

## Factorial

Given a natural number  $n \geq 1$ , its **factorial**  $n!$  is defined in a recursive fashion:

- $1! = 1$ , and
- $n! = n \cdot (n - 1)!$ .

### EXAMPLE 1.

The factorial of 5 is 120 because

- $5! = 5 \cdot 4!$
- $4! = 4 \cdot 3!$
- $3! = 3 \cdot 2!$
- $2! = 2 \cdot 1!$
- $1! = 1$

So  $5!$  reduces to  $5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

The factorial often appears in combinatorial problems. For instance, if you have  $n$  distinct elements, then they can be arranged in  $n!$  ways.

### EXAMPLE 2.

There are  $3! = 6$  ways to order  $a$ ,  $b$ , and  $c$ :

- $abc$
- $acb$
- $bac$
- $bca$
- $cab$
- $cba$

The factorial function grows very fast, even faster than an exponential function.

$n$	$2^n$	$n!$
1	2	1
2	4	2
3	8	6
4	16	24
5	32	120
6	64	720

Even a very fast growing exponential like  $10,000^n$  will eventually grow more slowly than the factorial, even though it grows more rapidly for small values of  $n$  (e.g.  $10,000^{10} = 10^{40} = 10^{40}$  is much larger than  $10! = 3,628,800$ ).