LORA: LOW-RANK ADAPTATION OF LARGE LANGUAGE MODELS

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Abstract

- As we pre-train larger models, full fine-tuning becomes less feasible
- Low-Rank Adaptation (LoRA)
 - greatly reducing the number of trainable parameters for downstream tasks
- LoRA performs on-par or better than fine-tuning in model quality

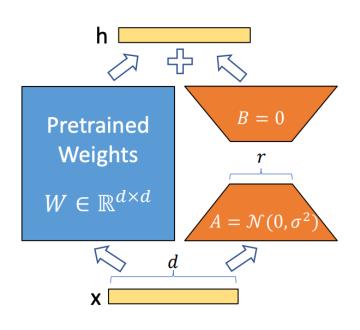
Introduction

- One large-scale, pre-trained LM -> multiple downstream applications
 - Fine-tuning: updates all the parameters of the pre-trained model
 - GPT-2, RoBERTa large -> mere inconvenience…
 - GPT-3 with 175 billion parameters -> critical deployment challenge!

Introduction

- Instead of fine-tuning,
 - Adapting only some parameters
 - Learning external modules
 - -> Greatly boosting the operational efficiency!
- However,
 - Inference latency
 - often fail to match the fine-tuning baselines

Introduction



- Inspiration (prior research)
 - over-parameterized models reside on a low intrinsic dimension
- LoRA
 - hypothesize that the change in weights also has a low "intrinsic dimension"
 - optimizing rank decomposition matrices of the dense layers' change

Problem Statement

Full fine-tuning

$$\max_{\Phi} \sum_{(x,y)\in\mathcal{Z}} \sum_{t=1}^{|y|} \log(P_{\Phi}(y_t|x,y_{< t}))$$

- Initialized to pre-trained weights Φ_0 and updated to $\Phi_0 + \Delta \Phi$
- $|\Phi_0| = |\Delta\Phi|$
- If the pre-trained model is large, storing and deploying many independent instances of fine-tuned models can be challenging

Problem Statement

- More parameter-efficient approach
 - $\Delta \Phi = \Delta \Phi(\Theta)$ is encoded by a much smaller-sized set of parameters Θ
 - The task of finding $\Delta\Phi$ becomes optimizing over Θ

$$\max_{\Theta} \sum_{(x,y)\in\mathcal{Z}} \sum_{t=1}^{|y|} \log(p_{\Phi_0 + \Delta\Phi(\Theta)}(y_t|x, y_{< t}))$$

• Use a low-rank representation to encode $\Delta\Phi$ that is both computeand memory-efficient

Existing solutions?

Adapter layers introduce inference latency

Batch Size	32	16	1		
Sequence Length	512	256	128		
$ \Theta $	0.5M	11 M	11 M		
Fine-Tune/LoRA	1449.4±0.8	338.0 ± 0.6	19.8±2.7		
Adapter ^L	1482.0±1.0 (+2.2%)	354.8±0.5 (+5.0%)	23.9±2.1 (+20.7%)		
Adapter ^H	1492.2±1.0 (+3.0%)	366.3±0.5 (+8.4%)	25.8±2.2 (+30.3%)		

Table 1: Infernece latency of a single forward pass in GPT-2 medium measured in milliseconds, averaged over 100 trials. We use an NVIDIA Quadro RTX8000. " $|\Theta|$ " denotes the number of trainable parameters in adapter layers. Adapter^L and Adapter^H are two variants of adapter tuning, which we describe in Section 5.1. The inference latency introduced by adapter layers can be significant in an online, short-sequence-length scenario. See the full study in Appendix B.

Existing solutions?

- Directly Optimizing the Prompt is Hard
 - Prefix tuning is difficult to optimize
 and performance changes non-monotonically in trainable parameters
 - Reserving a part of the sequence length for adaptation reduces the sequence length available to process a downstream task, which makes tuning the prompt less performant

- Low-Rank Parameterized Update Matrices
 - pre-trained weight matrix $W_0 \in \mathbb{R}^{d \times k}$ is updated to $W_0 + \Delta W = W_0 + BA$, where $B \in \mathbb{R}^{d \times r}, A \in \mathbb{R}^{r \times k}$ and $r \ll \min(d, k)$
 - W_0 is frozen, A and B contain trainable parameters
 - Forward pass:

$$h = W_0 x + \Delta W x = W_0 x + BAx$$

- a random Gaussian initialization for A, and zero for B, so $\Delta W = BA$ is zero
- scale ΔWx by α/r , where α is a constant in r

- Low-Rank Parameterized Update Matrices
 - A Generalization of Full Fine-tuning
 - More general form of fine-tuning: the training of a subset of the pre-trained parameters
 - Setting the LoRA rank r to the rank of the pre-trained weight matrices -> full fine-tuning
 - No Additional Inference Latency
 - compute and store $W = W_0 + BA$ and perform inference as usual
 - recover W_0 by subtracting BA and then adding a different B'A'

- Applying LoRA to Transformer
 - Transformer Architecture
 - Four weight matrices in the self-attention module ($W_q, W_k, W_v, W_o \in \mathbb{R}^{d_{model} \times d_{model}}$)
 - Two in the MLP module
 - In this study, only adapting the attention weights and freeze MLP modules
 - for simplicity and parameter-efficiency
 - leave future work

- Applying LoRA to Transformer
 - Practical Benefits: Reduction in memory and storage usage
 - reduce that VRAM usage by up to 2/3
 - the checkpoint size is reduced by roughly 10,000×
 - 25% speedup during training
 - This allows us to train with significantly fewer GPUs and avoid I/O bottlenecks
 - We can switch between tasks while deployed at a much lower cost by swapping the LoRA weights

Empirical Experiments

RoBERTa base/large, DeBERTa XXL

Model & Method		7007561548810.09m255	CCTA	MDDC	C-I A	ONLI	OOR	DTE	стс р	A
	Parameters	MINLI	331-2	MRPC	CoLA	QNLI	QQP	RTE	STS-B	Avg.
RoB _{base} (FT)*	125.0M	87.6	94.8	90.2	63.6	92.8	91.9	78.7	91.2	86.4
RoB _{base} (BitFit)*	0.1M	84.7	93.7	92.7	62.0	91.8	84.0	81.5	90.8	85.2
$RoB_{base} (Adpt^{D})^*$	0.3M	$87.1_{\pm .0}$	$94.2_{\pm.1}$	$88.5_{\pm 1.1} \\$	$60.8_{\pm.4}$	$93.1_{\pm.1}$	$90.2_{\pm.0}$	$71.5_{\pm 2.7}$	$89.7_{\pm.3}$	84.4
RoB_{base} (Adpt ^D)*	0.9M	$87.3_{\pm.1}$	$94.7_{\pm .3}$	$88.4_{\pm .1}$	$62.6_{\pm .9}$	$93.0_{\pm.2}$	$90.6_{\pm.0}$	$75.9_{\pm 2.2}$	$90.3_{\pm .1}$	85.4
RoB _{base} (LoRA)	0.3M	$87.5_{\pm .3}$	$\textbf{95.1}_{\pm .2}$	$89.7_{\pm.7}$	$63.4_{\pm1.2}$	$\textbf{93.3}_{\pm.3}$	$90.8_{\pm.1}$	$\textbf{86.6}_{\pm.7}$	$\textbf{91.5}_{\pm .2}$	87.2
RoB _{large} (FT)*	355.0M	90.2	96.4	90.9	68.0	94.7	92.2	86.6	92.4	88.9
RoB _{large} (LoRA)	0.8M	90.6 _{±.2}	$96.2 \scriptstyle{\pm .5}$	$\textbf{90.9}_{\pm 1.2}$	$\textbf{68.2}_{\pm 1.9}$	$\textbf{94.9}_{\pm.3}$	$91.6 \scriptstyle{\pm .1}$	$\textbf{87.4}_{\pm 2.5}$	$\textbf{92.6}_{\pm .2}$	89.0
RoB _{large} (Adpt ^P)†	3.0M	90.2 _{±.3}	96.1 _{±.3}	90.2 _{±.7}	68.3 _{±1.0}	94.8 _{±.2}	91.9 _{±.1}	83.8 _{±2.9}	92.1 _{±.7}	88.4
RoB _{large} (Adpt ^P)†	0.8M	90.5 _{±.3}	$96.6_{\pm .2}$	$89.7_{\pm 1.2}$	$67.8_{\pm 2.5}$	$\textbf{94.8}_{\pm.3}$	$91.7 \scriptstyle{\pm .2}$	$80.1_{\pm 2.9}$	$91.9_{\pm.4}$	87.9
RoB _{large} (Adpt ^H)†	6.0M	89.9 _{±.5}	$96.2_{\pm .3}$	$88.7_{\pm 2.9}$	$66.5_{\pm 4.4}$	$94.7_{\pm .2}$	$92.1_{\pm .1}$	$83.4_{\pm 1.1}$	$91.0_{\pm 1.7}$	87.8
RoB _{large} (Adpt ^H)†	0.8M	90.3 _{±.3}	$96.3_{\pm .5}$	$87.7_{\pm 1.7}$	$66.3_{\pm 2.0}$	$94.7_{\pm .2}$	$91.5_{\pm,1}$	$72.9_{\pm 2.9}$	$91.5_{\pm .5}$	86.4
RoB _{large} (LoRA)†	0.8M	$\textbf{90.6}_{\pm .2}$	$96.2_{\pm.5}$	90.2 $_{\pm 1.0}$	$68.2_{\pm 1.9}^{-}$	$\textbf{94.8}_{\pm.3}$	$91.6_{\pm.2}$	$\textbf{85.2}_{\pm 1.1}$	$\textbf{92.3}_{\pm.5}$	88.6
DeB _{XXL} (FT)*	1500.0M	91.8	97.2	92.0	72.0	96.0	92.7	93.9	92.9	91.1
DeB _{XXL} (LoRA)	4.7M	$91.9_{\pm .2}$	$96.9_{\pm.2}$	$\textbf{92.6}_{\pm.6}$	$\textbf{72.4}_{\pm 1.1}$	$\textbf{96.0}_{\pm.1}$	$\textbf{92.9}_{\pm.1}$	$\textbf{94.9}_{\pm.4}$	$\textbf{93.0}_{\pm.2}$	91.3

Empirical Experiments

• GPT-2 medium/large

Model & Method	# Trainable		E2I	E NLG Ch	allenge	
	Parameters	BLEU	NIST	MET	ROUGE-L	CIDEr
GPT-2 M (FT)*	354.92M	68.2	8.62	46.2	71.0	2.47
GPT-2 M (Adapter ^L)*	0.37M	66.3	8.41	45.0	69.8	2.40
GPT-2 M (Adapter ^L)*	11.09M	68.9	8.71	46.1	71.3	2.47
GPT-2 M (Adapter ^H)	11.09M	$67.3_{\pm .6}$	$8.50_{\pm.07}$	$46.0_{\pm.2}$	$70.7_{\pm.2}$	$2.44_{\pm .01}$
GPT-2 M (FT ^{Top2})*	25.19M	68.1	8.59	46.0	70.8	2.41
GPT-2 M (PreLayer)*	0.35M	69.7	8.81	46.1	71.4	2.49
GPT-2 M (LoRA)	0.35M	$\textbf{70.4}_{\pm.1}$	$\pmb{8.85}_{\pm.02}$	$\textbf{46.8}_{\pm .2}$	$\textbf{71.8}_{\pm.1}$	$\pmb{2.53}_{\pm .02}$
GPT-2 L (FT)*	774.03M	68.5	8.78	46.0	69.9	2.45
GPT-2 L (Adapter ^L)	0.88M	$69.1_{\pm .1}$	$8.68_{\pm.03}$	$46.3_{\pm.0}$	$71.4_{\pm .2}$	$\textbf{2.49}_{\pm.0}$
GPT-2 L (Adapter ^L)	23.00M	$68.9_{\pm.3}$	$8.70_{\pm.04}$	$46.1_{\pm.1}$	$71.3_{\pm.2}$	$2.45_{\pm.02}$
GPT-2 L (PreLayer)*	0.77M	70.3	8.85	46.2	71.7	2.47
GPT-2 L (LoRA)	0.77M	$\textbf{70.4}_{\pm.1}$	$\pmb{8.89}_{\pm.02}$	$\textbf{46.8}_{\pm .2}$	$\textbf{72.0}_{\pm.2}$	$2.47_{\pm.02}$

Empirical Experiments

• GPT-3 175B

Model&Method	# Trainable Parameters	WikiSQL Acc. (%)	MNLI-m Acc. (%)	SAMSum R1/R2/RL
GPT-3 (FT)	175,255.8M	73.8	89.5	52.0/28.0/44.5
GPT-3 (BitFit)	14.2M	71.3	91.0	51.3/27.4/43.5
GPT-3 (PreEmbed)	3.2M	63.1	88.6	48.3/24.2/40.5
GPT-3 (PreLayer)	20.2M	70.1	89.5	50.8/27.3/43.5
GPT-3 (Adapter ^H)	7.1M	71.9	89.8	53.0/28.9/44.8
GPT-3 (Adapter ^H)	40.1M	73.2	91.5	53.2/29.0/45.1
GPT-3 (LoRA)	4.7M	73.4	91.7	53.8/29.8/45.9
GPT-3 (LoRA)	37.7M	74.0	91.6	53.4/29.2/45.1

Which weight matrices in Transformer should we apply LoRA to?

	# of Trainable Parameters = 18M						
Weight Type Rank r	$\left egin{array}{c} W_q \ 8 \end{array} ight $	$\frac{W_k}{8}$	$W_v 8$	W_o	W_q, W_k 4	W_q, W_v 4	W_q, W_k, W_v, W_o
WikiSQL (±0.5%) MultiNLI (±0.1%)	A		73.0 91.0		71.4 91.3	73.7 91.3	73.7 91.7

• Adapting both W_a and W_v gives the best performance overall

What is the optimal rank r for LoRA?

	Weight Type	r = 1	r = 2	r = 4	r = 8	r = 64
Wilder (10 Fot)	$ W_q $	68.8	69.6	70.5	70.4	70.0
WikiSQL($\pm 0.5\%$)	W_q, W_v	73.4	73.3	73.7	73.8	73.5
	W_q, W_k, W_v, W_o	74.1	73.7	74.0	74.0	73.9
	$ W_q $	90.7	90.9	91.1	90.7	90.7
MultiNLI (±0.1%)	W_q, W_v	91.3	91.4	91.3	91.6	91.4
	W_q, W_k, W_v, W_o	91.2	91.7	91.7	91.5	91.4

- LoRA already performs competitively with a very small r
- This suggests the update matrix ΔW could have a very small "intrinsic rank"
 - do not expect a small r to work for every task or dataset

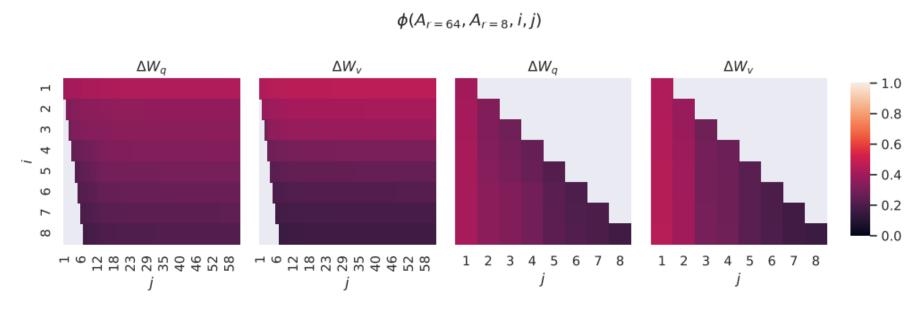
- What is the optimal rank r for LoRA?
 - Subspace similarity between different r
 - $A_{r=8}$, $A_{r=64}$: learned adaptation matrices with rank r=8,64
 - $U_{A_r=8}$, $U_{A_r=64}$: the right-singular unitary matrices by singular value decomposition

$$\phi(A_{r=8}, A_{r=64}, i, j) = \frac{||U_{A_{r=8}}^{i \top} U_{A_{r=64}}^{j}||_{F}^{2}}{\min(i, j)} \in [0, 1]$$

- Grassmann distance:

How much of the subspace spanned by the top i singular vectors in $U_{A_r=8}$ is contained in the subspace by the top j singular vectors of $U_{A_r=64}$

What is the optimal rank r for LoRA?

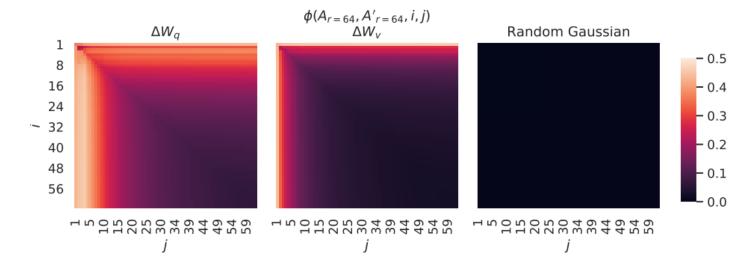


• Directions corresponding to the top singular vector overlap between $U_{A_r=8}$ and $U_{A_r=64}$, while others do not

- What is the optimal rank r for LoRA?
 - the top singular-vector directions of $U_{A_r=8}$ and $U_{A_r=64}$ are the most useful
 - ΔW_v and ΔW_q of $A_{r=8}$ and ΔW_v and ΔW_q of $A_{r=64}$ share a subspace of dimension 1 with normalized similarity > 0.5
 - -> reason why r = 1 performs quite well
 - the adaptation matrix can indeed have a very low rank

2024-Winter Study

- What is the optimal rank r for LoRA?
 - Subspace similarity between different random seeds



- ΔW_q appears to have a higher "intrinsic rank" than ΔW_v

- How does the adaptation matrix ΔW compare to W?
 - project W onto the r-dimensional subspace of ΔW by computing U^TWV^T
 - compare the Frobenius norm

		r=4	:	$egin{array}{ccc} r=64 \ \Delta W_q & W_q & { m Random} \end{array}$			
	ΔW_q	W_q	Random	$\mid \Delta W_q$	W_q	Random	
$ U^{\top}W_qV^{\top} _F =$	0.32	21.67	0.02	1.90	37.71	0.33	
$ W_q _F = 61.95$	<u> </u>	$ W_q _F$ =	= 6.91		$ W_q _F$ =	= 3.57	

• How does the adaptation matrix ΔW compare to W?

		r=4	:	$egin{array}{cccc} r=64 \ \Delta W_q & W_q & { m Random} \end{array}$			
	ΔW_q	W_q	Random	ΔW_q	W_q	Random	
$ U^{\top}W_qV^{\top} _F =$	0.32	21.67	0.02	1.90	37.71	0.33	
$ W_q _F = 61.95$		$ W_q _F =$	= 6.91		$ W_q _F$ =	= 3.57	

- ΔW has a stronger correlation with W compared to a random matrix
 - $\rightarrow \Delta W$ amplifies some features that are already in W
- ΔW only amplifies directions that are not emphasized in W
- the amplification factor is rather huge: $21.5 \approx 6.91/0.32$ for r = 4 (evidence for low-rank)

• How does the adaptation matrix ΔW compare to W?

		r=4	:	$egin{array}{cccc} r=64 \ \Delta W_q & W_q & { m Random} \end{array}$			
	ΔW_q	W_q	Random	ΔW_q	W_q	Random	
$ U^{\top}W_qV^{\top} _F =$	0.32	21.67	0.02	1.90	37.71	0.33	
$ W_q _F = 61.95$		$ W_q _F =$	= 6.91		$ W_q _F$ =	= 3.57	

 This suggests that the low-rank adaptation matrix potentially amplifies the important features for specific downstream tasks that were learned

Conclusion

- LoRA, an efficient adaptation strategy
 - eliminate inference latency
 - not reduces input sequence length while retaining high model quality
 - quick task-switching
 - generally applicable

Further Research

- QLoRA: Efficient Finetuning of Quantized LLMs
 - an efficient fine-tuning approach that reduces memory usage

