Olfactory inference using correlated priors with sister cells

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Neuroscience Interest Group
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Section 1

Overview

Outline

- Olfactory bulb, sister cells

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- Olfactory bulb, sister cells
- Top-down understanding of neural circuits

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- Olfactory inference

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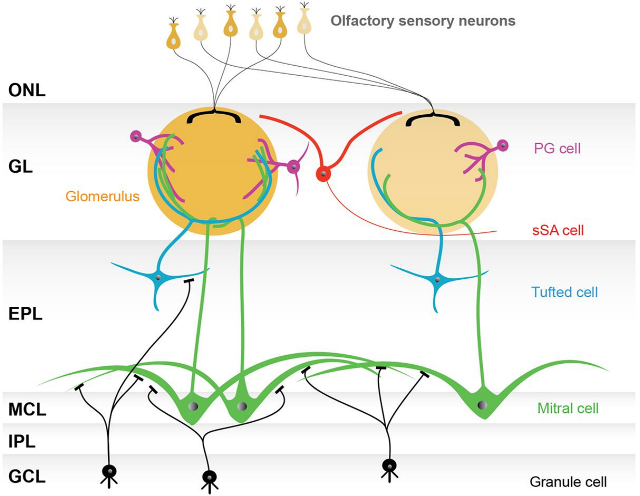
- Preprint at http://arxiv.org/abs/2505.14579

The olfactory bulb



Nagayama et al. 2014

Two approaches to understanding neural circuits

- Bottom-up

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    3. What is the implementation?

The computation: Olfactory inference

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- Computation: Find the odour with highest posterior probability

$$\hat{\mathbf{x}} = \max_{\mathbf{x}} p(\mathbf{x}|\mathbf{r}).$$

The algorithm: Hill climbing

- Our computation:

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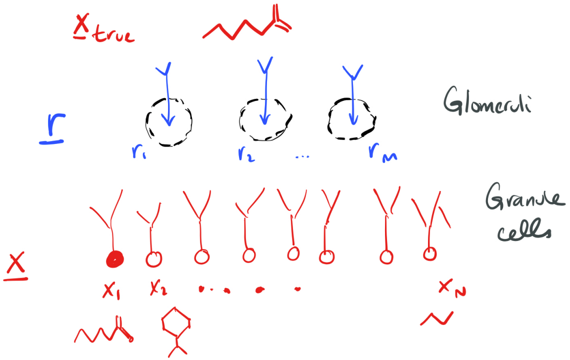
- Two contributions to the dynamics

$$\frac{d\hat{\mathbf{x}}}{dt} \propto \nabla \log p(\hat{\mathbf{x}}) + \nabla \log p(\mathbf{r}|\hat{\mathbf{x}}).$$

Implementation: Representations



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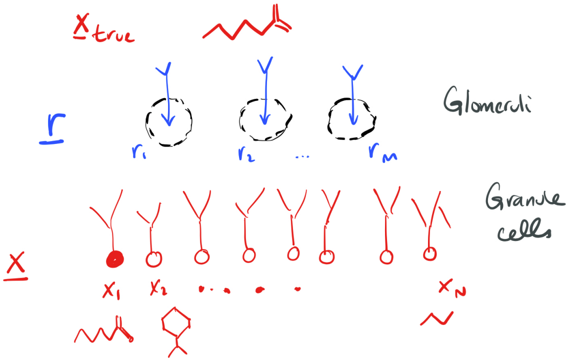

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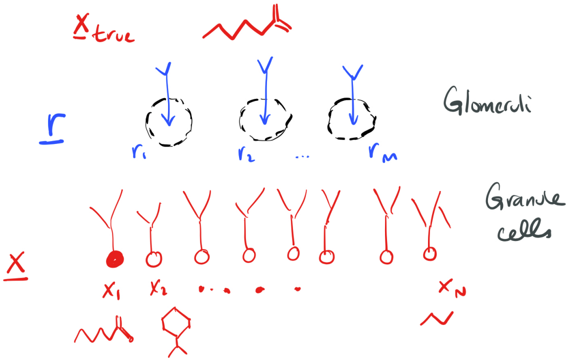

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- Assign one molecular species to one granule cell.

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- Suggests e.g.

$$p(x_i) \propto e^{-ax_i}.$$

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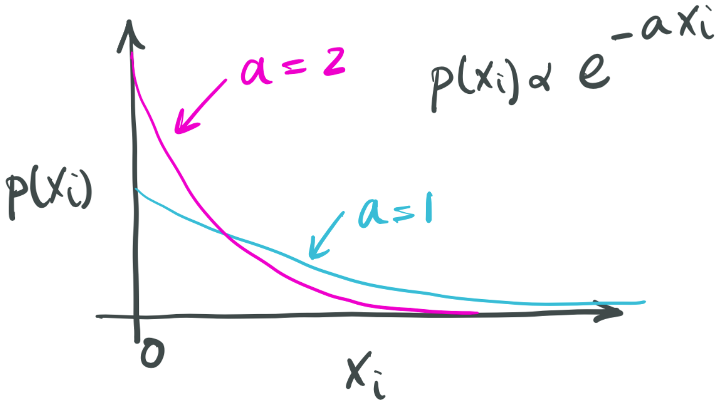

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- Plays nicely with dynamics

$$\frac{dx_i}{dt} \propto \frac{\partial \log p(\mathbf{x})}{\partial x_i} + \cdots = -a + \ldots.$$

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- Easy to incorporate into the prior, e.g.

$$p(\mathbf{x}) \propto e^{-C_{ij}x_i x_j}.$$

- Introduces interactions into the dynamics

$$\frac{dx_i}{dt} \propto \frac{\partial \log p(\mathbf{x})}{\partial x_i} + \cdots = -a - C_{ij}x_j + \ldots.$$

Evidence from receptor activations

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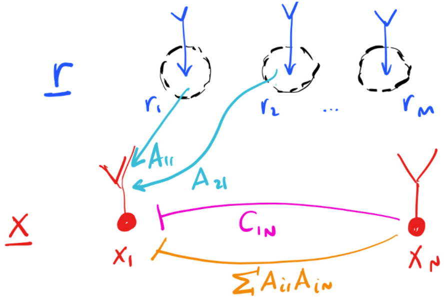
- We sum the evidence from all the receptors

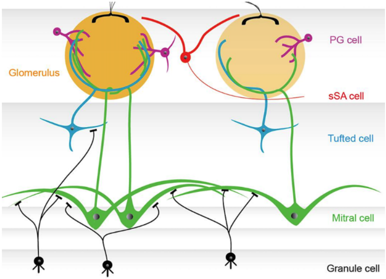
$$\log p(\mathbf{r}|\mathbf{x}) = -\sum_i (r_i - \sum_j A_{ij} x_j)^2.$$

Inference circuit, v. 1

- We have all the pieces we need to specify the dynamics:

$$\frac{dx_i}{dt} \propto \underbrace{-a - \sum_j C_{ij} x_j}_{\partial\text{prior}/\partial x_i} + \underbrace{\sum_k A_{ki} r_k - \sum_j \left[\sum_k A_{ki} A_{kj} \right] x_j}_{\partial\text{evidence}/\partial x_i} .$$





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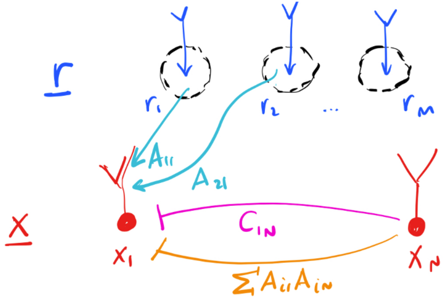
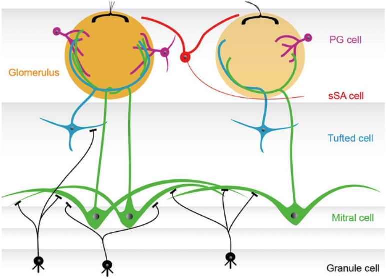
- But it doesn't look like the bulb...





Inference circuit, v. 2

- Drop the cross term in the prior

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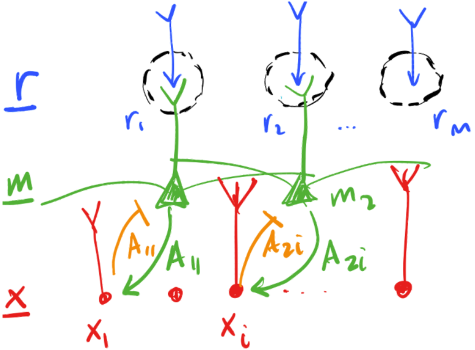
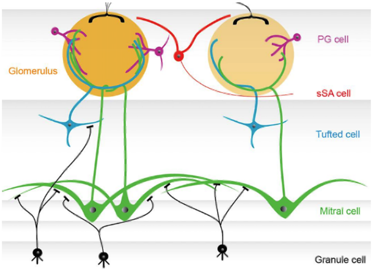
- Granule cells now only interact with mitral cells:

$$\frac{dx_j}{dt} = -a + \sum_k A_{kj} m_k.$$

Mission accomplished?





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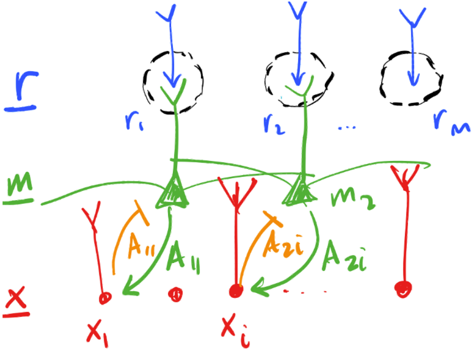

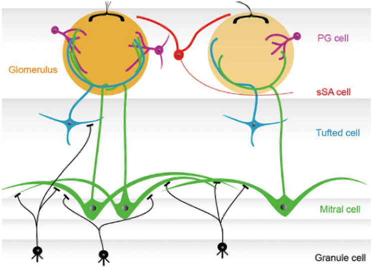

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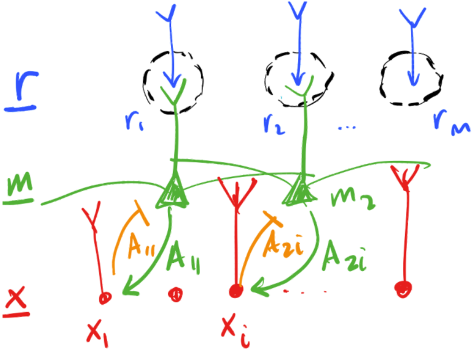

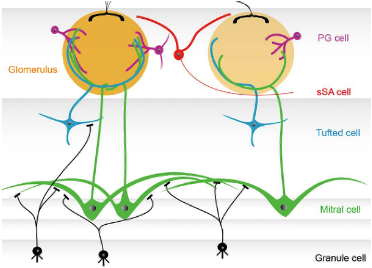

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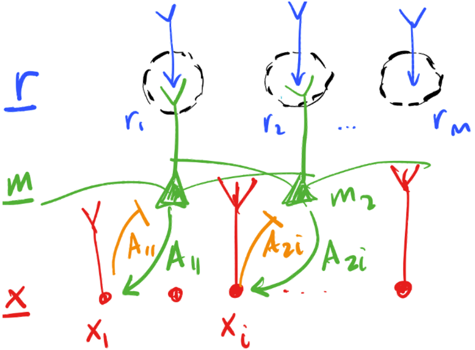

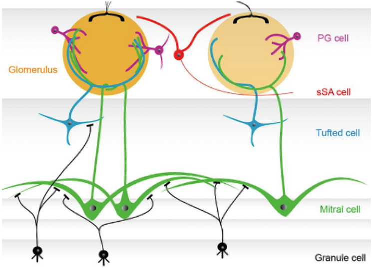

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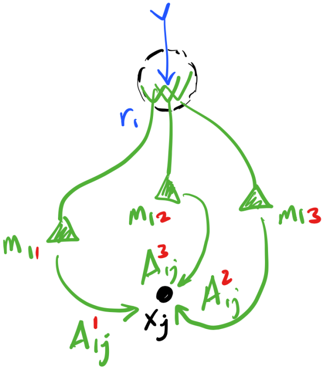
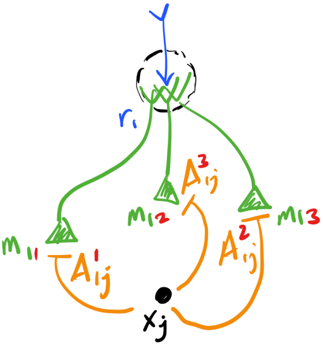
- But: using an independent prior
- No sister cells. . .

To include sister cells, just add 's' to the dynamics

$$\frac{dm_{is}}{dt} = -m_{is} + r_i - \sum_j A^s_{ij} x_j.$$

$$\frac{dx_j}{dt} = -a + \sum_{i,s} A^s_{ij} m_{is}.$$





The objective of the dynamics

- We *derived* the old dynamics to maximize the posterior.

$$L(\mathbf{x}) = \underbrace{-\sum_j a x_j}_{\text{log prior}} \underbrace{-\sum_i (r_i - A_{ij} x_j)^2}_{\text{log evidence}} .$$

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- How to interpret this?

Intrepreting the new objective

- The original objective maximized a log posterior:

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$$L(\mathbf{x}) = -\sum_j a x_j - \sum_{i,s} (r_i - A^s_{ij} x_j)^2 .$$

- ...we express it in the same form as the old

$$L(\mathbf{x}) = \underbrace{-\sum_j a x_j - \sum_{j,k} C_{jk} x_j x_k}_{\text{log prior}} \underbrace{-\sum_i S_i (r_i - \sum_j \overline{A_{ij}} x_j)^2}_{\text{log evidence}} .$$

Mean connectivity encodes affinity

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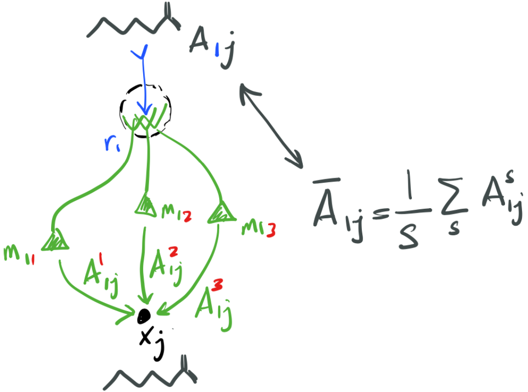

Covariance of connectivity encodes correlated priors

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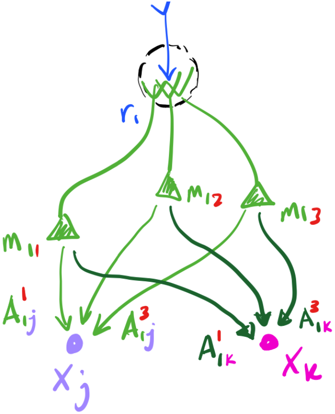


$$C^1_{jk} = \mathrm{Cov}_s(A^s_{1j}, A^s_{1k})$$

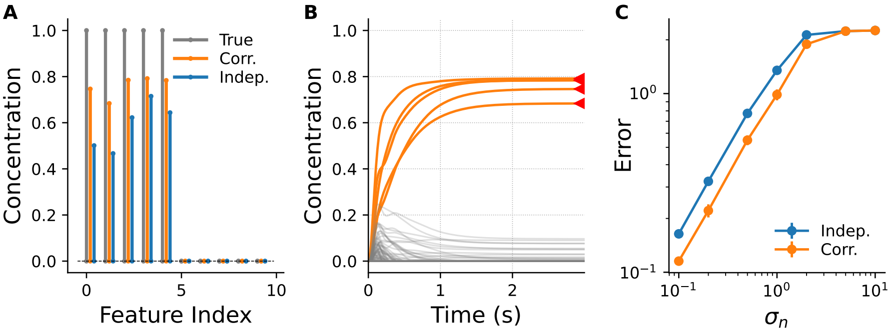
$$C_{jk} = \sum_i S_i C^i_{jk}$$

How many odours can we encode correlations for?

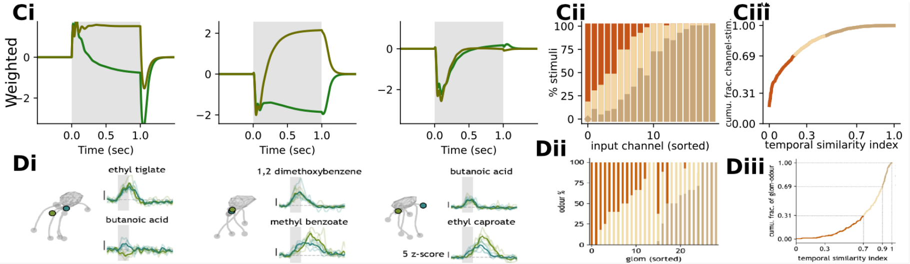
Connectivity solutions exist whenever

$$n \leq \underbrace{S}_{\text{total \# sisters}} - \underbrace{M}_{\text{\# glomeruli}}.$$

Simulations: Using correlated priors helps



Simulations: Comparison to experimental data



Data from Zhang et al. 2025

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- Details: `https://arxiv.org/abs/2505.14579`

Section 2

Thank you Schaefer Lab,
esp. Yuxin and Carles,
and you for your attention!