Factorial

Given a natural number $n \geq 1$, its **factorial** $n!$ is defined in a recursive fashion:

- $1! = 1$, and
- $n! = n \cdot (n-1)!$.

Example 1.
The factorial of 5 is 120 because

- $5! = 5 \cdot 4!$
- $4! = 4 \cdot 3!$
- $3! = 3 \cdot 2!$
- $2! = 2 \cdot 1!$
- $1! = 1$

So $5!$ reduces to $5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$, which is 120.

The factorial often appears in combinatorial problems. For instance, if you have n distinct elements, then they can be arranged in $n!$ ways.

Example 2.
There are $3! = 6$ ways to order a, b, and c:

- abc
- acb
- bac
- bca
- cab
- cba

Exercise 1.
Write down all possible ways of ordering a, b, c and d and confirm that this number is the same as $4!$.

The factorial function grows very fast, even faster than an exponential function.

n	2^n	$n!$
1	2	1
2	4	2
3	8	6
4	16	24
5	32	120

n	2^n	$n!$
6	64	720

Even a very fast growing exponential like $10,000^n$ will eventually grow more slowly than the factorial, even though it grows more rapidly for small values of n (e.g. $10,000^{10} = 10^{4^{10}} = 10^{40}$ is much larger than $10! = 3,628,800$).