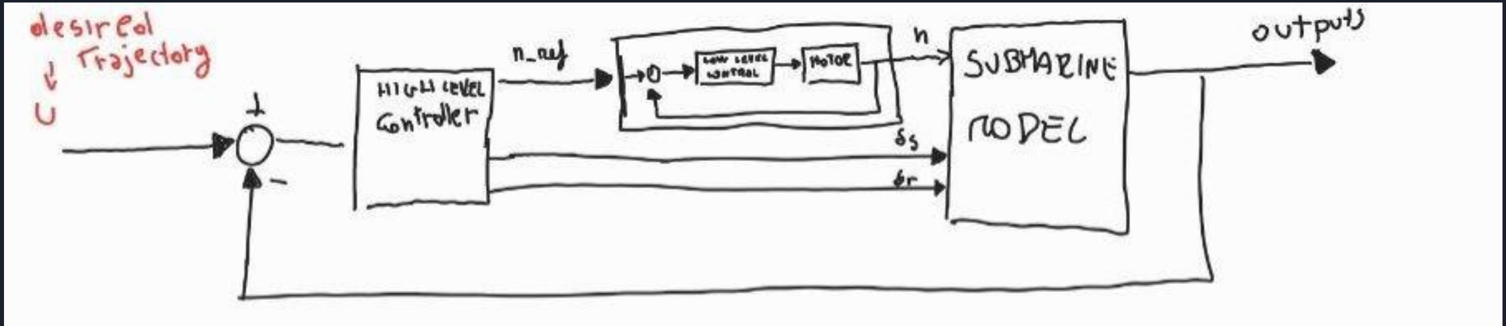
$$n(s) = h_{u_a}(s)\, u_a(s) - h_Q(s)\, Q(s)$$

where *s* is the Laplace variable and:

$$h_{u_a}(s) = \frac{K_1}{(1+T_1 s)(1+T_2 s)}; \qquad h_Q(s) = \frac{K_2(1+T_3 s)}{(1+T_1 s)(1+T_2 s)}$$

Here K_i (i = 1, 2) are two gain constants and T_i (i = 1, 2, 3) are three time constants depending on the parameters in the DC motor dynamic model equation

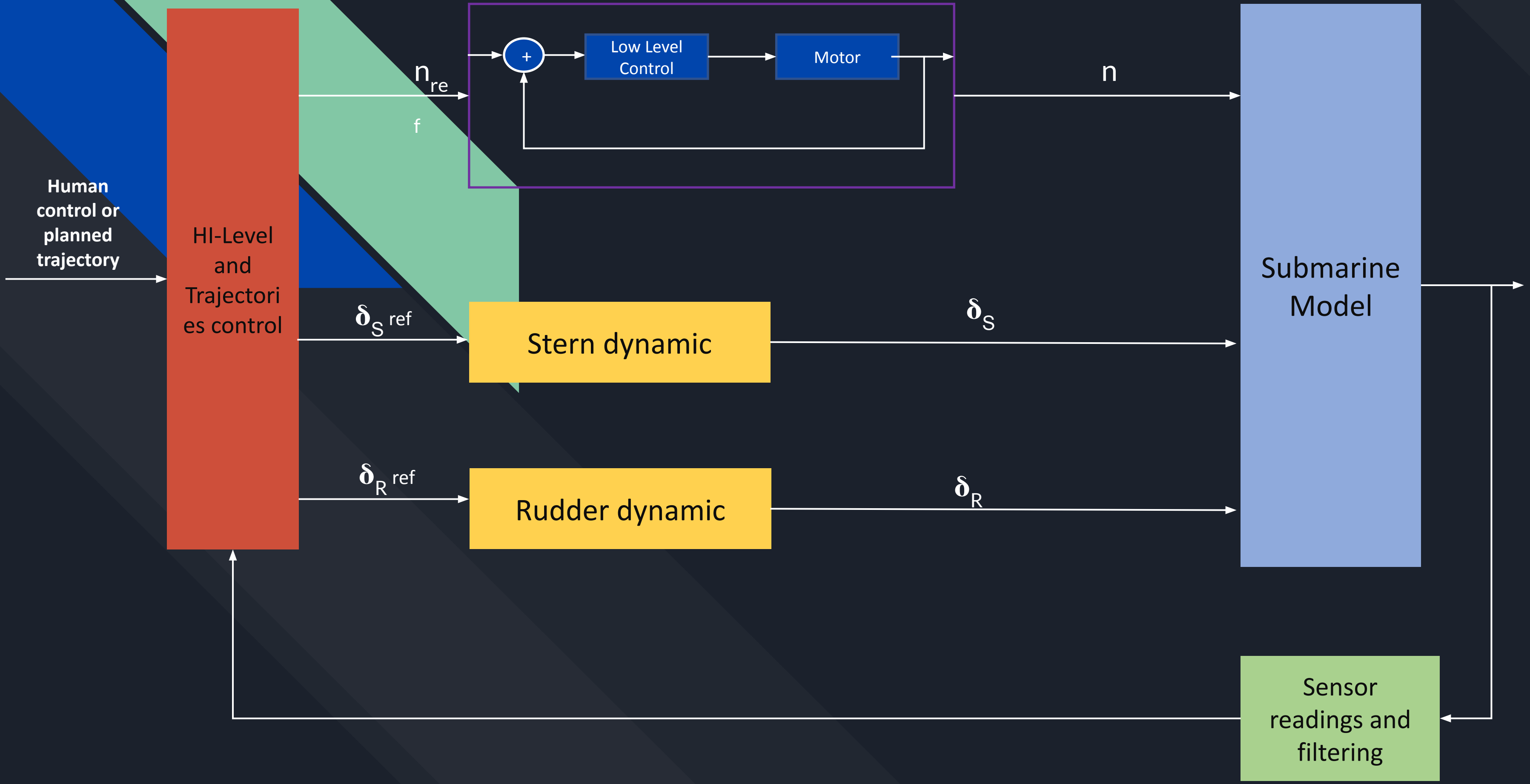
Proposed Control Scheme



The variables that the controller passes to the motor subsystem and to the submarine model are:

- $\boldsymbol{\delta}_S$: effective angle in radians of the stern planes;
- $\boldsymbol{\delta}_R$: effective angle in radians of the rudder planes;
- ***n***: velocity of the motor in revolutions per second

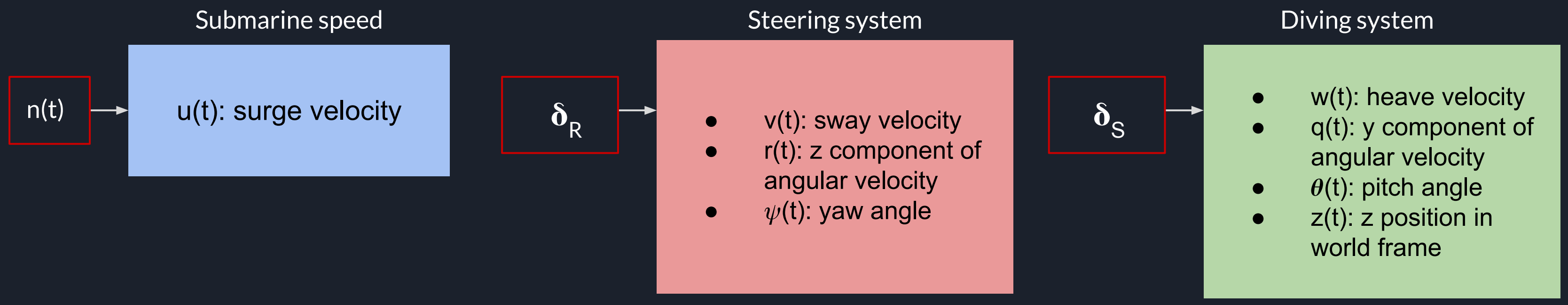
Conceptual scheme of control system



Decoupling of dynamics

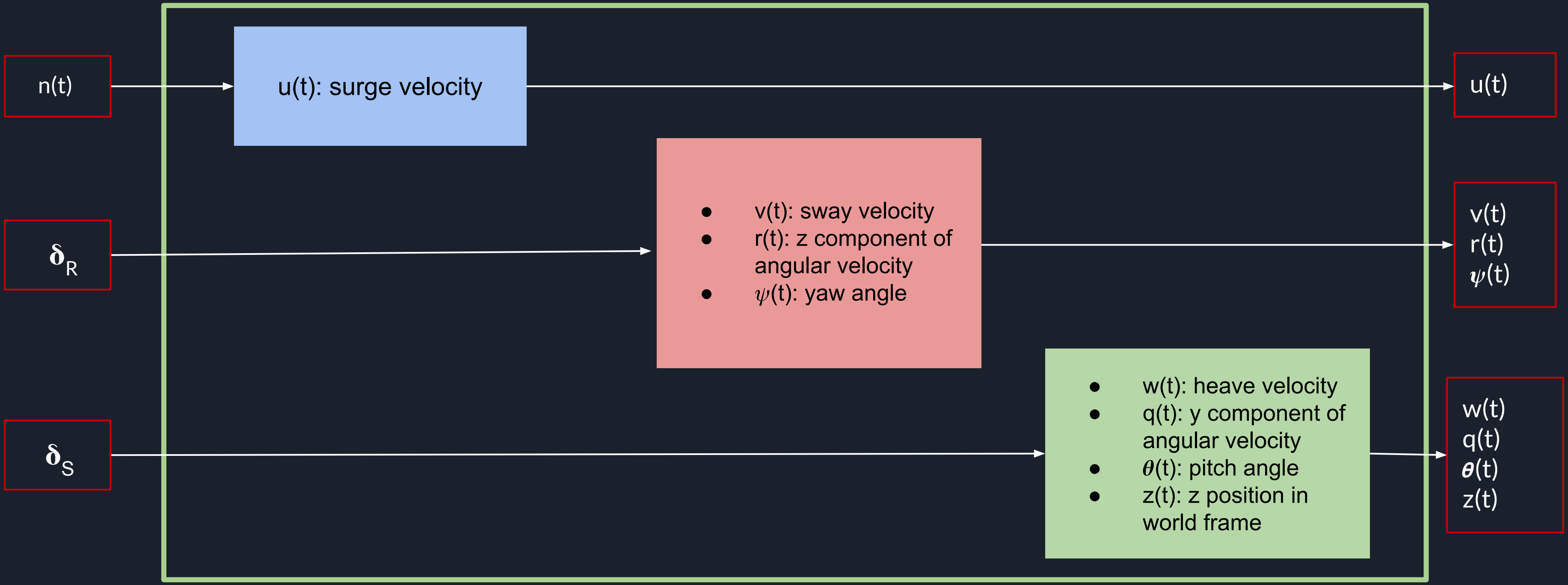
Given the 6 DOF linearized system, its control can be challenging.
An important simplification can be made dividing the model in three non-interacting subsystems: this means that all the interacting signals are neglected. The subsystems are related to the control of speed, steering and diving. The variable relative to the roll angle in both the frames are not considered in this case.

The model state variable involved are:



Decoupled System model

The 6DOF system can be decoupled in this way:



Forward speed control

The forward motion equation is:

$$(m - X_{\dot{u}})\,\dot{u} = X_{|u|u}\,|u|u + (1-t)\,T + X_{\text{ext}}$$

X_{ext} are the external disturbances due to waves and currents

The T is the thruster force, which depends from the propeller revolution n:

$$T = T_{|n|n}\,|n|n + T_{|n|V_a}\,|n|V_a$$

The final equation is:

$$\boxed{(m - X_{\dot{u}})\,\dot{u} = X_{|u|u}\,|u|u + X_{|n|n}\,|n|n + X_{\text{ext}}}$$

A type of control consists in the control of the internal motor system. It allows the reproduction at steady state of of a propeller revolution signal n_{ref}. This signal is set by a outer control loop, which is in charge of controlling the forward speed of the submarine at steady state.

Steering dynamics

The dynamics of steering maneuver is controlled by the rudder angle input $\boldsymbol{\delta}_R$. Taking the forward velocity as u_0 as constant. The equations relative to this dynamics are reported below:

$$\begin{bmatrix} m - Y_{\dot v} & m x_G - Y_{\dot r} & 0 \\ m x_G - N_{\dot v} & I_z - N_{\dot r} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot v \\ \dot r \\ \dot \psi \end{bmatrix} + \begin{bmatrix} -Y_v & m u_0 - Y_r & 0 \\ -N_v & m x_G u_0 - N_r & 0 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} v \\ r \\ \psi \end{bmatrix} = \begin{bmatrix} Y_\delta \\ N_\delta \\ 0 \end{bmatrix} \delta_R$$

The transfer function of the process is reported below:

$$\frac{\psi}{\delta_R}(s) = \frac{K(1 + T_3 s)}{s(1 + T_1 s)(1 + T_2 s)}$$

Diving equation

Assuming the linearization of the dynamics around zero pitch angle ($\theta_0 = 0$) and a constant forward speed u_0 = constant. The dynamics is controlled by the stern angle δ_S. The dynamics of the diving motion is expressed by:

$$\begin{bmatrix} m - Z_{\dot{w}} & mx_G - Z_{\dot{q}} & 0 & 0 \\ mx_G - M_{\dot{w}} & I_y - M_{\dot{q}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{w} \\ \dot{q} \\ \dot{\theta} \\ \dot{z} \end{bmatrix} + \begin{bmatrix} -Z_w & mu_0 - Z_q & 0 & 0 \\ -M_w & mx_G u_0 - M_q & \overline{BG}_z W & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & u_0 & 0 \end{bmatrix} \begin{bmatrix} w \\ q \\ \theta \\ z \end{bmatrix} = \begin{bmatrix} Z_\delta \\ M_\delta \\ 0 \\ 0 \end{bmatrix} \delta_S$$

The transfer function between the stern angle and the z position is reported below:

$$\frac{z}{\delta_S}(s) = \frac{b_1 s^2 + (b_2 a_{12} - b_1 a_{22} - b_2 u_0)s + (b_2 u_0 a_{11} - b_1 a_{21} u_0 - b_1 a_{23} + b_2 a_{13})}{s[s^3 - (a_{11} + a_{22})s^2 + (a_{11}a_{22} - a_{23} - a_{21}a_{12})s + (a_{11}a_{23} - a_{21}a_{13})]}$$

Simplified Model

Kinetics

The rigid-body kinetics can be derived using the *Newton-Euler formulation*, which is based on *Newton's second law*.

Back on the previous six equations we know that they could be expressed in a more compact form as:

$$M_{RB}\,\dot{\nu} + C_{RB}(\nu)\,\nu = \tau_{RB}$$

Here:

- $\mathbf{v} = [u, v, w, p, q, r]^T$ is the body-fixed linear and angular velocity vector;
- $\boldsymbol{\tau}_{RB} = [X, Y, Z, K, M, N]^T$ is a generalized vector of external forces and moment expressed in {*o*};
- $\boldsymbol{M}_{RB}$ is the rigid-body inertia/mass matrix;
- $\mathbf{C}_{RB}$ is the rigid-body Coriolis and centripetal matrix due to the rotation of {*o*} about the geographical frame {*n*};

Simplified Model

$$\dot{\boldsymbol{\eta}} = \mathbf{J}(\boldsymbol{\eta})\boldsymbol{\nu}$$

$$\mathbf{M}\dot{\boldsymbol{\nu}} + \mathbf{C}(\boldsymbol{\nu})\boldsymbol{\nu} + \mathbf{D}(\boldsymbol{\nu})\boldsymbol{\nu} + \mathbf{g}(\boldsymbol{\eta}) = \boldsymbol{\tau}$$

- **J:** inertial matrix
- **M:** hydrodynamic added mass matrix due to body acceleration
- **C:** added mass matrix due to Coriolis acceleration
- **D:** linear damping for a port-starboard symmetrical vehicle
- **g:** vector of generalized restoring forces

Simplified Model

Assuming that the gravitational force acts through the center of gravity (CG) defined by the vector $\mathbf{r}_g := [x_g, y_g, z_g]^T$ with respect to the coordinate origin $\{o\}$.

Similarly, the buoyancy force acts through the center of buoyancy (CB) defined by the vector $\mathbf{r}_b := [x_b, y_b, z_b]^T$: for most vehicles $y_g = y_b = 0$.

For a port-starboard symmetrical vehicle with homogeneous mass distribution, CG satisfying $y_g = 0$ and products of inertia $I_{xy} = I_{yz} = 0$, the system inertia matrix becomes:

$$\mathbf{M} := \begin{bmatrix} m - X_{\dot{u}} & 0 & -X_{\dot{w}} & 0 & mz_g - X_{\dot{q}} & 0 \\ 0 & m - Y_{\dot{v}} & 0 & -mz_g - Y_{\dot{p}} & 0 & mx_g - Y_{\dot{r}} \\ -X_{\dot{w}} & 0 & m - Z_{\dot{w}} & 0 & -mx_g - Z_{\dot{q}} & 0 \\ 0 & -mz_g - Y_{\dot{p}} & 0 & I_x - K_{\dot{p}} & 0 & -I_{zx} - K_{\dot{r}} \\ mz_g - X_{\dot{q}} & 0 & -mx_g - Z_{\dot{q}} & 0 & I_y - M_{\dot{q}} & 0 \\ 0 & mx_g - Y_{\dot{r}} & 0 & -I_{zx} - K_{\dot{r}} & 0 & I_z - N_{\dot{r}} \end{bmatrix}$$

where the hydrodynamic derivatives are defined according to SNAME (1950).

Simplified Model

The centripetal matrix is:

$$\mathbf{C}_{RB}(\boldsymbol{\nu}) = \begin{bmatrix} 0 & -mr & mq & mz_g r & -mx_g q & -mx_g r \\ mr & 0 & -mp & 0 & m(z_g r + x_g p) & 0 \\ -mq & mp & 0 & -mz_g p & -mz_g q & mx_g p \\ -mz_g r & 0 & mz_g p & 0 & -I_{xz} p + I_z r & -I_y q \\ mx_g q & -m(z_g r + x_g p) & mz_g q & I_{xz} p - I_z r & 0 & -I_{xz} r + I_x p \\ mx_g r & 0 & -mx_g p & I_y q & I_{xz} r - I_x p & 0 \end{bmatrix}$$

It is possible to note that this representation of $\mathbf{C}_{RB}(\boldsymbol{\nu})$ only depends on the angular velocities p, q and r and not the linear velocities u, v and r.

Coriolis

Coriolis force is an inertial force described by Gustave-Gaspard Coriolis in 1835: he showed that if the ordinary Newtonian laws of motion of bodies are to be used in a rotating frame of reference an inertial force, acting to the right of the direction of body motion for counterclockwise rotation of the reference frame or to the left for clockwise rotation, must be included in the equations of motion.

The effect of the Coriolis force is an apparent deflection of the path of an object that moves within a rotating coordinate system: the object does not actually deviate from its path, but it appears to do so due to the motion of the coordinate system.

$$\mathbf{C}_A(\boldsymbol{\nu}) = \begin{bmatrix} 0 & 0 & 0 & 0 & -a_3 & a_2 \\ 0 & 0 & 0 & a_3 & 0 & -a_1 \\ 0 & 0 & 0 & -a_2 & a_1 & 0 \\ 0 & -a_3 & a_2 & 0 & -b_3 & b_2 \\ a_3 & 0 & -a_1 & b_3 & 0 & -b_1 \\ -a_2 & a_1 & 0 & -b_2 & b_1 & 0 \end{bmatrix}$$

$$\begin{aligned} a_1 &= X_{\dot{u}}u + X_{\dot{w}}w + X_{\dot{q}}q \\ a_2 &= Y_{\dot{v}}v + Y_{\dot{p}}p + Y_{\dot{r}}r \\ a_3 &= Z_{\dot{u}}u + Z_{\dot{w}}w + Z_{\dot{q}}q \\ b_1 &= K_{\dot{v}}v + K_{\dot{p}}p + K_{\dot{r}}r \\ b_2 &= M_{\dot{u}}u + M_{\dot{w}}w + M_{\dot{q}}q \\ b_3 &= N_{\dot{v}}v + N_{\dot{p}}p + N_{\dot{r}}r \end{aligned}$$

Simplified Model

Linear damping for a port-starboard symmetrical vehicle takes the following form:

$$\mathbf{D} = -\begin{bmatrix} X_u & 0 & X_w & 0 & X_q & 0 \\ 0 & Y_v & 0 & Y_p & 0 & Y_r \\ Z_u & 0 & Z_w & 0 & Z_q & 0 \\ 0 & K_v & 0 & K_p & 0 & K_r \\ M_u & 0 & M_w & 0 & M_q & 0 \\ 0 & N_v & 0 & N_p & 0 & N_r \end{bmatrix}$$

Let $W = mg$ and $B = \rho g \nabla$ denote the weight and buoyance where m is the mass of the vehicle including water in free floating space, ∇ the volume of fluid displaced by the vehicle, g the acceleration of gravity (positive downward), and ρ the water density. Hence, the generalized restoring forces for a vehicle satisfying $y_g = y_o = 0$ becomes:

$$\mathbf{g}(\boldsymbol{\eta}) = \begin{bmatrix} (W - B)s\theta \\ -(W - B)c\theta s\phi \\ -(W - B)c\theta c\phi \\ (z_g W - z_b B)c\theta s\phi \\ (z_g W - z_b B)s\theta + (x_g W - x_b B)c\theta c\phi \\ -(x_g W - x_b B)c\theta s\phi \end{bmatrix}$$

Disturbances

Disturbances

The sea wave disturbances were introduced as a simplification of the Pierson-Moskowitz spectrum. As disturbances were introduced the signals Z and W, the are respectively the disturbances associated with the vertical forces of waves and pitch angle.

Both the signals are obtained by the approximated wave elevation signal v(t). This signal is obtained in the frequency domain as the output of the $G_4(s)$ transfer function, with a white noise as input.

$$\frac{v}{\xi} = G_4(s)$$

$$G_4(s) = \frac{K(s/\omega_M)^2}{[(s/\omega_M)^2 + s/\omega_M + 1]^2}$$

- ω_M = ¼ (0.8 B)
- B = 3.11/ h_{13}^2
- h_{13} = 3 m (significant waves height)
- K is a gain calculated such that the output (either peak magnitude or variance) of the filter corresponds to S(w), in this case K = 1.11

Disturbances

The value of the parameters *a*, *b*, *c* and *d* can be estimated initially from data obtained from the Pierson-Moskowitz spectrum:

$$a = \frac{2}{K}\left(\frac{\text{variance}\,(Z_\omega)}{\omega_M}\right)^{1/2}, \quad b = \text{mean}\,(Z_\omega)$$

$$c = \frac{2}{K}\left(\frac{\text{variance}\,(M_\omega)}{\omega_M}\right)^{1/2}, \quad \text{and} \quad d = \text{mean}\,(M_\omega)$$

This approximation requires further adjustments since some controllers, as H_∞, cannot deal with constant input disturbances: thus constant terms, associated with the second-order wave effects, are replaced by integrated white noise multiplied by the parameters *b* and *d*.

Disturbances

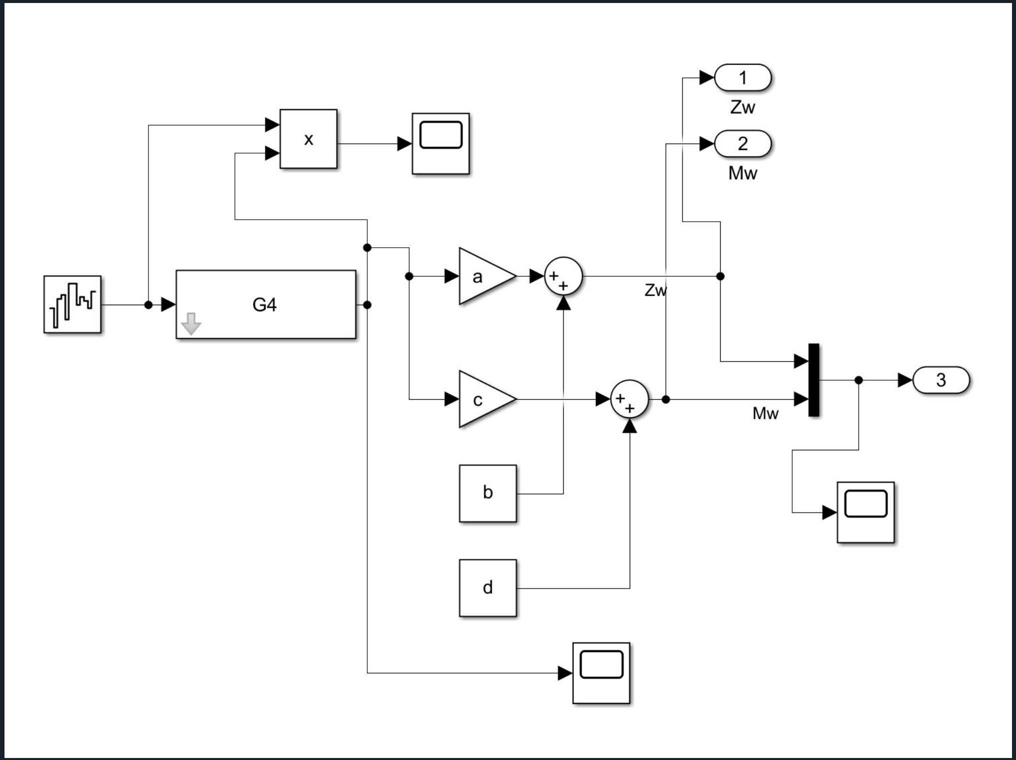
To get the signals Z and W, we used these equations:

$$Z_\omega = a \cdot v(t) + b$$
$$M_\omega = c \cdot v(t) + d$$

- a = 2*10$^{(-1)}$;
- b = -5*10$^{(-1)}$;
- c = 2*10$^{(3)}$;
- d = 2*10$^{(3)}$;

The disturbances were applied to the Z and W inputs of the system, as shown in the following simulink scheme

Disturbances - Simulink



Model Linearization

Model Linearization

To simplify our modelling we linearized our non linear model around an equilibrium point.

The dynamic of the linearized model is reported below:

$$\dot{\eta} = \left.\frac{\partial J(\eta)\nu}{\partial \eta}\right|_{\eta=0 \nu=0} \Delta\eta + \left.\frac{\partial J(\eta)\nu}{\partial \nu}\right|_{\eta=0 \nu=0} \Delta\nu$$

$$\dot{\nu} = \left.\frac{\partial(M^{-1}(-C(\nu)\nu - D(\nu)\nu - g(\nu))}{\partial \eta}\right|_{\eta=0 \nu=0} \Delta\eta + \left.\frac{\partial(M^{-1}(-C(\nu)\nu - D(\nu)\nu - g(\nu))}{\partial \nu}\right|_{\eta=0 \nu=0} \Delta\nu$$

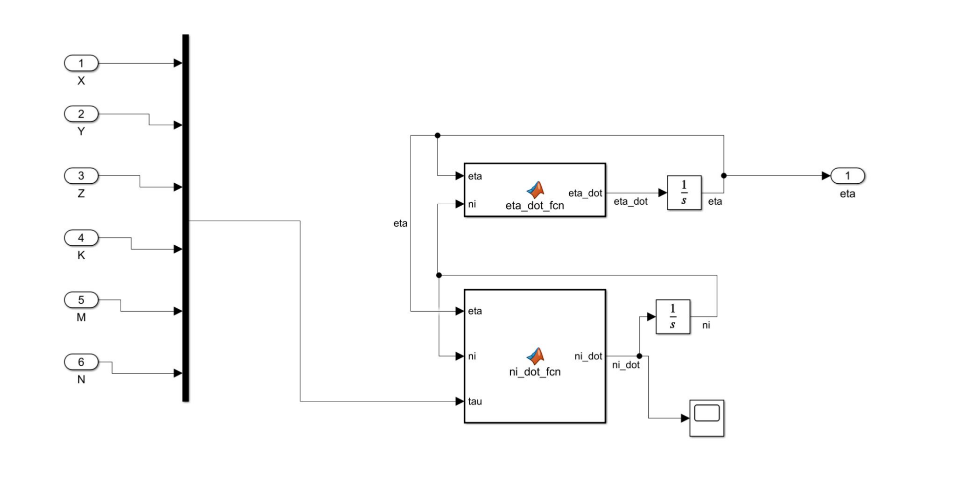
The variable Δη and Δv have the following expressions:

- Δη = η - η0
- Δv = v - v0

Where η0 and v0 are the equilibrium points. We choose for our project, η0 = 0 and v0 = 0.

Model Linearization

For the linearization we started from the non linear model that we implemented in Simulink:



Then we computed automatically the linear model around the equilibrium using the linearization manager available on Simulink

PID Controller

PID Controller - Introduction

The **PID Controller (P**roportional**-I**ntegral**-D**erivative) is a widely used controller in many fields of the industry: this kind of controllers use a control loop feedback mechanism to control process variables and are the most accurate and stable controller.

The classical PID Controller has the advantage that the gains (proportional, integral and derivative) are easy to choose through manual tuning or special techniques, as Ziegler-Nichols.

The input is given to the P, I and D at the same instance and their output is summed using an adder: this added input is so applied to the Process which sets the parameters accordingly.
These set parameters are then fed back to the system: this output and the original input are then summed to estimate the error which is again fed to the PID.
Now the loop can continue.

The basic mechanism used in PID controllers is control loop feedback.
A PID calculates the error by calculating difference between actual value and desired value and then sets the deciding parameters accordingly. This error is continuously being calculated until the process stops.

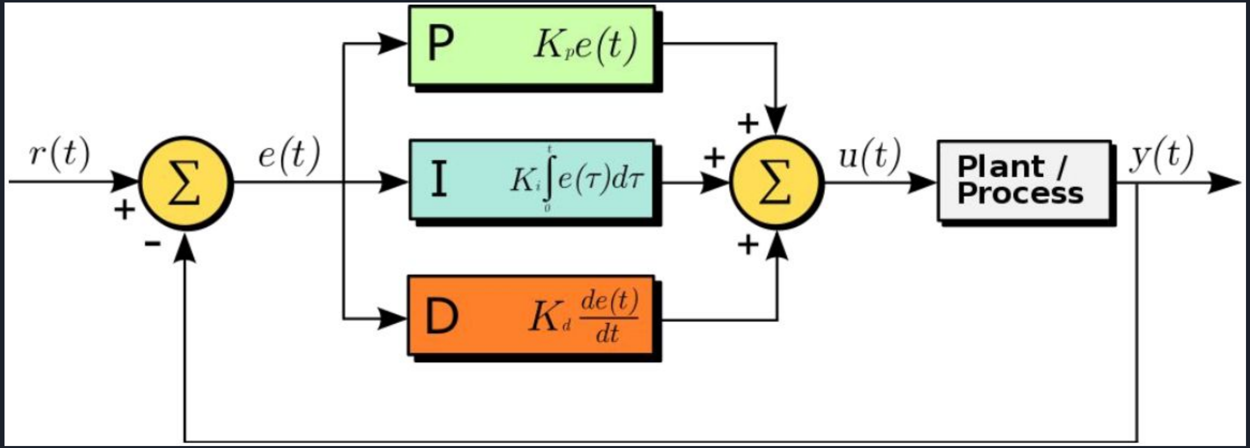
PID Controller

Proportional is used to find out the error between the desired value and actual value and is responsible for the corrective response.

Integral is applied to calculate all the past values of error and then integrate them to find out the Integral term: when error is expelled from the system this integral stops increasing.

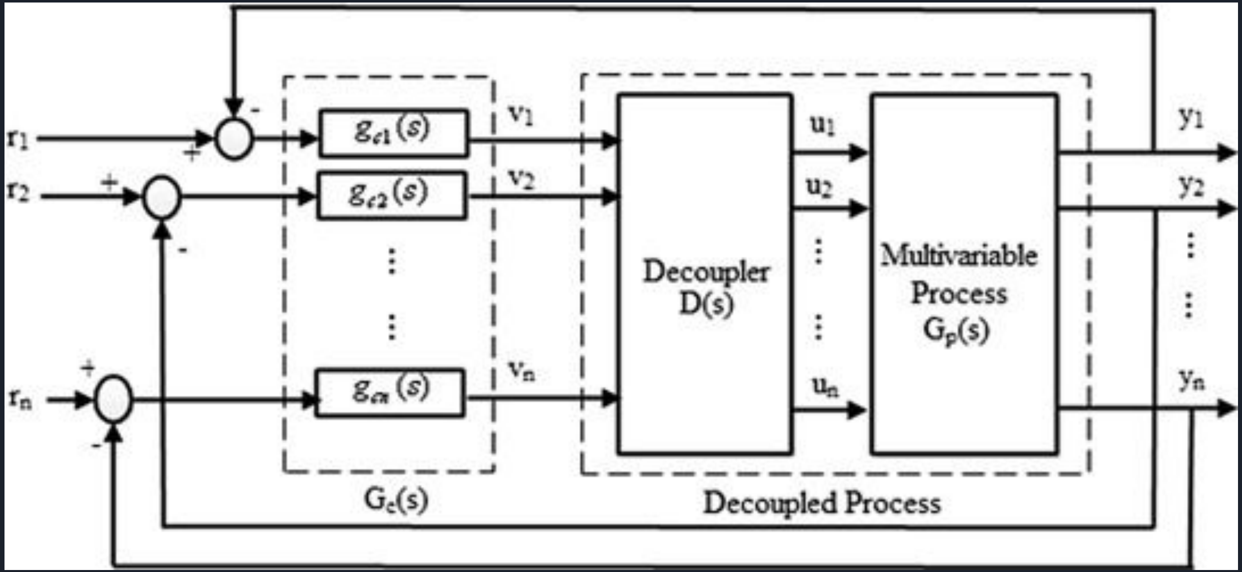
Derivative is used to predict the expected error values in future based on the present values. Controlling effect can be increased if the system has a rapid rate of change, which is also based on Derivative.

Combining all these three operations gives the name Proportional Integral Derivative (PID) Controller.



PID Control Scheme

The PID structure was conceived by 6 PID controller, one for input, plus a decoupling matrix. The control scheme is reported below. The transfer functions g_1,. . .,g_n are the transfer functions of our PID.



PID Data

Using the systune function, we found the optimal parameters for the PIDs and for the decoupling matrix:

PID1:

- Kp = 2.14e-6
- Ki = 3.01e-13
- Kd = 0.844
- Tf = 0.0145

PID2:

- Kp = 9.42e3
- Ki = 9.64
- Kd = 7.59e3
- Tf = 3.11e6

PID3:

- Kp = 3.31
- Ki = 0.119
- Kd = 396
- Tf = 6.13e5

PID4:

- Kp = 90.4
- Ki = 690
- Kd = 9.75
- Tf =2.83e-5

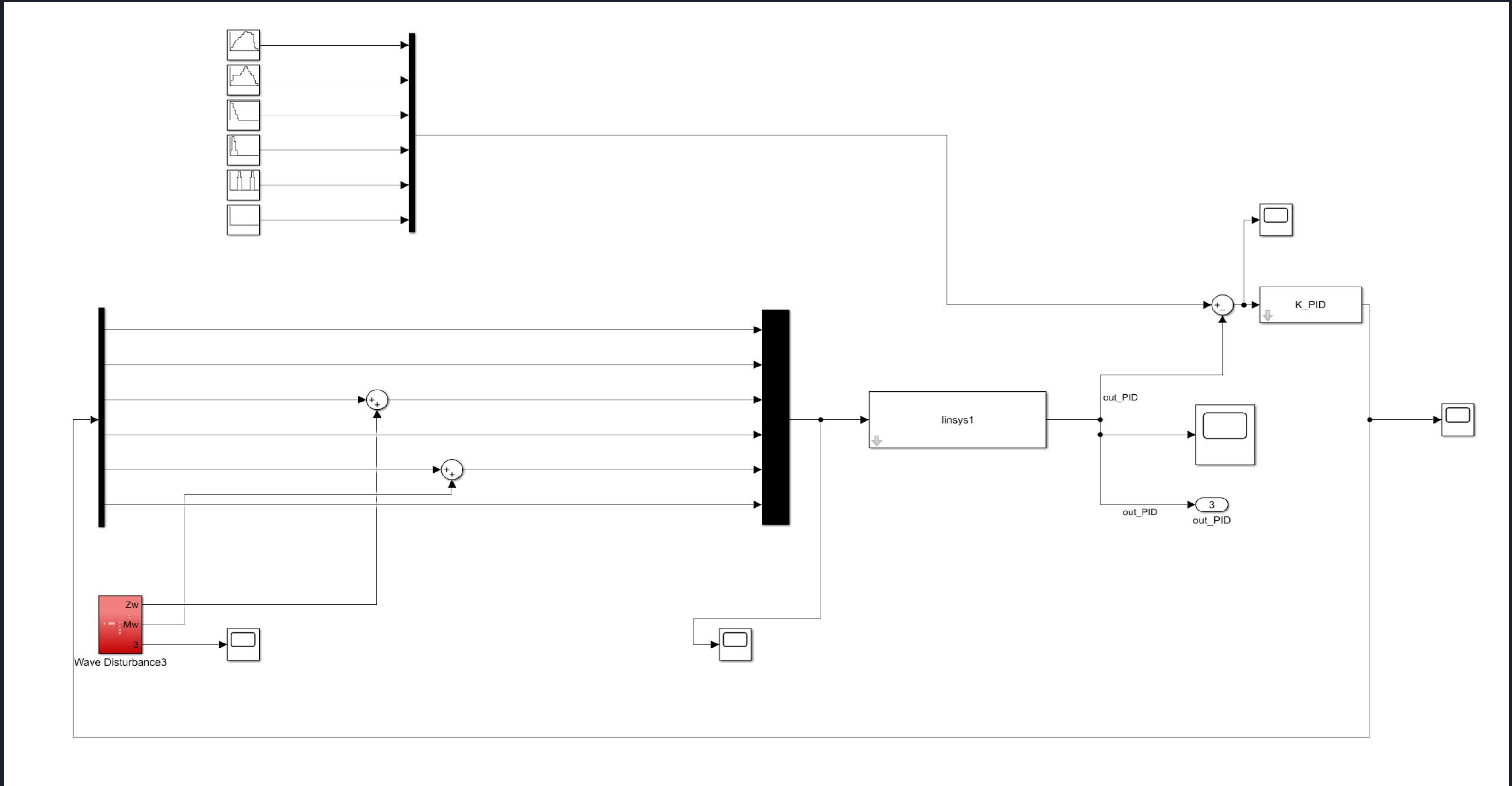
PID5:

- Kp = 12
- Ki = 1.63e3
- Kd = 6.26e3
- Tf = 0.000116

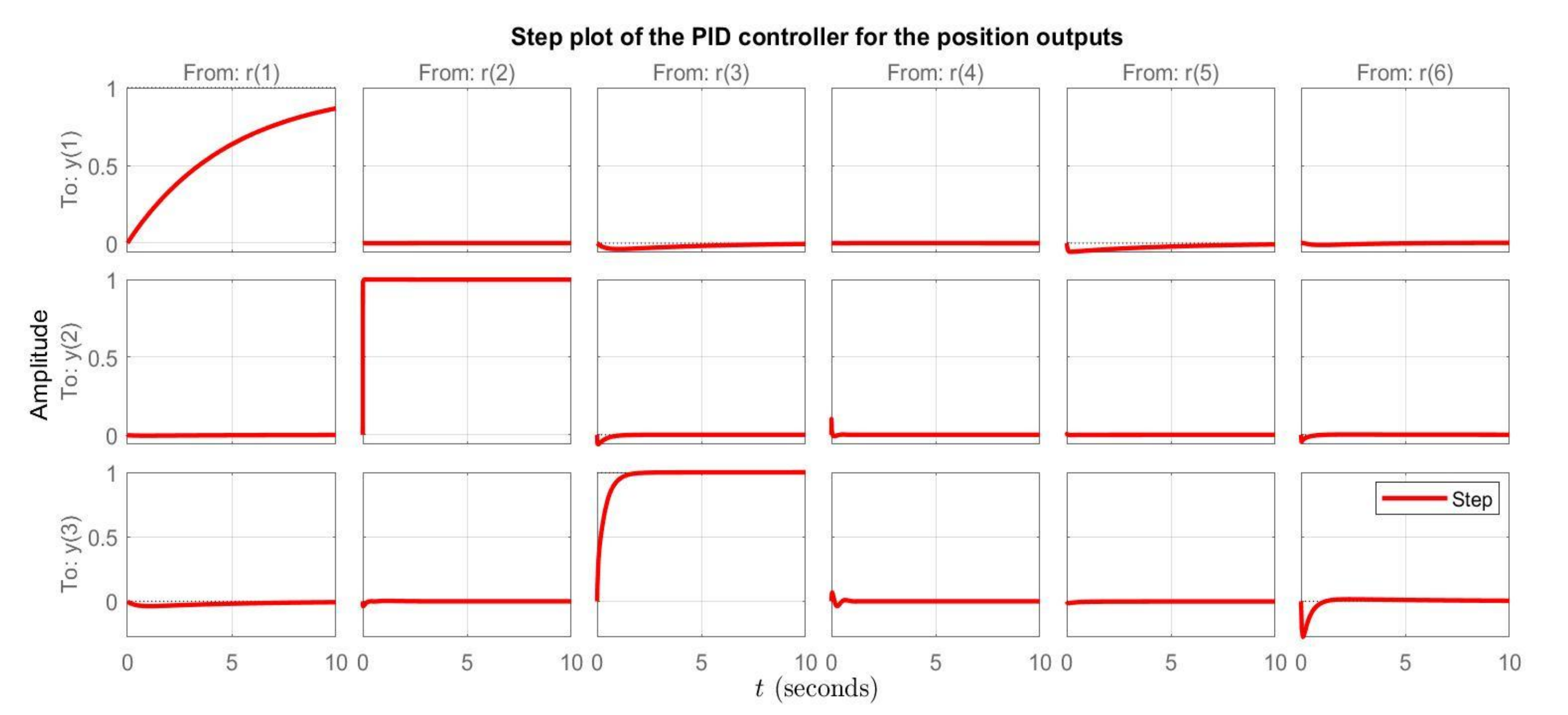
PID6:

- Kp = 8.67
- Ki = 0.000818
- Kd = 50.6
- Tf = 8.76e6

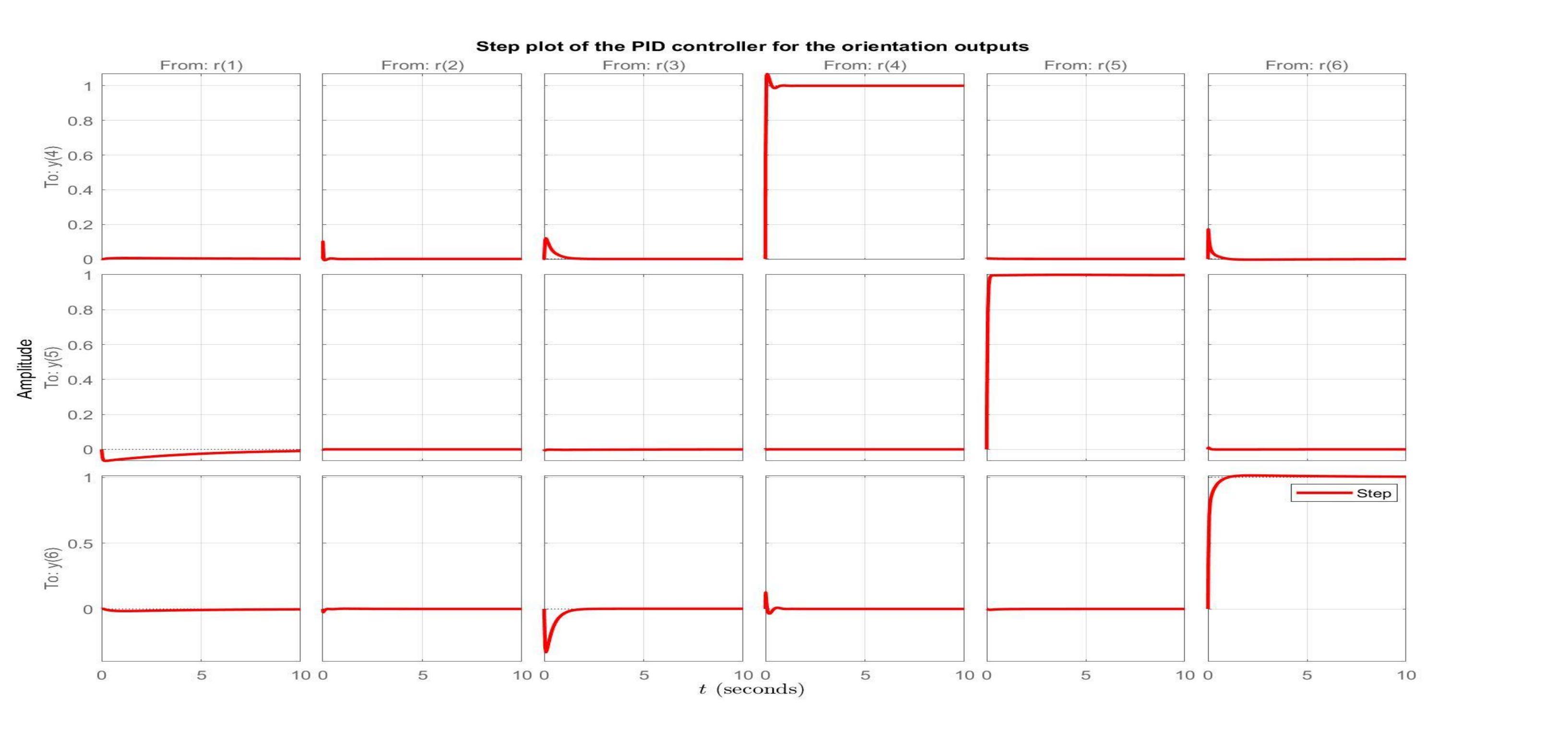
PID - Simulink Model



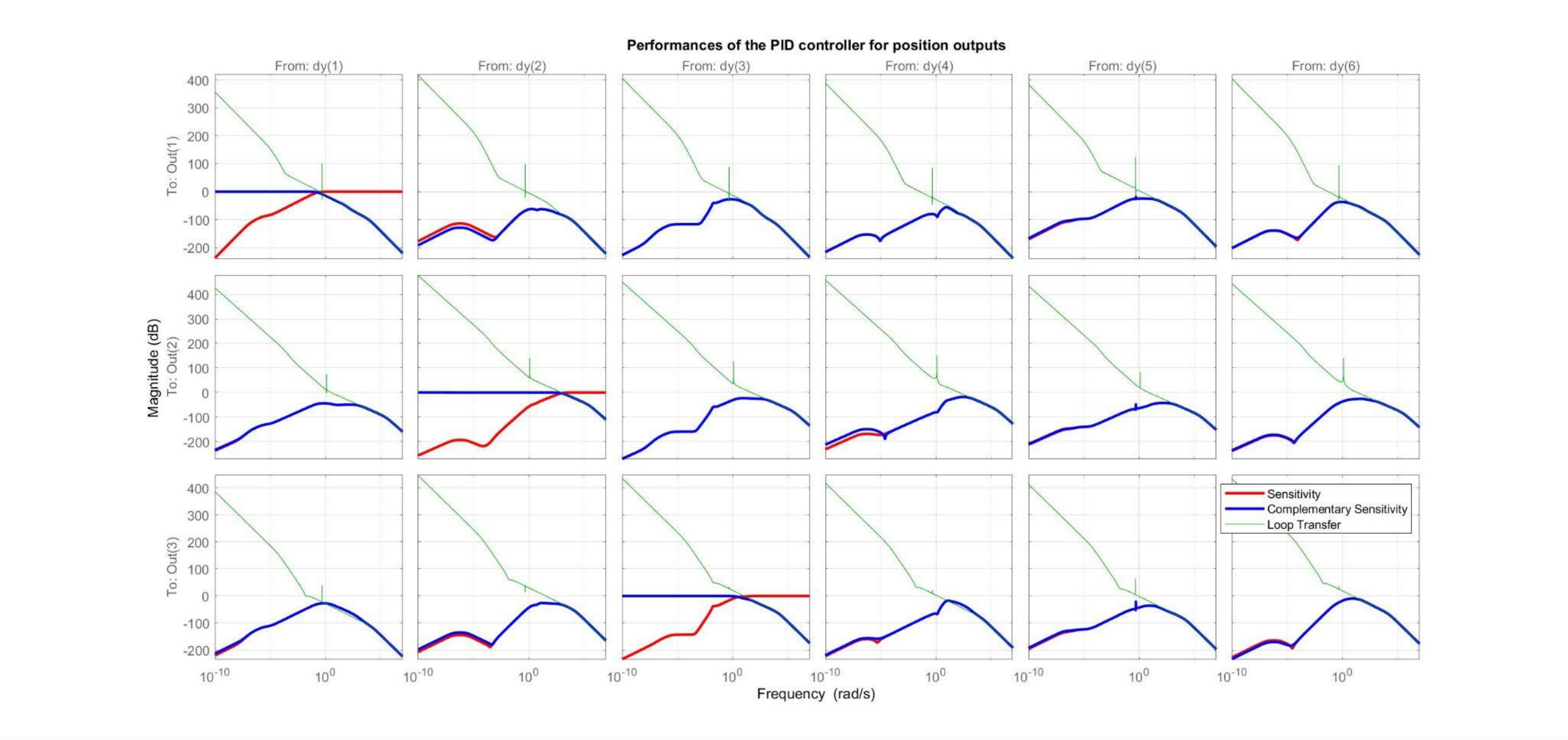
PID - Step Response



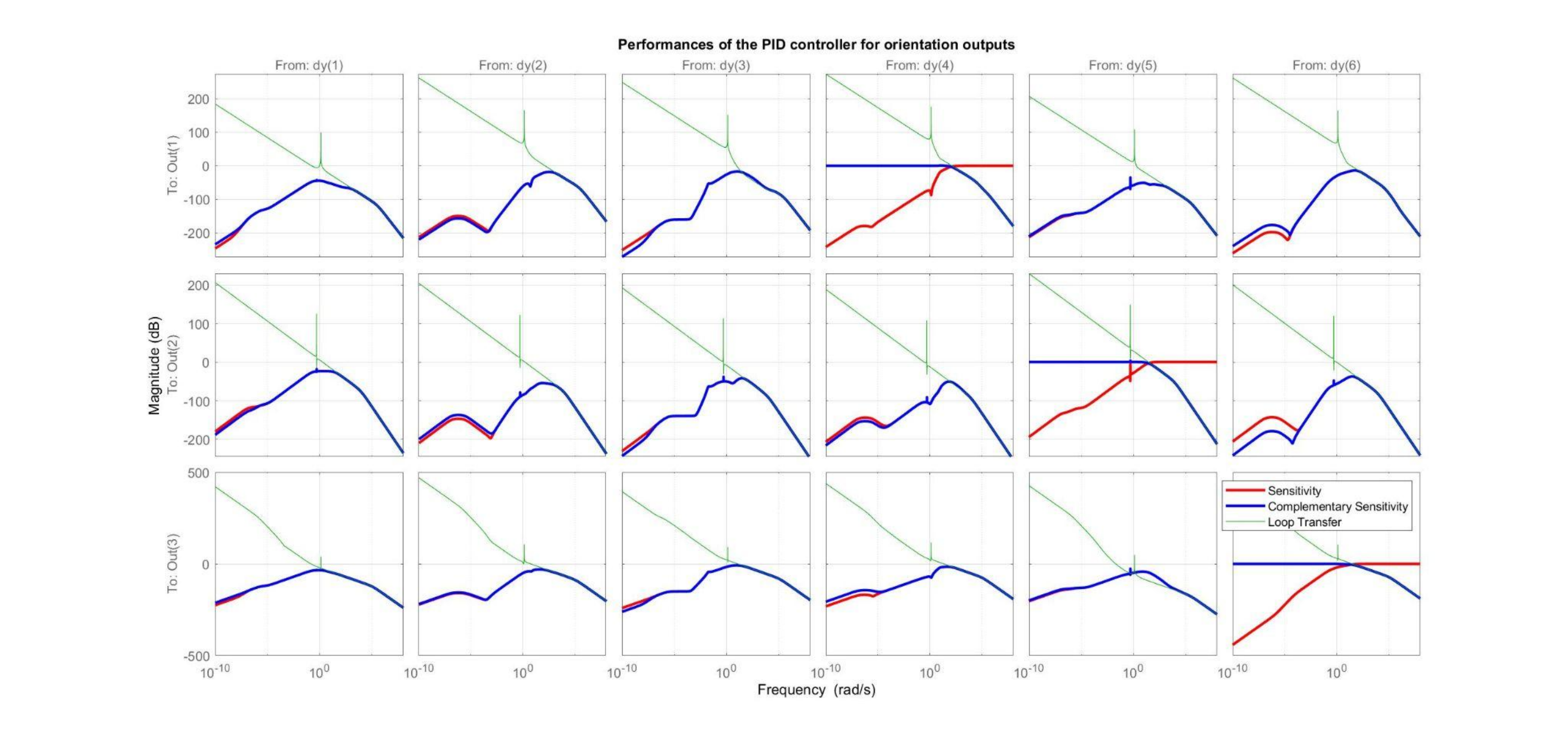
PID - Step Response



PID - Performances



PID - Performances



LQR Controller

LQR - Linear Quadratic Regulator

The Linear Quadratic Regulator (LQR), just like PID, is also considered as a feedback controller. it provides optimally controlled feedback gains to enable the closed-loop stable and high performance design of systems.

Nevertheless, LQR computes the control signals by modelling it as an optimization problem, so it tries to compute control signals that minimizes some cost function.

LQR problem is one of the most fundamental problems in control theory.

LQR - Linear Quadratic Regulator

Consider a linear system:

$$\dot{s} = As + Bu$$

where *s* is a *n*-dimensional vector that represents the state of the system and *u* is a m-dimensional vector that represents the control signal.

A and *B* are *n*-by-*n* and m-by-m matrices, respectively, that describes the dynamics of the system.

We also have a goal state s_g that we want to reach: our job is to find a sequence of control signals *u* that will take us to this goal state.

LQR - Linear Quadratic Regulator

The assumption we make in LQR is that the controller is a linear controller.
In most cases, a simple linear controller, in the form of $u = -ks$, works well to solve this problem. The question now is: what is and how do we find it in order to achieve our goal?

First we need to define a cost function that quantify the performance of the controller.
We defined it as a quadratic cost function:

$$C = \int_{t=0}^{t=H} ((s_t - s_g)^T Q (s_t - s_g) + u_t^T R u_t) dt$$

where t denotes time, while Q and R are positive definite matrices where off-diagonal elements in the matrix are all 0.

Each of the diagonal elements in Q reflect how much we care about a particular error signal. On the other hand, each element in R reflect how much we care about performing a particular action.

LQR - Linear Quadratic Regulator

So it is easy to understand that Q and R are something that we, as controller designer, manually choose depending on how we want the system to perform.

Even if there is no systematic method to choose Q and R it is possible to define them in this way:

- Q: square matrix which defines the number of the states;
- R: square matrix which defines the number of the inputs.

At this point the name "*LQR*" starts to make sense: it comes from the fact that

1. we assume a *linear* controller;
2. we assume a *quadratic* cost function;
3. we aim to *regulate* the system.

Kalman Filter

The Kalman filter or the **L**inear **Q**uadratic **E**stimation (LQE) is one of the most significant and common sensor and data fusion algorithms.

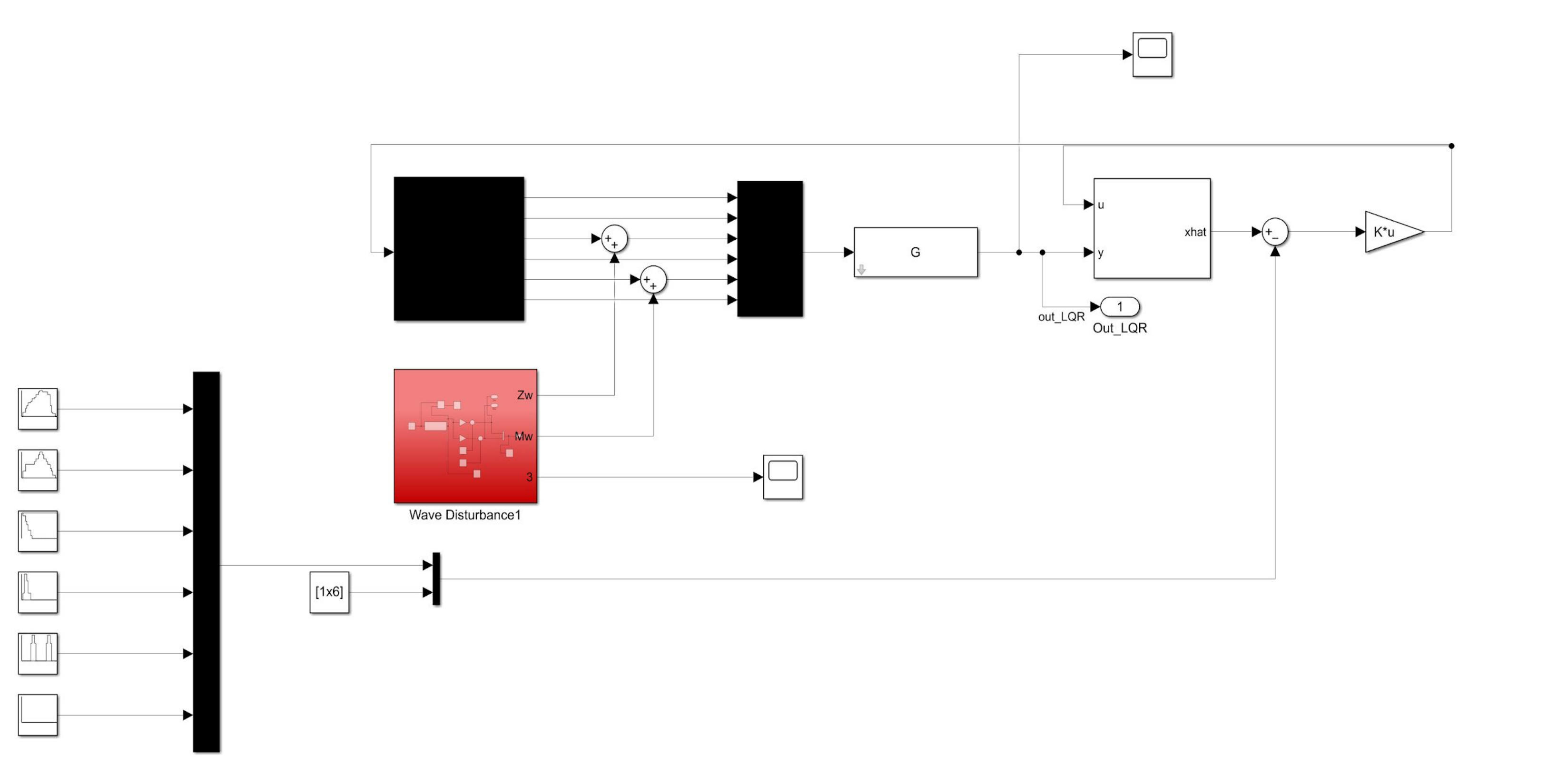
Its efficiency is due to its inexpensive computational requirements, well-designed recursive properties, representation of the optimal estimator for one-dimensional linear systems assuming Gaussian error statistics, and suitability for real-time implementation.

Kalman filter can be considered an extension of Gauss' original development of least squares to estimate unknown parameters of a model

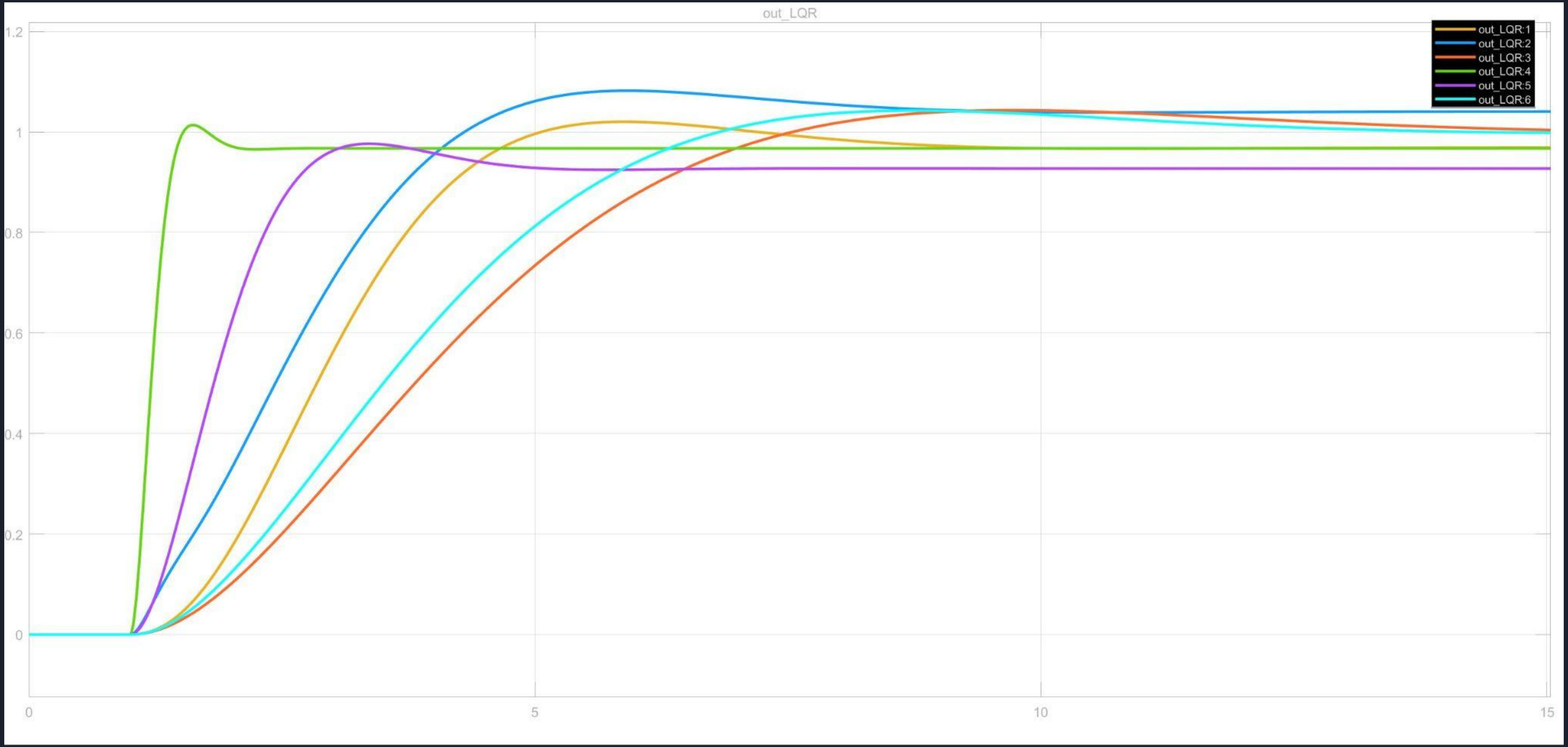
The most important concepts when using the Kalman filter are summarized as:

- Kalman filters are discrete: they rely on measurement samples taken between repeated but constant periods of time;
- Kalman filters are recursive: its prediction of the future relies on the state of the present (position, velocity, acceleration, etc.). Further, it presents a guess about external factors that may affect the situation;
- Kalman filters predict the future: this is applied by making measurements (such as by sensors) and then deriving an adjusted estimate of the state from both prediction and measurements.

LQR - Simulink Model



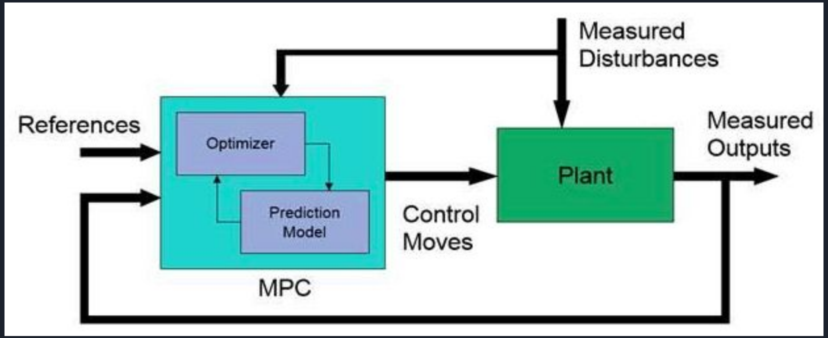
LQR - Step Response



Model Predictive Control

Model Predictive Control - Introduction

Model Predictive Control or MPC for short, is a method for controlling a process in a determined time slot. So it has an advantage over classical PID control, it operates using predictions of future behaviour of the process. In MPC the calculated control actions minimize a cost function for a constrained dynamical system over a finite, receding, horizon. The cost function is evaluated only in a limited number of coincidence points along the prediction horizon.



Model Predictive Control - Types

- Dynamic Matrix Control (DMC)
    - It uses a step response model and it’s applicable only to stable process without integrators and the disturbance is modeled as constant.
- Model Algorithmic Control (MAC)
    - It uses an impulse response model of the process so it’s applicable only in process without integrators, only the first N samples of the impulse response are considered and the disturbance is modeled as constant.
- Predictive Functional Control (PFC)
    - Uses a state space model and is applicable to almost every process even unstable ones.

Model Predictive Control - PFC

With our kind of problem we used Predictive Functional Control (PFC), so to build a PFC scheme we need an LTI state-space model:

$$\begin{cases} x(t) = Mx(t-1) + Nu(t-1) \\ y(t) = Qx(t) \end{cases}$$

We also need a disturbance model:

$$\hat{n}(t+k|t) = \hat{n}(t|t) = y_m(t) - \hat{y}(t|t), k = 1, \dots, p$$

$$\hat{y}(t|t) = Qx(t)$$

Model Predictive Control - Control Law

The control signal is structured as a linear combination of base functions and is:

$$u(t+k|t) := \sum_{i=1,\ldots,n_B} \mu_i(t) B_i(k)$$

μ is the coefficient of the base function.
B is the value of the base function.
n is the number of base functions.

Model Predictive Control - Prediction Horizon

The predicted output is the output of the base functions with initial state set to 0 and the response of the system to the control signal.
So we obtain:

$$u(t+k|t) = \sum_{i=1,\ldots,n_B} \mu_i(t) B_i(k)$$

$$\begin{cases} x_{B_i}(k) = M^{k-1}NB_i(0) + M^{k-2}NB_i(1) + \cdots + NB_i(k-1) \\ y_{B_i}(k) = QM^{k-1}NB_i(0) + QM^{k-2}NB_i(1) + \cdots + QNB_i(k-1) \end{cases}$$

$$\begin{cases} x(t+k) = M^k x(t) + \sum_{i=1,\ldots,n_B} \left(M^{k-1}NB_i(0) + M^{k-2}NB_i(1) + \cdots + NB_i(k-1)\right) \mu_i(t) \\ y(t+k) = QM^k x(t) + \sum_{i=1,\ldots,n_B} y_{B_i}(k)\, \mu_i(t) \end{cases}$$

$$\hat{y}(t+k|t) = QM^k x(t) + \sum_{i=1,\ldots,n_B} y_{B_i}(k)\, \mu_i(t) + \hat{n}(t+k|t)$$

Model Predictive Control - Cost Function (1)

The response is evaluated only on a set of coincident point and the predicted error e is obtained through least square minimization:

$$\hat{e}(t+k|t) = \hat{y}(t+k|t) - \widehat{w}(t+k|t)$$

So the cost function is defined as below, with u as the vector of future control and lambda as the weight of control effort minimization and error minimization.

$$J(\boldsymbol{u}) = \sum_{j=1}^{n_H} \left|\hat{e}\left(t+h_j|t\right)\right|^2 + \lambda \sum_{j=1}^{n_H} \left(\Delta u\left(t+h_j-1|t\right)\right)^2 = \hat{\boldsymbol{e}}^T \hat{\boldsymbol{e}} + \lambda \boldsymbol{u}^T \boldsymbol{u}$$

Model Predictive Control - Cost Function (2)

So with the objective in mind of minimizing the cost function we can divide the problem into two cases:
Lambda = 0 where we obtain the following cost function and optimal coefficients:

$$J(\boldsymbol{\mu}) = (\boldsymbol{Y_B\mu} - \boldsymbol{d})^T(\boldsymbol{Y_B\mu} - \boldsymbol{d}) = \boldsymbol{\mu}^T\boldsymbol{Y_B^T Y_B \mu} + \boldsymbol{d}^T\boldsymbol{d} - 2\boldsymbol{\mu}^T\boldsymbol{Y_B^T d}$$

$$\boldsymbol{Y_B\mu^*} = \boldsymbol{d} \Rightarrow \boldsymbol{\mu^*} = \left(\boldsymbol{Y_B^T Y_B}\right)^{-1}\boldsymbol{Y_B^T\,d}$$

And Lambda != 0:

$$J(\boldsymbol{\mu}) = (\boldsymbol{Y_B\mu} - \boldsymbol{d})^T(\boldsymbol{Y_B\mu} - \boldsymbol{d}) + \lambda\boldsymbol{u}^T\boldsymbol{u}$$

$$\boldsymbol{\mu^*} = \left(\boldsymbol{Y_B^T Y_B} + \lambda\boldsymbol{I}\right)^{-1}\boldsymbol{Y_B^T\,d}$$

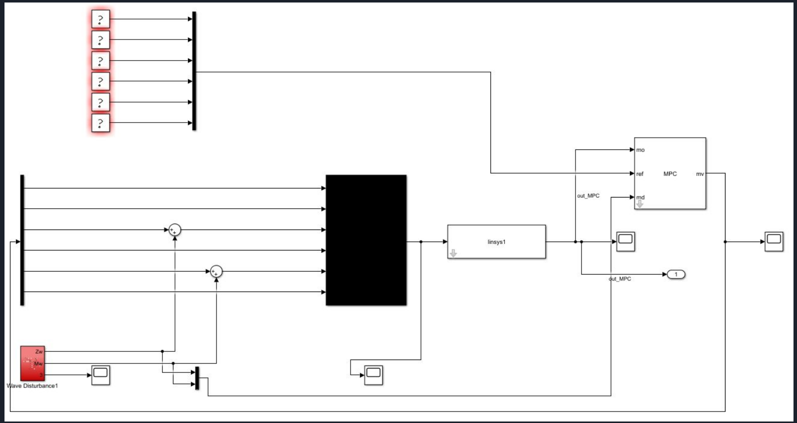
The first control action set is given by:

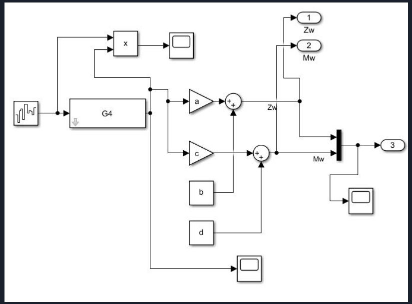
$$u^*(t|t) := \sum_{i=1,\dots,n_B} \mu_i^*(t)B_i(0)$$

Model Predictive Control - Implementation

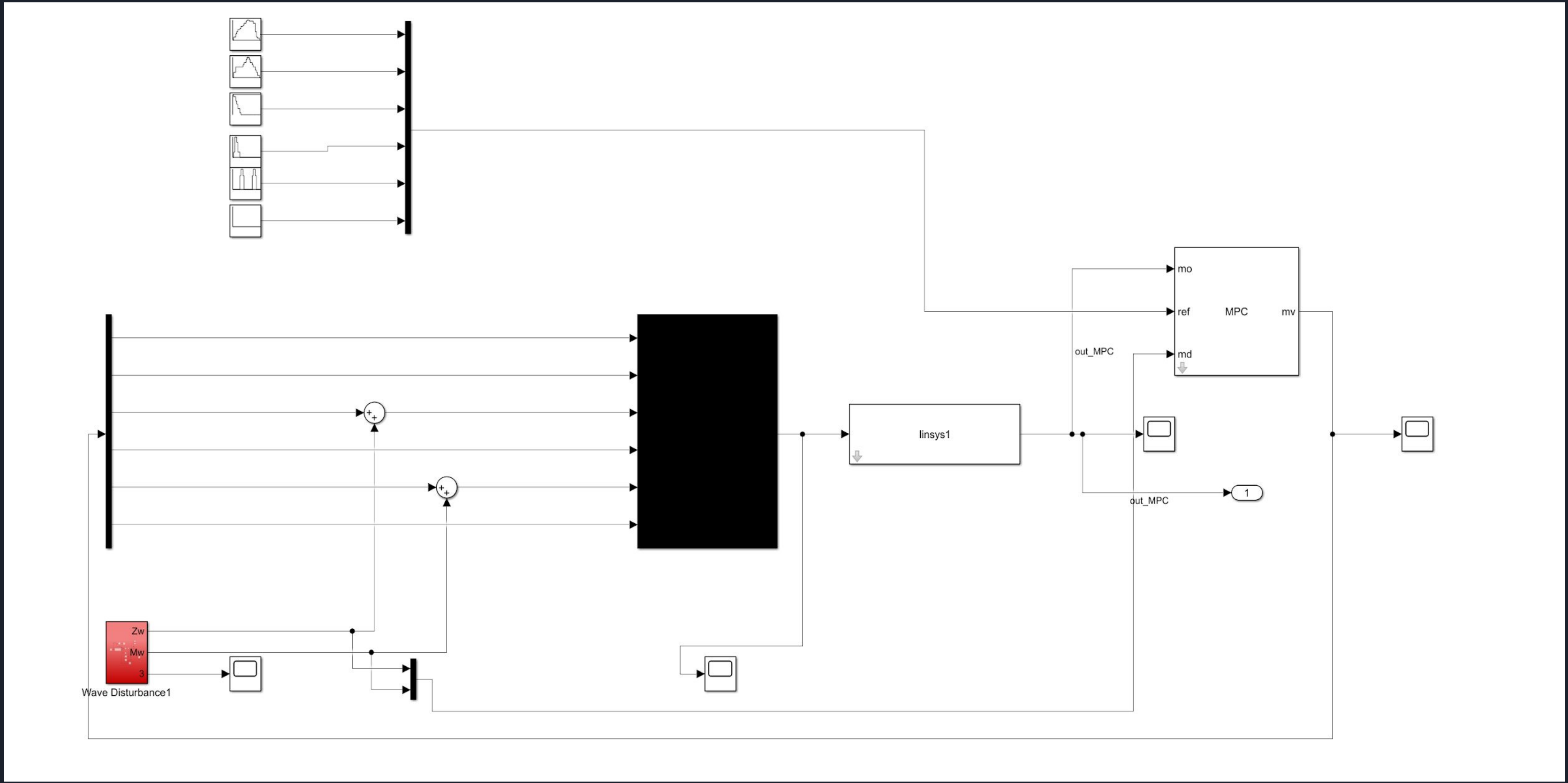
To control the submarine through an mpc scheme we built this model in matlab, we used the linearized system, and the model of the disturbances as we can see in the second photo. Then we use a matlab script to load all the data and plot the results of the simulation. For the MPC settings we found that optimal values for our problem are:

- Prediction Horizon = 15
- Control Horizon = 1
- Sample Time = 0.1
- Weight on OV (Output Variable) = [100 100 100 100 100 100]*4.2207

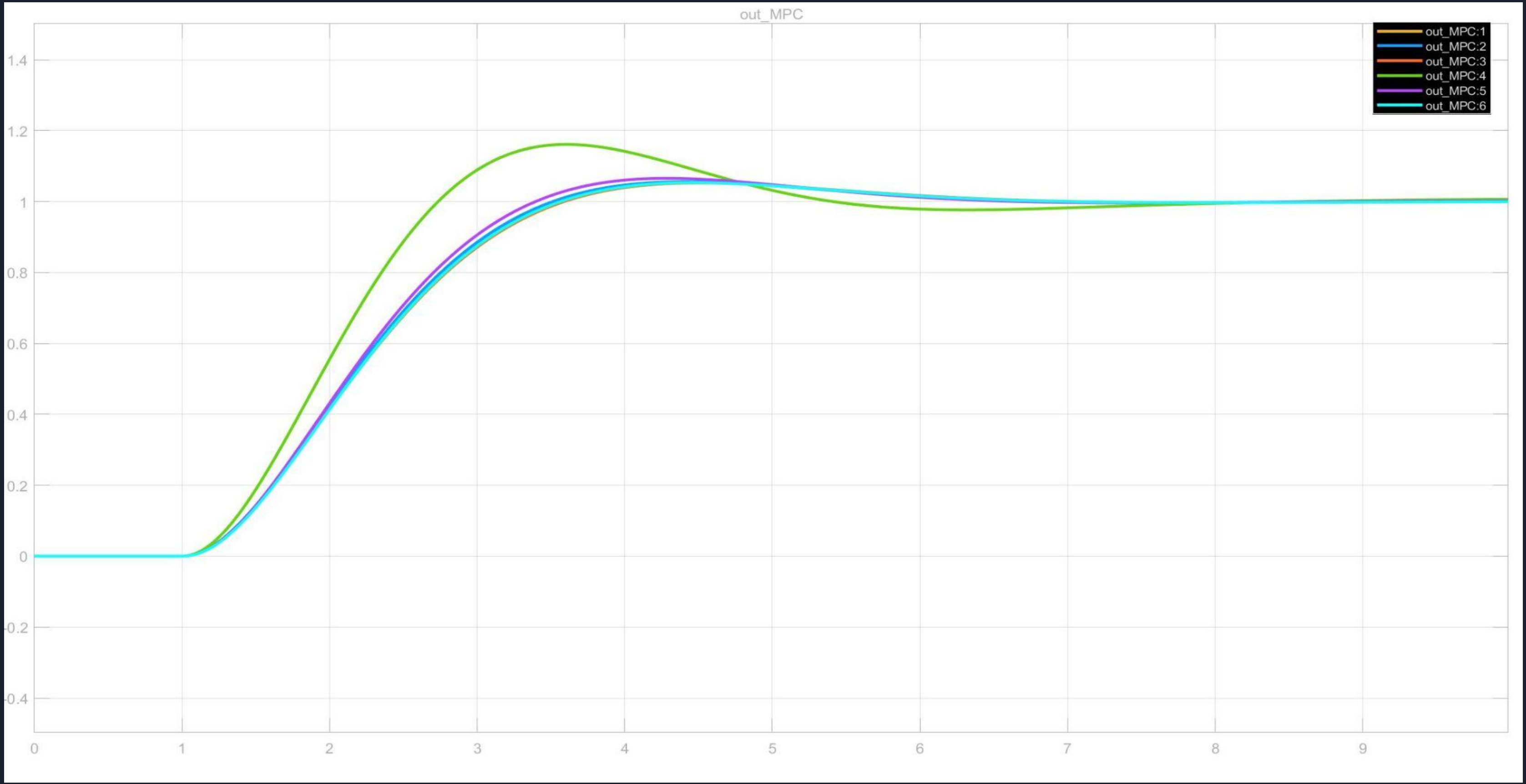




Model Predictive Control - Simulink Model



Model Predictive Control - Step Response



H_∞ synthesis controller

H_∞ control

***H_∞* methods** are used in control theory to synthesize controllers to achieve stabilization with guaranteed performance: to use this methods, a control designer expresses the control problem as a mathematical optimization problem and then finds the controller that solves this optimization.

H_∞ techniques have the advantage to be readily applicable to problems involving multivariate systems with cross-coupling between channels; instead disadvantages of *H_∞* techniques include the level of mathematical understanding needed to apply them successfully and the need for a reasonably good model of the system to be controlled.

It is important to keep in mind that the resulting controller is only optimal with respect to the prescribed cost function and does not necessarily represent the best controller in terms of the usual performance measures used to evaluate controllers such as settling time, energy expended, etc. Also, non-linear constraints such as saturation are generally not well-handled.

H_∞ control

H_∞ techniques can be used to minimize the closed loop impact of a perturbation: depending on the problem formulation, the impact will either be measured in terms of stabilization or performance.

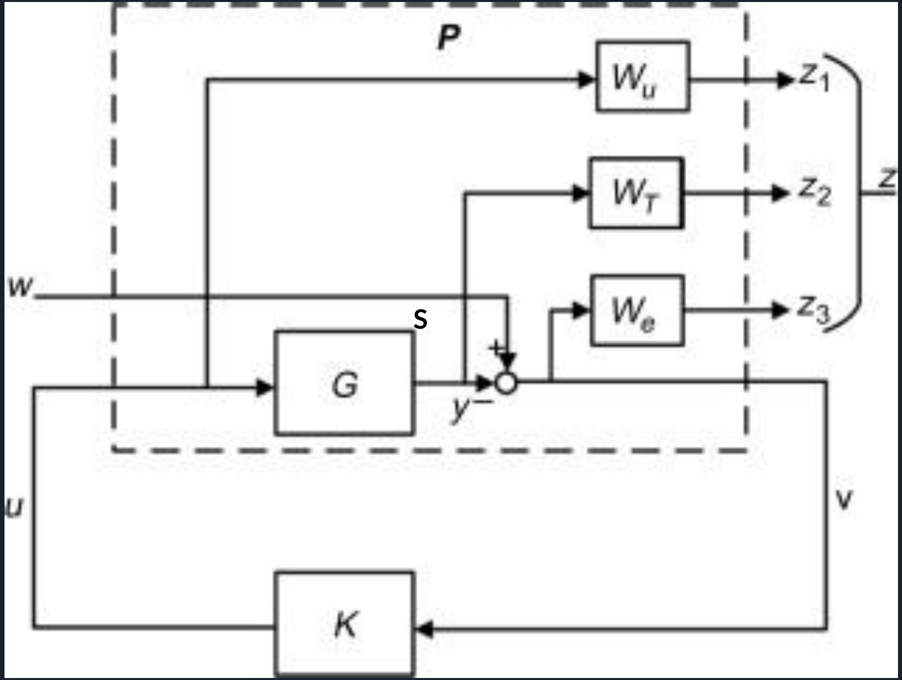
The standard H_∞ optimal control problem is to find all stabilizing controllers K which minimize

$$||F_l(P,K)||_\infty = \max_\omega \bar{\sigma}(F_l(P,K)(j\omega))$$

The H_∞ norm has several interpretations in terms of performance: one is that it minimizes the peak of the maximum singular value of $F_l(P(j\omega), K(j\omega))$.

H_∞ control

The H_∞ control scheme is reported below. The weights adopted are: W_u, W_T, W_e. We introduced the same weights in our project and we computed the H infinity controller.



H_∞ control weights

To build the controller we introduced the following weights:

$$W_s = \frac{\frac{s}{M_s} + w_{BS}}{s + w_{BS} A_s}$$

To reproduce, at steady state, a constant reference is necessary to lower as much as possible the S(jω) function. Also S($j\omega$) must have a cut-off frequency of wc >= w_{BS} = 100 rad/s to reject the sinusoidal disturbances. For these reasons has been introduced the following sensitivity weight. $A_s = 10^{-4}$, which is the upper bound for sensitivity function gain. $M_s = 5$, is the maximum value of the sensitivity function peak.

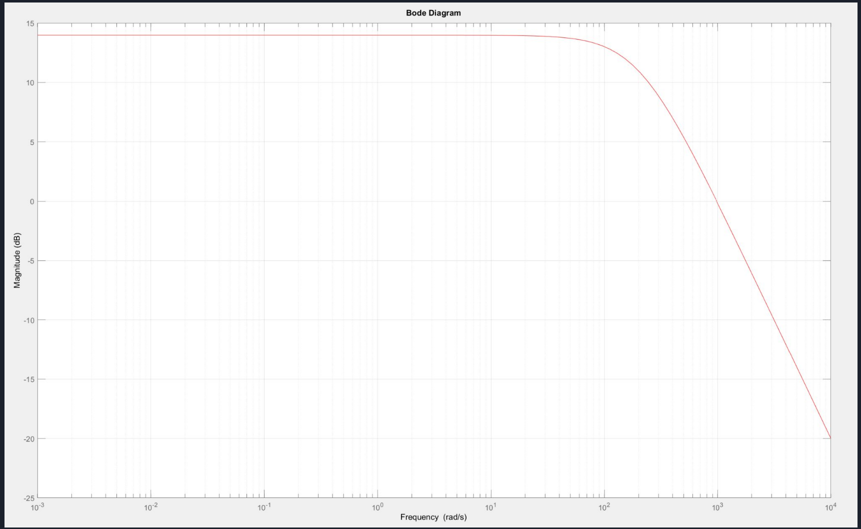
$$W_t = \frac{s + \frac{w_{BT}}{M_t}}{A_t s + w_{BT}}$$

In presence of a measurement noise (not included in this model), the specifications requires the rejection of measurement disturbances at high frequencies. T(jω) must have a cut-off frequency of wc <= w_{BT} = 1000 rad/s to reject the sinusoidal disturbances. For these reasons has been introduced the following sensitivity weight. $A_t = 10^{-4}$, which is the upper bound for sensitivity function gain.
$M_t = 5$, is the maximum value of the complementary sensitivity function peak.

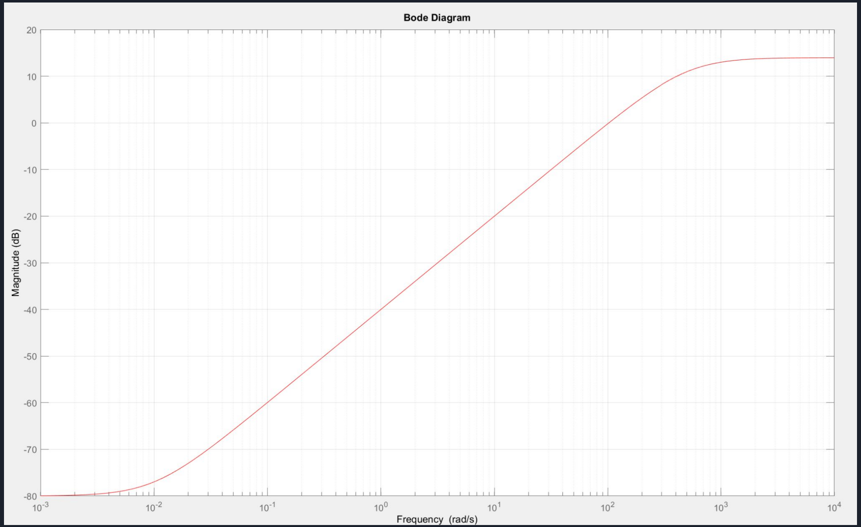
The control effort can be limited introducing a constant upper bound W_u to the control sensitivity function.
To limit this function to a maximum value of 60dB, can be chosen a value $W_u = 10^3$

H_∞ control weights

For simplicity the same weights were applied to all the channels. Therefore, the real weights are 6 by 6 matrices, with the singular weights reported on the main diagonal.

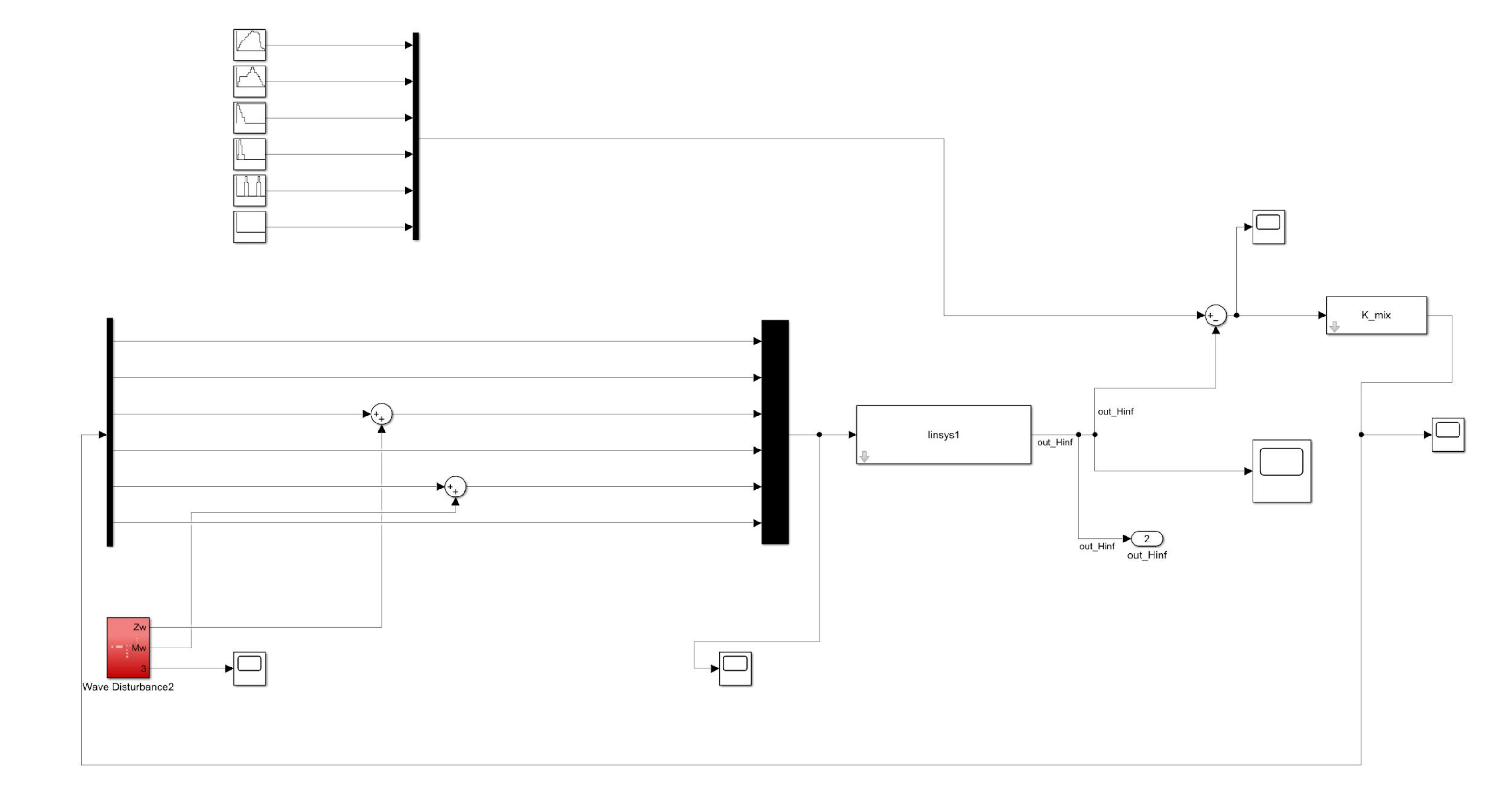


Complementary Sensitivity Function Weight

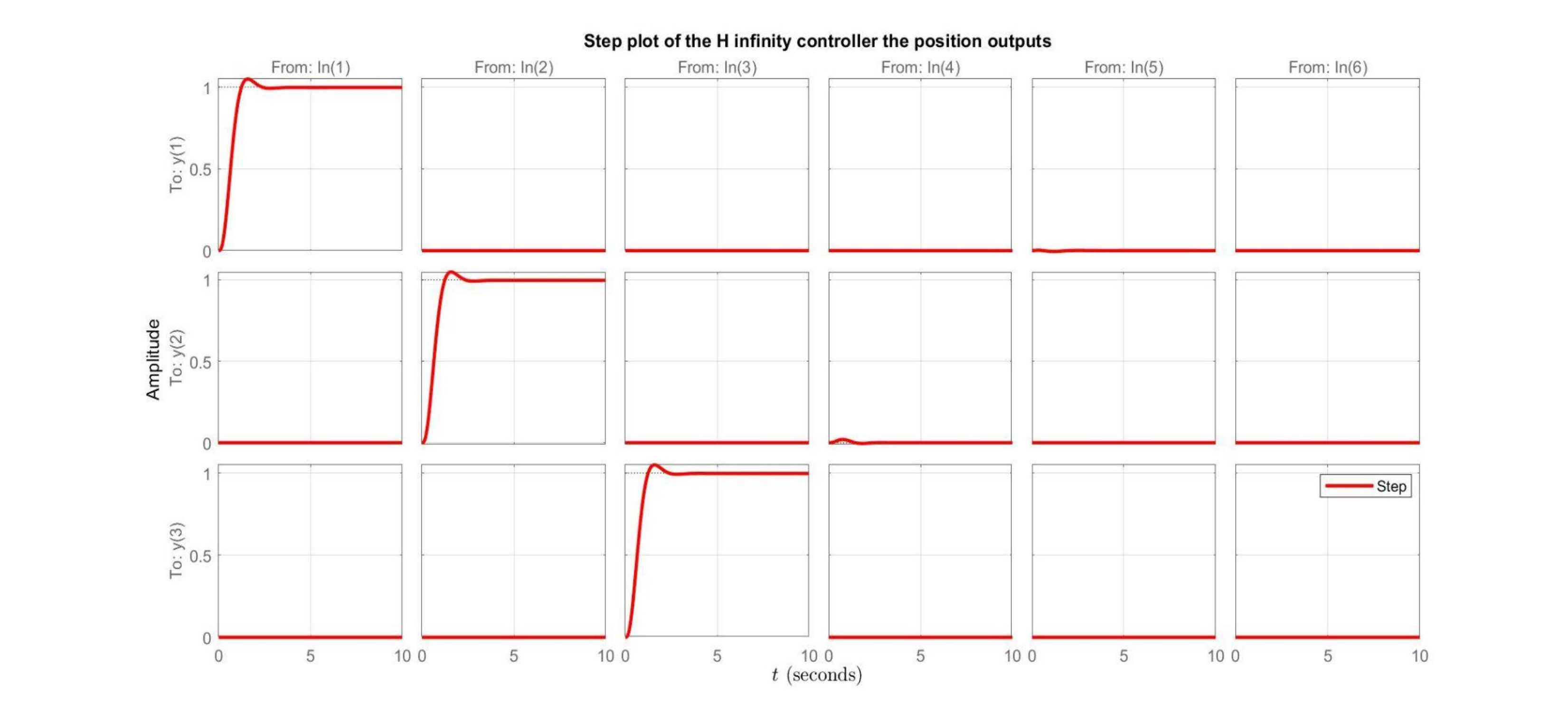


Sensitivity Function Weight

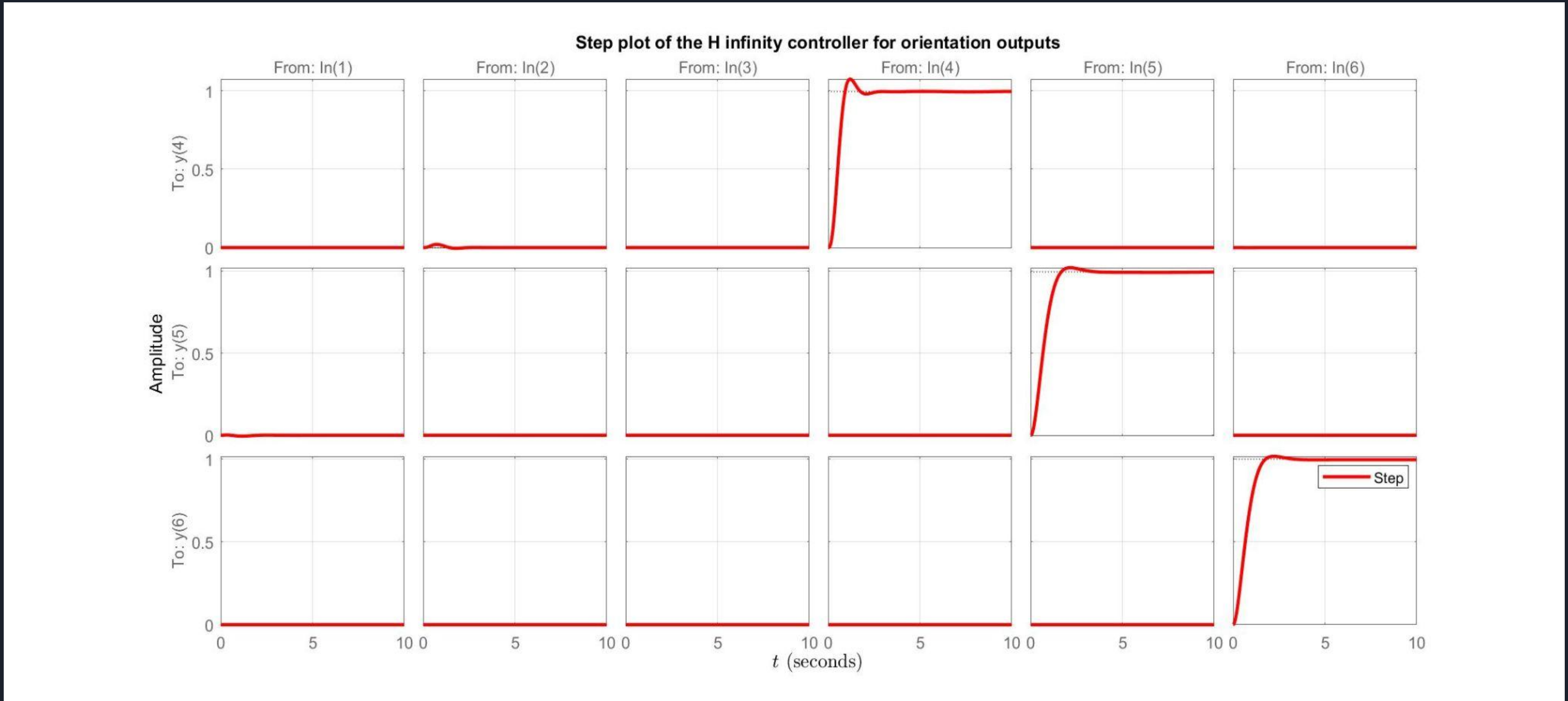
H_∞ - Simulink Model



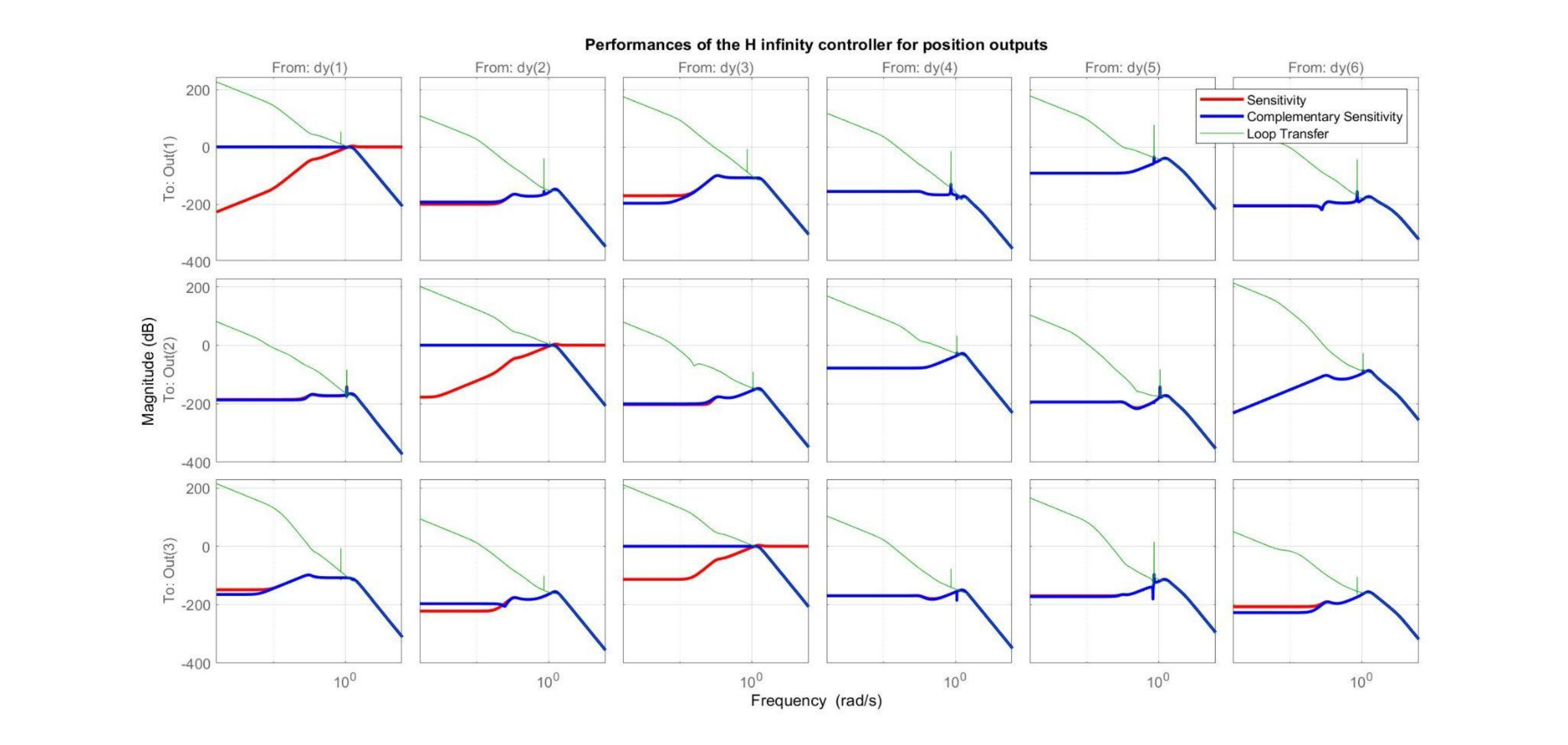
H_∞ - Step Response



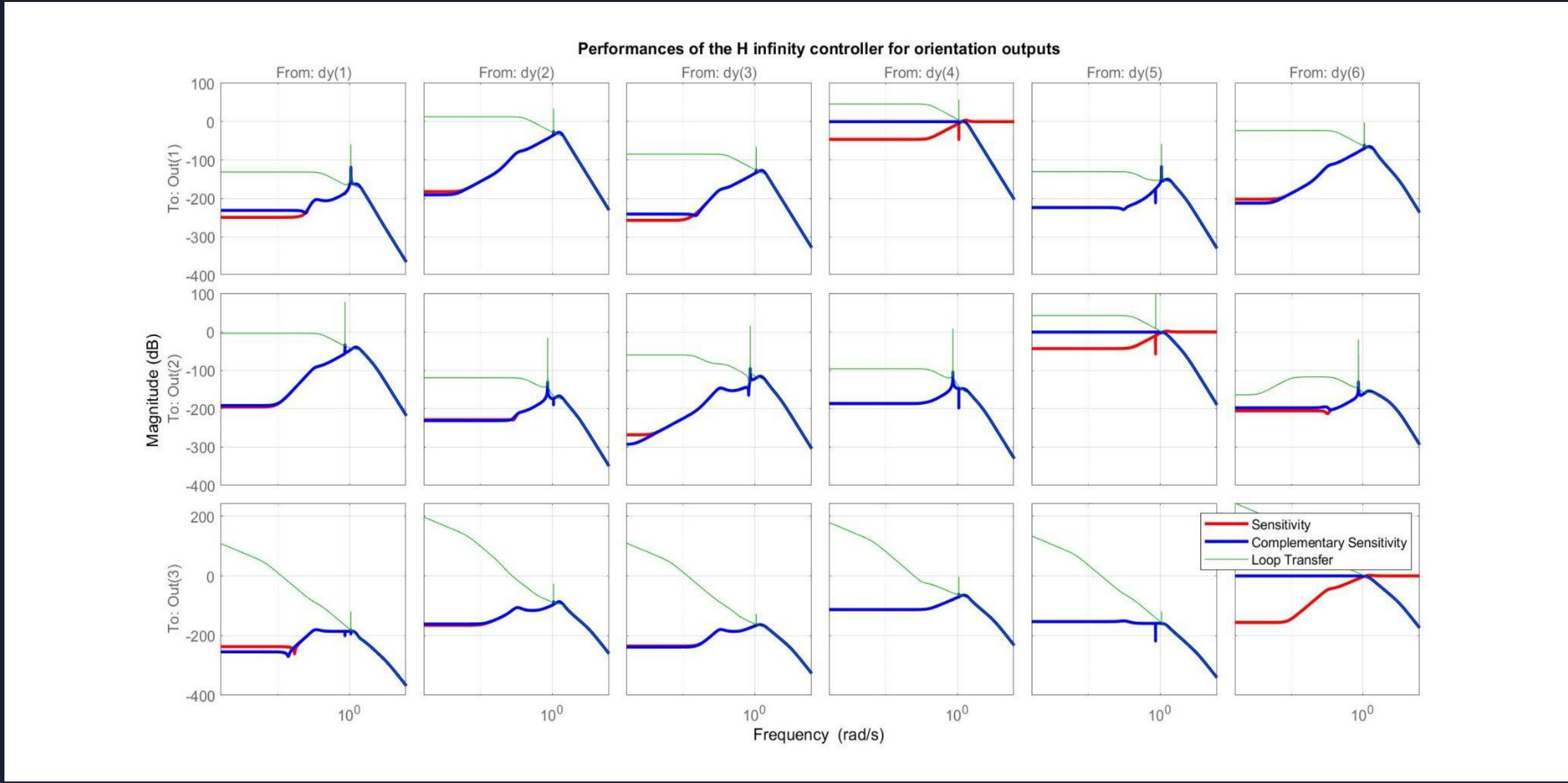
H_∞ - Step Response



H_∞ - Performances



H_∞ - Performances



Trajectories

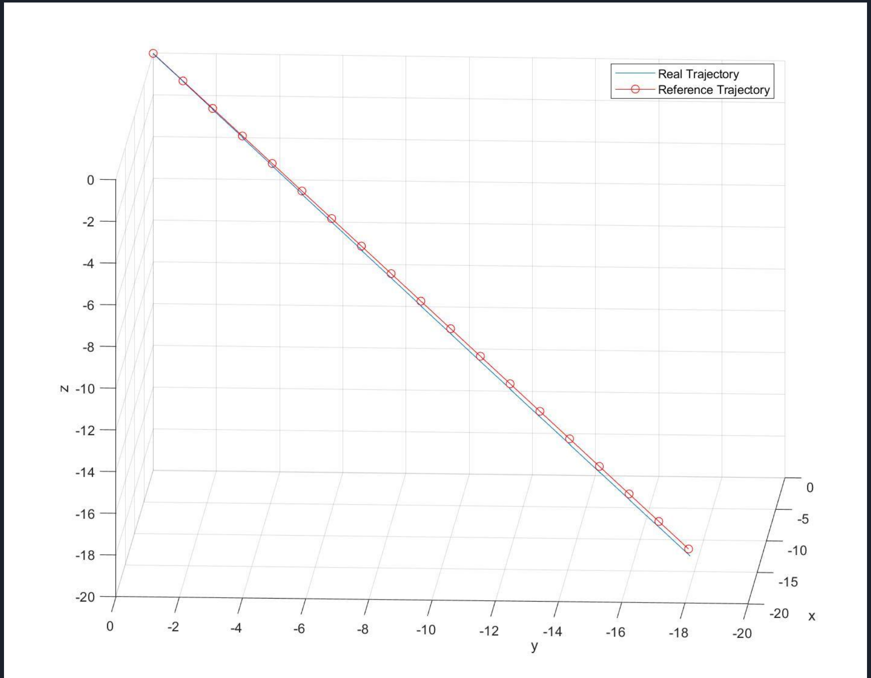
Test Results

Once obtained a controller, that is able to reproduce a step reference, even in presence of the previously mentioned disturbances, we submitted to our controller more technical issues to evaluate it.
We have enforced our controllers, by trying them on new trajectories, that change the position and orientation of the submarine.

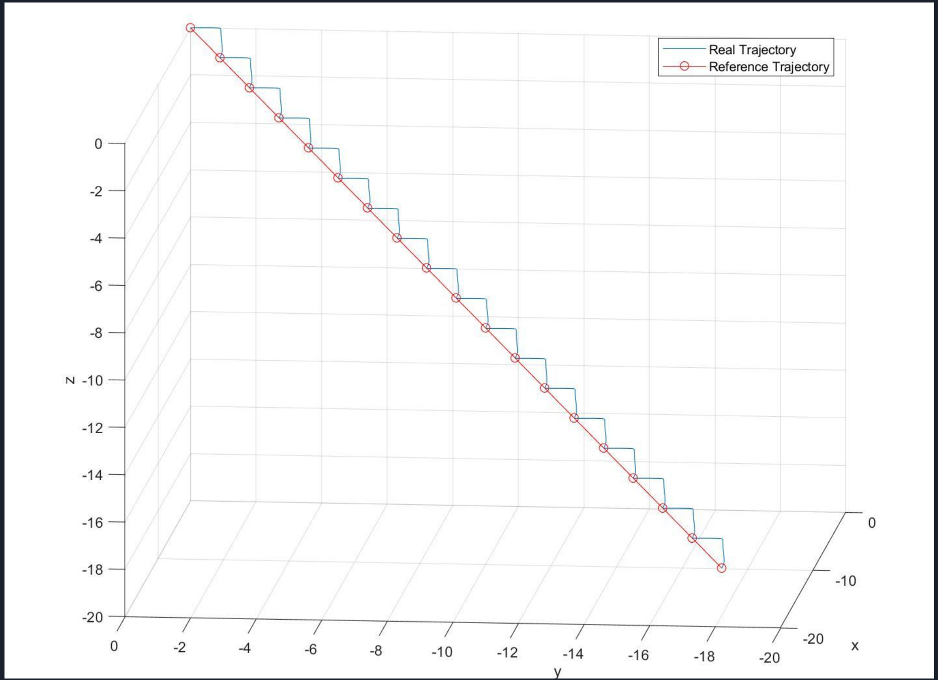
The trajectories are:

- Descended trajectory;
- Descended trajectory avoiding obstacles;
- Poseidon (Descending and Ascending);
- Mariana Trench

Descended Trajectory

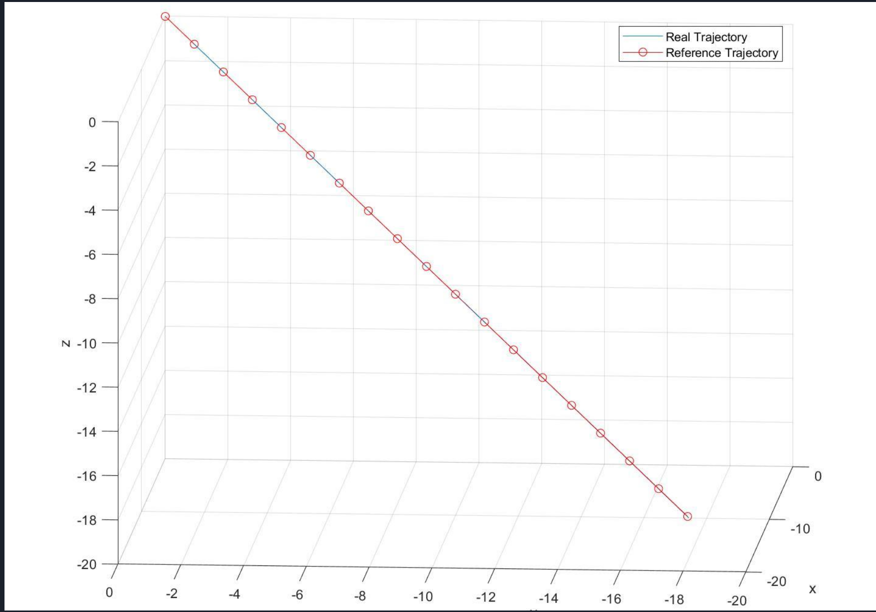


MPC controller trajectory

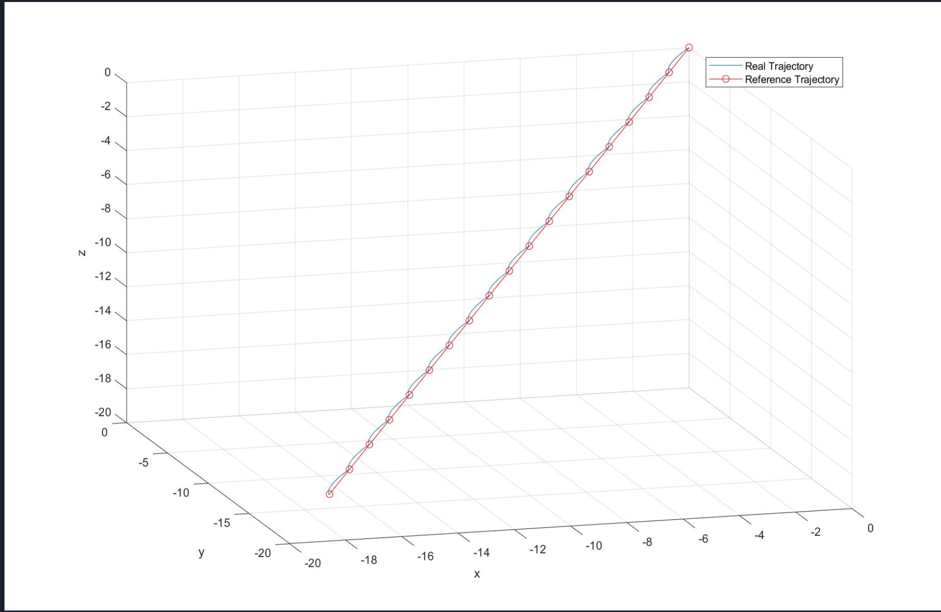


PID+Decoupler controller trajectory

Descended Trajectory

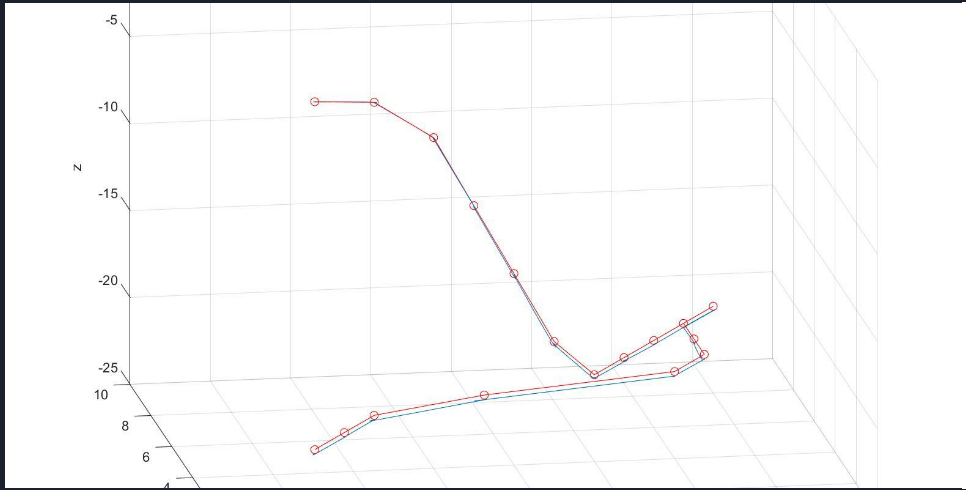


H_∞ controller trajectory

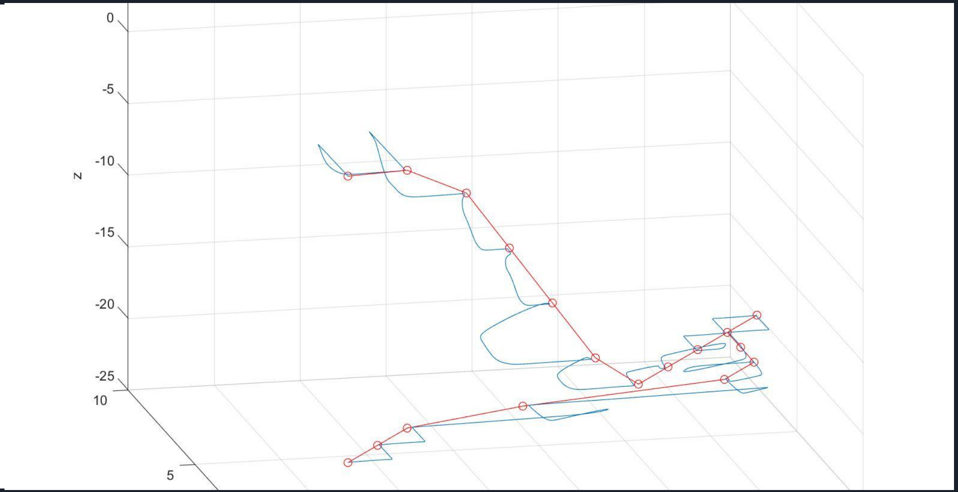


LQR controller trajectory

Descended Trajectory Avoiding Obstacles

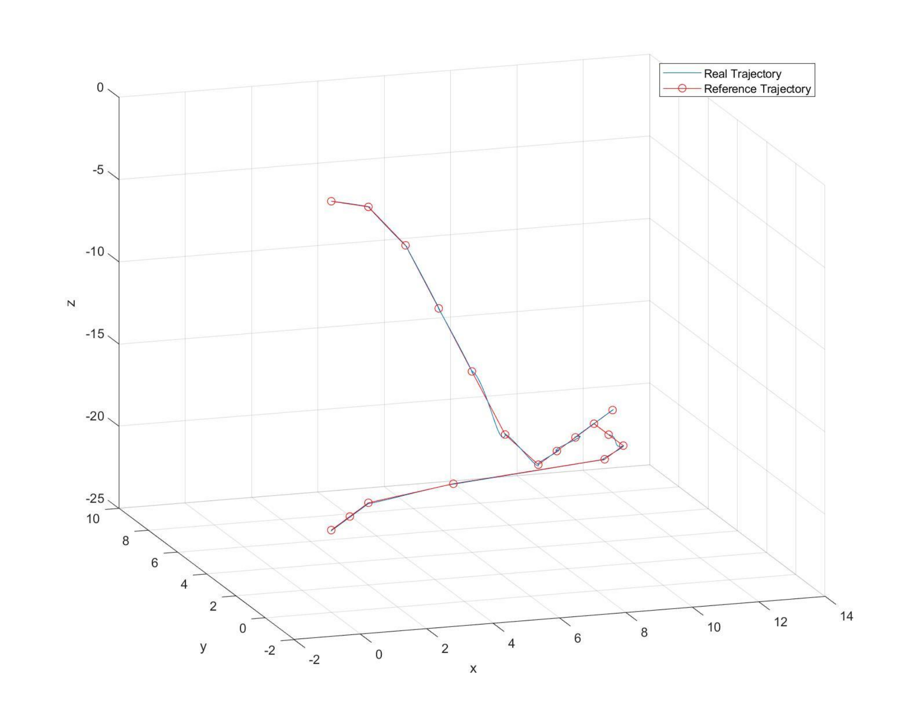


MPC controller trajectory

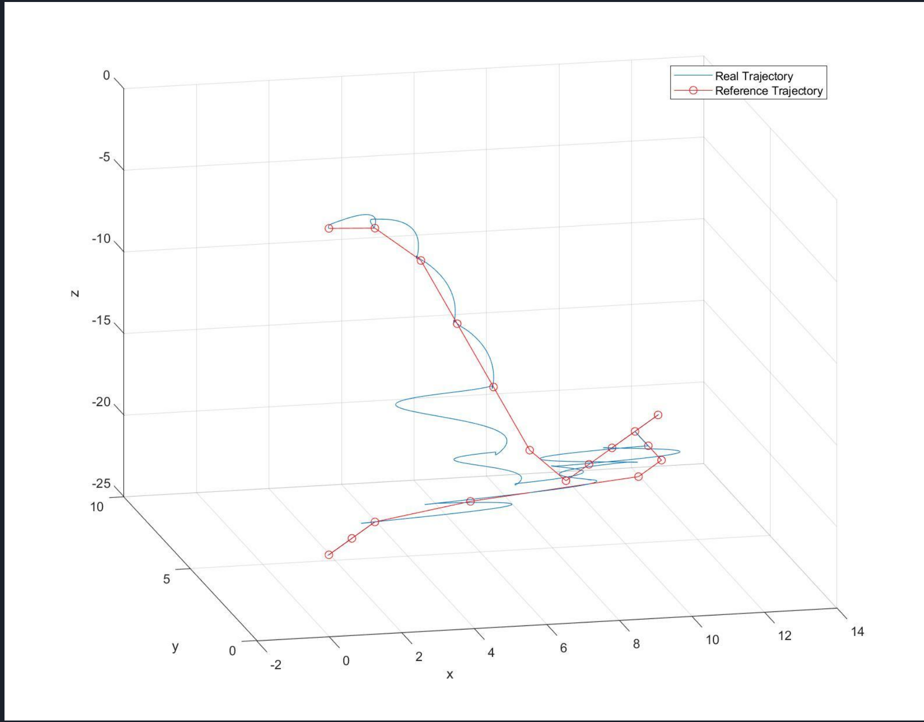


PID+Decoupler controller trajectory

Descended Trajectory Avoiding Obstacles

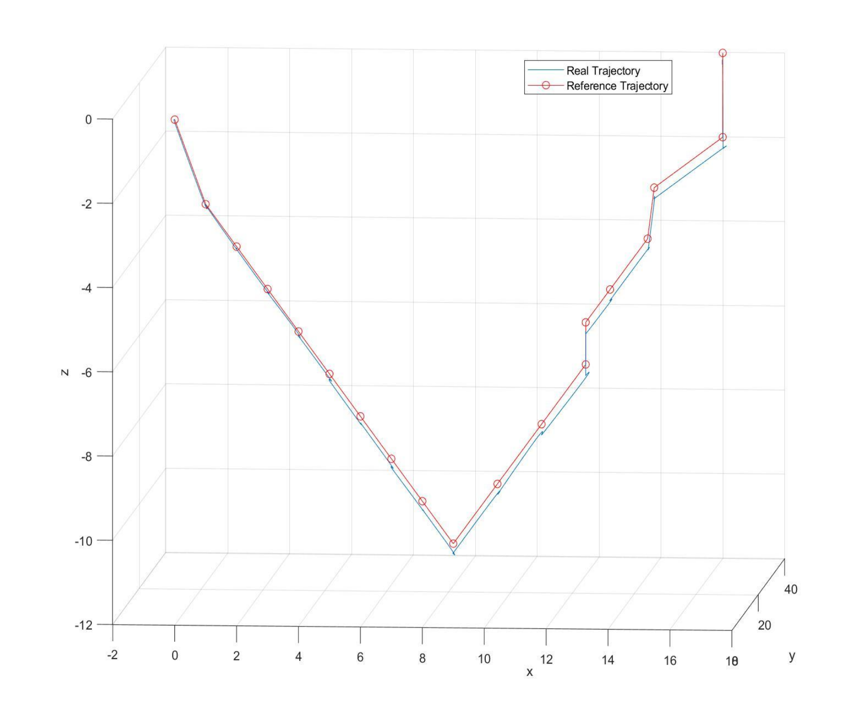


H_∞ controller trajectory

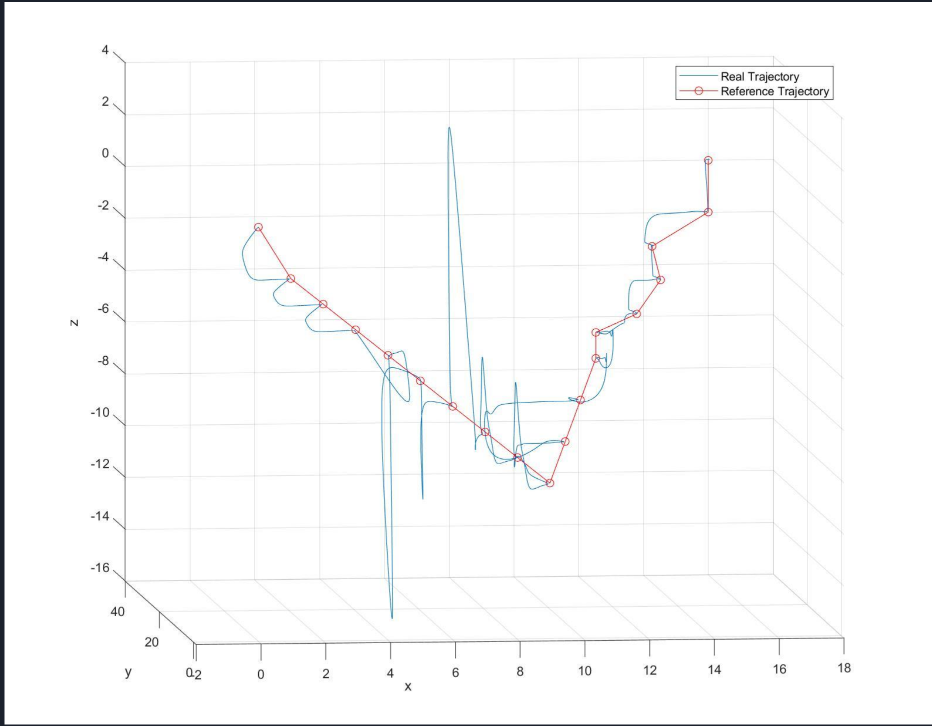


LQR controller trajectory

Poseidon

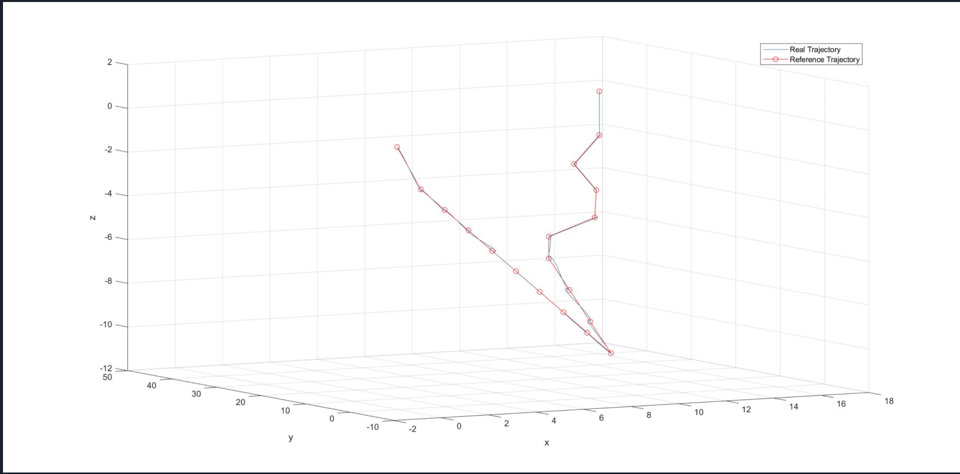


MPC controller trajectory

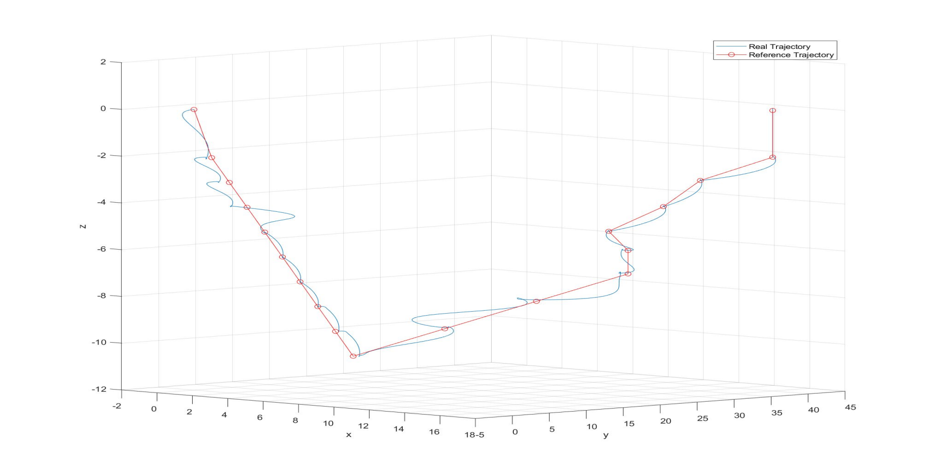


PID+Decoupler controller trajectory

Poseidon

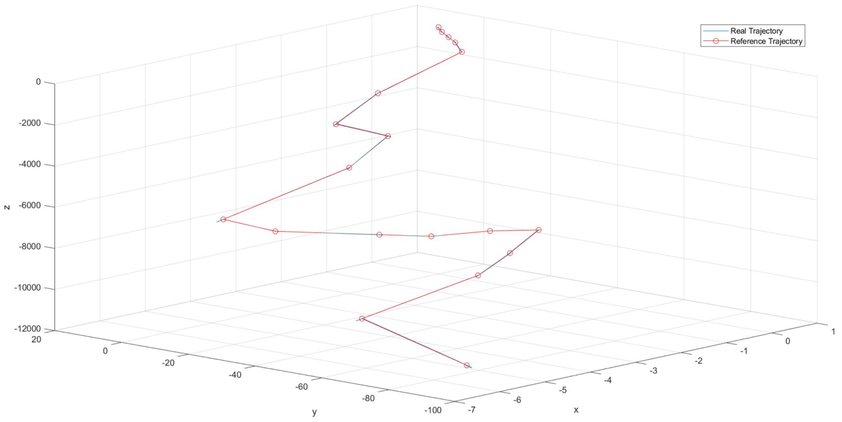


H_∞ controller trajectory

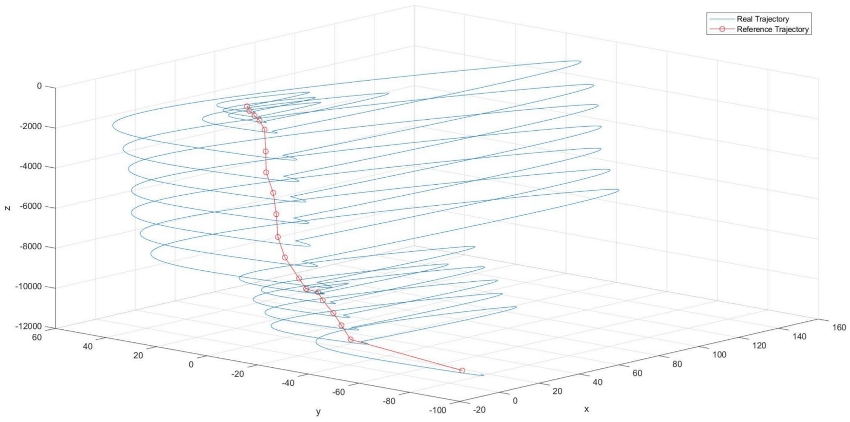


LQR controller trajectory

Mariana Trench

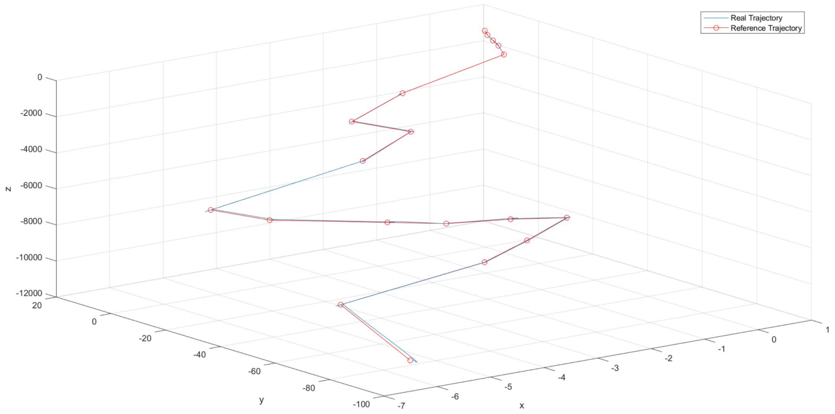


MPC controller trajectory

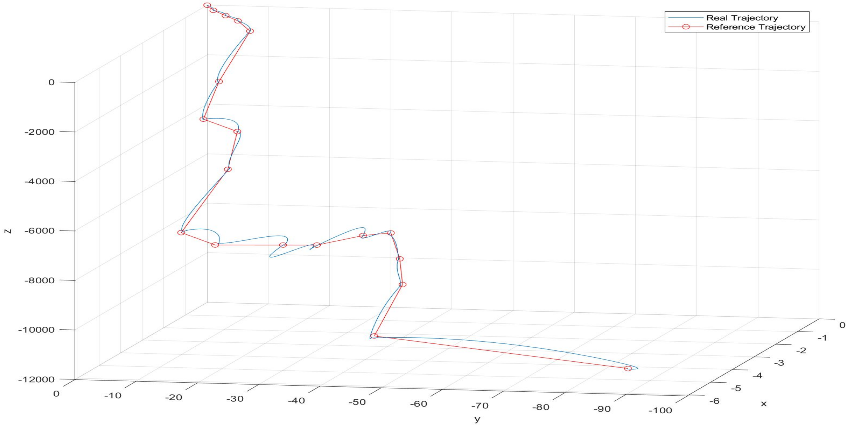


PID+Decoupler controller trajectory

Mariana Trench



H_∞ controller trajectory



LQR controller trajectory

Conclusions: MPC and H_∞ vs. PID and LQR

In the simulated trajectories turned out that the MPC and H_∞ controllers performed better than LQR and PID+Decoupler.

The MPC is capable to optimize a cost function, based on the given weights, in order to perform a control action based even on the future events: this allows the controller to react to the wave disturbances.

The H_∞ has the objective to make the MIMO system transfer function equal to a reference transfer function, so in this way the input-output behaviour results highly decoupled. This resulted in a very low error in the reproduction of the reference trajectory.

On the other hand, PID and LQR have shown a reduced capacity of handling a MIMO system with a not decoupled input-output behaviour.

References

[1] Guidance and Control of Ocean Vehicles, [Thor I. Fossen]

[2] Submarine Hm Depth Control Under Wave Disturbances, [Eduardo Liceaga-Castro] & [Gerrit M. van der Molen]

[3] Submarine Low-Noise Optimum Depth Controller Design Based on LQR , [Bangjun Lv, Bin Huang, Likun Peng] & [Kun Bi]

[4] H_∞ Submarine Depth and Pitch Control, [Gerrit M. van der Molen] & [Michael J. Grimble]

[5] Mathematical Models of Ships and Underwater Vehicles, [Thor I. Fossen]