Stop word removal as a function

PREREQUISITES

- functions (domain terminology)

This unit illustrates how one might define stop word removal as a mathematical function *del* (read *delete*).

First, we fix some alphabet Σ and let S be some finite set of symbols drawn from Σ. For every such S, we define a deletion function del_S that maps strings over Σ to strings over $\Sigma - S$. In mathematical notation, $del_S : \Sigma^* \to (\Sigma - S)^*$.

This only tells us the domain and co-domain of del_S, but not how exactly inputs and outputs are connected to each other. For any string of the form $u_1 \cdots u_n$ (where $n \geq 0$ and each u_i is a symbol drawn from Σ), we define

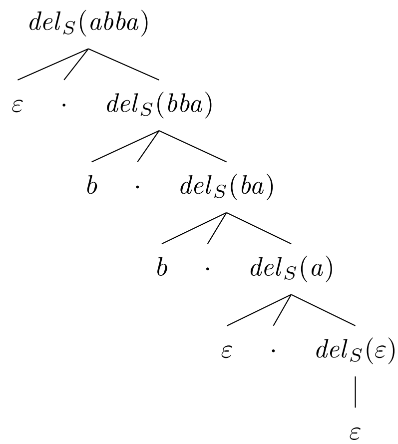
$$del_S(u_1 \cdots u_n) := \begin{cases} \varepsilon & \text{if } u_1 \cdots u_n = \varepsilon \\ del_S(u_2 \cdots u_n) & \text{if } u_1 \in S \\ u_1 \cdot del_S(u_2 \cdots u_n) & \text{otherwise} \end{cases}$$

EXAMPLE 1.
Suppose $\Sigma := \{a, b\}$ and $S := \{a\}$. Let $s := abba$. Then $del_S(s)$ should yield bb. To this end, we compute $del_S(s)$ in a stepwise fashion:

$$\begin{aligned} del_S(s) &= del_S(abba) \\ &= del_S(bba) \\ &= b \cdot del_S(ba) \\ &= b \cdot b \cdot del_S(a) \\ &= b \cdot b \cdot del_S(\varepsilon) \\ &= b \cdot b \cdot \varepsilon \\ &= b \cdot b \\ &= bb \end{aligned}$$

So $del_S(abba) = bb$, as expected.

As you can see, del_S is partially defined in terms of itself: the value of $del_S(abba)$ is inferred from the value of $del_S(bba)$. This is called a **recursive** definition. We can visualize the computation of this recursive function as below:



Every recursive function has one or more **base cases** and a **recursion step**. The base cases are those where the value of the function can be determined without recursion. For *del*, there is only the base case $del_S(\varepsilon) = \varepsilon$. Notice how in the graph above $del_S(\varepsilon)$ does not contain any further instances of del_S. Instead, we immediately get ε as the output. The recursion step defines the function in terms of the function itself. In the graph above, that's every instance of del_S which has another instance of del_S below it.

Exercise 1.
Here is another recursively defined function.

$$f(x, y) := \begin{cases} x & \text{if } y \leq 1 \\ x + f(x, y-1) & \text{otherwise} \end{cases}$$

What does this function do? Is there a commonly used name for it?

Exercise 2.
This continues the previous exercise. Draw a diagram like the one above for $f(5, 4)$.

Exercise 3.
Give a recursive definition of a function that takes two arguments: a string $u := u_1 \cdots u_n$ over alphabet Σ, and a set S of symbols drawn from Σ. The function returns 1 if at least one member of S occurs in u, and 0 otherwise.

Exercise 4.
This continues the previous exercise. Draw a diagram like the one above for

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$f(aaba, \{b\})$ and $f(aaba, \{c, d, e\})$.