Notation: Big operators (Solutions)

Exercise 1.
Write each one of the following expressions as a formula with the addition operator $+$.

$$5 - \sum_{1 \leq i \leq 3} i$$

$$\sum_{i=1} i + i + i$$

Solution

$$5 - \sum_{1 \leq i \leq 3} i = 5 - (1 + 2 + 3)$$

$$\sum_{i=1} i + i + i = 1 + 1 + 1$$

Exercise 2.
Express each formula below in a more succinct format using the sum operator $\sum$.

1. $20 + 21 + 22 + 23 + 24 + 25 + 26 + 27 + 28 + 29$
2. $2 + 4 + 6 + 8 + 10$
3. $30 + 15 + 3 + 10 + 2 + 20$
4. $-10 - 9 - 8 - 7$

Solution

1. $\sum_{20 \leq i \leq 29} i$
2. $\sum_{i \in \{2,4,6,8,10\}} i$
3. $\sum_{i \in \{2,3,10,15,20,30\}} i$
4. not well-defined because subtraction isn't associative; but $(-10) + (-9) + (-8) + (-7)$ would be $\sum_{7 \leq i \leq 10} -i$

Exercise 3.
Rewrite the formula below so that it uses the subscript/superscript format.

$$5 - \sum_{1 \leq i \leq 3} i$$

Solution

$$5 - \sum_{i=1}^{3} i$$

Exercise 4.
Consider once more the formulas below.

- $20 + 21 + 22 + 23 + 24 + 25 + 26 + 27 + 28 + 29$
- $2 + 4 + 6 + 8 + 10$
- $30 + 15 + 3 + 10 + 2 + 20$
- $-10 - 9 - 8 - 7$

Combine the sum operator with other operations in order to describe each formula in a more systematic way. Keep in mind that there are infinitely many ways this can be done, just pick the one that seems most plausible to you.

Solution

1. $\sum_{0 \leq i \leq 0}(20 + i)$
2. $\sum_{1 \leq i \leq 5} 2i$
3. $\sum_{i \in \{2,3\}}(i + 5i + 10i)$
4. still undefined

Exercise 5.
Suppose that $\mathcal{S} := \{\{0, 3\}, \{9, 23\}, \{2, 9\}\}$ as before. What is the value of the formula below, assuming that $i \oplus j = ij$ if $i \geq j$ and ji otherwise (for example, $8 \oplus 7 = 87$ and $3 \oplus 15 = 153$).

$$\prod_{S \in \mathcal{S}} \bigoplus_{n \in S} n$$

Solution

$$\begin{aligned}
\prod_{S \in \mathcal{S}} \bigoplus_{n \in S} n &= \bigoplus_{n \in \{0,3\}} n \times \bigoplus_{n \in \{9,23\}} n \times \bigoplus_{n \in \{2,9\}} n \\
&= 0 \oplus 3 \times 9 \oplus 23 \times 2 \oplus 9 \\
&= 30 \times 239 \times 92 \\
&= 659,640
\end{aligned}$$