European Journal of Operational Research 218 (2012) 775–788

Contents lists available at SciVerse ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Innovative Applications of O.R.

Modeling wildfire propagation with Delaunay triangulation and shortest path algorithms

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ARTICLE INFO

Article history:
Received 8 February 2010
Accepted 22 November 2011
Available online 31 December 2011

Keywords:
Natural resources
Delaunay triangulation
Shortest path algorithms
Minimum travel time
Wildfire modeling

ABSTRACT

In this paper, a methodology for modeling surface wildfire propagation through a complex landscape is presented. The methodology utilizes a Delaunay triangulation to represent surface fire spread within the landscape. A procedure to construct the graph and estimate the rate of spread along the edges of a network is discussed. After the Delaunay data structure is constructed, a two pass shortest path algorithm is incorporated to estimate the minimum travel time paths and fire arrival times. Experimental results are also included.

1. Introduction

During the years 2000–2004, the National Interagency Fire Center (NIFC) reported in excess of 459 thousand fires on 9.6 million hectares of land. The cost of fire extinction was estimated to be around 6.2 billion US dollars (USD). In other words, an average of more than three million US dollars were spent daily on fire extinction during 2000–2004; 108 firefighters lost their lives during these operations. Federal agencies reported 327,822 fires during the period 2005–2008. Wildfires affected 13.4 million hectares (National Interagency Fire Center, 2007). In addition to loss of life and financial drain, wildfires can cause significant damage to infrastructure located in wildland-urban interfaces. For instance, in 2002, the Rodeo–Chedeski fire damaged $329 million worth of property, and its suppression cost almost $22 million dollars. Wildfire struck the South East Oklahoma and Central Texas area on December 27, 2005, destroying 125 houses and taking the lives of five people (National Interagency Fire Center, 2006). Wildfire is a very tough phenomenon to fight, as its behavior depends on weather and wind conditions, fuel and terrain characteristics. To respond effectively to a wildfire threat, effective analytical models of fire behavior/propagation are required to assess potential risk to human lives, property, infrastructure and facilities in a wildland-urban interface. Such models are necessary also to provide realistic and timely prediction of spatial and temporal expansion of a fire front, its intensity and rate of spread.

1.1. Objectives of research

The purpose of this paper is to design a methodology to model surface fire propagation over a complex heterogeneous landscape. This problem is an intricate one as the fire propagation velocity and direction depends on diverse dynamic factors, such as weather conditions, wind speed, fuel (combustible vegetation and debris) and static factors, including terrain, fuel distribution and types. Accurate modeling of fire spread as a continuous phenomenon has to take these numerous factors into consideration, which may require substantial computations and time to perform the analysis. We develop a method for modeling fire propagation using a discrete Delaunay (also known as a triangulated irregular) network that is refined by a two-pass shortest path algorithm. The suggested methodology is tailored for the fast evaluation of minimum wildfire travel time from ignition sources (points) to specific points of interest or destination points, such as human settlements and infrastructure. Therefore we contribute to the areas of emergency management and wildfire modeling by proposing and evaluating this Delaunay graph-based approach, which is computationally effective. After this construction, fire perimeter iso-lines can be interpolated based on time-arrivals at specific landscape points. We illustrate the suggested approach with a case study and experimental results.

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doi:10.1016/j.ejor.2011.11.031

1.2. Outline of paper

Section 2 of the paper describes the overall problem and gives an historic overview and taxonomy/classification of fire growth models. Section 3 outlines the framework for the methodology. Section 4 presents a solution methodology and algorithm. A case study and experimental results are presented in Section 6. Section 7 concludes the paper.

2. Problem formulation

2.1. Background

Wildfire, spreading through a landscape, is a very complex phenomenon. Though wildfire is an important element of ecosystems, it can impose significant risk to people and property within wildlife-urban interfaces (Cova et al., 2005). Fire growth models, which estimate spatial and temporal dynamics of fire spread, may aid in assessing risks which wildfires impose on communities and also in defining suitable actions to protect the well-being of people. Modelling wildfire propagation can also assist evacuation planning, as well as in locating proper evacuation/egress routes.

Fire behavior depends on the following key factors: fuel, topography and weather conditions. These three factors constitute *the fire environment* (Countryman, 1972). Studies by Albini, Rothermel (Albini, 1976; Rothermel, 1972) and others postulate that fire behavior can be considered constant within a homogeneous fire environment, where there are no changes in physical and chemical content of fuel, moisture content and slope. Therefore, fire behavior characteristics can be viewed as a function of fire environment. Fig. 1 illustrates the last statement: the fire spread velocity changes at points A, B, C and D due to alteration of one or several factors/parameters. The decrease in the fire propagation speed at point A is caused by changes in the weather conditions. At point B, the fire starts moving faster due to an additional draft, which is generated by the slope. The orientation of the area toward the North (so called aspect) may be another contributing factor, as it relates to the amount of solar radiation to which fuel has been exposed prior to the fire. To simplify the analysis, the effective wind speed and direction may be used to account for terrain changes. It refers to adjusted wind speed and direction which will reproduce the same fire behavior, but on the flat area.

To compute the dynamics of fire perimeter extension, such fire growth models utilize two components (specific to a particular model): a fire perimeter growth construction method and a fire spread model. The first component defines a procedure for locating the fire perimeter based on the spatial dependency of the previous fire perimeter location. The second component, a fire spread model, evaluates the maximum rate of fire spread and its direction, fire intensity and other parameters of fire front propagating in one direction through a uniform fire environment. McArthur (1966) suggests a model of fire spread which was derived from several hundred cases of prescribed fires. In McArthur's model, the rate of fire spread R depends on the wind speed, fuel properties, relative humidity and air temperature (Perry, 1998; Noble et al., 1980). Rothermel (1972) suggested a method to compute the head rate of fire spread R for an area with constant fuel type, topography and constant weather conditions. Rothermel's fire spread model can be classified as semi-physical, as it is based on energy conservation principles with parameters calibrated from empirical data. The fuels comprise a fuel bed, containing fuel particles with specific physical and chemical properties. Rothermel's method considers fire as a sequence of ignitions among fuel particles in the fuel layer. A fuel particle releases heat during combustion, part of which is transferred to the adjacent particles. If the heat energy is enough to ignite the neighboring particle, then fire spread continues. Eq. (1) summarizes Rothermel's model as

$$R = \frac{I_R \xi (1 + \phi_w + \phi_s)}{\rho_b \varepsilon Q_{ig}} \quad (1)$$

where:

I_R - reaction intensity
ξ - no wind propagation flux ratio
ϕ_w- additional propagating flux produced by wind
ϕ_s- additional propagating flux produced by slope
ρ_b - bulk density (fuel particle density)
ε - effective heating number
Q_{ig} - the heat of preignition (the energy per unit mass required for ignition)

Another fire spread model, also based on empirical observations and which is a part of the Canadian Fire Behavior Prediction system was developed. The statistical relation between primary fire characteristics, such as rate of spread, head fire intensity, flame length and fuel, weather conditions, moisture content and topography were derived based on the historical data of 400 forest fires (FDP, 1992).

Fire spread models use fuel models as one of the input parameters. The fuel model is a standard description of fuel properties, including, among others, density of materials, depth, particle size, and surface to volume ratio. Anderson (1982) explored and documented a set of 13 fuel models which map vegetation types to standard fuel models used in mathematical fire spread models. Scott and Burgan (2005) extended the list of fuel models to 40 vegetation types.

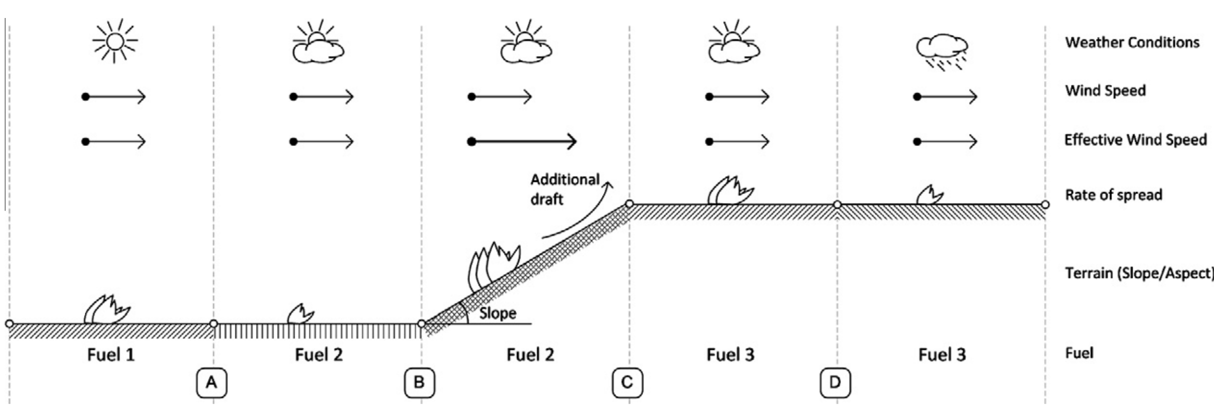


Fig. 1. Effect of fire environment on fire propagation.

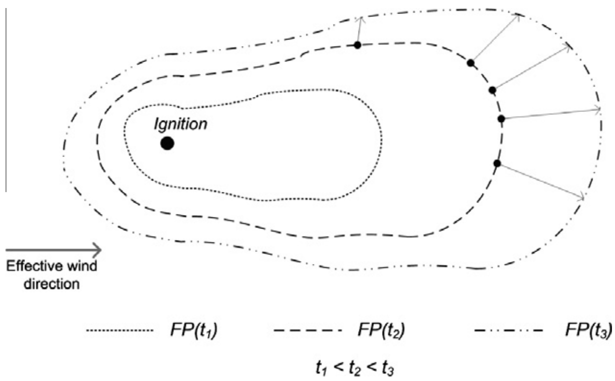


Fig. 2. Fire perimeter expansion $FP(t)$.

Fire growth models analyze fire perimeter shape after ignition. Fons (1946) noted that the tracks of many historic fires can be approximated by an elliptical shape. Van Wagner (1969) suggested that a fire shape can be explained with an elliptical equation, where an ignition point is located at a focus of the ellipse. Coupling this with a fire spread model, an elliptical fire model is able to compute fire perimeters for intervals of time after ignition for an area with constant fire environment. A fire spread model evaluates the maximum rate of spread and its direction, which overlays with the major axis of an ellipse. Anderson (1982) suggested a procedure of evaluating ellipse parameters for particular fuel environments. Catchpole et al. (1982) extended the traditional elliptical fire model to estimate rate of spread in any direction β, if maximum velocity of fire spread is known. Computer applications, *BEHAVE* and the succeeding *BehavePlus* utilize Rothermel's fire spread model, an elliptical fire model, and standard fuel models as an input (Burgan and Rothermel, 1984).

In general, the traditional fire spread models assume that fire propagates within a homogeneous fire environment and thus, these traditional models cannot be applied directly for modeling large scale wildfires propagating through heterogeneous terrain and areas with deviations in fuel, topology and weather conditions. Therefore, two essential steps in computing the spatial and temporal extension of a fire perimeter *FP* are: *i)* to evaluate the rate of spread and direction of the current fire perimeter $FP(t_1)$ at time t_1 and *ii)* to construct the new fire perimeter $FP(t_2)$ for time $t_2 > t_1$ (Fig. 2). Step *ii)* utilizes deterministic or stochastic techniques (Perry, 1998; Pastor et al., 2003). Fire perimeter construction techniques can be divided into *discrete* and *continuous*. In the discrete representation, a landscape is viewed as set of adjacent elements with a regular or irregular structure, for example, a grid or lattice. Therefore, the fire perimeter line is composed of adjacent cells with the same fire arrival time value. Fire spread is considered as an event shift from ignited cells to neighboring cells in accordance with some pre-defined rules. Cellular automata (*CA*) and site percolation are techniques used on grids (Clarke and Olsen, 1996; Vasconcelos and Guertin, 1992). If a landscape is presented as a lattice, then bond percolation can be used for modeling fire propagation. The second group of techniques are based on Huygens' wave propagation principle (Wallace, 1993; Richards and Bryce, 1995; Richards, 1999; Coleman and Sullivan, 1995; Tymstra et al., 2009). A fire perimeter *FP(t)* is treated as a continuous line. A set of points is located on the fire perimeter line, after that each point is considered as an independent ignition source. An expanded fire perimeter line encompasses areas affected by fire growth from each of ignition points. To satisfy the model's assumption about a homogeneous area, the complex landscape is usually tessellated into a set of adjacent cells of regular or irregular shape with a constant fire environment. In this sense, fire spread and growth models work over a discrete representation of the landscape. A particular combination of a fire spread model and a fire perimeter construction technique defines the essential features of the fire growth model (Fig. 3). The majority of calculation systems use simulation to compute the fire perimeter expansion. Kourtz et al. (1977) suggested using shortest path algorithms to evaluate fire arrival times at grid cells. *FlamMap* (Finney, 2006), a counterpart of *FARSITE*, also developed by Finney (2002), takes a regular grid (where fuel model, slope, aspect and weather conditions are

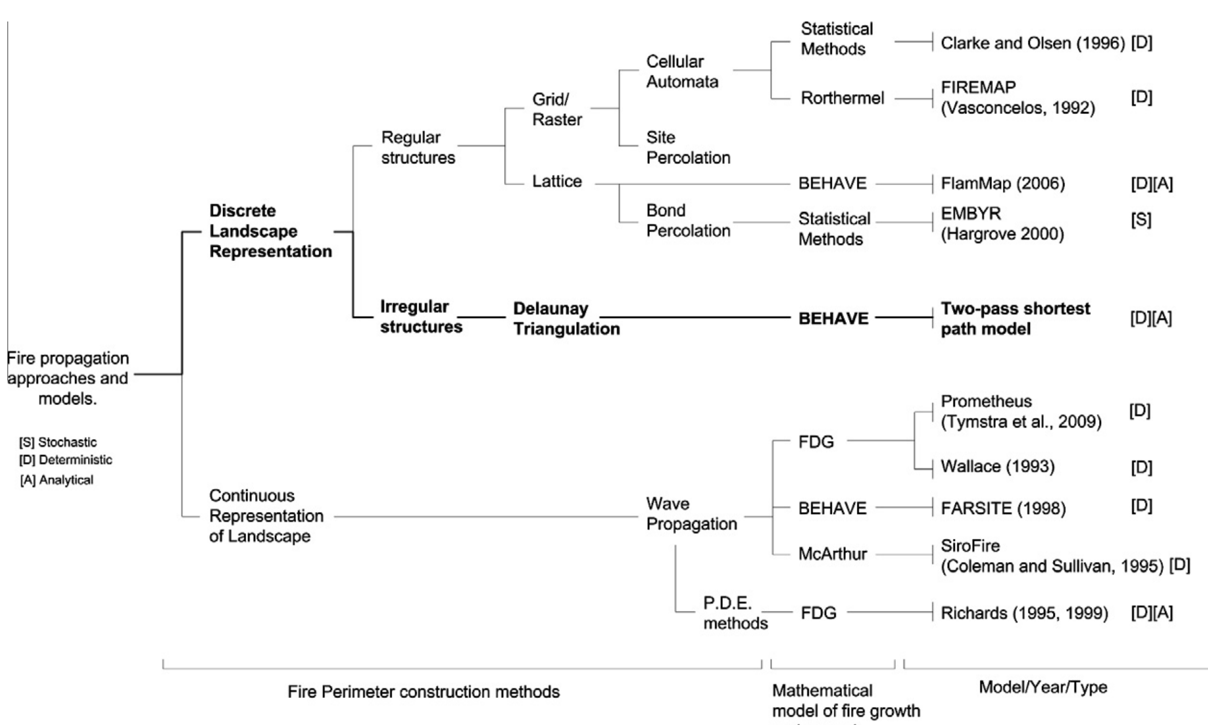


Fig. 3. Classification scheme of fire growth models.

specified for each cell) and computes the rate of fire spread for each cell using a Rothermel-based module. Cova et al. (2005) extended this approach, representing a grid as a lattice network, where rates of fire spread on arcs computed with *FlamMap*, and used a Dijkstra shortest path algorithm to compute the minimum propagation time from an ignition point to points of interest. An approach developed by Richards (1995) and Richards (1999) solves systems of partial differential equations to construct a fire perimeter and can be viewed as a continuous fire perimeter construction technique.

Most models operate under assumptions that fuel properties are constant for some area and weather conditions are stable for the study area and for the period of modeling (though *FARSITE* allows one to change the weather conditions during modeling). *EMBYR* uses expert-defined probabilities of fire transition from an ignition cell to adjacent cells (Hargrove et al., 2000). Catchpole et al. (1989) considered fire propagation as a Continuous Time Markov Chain, but the model implementation was limited to a one-dimensional representation of modeling area (chain of fuel cells). Fig. 3 gives an overview of the types of existing models as well as an emboldened branch to show our approach to the problem.

The focus of this paper is to explore the application of a shortest path algorithm on irregular networks.

2.2. Problem definition

The problem can be viewed as finding a shortest path or minimum travel time paths between fire ignition sources (ignition points) and destination points through a complex heterogeneous landscape. This particular surface wildfire propagation problem can be viewed as a minimum travel time problem on a Delaunay triangulation network $G(V,E)$ overlaid over the heterogeneous area. Traversal time along any edge of the Delaunay triangulation is a cost function which depends on the underlying fire environment. The problem embodies three research issues i) how to represent a homogeneous continuous landscape with discrete irregular network, ii) how to model propagation of a continuous fire front as the movement of fire along edges of a network and iii) how to evaluate first passage times of the fire event at any point of the landscape.

In the next few sections we present the suggested fire modeling approach which has two distinguishing features. The first feature is that it employs a discrete irregular network structure, in particular the Delaunay triangulation, to model surface fire propagation. The second feature is that it uses a two-pass minimum travel time (shortest path) algorithm to evaluate the spatial and temporal extension of fire.

2.3. Notation

The following notation is utilized to model the problem.

$G(V,E)$	:= Delaunay triangulation containing a finite set of nodes V and finite set of edges E
v_i	:= a node $i \in V$
e_{ij}	:= a directed edge connecting nodes v_i and v_j
v_s	:= an ignition node
v_t	:= a terminal node (point of interest)
$\tau(v_i)$	:= traversal time for a hazardous event reaching node v_i starting at node v_s
$\tau(e_{ij})$	:= a time for an event to traverse edge e_{ij}
r_i^{max}	:= maximum or head rate of spread from node v_i
r_i^{back}	:= back rate of spread from node v_i (in direction opposite to r_i^{max})
d_i^{max}	:= direction of maximum rate of spread originating from node v_i (this is an angle from the North).
r_{ij}	:= a rate of spread along edge e_{ij}. This is the projected rate r_i^{max} onto emanating edge e_{ij}.
d_{ij}	:= direction of edge e_{ij} (an angle from the North)
$SP(V',E')$	:= a shortest path from node v_s to node v_t. This is a subgraph of the network $G(V,E)$.
V'	:= an ordered set of nodes $V' = \{v'_1, v'_2, , v'_k, ., v'_n\}$, where v'_1 and v'_n correspond to the start and terminal nodes respectively
v'_g	:= a new generated/introduced node
$\mathscr{E}_i$	:= an ellipsoid with back focus at the ignition node v_i
β_i^{max}	:= an angle defining a wedge of heading sector of an ellipse E_i. This value ensures that any vector of fire spread r with direction $d_i^{max} \pm \beta_i^{max}$
$\mathscr{E}_i^{\beta}$	:= heading sector of the ellipse $\mathscr{E}_i$ with angle/wedge β_i^{max}
k	:= an ordered index of nodes of the shortest path SP, where $k = 1$ corresponds to starting node
β_{ij}	:= an angle between front direction d_i^{max} and edge e_{ij} ($\beta_{ij} = \|d_i^{max} - d_{ij}\|$)
f_i	:= semi-major axis of ellipse $\mathscr{E}_i$
h_i	:= semi-minor axis of ellipse $\mathscr{E}_i$
g_i	:= distance between the back focus of the ellipse and its center
ϵ_i	:= eccentricity parameter for the fire-growth model ellipse $\mathscr{E}_i$
n	:= size of set V' or number of nodes in the shortest path $SP(V',E')$
S_T	:= a study area
P	:= set of polygons into which study area S_T is tessellated. We assume that each polygon $p \in P$ has the constant fire environment.
p_i	:= a polygon $i \in P$, with centroid node v_i. Any $p_i \in P$ has a set of constant attributes, such as fuel type, slope and aspect).
$DT(V)$	:= Delaunay Triangulation on set of nodes V.The result of the operation is a network $G(V,E)$
$L_{name}^{type}(attr)$	:= a spatial layer with a set of attributes "attr". The following types are legitimate: polygon, line and point.
$\times$	:= an operation of spatial overlay or union on two spatial layers

3. Solution framework

The problem, as formulated in the previous section, has fire modeling, computational geometry and operations research facets. To analyze and model the problem, to perform numerical experiments, as well as to gain some insights, we utilize the following framework (Fig. 4). There are three major components in the system: representation module, analytical module and visualization module. The first, the representation module, is responsible for the construction of the fire propagation graph $G(V,E)$ which includes fuel, topographic properties, moisture and wind conditions of a modeled area S_T. This module creates and maintains a discrete representation of a heterogeneous continuous landscape S_T with $G(V,E)$. We discuss the representation procedure in Section 4.1 in detail. Submodule 1a ("tesselation

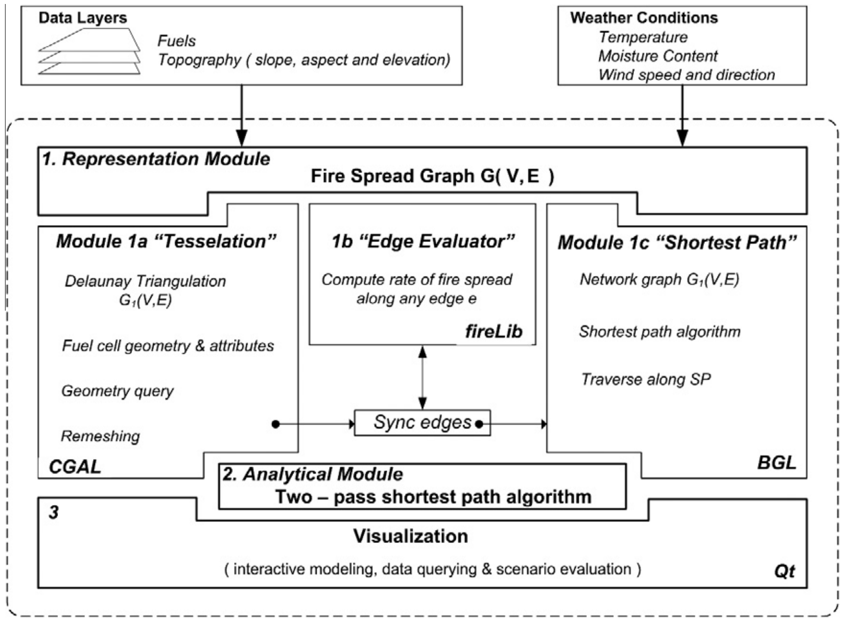


Fig. 4. Solution framework.

module") addresses a geometric component of the problem. It maintains a Delaunay Triangulation $G_1(V,E)$ of the fire propagation graph G, stores underlying fuel cell geometry and properties, and allows one to perform queries of area S_T. Importantly, the submodule provides several remeshing methods or ways for a modeller to add new nodes into the triangulation. Submodule 1*b* ("edge evaluator") provides functionality of the elliptical fire model to evaluate a rate of fire spread along any edge of graph G. The model uses a procedure to compute the rate of fire spread (in one particular direction) across several fuel polygons, taking into account their fire environment. Finally, submodel 1*c* ("shortest path module") deals with an optimization facet of the problem. This submodule maintains the network representation of fire graph G and treats traversal time along edges as costs. Any changes in geometric representation of graph G lead to automatic updates of the network representation.

The second, analytical module, which is a core of the framework, estimates arrival times for a given set of sources (ignition) and destination nodes. This module employs a two-pass shortest path algorithm, which in the first pass computes the shortest paths on G for ignition/destination pairs and, in the second pass, augments those paths to improve arrival time estimates. The augmenting process introduces new nodes to capture the fastest fire spread and remeshes/updates the fire propagation graph $G(V,E)$.

Module 3 provides visualization of the study area S_T, fuel cells, fire spread graph and shortest paths. This module is responsible for user interaction and allows a modeller to specify a set of vertices for analysis, to choose remeshing procedures and to design the

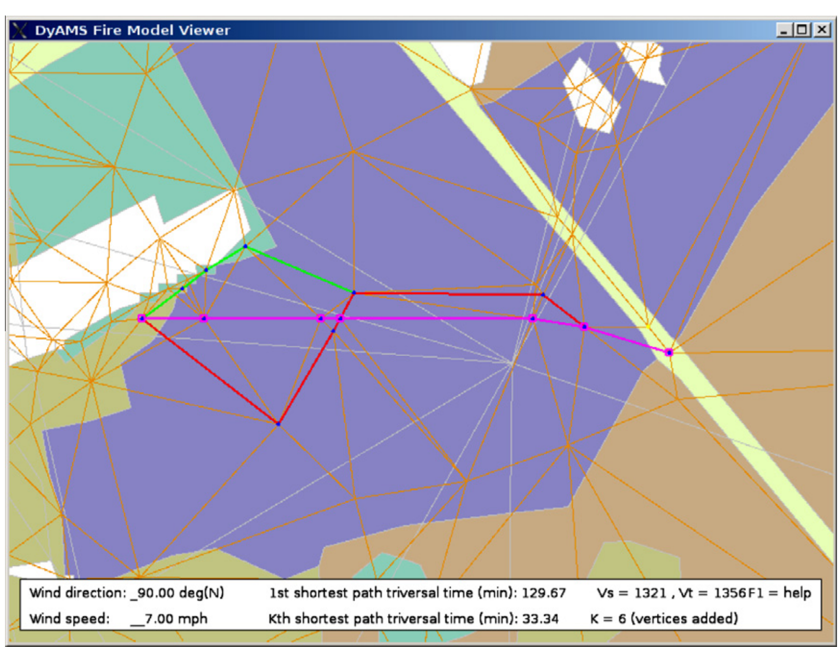


Fig. 5. An illustration of developed software application, which implements two-pass shortest path algorithm with Delaunay triangulation.

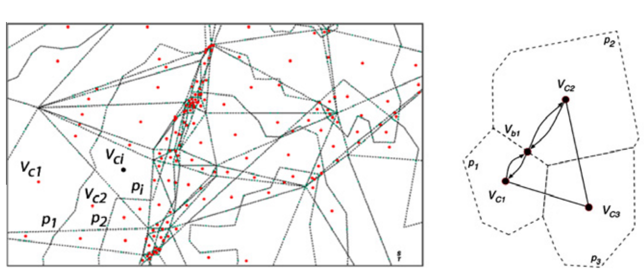
algorithm for module 2. The framework was implemented as a cross-platform C++ application "Dyams Fire Propagation Model" (Fig. 5). Submodule *1a* uses the *Computational Geometry Algorithms Library* (CGAL) (Fabri and Pion, 2009). Module *1b* is based on the *fireLib* library, developed by Systems for Environmental Management (Bevins, 2004), to evaluate rate of fire spead and traversal time along edges. The *Boost Graph Library*(BGL) is utilized to provide an operational research functionality within module 1*c* (Siek et al., 2002). Interactive interface and visualization functions are implemented with the *Qt* library (Blanchette and Summerfield, 2008). Some GIS functionality, such as zoom, pan, fuel cell queries and layer manipulations are based on the *CGAL* interface to *Qt* library. The application allows a modeller to load an original dataset, build a graph *G*, and to specify ignition and destination points interactively or from a file. During execution, information on wind conditions, shortest path length and time are provided on a dash board. Finally, the application takes a video capture for post-analysis of the algorithm.

4. Solution methodology

Our solution methodology approaches fire propagation through a complex landscape as the movement of a fire event through a network $G(V,E)$ connecting centroids of cells *P* with constant fire environment properties. A two-pass shortest path algorithm estimates the arrival time of fire at particular nodes. Fire perimeter iso-lines $FP(t)$ are constructed with spatial interpolation techniques. Our methodology has three distinctive stages: i) representation, ii) analysis and iii) synthesis. The first, representation stage, transforms a continuous complex landscape into a discrete graph $G(V,E)$ in such way that it captures the major terrain and fuel characteristics, as well as the rate of fire spread along edges *E*. The second stage, analysis, uses potential methods to evaluate shortest (minimum travel time) paths. The third stage introduces an adaptable shortest path algorithm to more accurately compute fire event arrival times.

4.1. Representation of fire propagation

We use $G(V,E)$ to model fire event proliferation through a heterogeneous terrain S_T (Fig. 6). The set of nodes *V* represents centroids of adjacent polygons *P* with a constant fire environment, into which the terrain S_T is tessellated (Fig. 6(a)). Our partitioning method ensures that polygons have constant properties in terms of fuel and topography. We also assume that wind and moisture conditions are assigned to polygons *P* and that they are constant for some time period *t*. The set of edges *E* represents the potential directions of the fire event propagation from affected nodes to adjacent nodes or from a current polygon to neighboring polygons. We assume that fire behavior does not change within a particular polygon $p_i \in P$, as it has a constant fire environment; however, the rate of spread changes when the event propagates into another polygon/cell. Nodes V_b represent points along edges where fire propagation speed changes. Fig. 6(b) illustrates that such nodes are located at the intersection of polygons *P* and edges of the Delaunay graph. Therefore, we view propagation of the continuous fire front $FP(t)$ through a continuous space S_T as discrete event movements from ignited nodes to adjacent nodes *V* along edges *E* of graph $G(V,E)$.



(a) Polygons P and a set of nodes V

(b) Network edges E defined with Delaunay Triangulation $DT(V)$

Fig. 6. Tessellation of a landscape: (a) Polygon boundaries (—) and centroids (•); (b) corresponding Delaunay graph.

We use a Geographic Information System (*GIS*) to create a spatial model of the region, which includes data on fuel, topography, barriers and points of interest. Then, *GIS* functionality is used to partition the heterogeneous region S_T into adjacent polygons with homogeneous fuel and topographic properties. Wind and moisture conditions are defined for each cell. We assume that wind and moisture conditions are constant for some time period *t* to ensure constant fire environments for the polygons. We use the Delaunay triangulation $DT(V)$ to connect centroids of polygons (Algorithm 1).

Algorithm 1. Representation of fire spread graph $G(V,E)$

input: fuel, slope, aspect and elevation layers, wind conditions and fuel moisture
output: Delaunay triangulation $G(V,E)$ with computed traversal time τ_{ij} for all $e_{ij} \in E$ and (r_i^{max}, d_i^{max}) for all $v_i \in V$
$L_{fuel}(Fm) \leftarrow$ slope layer
$L_{slope}(Sl) \leftarrow$ slope layer
$L_{asp}(Asp) \leftarrow$ aspect layer
$L_{HFTE}(Fm,Sl,Asp) \leftarrow L_{fuel}(Fm) \times L_{slope}(Sl) \times L_{asp}(Asp)$ ◁ Compute layer with homogeneous fuel and topographical properties
$P \leftarrow GetPolygons(L_{HFTE}(Fm,Sl,Asp))$
$V \leftarrow GetCentroids(P)$
for all polygons $p_i \in P$ **do** ◁ define polygons with homogeneous fire environment
  $p_i \leftarrow$ fuel moisture
  $p_i \leftarrow$ wind direction W_d
  $p_i \leftarrow$ midflame wind speed W_s
end for
for all nodes $v_i \in V$ **do**
  $p_i \leftarrow GetUnderlyingPolygon(v_i,P)$
  $r_i^{max} \leftarrow MaxRateOfSpread(p_i)$
  $d_i^{max} \leftarrow DirOfHeadFire(p_i)$
end for
for all edges $e_{ij} \in E$ **do** ◁ Precompute traverse time τ along edges *E*
  $(\tau_{ij}, \tau_{ji}) \leftarrow ComputeTraverseTime(e_{ij},P)$
end for
return $G(V,E)$ and *P*

The triangulated network provides an effective and economical representation of the modelled region in comparison to regular grid structures (Mark, 1975; Okabe et al., 1992) and has found widespread use in modelling surfaces, terrains in computer visualization and GIS. The size of the irregular network is at least 14 times less than the size of the grid network for the same area (Okabe et al., 1992, pg. 352). Density of nodes of the graph $DT(V)$ corresponds to a rate of changes occurring in the area.It would be worth mentioning that the Minimum Spanning Tree is a subset of the Delaunay Triangulation, as MST may be used as a backbone circulation network for many *OR* applications (Garcia-Diaz and MacGregor Smith, 2007, pg. 256). To compute traversal time $\tau(e_{ij})$ along edges $e_{ij} \in E$ (in both directions), the edge is viewed as a set of segments, with break points on the boundaries of polygons *P* (Fig. 7(a)). Therefore each segment lies within a polygon with constant fire environment and rate of spread along the segment can be

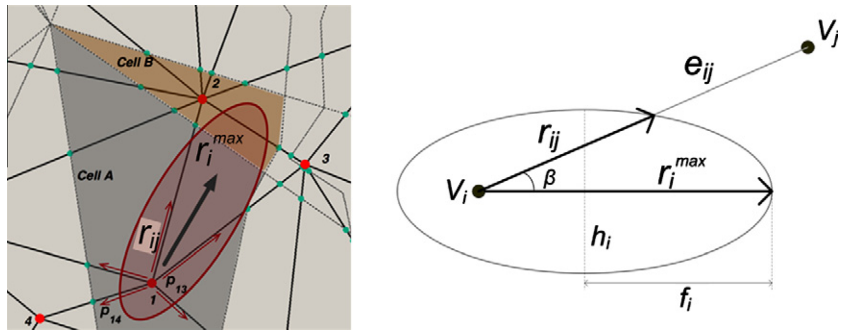


Fig. 7. Defining projected rate of spread on edges with the elliptical fire model.

evaluated with the elliptical fire model (Fig. 7(b)). Then the traversal time $\tau(e_{ij})$ along edge e_{ij} is computed as an aggregate time (Algorithm 2) to traverse all segments (Fujioka, 1985). The head fire spread rate, back fire velocity and eccentricity coefficient ϵ_i are computed for each starting node $v_i \in V$. Then, the semimajor axis f_i is evaluated as: $f_i = (r_i^{max} + r_i^{back})/2$. The following equation is used to calculate fire velocity along the edge e_{ij} (Catchpole et al., 1982).

$$r_{ij} = \frac{f_i(1-\epsilon^2)}{1-\epsilon\cos(\beta_{ij})} \tag{2}$$

This equation defines the rate of fire spread r_{ij} with polar coordinates and back focus as the point of origin. We use *fireLib* library to compute the rate of spread, intensity of fire for specific combinations of fuel, slope, aspect, moisture, wind direction and midflame wind speed.

4.2. Analysis of fire propagation with G(V,E)

Given the network graph $G(V,E)$ and an ignition node $v_s \in V$, how could we evaluate time $\tau(v_t)$ when the fire event reaches some node of interest v_t? Time $\tau(e_{ij})$ to traverse an edge e_{ij}, which connects nodes (v_i, v_j) can be computed a

Algorithm 2. Defining traversal time on edges

input: an edge $e \in E$, polygons with constant fire environment P
output: traversal time τ along edge e in both directions
function ComputeTraverseTime e_{ij}, P
  $d_{ij} \leftarrow$ GetDirection (e_{ij})
  $d_{ji} \leftarrow -d_{ij}$
  $\tau_{ij} = \tau_{ji} \leftarrow 0$
  for all polygons $p \in P$ **do**
    if $e \cap p \neq \emptyset$ **then**
      Segment $s \leftarrow e \cap p$
      $r_s \leftarrow$ RateOfSpread (d_{ij}, p)
      $r'_s \leftarrow$ RateOfSpread (d_{ji}, p)
      $\tau_{ij} \leftarrow \tau_{ij}$+length $(s)/r_s$
      $\tau_{ji} \leftarrow \tau_{ji}$+length $(s)/r'_s$
    end if
  end for
return (τ_{ij}, τ_{ji})
end function

A conventional approach, one may suggest, is to utilize a shortest path algorithm (SP), where the shortest path would represent the fastest path of fire propagation. However, in the case of strong winds and when vector (v_s, v_t) aligns with wind direction, the shortest path techniques cannot be used directly to compute minimum travel time on $G(V,E)$ for two reasons. The first reason is that the rate of spread at specific direction/azimuth β depends greatly on the value of the angle β as well as on fuel, moisture, direction of slope and wind speed. Fig. 8 shows how the projected rate of fire spread r_{ij} along an edge e_{ij} changes with changes in angle between maximum/head rate and vector e_{ij}. The second reason is that, by definition, the proposed Delaunay graph $G(V,E)$ is an irregular structure, where spatial orientation of the defined arcs is independent of the wind direction. We will clarify this statement with an example, which presents an extreme but possible situation. Let us suppose that the graph presented in Fig. 9 is a subset of the graph $G(V,E)$. Consider the task of computing the minimum propagation time $\tau(v_f)$ from an ignition node v_a to node v_f. We assume that maximum rate of spread r_i^{max} from each node $v_i \in V$ was computed with a fire spread calculation module (for given fire environment), and the projected rate of spread r_{ij} was computed for each outbound edge e_{ij} with an elliptical growth model. Travel time $\tau(e_{ij})$ along edge e_{ij} was estimated for each edge as well. An attempt to use the shortest path technique directly would estimate $\tau(v_f)$ as the shortest/quickest path. For the sake of argument, let us assume that the shortest path is $v_a - v_b - v_c - v_e - v_f$, i.e.

$$\tau(v_f) = \tau(e_{ab}) + \tau(e_{bc}) + \tau(e_{ce}) + \tau(e_{ef})$$

If we instead used an elliptical growth model in continuous space, we would note that the fire front perimeter $FP(t)$ reaches nodes v_e and v_c at the same time (see perimeter of largest ellipse in Fig. 9), although the shortest path method estimated $\tau(e_{be})$ as $\tau(e_{bc})$ + $\tau(e_{ce})$.

To overcome this problem and improve the time estimation of fire propagation, we introduce new nodes to $G(V,E)$. Then, local re-meshing is done to establish inbound and outbound edges to the new nodes. In general, locating new nodes can be done with heuristics or can be viewed as an optimization sub-problem, since we need to capture the direction of the fastest fire propagation and, at the same time, minimize the number of newly added edges. In the presented research, we use a heuristic to locate new nodes.

4.3. Synthesis step

In this section, we propose a 2-pass shortest path algorithm on graph $G(V,E)$ to compute arrival times of a fire event. Our suggested algorithm synthesizes a representation of fire propagation with $G(V,E)$ and concepts of local remeshing as discussed in Section 4.2. During the first pass, our algorithm defines the shortest path $SP(v_s, v_t)$ between an ignition point v_s and a terminal point v_t. This is the first approximation of fire arrival time to all nodes. As we discussed earlier, use of "classical" shortest path algorithms without local remeshing is prone to over-estimation of fire arrival times. To improve the accuracy of the fire arrival estimation, in the second pass, the algorithm introduces new nodes in the direction of the front fire propagation and performs local remeshing of the graph $G(V,E)$. Let us suppose that we need to compute the propagation time of a fire to the node v_f given that the fire started at node v_a. The Delaunay graph $G(V,E)$ and traversal times $\tau(e_{ij})$ are defined

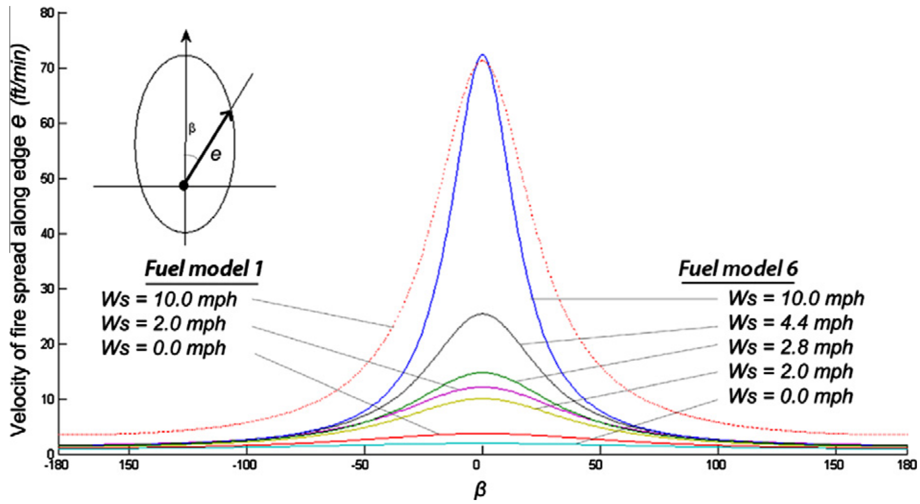


Fig. 8. Rate of wildfire spread along edge e in particular direction β.

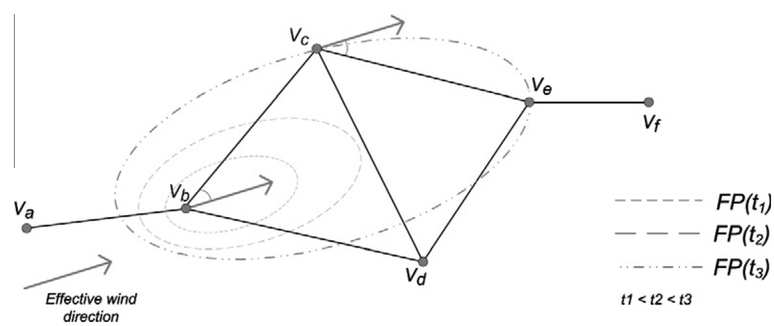


Fig. 9. Illustration of the difference in estimating a fire arrival time with an elliptical growth model and first shortest path algorithm.

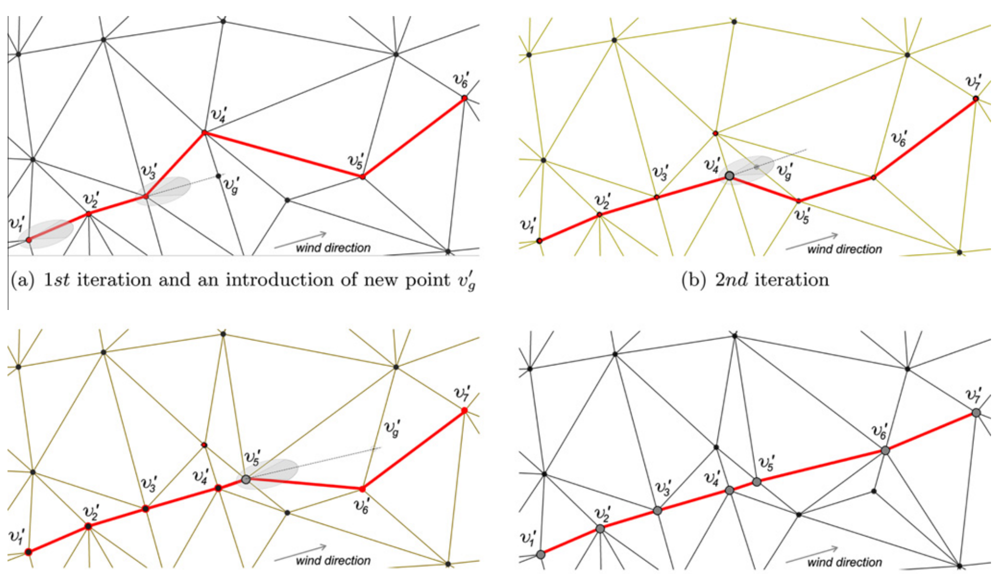


Fig. 10. Illustration of algorithm's iterations and remeshing points found during the second pass.

for all edges E. During the first pass, the minimum travel time path $SP(v_s, v_t)$ is defined. Let us suppose that it will take $\tau(v_t)$ for an fire event to propagate from node v_s to v_t along the path $V' = \{v'_1 - \cdots - v'_7\}$ (Fig. 10).

During the second pass, the algorithm checks if any improvements for $\tau(v_t)$ are possible. For every node $v_i \in V_{SP}$, the algorithm checks if the outbound edge e_{ij} is out of the "front sector" (the angle between head rate r_i^{max} and d_{ij} is greater than a specific value). If there is such an edge, then the algorithm introduces the new point v'_g (Fig. 10(b)). We propose to use the following heuristic to locate a new point v'_g. The new node v'_g can be located at the intersection of vector r_i^{max} and an edge emanating from node v'_{i+1}. The new node can be also added in the direction of vector (v_i, v'_{i+2}), if there is no edge connecting nodes v_i and v'_{i+2} of the path. The rationale for this heuristic is that we can find the fastest segment *from* node v_i and introduce a shorter detour $v'_i - v'_{i+1} - v'_{i+2}$.

This algorithm creates a new local mesh by adding new edges e_{ig} and $e_{g(i+2)}$ (Fig. 10(c)). Re-computation of the new shortest path SP_{aug} and minimal propagation time $\tau_{aug}(v_t)$ concludes the iteration. If there is an improvement in minimum travel time due to the introduction of a new node and remeshing, then the augmented shortest path SP_{aug} is set as the new shortest path. (Fig. 10(b)). Finally, the next node in the shortest path is analyzed to improve the path. Fig. 10(d) presents the final shortest path.

4.4. Description of the algorithm

The suggested algorithm to estimate traversal time $\tau(v_t)$ for a given $G(V,E)$ is described by the following pseudo-code (Algorithm 3).

Algorithm 3. Two-pass shortest path algorithm

input: Delaunay triangulation $G(V,E)$ with computed r_{ij} for all $e_{ij} \in E$ and (r_i^{max}, d_i^{max}) for all $v_i \in V$, ignition node v_s and terminal node v_t
output: traversal time $\tau(v_t)$
function MinTraversalTime $G(V,E), v_s, v_t$
  $k \leftarrow 1$
  $SP(V',E') \leftarrow FindShortestPath(G(V,E),\ v_s, v_t)$ ◃ The 1*st* pass
  $\tau(v_t) \leftarrow TraversalTime(SP(V',E'))$ ◃ Wildfire propagation time
  set $LIST \leftarrow \{V'\} - v'_{n-1} - v'_n$
  while set $LIST$ is not $\emptyset$ **do**
    for all nodes $v'_k \in LIST$ **do**
      if d_k^{max} is within head sector $\mathscr{E}_k$ **then**
        continue ◃ Analyze next node
      else
        $v'_g \leftarrow FindIntersection(r_k^{max}$ and emanating edge from $v'_{k+1})$ ◃ Improve time estimate $\tau(v_t)$
        $V \leftarrow \{V\} \cup v'_g$ ◃ new node added
        Perform local re-meshing adding outbound e_{gj} and inbound e_{jg} edges for node v'_g
        Compute rate of spread r_{gj} and r_{jg} for all new edges
        Augment the shortest path $SP : e'_{kg} \in SP$
        Compute $\tau_{aug}(v_t)$
        if $\tau_{aug}(v_t) < \tau(v_t)$ **then**
          $\tau(v_t) \leftarrow \tau_{aug}(v_t)$
          $SP(V',E') \leftarrow SP_{aug}(V',E')$
          index set V'
          set $LIST \leftarrow \{V'\} - v'_k - v'_{n-1} - v'_n$
        else ◃ there is no improvement
          Remove v'_g from set V: $V \leftarrow \{V\} - v'_g$
          Remove newly added edges e' ◃ Analyze the next node in V'
        end if
        $k \leftarrow k + 1$
      end if
    end for
  end while
  return $\tau(v_t)$
end function

In summary, the proposed approach uses the Delaunay-based network $G(V,E)$ as a tool to model fire propagation over a complex terrain. The module, which is based on *fireLib* (Bevins, 2004), computes traversal time along edges E. The 2-pass shortest path algorithm is programmed to evaluate the temporal and spatial extent of wildfire spread. Fire perimeters $FP(t)$ are interpolated based on arrival times at nodes of $G(V,E)$.

5. Complexity analysis

In this section, we gauge the performance of the proposed algorithm. We will use the standard worst case performance metric and big-O notation. To evaluate the overall complexity of the algorithm, we will analyze the computational performance of each sub-process, which includes construction of the Delaunay triangulation $G(V,E)$, computation of fire spread rate R along the edges E, finding the shortest path SP between any two nodes v_s and v_t, and a process of augmenting the shortest path with refinements of the Delaunay triangulation. The Delaunay triangulation on set V can be performed with $O(n \log(n))$ operations, where n is a number of nodes in the set V. There are several highly effective *polynomial time* algorithms to establish a Delaunay network (Okabe et al., 1992). As the Delaunay network has a special structure, the number of edges m is limited by $\lceil 3n \rceil$, where n is number of nodes. Hence, to evaluate traversal time along the edges we need $\lceil cm \rceil$ or $\lceil 3cn \rceil$ calls of the procedure, which is $O(n)$. As a result, the traversal time along edges for the Delaunay structure can be computed in *linear time*. The next stage of the algorithm, the computation of a shortest path $SP(v_s, v_t)$, will require at maximum $O(n^2)$ steps. Utilization of the special structure/properties of Delaunay triangulations can improve this estimate up to $O(m + n \log(n))$ (Ahuja et al., 1993). Hence, the shortest path computations can be performed in *polynomial* time. Let n^* denote the number of nodes in the shortest path SP. Then, in the worst-case we will need to perform $(n^* - 1)$ refinements of the Delaunay graph. Each sub-operation of local remeshing implies adding a new node, destroying edges/triangles which are no longer Delaunay, re-meshing or adding new edges, and, finally, the computation of rate of spread along the new edges (Shewchuk, 2002). Hence, augmentation of the shortest path can be done in linear time or limited by $O(n^*)$ operations (Ruppert, 1995; Shewchuk, 1996). After gauging the stages of the proposed algorithm, we can conclude that finding a minimum travel path for a fire event with the Delaunay triangulation and shortest path algorithms is bounded by $O(n^3)$ and can be performed in *polynomial time*. We believe that it can be computed even faster for average-case or empirical instances, as the SP algorithm can benefit from the special structure of the Delaunay triangulation.

To illustrate performance of the Delaunay method with two-passes in comparison with lattice-based approaches, let $\tilde{m}\tilde{n}$ denote the number of nodes required for lattice-based graph representation of the modeled area (where $\tilde{m}$ and $\tilde{n}$ are number of nodes in horizontal and vertical directions respectively). The number of edges of such a graph can be approximated as $\tilde{c}(\tilde{m}-1)\tilde{n} + (\tilde{n}-1)\tilde{m} \approx \tilde{c}\tilde{n}^2$. As we pointed out in the literature review, usually it is the case that $n \ll \tilde{n}$. Therefore a reader may see that for calculating the fire arrival time to the same destination nodes (points of interest), the suggested methodology will evaluate $\tilde{c}\tilde{n}^2/cn$ times fewer edges than if a lattice-based graph would be used.

6. Experimental results

In this section of the paper, a case study for the Montague Plain Wildlife Management Area (*MPWMA*) is used to demonstrate the methodology. We selected this area due to the availability of detailed fire modeling data, which was provided by Duveneck (2005). In general, the availability of complete historical data for forest fire modeling may be problematic, as it would be almost impossible to recreate a fuel cover classification after it was destroyed by a wildfire, and conversely, there might be no fires in well classified and measured areas.

To evaluate fire spread from an ignition point, we will construct the Delaunay graph $G(V,E)$ for the study area, analyze fire spread rates R along edges and estimate traversal time for fire event to reach some points of interest.

Due to the lack of substantial historical empirical data documenting a large wildfire and its propagation footprint for *MPWMA*, we will use results simulated with the *FARSITE* model as a proxy and an approximation for such historical observation. Though this is not ideal, the decision may be justified for the following reasons. First, *FARSITE* is a well-recognized simulation program in the wildfire modeling community (the results of which have been tested against historical observations). Second, *FARSITE*'s calculations of fire behavior within a homogenous grid cell are based on Rothermel's fire spread model. This (Rothermel's) model is a foundation of the *fireLib* library, which we use in our approach to evaluate fire behavior along an edge of Delaunay Triangulation.

Our intention is to use simulated results as a reality check and to demonstrate running time benefits of using the Delaunay triangulation, and to avoid any direct comparison with *FARSITE*'s accuracy. For example, *FARSITE* uses wind speed specified at 6 meter (20 feet) height, while *fireLib* requires midflame wind speed, so wind speed should be adjusted. Resolution of input grids and steps in perimeter construction can also affect the results.

6.1. Study area and required data

MPWMA is located in the Town of Montague, in Franklin County of Western Massachusetts (Fig. 11(b)). The *MPWMA* property extends about 3.2 × 4.8 kilometer and embodies about 610 hectares (Clark and Patterson, 2003).

The vegetation layer, which comprises 8 major types and covers about 90% of area, is wildfire prone (Fig. 11(a)). Table 1 summarizes major fuel types and corresponding fuel models for the area. Elevation changes from 40 to 170 meters above the sea level occur and even some areas have very steep slopes. According to the Fire Management Plan, the south and south-southwest winds prevail during the spring, summer and fall periods. During the winter period, the north and west-northwest winds are dominant. An average wind speed during a year is about 14 kilometer/hour (Clark and Patterson, 2003). Detailed weather observations recorded at Westover Air Force Base, which is located about 32 kilometer south of the modeled area, can be used as proxy data on weather conditions (Clark and Patterson, 2003). Table 2 summarizes data types used for the modeling in this case study. In the following section, we will evaluate speed of fire propagation from an ignition point v_s to points of interest v_j (Fig. 13). We assume that the wind blows from the West and its bearing direction W_d is 270 degrees. The average wind speed W_S is 16 kilometer/hour measured at 6.1 meter. The fire starts at t_0 = 8 am. We are interested in how soon the fire will reach points of interest v_j (in other words we need to define the minimum travel time $\tau(v_j)$ or first passage time). We will unfold this case study in three stages (as we described in the Section 4). First, we will illustrate the process of representing the area with the Delaunay graph. Second, we will demonstrate the process of assigning corresponding traversal time $\tau(e_{ij})$ along edges e_{ij}. Finally, we will use our two-pass shortest path algorithm to evaluate the time when the fire event will reach the point of interest $\tau(v_j)$ and to estimate the fire perimeter $FP(t)$ until 5 h after ignition.

Table 1
Land use/vegetation type classification and fuel models (adapted from Montague plain wildlife management area fire management plan).

Land use code	Description	Corresponding fuel models	Area (*ha*)	Number of polygons
1	Pitch pine forest	9	182.5	14
2	Hardwood forest	9	267.9	36
3	Shrub oak thicket	6	36.8	4
4	Mixed pine-hardwood forest	8,10	96.3	43
5	Grass	1	23.9	22
6	Gravel pit	n/a	3.2	8
7	Residential and surroundings	1		3
8	Pine plantation	8	1.2	1

6.2. Representation of the modeled area with $G(V,E)$

We determine the areas with constant fire environments P by a pairwise overlay of topography and fuel layers, which is followed by an assignment of midflame wind speed, wind direction and moisture content to each polygon $p \in P$. The operation, summarized by Eqs. (3) and (4), tessellates the modeling area S_T into a set layer $L_{CFE}(Fm,Z,Sl,Asp)$ of adjacent polygons P with constant fuel and topographical properties by spatially overlaying/intersecting elevation layer L_{elev}, slope layer L_{slope}, aspect layer L_{asp} with fuel layer L_{fuel}. Then, the adjusted value of midflame wind speed and wind direction should be assigned to the polygons. It will ensure that each polygon p has a constant fire environment and inherits distinctive combinations of fuel, slope, aspect, moisture content, wind direction and wind speed. For the case study the coefficient of 0.4 was used to adjust wind speed for unsheltered fuels; the same wind direction was used for the whole study area.

$$L_{CFE}(Fm,Z,Sl,Asp) \leftarrow L_{elev}(Z) \times L_{slope}(Sl) \times L_{asp}(Asp) \times L_{fuel}(Fm) \qquad (3)$$

$$P \leftarrow \{\text{polygons of } L_{CFE}\} \qquad (4)$$

The set of nodes V is defined as centroids of the polygons P and located in their interior. Subsequently, the graph $G(V,E)$ is con-

(a) Fuel types

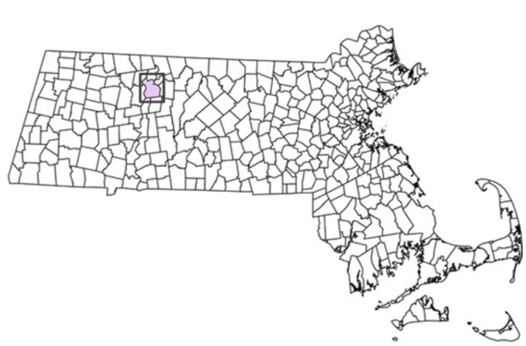

(b) Locus map

Fig. 11. Montague plain wildlife management area.

Table 2
Input topographic, fuel and meteorological data required for construction of *DG* for MPWMA.

Id	Layer type	Attribute (s)	Attr. notation	Layer notation	Number of features	Source
1	Digital elevation Model (DEM)	Elevation (m)	Z 10x10m	$L_{DEM}(Z)$		MassGIS
2	Elevation layer	Elevation ranges	Z	$L_{elev}(Z)$	213	Derived from DEM
3	Slope layer	Slope %	Sl	$L_{slope}(Sl)$	1058	DEM
4	Aspect layer	Aspect (deg)	Asp	$L_{asp}Asp$	1058	DEM
5	Fuel models layer	Fuel model id	Fm	$L_{fuel}(Fm)$	90	Duveneck (2005)
6	Weather and moisture data					Clark and Patterson (2003)
7	Wind direction/speed		W_S, W_d			Clark and Patterson (2003)

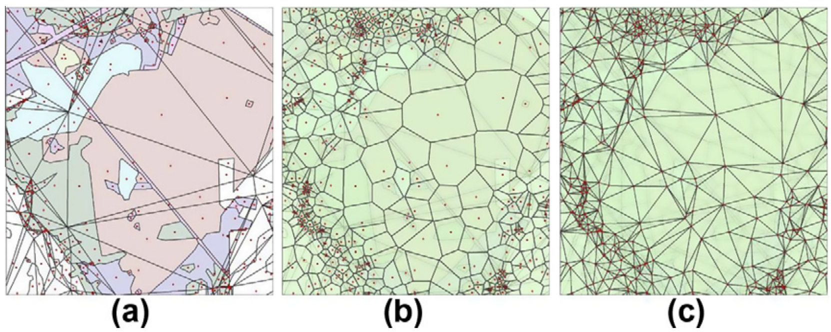


Fig. 12. (a) Cell centroids, (b) Voronoi Polygons and (c) Delaunay triangulation $G(V,E)$.

structed with the Delaunay Triangulation over the set of nodes V. This graph represents the movement of fire events among centers V of the fuel cells. To take into account changes in rate of fire spread (as an fire event moves from one polygon into another) we overlay $G(V, E)$ with polygons P. At this stage we have constructed $G(V,E)$ which connects centers of homogenous fuel cells of the modeled area (Fig. 12). Fig. 12(b,c) acknowledges the dual relationship between the Delaunay Triangulation and Voronoi Polygon structures.

6.3. Analysis of fire spread rates

At this stage, we define the fire spread rates along the edges E and corresponding traversal time $\tau(e_{ij})$. The procedure *ComputeTraversalTime* is invoked for every edge e_{ij} with corresponding parameters of underlying polygons: fuel, moisture, wind speed, wind direction, slope and aspect. As specified above, the module *ComputeTraversalTime* utilizes the *fireLib* library. In summary, the constructed $G(V,E)$ has 3,022 nodes and 17,000 directed edges and represents the continuous complex landscape with changing topography and fuel types.

6.4. Computation of minimum travel time paths

We utilize the two-pass shortest path algorithm to evaluate the minimum travel time $SP(v_s, v_j)$ for each node of interest v_j. During the first pass, the algorithm evaluates fire arrival times at each node v_j. The results of the first pass are presented in Table 3. Fire spatial extension was computed with spatial interpolation. Fig. 13(b) depicts fire progression during the first 5 h. Each shaded area in Fig. 13(b) depicts a one-hour fire growth area. During the second pass, the algorithm attempts to increase accuracy by augmenting the first shortest path for each pair of (v_s, v_j). We have tested two re-meshing heuristics. The first approach introduces new nodes in the direction of maximum fire spread. The second heuristic also introduces nodes in the direction of the third node in the sequence from a current node in the shortest path. New points are introduced only when an improvement in the time estimate is possible. In the first approach, 27 new nodes were introduced into the Delaunay

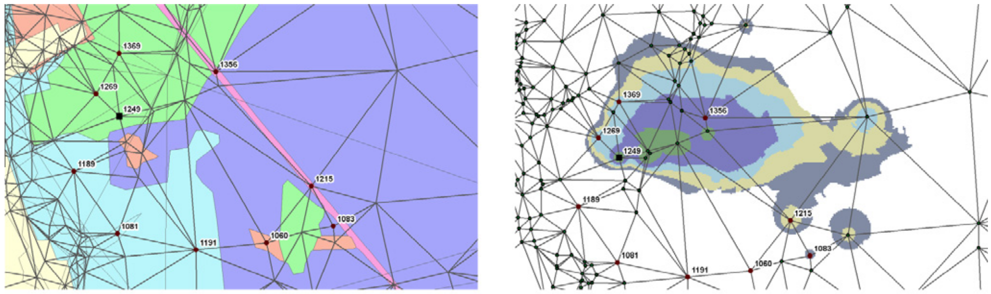
Table 3
Computed fire arrival times. Ignition point is node 1249.

Destinations (node id)	Fire arrival time (min)			
	First pass	Alg 1	Alg 2	FARSITE
1060	800	726	650	795
1081	1141	1141	1103	1155
1083	211	188	183	268
1189	682	682	643	729
1191	1182	1182	783	803
1215	136	113	108	182
1249	0	0	0	0
1269	121	121	121	150
1356	78	78	40	51
1369	146	146	141	148

triangulation. In the second approach, 59 new nodes were introduced to the graph $G(V,E)$, allowing us to track the flank fire propagation more precisely. Fig. 14 depicts the fire propagation for a period of 5 h, computed with the second heuristic. Each shaded area in the figure represents a one-hour fire extension. The same experiment was conducted with the *FARSITE* package. Table 3 summarizes event arrival times for the described experiments. The results, iso-chrons of fire spread or time line of fire spread with one hour interval, are presented in Fig. 15. Please notice that our main task is to evaluate the arrival times at points of interest, which are important for a decision maker. Interpolating fire perimeters is an auxiliary task and used mainly for visualizing results.

6.5. Analysis of the results

Observe that the general shape of the fire propagation computed using our method is similar to the one produced by *FARSITE*. We need to note that the second pass of our algorithm is required when destination points are located in the main direction of front fire (or in the effective wind direction). The second pass is not required when destination points are approached by flank fires. In addition, the use of the Delaunay triangulation provides an



(a) test area (b) 1st pass

Fig. 13. (a) Location of ignition node (■) and destination nodes (•). Colored areas represent different fuel types; (b) an estimation of fire propagation after the first pass: colored polygons represent one hour interval.

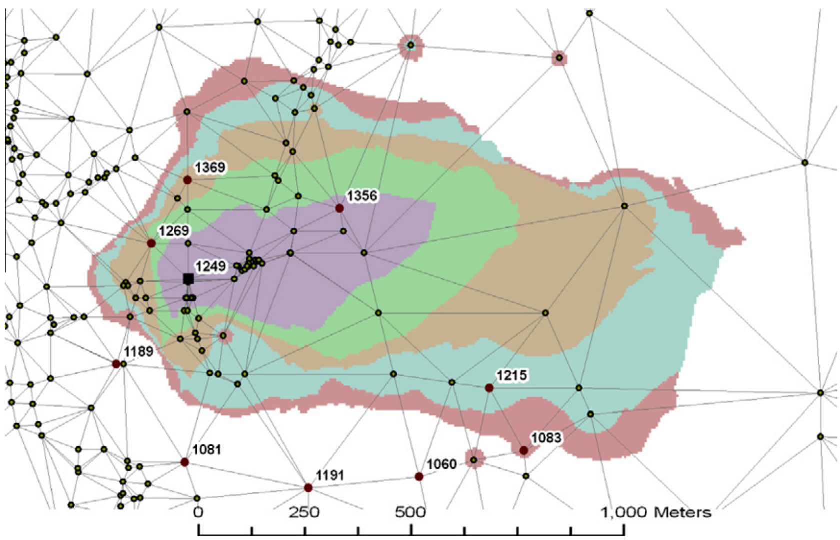


Fig. 14. Fire perimeters $FT(t)$ after second pass.

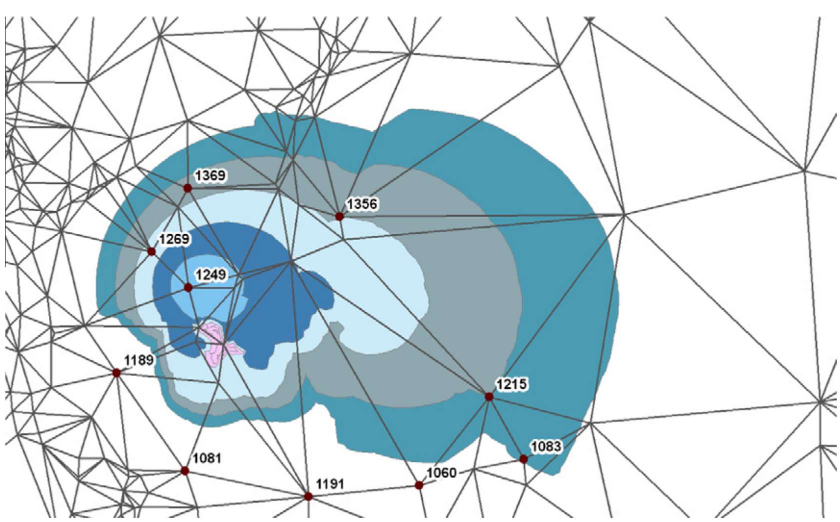


Fig. 15. Fire perimeter $FP(t)$ evaluated with *FARSITE*.

effective representation of the modeled area. As we stated in Section 6, the smaller number of edges in the graph translates into a smaller number of computing operations required to evaluate the projected rate of spread along edges as well as in faster computations of shortest paths. The construction time for a regular lattice structure can be performed in polynomial time $O(\tilde{n}\tilde{m})$, where $\tilde{n} \times \tilde{m}$ is a dimension of the original grid structure. The number of corresponding edges (in a case of just rectangular connections) can be estimated as $(\tilde{n}-1)\tilde{m} + (\tilde{m}-1)\tilde{n}$. Although the suggested approach also runs in polynomial time, the number of nodes V is

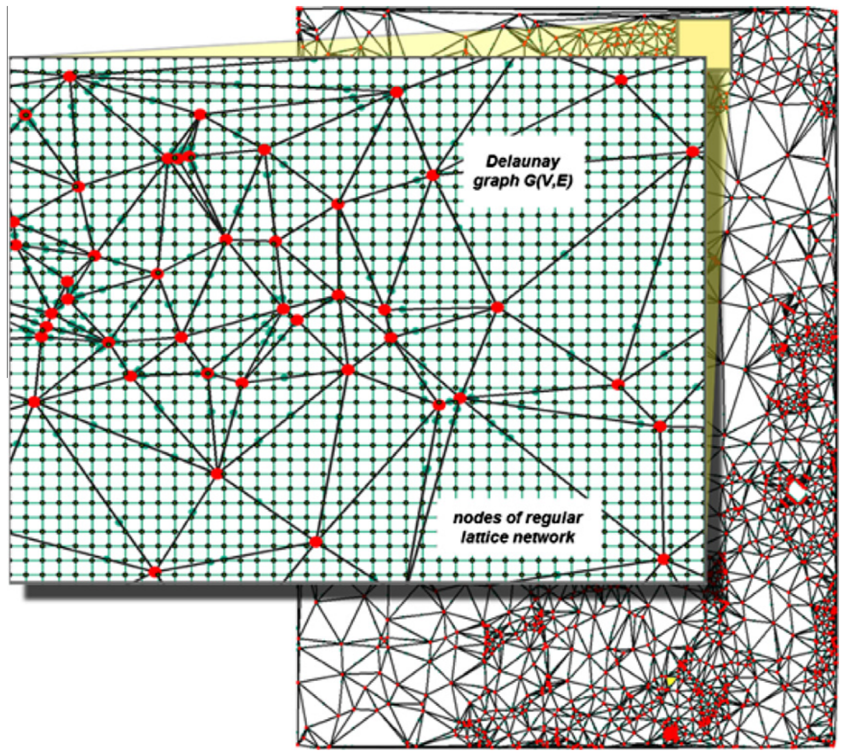


Fig. 16. Size comparison of regular lattice network and proposed irregular Delaunay graph (a subset of modelled area).

Table 4
Comparison of number of edges and vertices for two representation types.

$G(V,E)$ based representation (bidirectional graph)	Grid based representation (bidirectional graph [lattice])
	Cell size = 10 meter
	Number of rows in the grid $\tilde{n} = 519$
	Number of columns $\tilde{m} = 524$
Number of nodes V = 3,022	Total number of nodes V = 271,956
Number of edges E = 17,000	Number of arcs E = 542,869

significantly smaller than $\tilde{m} \times \tilde{n}$ and the number of edges is limited by $3n$. Just this difference gives some advantages in terms of computational time to the Delaunay triangulation approach. For example, the lattice (regular) graph representing the same modeled area S_T has 271,956 nodes and 542,869 edges (see Fig. 16 and Table 4).

7. Summary and conclusions

Our primary motivation for this research originated from a need for a fast analytical (not simulation-based) approach to evaluate surface fire propagation over a vast complex landscape. The second motive stems from a fascination with the success of Operations Research (OR) applications based on Voronoi Polygons and Delaunay Triangulation in such areas as computer graphics, computational geometry, terrain and surface modeling. We have proposed a method to capture the essential properties of complex landscapes, which are important for wildfire modeling, and representing them as a Delaunay Triangulation. We also outlined limitations of conventional Shortest Path algorithms to assess fire event arrival times and we suggested a modification of the shortest path algorithm to gauge fire propagation time fast and effectively.

In summary, we have introduced a polynomial time algorithm based upon the Delaunay triangulation, shortest path algorithms and mesh refinement, for evaluating surface wildfire propagation through a complex heterogeneous landscape. Our approach uses a Delaunay triangulation as a special data structure to compute minimum travel time path for a fire event and serves as a first approximation to evaluate wildfire propagation time. As fire growth depends greatly on the non-linear direction of fire perimeter expansion, the Delaunay triangulation refinement is utilized to capture fire expansion in the direction of the fastest spread. The proposed technique for fire propagation modeling is scalable, adaptive, and effective, as it benefits from the special computational and algorithmic properties of Delaunay triangulations. Finally, we demonstrated our approach on a non-trivial example.

In future research, we will address some of the limitations of the suggested methodology. The suggested algorithm implies constant weather (moisture and wind) conditions during a modeling period. The extension of the method will divide a large planning horizon into a sequence of time periods each with constant weather conditions and running the model iteratively for each time period and considering destination points as ignition points for the next time periods. It would be interesting to explore an optimization procedure for locating new points during re-meshing (notice that currently we use heuristics). A reader may notice that computation of traversal time along each edge (as a function of the underlying fire environment) can be done independently for each polygon. Application of graphic processing units (GPU) to parallelize the operation is another very promising direction (Harish and Narayanan, 2007; Buluç et al., 2010; Bleiweiss, 2008; Sud et al., 2007). Examining stochasic first passage time will be considered as well. One of the crucial tasks for future research will be testing the algorithm against available empirical data on wildfires.

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