Chapter 7

Exercise 7.1
(a)

$$\min_{\theta} \frac{1}{2}\|y-\theta\|_2^2 + \frac{1}{2}\lambda^2\|\theta\|_0 = \sum_i \min_{\theta_i} \frac{1}{2}(y_i-\theta_i)^2 + \frac{1}{2}\lambda^2\|\theta_i\|_0.$$

If $\theta_i = 0$, then we incur a cost of $y_i^2/2$. If $\theta_i \neq 0$, then then the ℓ_0 term contributes a constant cost of $\lambda^2/2$ no matter the value of θ_i, so we set it to y_i, and incur a cost of $\lambda^2/2$. Therefore we minimize the cost if we set

$$\hat{\theta}_i = \begin{cases} y_i & \text{if } y_i^2 > \lambda^2 \\ 0 & \text{otherwise} \end{cases} = H_\lambda(y_i).$$

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