Cost-time trade-off (using LP)

Project management

Rob Broekmeulen

1

Optimal Minimal Cost Due Date Crashing

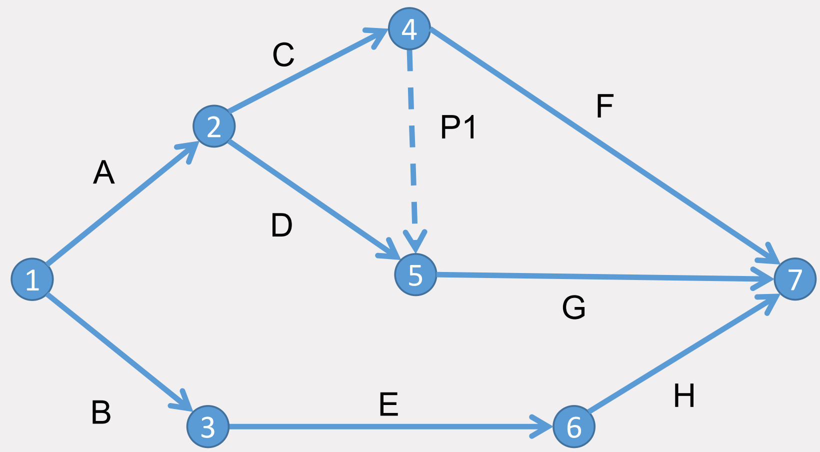
CPM heuristic does not guarantee optimal solution

Optimal solution requires linear programming

LP formulation is based on Activity on Arc (AOA) network

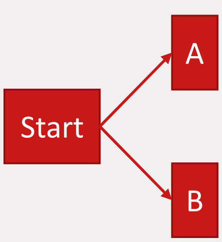
2

Activity network: AOA format

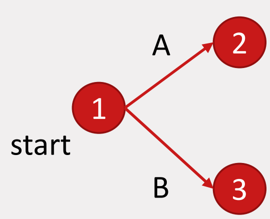


Sample of network construction

AON

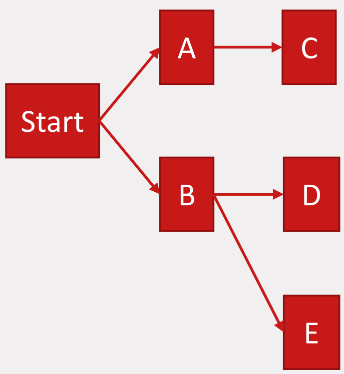


AOA

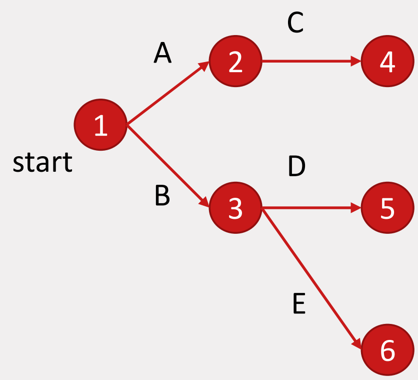


Sample of network construction

AON



AOA

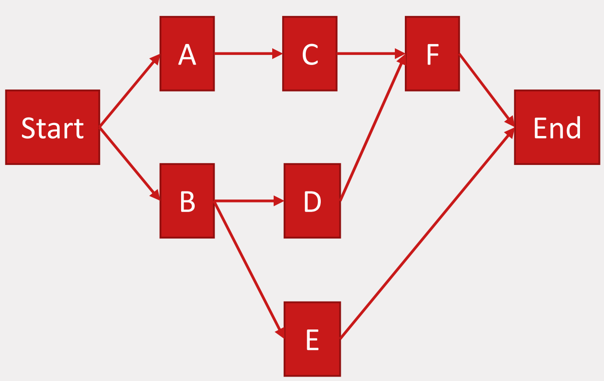


5 PM - Cost-time trade-off LP

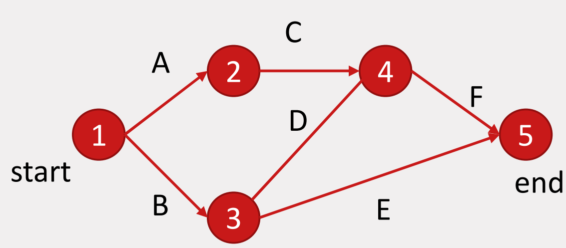
5

Sample of network construction

AON



AOA

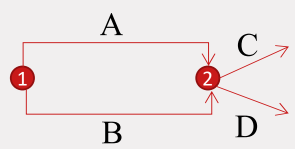


6 PM - Cost-time trade-off LP

6

Networking concurrent activities in AOA

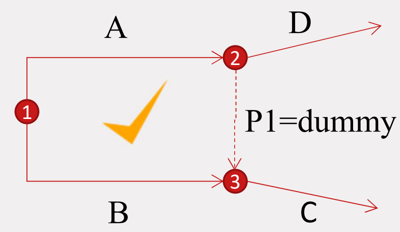
Act	Prev
A	-
B	-
C	A,B
D	A



Precedence:
- C after A <u>and</u> B
- D after A <u>only</u>

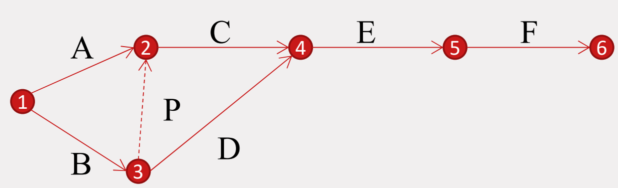
Combination start and end node must be unique (for LP)

A = (1,2)
B = (1,2)



A = (1,2)
B = (1,3)
P1 = (2,3)

A simple AOA network



Act	Prev
A	-
B	-
C	A,B
D	B
E	C,D
F	E

Difficulties in drawing an AOA

AOA: many representations possible, depending on the number of dummies

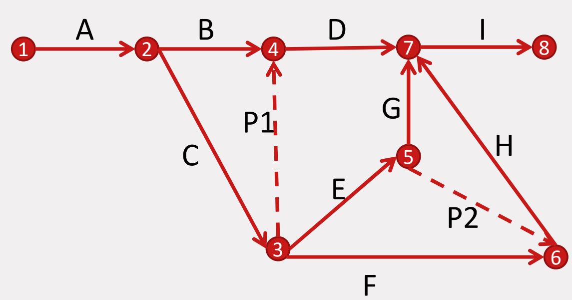
With AON: always just one representation

Objective: use as few dummies as possible

Trick: look at the overlap in the clusters of precedence constraints to determine the number of dummies needed

Exercise AOA network (Nahmias Ex 10.4)

Act	Prev
A	-
B	A
C	A
D	B, C
E	C
F	C
G	E
H	E, F
I	D, G, H



Notations in AOA

Activity A is an arc between nodes i and j with duration T_{ij}

The earliest start time from node i is x_i and from node j is x_j

Basic Linear Programming formulation

Given a network *G(V,A)*, find values for the start times x_i at each node *i* such that

$$\sum_{i=1}^{m} x_i$$

Earliest Start Time Problem

Is minimized, subject to

- $\forall (\boldsymbol{i}, \boldsymbol{j}) \in \boldsymbol{A}$: $\quad \boldsymbol{x_i + T_{ij} \leq x_j}$
- $\forall \boldsymbol{i} \in \boldsymbol{V}$: $\quad \boldsymbol{x_i \geq 0}$

$\Rightarrow x_i - x_j \leq -T_{ij}$
$\Rightarrow -x_i + x_j \geq T_{ij}$

Basic Linear Programming formulation

Given a network G(V,A), find values for the start times x_i at each node i such that

$$\sum_{i=1}^{m} x_i$$

Canonical notation (constant RHS = Right Hand Side)

Is minimized, subject to

$$\forall(\boldsymbol{i},\boldsymbol{j}) \in \boldsymbol{A}: \quad \boldsymbol{x_j} - \boldsymbol{x_i} \geq \boldsymbol{T_{ij}}$$
$$\forall \boldsymbol{i} \in \boldsymbol{V}: \quad \boldsymbol{x_i} \geq \boldsymbol{0}$$

13 PM - Cost-time trade-off LP

13

Basic Linear Programming formulation

Given a network G(V,A), find values for the start times x_i at each node i such that

$$m \cdot x_m - \sum_{i=1}^{m-1} x_i$$

Latest Start Time Problem

Is minimized, subject to

$$\forall(\boldsymbol{i},\boldsymbol{j}) \in \boldsymbol{A}: \quad \boldsymbol{x_j} - \boldsymbol{x_i} \geq \boldsymbol{T_{ij}}$$
$$\forall \boldsymbol{i} \in \boldsymbol{V}: \quad \boldsymbol{x_i} \geq \boldsymbol{0}$$

14 PM - Cost-time trade-off LP

14

Additional parameters for crashing

Due date $\boldsymbol{DD}$

Indirect costs $\boldsymbol{C^{Indirect}}$ on total project duration ($= \boldsymbol{x_m}$)

For each activity $(\boldsymbol{i}, \boldsymbol{j}) \in \boldsymbol{A}$:

- Nominal duration $\boldsymbol{N_{ij}}$
- Full crash duration $\boldsymbol{M_{ij}}$
- Cost/time slope $\boldsymbol{C_{ij}^{Direct}} = \frac{\boldsymbol{Crash\ cost - Normal\ cost}}{\boldsymbol{N_{ij} - M_{ij}}}$
- Upper bound on time reduction: $\boldsymbol{U_{ij}} = \boldsymbol{N_{ij}} - \boldsymbol{M_{ij}}$

15

15

LP formulation due date crashing

Given a network G(V,A), find values for the start times x_i at each node i and crash activity times t_{ij} such that

$$C^{Indirect} x_m + \sum_{(i,j) \in A}^{m} C_{ij}^{Direct} t_{ij}$$

Is minimized, subject to

$$\begin{aligned} \forall (i,j) \in A: \quad & x_j - x_i + t_{ij} \geq N_{ij} \\ & x_m \leq DD \\ \forall (i,j) \in A: \quad & 0 \leq t_{ij} \leq U_{ij} \\ \forall i \in V: \quad & x_i \geq 0 \end{aligned}$$

This formulation differs from Nahmias & Olsen (2015)

16 PM - Cost-time trade-off LP

16

LP formulation due date crashing + delay

Given a network G(V,A), find values for the start times x_i at each node i and crash activity times t_{ij} such that

$$C^{Indirect} x_m + \sum_{(i,j)\in A}^{m} C_{ij}^{Direct} t_{ij} + Fd$$

Allowing a delay *d* with fine *F*

Is minimized, subject to

$$\forall (i,j) \in A: \quad x_j - x_i + t_{ij} \geq N_{ij}$$
$$x_m - d \leq DD$$
$$\forall (i,j) \in A: \quad 0 \leq t_{ij} \leq U_{ij}$$
$$\forall i \in V: \quad x_i \geq 0$$

Summary of topics covered

Convert a WBS into a AON or AOA network

Calculate the critical path using CPM

Resource loading, leveling and scheduling

Cost-time trade-off for crashing using heuristic

LP formulation for optimal due date crashing