

## Picking maxima and minima

Four commonly encountered functions are  $\max$ ,  $\min$ ,  $\operatorname{argmax}$ , and  $\operatorname{argmin}$ . Each one is fairly intuitive.

### 1 $\max$ and $\min$

The  $\max$  function returns the largest element of a set.

**EXAMPLE 1.**

Let  $S := \{-5, 7, 23\}$ . Then  $\max(S) = \max(\{-5, 7, 23\}) = 23$ . Some authors just write  $\max(-5, 7, 23)$  instead of  $\max(\{-5, 7, 23\})$ .

In most cases,  $\max$  is used with numbers. But the function can be generalized to any structure that is a linear order.

**EXAMPLE 2.**

Consider **2**, the lattice with  $F < T$ . Then  $\max(\{F, T\}) = T$ .

Note that the order must be a linear order. With weak partial orders that aren't also linear orders,  $\max$  may not be defined for all cases.

**EXAMPLE 3.**

Consider a case hierarchy with  $\text{Nom} \leq \text{Acc}$  and  $\text{Nom} \leq \text{Gen}$ , but  $\text{Acc}$  and  $\text{Gen}$  are unordered with respect to each other. Then  $\max(\{\text{Acc}, \text{Gen}\})$  is undefined.

The opposite of  $\max$  is  $\min$ . It returns the smallest member of a set.

**EXAMPLE 4.**

While  $\max(\{-5, 7, 23\}) = 23$ ,  $\min(\{-5, 7, 23\}) = -5$ . And assuming  $F < T$ ,  $\min(\{T, F\}) = F$ .

### 2 $\operatorname{argmax}$ / $\operatorname{argmin}$

will be added at a later point