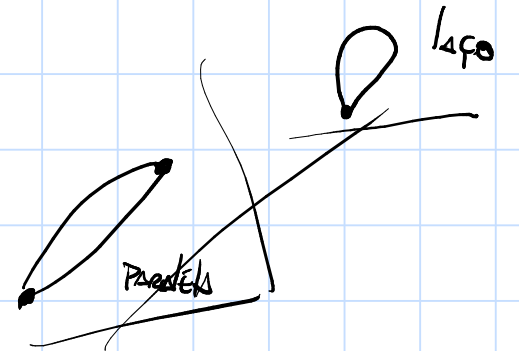


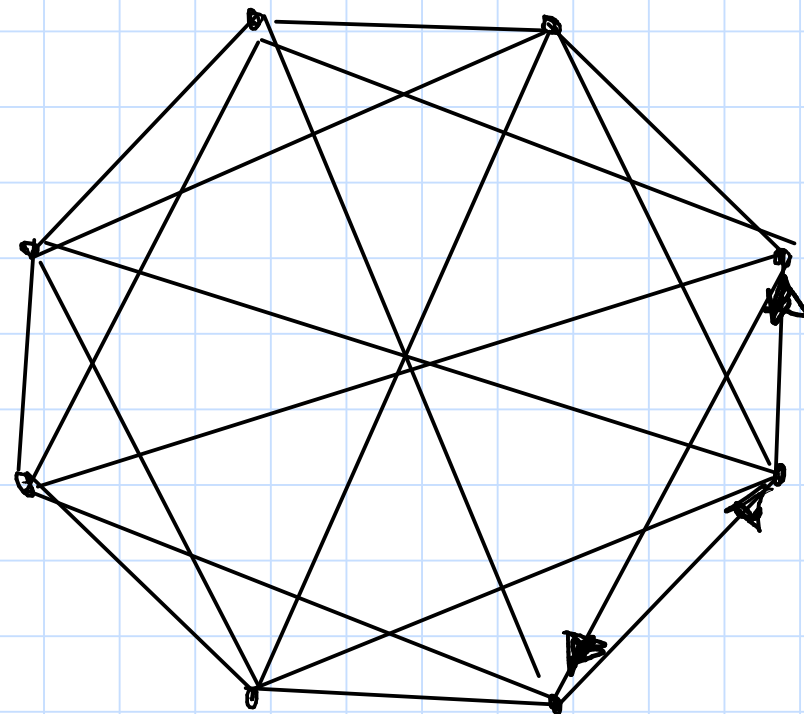
GRAFOS

SIMPLES

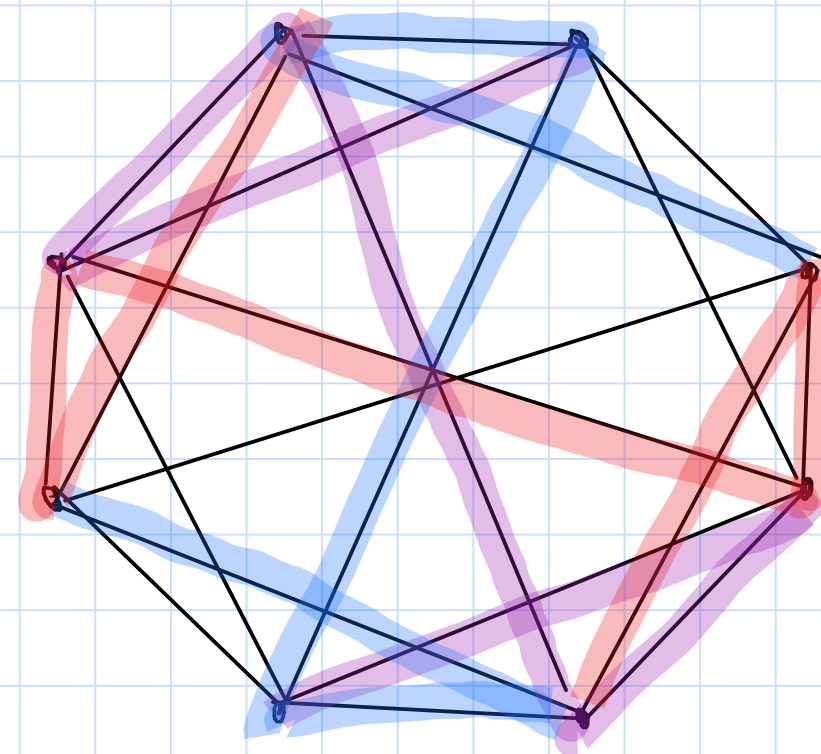
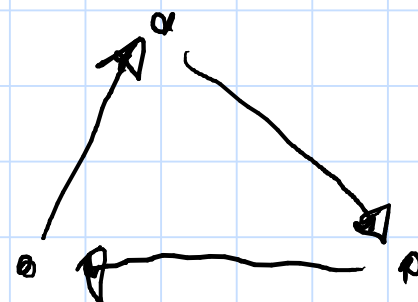
MULTIGRAFOS



K_6 COMPLETO



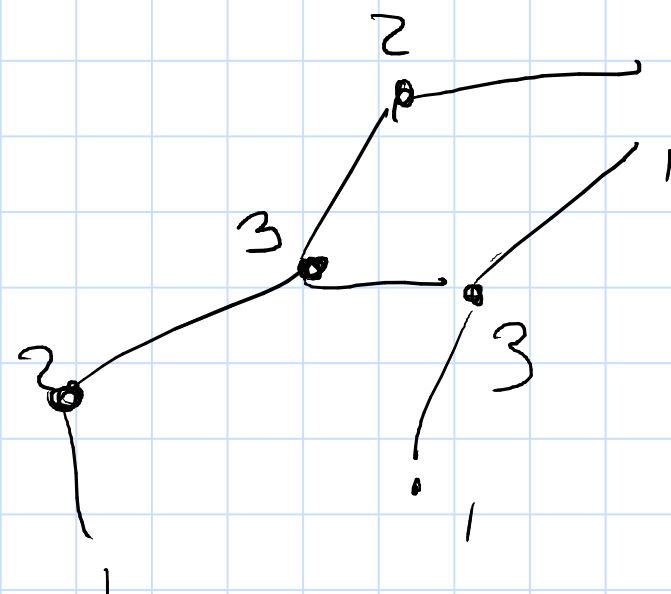
\rightarrow n° DE ARESTAS
 $2m$



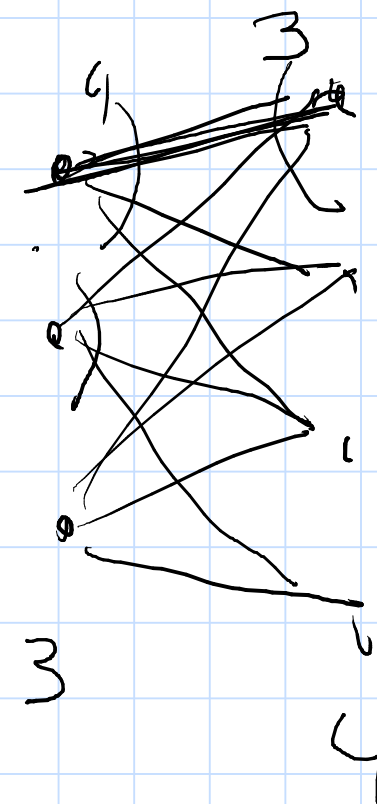
G é localmente irregular

CONJECTURA:

3 CORES



BIPARTIDO COMPLETO



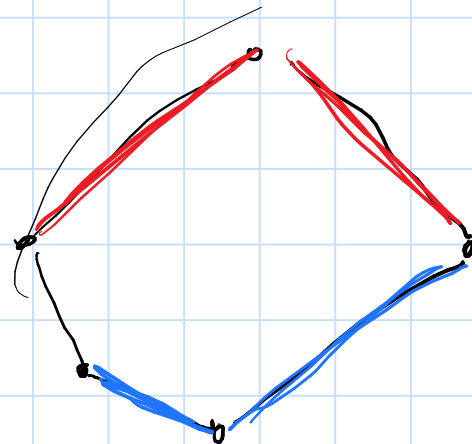
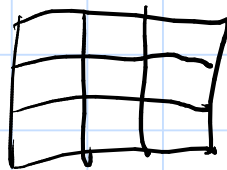
CONJ.: $\chi'_{\text{IRR}}(G) \leq 3$

MENOR # CORES EM UMA
COLORAÇÃO LOC. INEG

$$f: E(G) \rightarrow [1, \dots, c]$$

$$c \leq 3$$

G_1



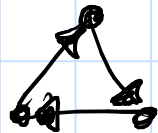
2x1

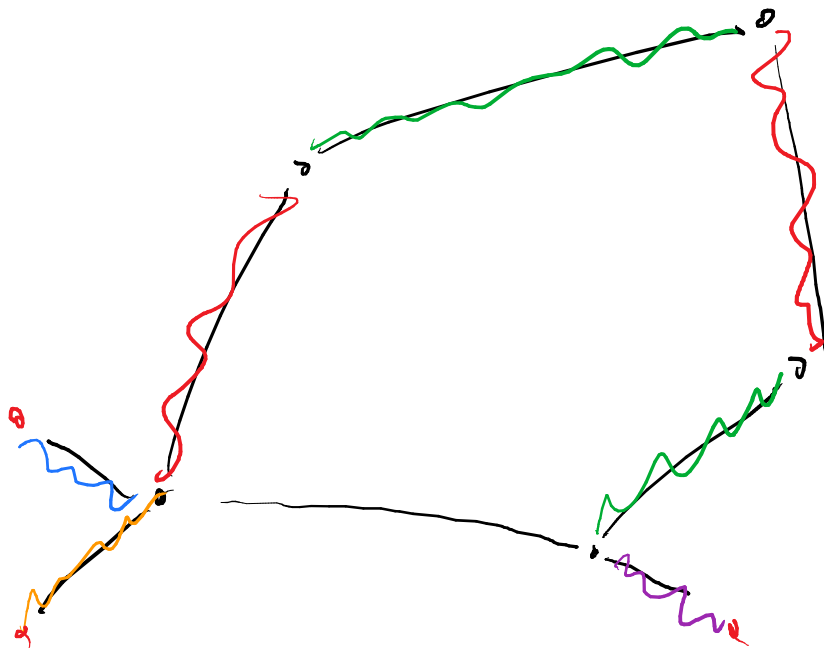
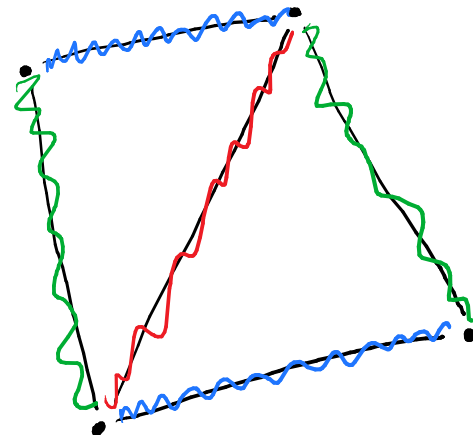
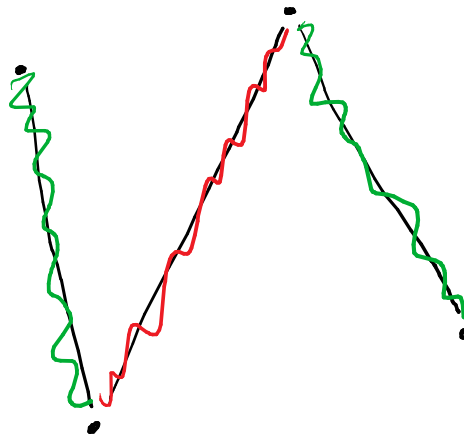
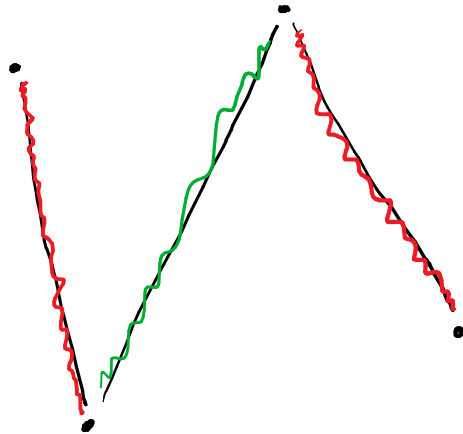
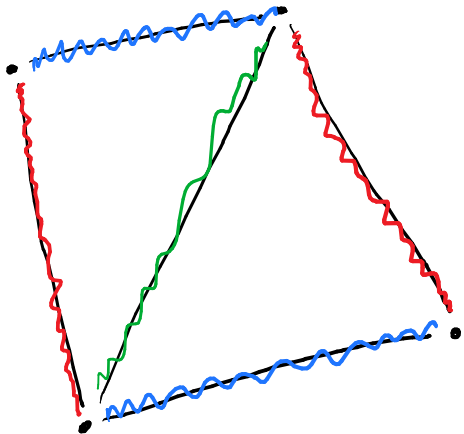
$$D(G, C_3^Q) \leq 2^{\lfloor \frac{n^2}{4} \rfloor}$$

n é nº de vértices de G

$$D(G, C_3^Q) = 2^{\lfloor \frac{n^2}{4} \rfloor}$$

$$\Leftrightarrow G = K_{\lfloor \frac{n}{2} \rfloor, \lfloor \frac{n}{2} \rfloor}$$



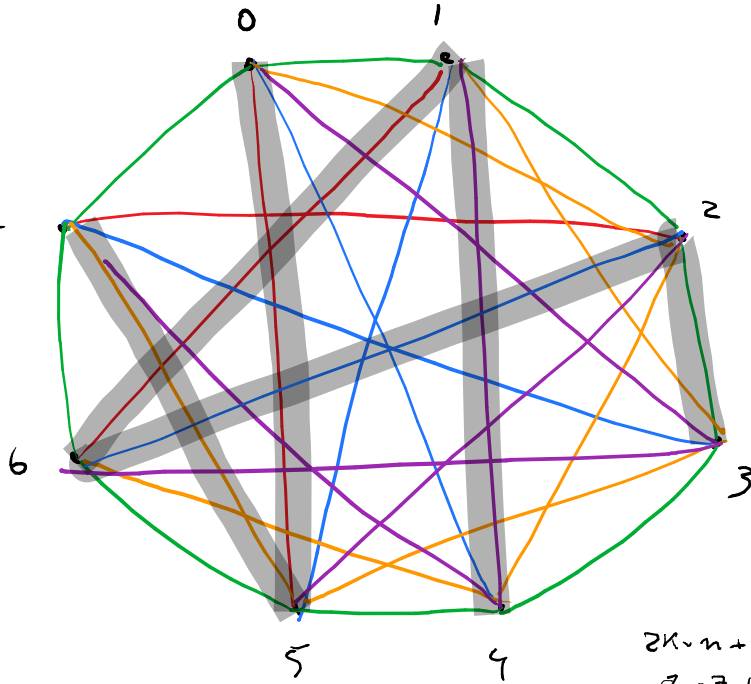


$$n - k - 1$$

0 5 7 3 2 6 1 4
5 2 4 1 4 5 3

6 6 7

$k=6$: 6 | 6 5 4 3 2 1
 $k=5$: 5, 4 | 5 4 3 2 1
 $k=4$: 4, 3, 2 | 4, 3, 2, 1



$$k=5$$

4

4 0 5 1 6 2 7 3

$$k=6$$

$$k=4$$

$$| 2 - 7 + 1$$

$$2k - n + 1$$

$$8 - 7 + 1 = 2$$

6

$$n - k - 1$$

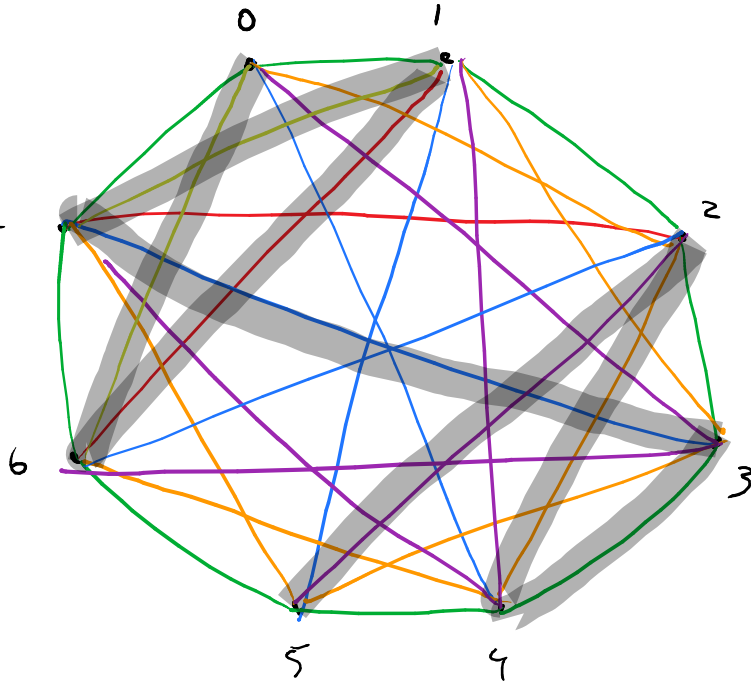
0 5 7 3 2 6 1 4
5 2 4 1 4 5 3

6 6 7

$k=6$: 6 1 6 5 4 3 2 1
 $k=5$: 5 4 5 4 3 2 1
 $k=4$: 4, 3, 2 | 4, 3, 2, 1

6 1 4 6
0 6 5 1 7

0 1 5 6 7
1 4 1 1



0 6 1 7 3 2 4
6 5 6 4 1 2

5
3

0 3 4 2 5 1 6 9
3 1 2 3 4 5 3

0 6 7 1

0 6 7 3 4 2 5
6 5 6 4 1 2 3

0 1

$$n - k - 1$$

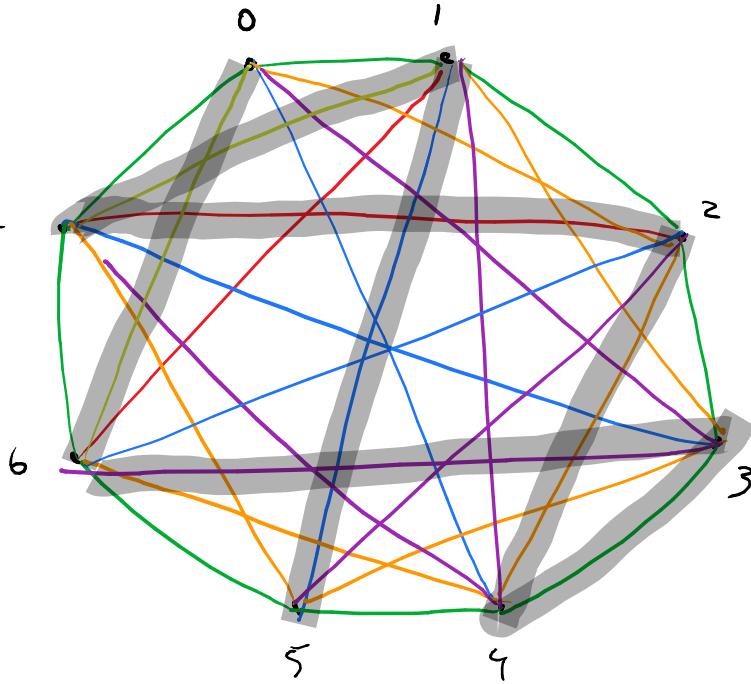
0 5 7 3 2 6 1 4
5 2 4 1 4 5 3

6 6 7

$k=6$: 6 6 5 4 3 2 1
 $k=5$: 5 4 5 4 3 2 1
 $k=4$: 4, 3, 2 | 4, 3, 2, 1

6 1 4 6
0 6 5 1 7

0 1 5 6 7
1 4 1 1



0 6 3 4 2 7 1 5
6 3 1 2 5 6 4

0 6 1 7 3 2 4
6 5 6 4 1 7

5
3

0 3 4 2 5 1 6 9
3 1 2 3 4 5 3

0 6 7 1

0 6 7 3 4 2 5
6 5 6 4 1 2 3

0 1

$$n - k - 1$$

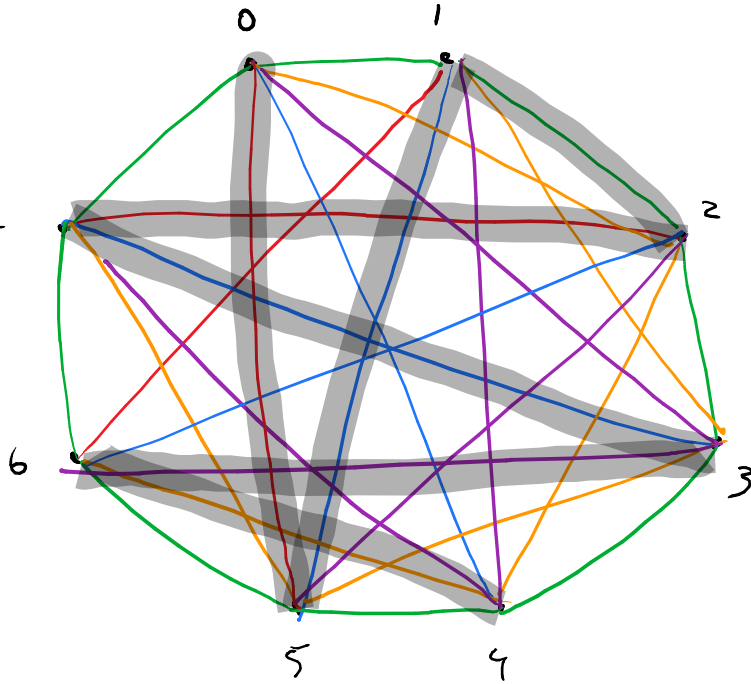
0 5 7 3 2 6 1 4
5 2 4 1 4 5 3

6 6 7

$k=6$: 6 | 5 4 3 2 1
 $k=5$: 5, 4 | 5 4 3 2 1
 $k=4$: 4, 3, 2 | 4, 3, 2, 1

6 1 4 6
0 6 5 1 7

0 1 5 6 7
1 4 1 1



0 6 3 4 2 7 1 5
6 3 1 2 5 6 4

0 6 1 7 3 2 4
6 5 6 4 1 7

5
3

0 3 4 2 5 1 6 9
3 1 2 3 4 5 3

0 6 7 1

0 6 7 3 4 2 5
6 5 6 4 1 2 3

0

5

0 1

0

4

0 5 1 2 7 3 6 4
5 4 1 5 4 3 2

$n - k - 1$

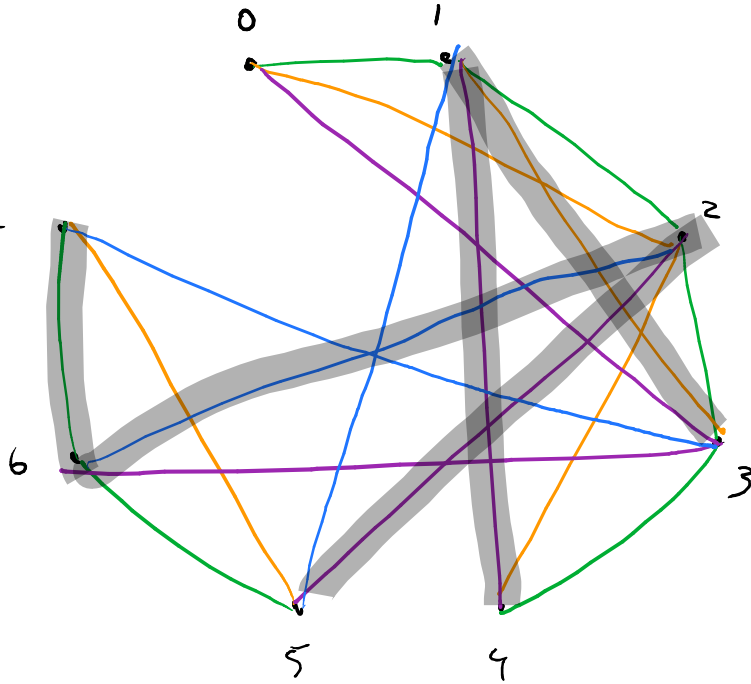
0 5 7 3 2 6 1 4
5 2 4 1 4 5 3

6 6 7

$k=6$: 6 1 5 4 3 2 1
 $k=5$: 5 4 1 5 4 3 2 1
 $k=4$: 4 3 2 1 4 3 2 1

6 1 4 6
0 6 5 1 7

0 1 5 6 7
1 4 1 1



0 6 4 2 7 1 5
6 3 1 2 5 6 4

0 4 1 3 5 2 6 7
4 3 2 2 3 4 1
— A —

0 6 1 7 3 2 4
6 5 6 4 1 2 3

0 3 4 2 5 1 6 9
3 1 2 3 4 5 3

0 6 7 1

0 6 7 3 4 2 5
6 5 6 4 1 2 3

0 1

0

5

0

4

0 5 1 2 7 3 6 4
5 4 1 5 4 3 2

5 4 4 5

$$2K - n + 1 \quad \begin{matrix} K = \frac{n}{2} \\ \Delta \end{matrix} \quad n - n + 1$$

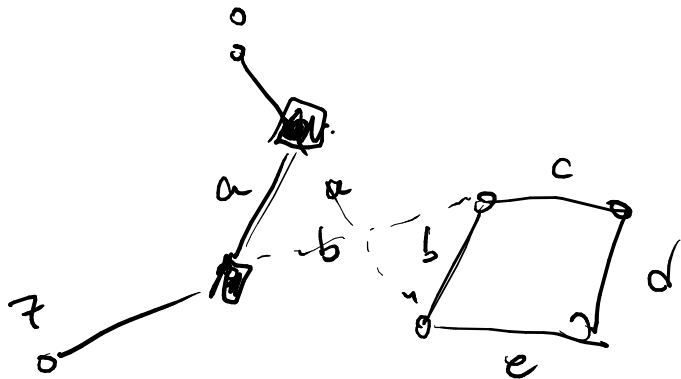
$$\hookrightarrow$$

$$K = n - 1$$

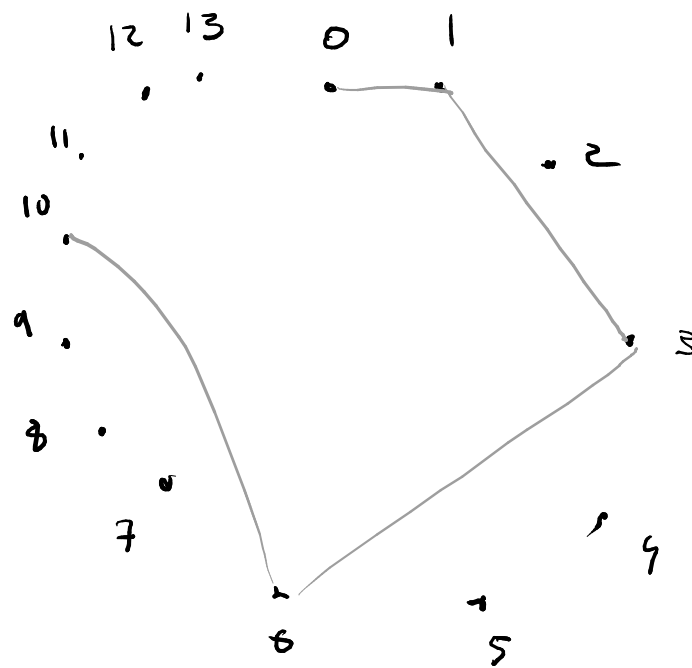
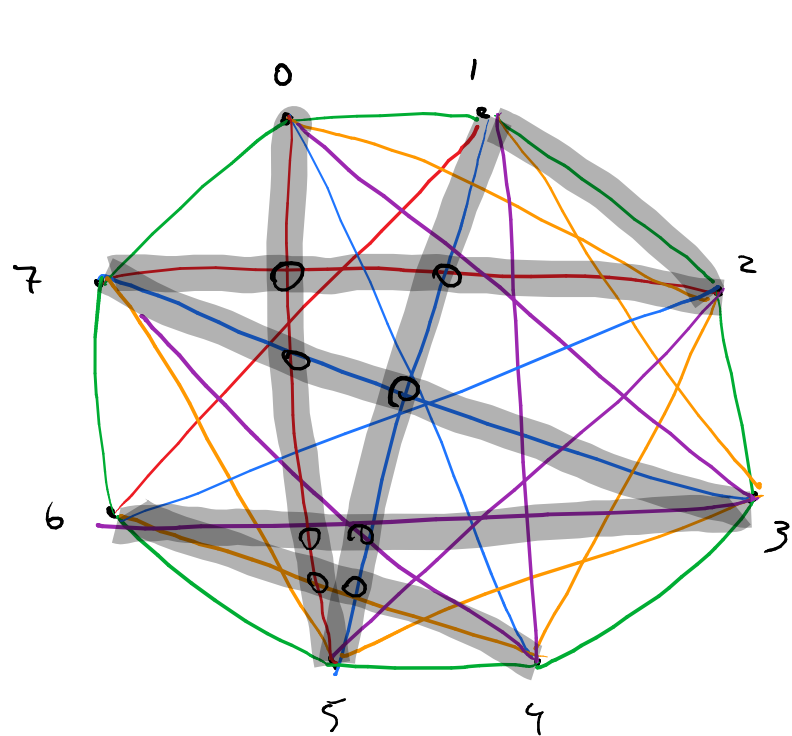
$$2(n - 1) - n + 1$$

$$2n - 2 - n + 1$$

$$n - 1$$



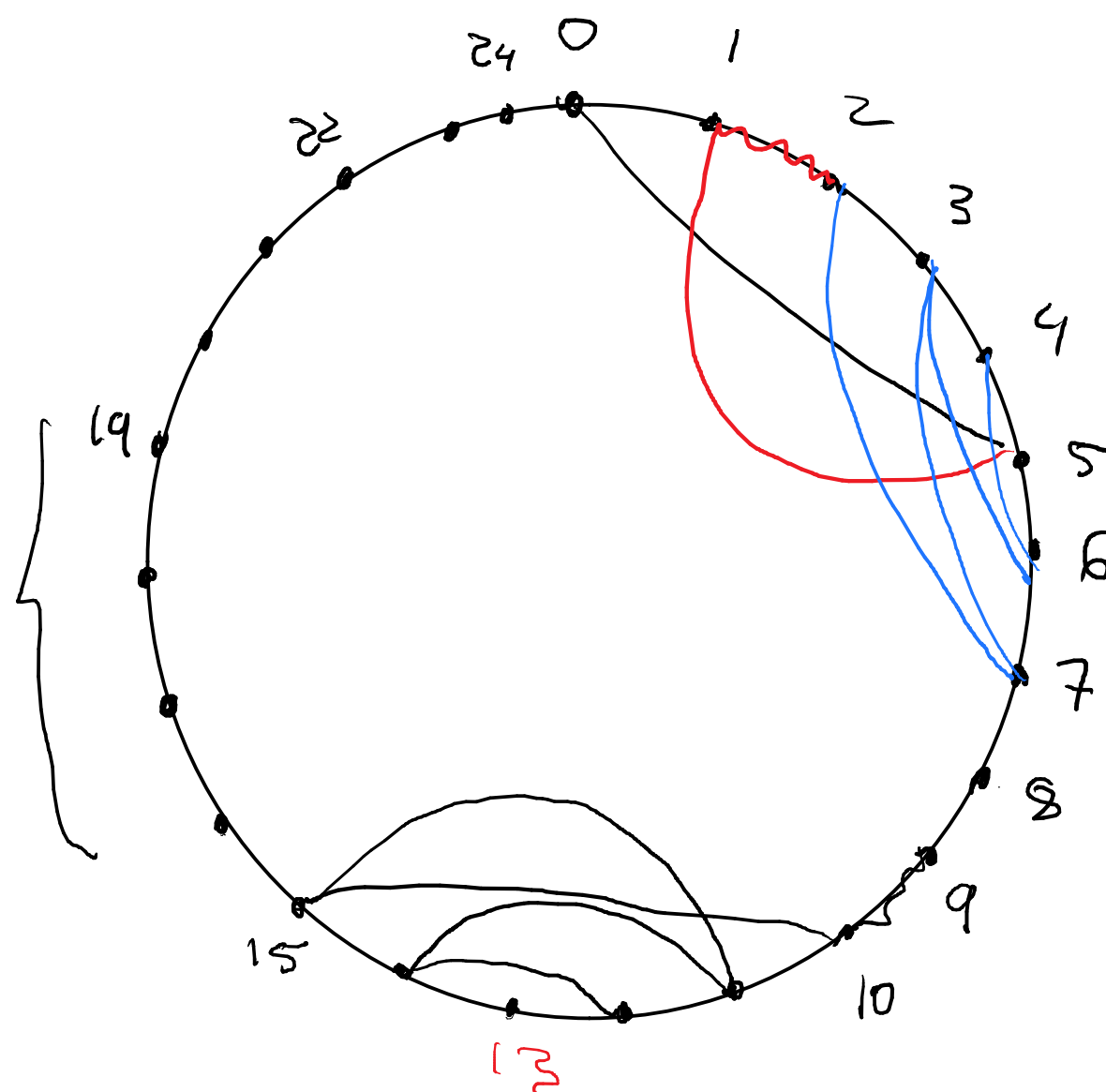
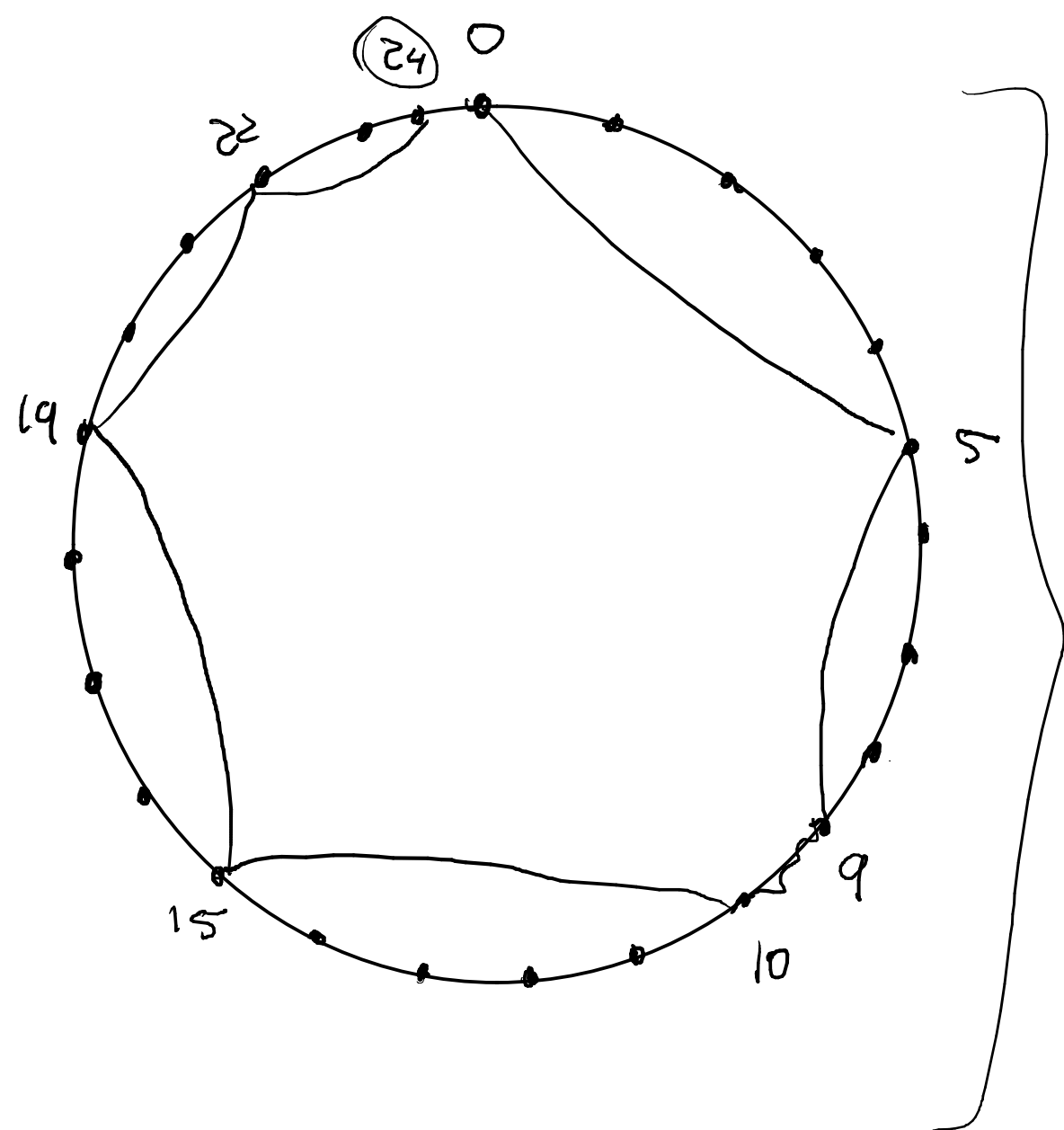
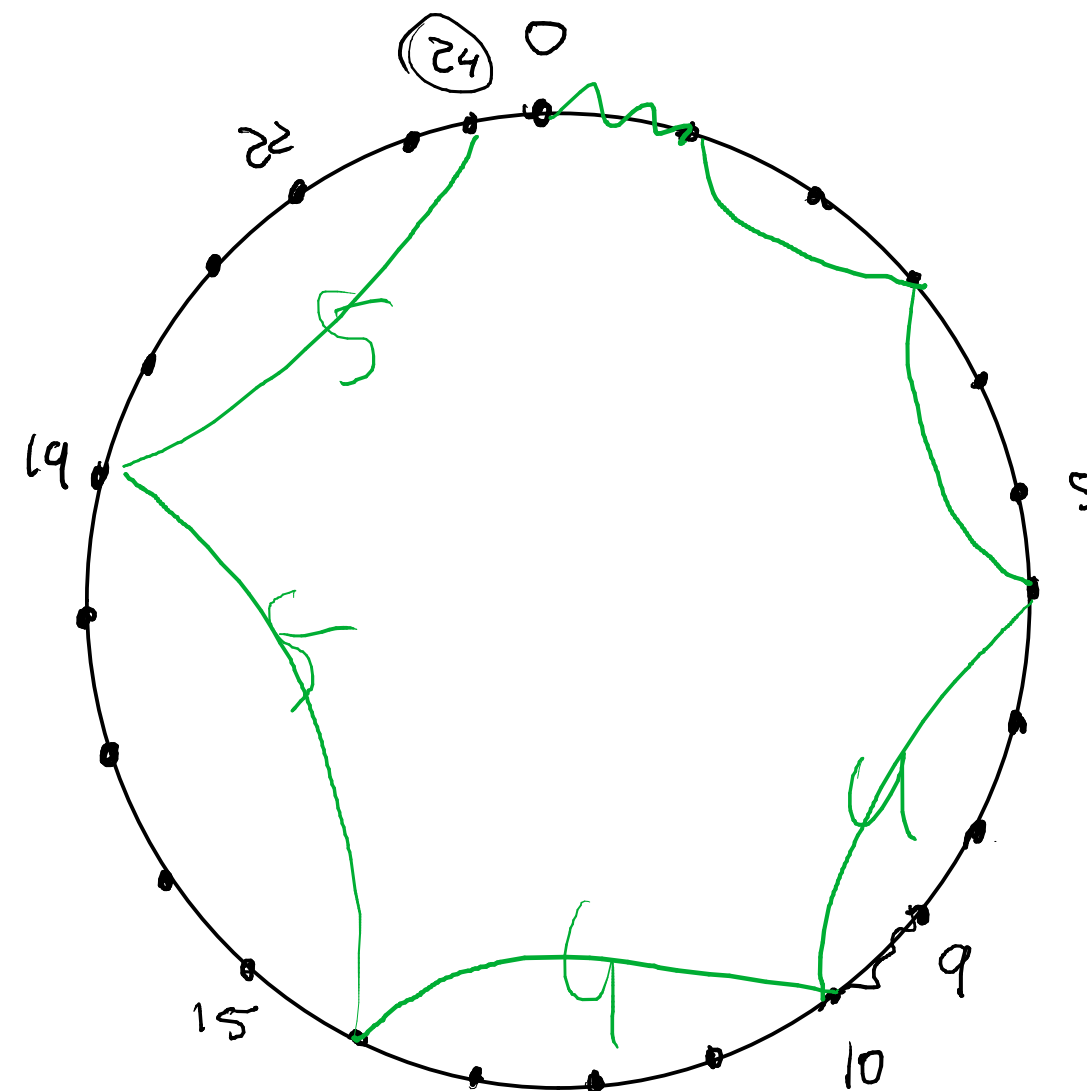
① 3 2 4 3 2 4



0 5 1 2 7 3 6 4
 5 4 1 5 4 3 2 ~ }
 0 5 9 10 15 19 22 24

$$0 + 1 + 2 + 3 + 4 + 4 + 5 + 5 = 24$$

0 5 1 2 7 3 6 4



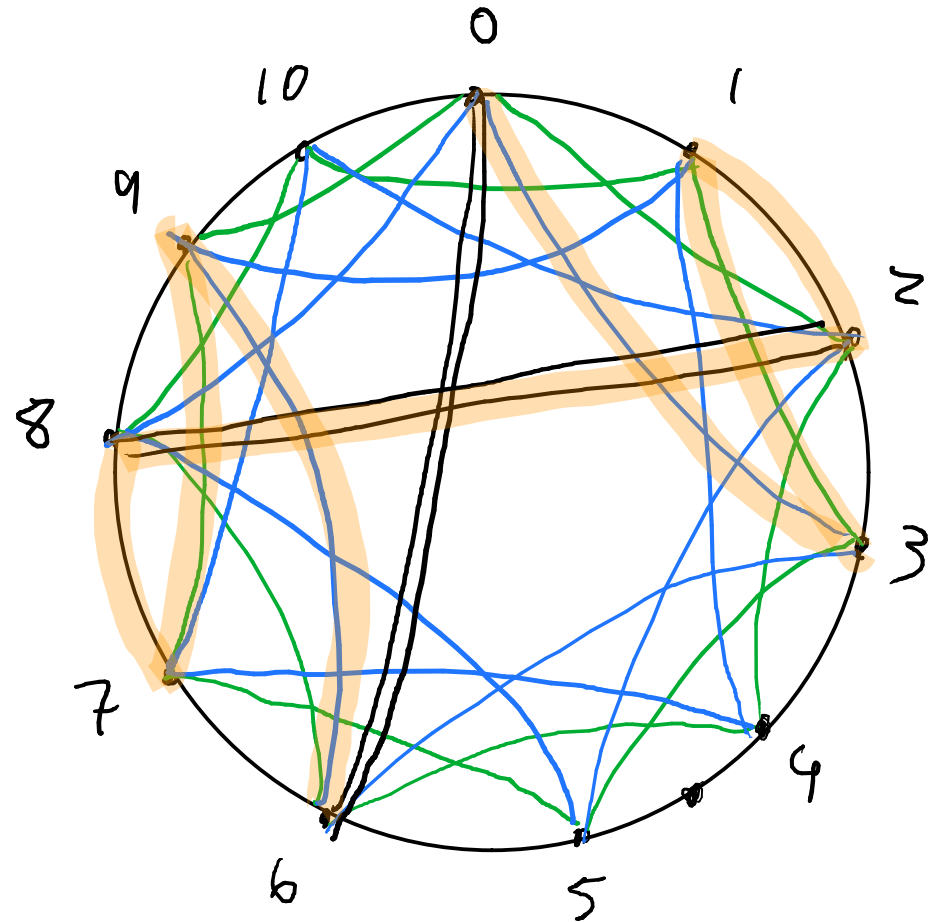
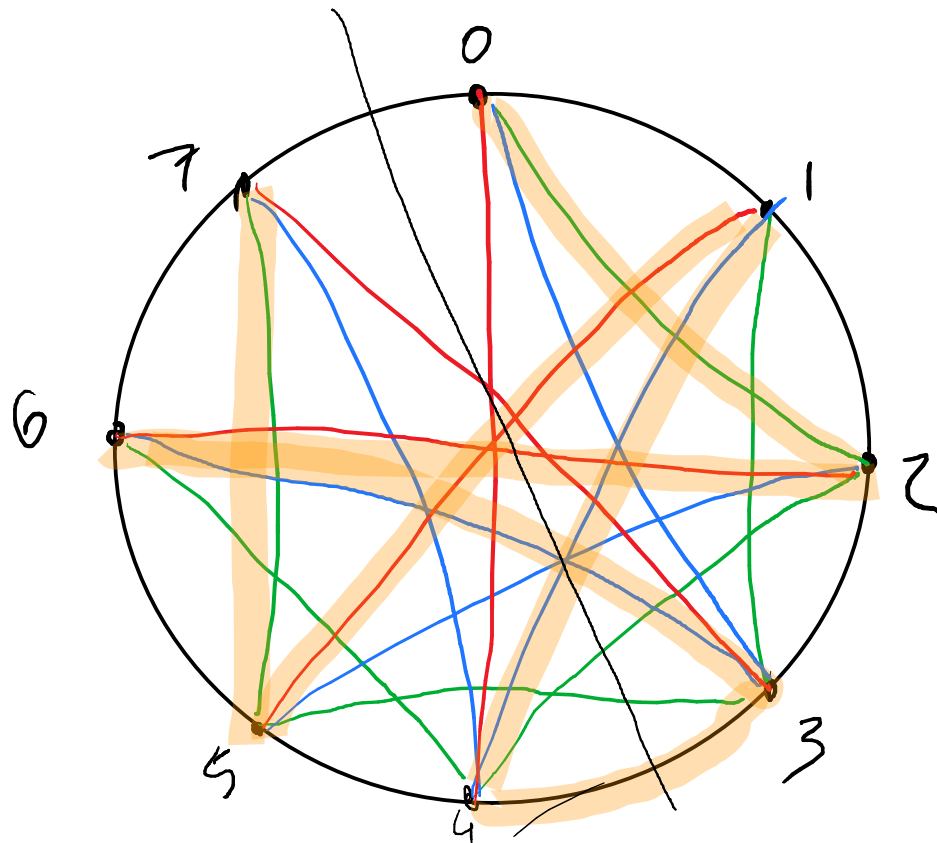
$$\frac{n}{2} \leq k \leq n$$

REPETIDOS : $2k - n + 1 \dots k$

EX: $k=5$ $n=7$ 4, 5

$k=6$ $n=7$ 6

$k=4$ $n=7$ 2 ... 4



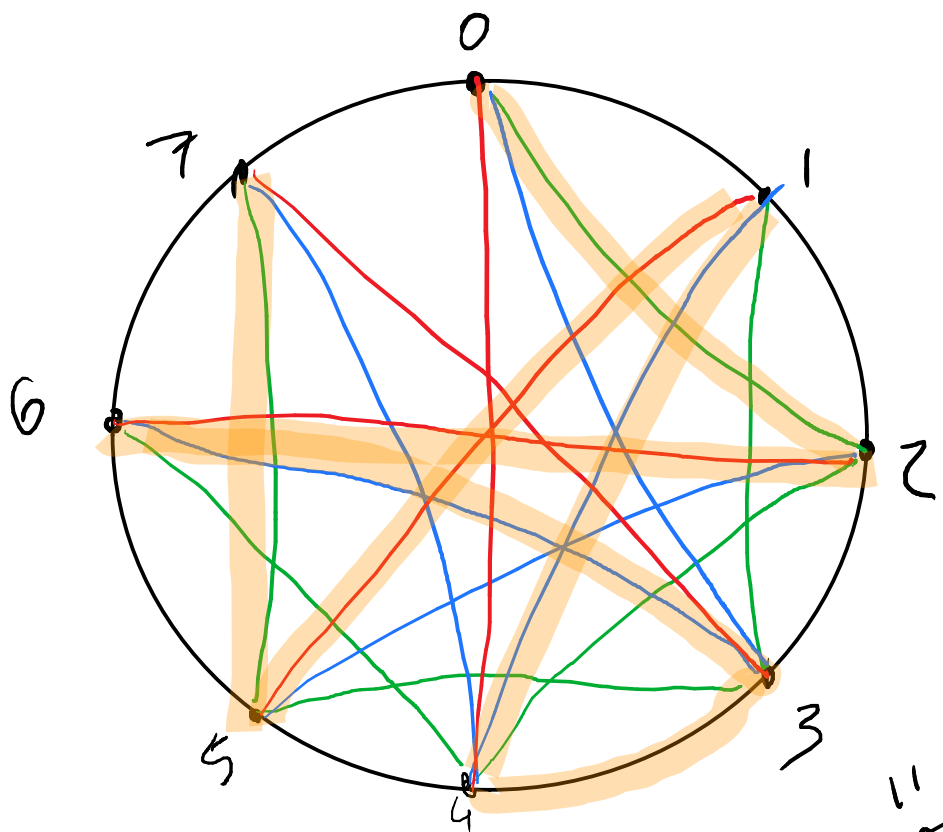
x_0

$7x_1, x_2 \dots x_7$

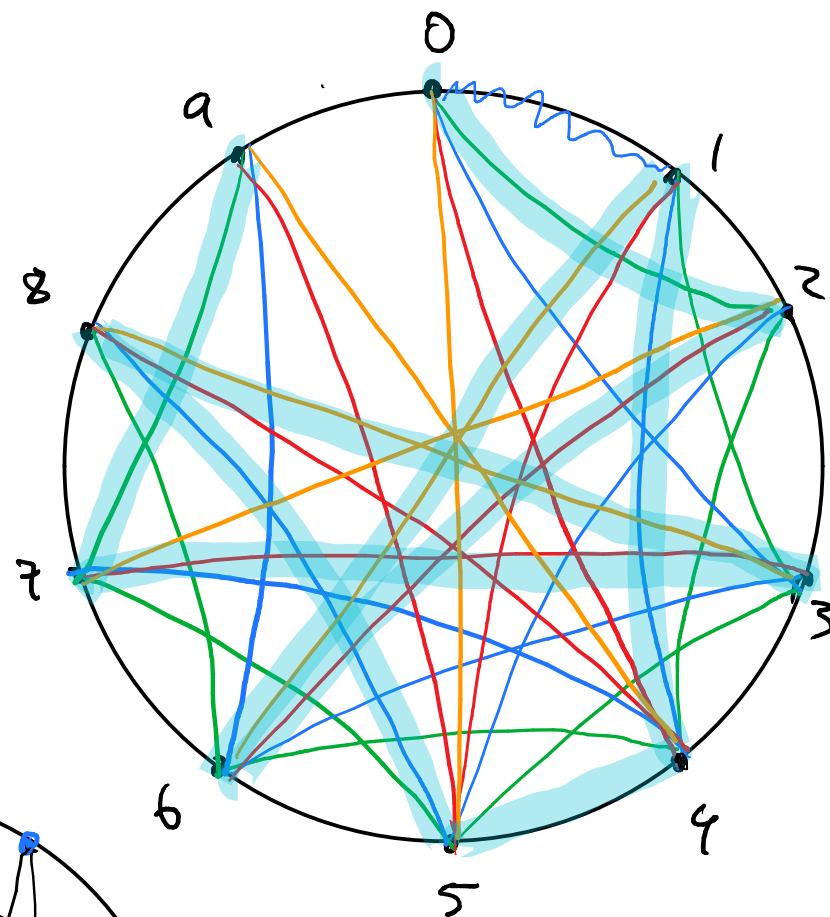
$07-x_1, 7x_2 \dots 7-x_7$

H-DECOMP. G

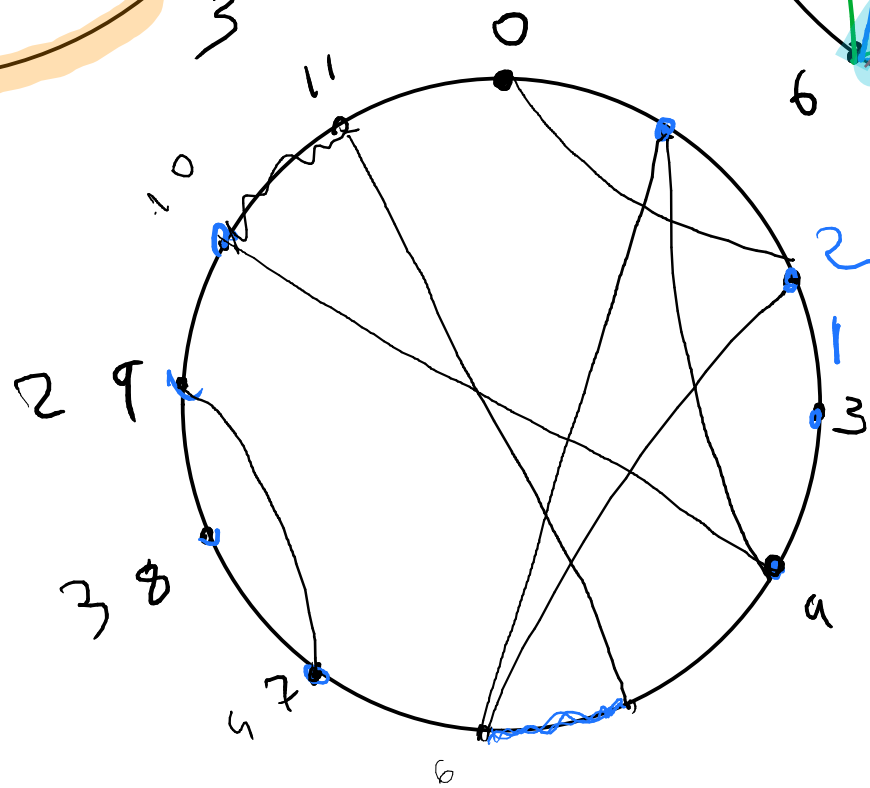
$y_i = 7-x_i$ $\mathcal{D} = \{H_1, \dots, H_{n-1}\}$



0263
 243
 ~~~~~



026145  
 2453  
 ~~~~~  
 65

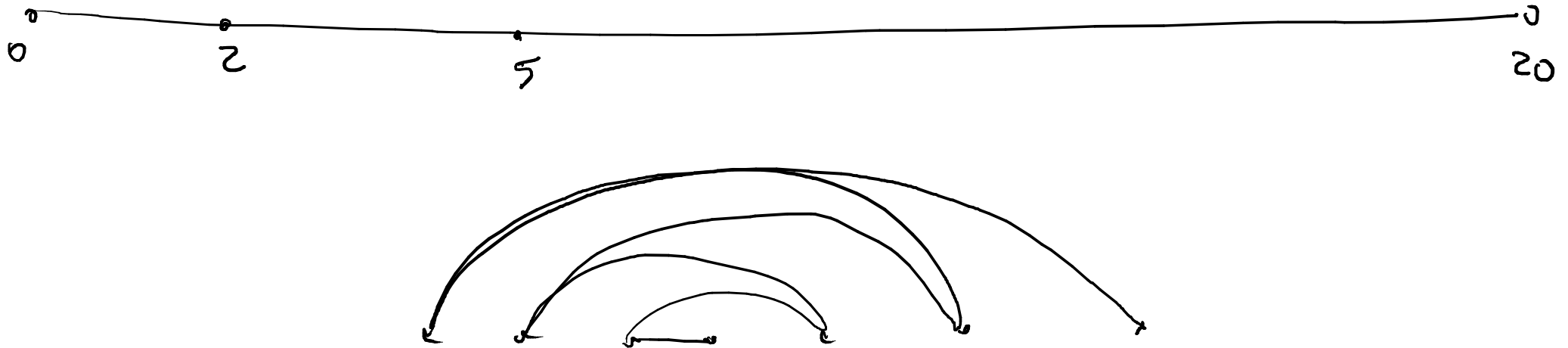


$$n = 11 \quad k = 6$$

$$2k - n + 1 = 2 \quad \dots \quad 6 = k$$

$$1 + 2 \cdot \sum_{i=2}^6 i \quad 1 + 2 \cdot 20 = 41$$

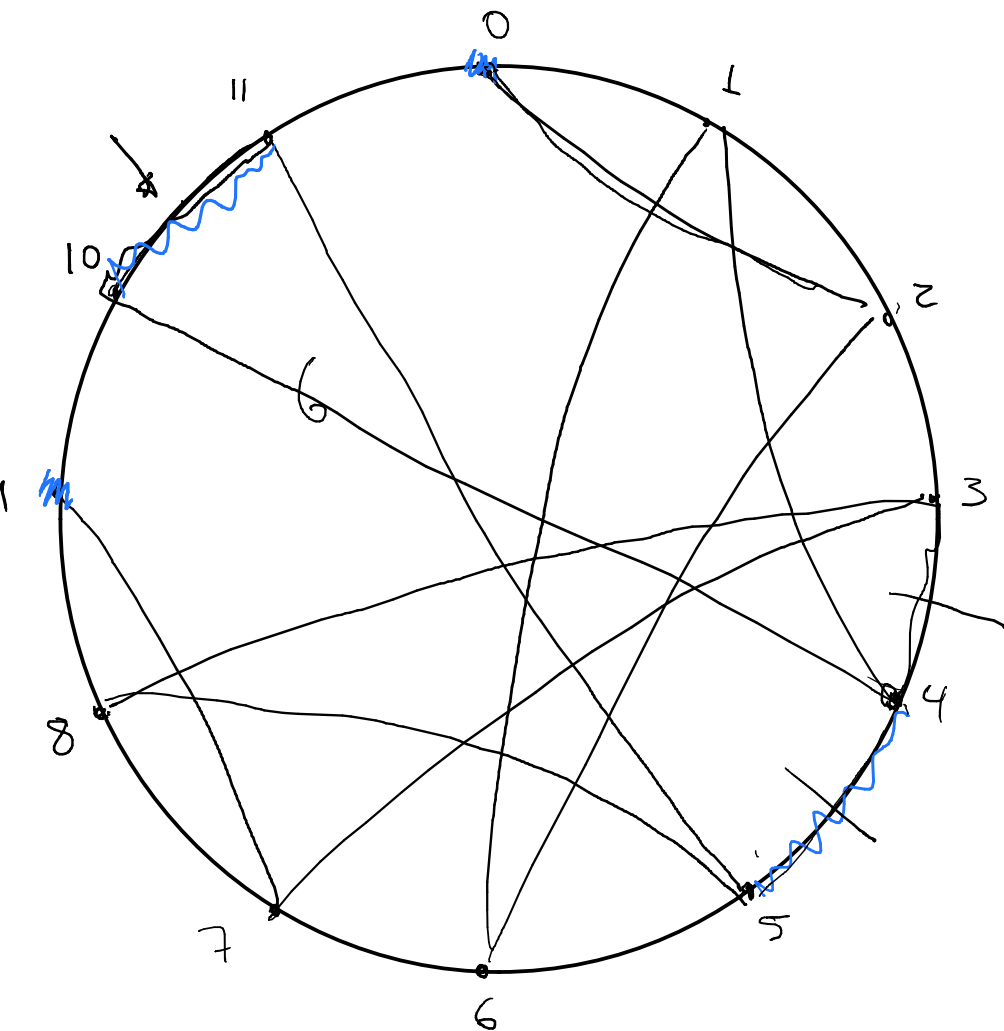
2 3 4 5 6



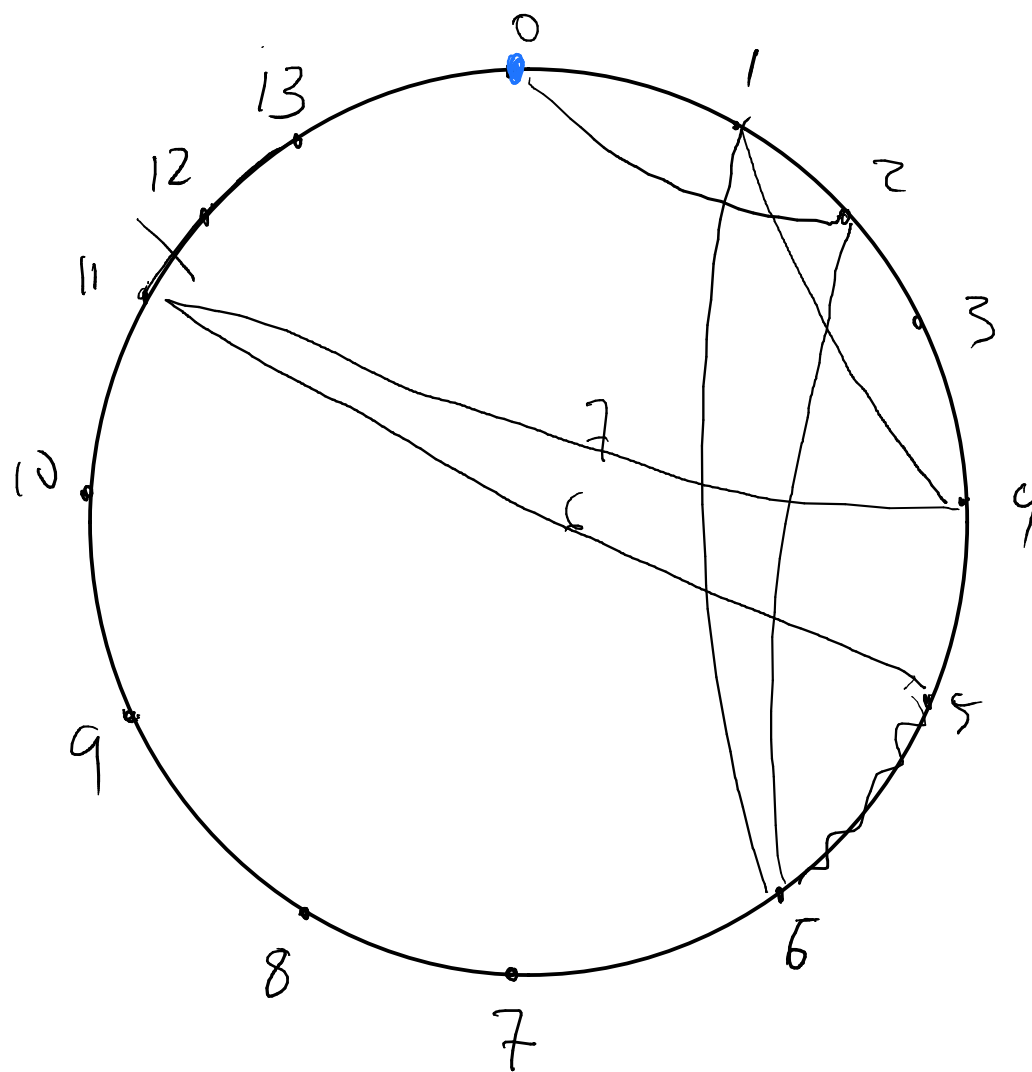
$$n=11$$

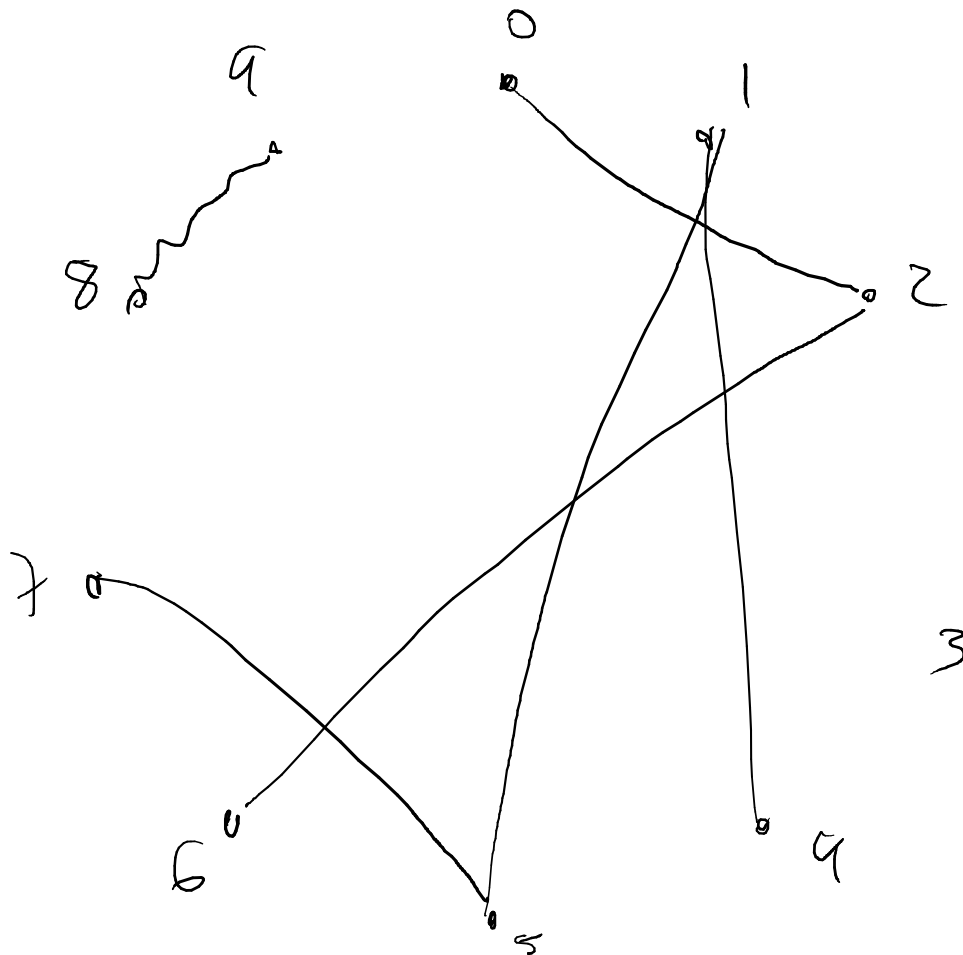
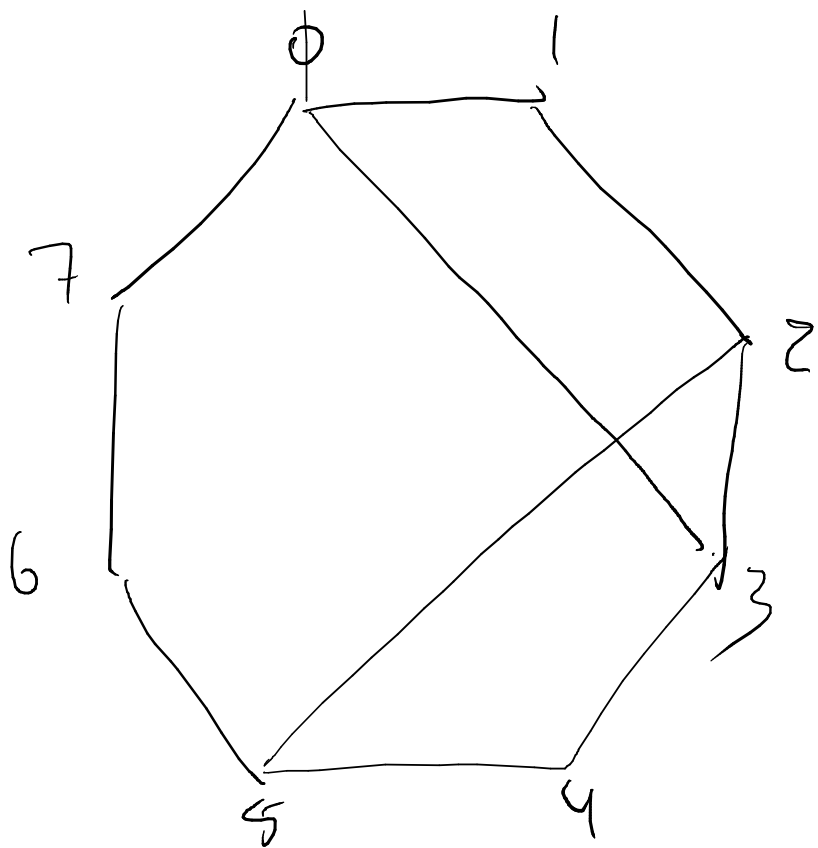
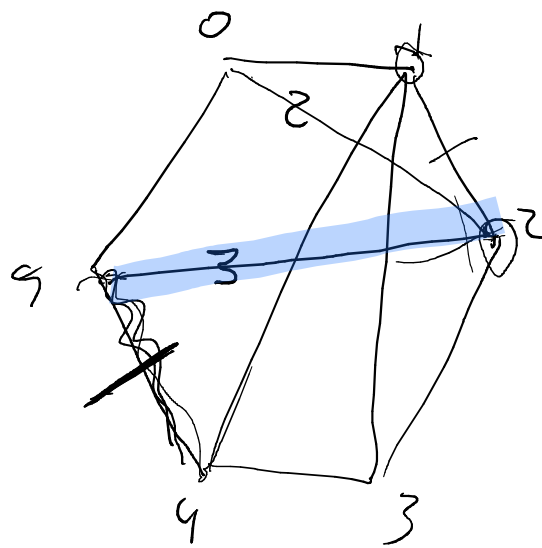
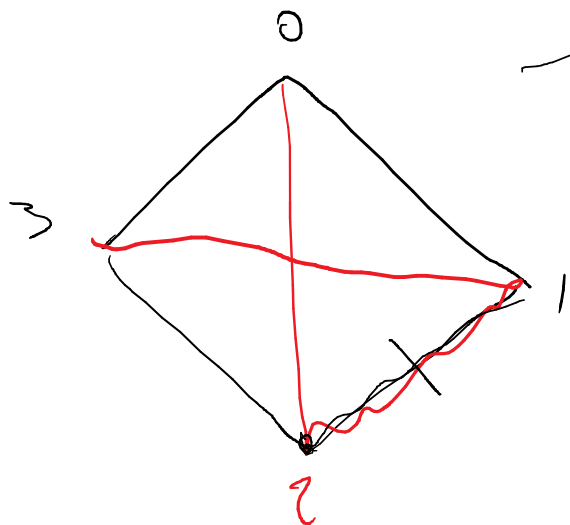
$$K=6$$

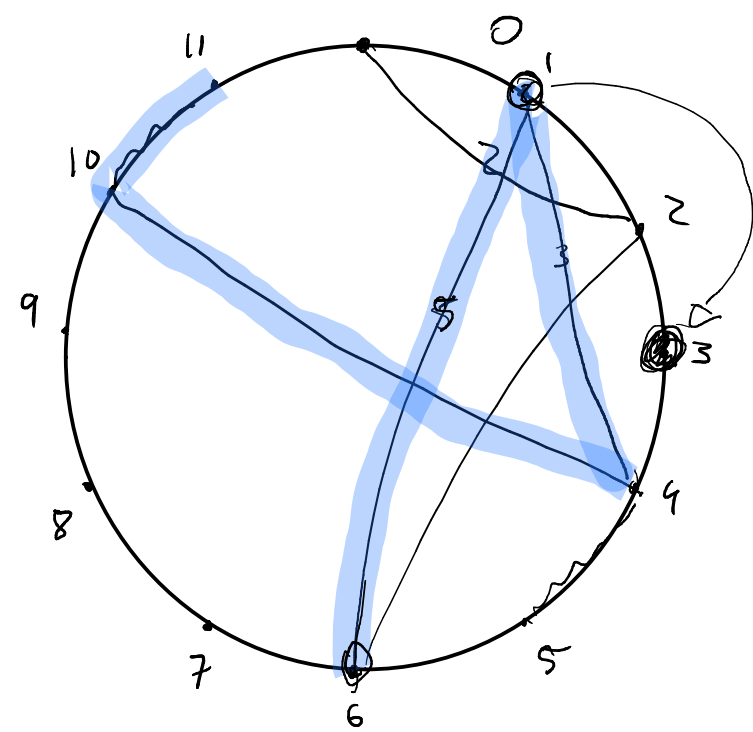
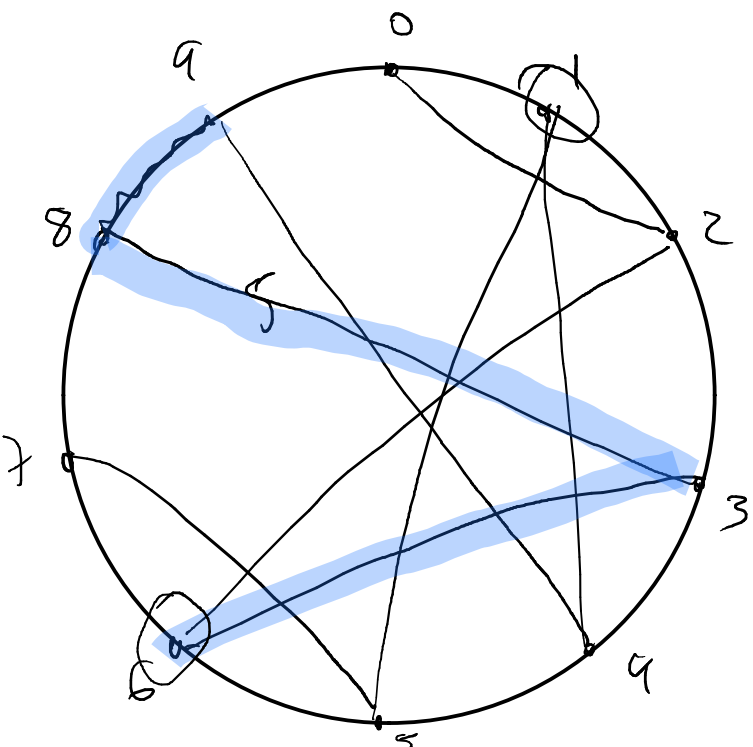
0 2 6 1 4 10 11



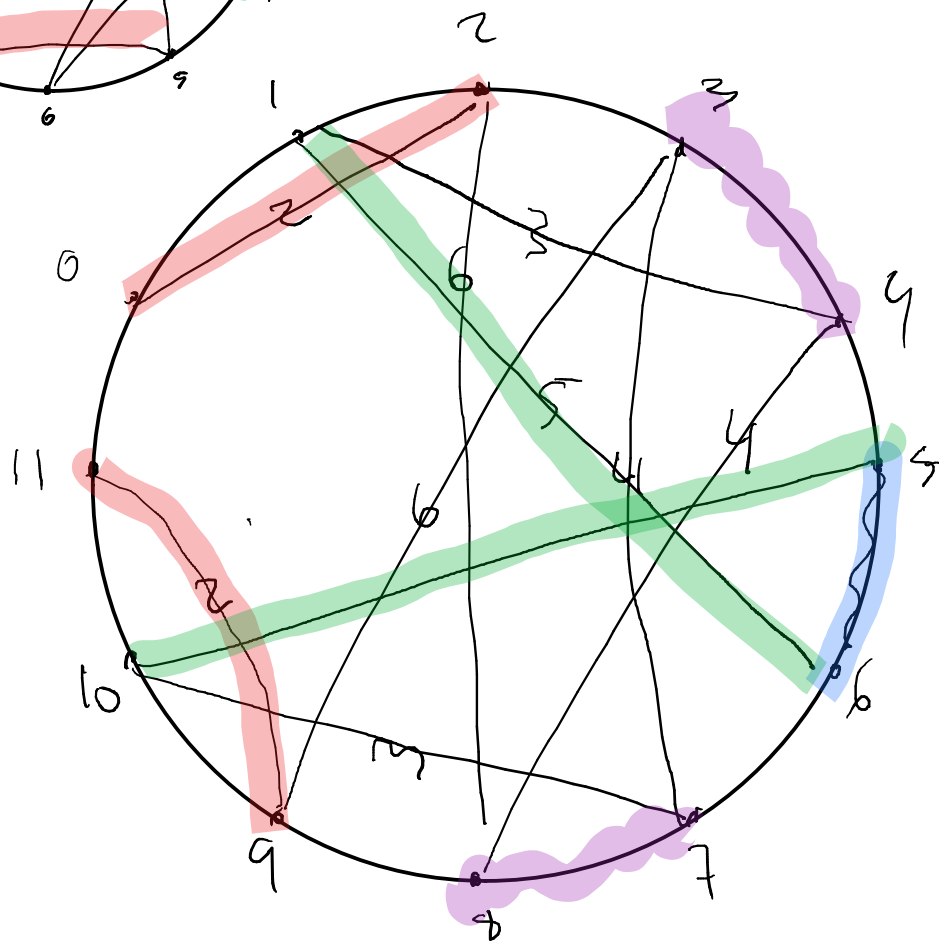
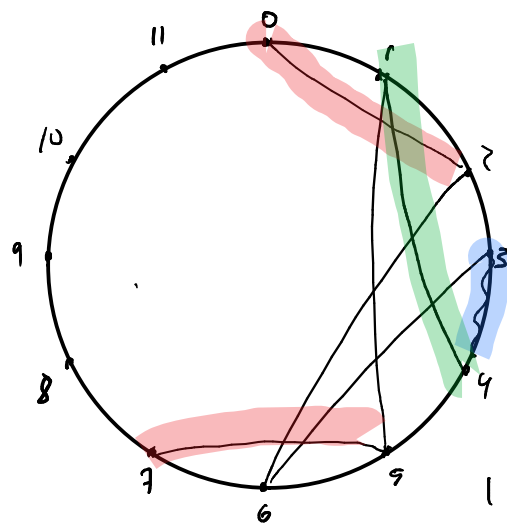
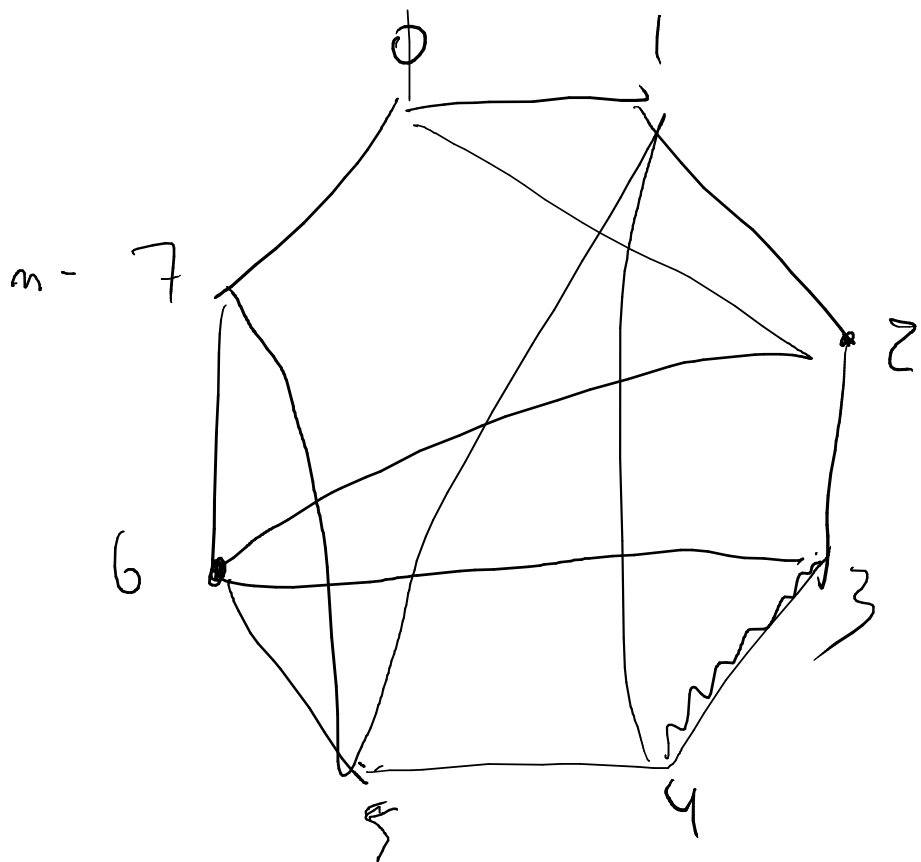
0 3 1 7 2 6 5 9 4 10 8 11
3 2 6 5 4 1 4 5 6 2 3





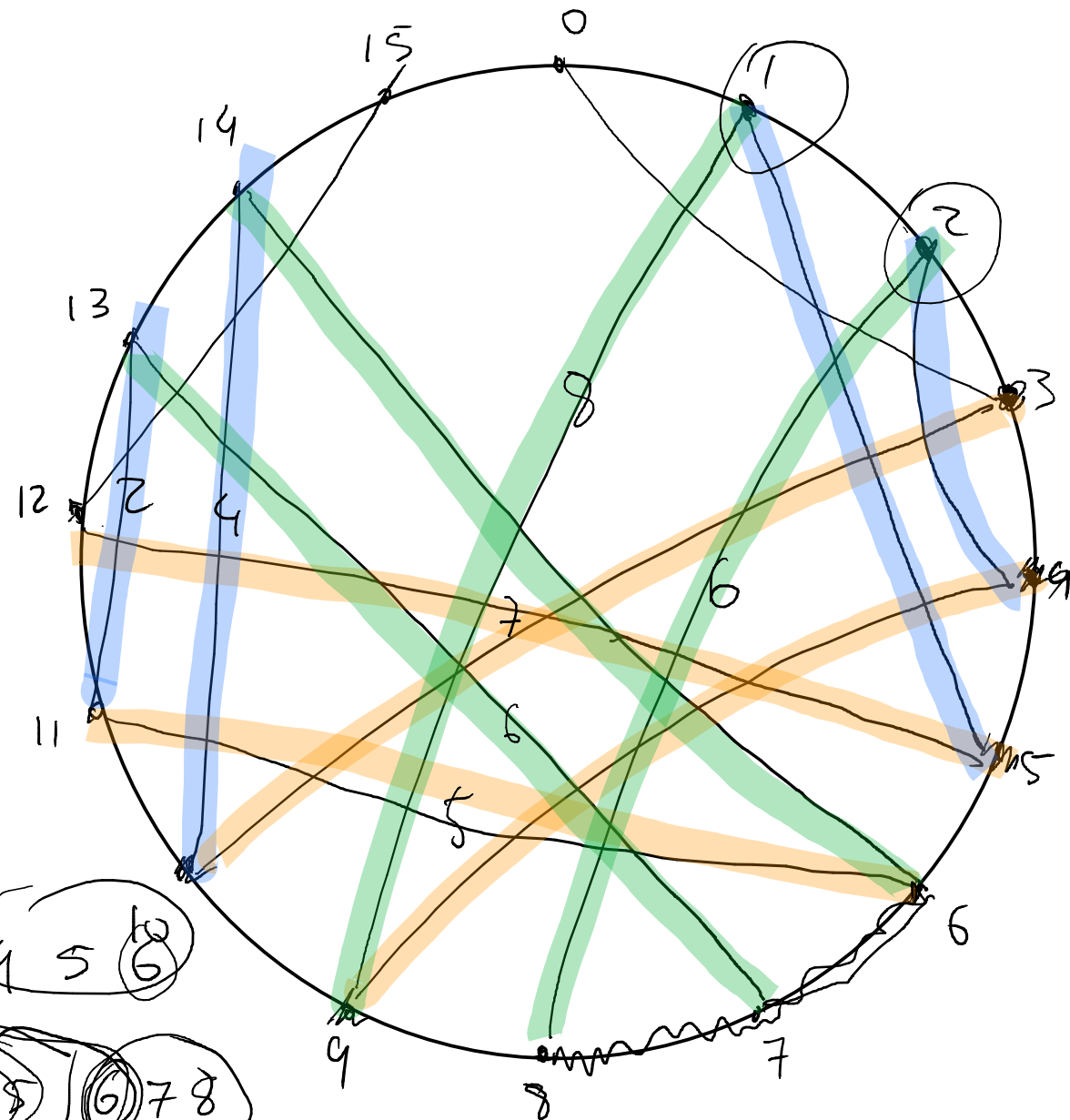
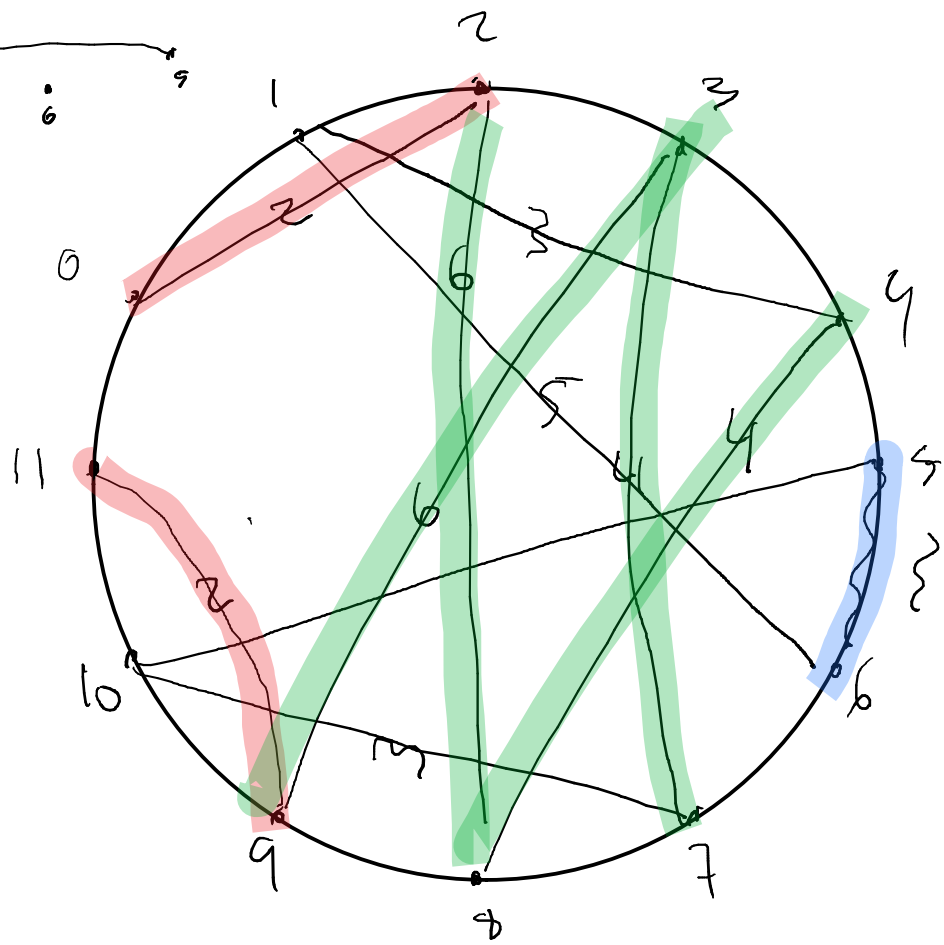


3 5 1
5 3 6 1



②4 ⑤7 ⑥8

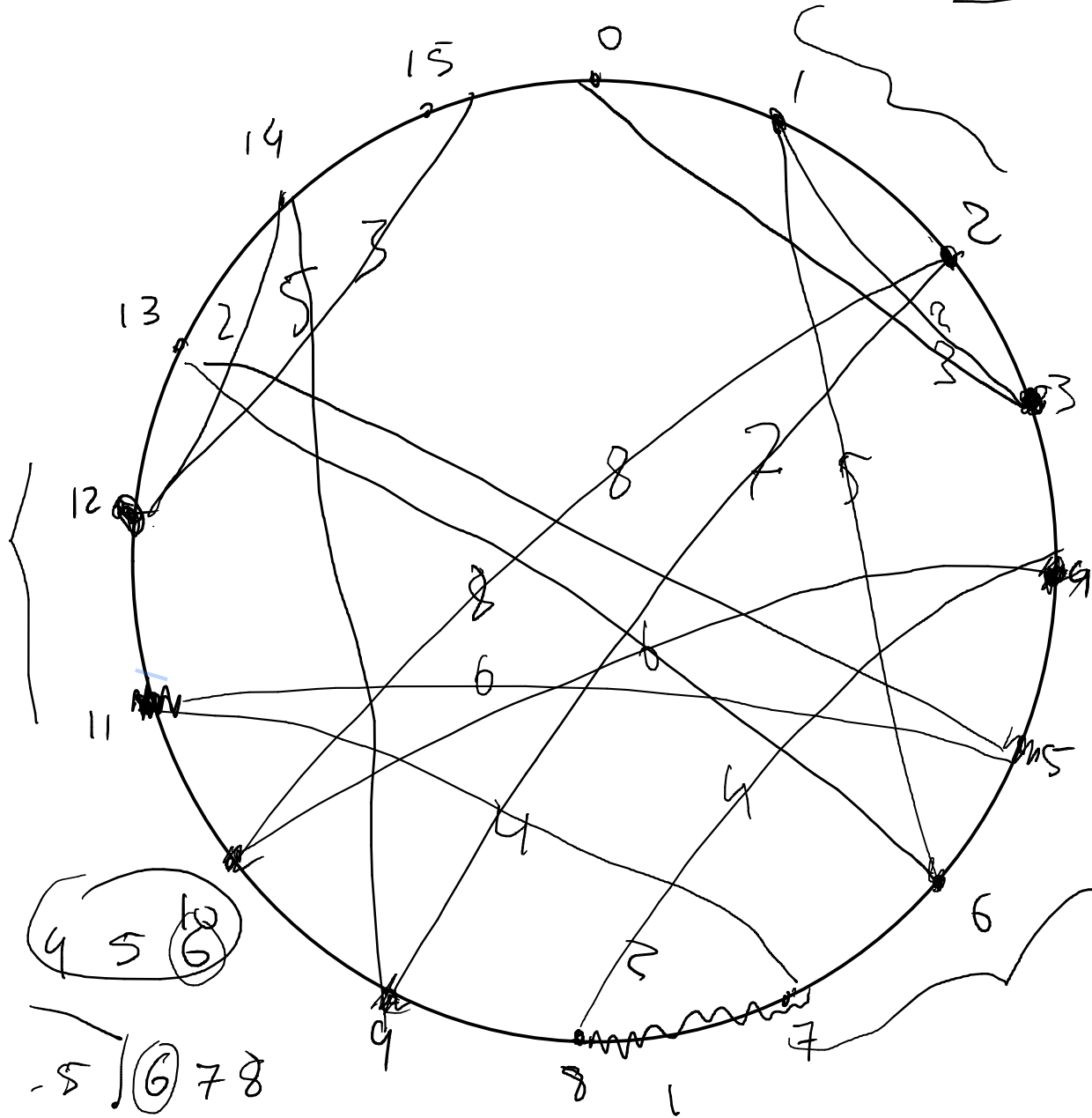
~~1~~ ~~2~~ ~~3~~ ~~4~~ ~~5~~ ~~6~~ ~~7~~ ~~8~~

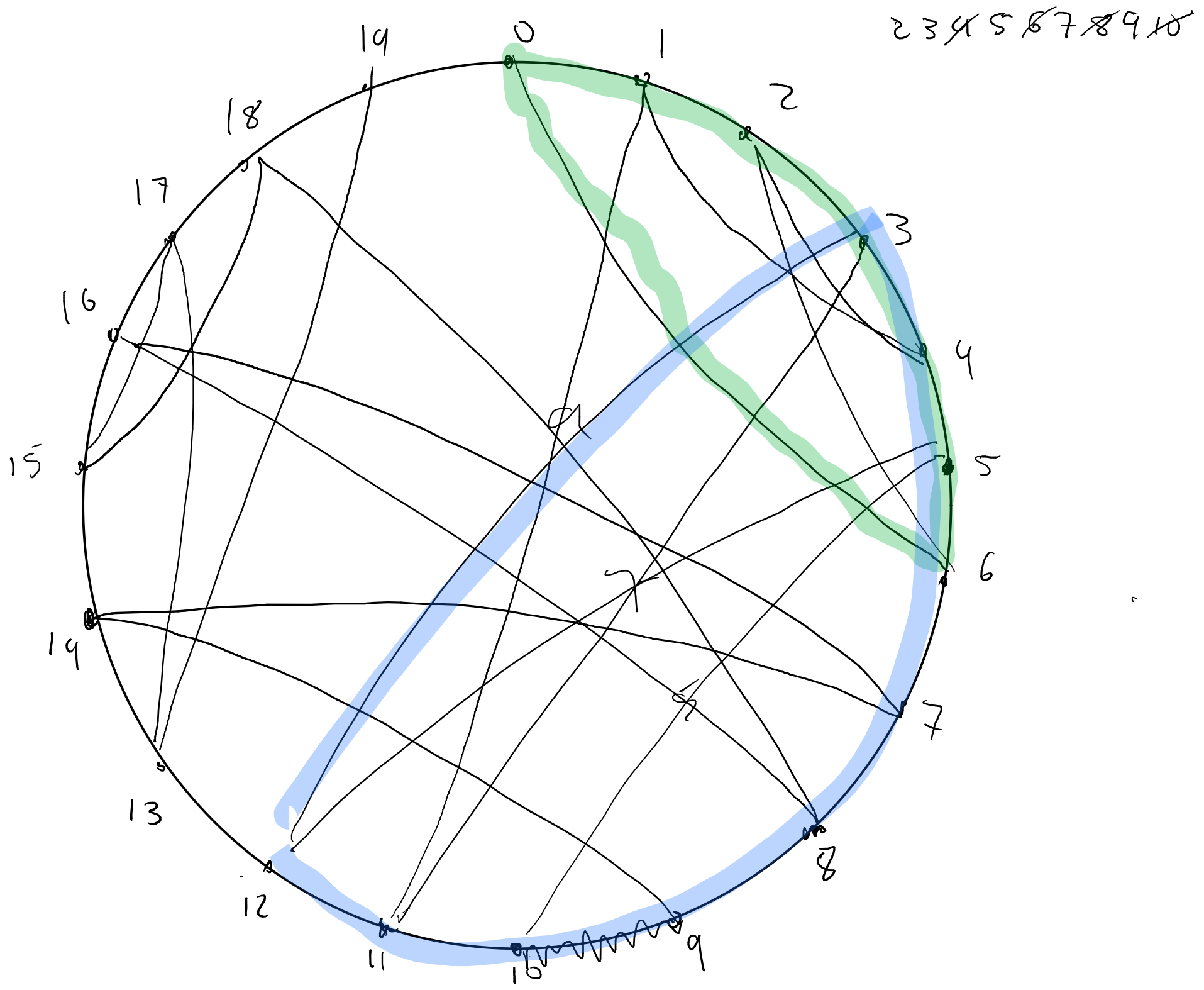


①②③ | ④ ⑤ ⑥

①②③ | ④ ⑤ ⑥
1 2 3 | ④ ⑤ | ⑥ ⑦ ⑧

2 (3) 4 5 6 7 8





$$n = 15$$

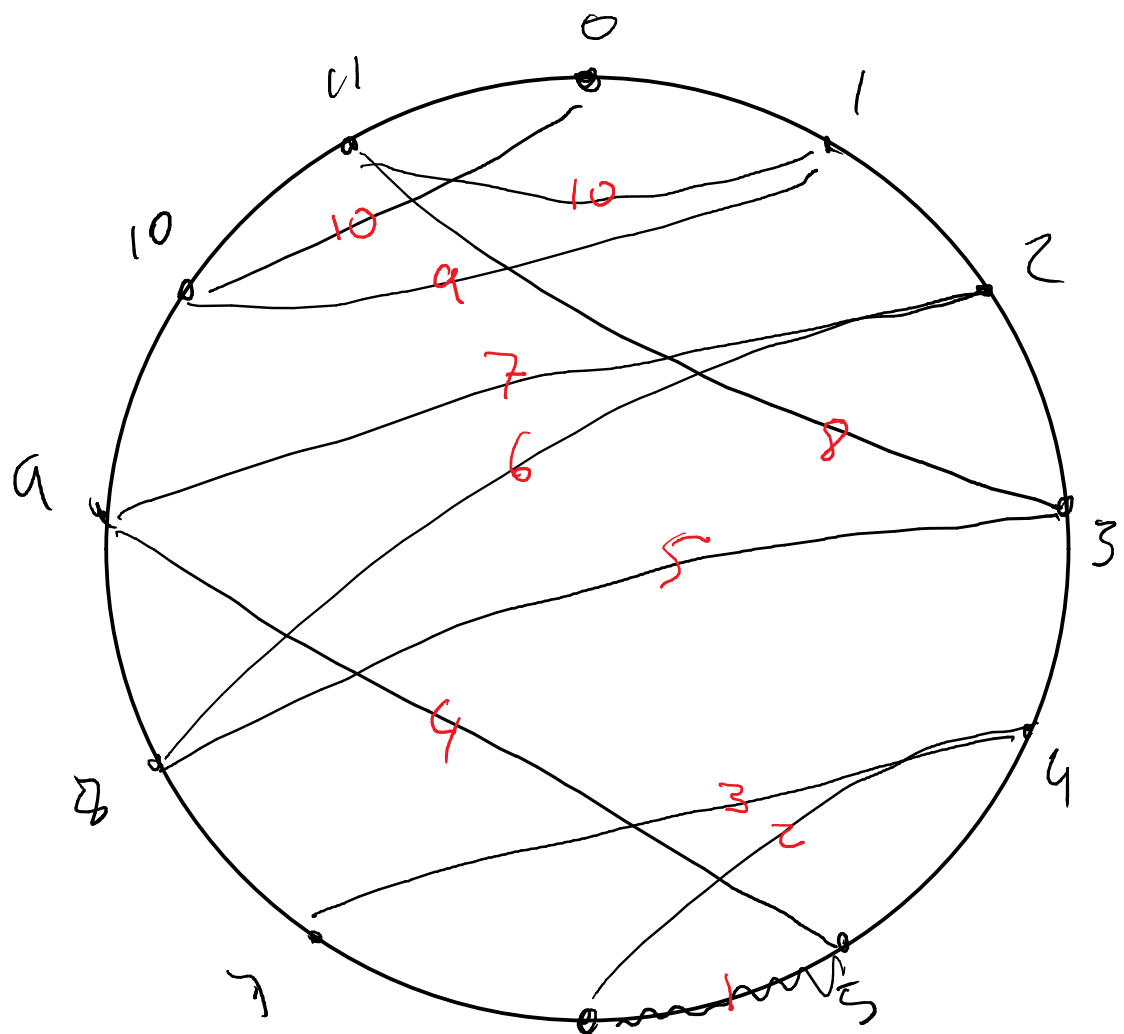
$$K = \frac{n+1}{2}$$

1	2	3	4	5	6	7	8	}
	2	3	4	5	6	7	8	

$$K = \frac{n+3}{2}$$

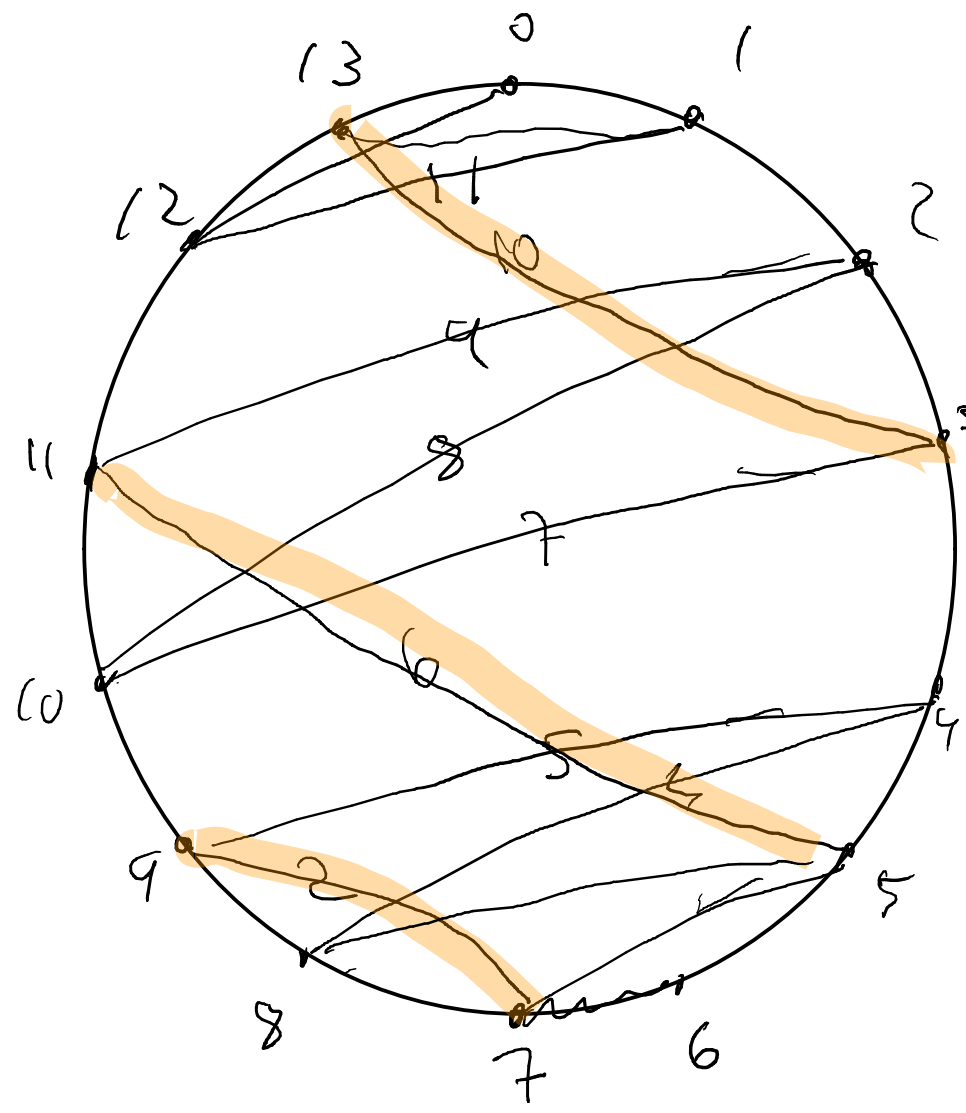
1	2	3	4	5	6	7	8	9
			4	5	6	7	8	9

*



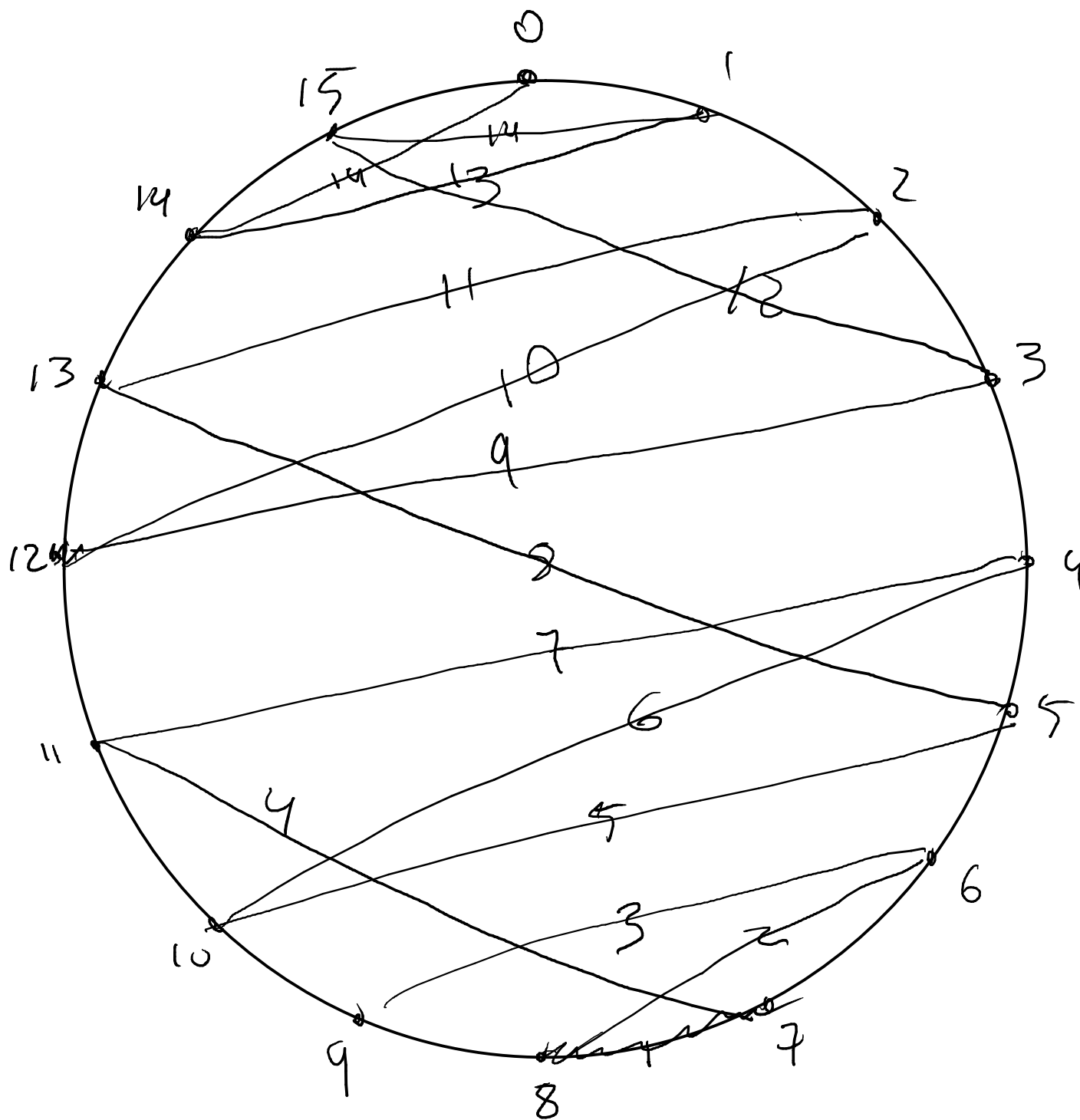
K

~~1~~ ~~2~~ ~~3~~ 4 ~~5~~ 6 ~~7~~
 8 ~~9~~ ~~10~~ ~~11~~ ~~12~~ ~~13~~



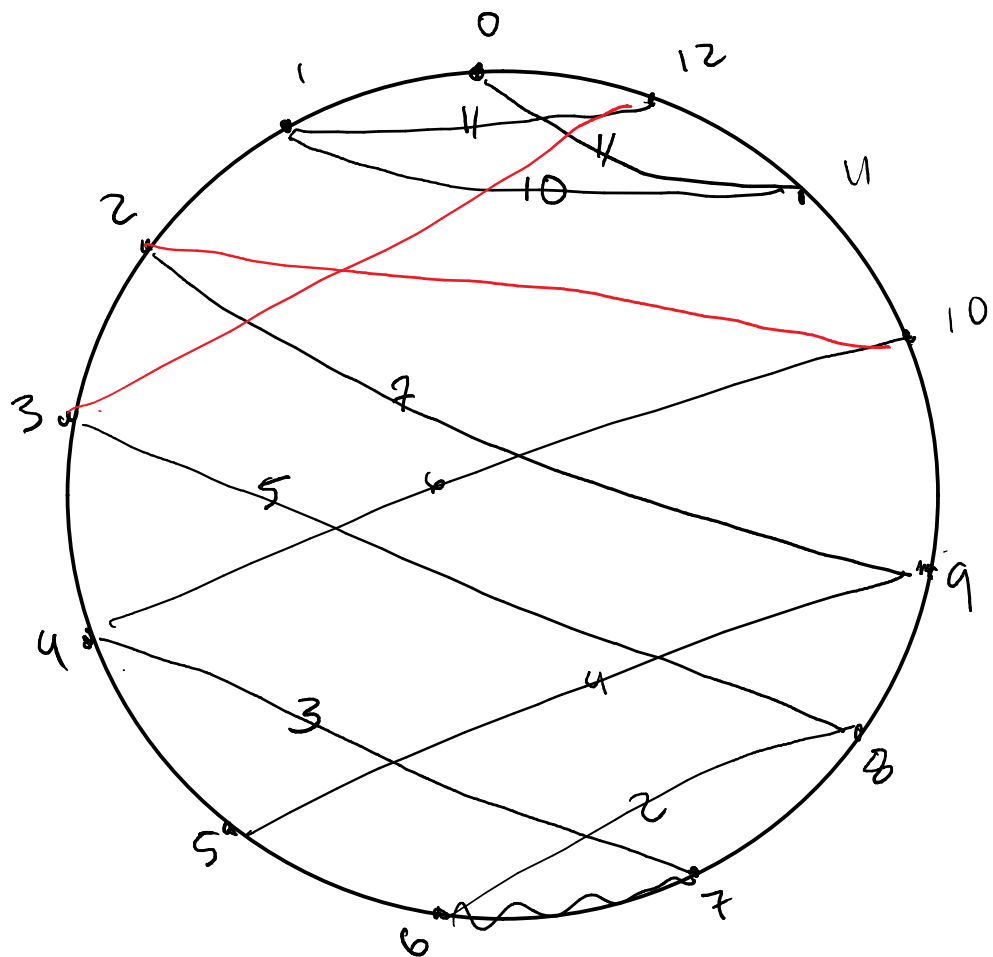
b

$$\frac{13-3}{2} = 5$$

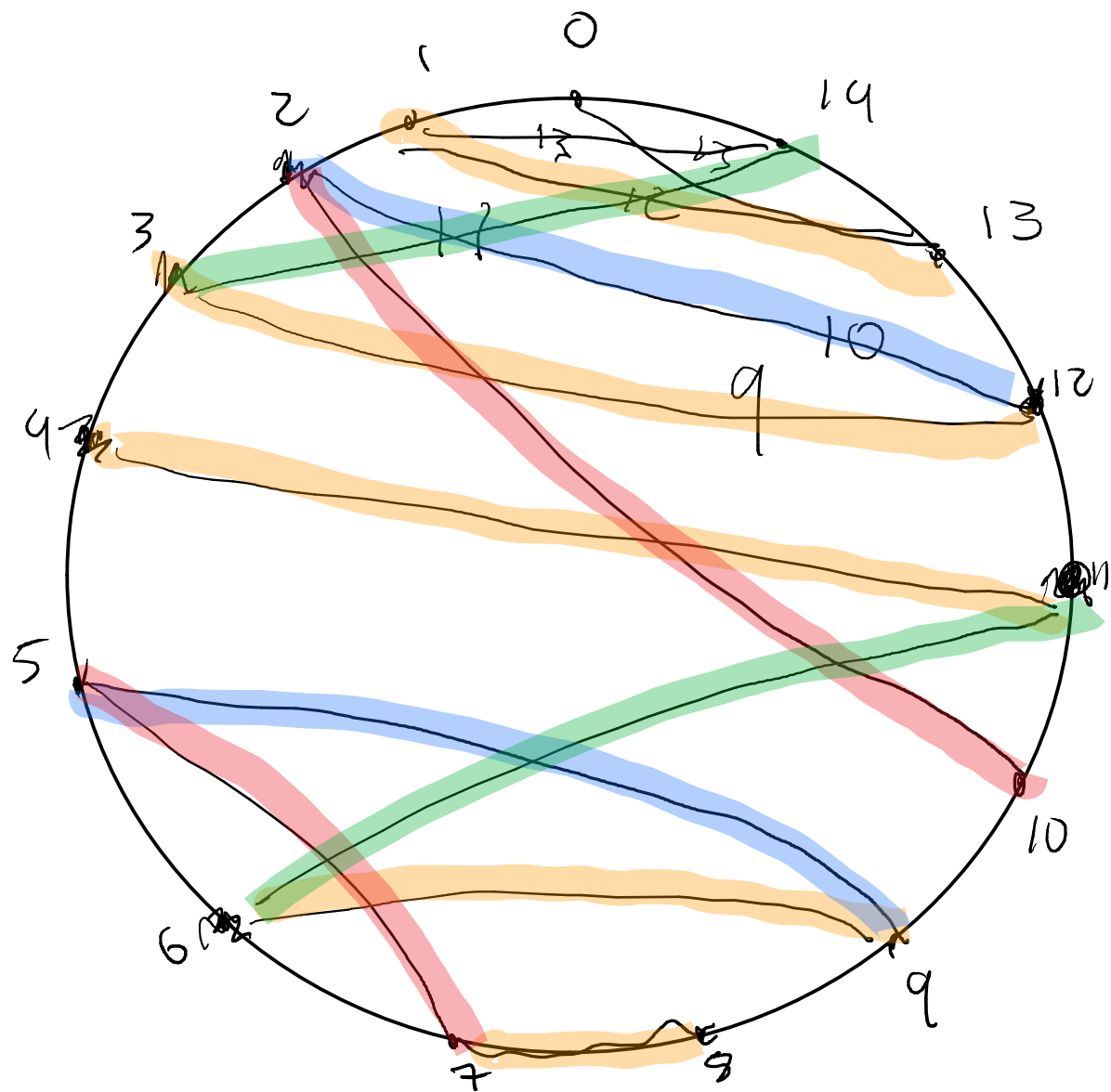


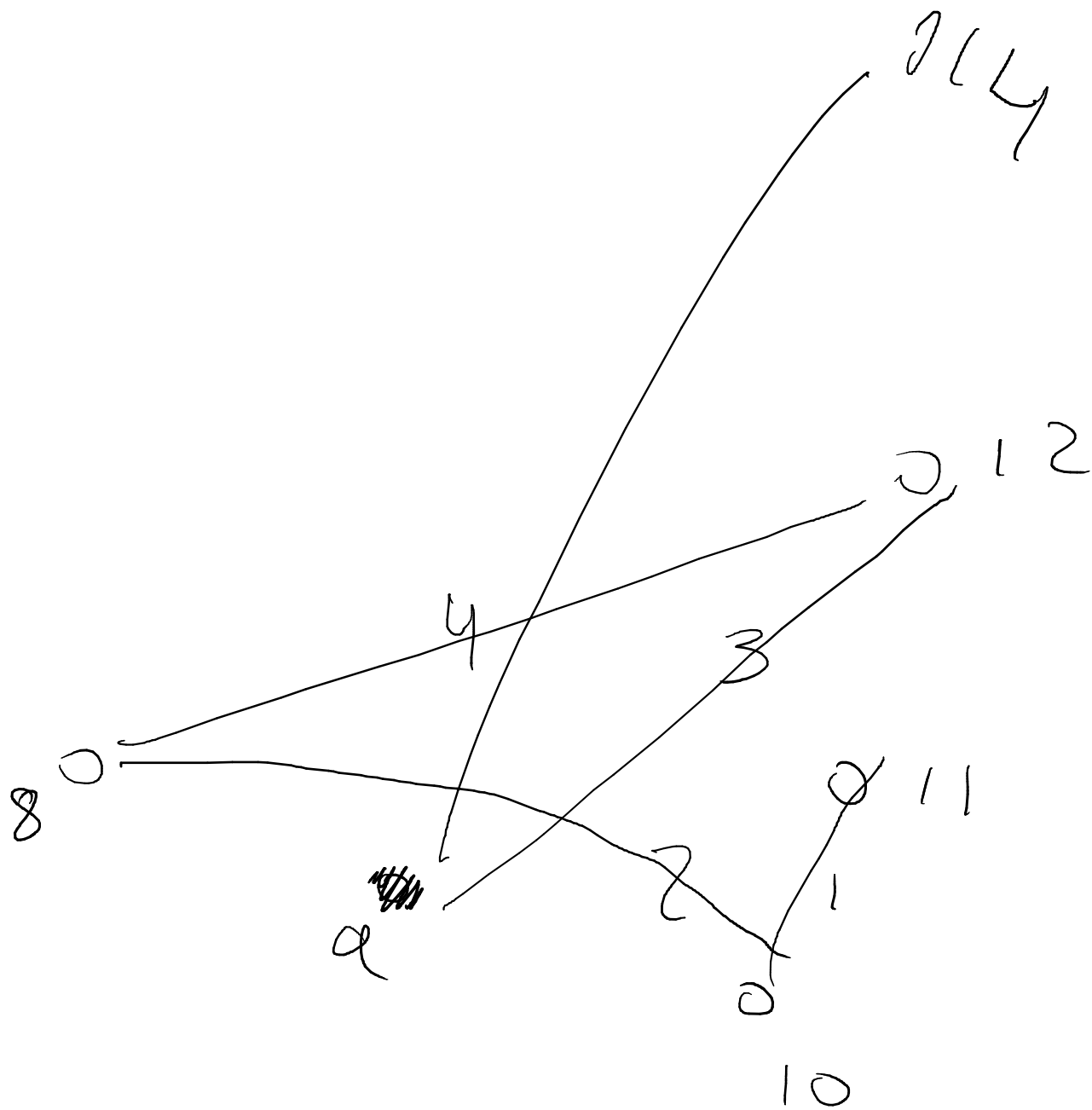
$$n = 15$$

$$\frac{n-3}{2}$$



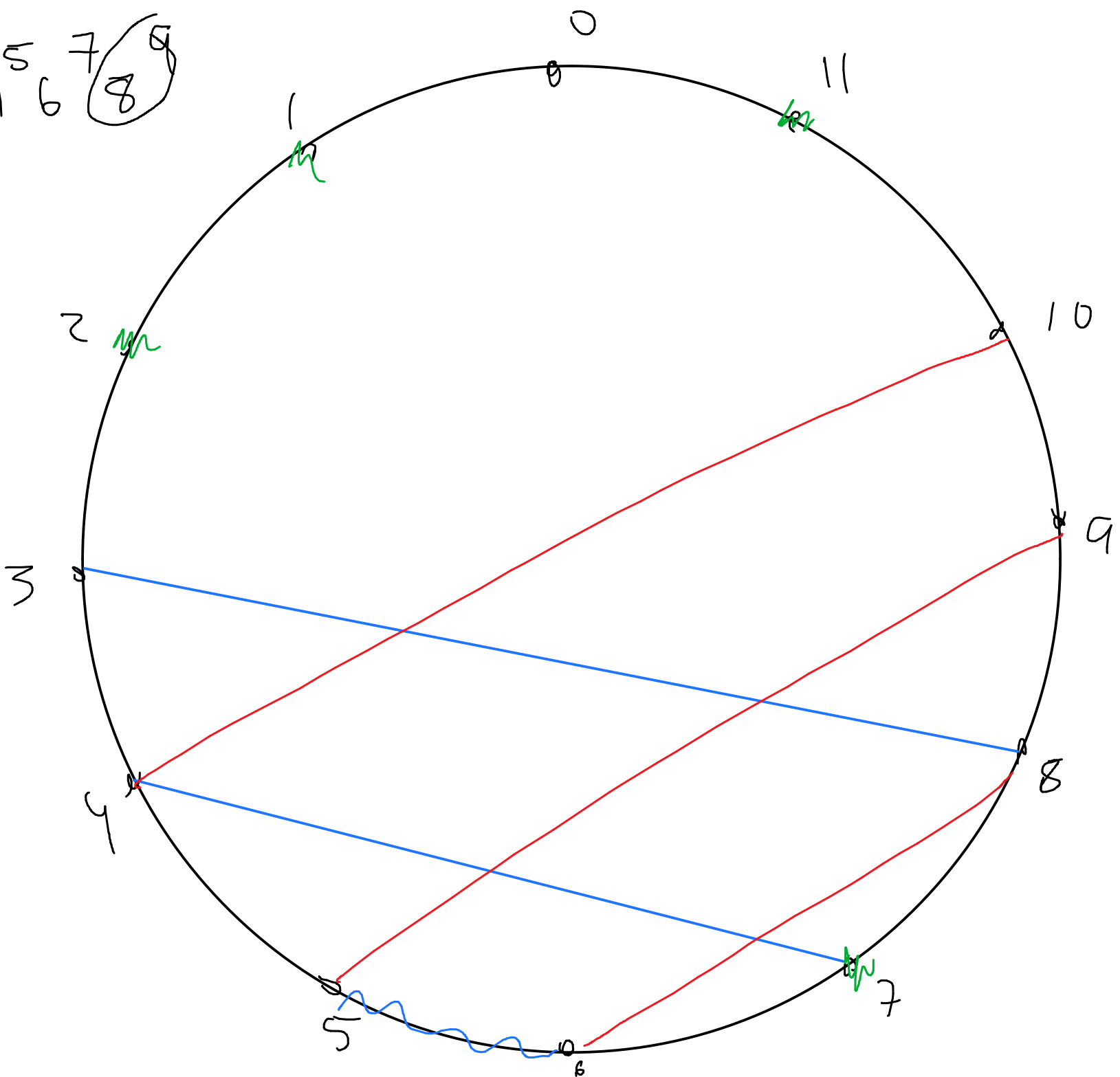
11 10 9





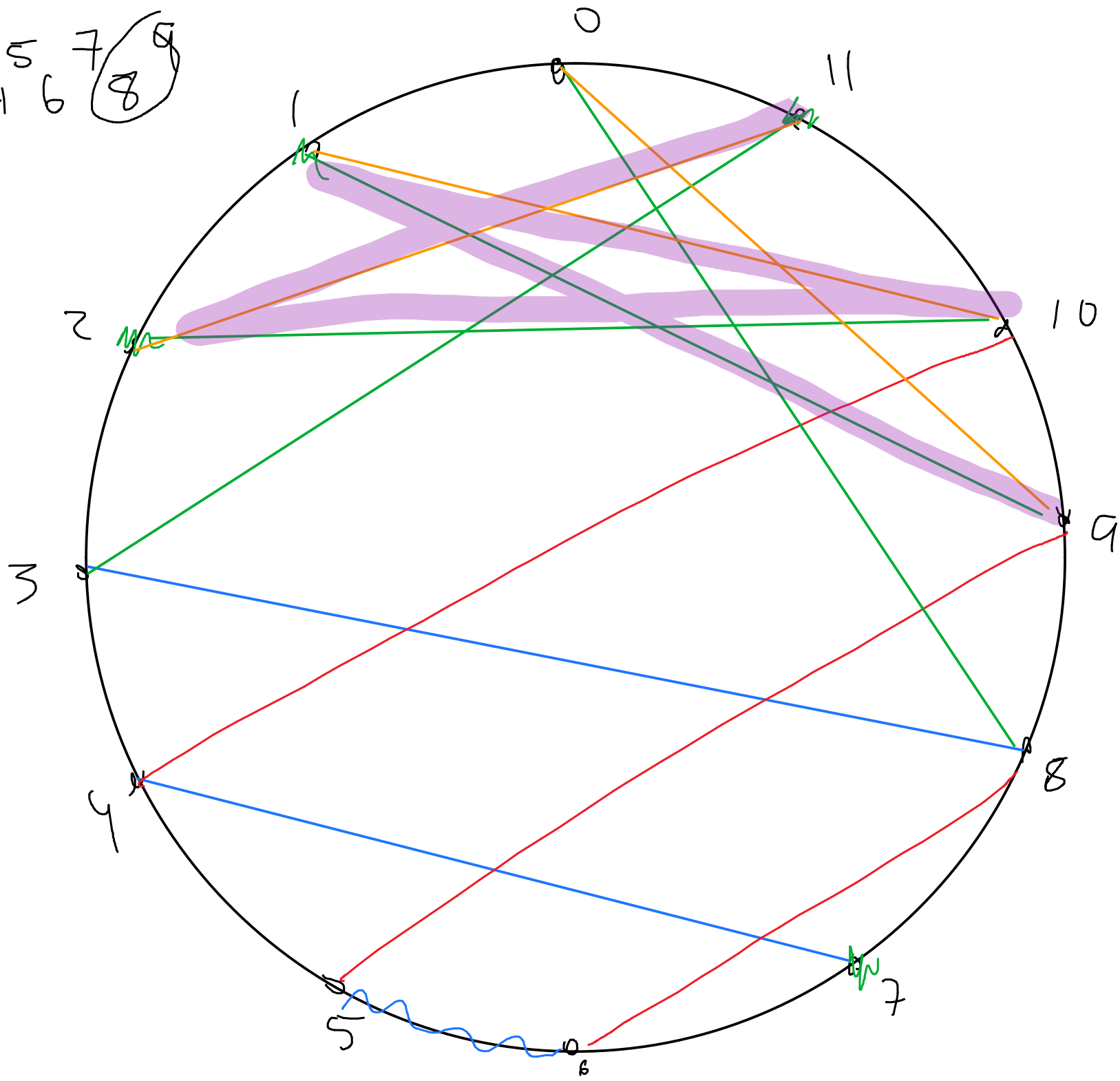
1, 2, 3

1 3 5 7 9
2 4 6 8



9
7
5
3
1

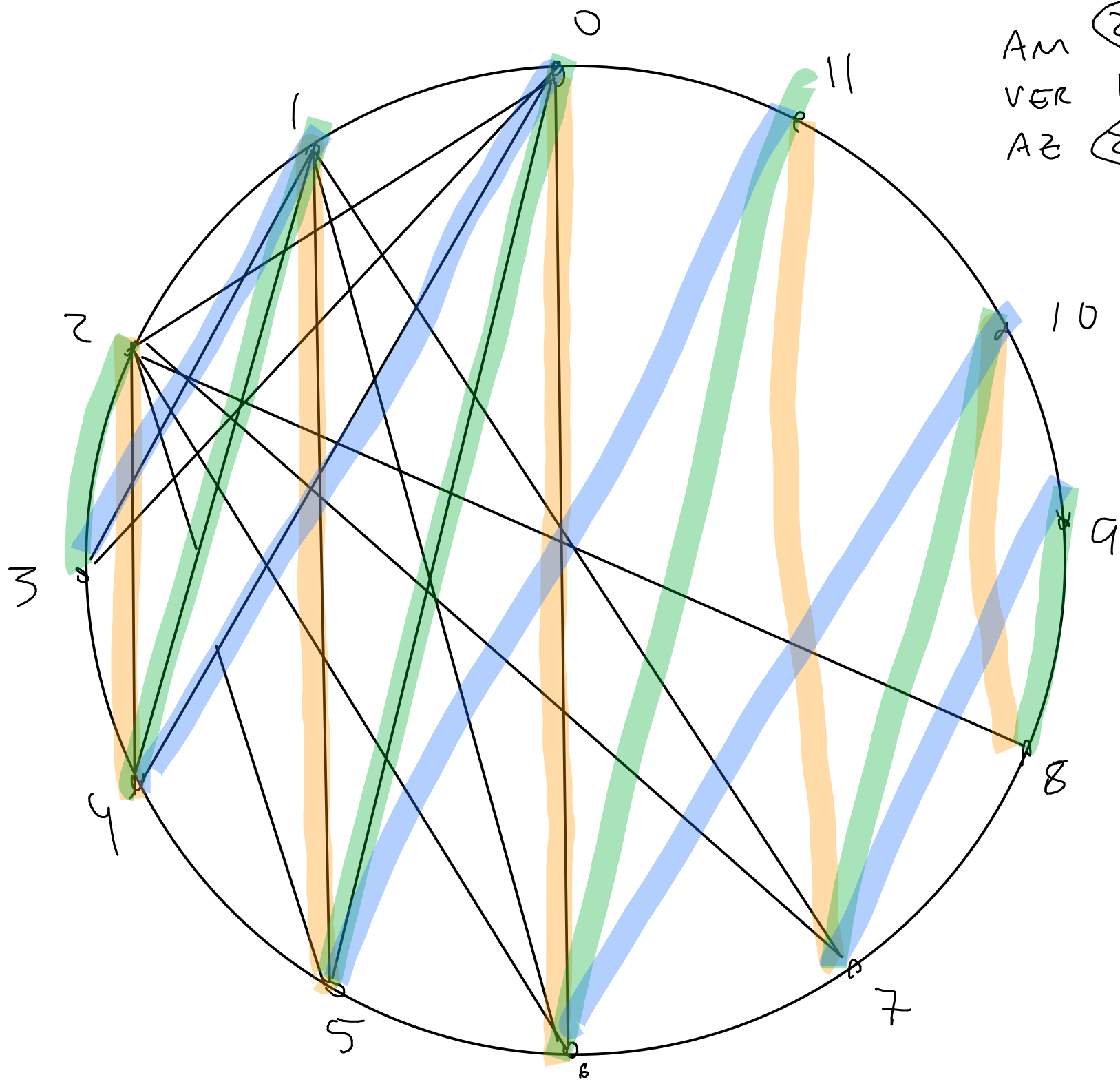
1 3 5 7 9
2 4 6 8



9
7
5
3
1

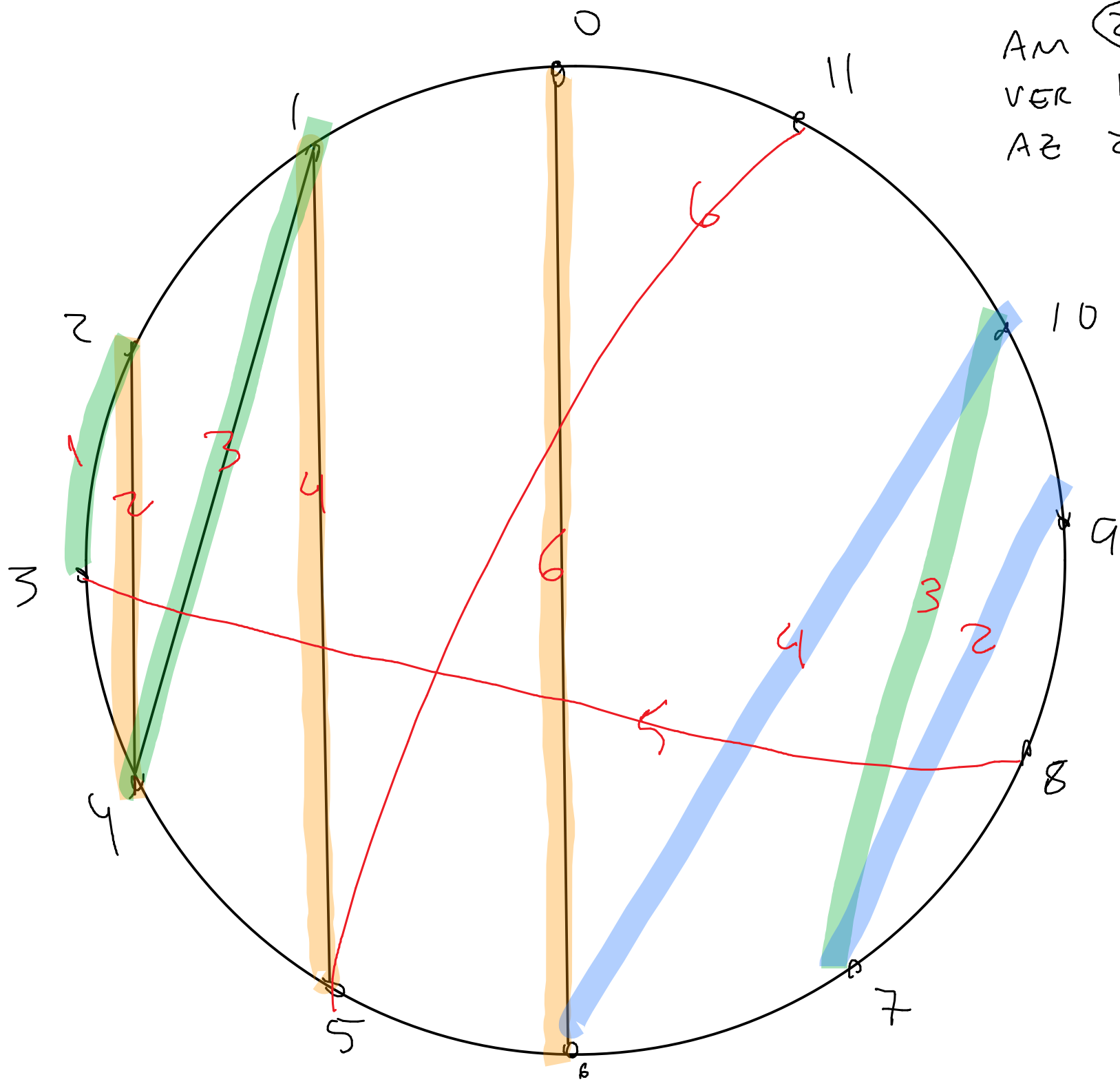
$m=11$
 $K=6$

Am	2	4	6	4	2	
Ver	1	3	5	5	3	1
Az	2	4	6	4	2	

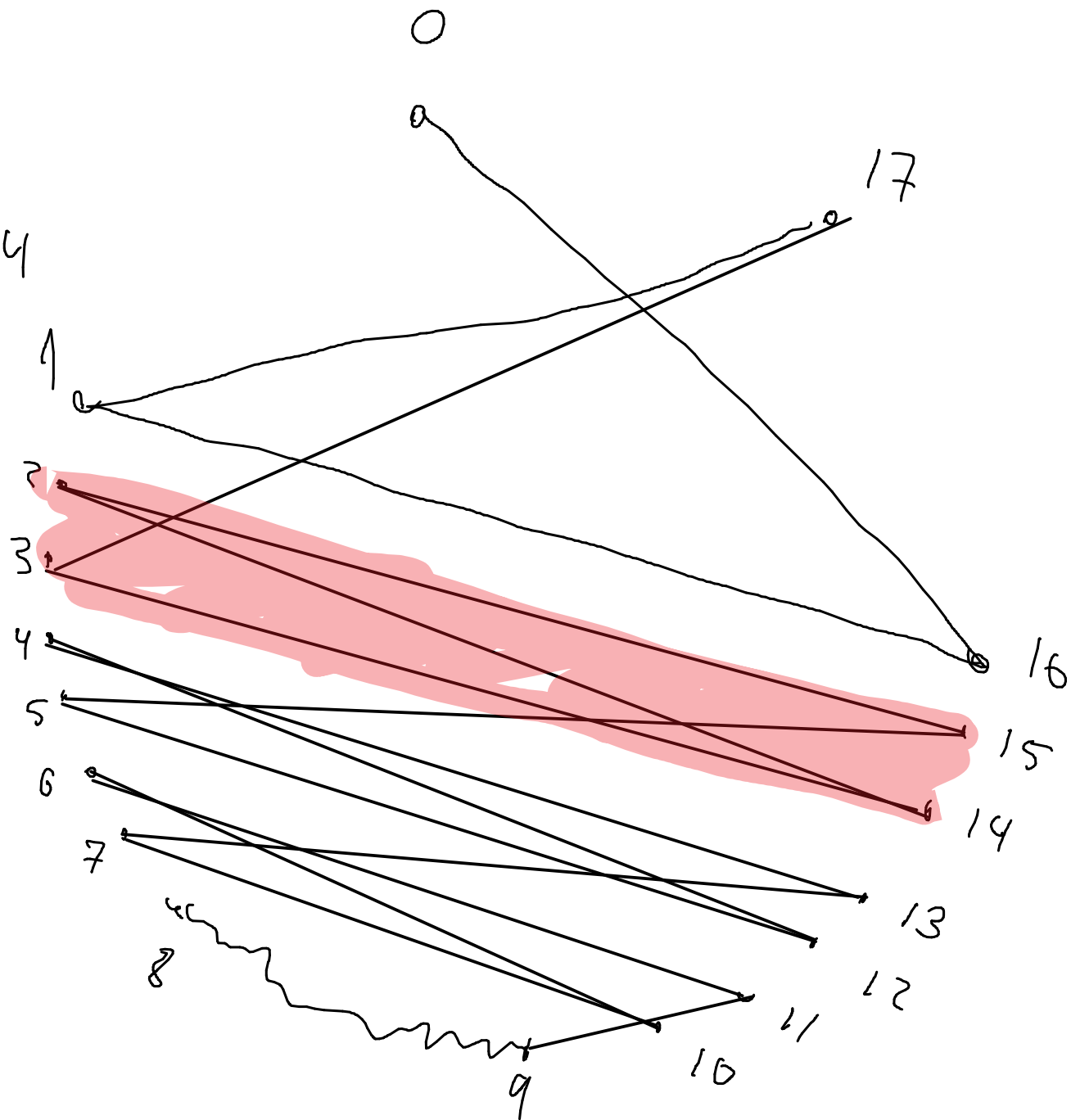
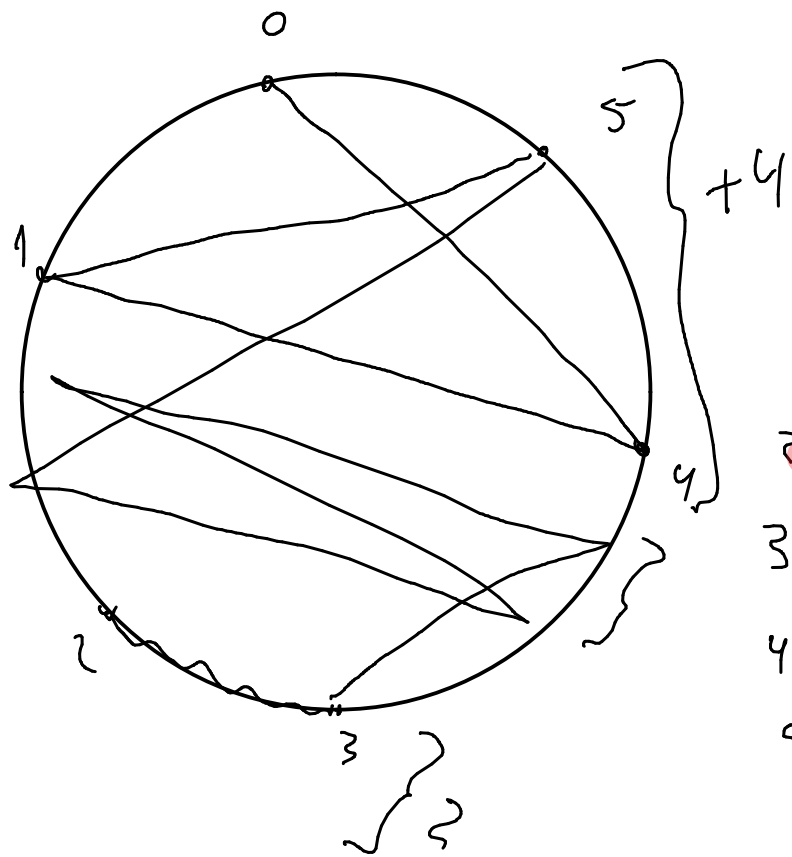


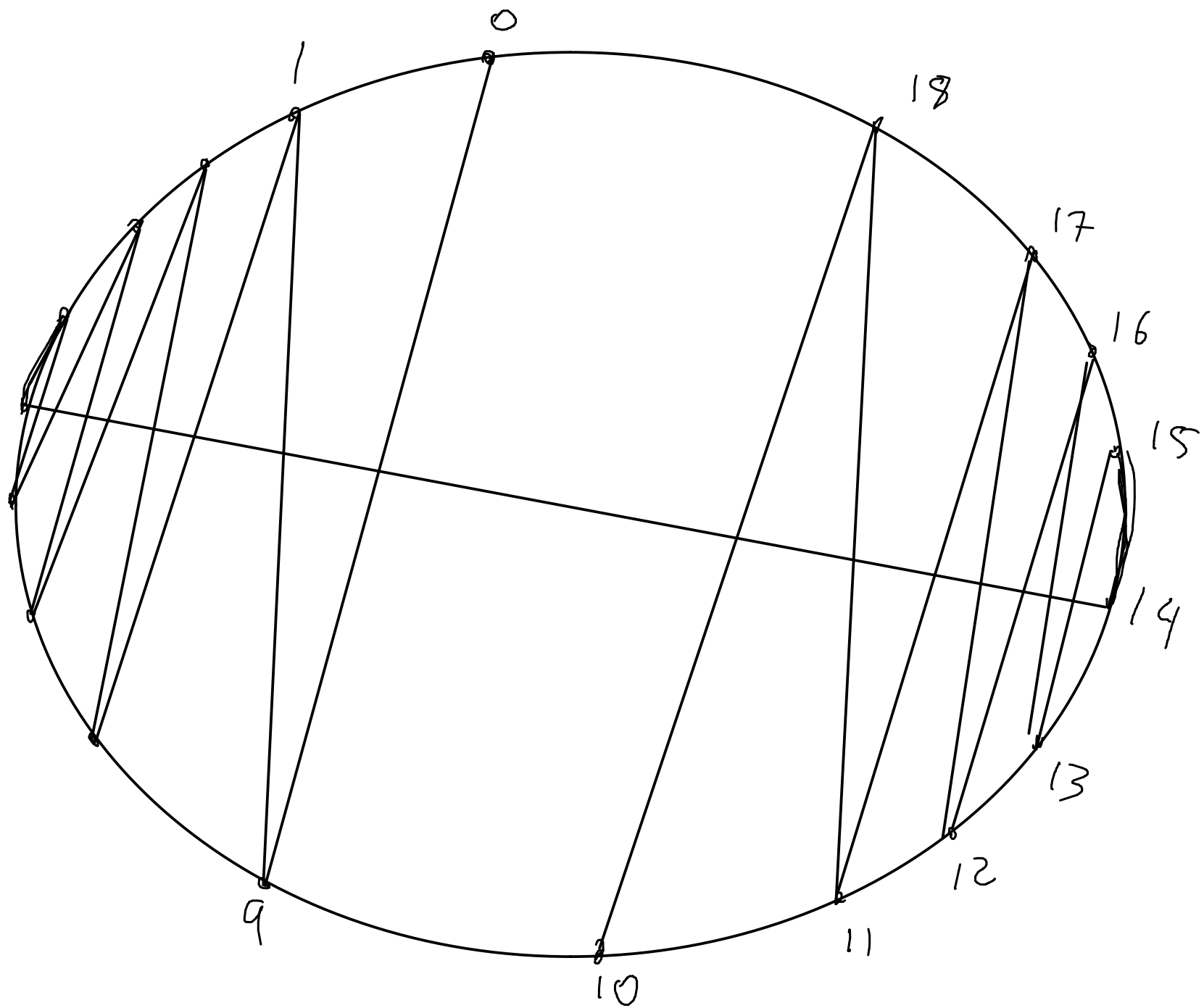
$m=11$
 $K=6$

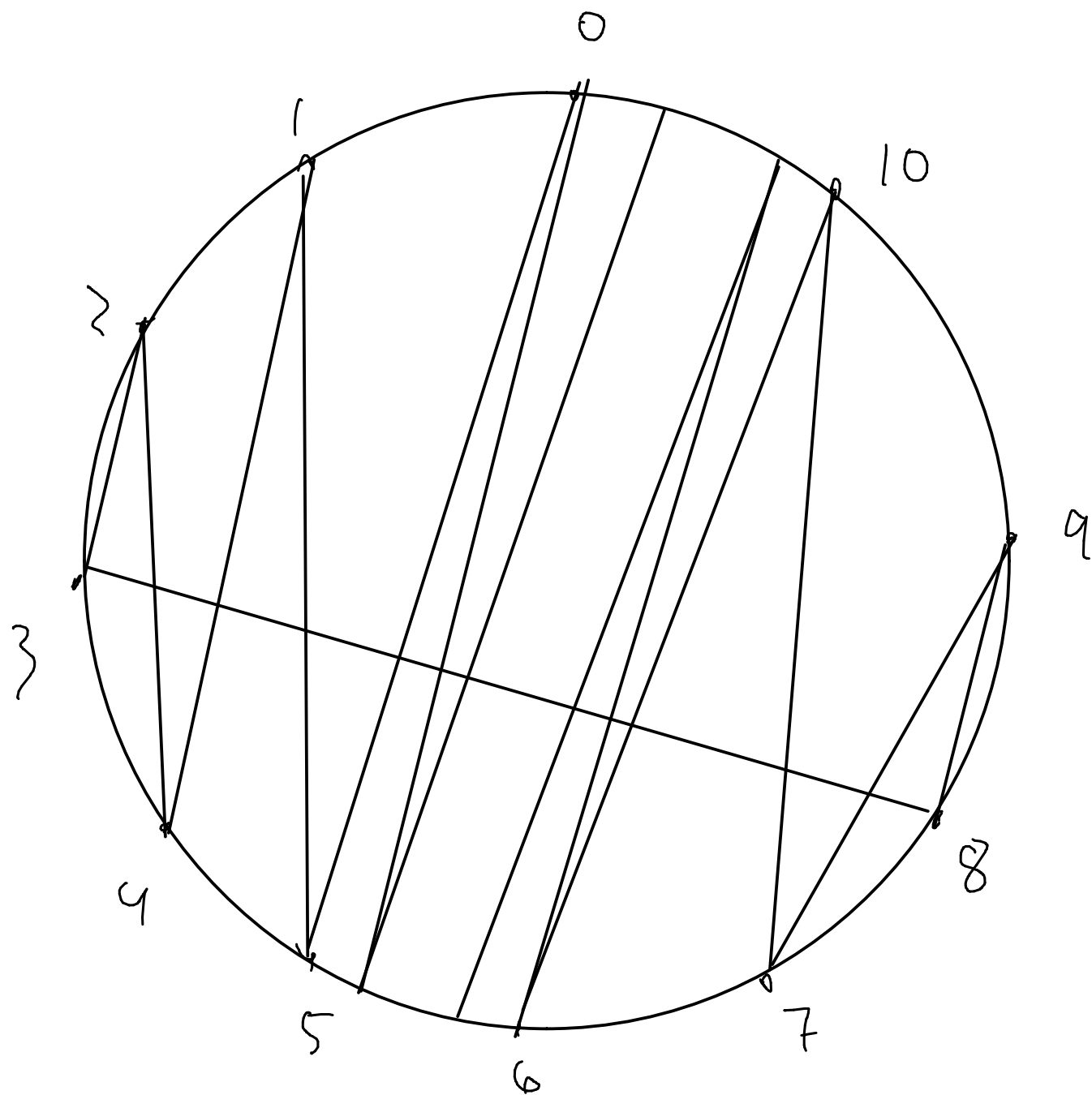
Am	2	4	6	4	2
Ver	1	3	5	5	3
Az	2	4	6	4	2

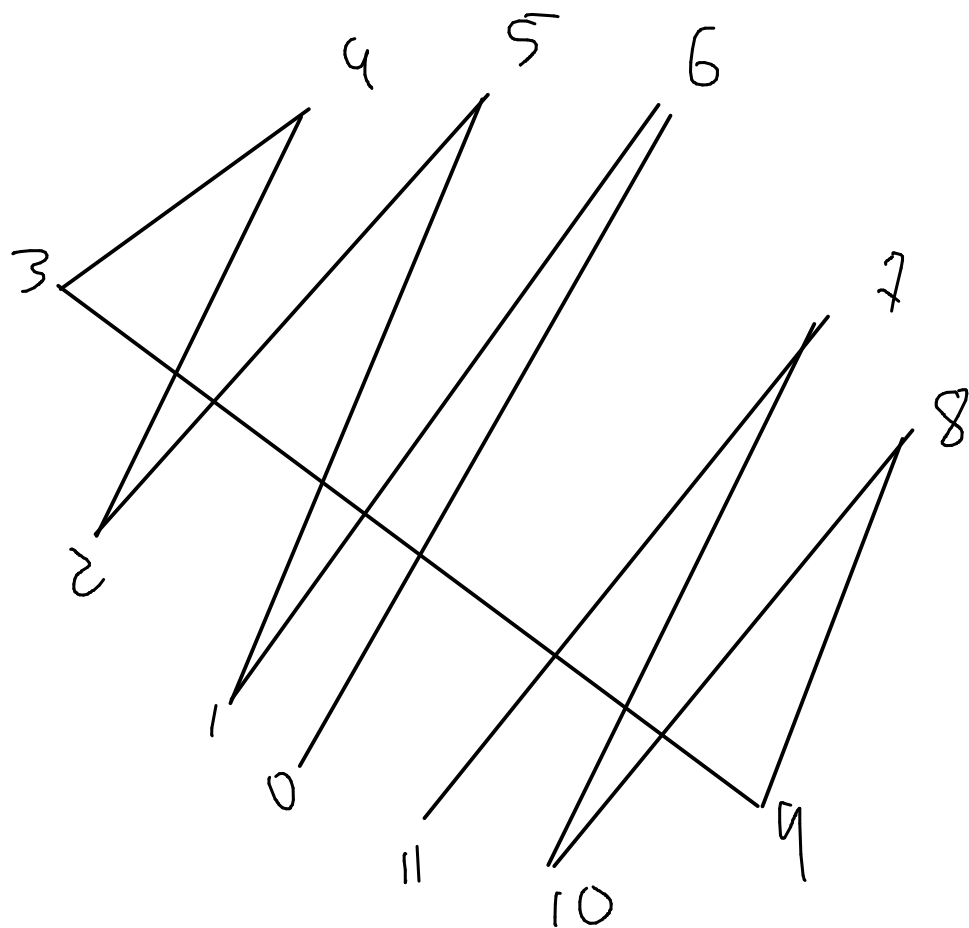


6 5 5









$n = 10$

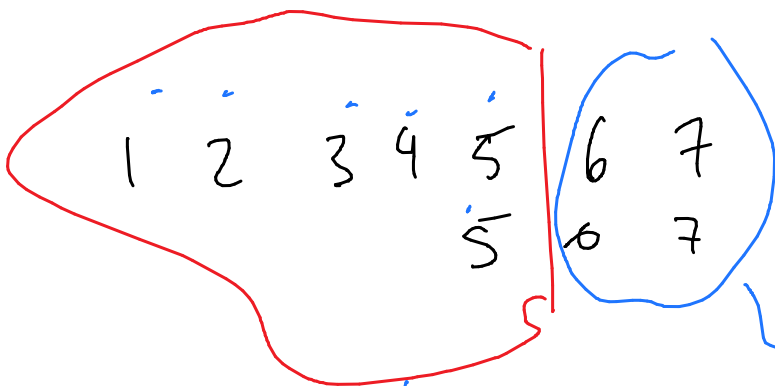
0 1 2 3 4 5 6 7 8 9 10

$k = 5$

1 2 3 4 5

1 2 3 4 5

$k = 7$



$n = 28$
 $k = 14$

$n = 6$
 $k = 5$

$n = 4$

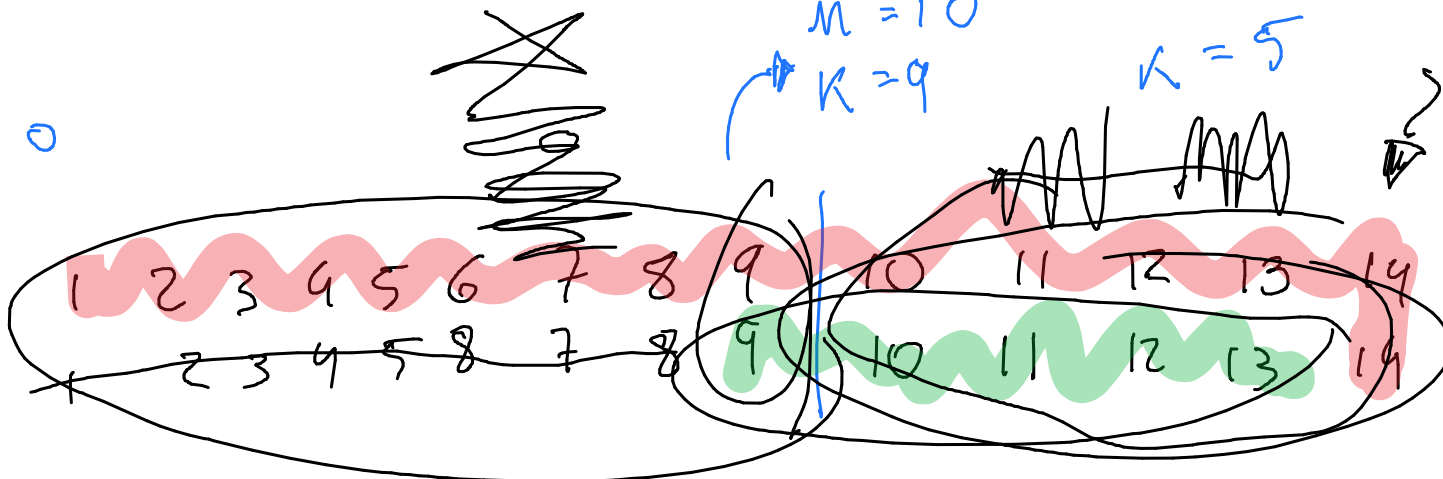
$k = 2$

$n = 10$
 $k = 9$

$n = 10$
 $k = 5$

16, 17, 18, 19, 20

$k = 14$



20

$n = 15$
 $k = 14$

$$(n, \kappa)$$

$$(n, \kappa, e)$$

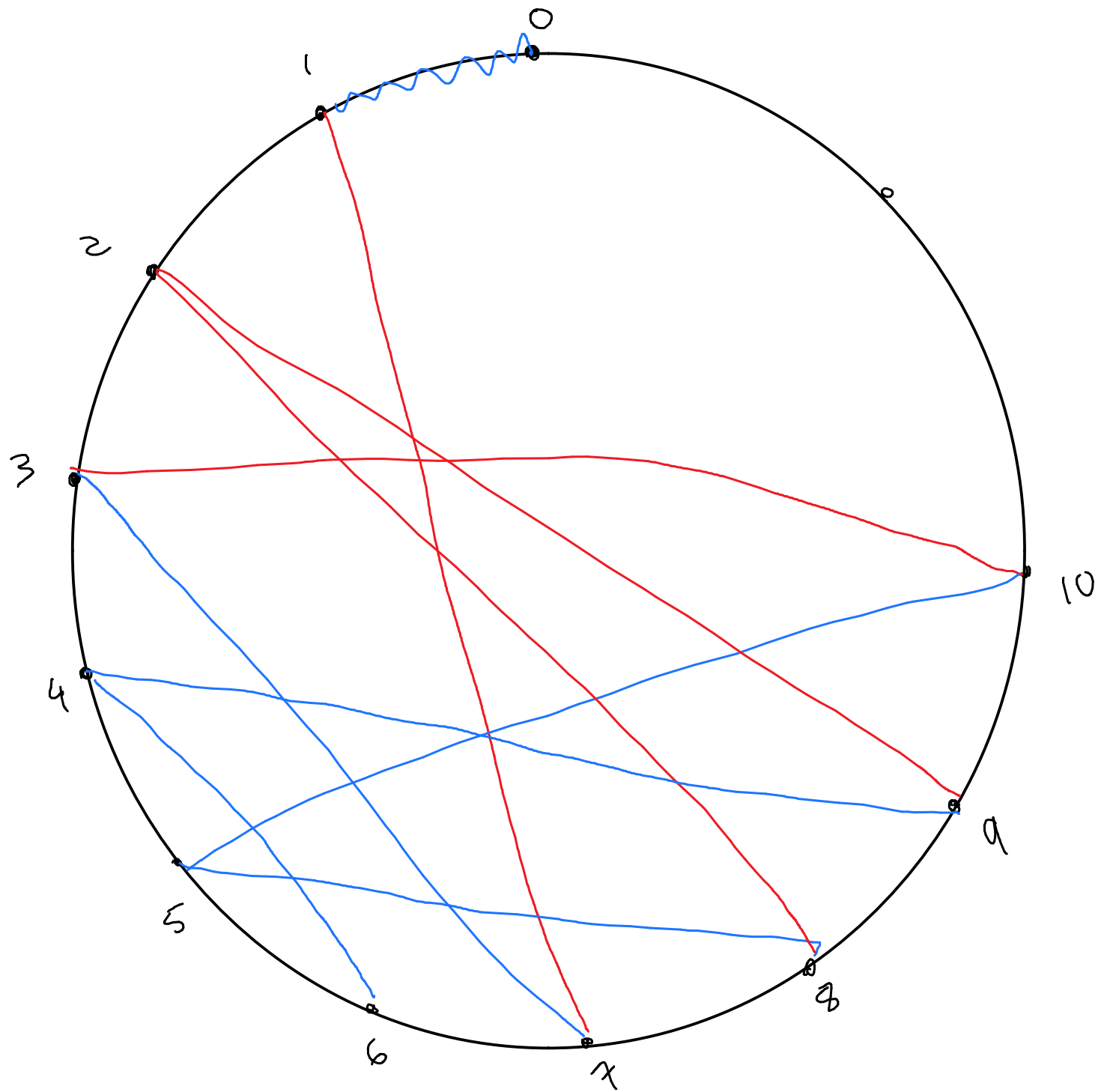
$$\hookrightarrow |e| \leq \kappa$$

$$n=12$$

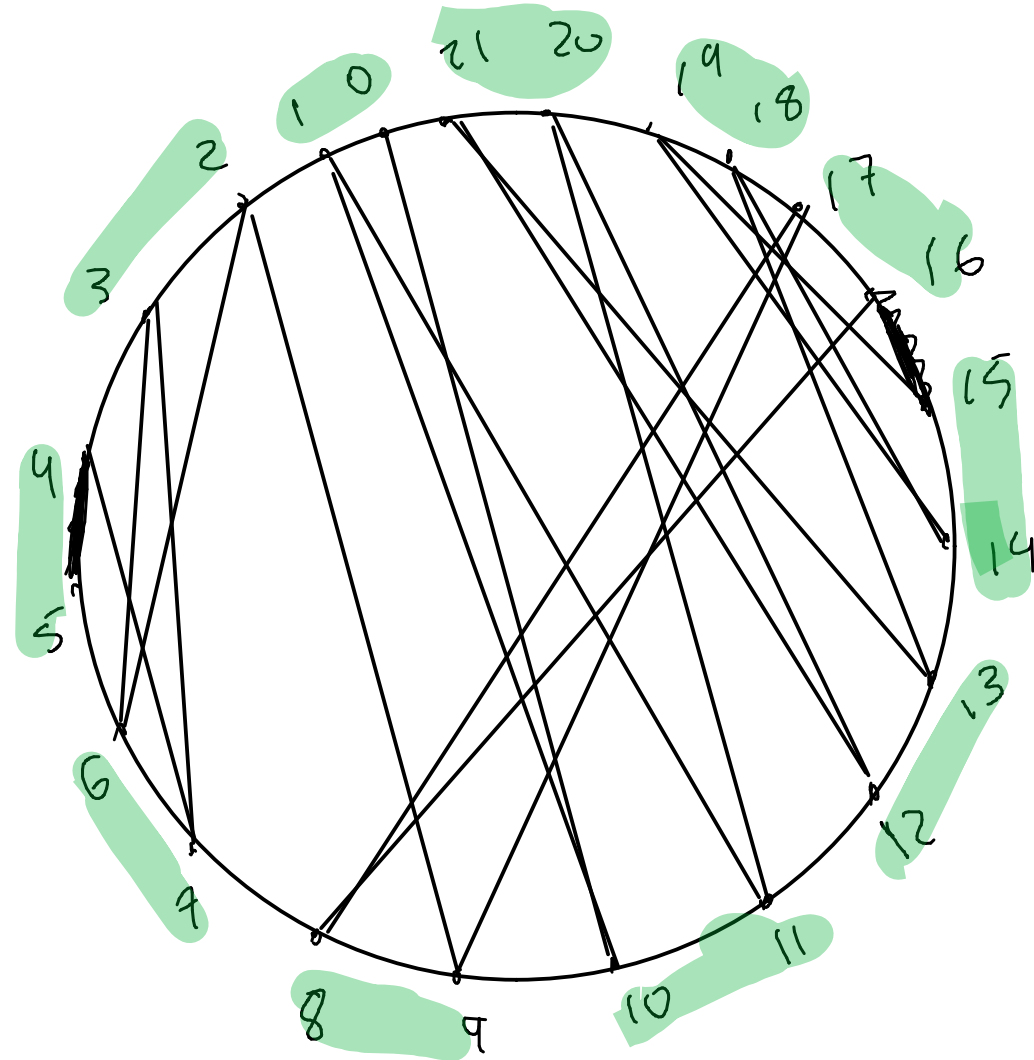
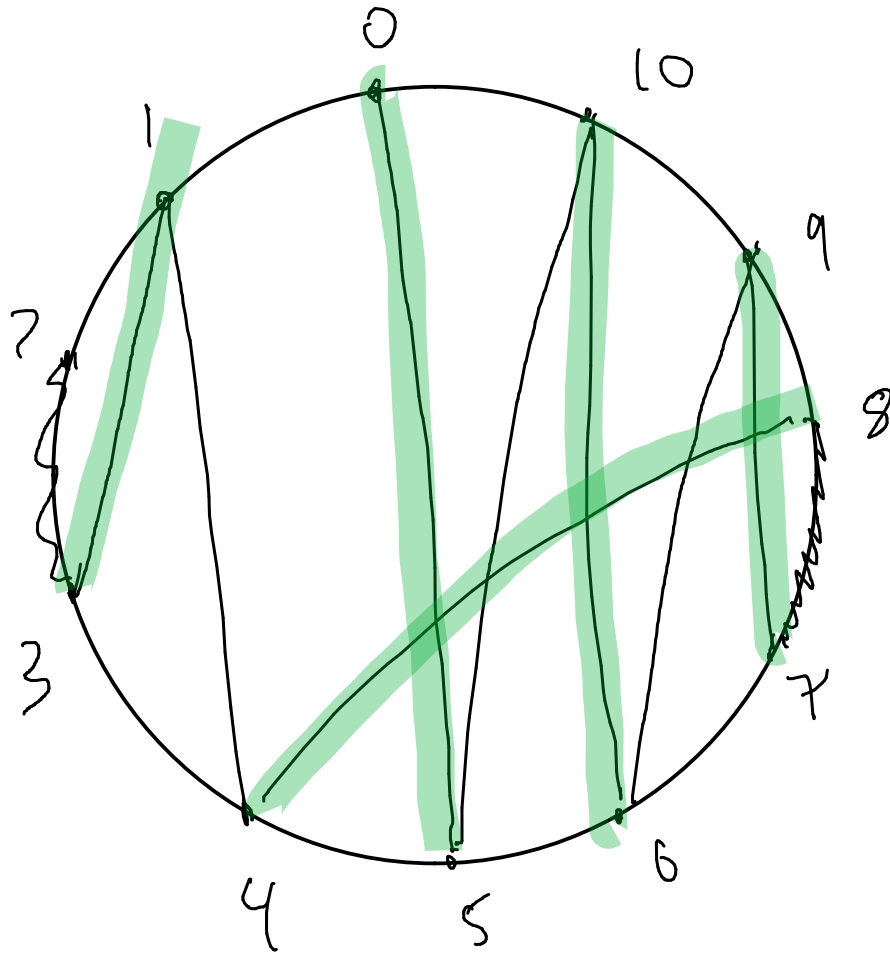
$$\kappa=6$$

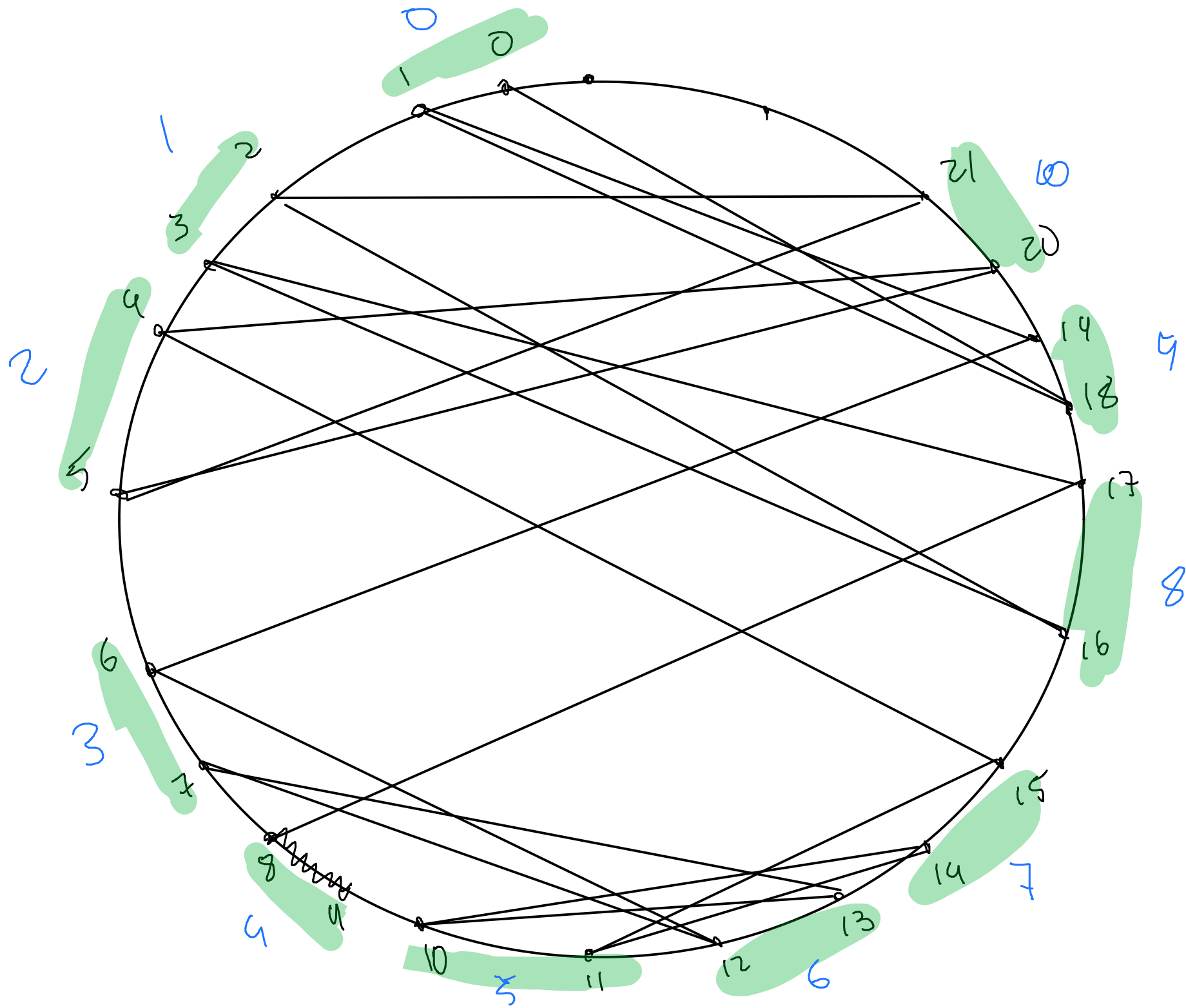
$$e=?$$

$$(10, 5, (5, 6))$$

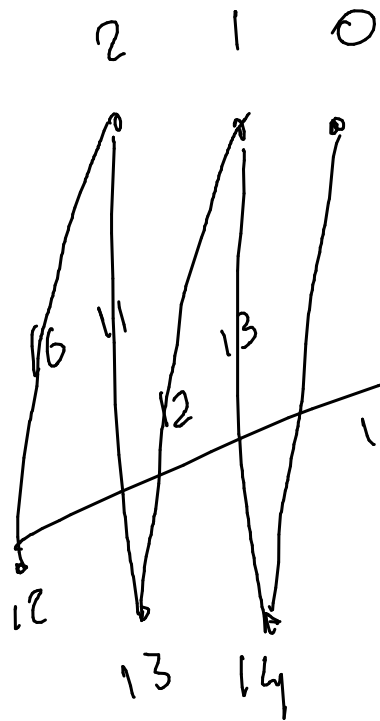


$$i \rightsquigarrow z_i, z_{i+1}$$

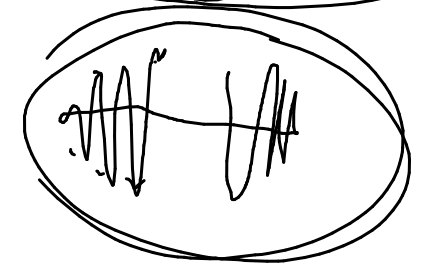
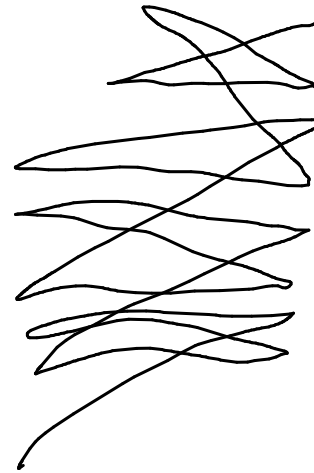
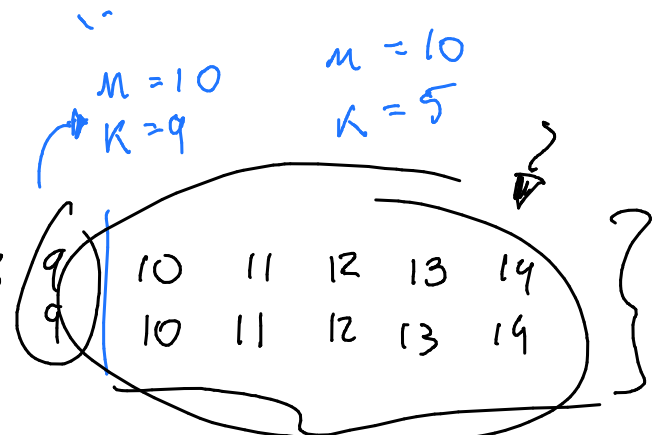




$$n = 20$$



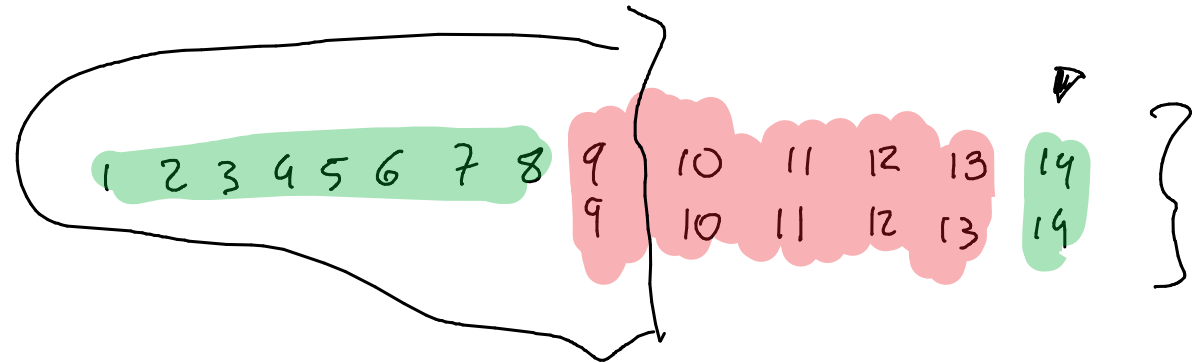
1 2 3 4 5 6 7 8



$$n = 10$$

$$k = 9$$

$$n = 20$$



MIN. Paddles

$$\pi = 0 \quad \dots \quad n \quad \left\{ \begin{array}{l} 1 \cdot (n-1) \cdot (n-1) \cdot \dots \cdot 1 = (n-1) (n-1)! \\ = n! - (n-1)! \end{array} \right.$$

$$\pi' = (|\pi_1 - \pi_0|, \dots, |\pi_n - \pi_{n-1}|)$$

$$\text{COUNT}_n(\pi') = (c_1^{\pi'}, \dots, c_k^{\pi'}, 0, \dots, 0) \cdot c_i^{\pi'} = \left| \left\{ j : |\pi_j - \pi_{j-1}| = i, 1 \leq j \leq n \right\} \right|$$

$$\left. \begin{array}{c} \sum c_i^{\pi'} = n \end{array} \right\}$$

QUEEREMOS
ENCONTRAR

$$(1, 1, \dots, 1, \underset{\substack{\uparrow \\ 2k-n+1}}{2}, 2, \dots, \underset{\substack{\uparrow \\ 0 \dots n}}{(2)}, 0, \dots, 0)$$

$$S_n \subseteq E_{n-1} = \{0, \dots, n\}^{n-1}$$

$$\parallel \{x \in E_{n-1} : |x| = n\}$$

$$|S_n| = \sum_{i=1}^n$$

$$(i, \dots, \underbrace{\dots}_{S_{n-2}}_{S_{n-i}})$$

$$S_n^{n-1} \subseteq E_{n-1} = \{0, \dots, n\}^{n-1}$$

$$\parallel$$

$$\{x \in E_{n-1} : |x| = n\}$$

$$0 \dots n$$

$$|S_n| = \sum_{i=1}^n$$

$$(i, \underbrace{\dots}_{n-2}, \dots)$$

$$S_{n-i}^{n-2}$$

$$S_n^k = |S_n^k|$$

$$S_m^1 = m \quad \forall m$$

$$S_n^k = \sum_{i=0}^n S_{n-i}^{k-1}$$

$$S_n^k = \left\{ x \in \mathbb{N}^k : x = (x_1, \dots, x_k) \in \sum x_i = n \right\}$$

$$S_n^k = |S_n^k|$$

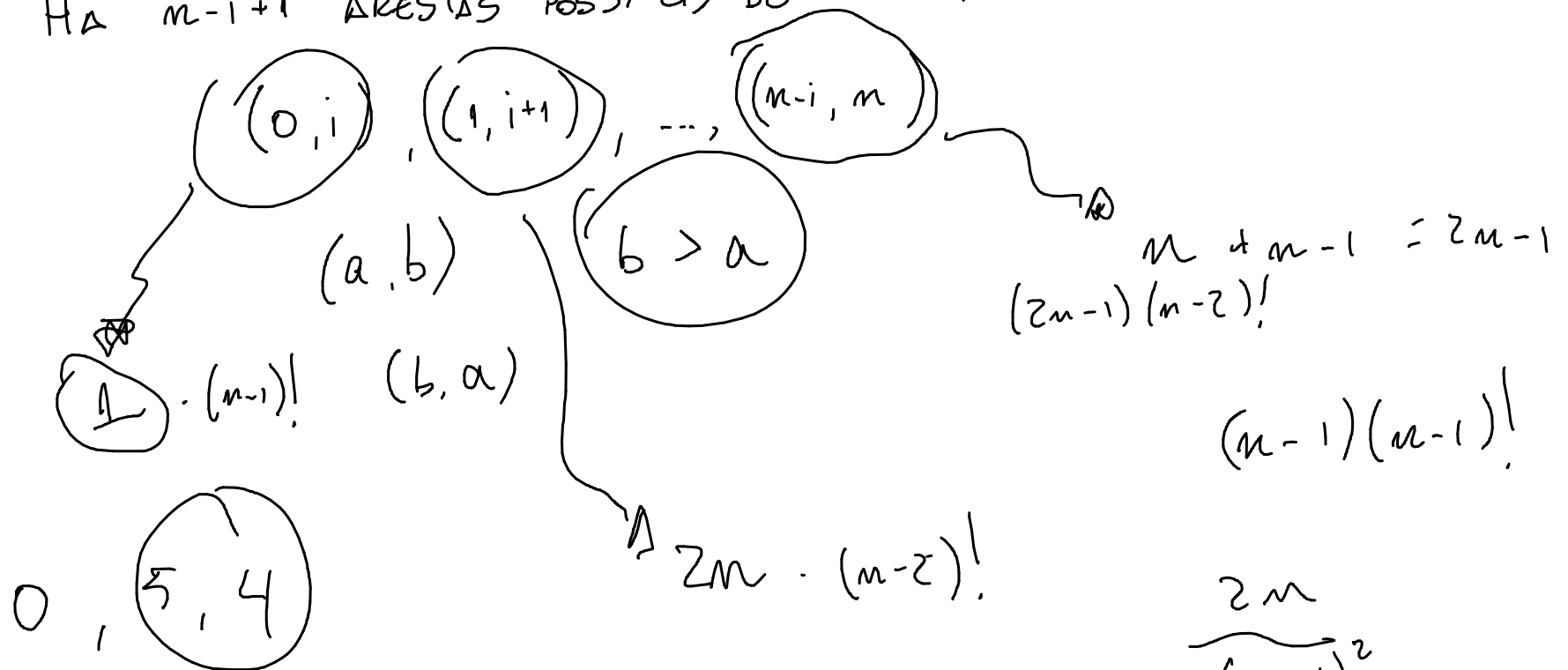
Π_0^m

9.8.7.. 45

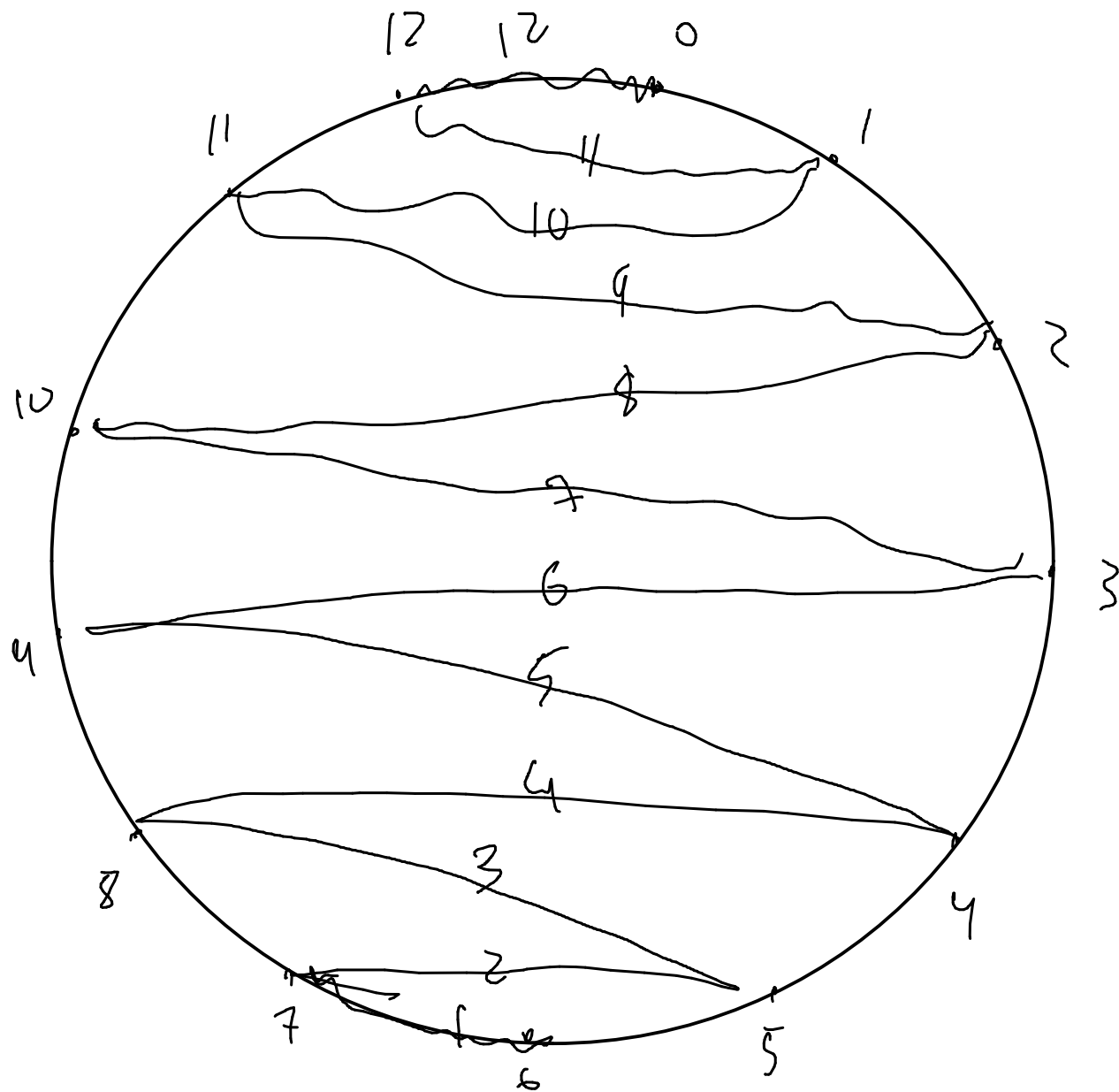
DADO i , QUANTOS CAMINHOS POSSUEM PELO MENOS
UMA i -ARESTA?

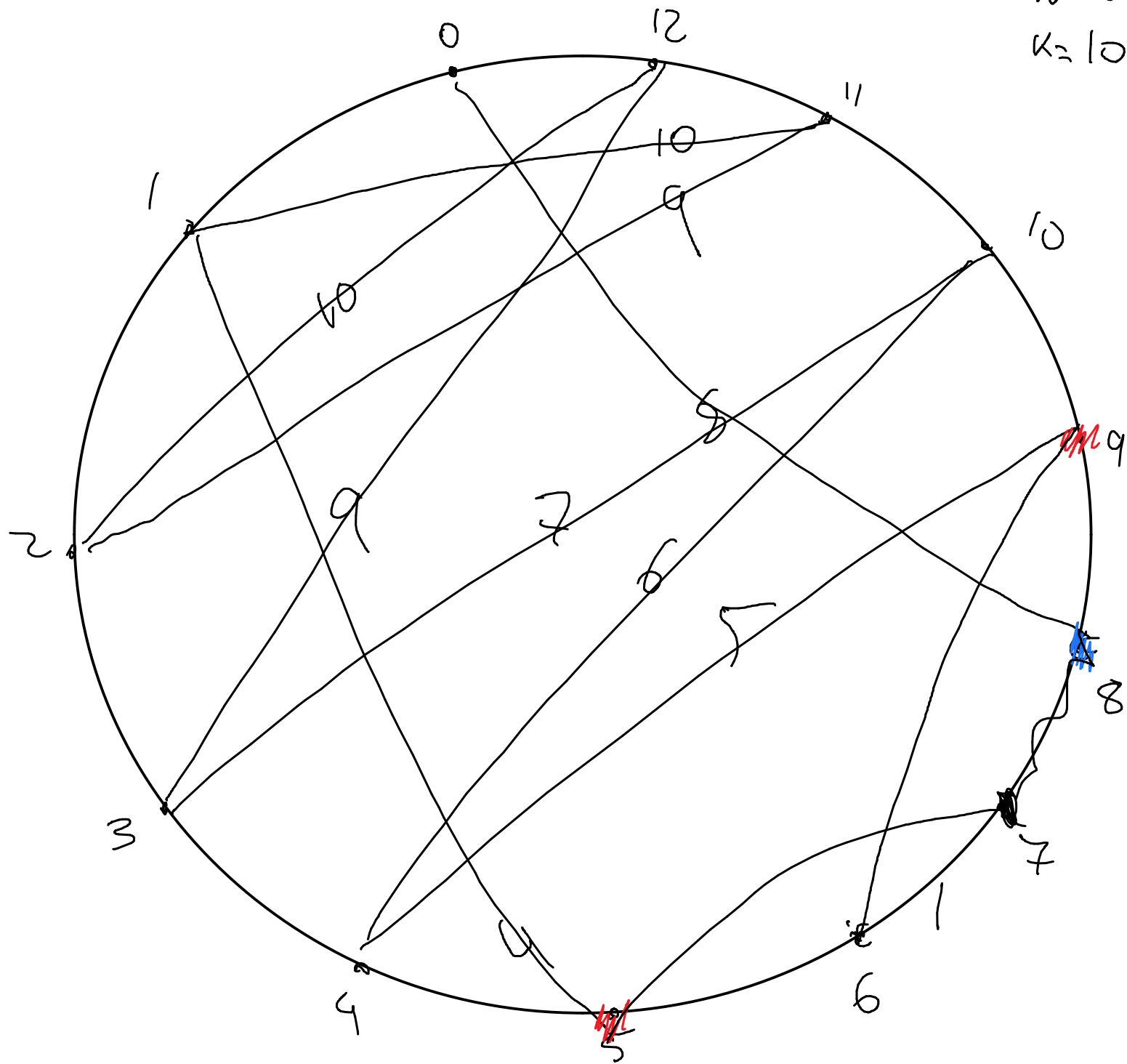
1, ..., ~~4, 5~~ 11

HÁ $m-i+1$ ARESTAS POSSÍVEIS DO TIPO i



$$\geq (m-1)(m-1)! - (m-1)! - 2m(m-2)! - (2m-1)(m-2)! = ((m-1)^2 - (m-1) - 2m - (2m-1))(m-2)!$$





$n=2$
 $k=10$

3, 2, 1

