

# Parts of strings

## 1 Substrings

A **substring** is a continuous part of a string.

### EXAMPLE 1.

The string *abcd* has 11 substrings:

- $\varepsilon$
- *a*
- *b*
- *c*
- *d*
- *ab*
- *bc*
- *cd*
- *abc*
- *bcd*
- *abcd*

Note that

1. the empty string is a substring of every string, and
2. every string is a substring of itself.

A substring *u* of *v* is **proper** iff  $u \neq v$ .

### EXAMPLE 2.

All the strings listed above are proper substrings of *abcd*, except *abcd* itself.

### EXERCISE 1.

For each one of the gaps below, enter  $\sqsubseteq$ ,  $\sqsubset$ , or  $\not\sqsubseteq$  depending on whether the first string is a substring of the second string, a proper substring, or neither:

- *a\_aaaa*
- *a\_b*
- $\varepsilon_b$
- $\varepsilon_\varepsilon$
- *aa\_abbcca*
- *bc\_abbcca*
- *cb\_abbcca*

## 2 Subsequence

A **subsequence** is a discontinuous part of a string that preserves the order between the symbols.

**EXAMPLE 3.**

The string  $abcd$  has subsequences:

- $\varepsilon$
- $a$
- $b$
- $c$
- $d$
- $ab$
- $ac$
- $ad$
- $bc$
- $bd$
- $cd$
- $abc$
- $abd$
- $bcd$
- $abcd$

Note that  $ca$  is not a subsequence of  $abcd$ , but it is a subsequence of  $abcda$ .

Just like substrings, a subsequence  $u$  of  $v$  is proper iff  $u \neq v$ .

**EXERCISE 2.**

For each one of the gaps below, enter  $\sqsubseteq$ ,  $\subsetneq$ , or  $\not\sqsubseteq$  depending on whether the first string is a subsequence of the second string, a proper subsequence, or neither:

- $a\_aaaa$
- $a\_b$
- $\varepsilon\_b$
- $\varepsilon\_ \varepsilon$
- $aa\_abbbca$
- $bc\_abbbca$
- $cb\_abbbca$

**EXERCISE 3.**

Say whether the following is True or False: Every substring of some string  $s$  is also

a subsequence of  $s$ , but not the other way round. Justify your answer.