

PARAMETERS

$n$  : number of rows and columns

$t_{ij} : \begin{cases} 1, & \text{if there is a tree at row } i, \text{ column } j \\ 0, & \text{otherwise} \end{cases}$

$i \in \{1, 2, \dots, n\}$

$j \in \{1, 2, \dots, n\}$

$R_i$  : Number of tests that must be placed at row  $i$   
 $i \in \{1, 2, \dots, n\}$

$C_j$  : Number of tests that must be placed at column  $j$   
 $j \in \{1, 2, \dots, n\}$

DECISION VARIABLES

$x_{ij} : \begin{cases} 1, & \text{if there is a test at row } i, \text{ column } j \\ 0, & \text{otherwise} \end{cases}$

$i \in \{1, 2, \dots, n\}$

$j \in \{1, 2, \dots, n\}$

OBJECTIVE FUNCTION

①  $\max \quad 1$

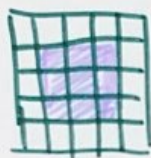
# CONSTRAINTS

2

$$(1) \quad t_{ij} + x_{ij} \leq 1 \quad i, j \in \{1, 2, \dots, n\}$$

$$(2) \quad \sum_{i=1}^n \sum_{j=1}^n x_{ij} = \sum_{i=1}^n \sum_{j=1}^n t_{ij}$$

$$(3) \quad x_{(i-1)j} + x_{i(j+1)} + x_{(i+1)j} + x_{i(j-1)} \geq t_{ij}$$



$$i, j \in \{2, 3, \dots, n-1\}$$

$$(4) \quad x_{12} + x_{21} \geq t_{11} \quad (5) \quad x_{1(n-1)} + x_{2n} \geq t_{1n}$$

$$(6) \quad x_{(n-1)1} + x_{n2} \geq t_{n1} \quad (7) \quad x_{n(n-1)} + x_{(n-1)n} \geq t_{nn}$$



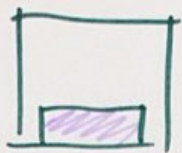
$$(8) \quad x_{1(j-1)} + x_{2j} + x_{1(j+1)} \geq t_{1j} \quad j \in \{2, \dots, n-1\}$$



$$(9) \quad x_{(i-1)n} + x_{i(n-1)} + x_{(i+1)n} \geq t_{in} \quad i \in \{2, \dots, n-1\}$$



$$(10) \quad x_{n(j-1)} + x_{(n-1)j} + x_{n(j+1)} \geq t_{nj} \\ j \in \{2, \dots, n-1\}$$



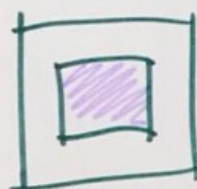
$$(11) \quad x_{(i-1)1} + x_{i2} + x_{(i+1)1} \geq t_{i1} \\ i \in \{2, \dots, n-1\}$$



$$(12) \quad \sum_{j=1}^n x_{ij} = R_i \quad i \in \{1, 2, \dots, n\}$$

$$(13) \quad \sum_{i=1}^n x_{ij} = C_j \quad j \in \{1, 2, \dots, n\}$$

$$(14) \quad x_{(i-1)(j-1)} + x_{(i-1)j} + x_{(i-1)(j+1)} + x_{i(j+1)} + x_{(i+1)(j+1)} \\ + x_{(i+1)j} + x_{(i+1)(j-1)} + x_{i(j-1)} \leq 8(1 - x_{ij}) \\ i, j \in \{2, 3, \dots, n-1\}$$



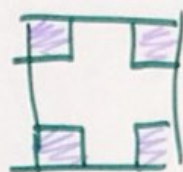


$$(15) \quad x_{12} + x_{21} + x_{22} \leq 3(1 - x_{11})$$

$$(16) \quad x_{1(n-1)} + x_{2(n-1)} + x_{2n} \leq 3(1 - x_{1n})$$

$$(17) \quad x_{(n-1)1} + x_{(n-1)2} + x_{n2} \leq 3(1 - x_{n1})$$

$$(18) \quad x_{n(n-1)} + x_{(n-1)(n-1)} + x_{(n-1)n} \leq 3(1 - x_{nn})$$



$$(19) \quad x_{1(j-1)} + x_{2(j-1)} + x_{2j} + x_{2(j+1)} + x_{1(j+1)} \leq 5(1 - x_{1j})$$

$$j \in \{2, \dots, n-1\}$$



$$(20) \quad x_{(i-1)n} + x_{in} + x_{(i-1)(n-1)} + x_{(i+1)(n-1)} + x_{(i+1)n} \leq 5(1 - x_{in})$$

$$i \in \{2, \dots, n-1\}$$



$$(21) \quad x_{n(j-1)} + x_{(n-1)(j-1)} + x_{(n-1)j} + x_{(n-1)(j+1)} + x_{n(j+1)} \leq 5(1 - x_{nj})$$

$$j \in \{2, \dots, n-1\}$$



(22)

$$X_{(i-1)1} + X_{(i-1)2} + X_{i2} + X_{(i+1)2} + X_{(i+1)1} \leq 5(1 - X_{i1}) \quad \boxed{5}$$

$$i \in \{2, 3, \dots, n-1\}$$



(23)

$$X_{ij} \in \{0, 1\} \quad i, j \in \{1, 2, \dots, n\}$$

### EXPLANATIONS

① Objective function can be anything

① In each grid, there can be at most one tree or one tent

② total number of tents and trees must be equal.  
this constraint cannot be used. The constraints 12 and 13 are adequate.

③ Main reason to have such a constraint is to ~~ensure~~ place a tent near to a tree. We are interested in inner grids.

④ ⑤ ⑥ ⑦ Same procedure with the constraint 3. However, in this case, we are interested with the corner grids.

⑧ ⑨ ⑩ ⑪ Same procedure with constraint 6  
3. However, in this case, we are interested  
in the boundary grids.

⑫ ⑬ we try to have given number of total texts  
for each row and column

⑭ If we have text in a grid, we try to not have  
a near text to it. We are interested in the inner  
grids

⑮ ⑯ ⑰ ⑱ Same procedure with the constraint 14.  
However, in this case, we are interested  
in the corner grids

⑲ ⑳ ㉑ ㉒ Same procedure with constraint ~~14~~ 14.  
However, in this case, we're interested  
in the boundary grids

㉓ Binary variables

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