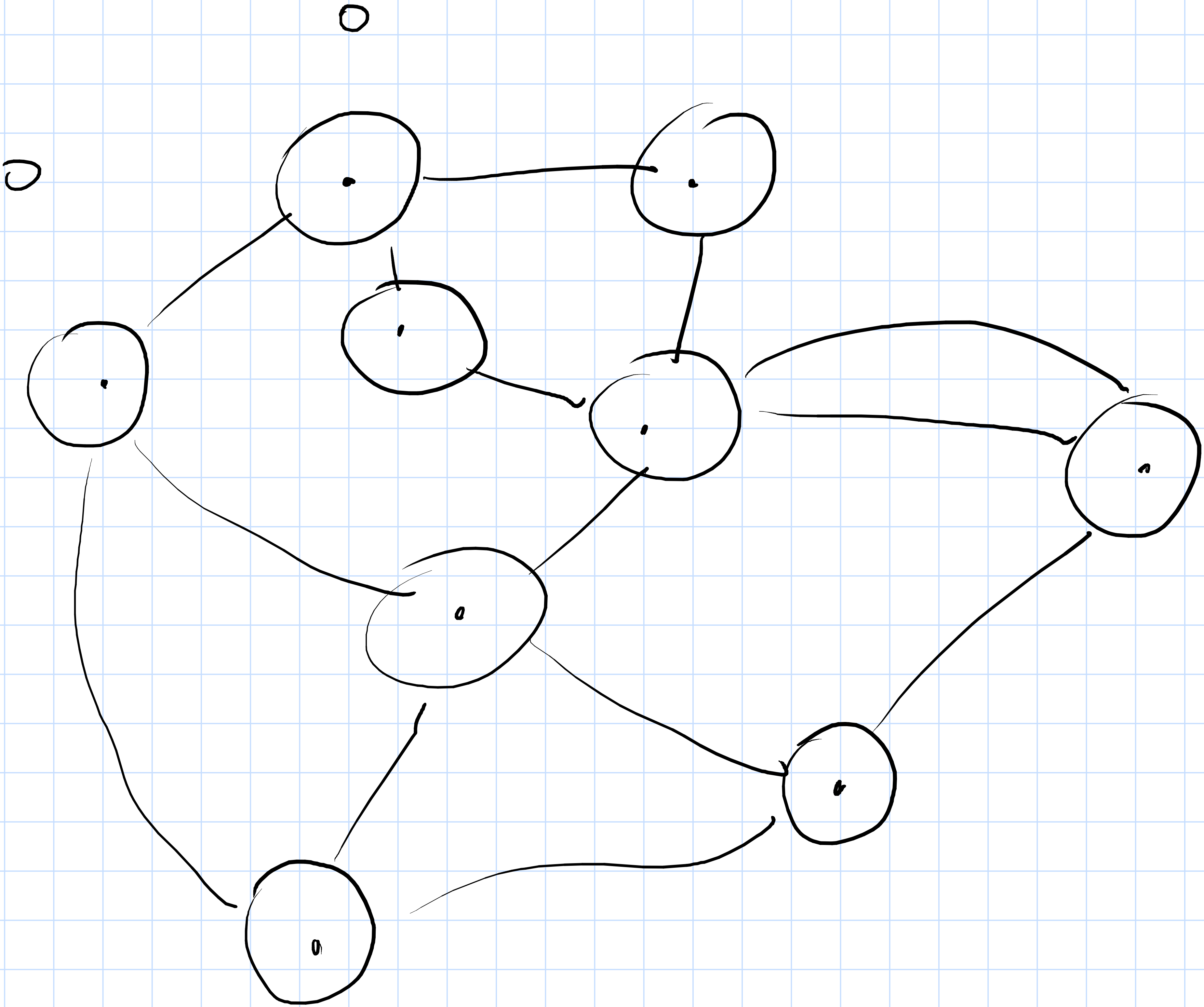
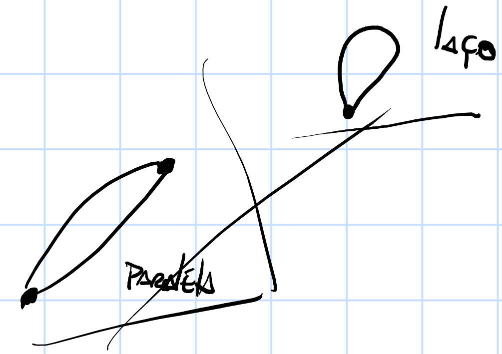
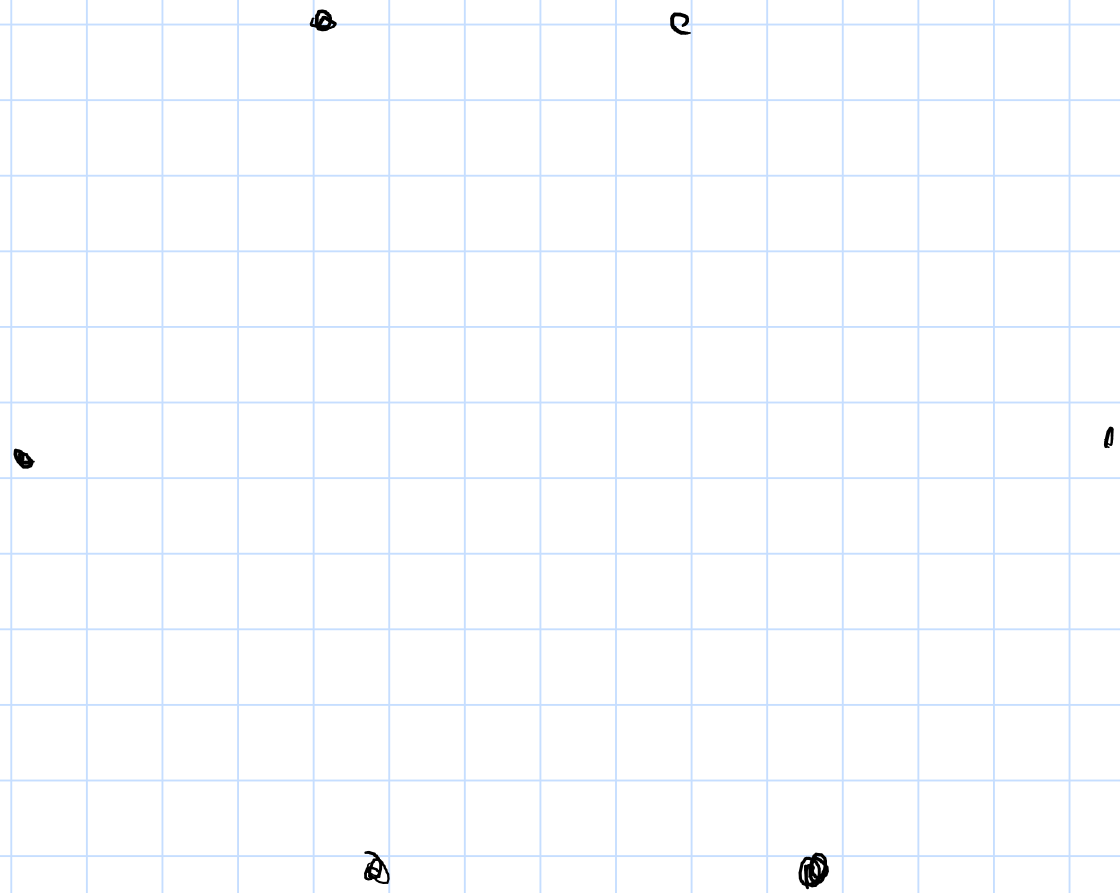


GRAFOS SIMPLES

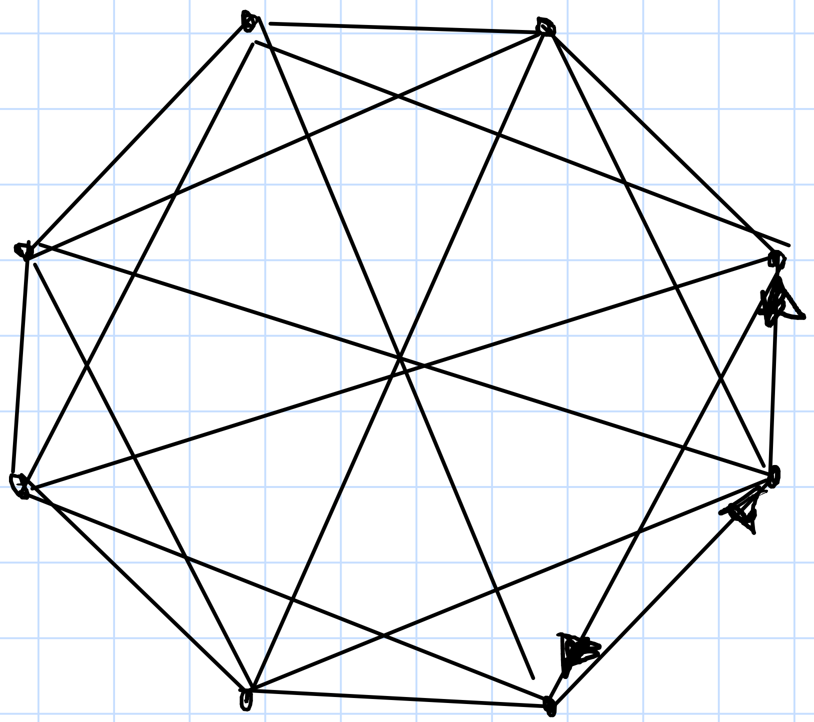
MULTIGRAFOS

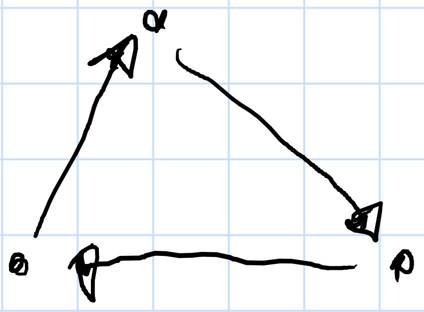


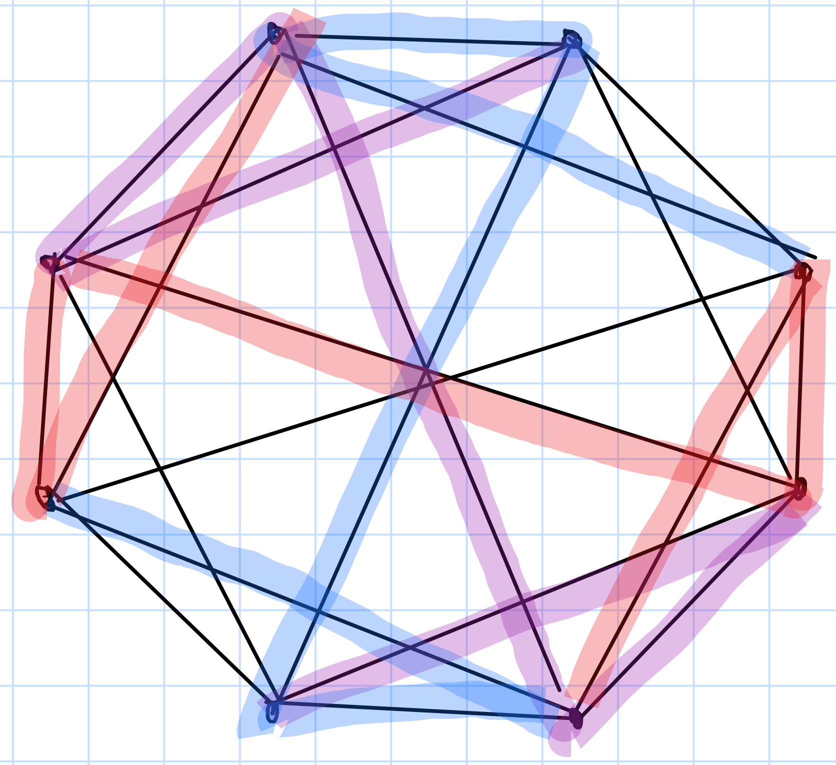


K_6 COMPLETO

2^m → nº DE ARESTAS

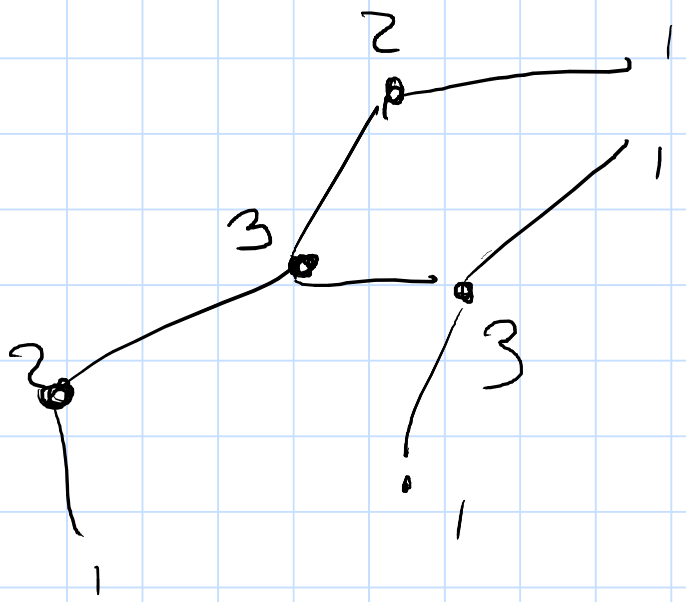




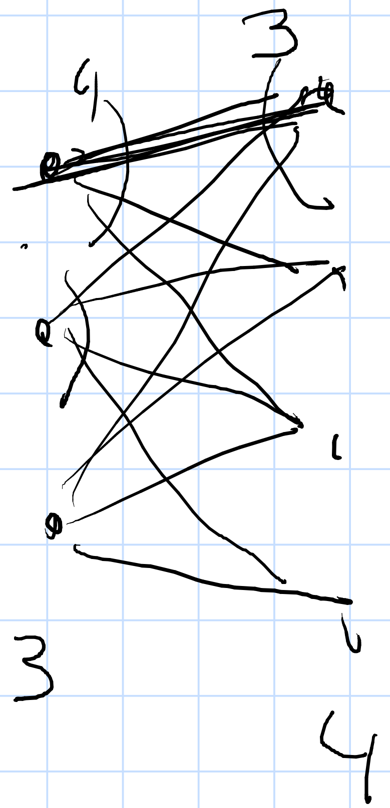


G É LOCALMENTE IRREGULAR

CONJECTURA: 3 CORES



BIPARTIDO COMPLETO



CONJ.: $\chi'_{IRR}(G) \leq 3$

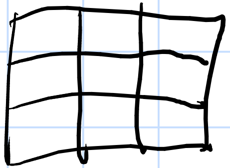
$\|$

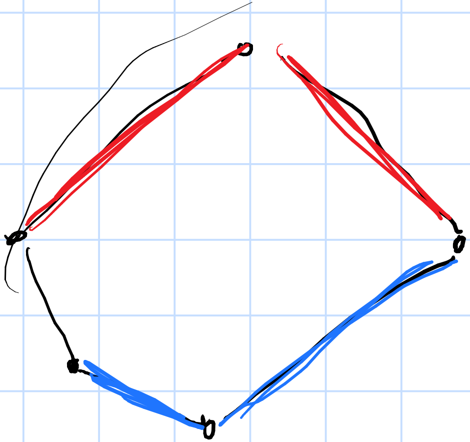
MENOR # CORES EM UMA COLORAÇÃO LOC. IRREG

$c \leq 3$

$$f : E(G) \to [1, \dots, c]$$

G_1



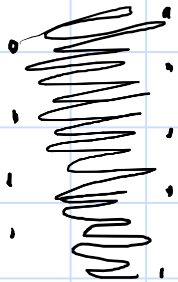


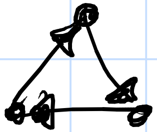
2×1

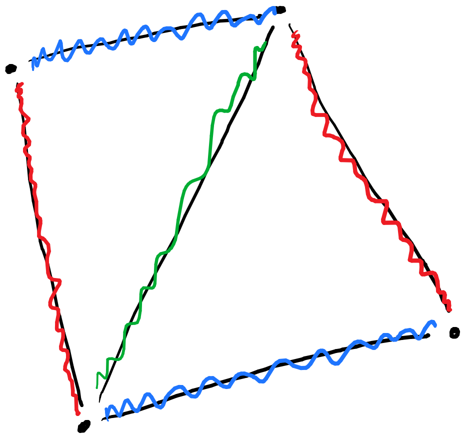
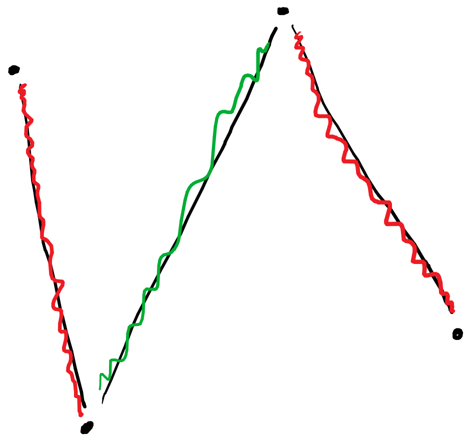
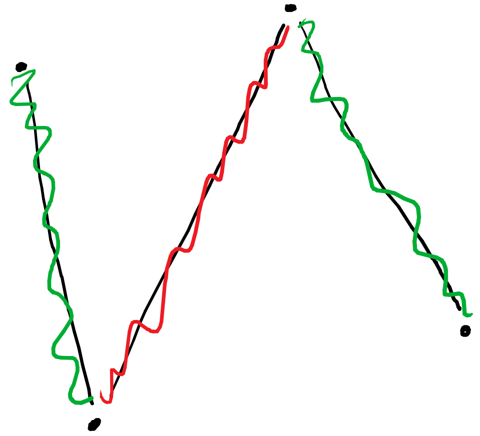
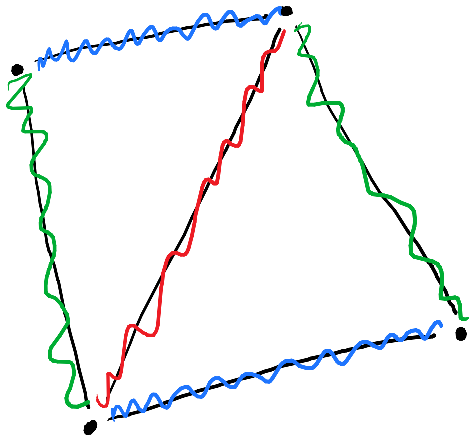
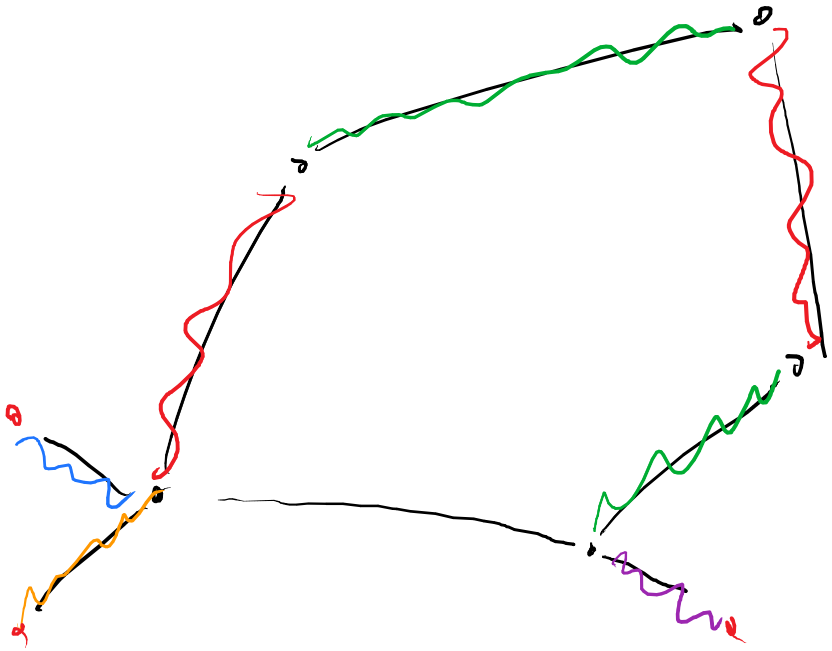
$$D(G, C_3^{\circlearrowright}) \leq 2^{\lfloor \frac{n^2}{4} \rfloor}$$

n é nº de vértices de G

$$D(G, C_3^{\circlearrowright}) = 2^{\lfloor \frac{n^2}{4} \rfloor} \iff G = K_{\lfloor \frac{n}{2} \rfloor, \lceil \frac{n}{2} \rceil}$$





$n - k - 1$

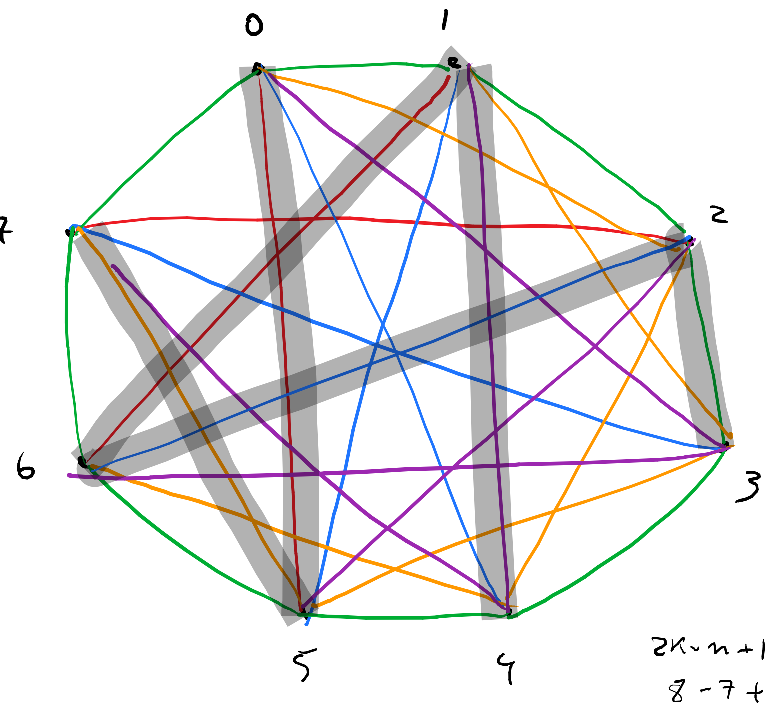
0 5 7 3 2 6 1 4
5 2 4 1 4 5 3

6 6

K=6 : 6 | 6 5 4 3 2 1

K=5 : 5,4 | 5 4 3 2 1

K=4 : 4,3,2 | 4,3,2,1



K = 5

4

4 0 5 1 6 2 7 3

K = 6

K = 4

12 - 7 + 1

6

$2k - n + 1$

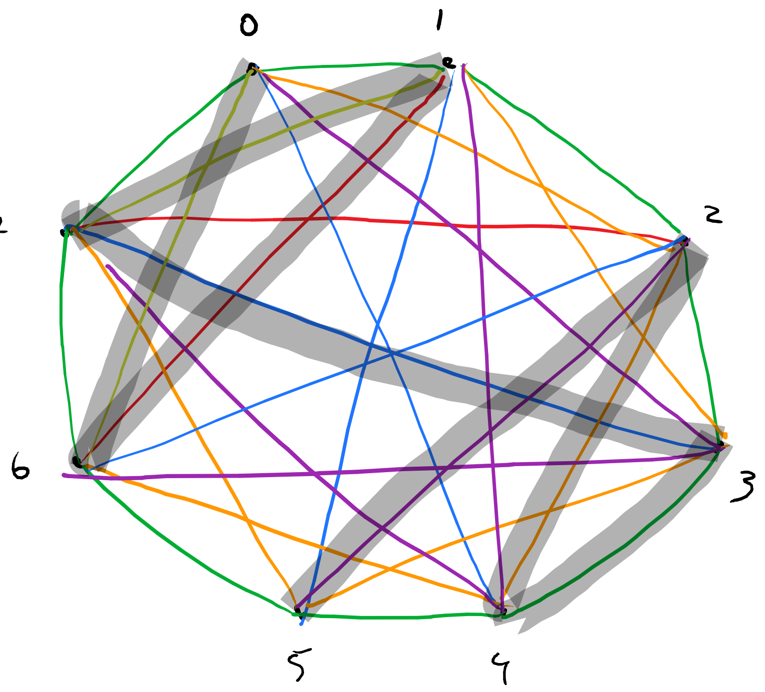
$8 - 7 + 1 = 2$

n − k − 1

0 5 7 3 2 6 1 4
5 2 4 1 4 5 3

6 6

k=6 : 6 | 6 5 4 3 2 1
k=5 : 5,4 | 5 4 3 2 1
k=4 : 4,3,2 | 4,3,2,1



6 1 4 6
0 6 5 1 7

0 1 5 6 7
1 4 1 1

5

0 6 1 7 3 2 4
6 5 6 4 1 2

3

0 3 4 2 5 1 6 9
3 1 2 3 4 5 3

0 6 7 1

0 6 1 7 3 4 2 5
6 5 6 4 1 2 3

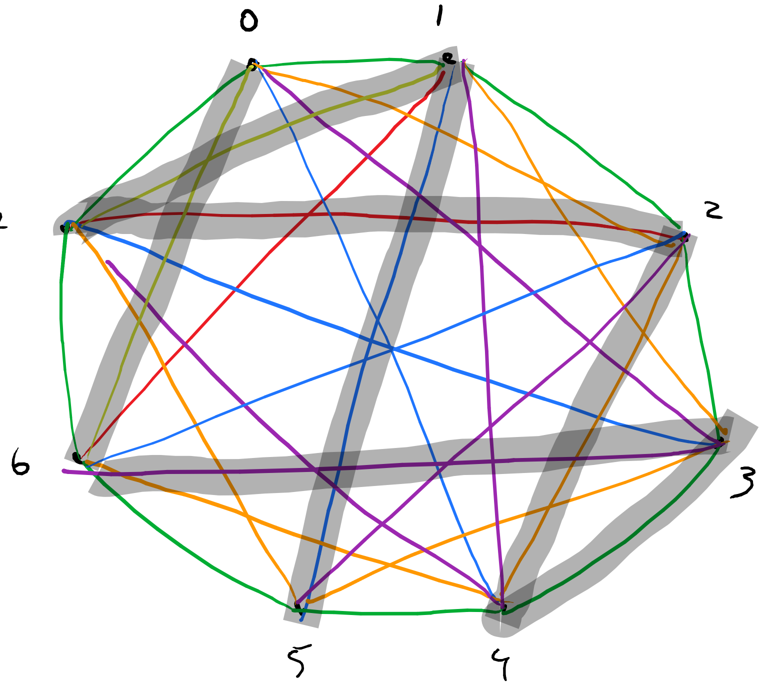
0 1

n - k - 1

0 5 7 3 2 6 1 4
5 2 4 1 4 5 3

6 6

K=6 : 6 | 6 5 4 3 2 1
K=5 : 5,4 | 5 4 3 2 1
K=4 : 4,3,2 | 4,3,2,1



5

0 6 1 7 3 2 4
6 5 6 4 1 2

3

0 3 4 2 5 1 6 9
3 1 2 3 4 5 3

06 71

0 6 1 7 3 4 2 5
6 5 6 4 1 2 3

0 1

6 1 4 6
0 6 5 1 7

0 1 5 6 7
1 4 1 1

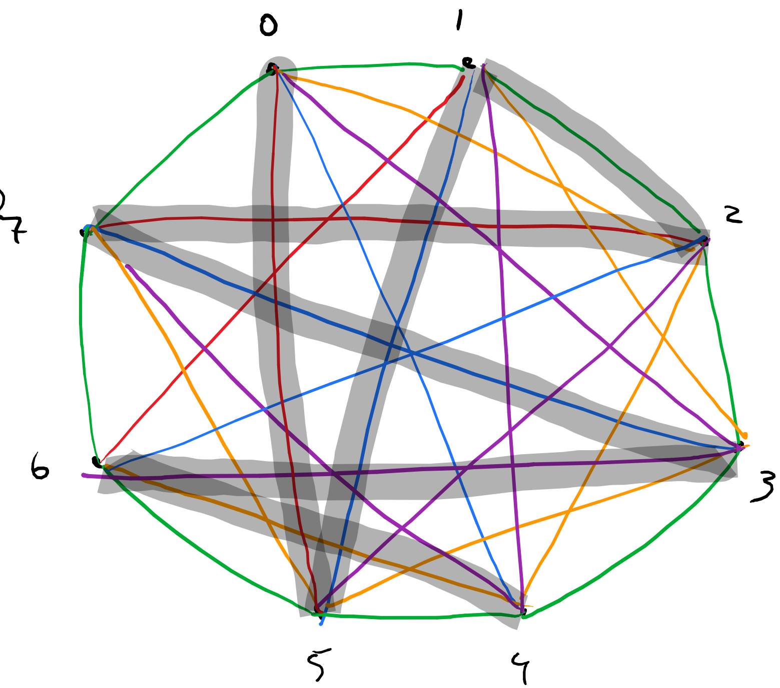
0 6 3 4 2 7 1 5
6 3 1 2 5 6 4

n - k - 1

0 5 7 3 2 6 1 4
5 2 4 1 4 5 3

6 6

K=6 : 6 | 6 5 4 3 2 1
K=5 : 5,4 | 5 4 3 2 1
K=4 : 4,3,2 | 4,3,2,1



6 1 4 6
0 6 5 1 7

0 1 5 6 7
1 4 1 1

0 6 3 4 2 7 1 5
6 3 1 2 5 6 4

5

0 6 1 7 3 2 4
6 5 6 4 1 2

3

0 3 4 2 5 1 6 9
3 1 2 3 4 5 3

0 6 7 1

0 6 1 7 3 4 2 5
6 5 6 4 1 2 3

0 5

0 1

0 4

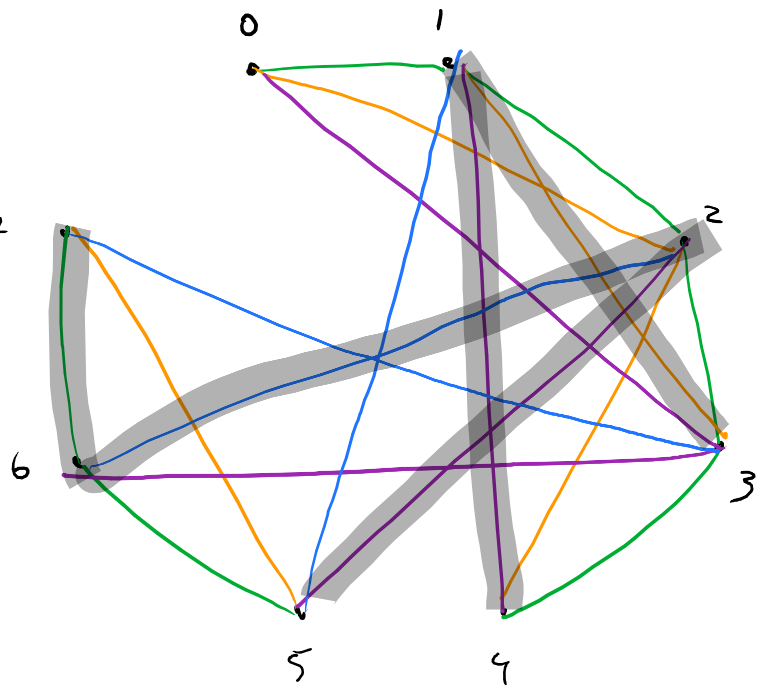
0 5 1 2 7 3 6 4
5 4 1 5 4 3 2

n - k - 1

0 5 7 3 2 6 1 4
5 2 4 1 4 5 3

6 6 7

k=6 : 6 | 6 5 4 3 2 1
k=5 : 5, 4 | 5 4 3 2 1
k=4 : 4, 3, 2 | 4, 3, 2, 1



6 1 4 6
0 6 5 1 7

0 1 5 6 7
1 4 1 1

5
0 6 1 7 3 2 4
6 5 6 4 1 2
3

0 3 4 2 5 1 6 9
3 1 2 3 4 5 3

0 6 7 1

0 6 1 7 3 4 2 5
6 5 6 4 1 2 3

0 5
0 1
0 4

0 5 1 2 7 3 6 4
5 4 1 5 4 3 2

0 6 4 2 7 1 5
6 3 1 2 5 6 4

0 4 1 3 5 2 6 7
4 3 2 2 3 4 1

5 4 4 5

$K = \frac{n}{2}$

$n - n + 1$

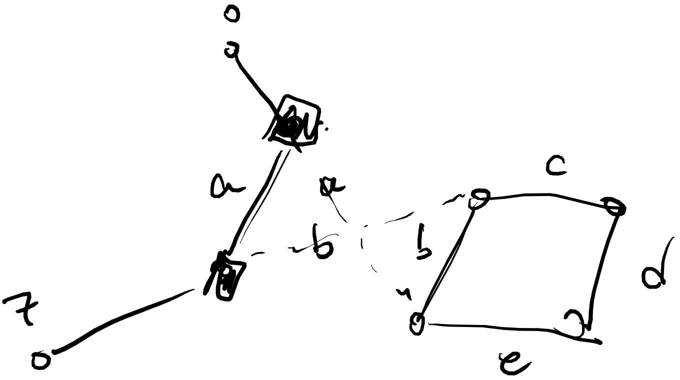
$2K - n + 1$

$K = n - 1$

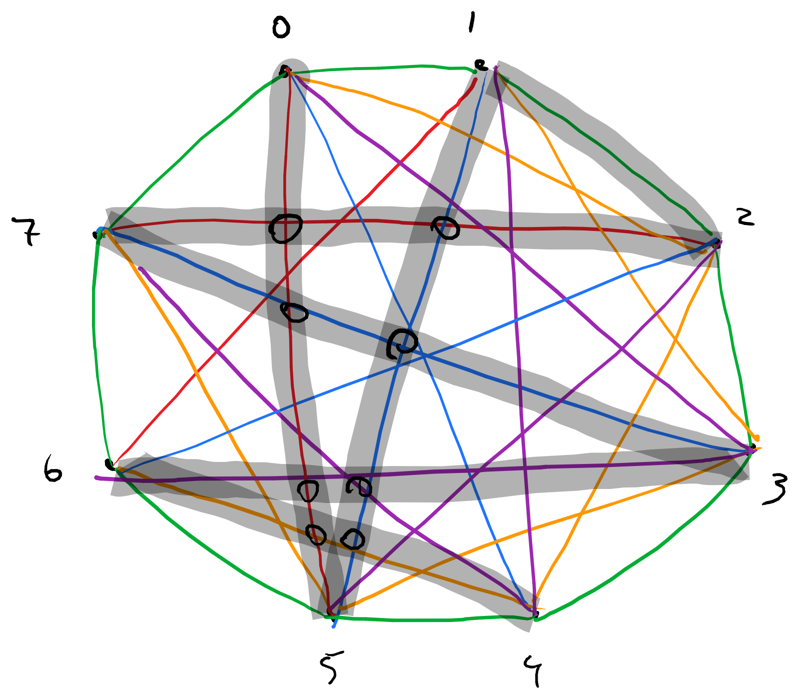
$2(n-1) - n + 1$

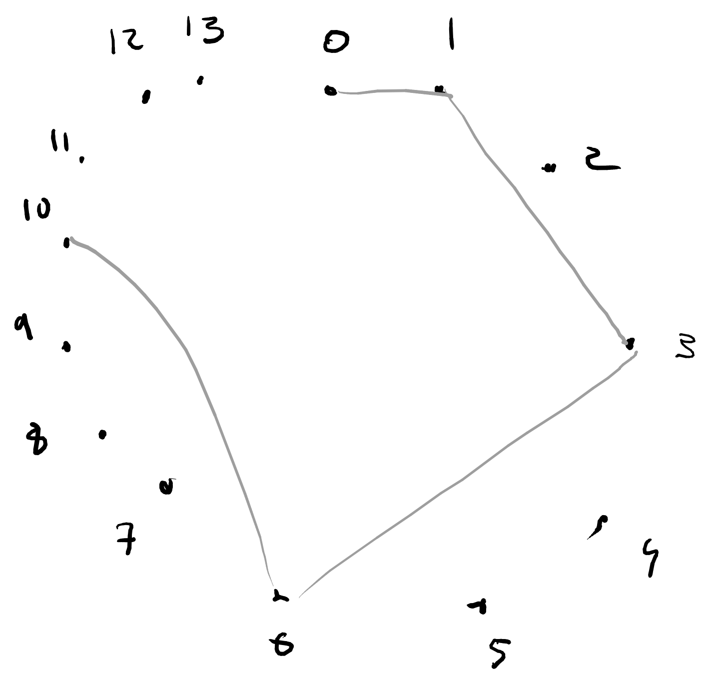
$2n - 2 - n + 1$

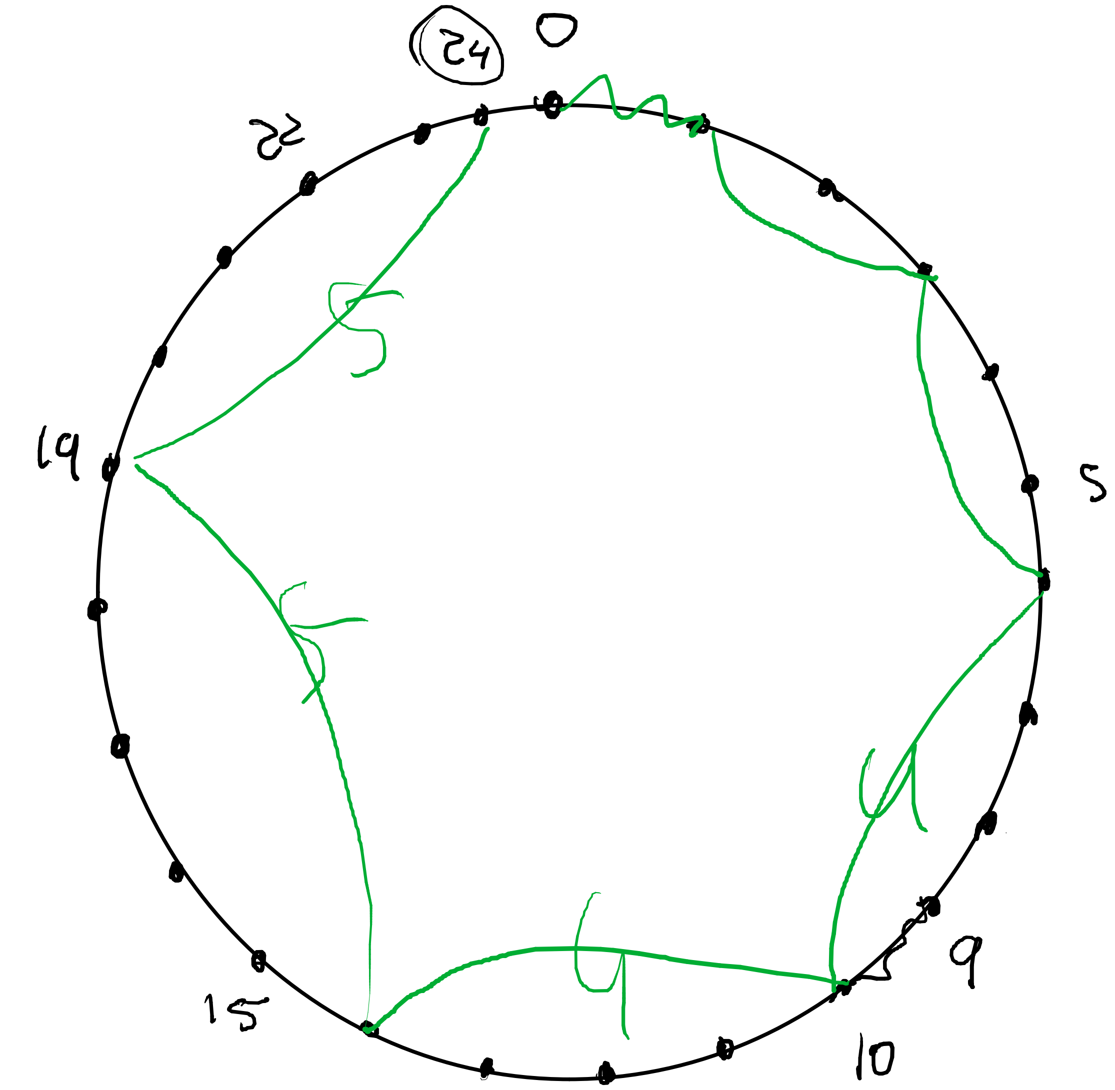
$n - 1$



1 3 2 4 3 2 4







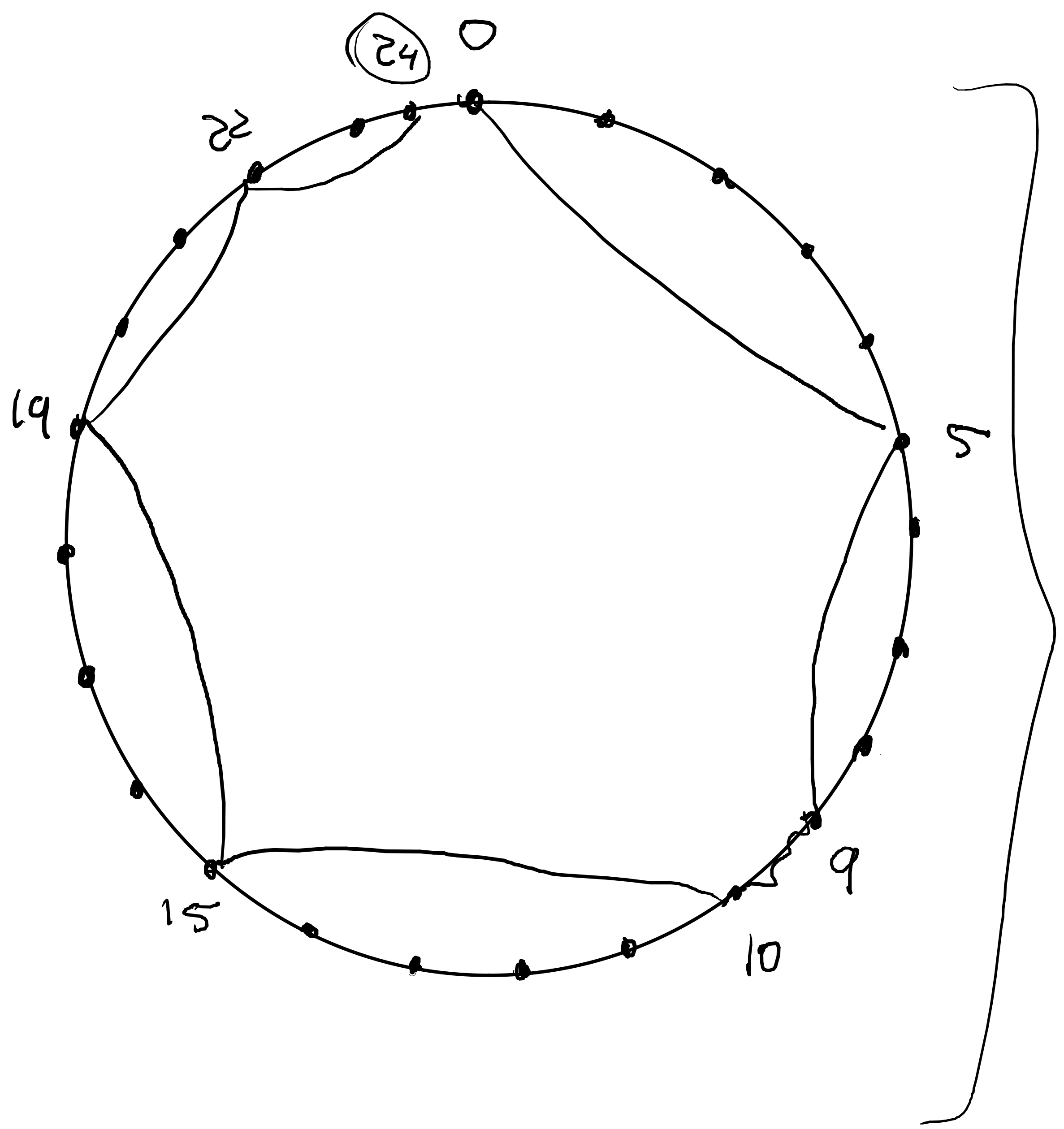
0 5 1 2 7 3 6 4

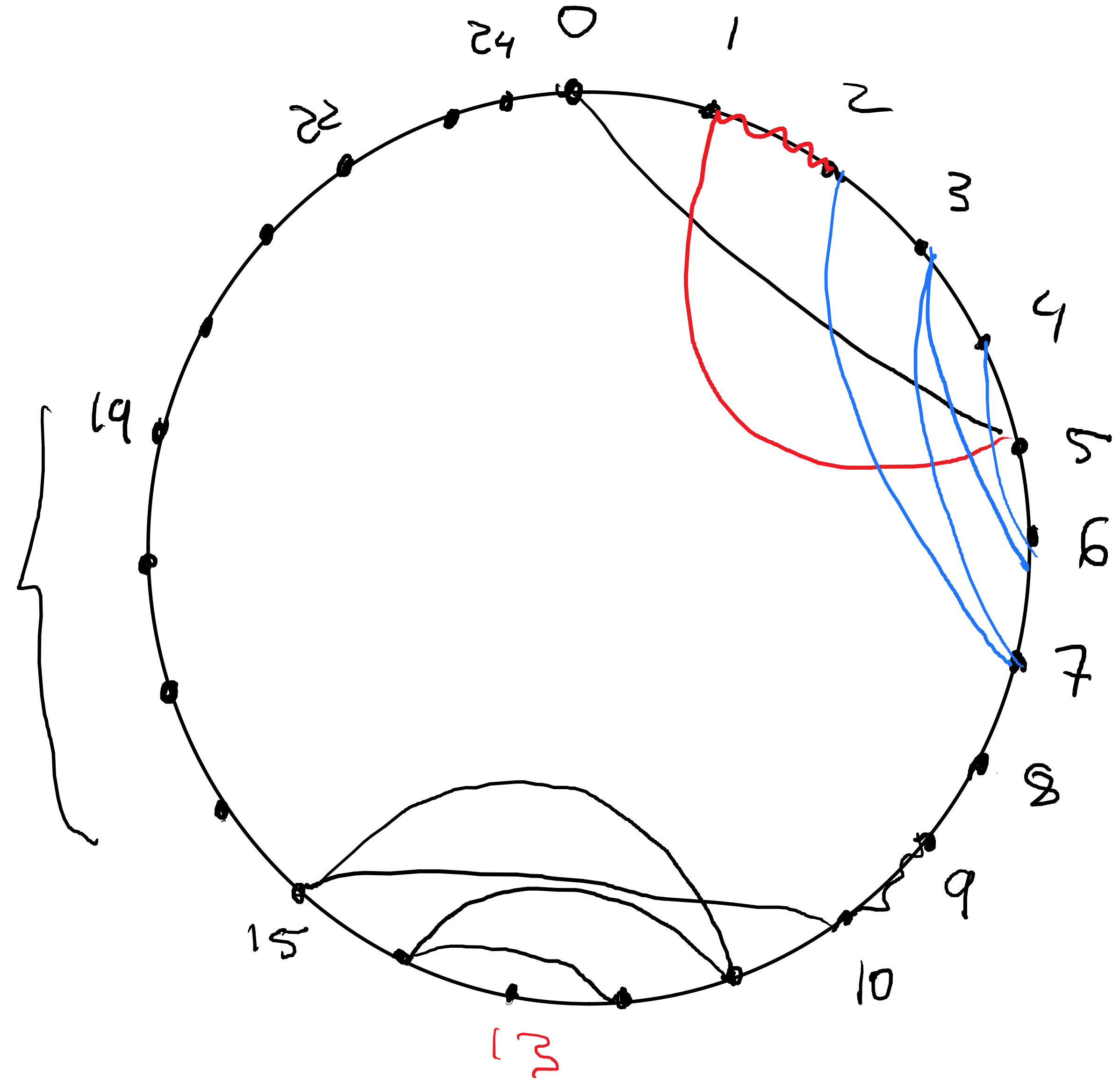
5 4 1 5 4 3 2 ~→ }

0 5 9 10 15 19 22 24

$0+1+2+3+4+4+5+5 = 24$

0 5 1 2 7 3 6 4





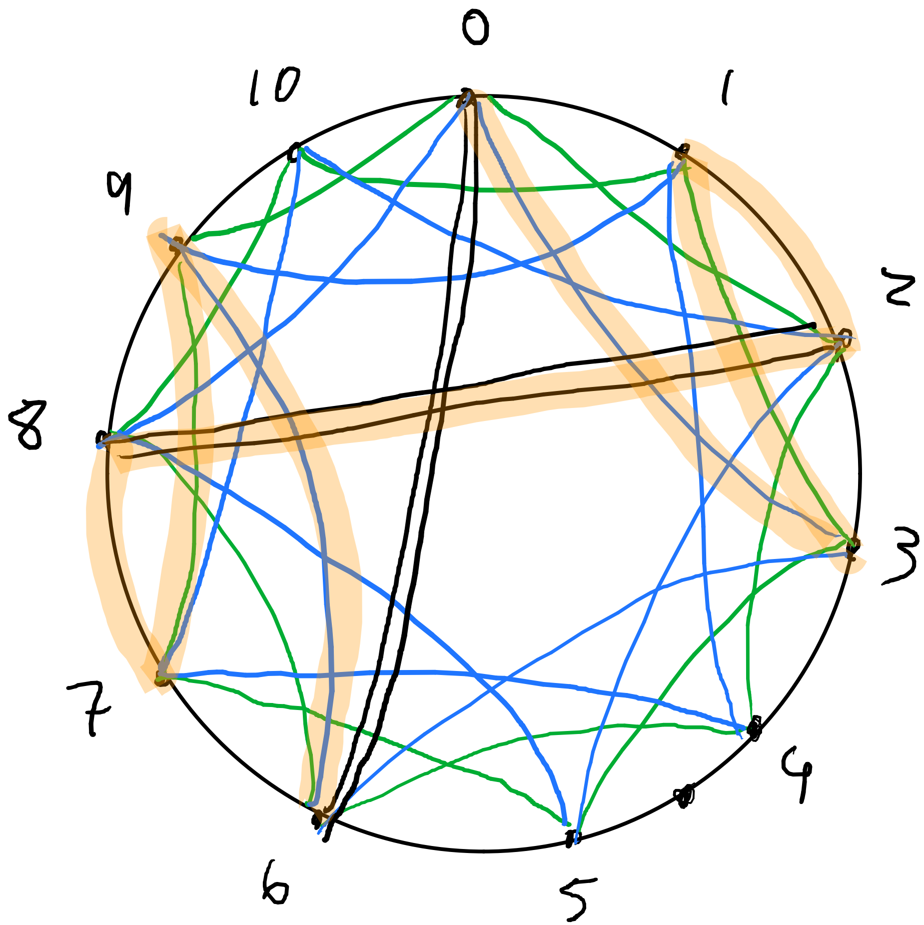
$\frac{n}{2} \leq k \leq n$

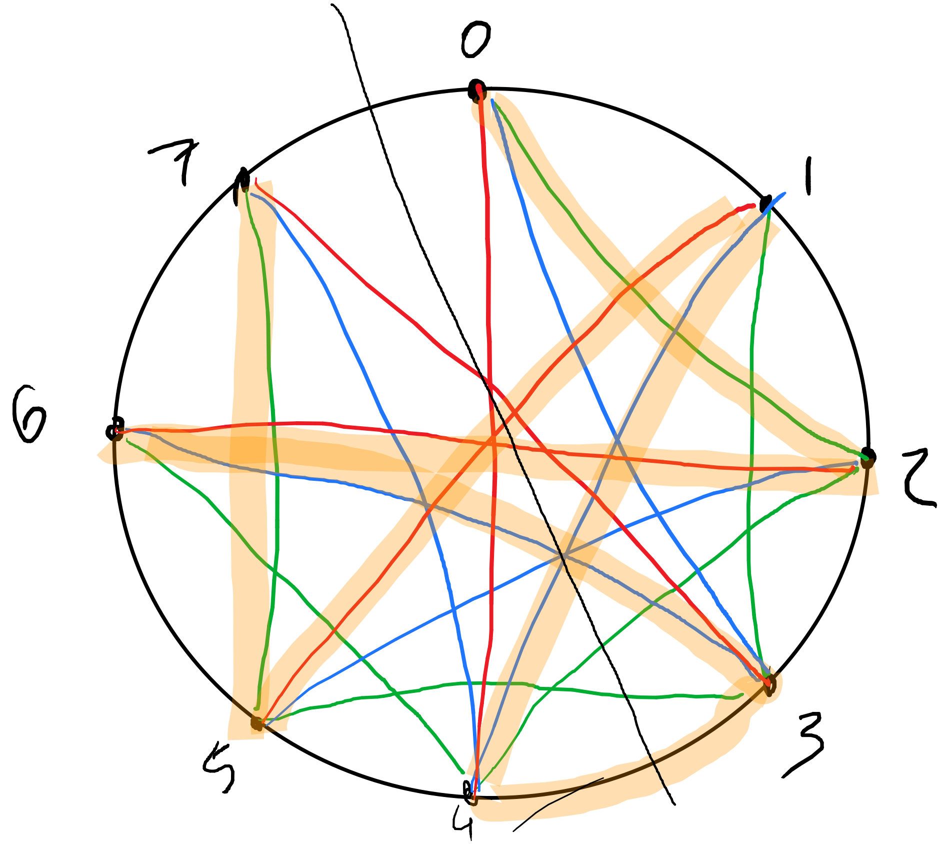
REPETIDOS : $2k - n + 1 \quad \cdots \quad k$

EX: $k=5 \quad n=7 \qquad 4,5$

$k=6 \quad n=7 \qquad 6$

$k=4 \quad n=7 \qquad 2 \ \ldots \ 4$





X_0

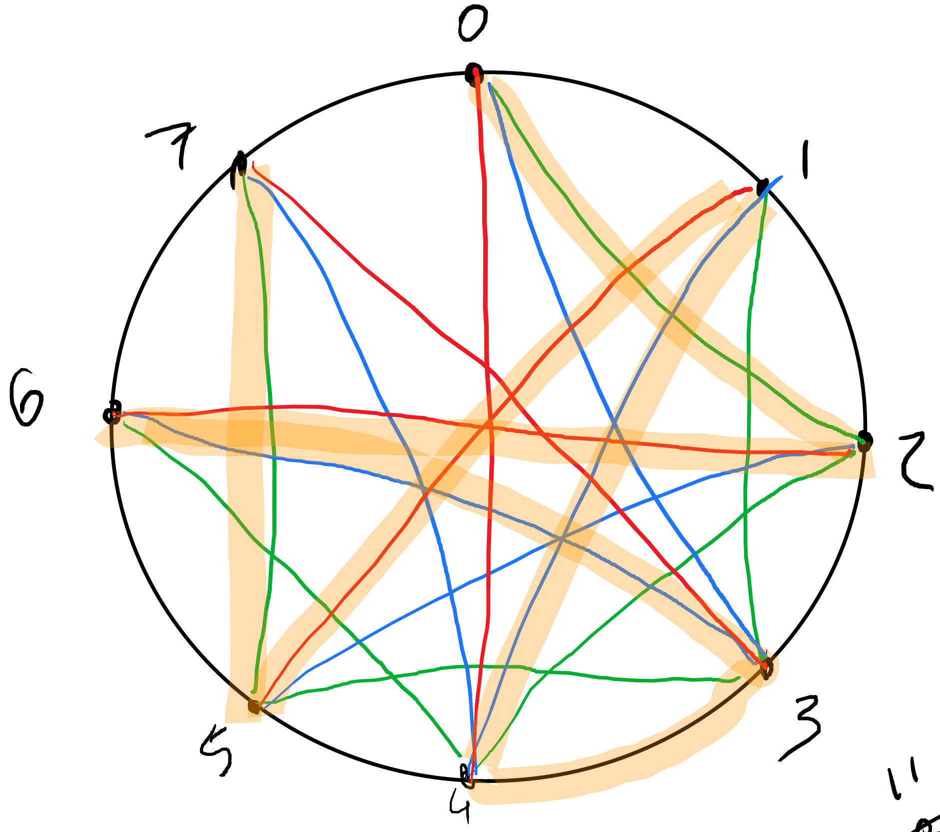
$7 \ X_1 \ X_2 \ \cdots \ X_7$

$0 \ 7-X_1 \ 7-X_2 \ \cdots \ 7-X_7$

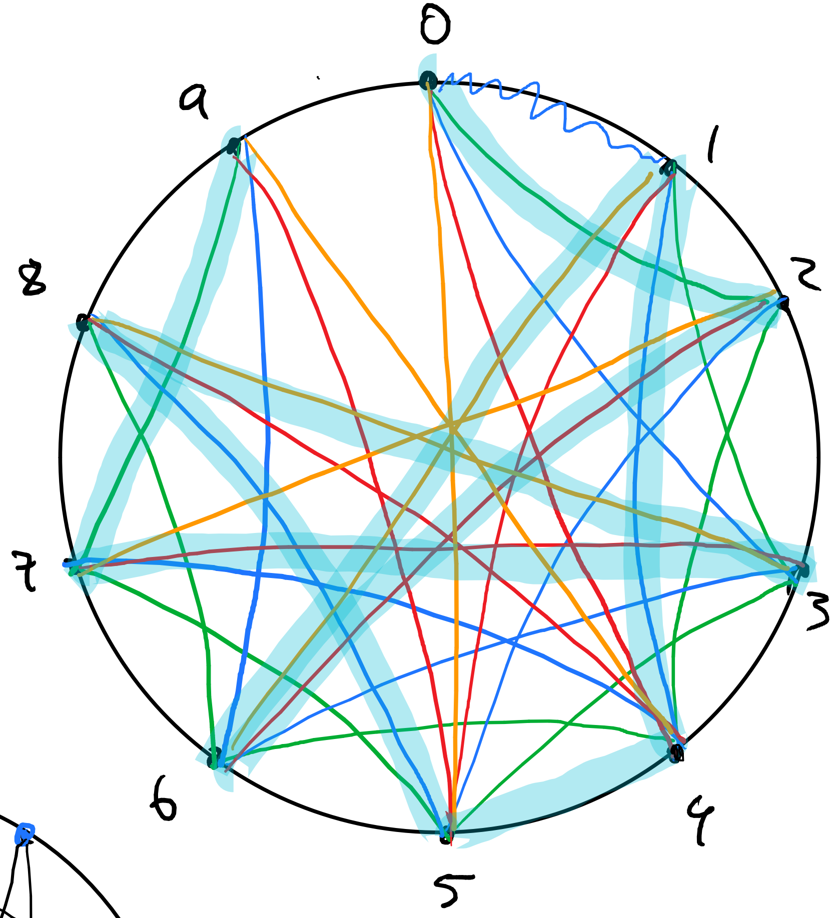
H-DECOMP. G

$Y_i = 7 - X_i$

$D = \{H_1, \ldots, H_k\}$

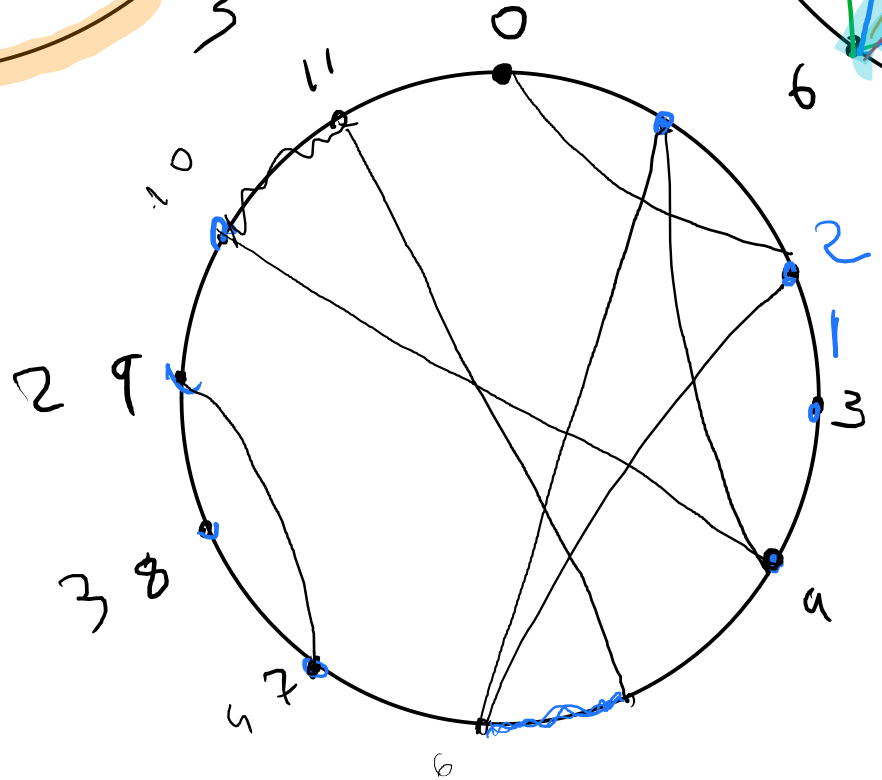


0 2 6 3
2 4 3



0 2 6 1 4 5
2 4 5 3

65

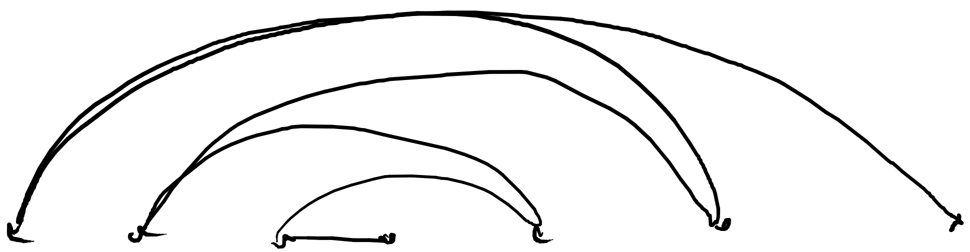


$n = 11 \qquad k = 6$

$2k - n + 1 = 2 \quad \ldots \quad 6 = k$

$1 + 2 \cdot \sum_{i=2}^{6} i \qquad 1 + 2 \cdot 20 = 41$

$2 \quad 3 \quad 4 \quad 5 \quad 6$



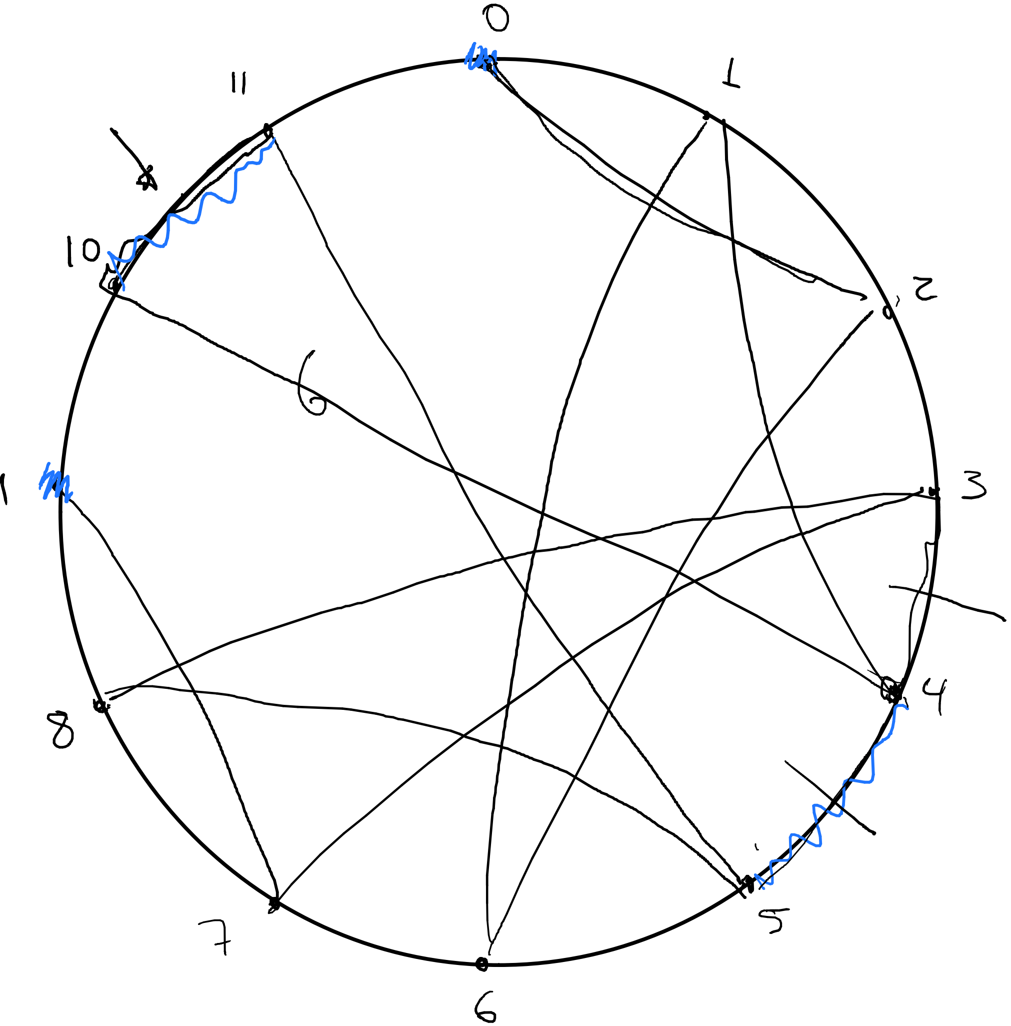
$n = 11$

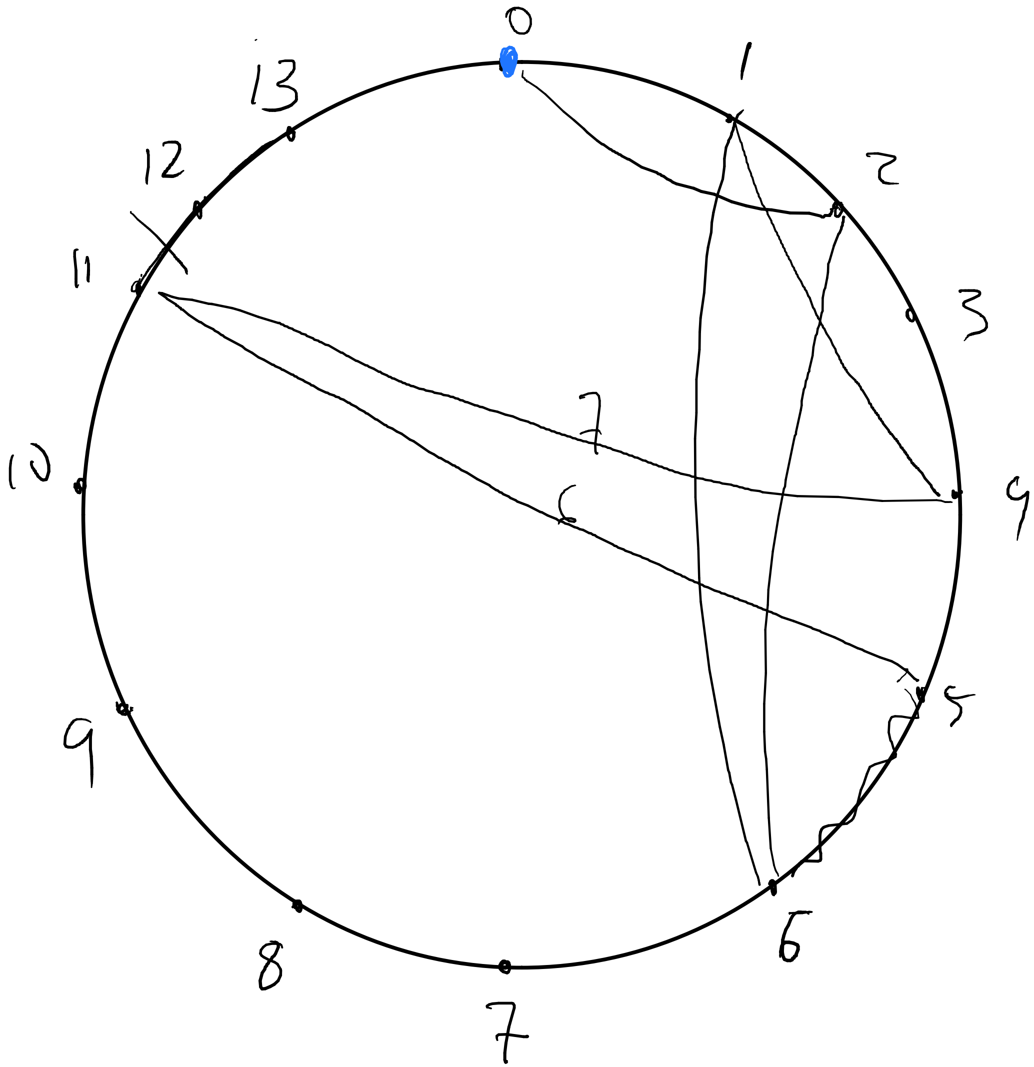
$K = 6$

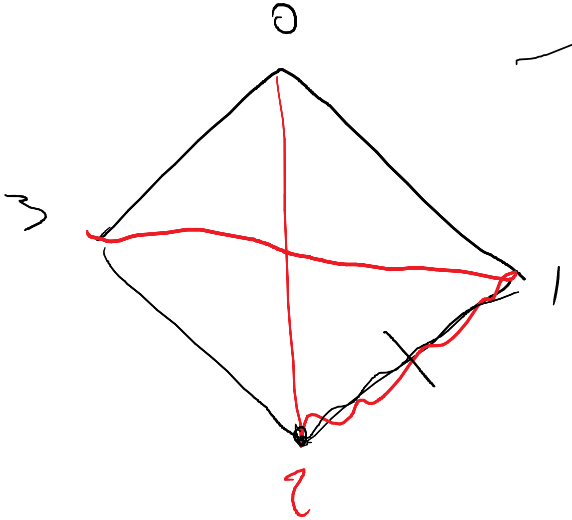
0 2 6 1 4 10 11

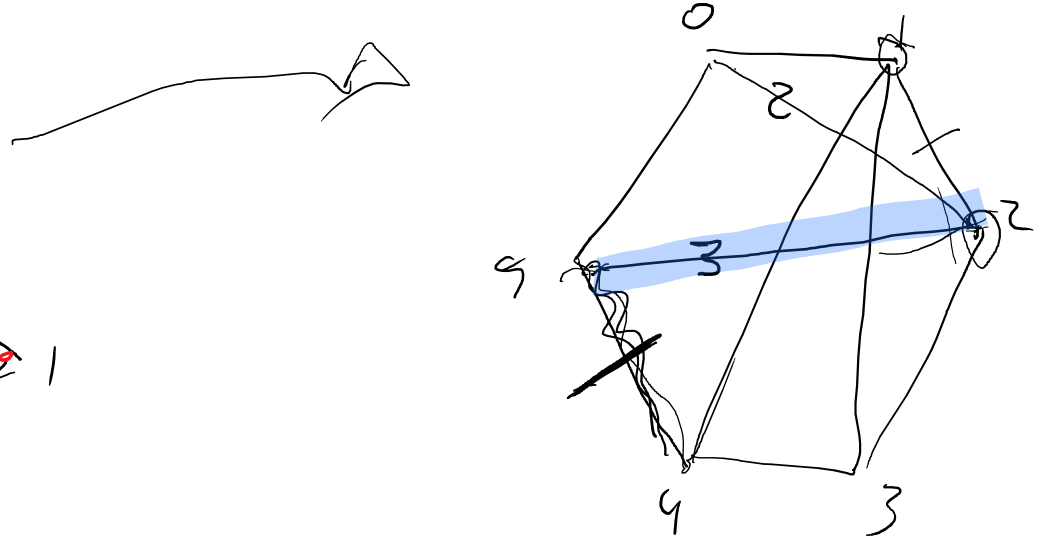
0 3 1 7 2 6 5 9 4 10 8 11

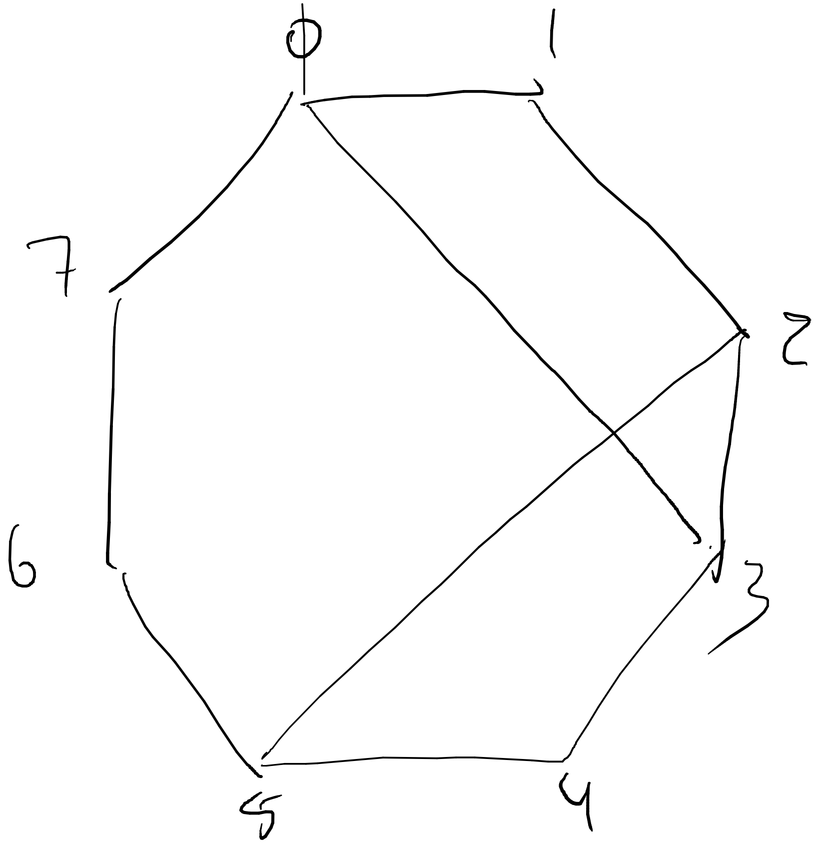
3 2 6 5 4 1 4 5 6 2 3

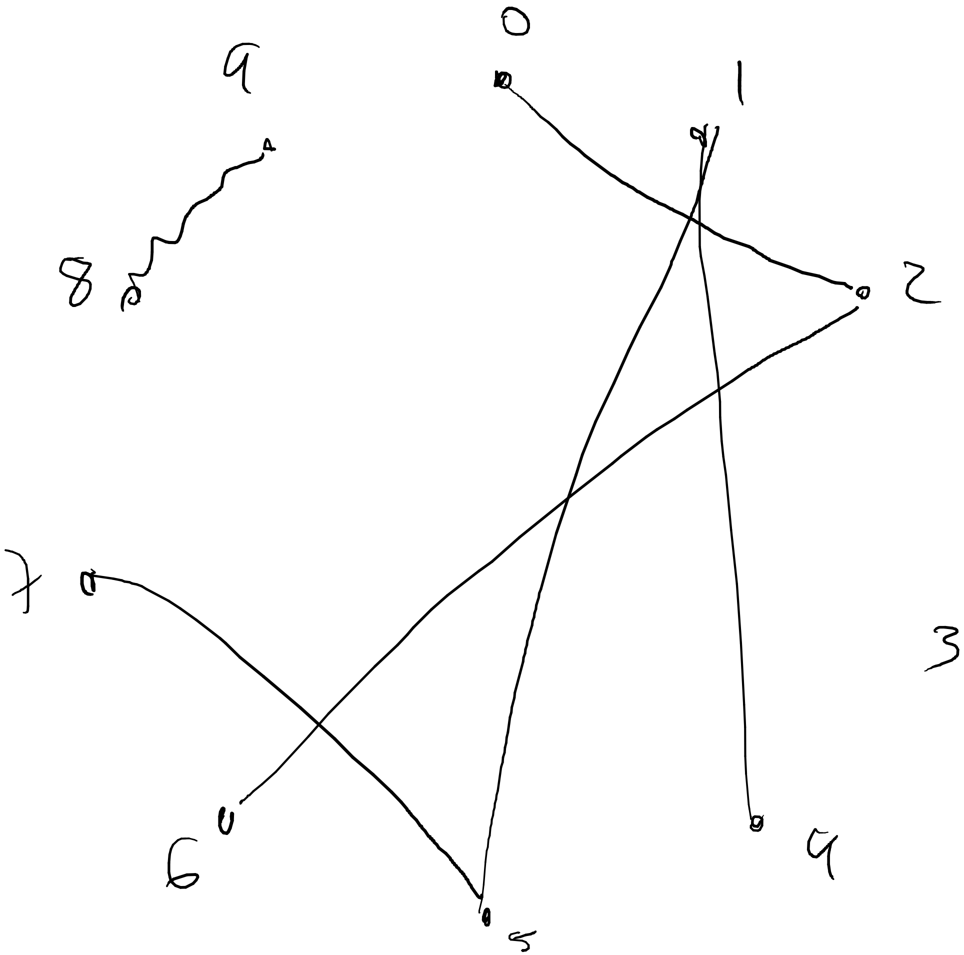


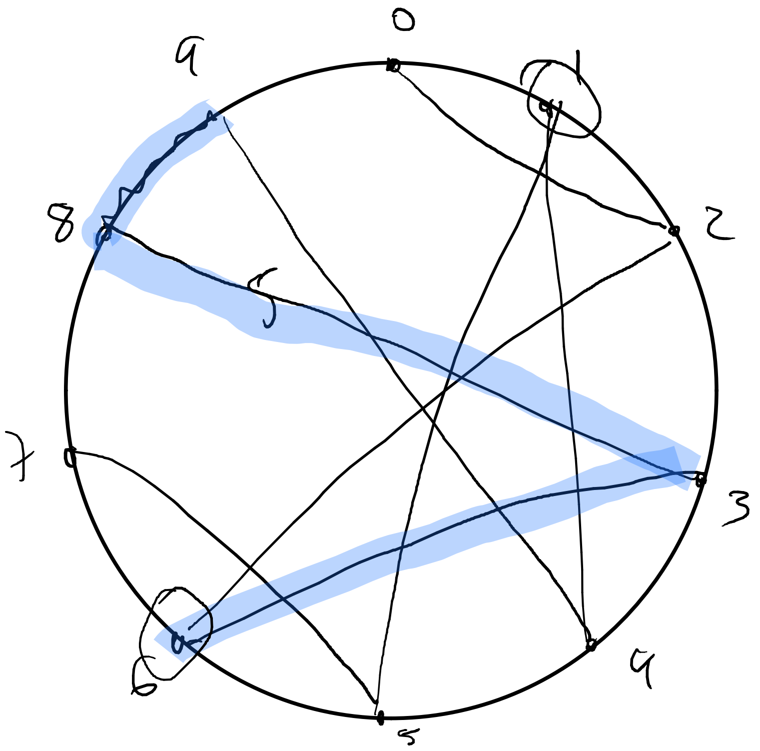


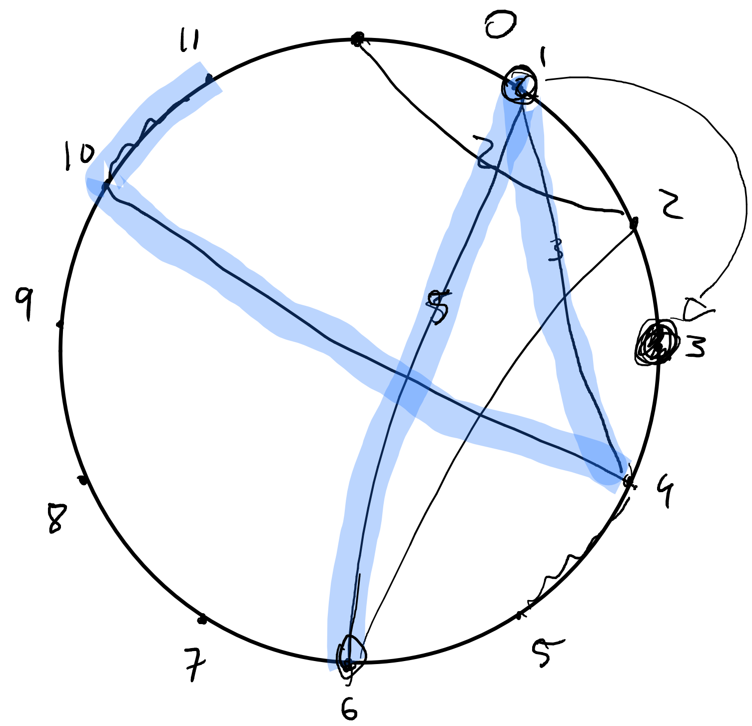

0
1
2
3


0
1
2
3
4
5
2
3


0
1
2
3
4
5
6
7

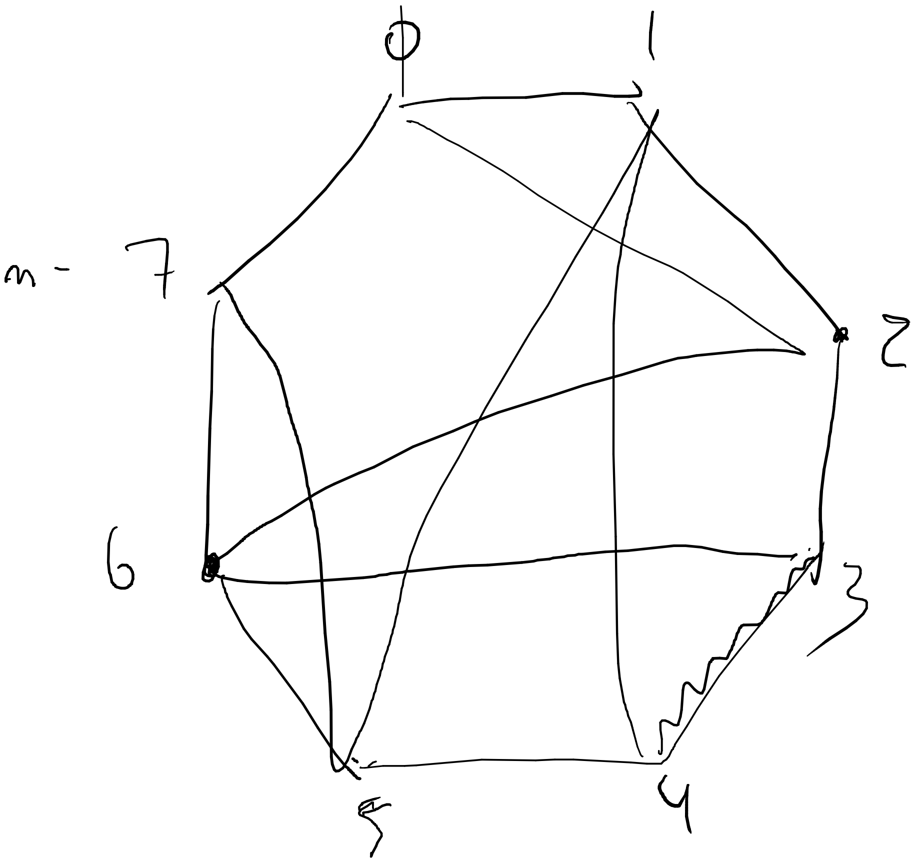

0
1
2
3
4
5
6
7
8
9

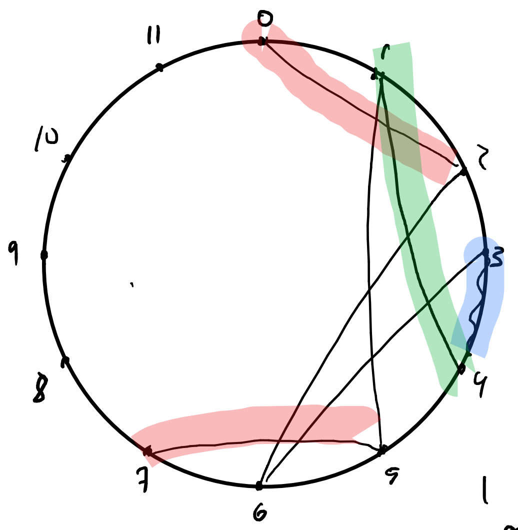


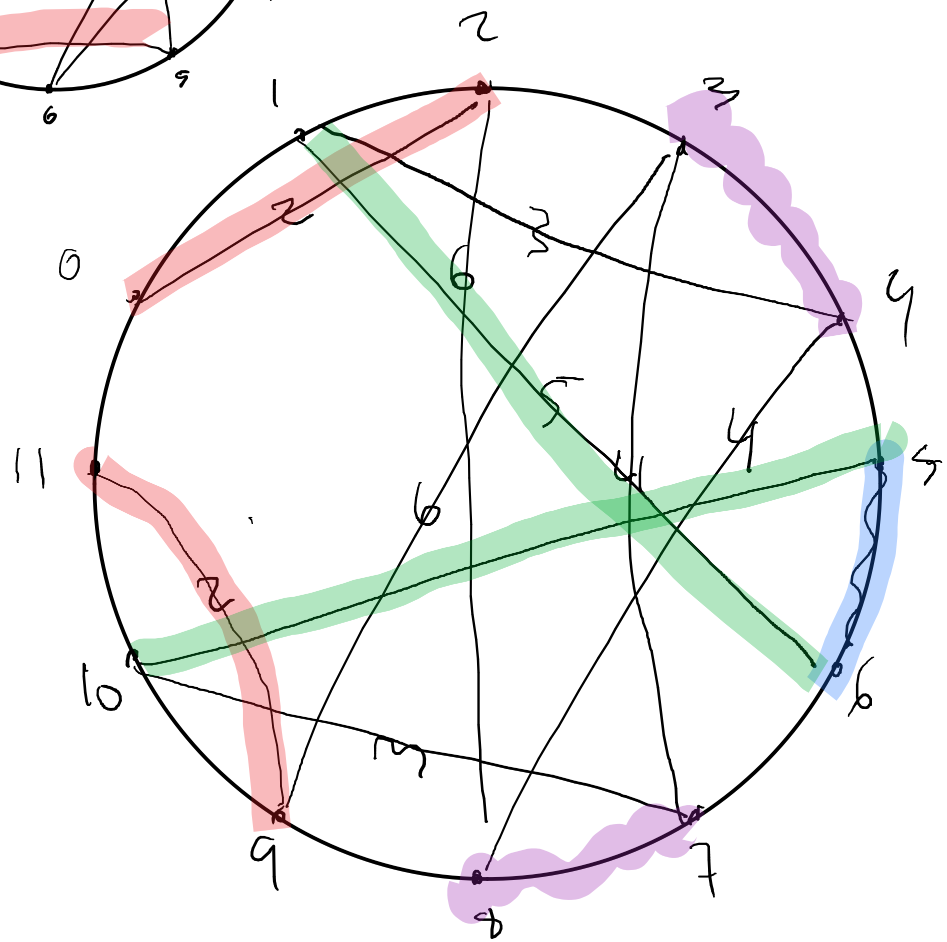


3 5 1

5 3 6 1

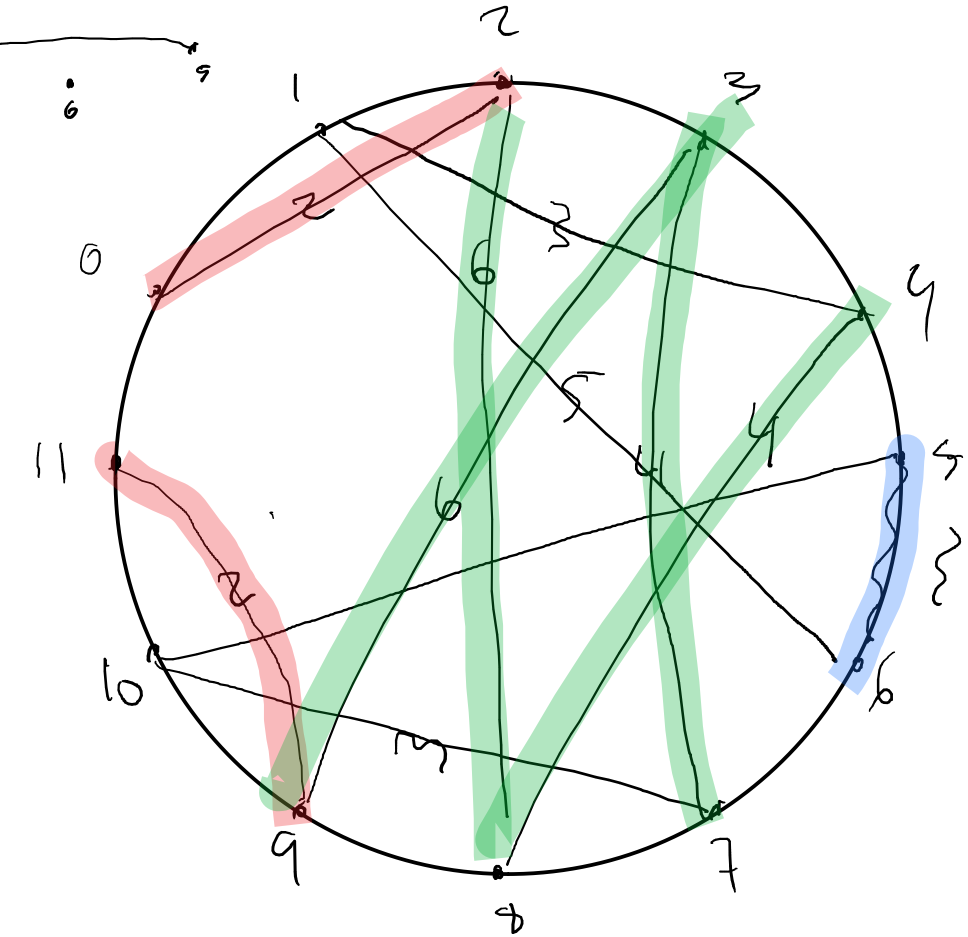

0
1
2
3
4
5
6
7

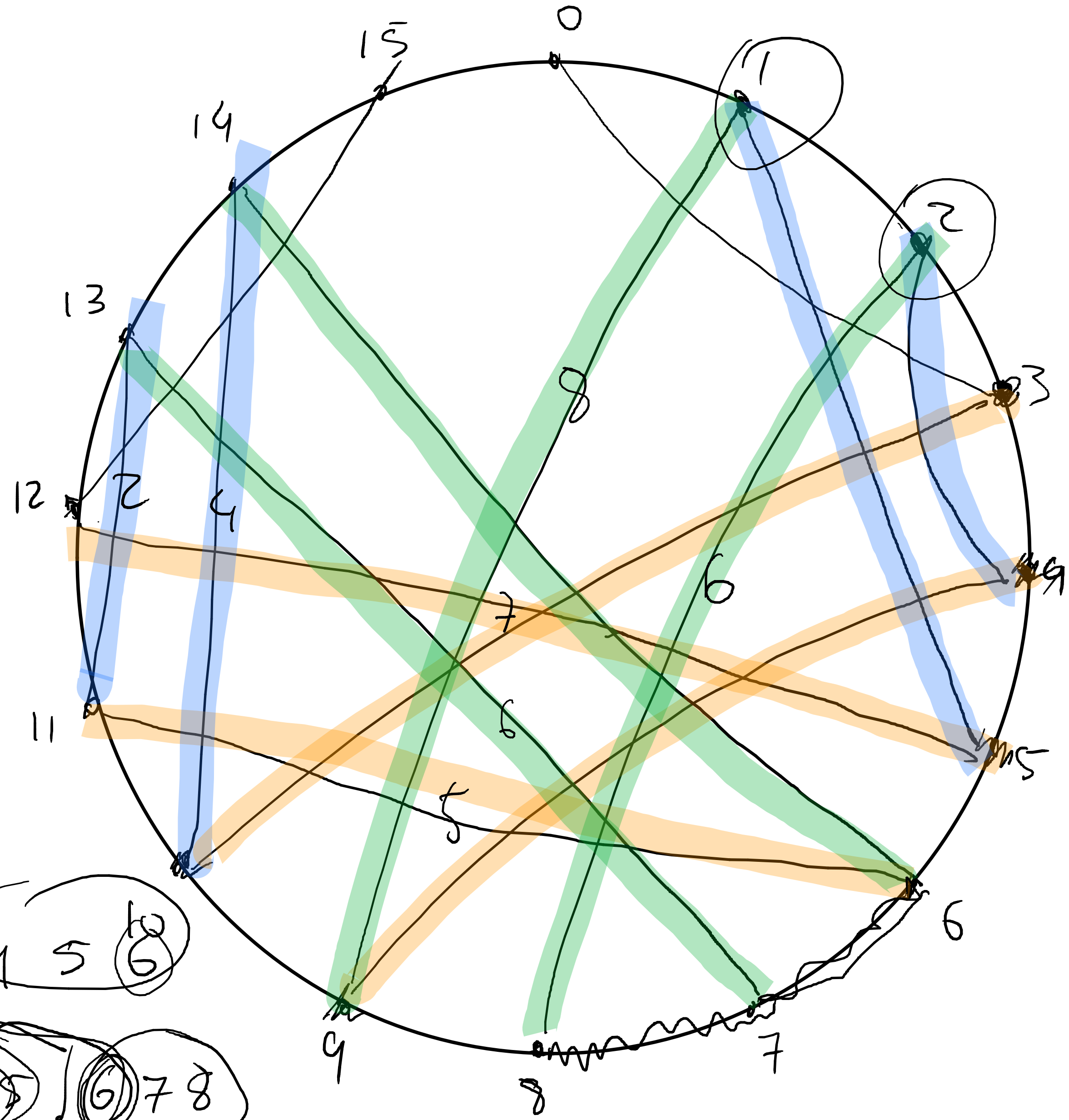

0
1
2
3
4
5
6
7
8
9
10
11


0
1
2
3
4
5
6
7
8
9
10
11

2 4   5 7   6 8

~~1~~ ~~2~~ 3 ~~4~~ 5 6 ~~7~~ ~~8~~



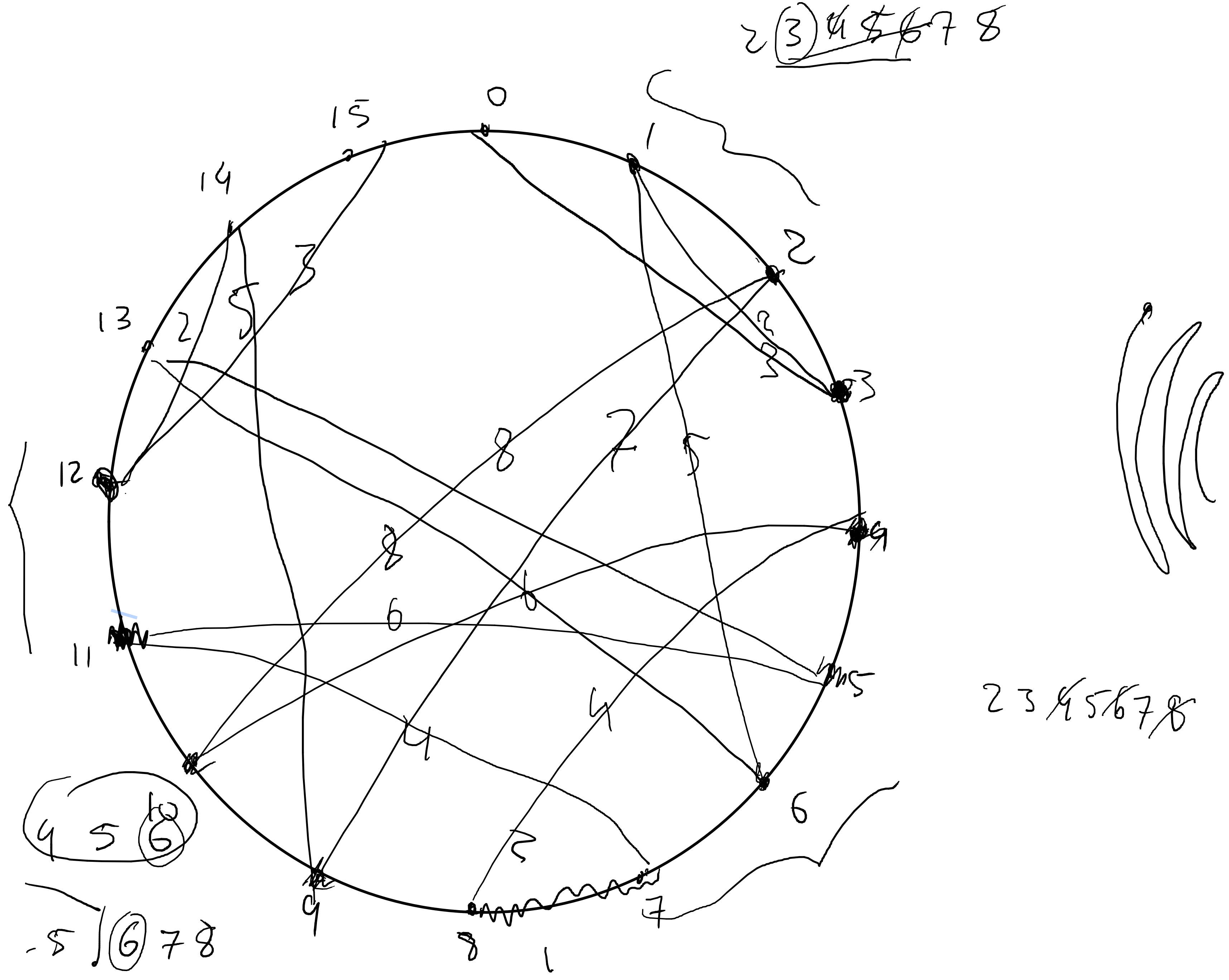


A

1 2 3 | 4 5 6

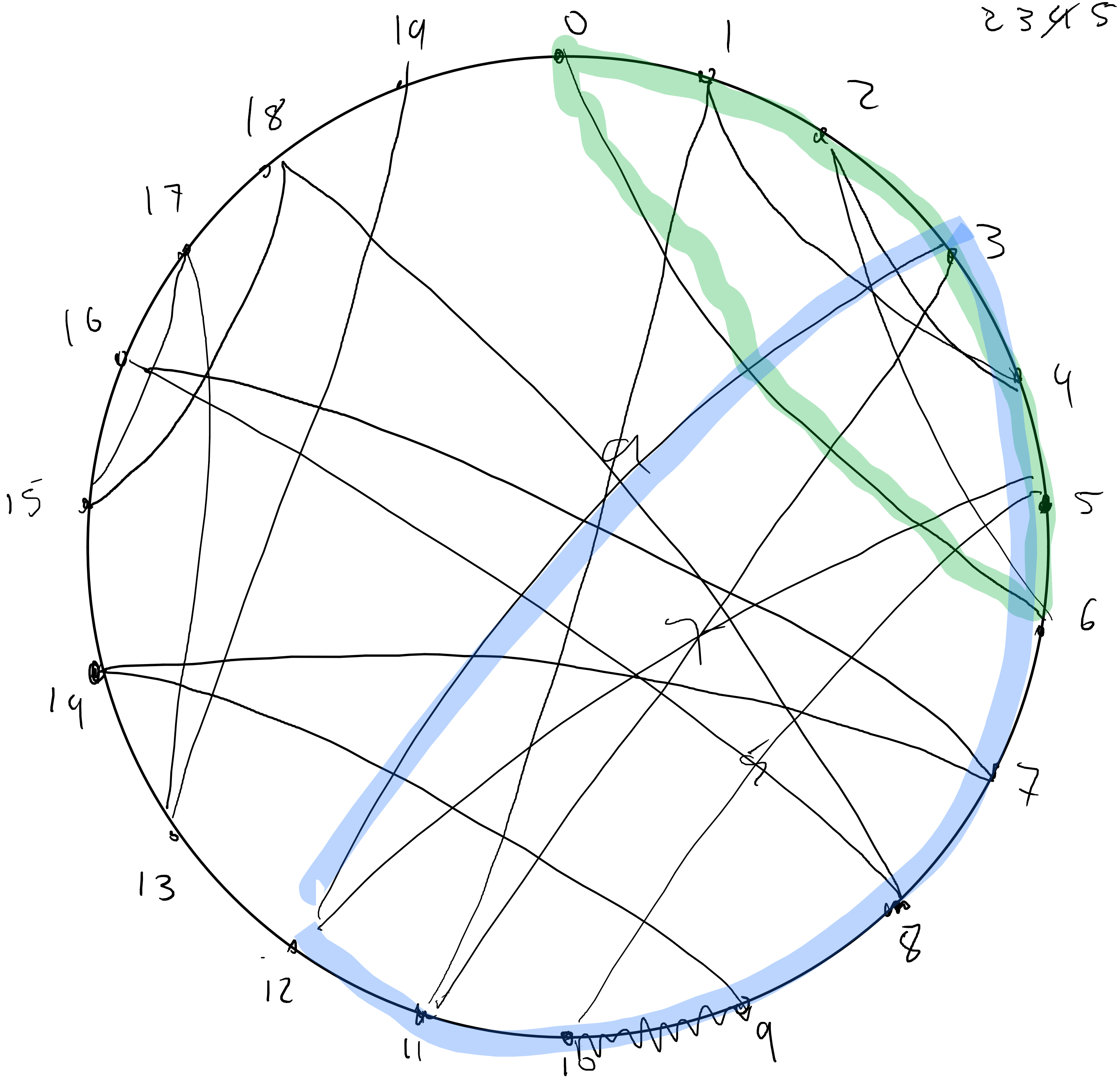
1 2 3 | 4 5 6

1 2 3 | 4 5 | 6 7 8



2 3 4 5 6 7 8
0
1
2
3
9
5
6
7
8
9
11
12
13
14
15
2 3 4 5 6 7 8
4 5 6 10
5 6 7 8

2 3 ~~4~~ 5 ~~6~~ 7 ~~8~~ 9 ~~10~~



$n = 15$

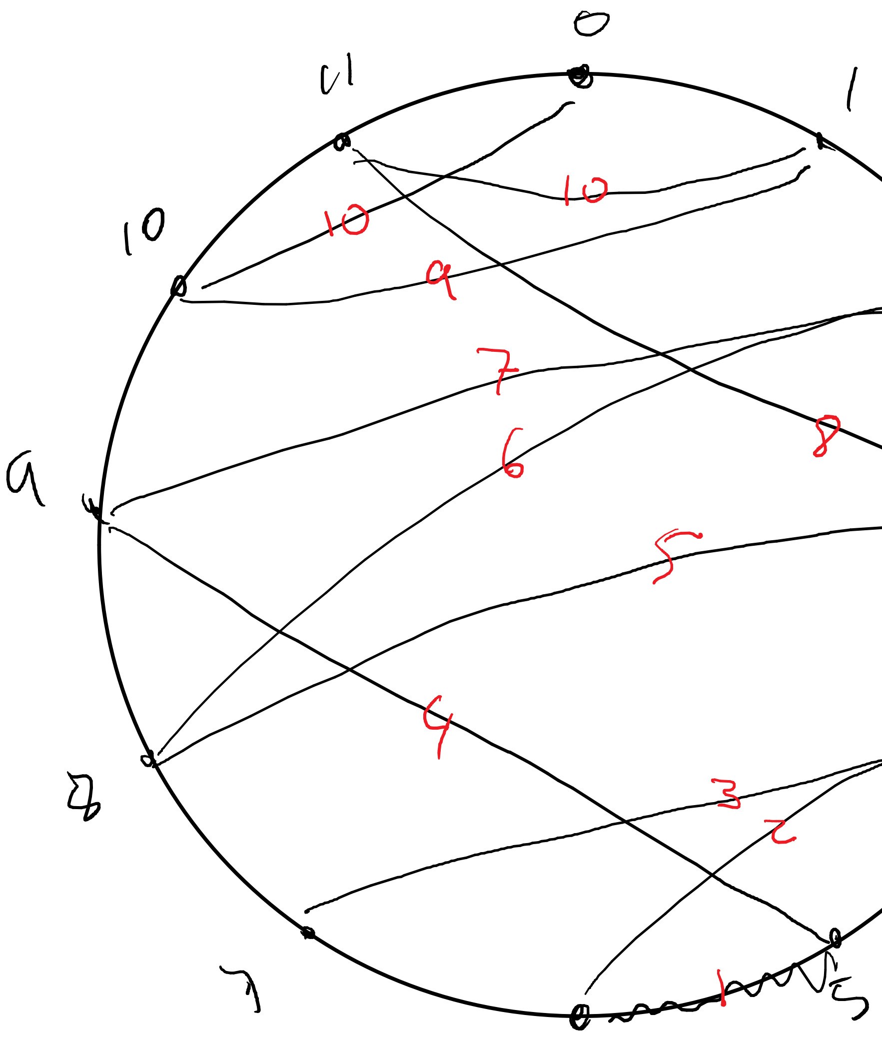
$K = \frac{n+1}{2}$ 1 2 3 4 5 6 7 8
2 3 4 5 6 7 8

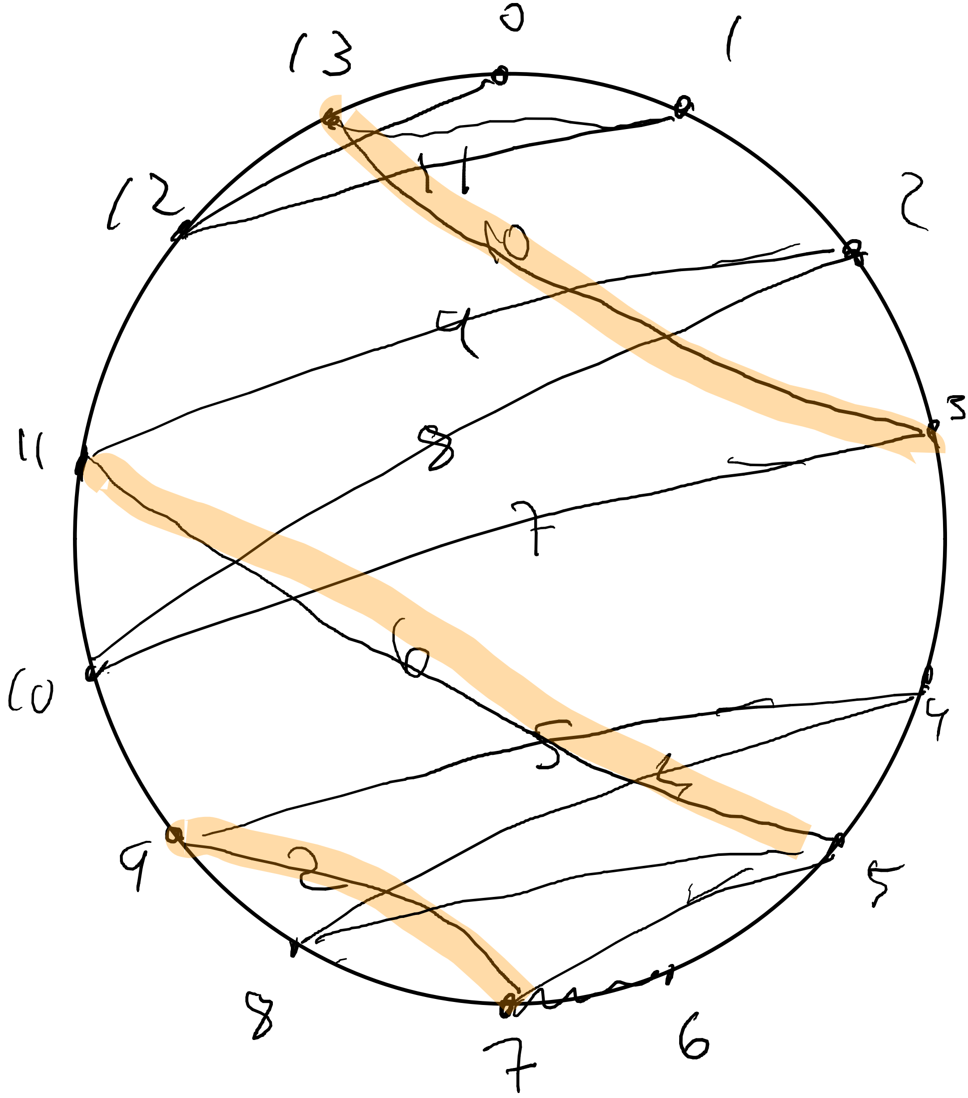
$K = \frac{n+3}{2}$ 1 2 3 4 5 6 7 8 9
4 5 6 7 8 9

K

~~1~~ ~~2~~ ~~3~~ 4 ~~5~~ 6 ~~7~~

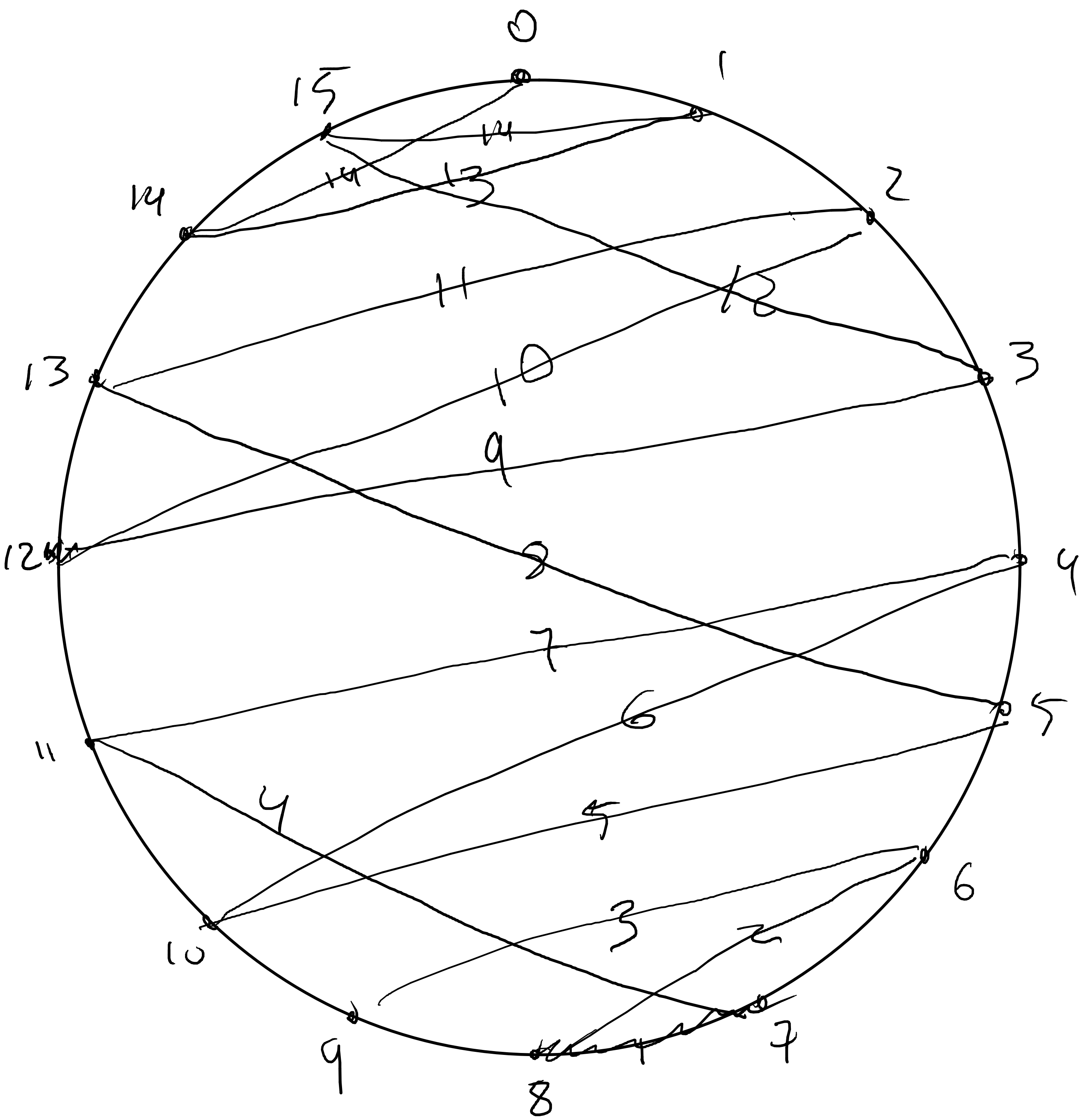
8 ~~9~~ ~~10~~ ~~11~~ ~~12~~ ~~12~~





6

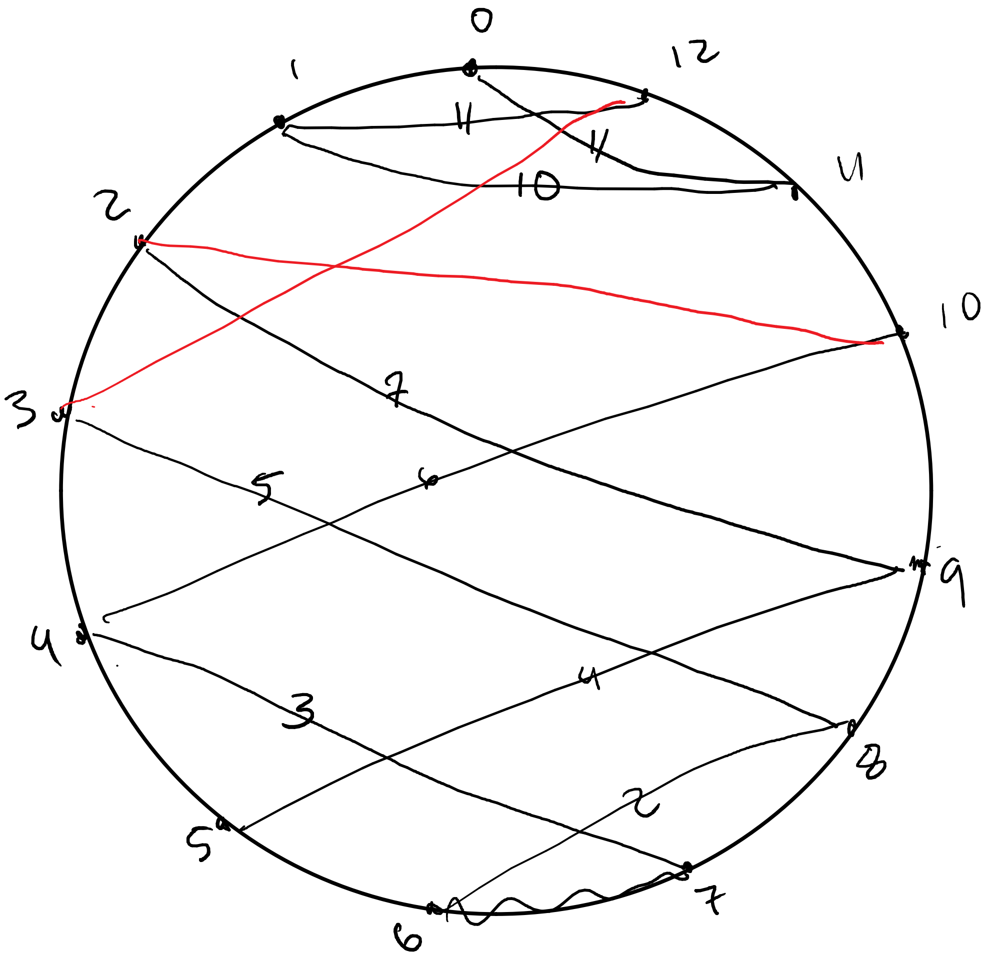
$$\frac{13-3}{2} = 5$$

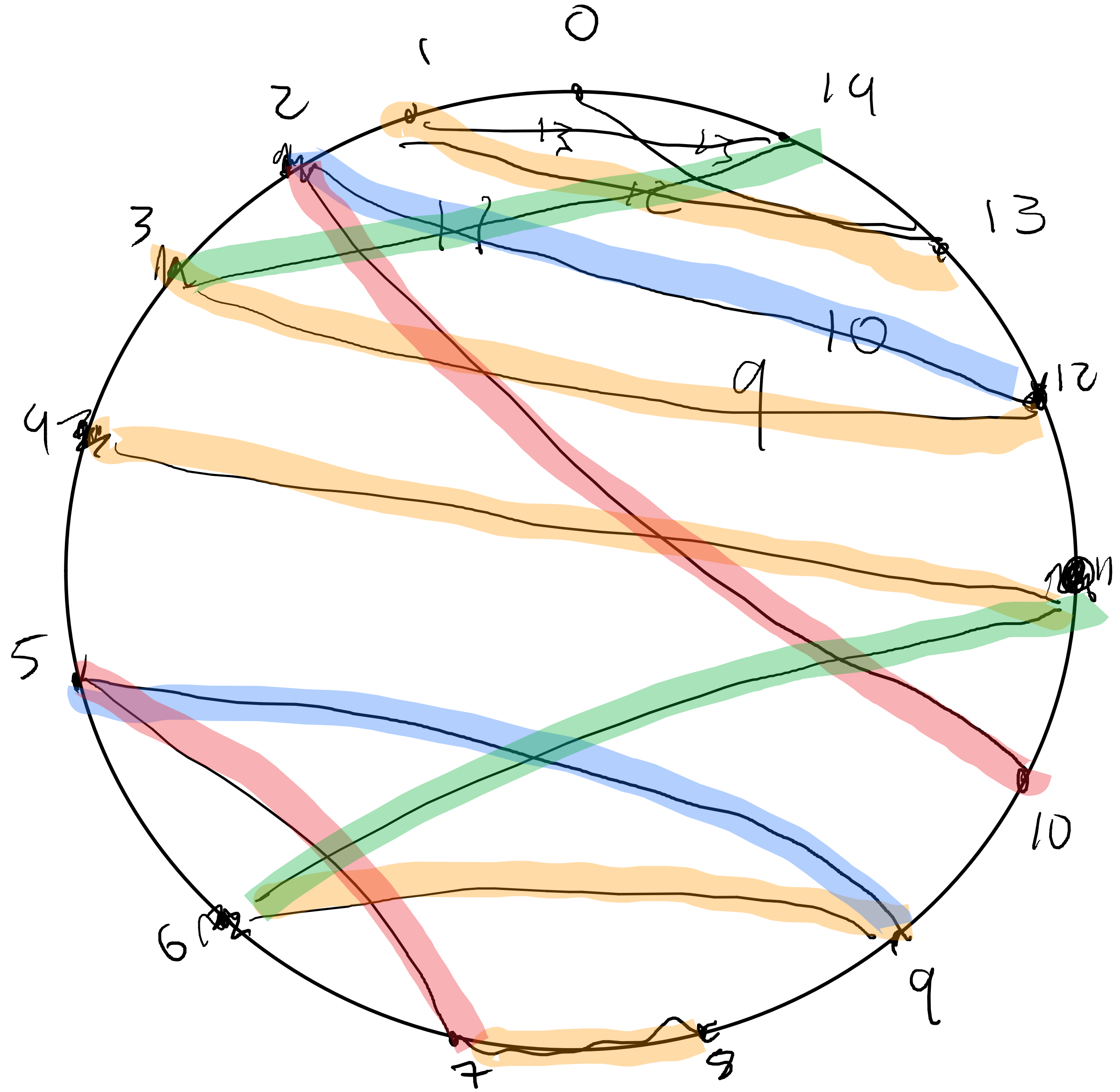


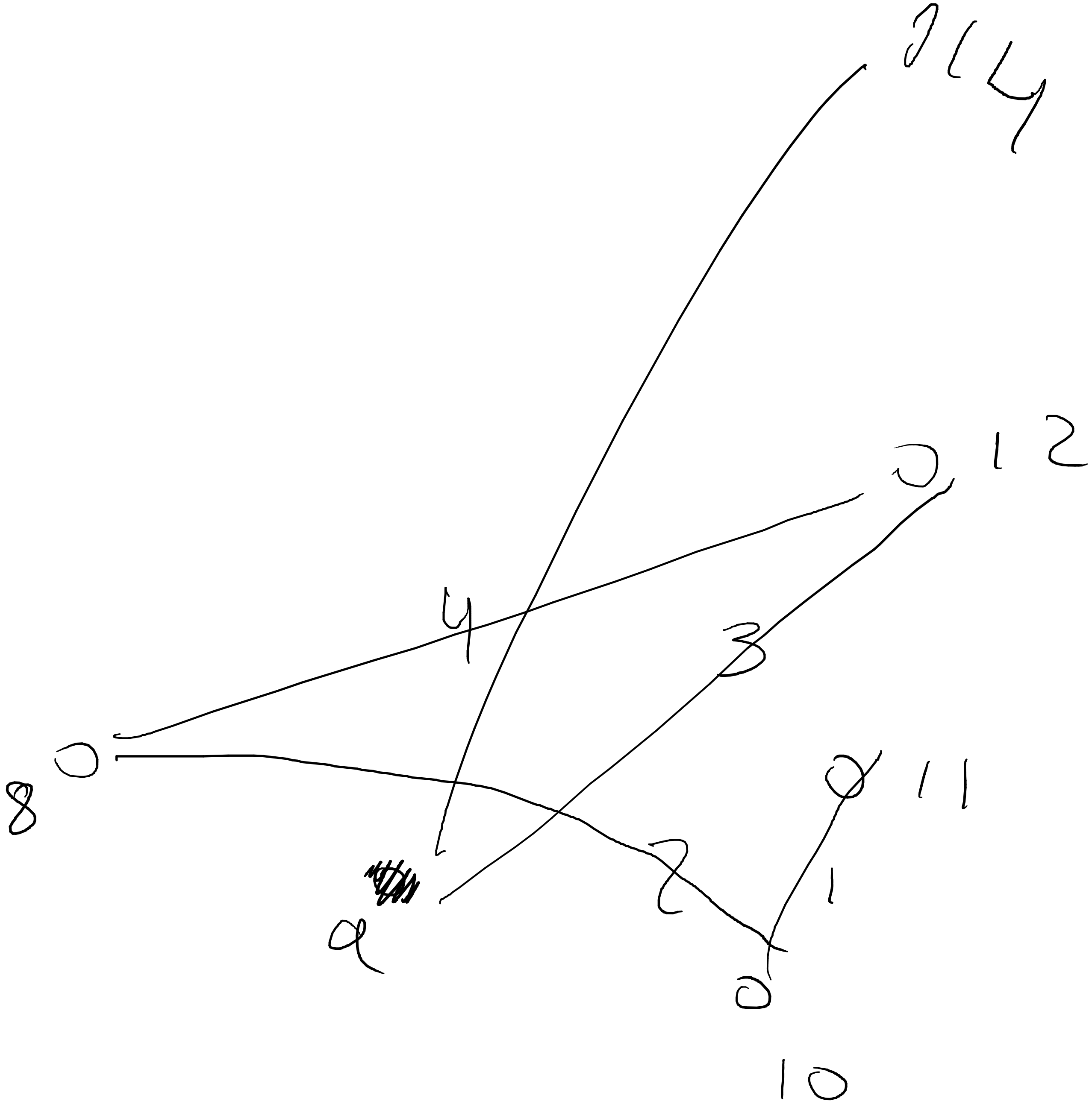
$n = 15$

$$\frac{n-3}{2}$$

11 10 9

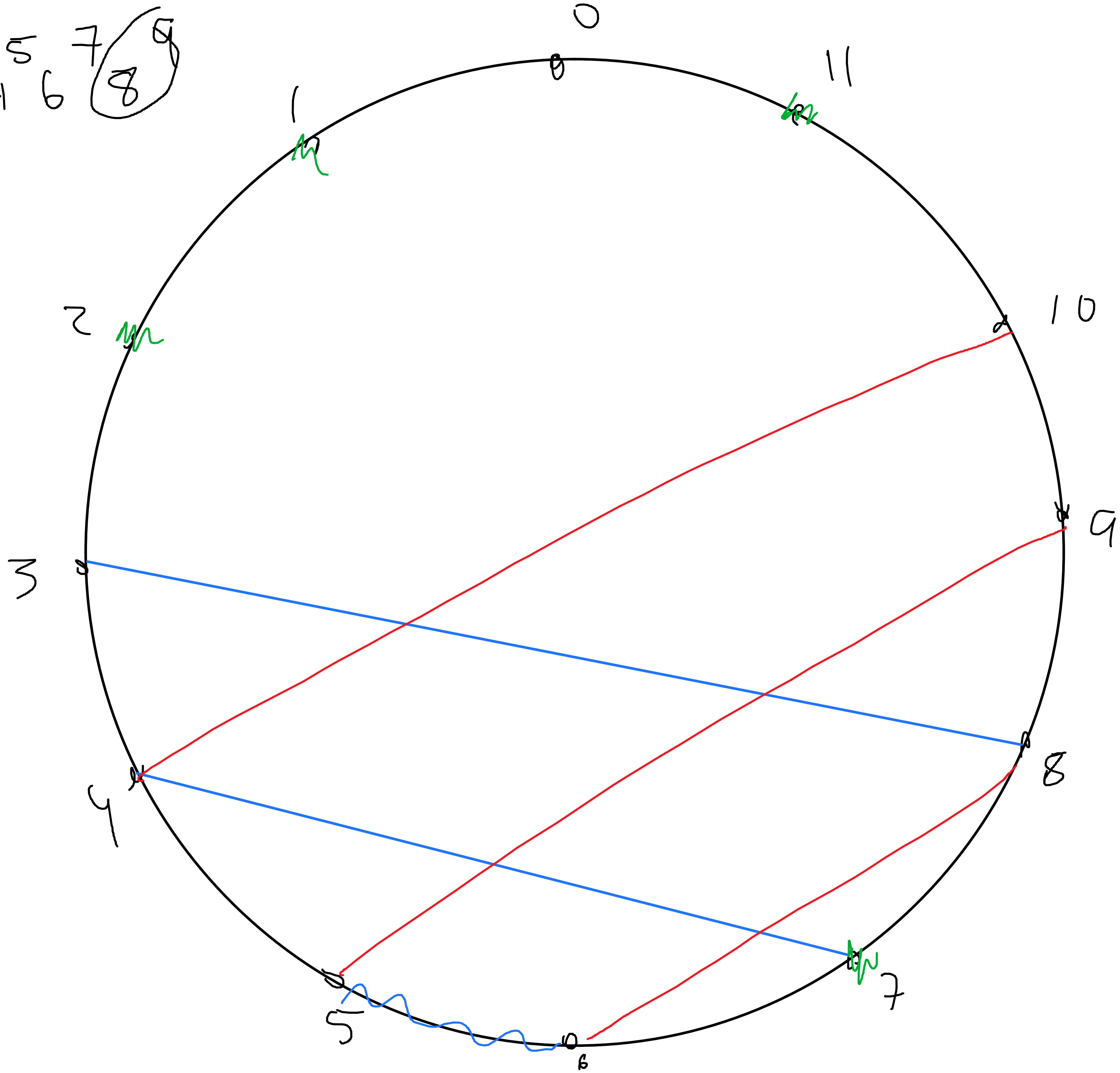






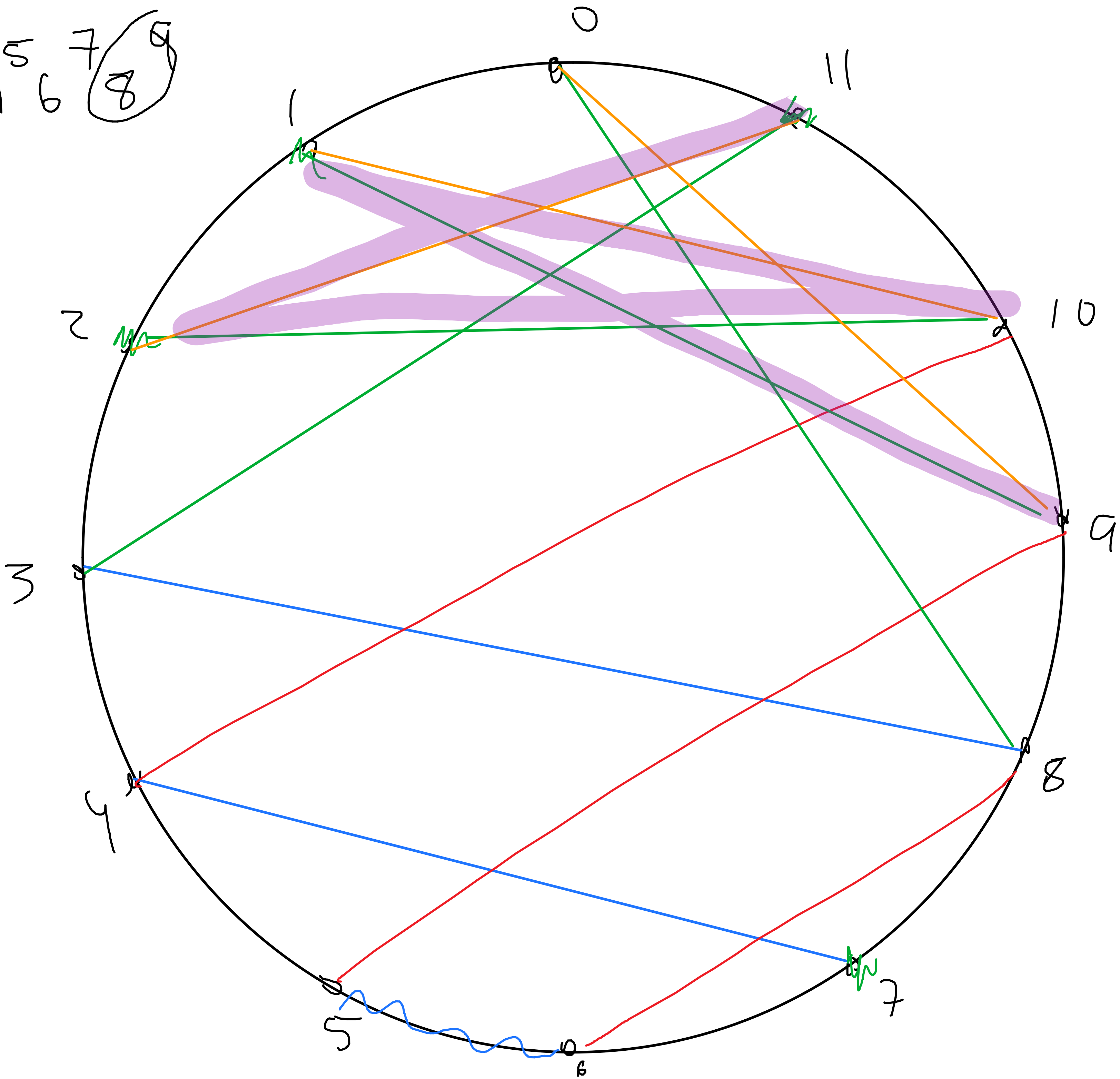
1, 2 3

1 3 5 7 9
2 4 6 8



9
7
5
3
1

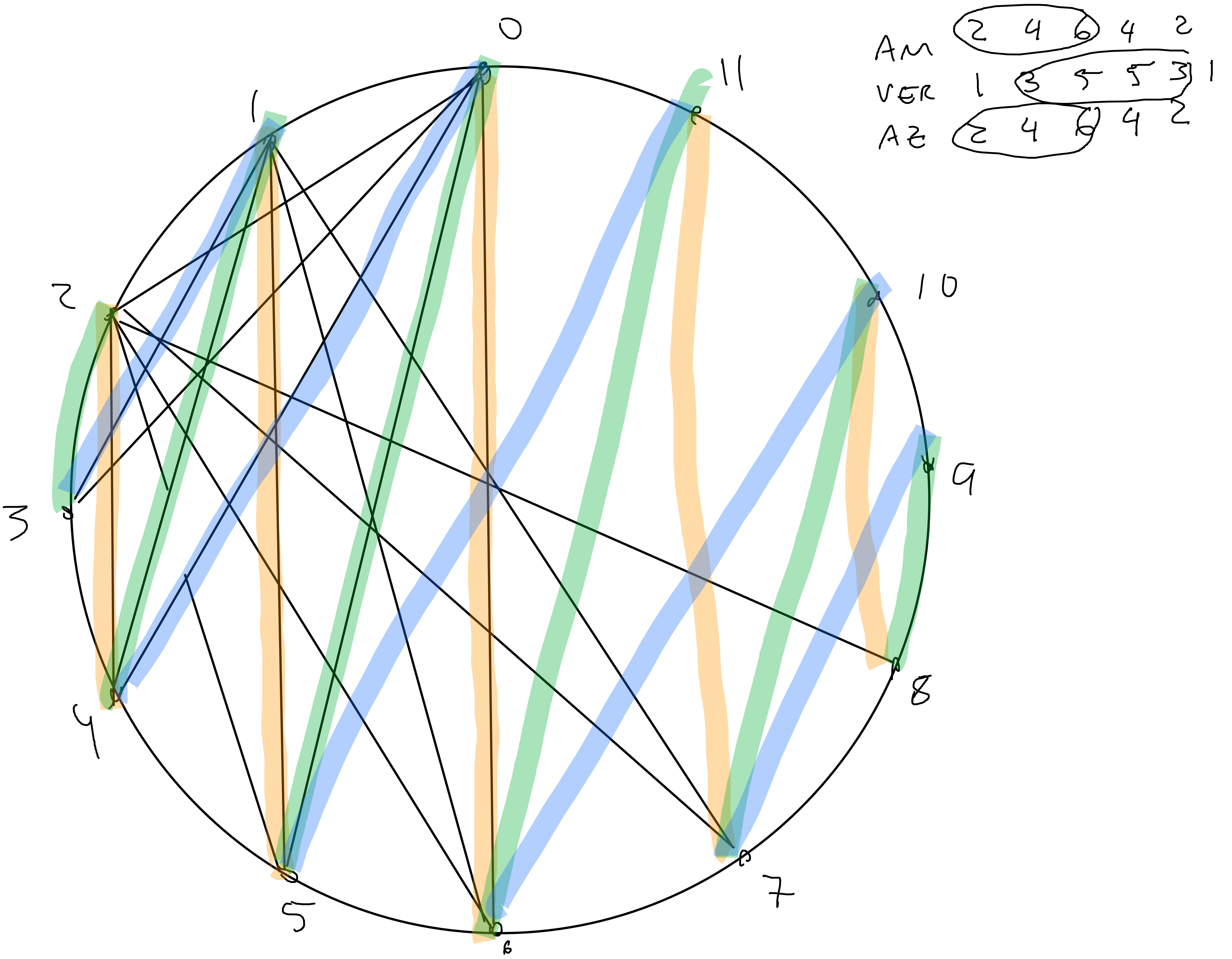
1 3 5 7 9
2 4 6 8



9
7
5
3
1

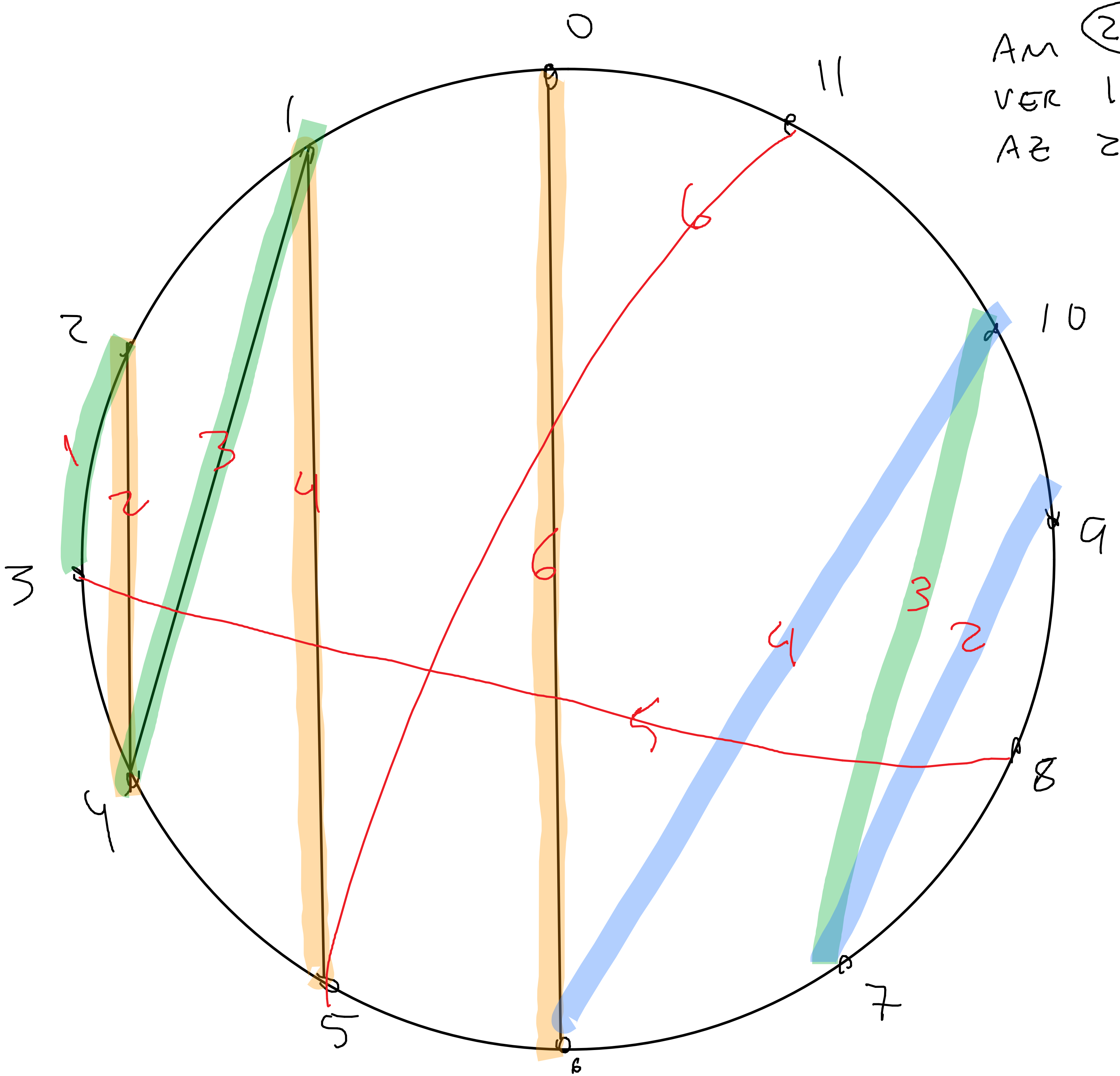
$n = 11$

$k = 6$



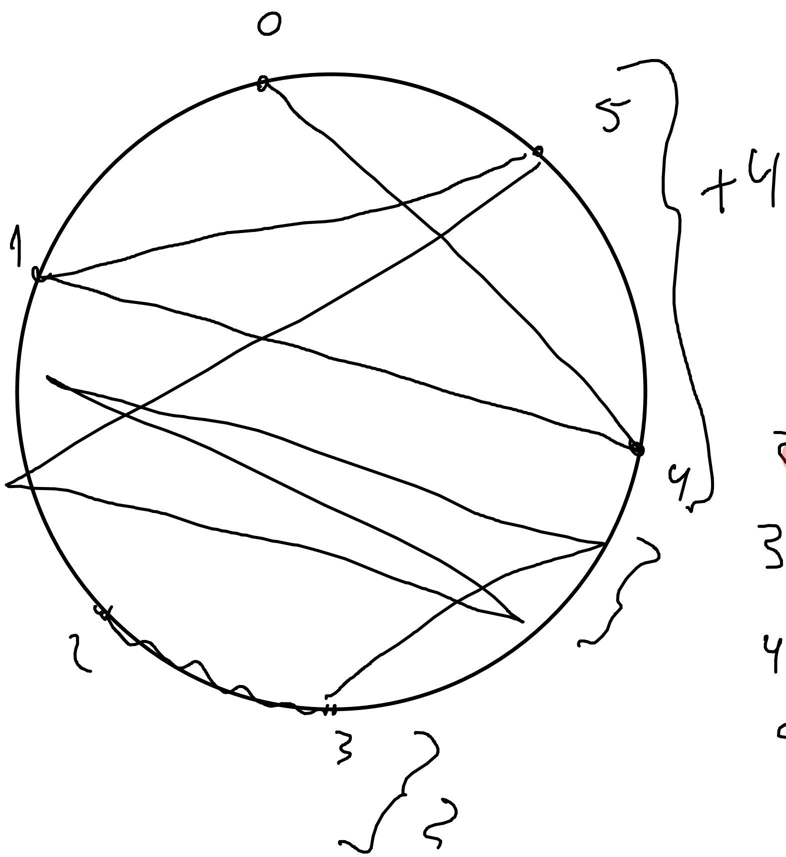
$n = 11$

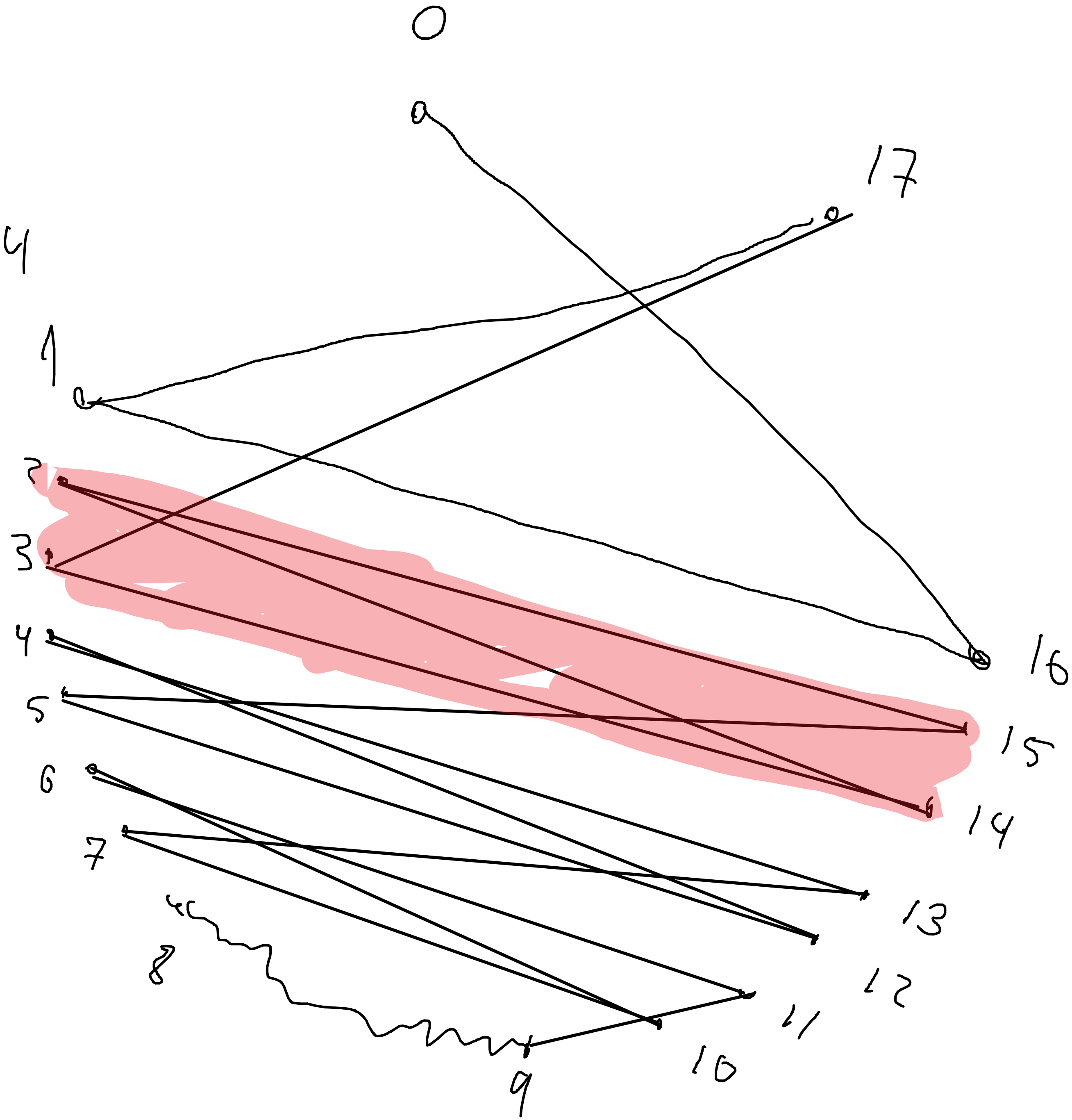
$k = 6$

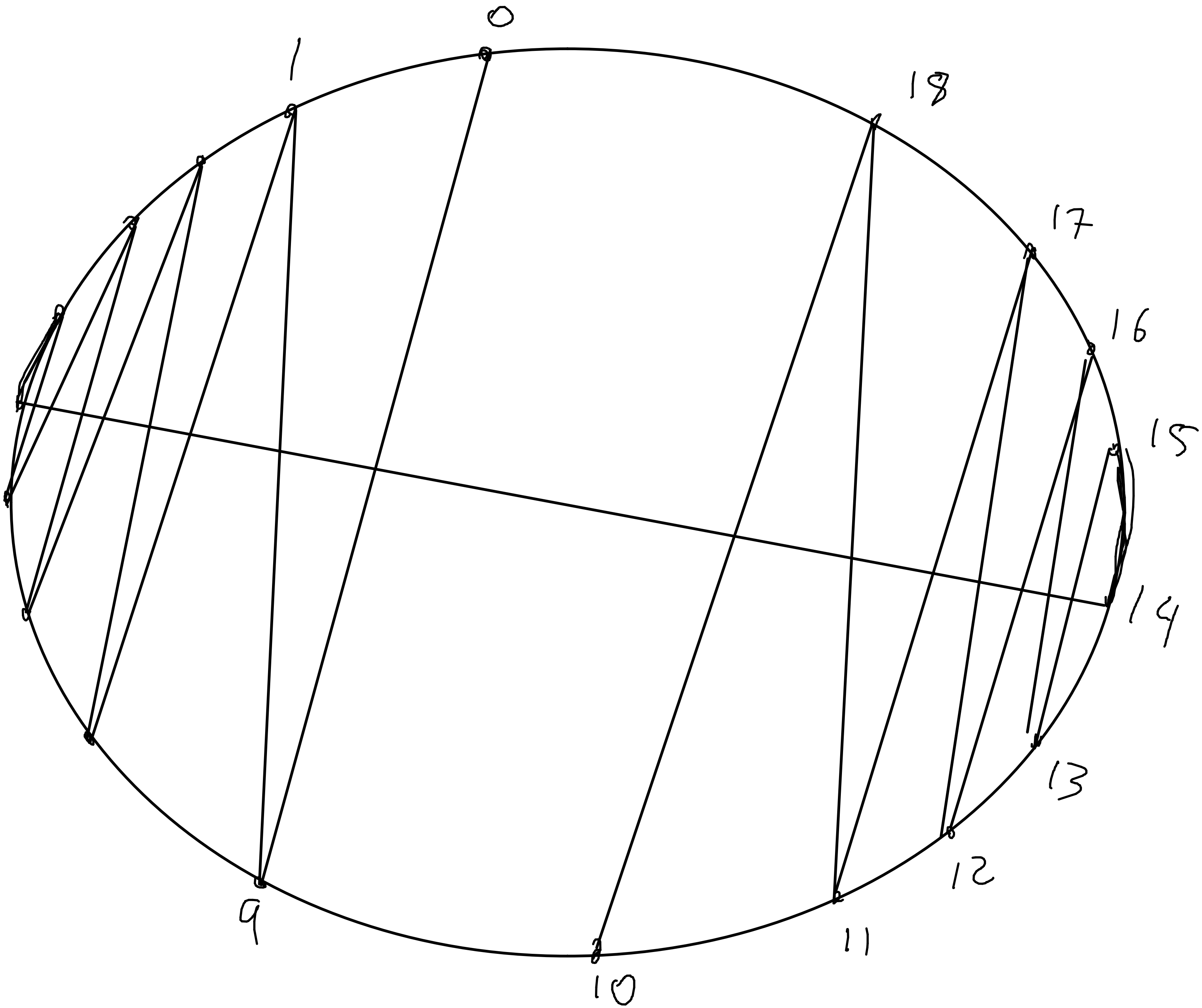


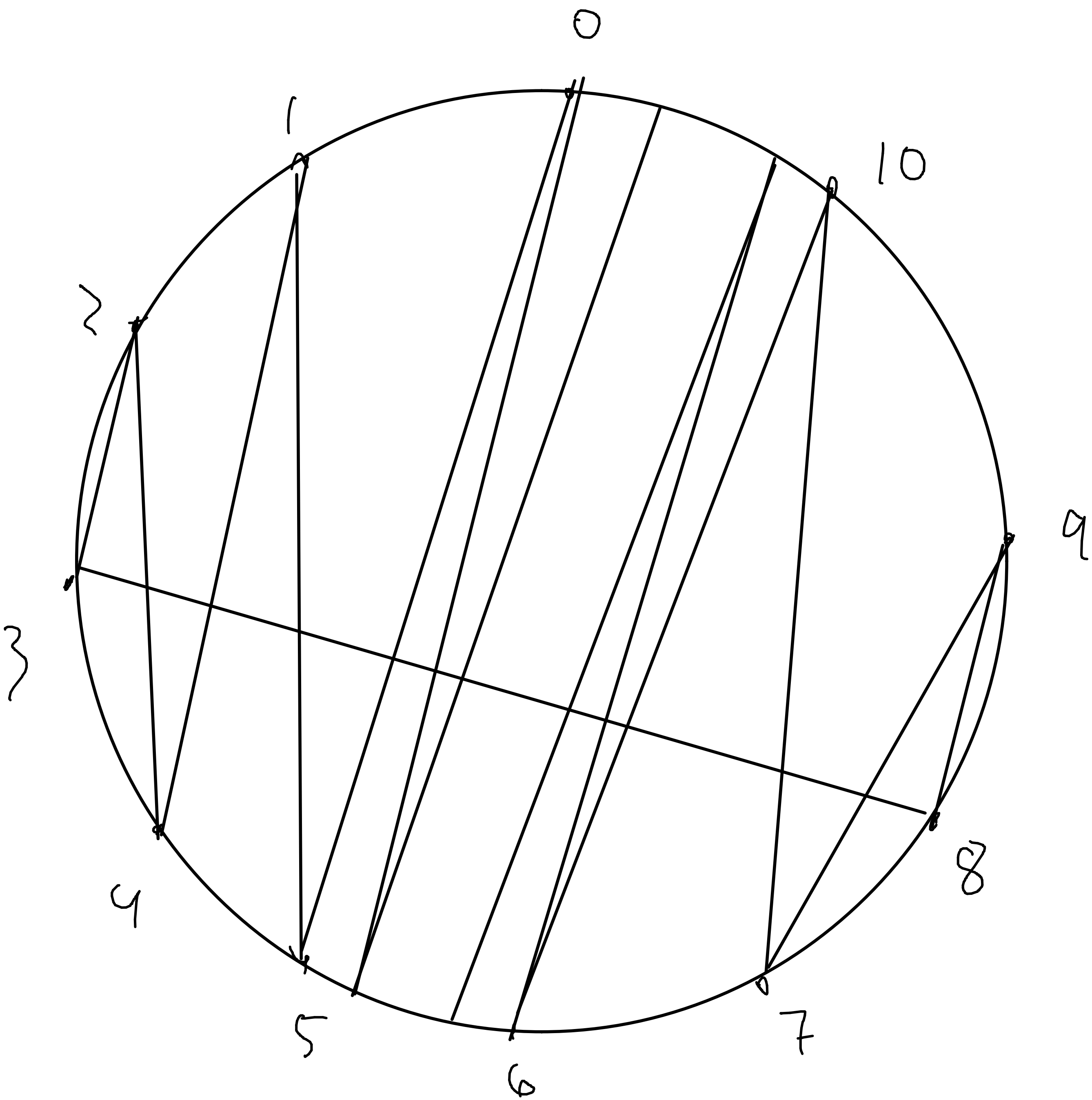
AM	2	4	6	4	2	
VER	1	3	5	5	3	1
AZ	2	4	6	4	2	

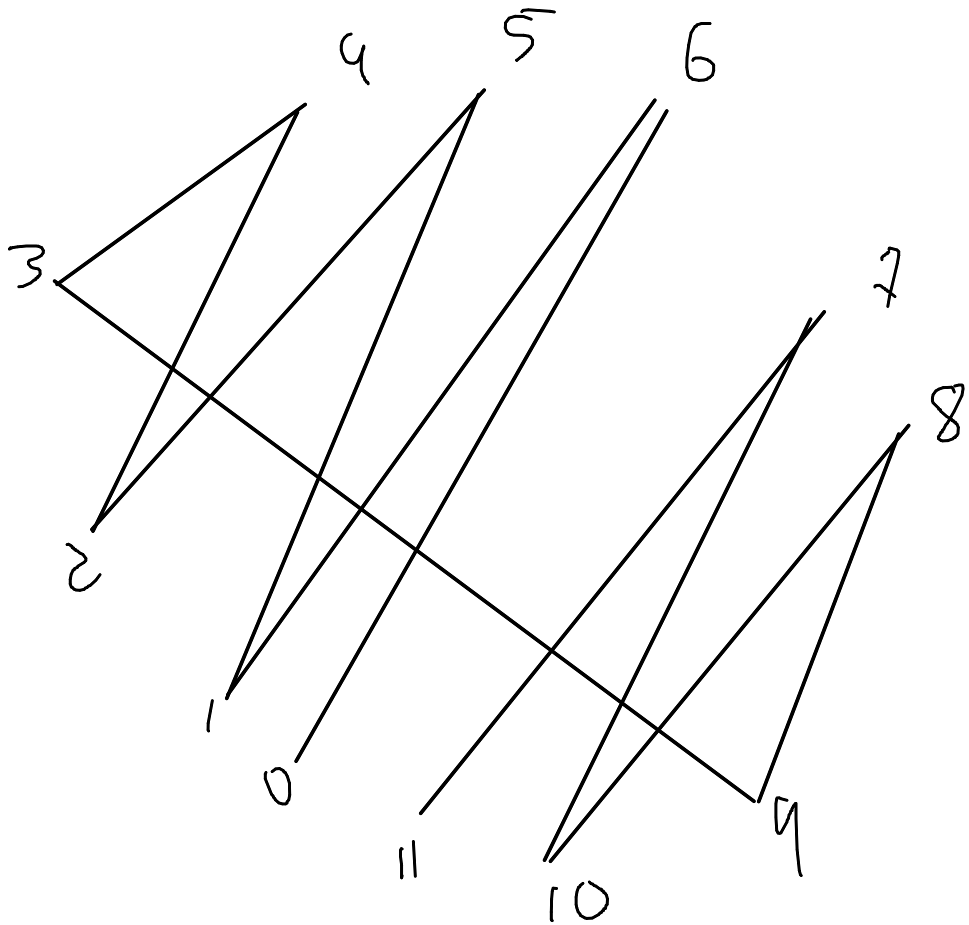
6 5 5


0
5
+4
1
4
2
3
2


0
17
1
2
3
4
5
6
7
8
16
15
14
13
12
11
10
9


0
1
18
17
16
15
14
13
12
11
10
9


0
1
2
3
4
5
6
7
8
9
10



0
1
2
3
4
5
6
7
8
9
10
11

n = 10   0   1   2   3   4   5   6   7   8   9   10

k = 5   1   2   3   4   5

1   2   3   4   5

k = 7   1   2   3   4   5 | 6   7

5 | 6   7

n = 28
k = 14

n = 6
k = 5

n = 4
k = 2

n = 10
k = 9

n = 10
k = 5

16, 17, 18, 19, 20

0

20

k = 14   1   2   3   4   5   6   7   8   9 | 10   11   12   13   14

1   2   3   4   5   8   7   8   9 | 10   11   12   13   14

n = 15
k = 14

(n, k)

(n, k, e)

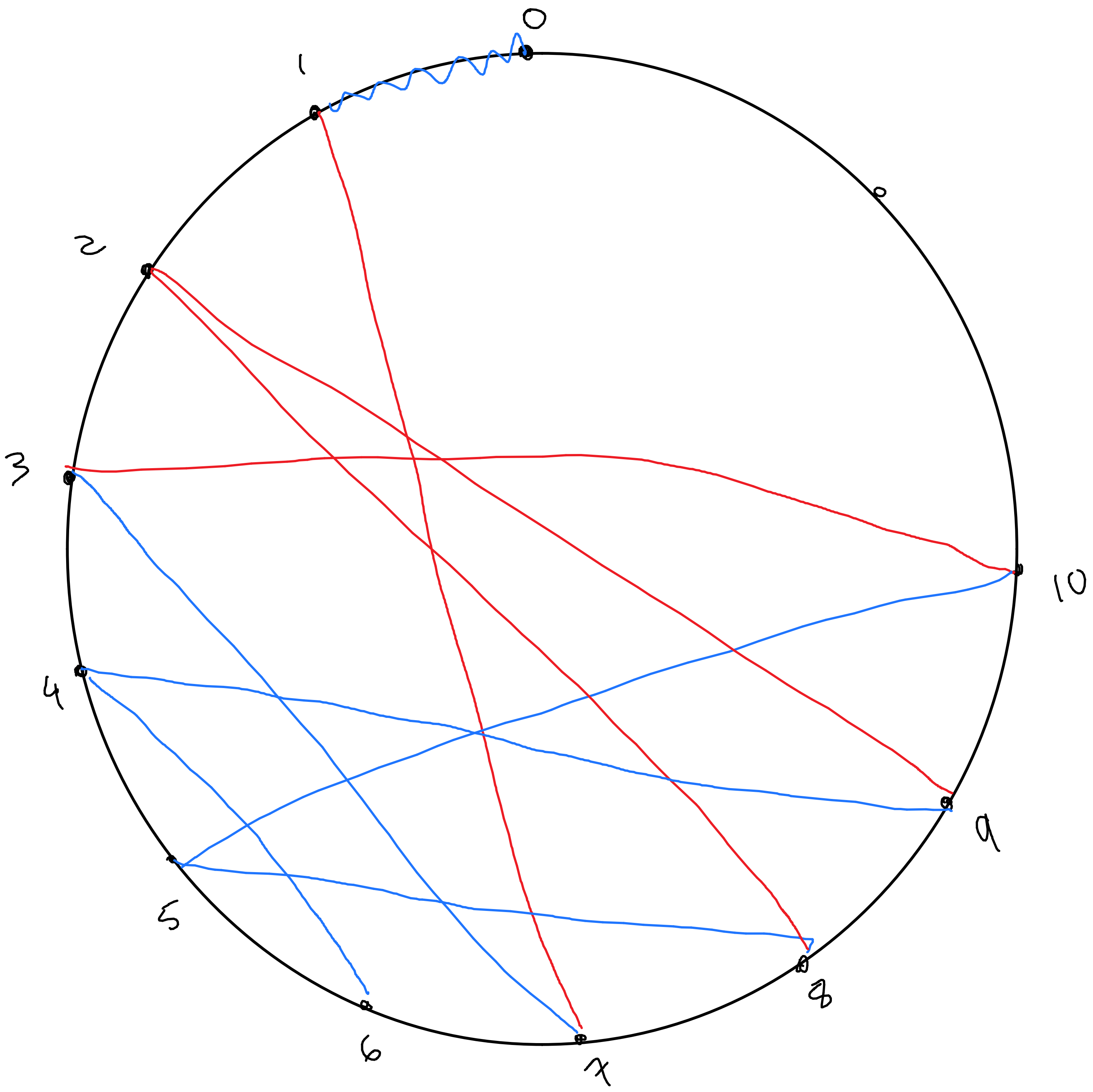
$\hookrightarrow \quad |e| \leqslant k$

$n = 12$

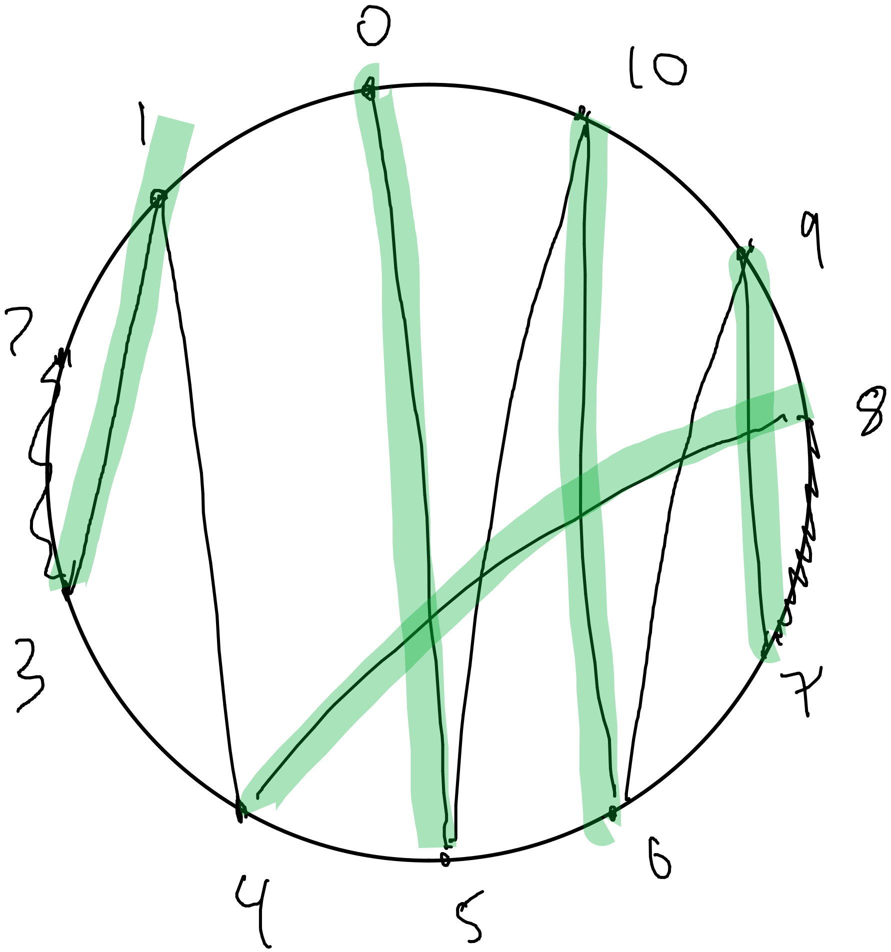
$k = 6$

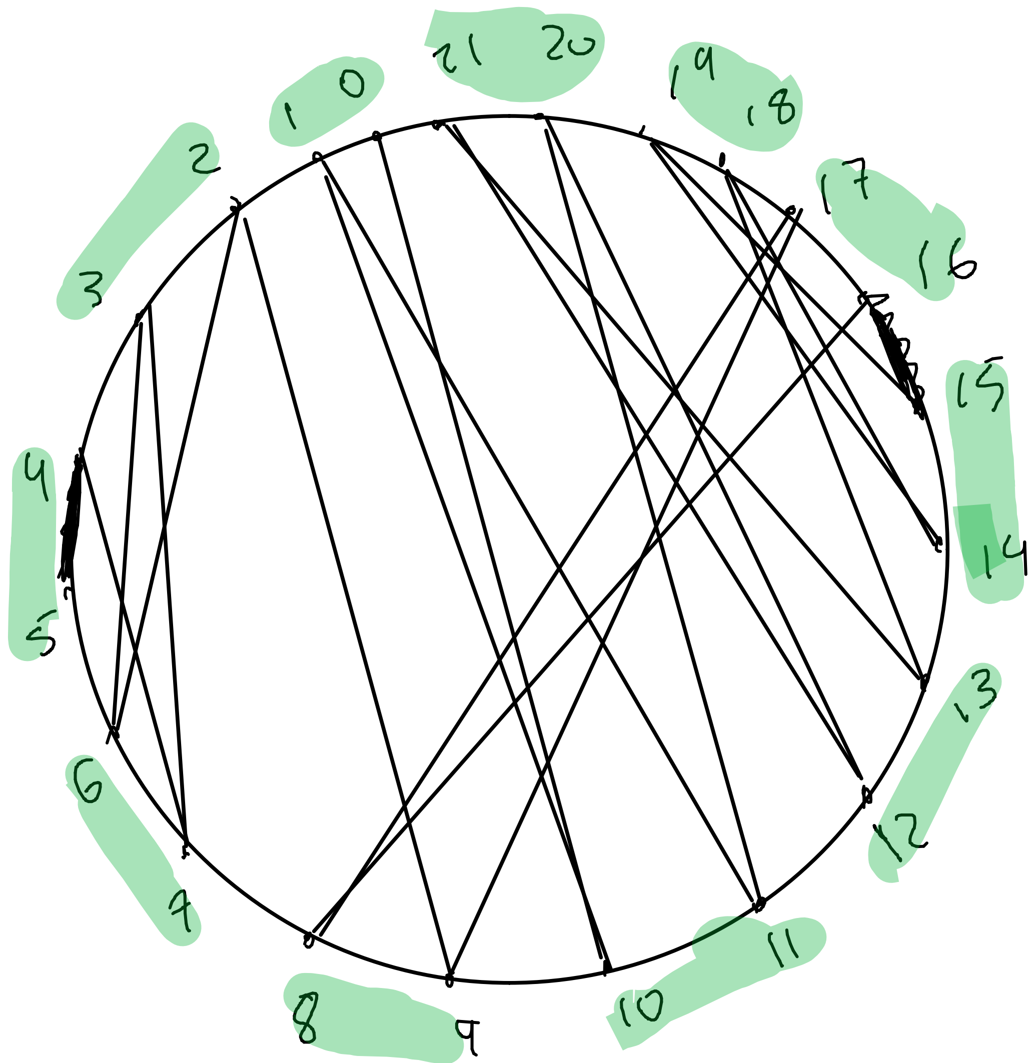
$e = ?$

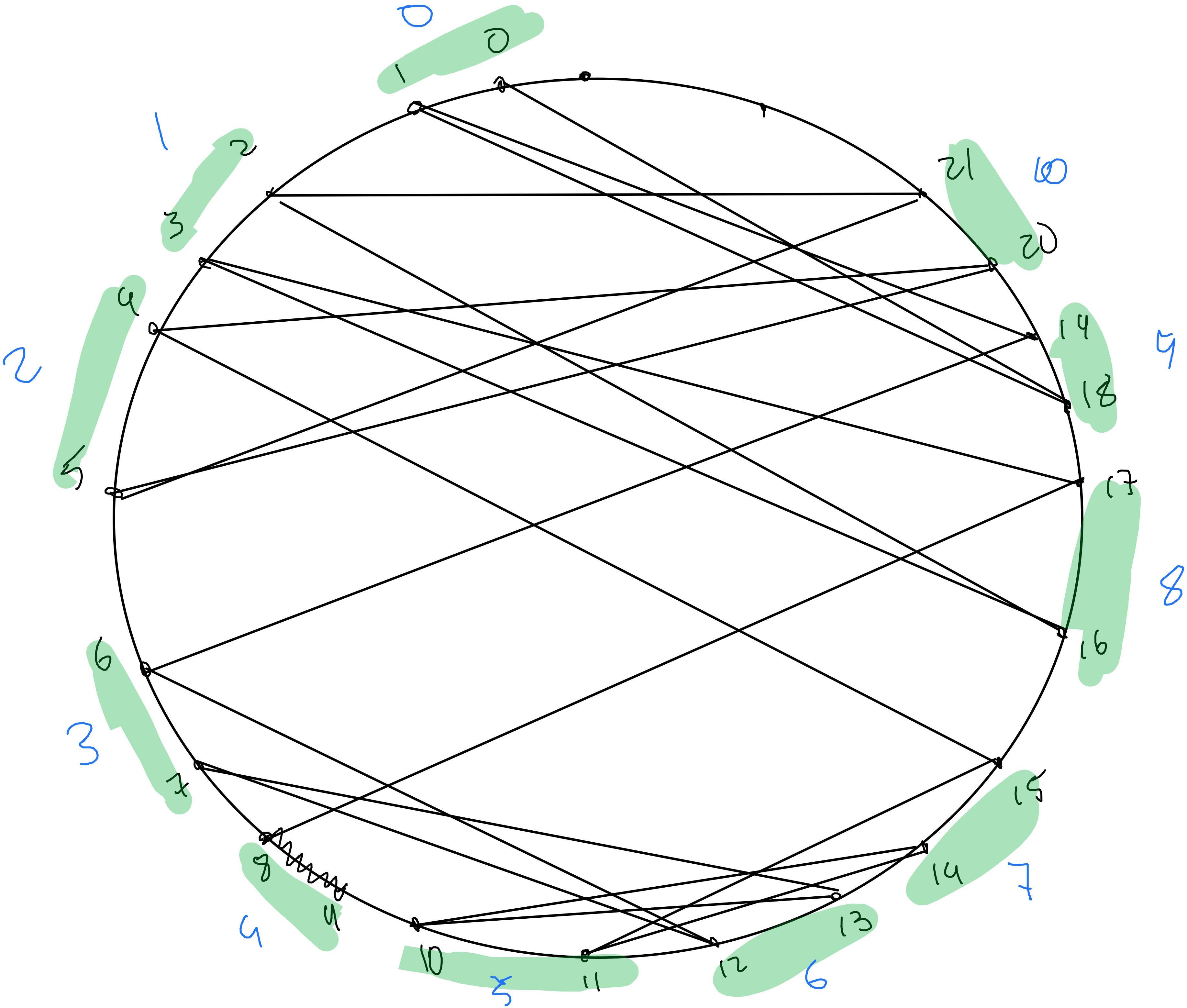
$(10, 5, (5, 6))$


0
1
2
3
4
5
6
7
8
9
10

$i \rightsquigarrow 2i, 2i+1$







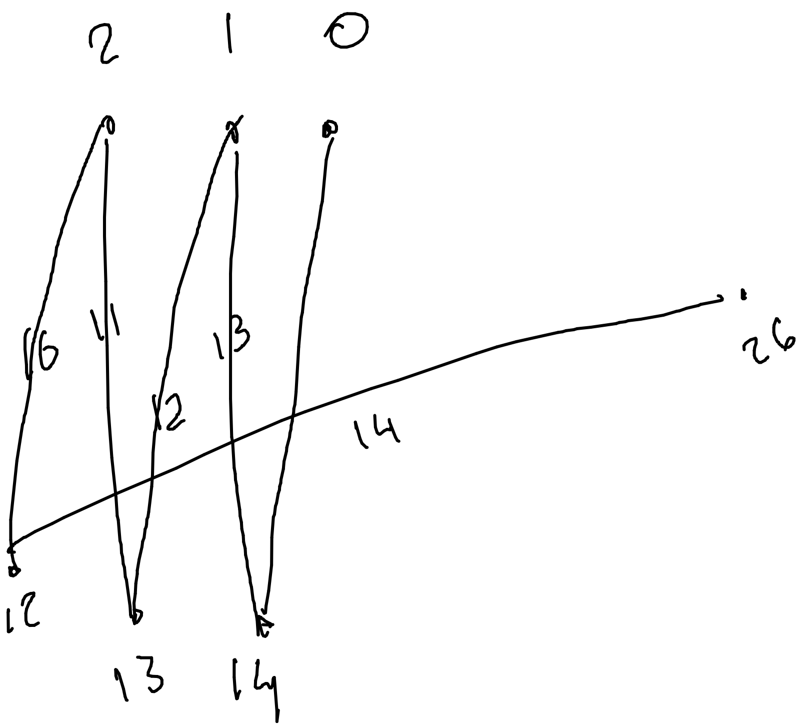
0
0
1
1
2
3
2
4
5
3
6
7
4
8
9
5
10
11
6
12
13
7
14
15
8
16
17
9
18
14
10
20
21

$n = 20$

$n = 10$
$k = 9$

$n = 10$
$k = 5$

1 2 3 4 5 6 7 8 9 | 10 11 12 13 14
9 | 10 11 12 13 14



$n = 10$
$k = 9$

$n = 20$

1 2 3 4 5 6 7 8 9 | 10 11 12 13 14
9 | 10 11 12 13 14

MIN. Parallels

$\pi = 0 \quad \dots$

$1 \quad \dots \quad n$

$1 \cdot (n-1) \cdot (n-1) \cdot \dots \cdot 1 = (n-1)(n-1)! = n! - (n-1)!$

$$\pi' = \left(|\pi_1 - \pi_0|, \dots, |\pi_n - \pi_{n-1}|\right)$$

$$\mathrm{COUNT}_{n-1}(\pi') = \left(c_1^{\pi'}, \dots, c_k^{\pi'}, 0, \dots, 0\right) \quad c_i^{\pi'} = \left|\{ j : |\pi_j - \pi_{j-1}| = i,\ 1 \leq j \leq n\}\right|$$

$$\sum c_i^{\pi'} = n$$

QUEREMOS ENCONTRAR

$$(1, 1, \dots, 1, \underset{\substack{\uparrow \\ 2k-n+1}}{2}, 2, \dots, 2, 0, \dots, 0)$$

$0 \dots n$

n^{n-1}

$$(i, \underbrace{\dots}_{S_{n-i}^{n-2}})$$

$$S_n^{n-1} \subseteq E_{n-1} = \{0, \dots, n\}^{n-1}$$

$$S_n^{n-1} = \{x \in E_{n-1} : |x| = n\}$$

$$|S_n| = \sum_{i=1}^{n}$$

$$S_n^{n-1} \subseteq E_{n-1} = \{0, \dots, n\}^{n-1}$$

$0 \dots n$

$$(i, \underbrace{\dots}_{S_{n-i}^{n-2}})$$

$$\| \quad \{x \in E_{n-1} : |x| = n\}$$

$$|S_n| = \sum_{i=1}^{n}$$

$$s_m^k = |S_m^k|$$

$$s_m^1 = m \qquad \forall m$$

$$s_n^k = \sum_{i=0}^{n} s_{n-i}^{k-1}$$

$$S_m^k = \{x \in \mathbb{N}^k : x = (x_1, \dots, x_k) \in \sum x_i = m\}$$

$$s_m^k = |S_m^k|$$

Π_0^n

$\supset 9 \cdot 8 \cdot 7 \ldots 4\,5$

DADO i, QUANTOS CAMINHOS POSSUEM PELO MENOS UMA i-ARESTA?

$1, \ldots \cancel{4, 5} \quad 11$

HÁ $n-i+1$ ARESTAS POSSÍVEIS DO TIPO i

$(0, i), (1, i+1), \ldots, (n-i, n)$

(a, b) $\quad b > a$

(b, a)

$1 \cdot (n-1)!$

$n + n - 1 = 2n - 1$

$(2n-1)(n-2)!$

$(n-1)(n-1)!$

$2n \cdot (n-2)!$

$0, 5, 4$

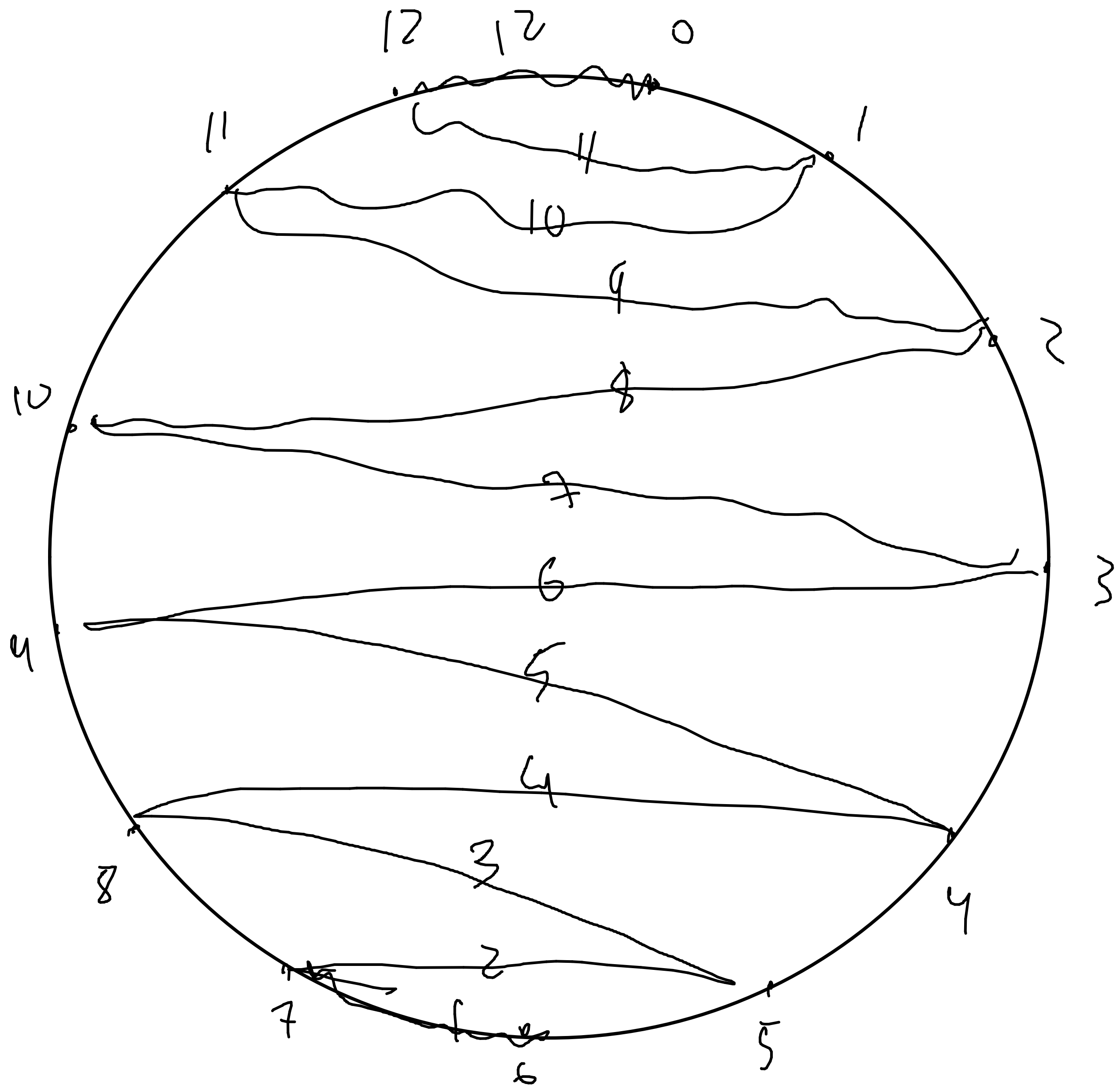
$2n$

$(n-1)^2$

$4\,5$

$0 \underbrace{i \ldots}\, 5\; 4 \underbrace{\ldots}$

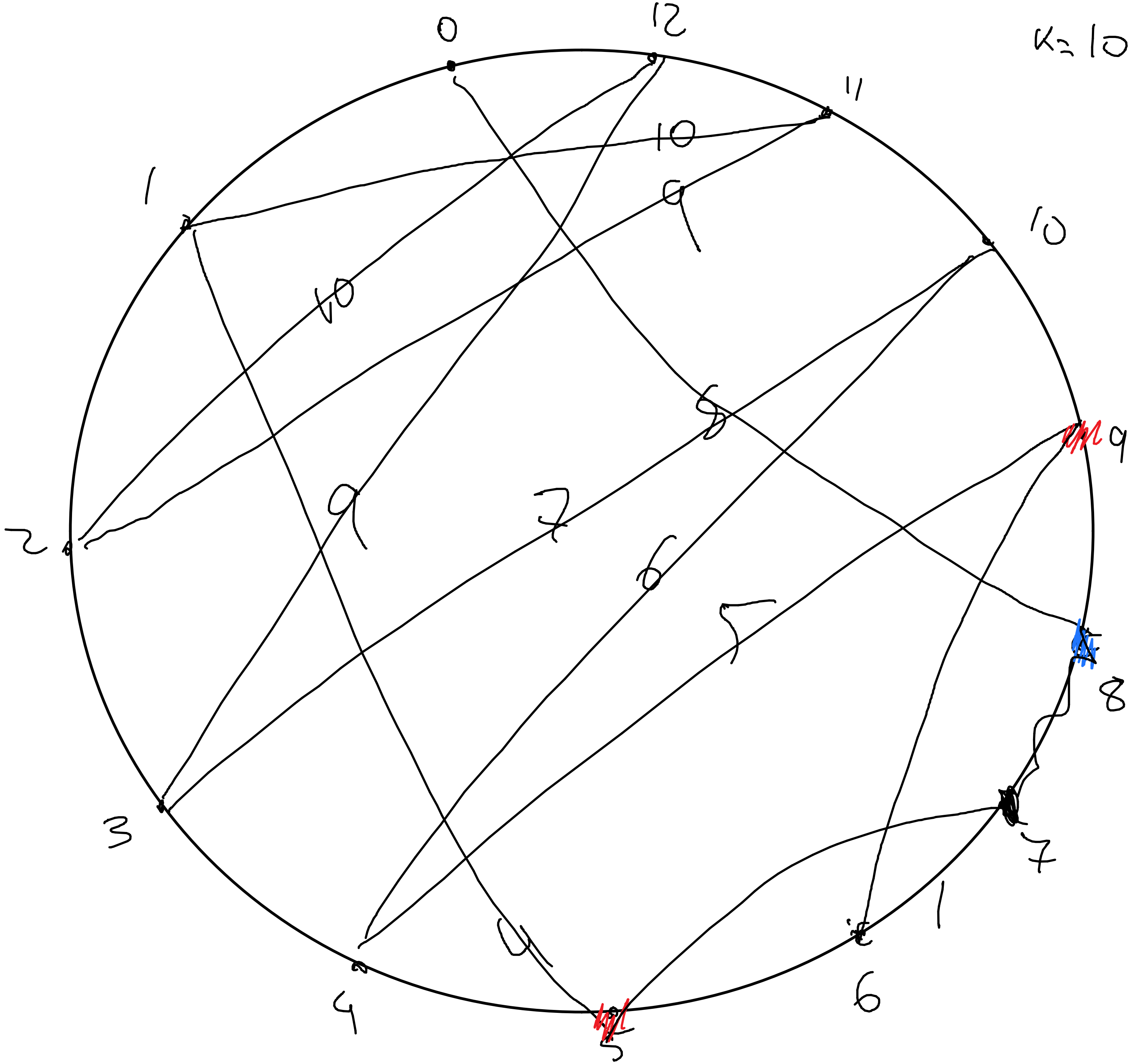
$\geq (n-1)(n-1)! - (n-1)! - 2n(n-2)! - (2n-1)(n-2)! = \left((n-1)^2 - (n-1) - 2n - (2n-1)\right)(n-2)!$



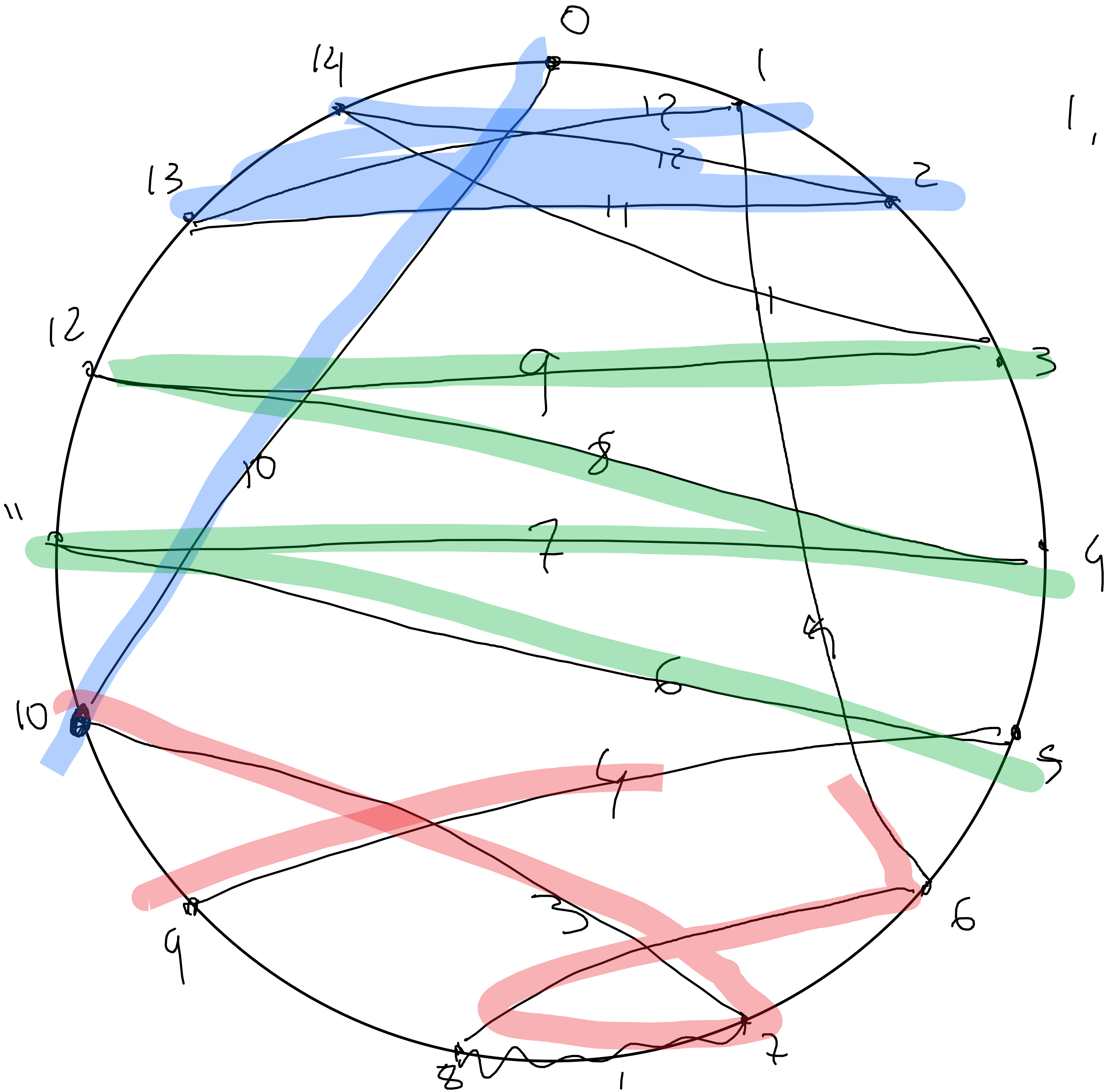
12
12
0
1
11
11
10
9
2
10
8
7
6
3
4
5
4
8
3
4
2
7
6
5

n -2

k=10



3, 2, 1



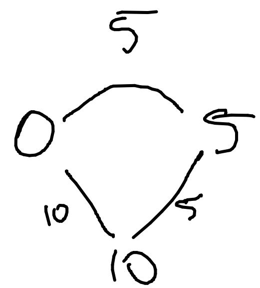
1, 2, 3, 4, 5, 6

S_9

$|S_{10}| = 11 \cdot |S_9|$

$= 11!$

$S_{10} = \{ P_1 \; 10 \; P_2 \; : \; P_1 \, P_2 \in S_9 \}$



SIGN $(P) = (x_1, \dots, \;)$

ONDE $x_i = \# \{ y \in D_{IFF}(P) : y = i \}$

EX: $P = [0, 1, 2, \dots, n]$

SIGN $(P) = [n, 0, \dots, 0]$

$a \quad 10 \quad b \rightarrow a \; b$

$10-a, \; 10-b \rightarrow |a-b|$

$S_1 = \{0\ 1\}$

0 1 2
[2, 0]

0 2 1
[1, 1]

0 1 2 3
3 0 0

0 1 3 2
2 1 0

0 3 1 2
1 1 1

0 2 1 3
1 2 0

0 2 3 1
1 2 0

0 3 2 1
2 0 1

1, 2, 1, 1

$\cancel{5}$ $\cancel{5}$ 5

0 3 1 4 2
3 2 3 2

K = 3

1 2 2 0
1 2 3 4

0 1 2 3 4 5

1 1 1 1 1

0 4 3 2 1

0 1 5 4 3 2

0 4 2 3 1 5

1 4 1 1 1

4 2 1 2 4

0 1 5 2 4 3

1 4 3 2 1

0 4 5 2 1 3

4 1 3 1 2

11120

0 5 1 4 2 3

5 4 3 2 1

1 1 1 2 0

1 2 2 0 0

0 4 1 5 3 2

4 3 4 2 1

0 1 4 5 3 2

1 3 1 2 1

0 1 4 3 5 2

1 3 1 2 3

0 2 5 3 4 1

2 3 2 1 3

0 10 1 9 2 8 3 7 4 6 5 Kmax +1

0 9 1 10 3 8 2 6 4 7 5

0 9 1 10 3 8 2 6 5 7 4

0 2 8 3 10 1 9 6 5 7 4

2 6 5 7 9 2 3 1 2 3

0 20 1 19 2 18 3 17 4 16 5 15 6 14 7 13 8 12 9 11 10

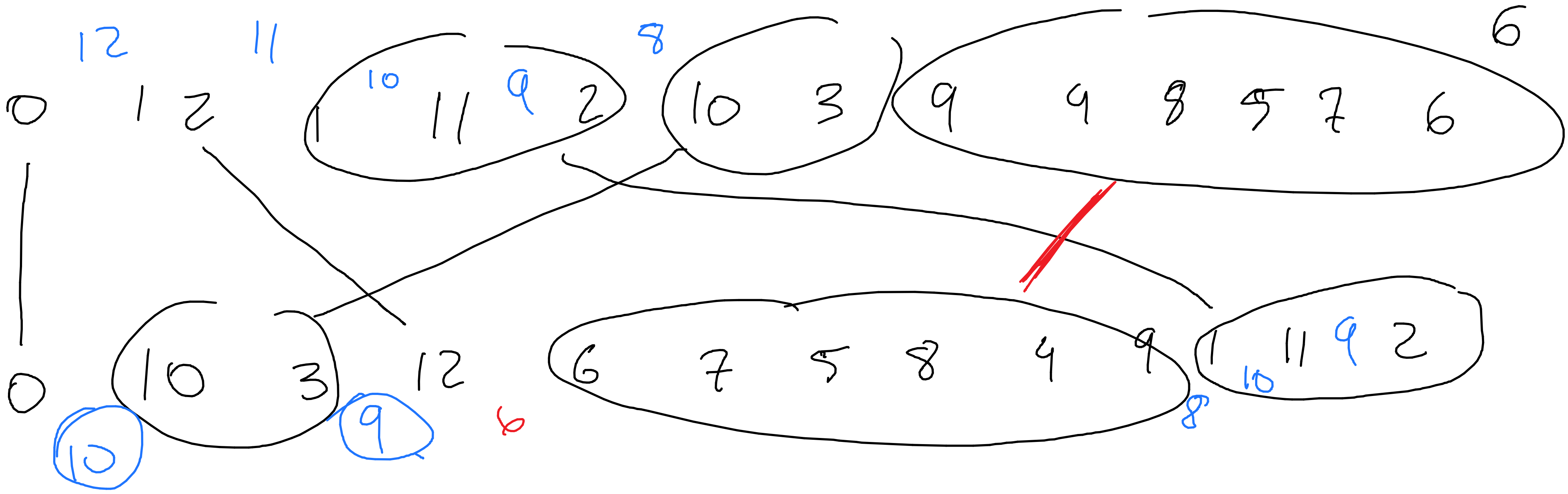
0 29 1 30 ✓

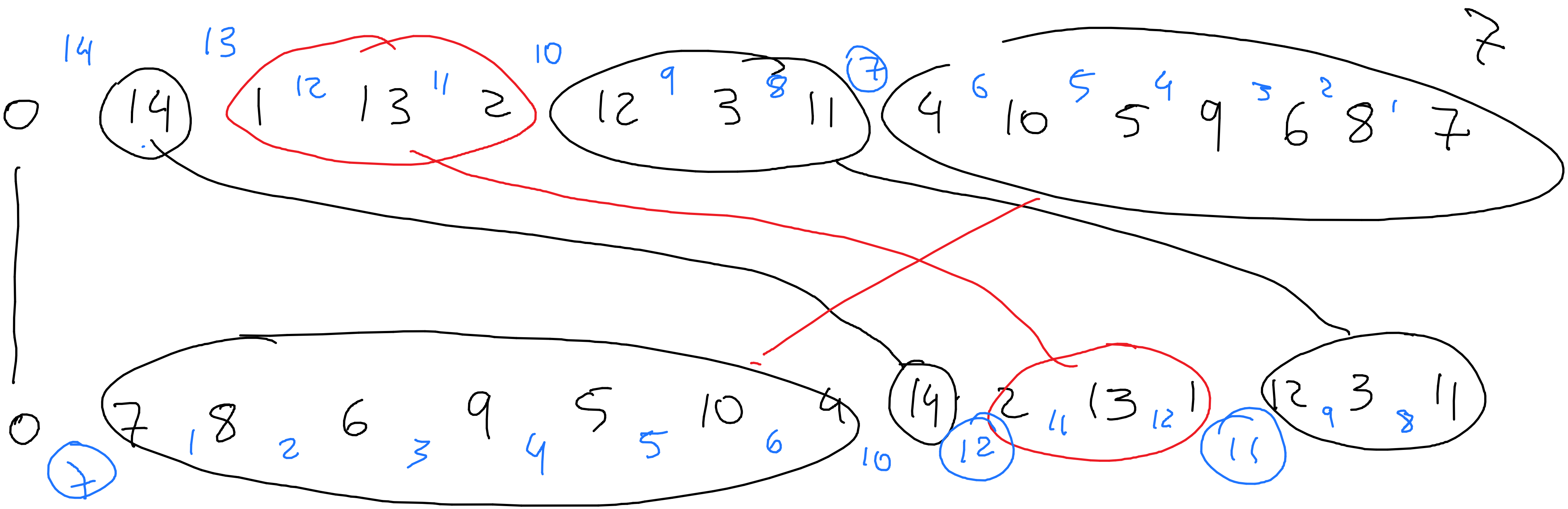
3 27 2 28 ✓

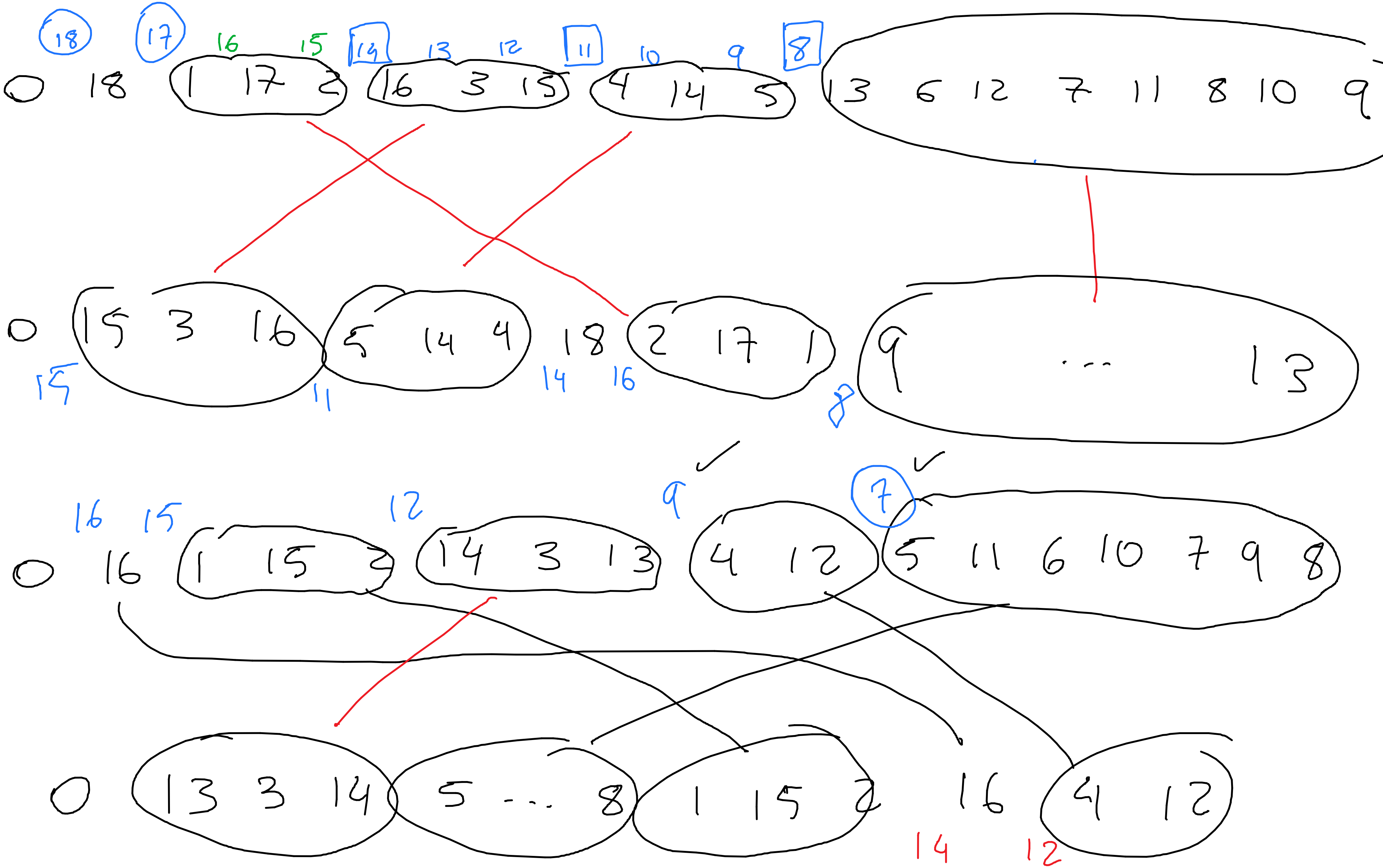
5 25 4 26 ✓

12 18 13 17 14 16 15

13 17 12 18 15 16 14






18 17 16 15 14 13 12 11 10 9 8
0 18 1 17 2 16 3 15 4 14 5 13 6 12 7 11 8 10 9
0 15 3 16 5 14 4 18 2 17 1 9 ... 13
15 11 14 16 8
16 15 12 9 7
0 16 1 15 2 14 3 13 4 12 5 11 6 10 7 9 8
0 13 3 14 5 ... 8 1 15 2 16 4 12
14 12

20 14 16 13 8 5 2

0 20 1 19 2 18 3 17 4 16 5 15 6 14 7 13 8 12 9 11 10

17 8 13 18 16

0 17 3 18 10 ... 14 1 19 2 20 4 16 5 15 6

8 ✓

13 ✓

16

17 ✓

18 ✓(20, 2)

$K = n - 2$

0 22 1 21 2 20 3 19 4 18 5 17 6 16 7 15 8 14 9 13 10 12

~~18~~ ~~20~~

15 ~~19~~

12

9

6

20 : 22-2 , 20-0

19 : 19-0

18 : 22-4,

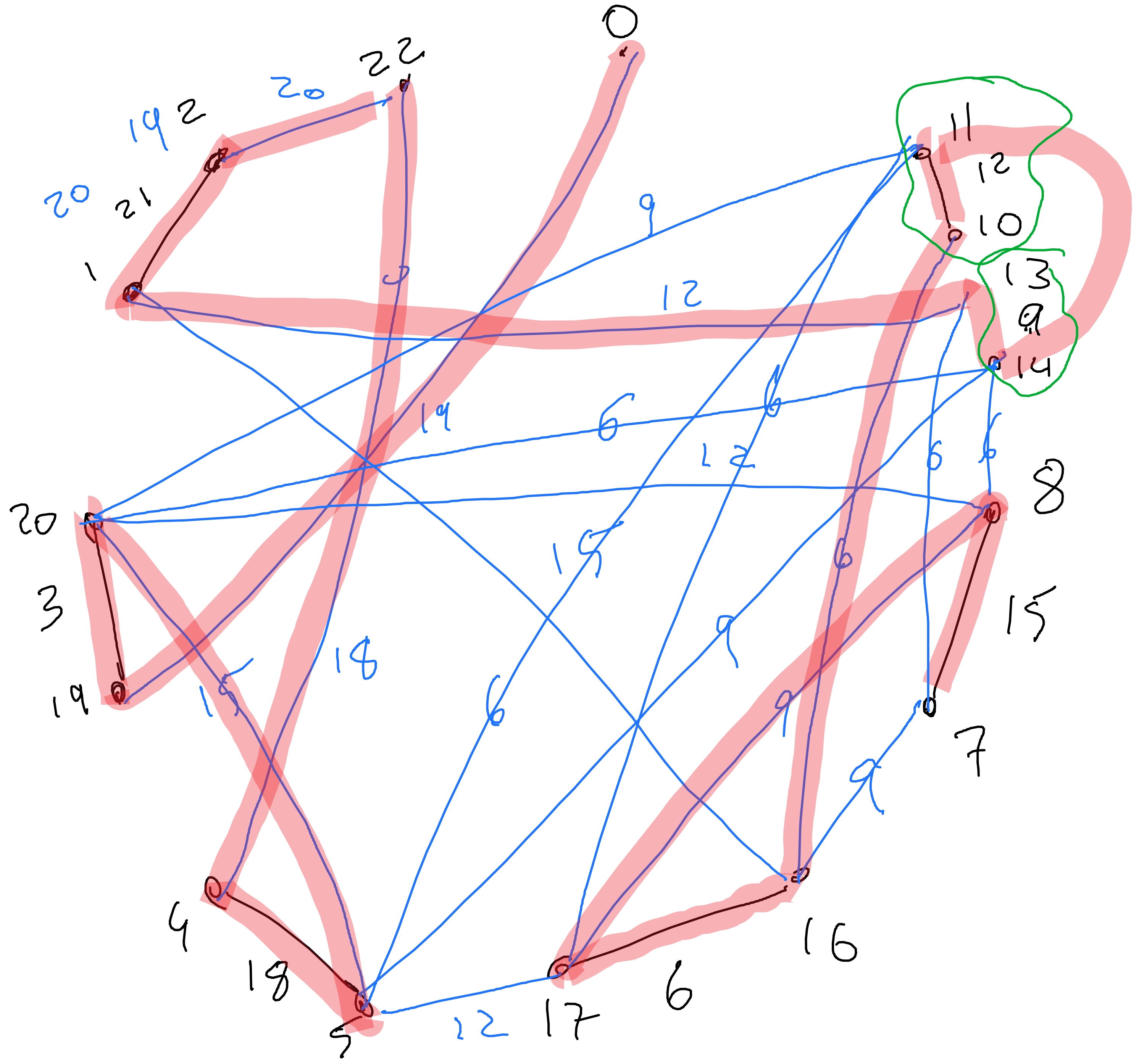
15 = 20-5

16-1

12 : 20-8

17-5

13-1



20

18

20

15

$k = n - 2$

0 22 (1 21 2)(20 3 19)(4 18 5)(17 6 16)(7 15 8)(14 9 13)(10 12 11)

0 (19 3 20)(5 18 4) 22 (2 21 1)(13 9 14)(11 12 10)(16 6 17)(8 15 7)

18 20 12 6

~~18~~ ~~20~~

15 ~~19~~

12

9

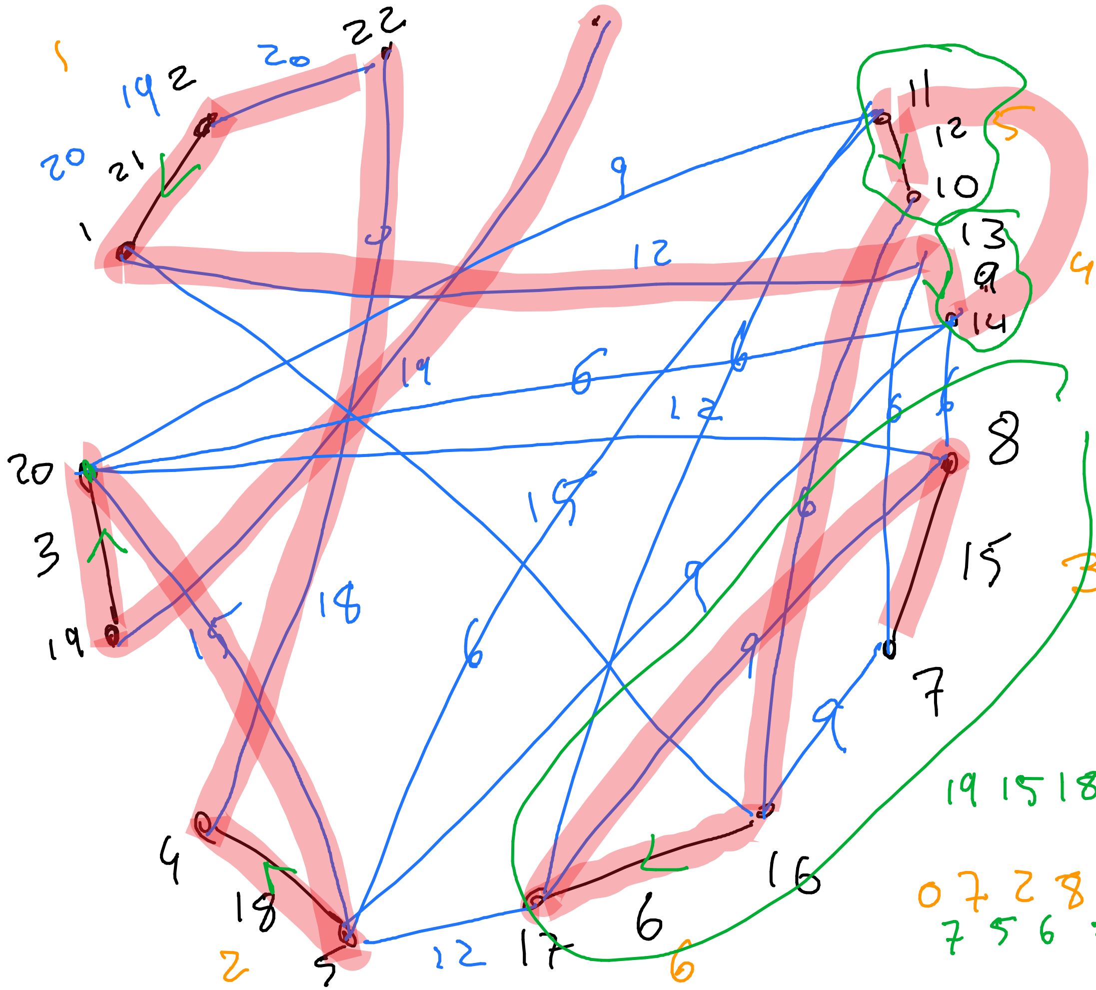
6

20 : 22-2 , 20-0

19 : 19-0

18 : 22-4,

7



18

20

15

15 = 20-5

16-1

12 : 20-8

17-5

13-1

19 15 18 20 12 3 6 9

0 7 2 8 1 4 5 6 3

7 5 6 7 3 1 1 3

0 34 1 33 2 32 3 31

O QUE QUEREMOS PROVAR:

$\left.\begin{array}{l} \text{1) } \forall i \text{, EXISTE } n_0 \text{ T.q. SE } n \geq n_0, \\ \text{ENTÃO HÁ SOLUÇÃO PARA } (n, n-i) \end{array}\right\}$ MAIS GERAL

2) $\forall i$ EXISTE C E n_0 T.q. SE $n = n_0 + K \cdot C$

ENTÃO HÁ SOLUÇÃO PARA $(n, n-i)$

(n, n) : $(0, n, 1, n-1, 2, n-2, 3, (n-3, 4, n-4) (\quad)$

$(n, n-1)$ $(0 \ast \ast \ast, 3, n-2, 2$ $(n-4, 4, n-3))$ 0

$n = 20$ no 19 $K = n-1$

0 20 (20 19 1 18 19) 17 2 16 18 15 3) 14 17 13 4 12 16 11 5 15 ...

0 (19 1 20) (3 18 2) 14 (16 4 17) ....

$n=10 \quad K=9=10-1$

$0 \; (10 \; 1 \; 9)(2 \; 8 \; 3)(7 \; 9)(6 \; 5)$

$0 \; (9 \; 1 \; 10)(3 \; 8 \; 2)(6 \; 5 \; 7 \; 4)$

$n=10 \quad K=8=10-2$

$0 \; 10 \; (1 \; 9 \; 2)(8 \; 3 \; 7)(4 \; 6 \; 5)$

$0 \; (10 \; (2 \; 9 \; 1))((7 \; 3 \; 8)(5 \; 6 \; 4))$

$0 \; (7 \; 3 \; 8)(5 \; 6 \; 4) \; (10 \; (2 \; 9 \; 1))$

$n=10 \quad K=7$

$0 \; 10 \; 1 \; (9 \; 2 \; 8)(3 \; 7 \; 4)(6 \; 5)$

$(8 \; 2 \; 9)(4 \; 7 \; 3) \; 10 \; (5 \; 6) \; 0$

$0 \; 5 \; (3 \; 7 \; 4) \; 10 \; (9 \; 2 \; 8) \; 1 \; 6$

0 34^{34} 1^{33} 33 2 ((32 3 31)) (4 30 5)) ((29 6 28) (7 27 8))
((26 9 25) (10 24 11)) 23

- 34 - 33

+32 + 31

0 34^{34} $(1^{33}$ 33 2) $(32^{30}$ 3 $31)^{27}$ (4 30 5^{2})

0 $(31^{31}$ 3 32) $(5^{27}$ 30 4) 34^{30} $(2^{32}$ 33 1) 25 9 26

$i = 3$ $\qquad 0 \quad 75 \; (\quad 24 \quad)(\quad 24 \quad)$

$n = 74$

$- \quad , 74, 73, 72 \qquad (\quad 24 \quad)$

$+ \quad , 71, 70, 69$

$0 \; (70 \; 4 \; 71)(2 \; 72 \; 3)(74)(1)(73)$

$(69 \; 6 \; 68)(4 \; 70 \; 5)$

$(71 \; 4 \; 70)$

70 69 70 69 71 64 68

71 | 74-3, ~~73-2~~, 72-1, ~~71-0~~

70 | 74-4, 73-3, ~~72-2~~, ~~71-1~~, 70-0

69 | ~~74-5~~, 73-4, 72-3, ~~71-2~~, 70-1, 69-0

74 73 72

$0 \; (74)(73 \; 2 \; 72)$

3 6 24th 12 (4) 24

0 ((n)))( ))( )( )

12th

0 1 2 3 4 5

3 4 5 0 1 2

[0, 74, 1, 73, 2, 72, 3, 71, 4, 70, 5, 69, 6, 68, 7, 67, 8, 66, 9, 65,
10, 64, 11, 63, 12, 62, 13, 61, 14, 60, 15, 59, 16, 58, 17, 57, 18, 56,
19, 55, 20, 54, 21, 53, 22, 52, 23, 51, 24, 50, 25, 49, 26, 48, 27, 47,
28, 46, 29, 45, 30, 44, 31, 43, 32, 42, 33, 41, 34, 40, 35, 39, 36, 38,
37]

i = 2

-74, 73

+72, 71

0 10 (1 9 2) (8 3 7) (4 6 5)

0 (7 3 8) (5 6 4) 10 (2 9 1)

15 12 9 6 3

0 (15 1 14)(2 13 3)(12 4 11)(5 10 6)(9 7 8)

2

0 5 1 4 2 3

14 12 11 9 6 3

0 ((14 1 15)(3 13)2)(11 4 12)(6 10 5)(8 7 9)

13 14 13 12

0 13 3 15 1 14 2 11 4 12 6 10 5 8 7 9

15 14 10 5

0 (15 1 14 2 13)(3 12 4 11 5)(10 6 9 7 8)

2

13 13 14 13 9 5

0 (13 2 15 1 14)(5 3)(8 10)

16 15 12 9 6 3

0 16 (1 15 2)(14 3 13)(4 12 5)(11 6 10)(7 9 8)

16 14 12 10 9 6 5 3 1

0 16 (2 15 1)(13 3 14)(5 12 4)(10 6 11)(8 9 7)

13 10 5

0 (13 1 15 2 16)(6 10 4 12 5 14 3)(8 11)

13 12 15 8 3 6

0 (13 3 14)(2 15 1) 16 (8 9 7)(10 6 11)(5 12 4)

10 9

0 (10 1 9 2 8)(3 7 4 6 5)

8 7

0 8 1 (7 2 6)(3 5 4)

8 5

0 (8 2 9 1 10)(5 6 4 7 3)

0 (6 2 7) 1 (4 5 3) 8

14 13 7

0 (14 1 13 2 12 3 11)(4 10 5 9 6 8 7)

11 10 13 7

0 (11 3 12 2 13 1 14)(7 8 6 9 5 10 4)

14 12 10 7 4 2

0 14 (2 13 1)(11 3 12)(5 10 4)(8 6 9) 7

18 9

0 (18 1 17 2 16 3 15 4 14)(5 13 6. 12 7 11 8 10 9)

14 9

0 (14 4 15 3 16 2 17 1 18)(9 10 8 11 7 12 6 13 5)

20 10

0 (20 1 19 2 18 3 17 4 16 5)(15 6 14 7 13 8 12 9 11 10)

5 10

0 (5 16 4 17 3 18 2 19 1 20)(10 11 9 12 8 13 7 14 6 15)

26 13

0 (26 1 25 2 24 3 23 4 22 5 21 6 20)(7 19 8 18 9 17 10 16 11 15 12 14 13

20 20 13

0 (20 6 21 5 22 4 23 3 24 2 25 1 26)(13 14 12 15 11 16 10 17 9 18 8 19 7)

$$\begin{array}{ccccccccccccccccc}
 & n & n-1 & n-2 & & & & & n-7 & & & & & & & \\
0 & (n & 1 & n-1 & 2 & n-2 & 3 & n-3) & (4 & n-4 & 5 & n-5 & 6 & n-6 & 7)
\end{array}$$

$$\begin{array}{cccccccccccc}
 & n-3 & n-5 & & & n-2 & n-1 & & n-7 & & \\
0 & (n-3 & 3 & n-2 & 2 & n-1 & 1 & n) & (7 & \cdots & 4)
\end{array}$$

$$\begin{array}{ccccccccccccccccc}
 & n & n-1 & n-2 & & n-3 & n-4 & n-6 & & & n-9 & & & & & \\
0 & (n & 1 & n-1 & 2 & n-2 & 3 & n-3 & 4 & n-4) & (5 & n-5 & 6 & n-6 & 7) \\
 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & & & & & & & \\
 & & & & & & & & & & 5 & & 6 & 7 & 8 & 9
\end{array}$$

$$\begin{array}{cccccccccccccc}
 & n-4 & & & & n-3 & n-2 & & n-3 & n-2 & n-9 & & \\
0 & (n-4 & 4 & n-3 & 3 & n-2 & 1 & n-1 & 2 & n) & (9 & \cdots & 5)
\end{array}$$

$n \quad n-1 \quad n-2 \qquad n-5$

$0 \quad n \quad 1 \quad n-1 \quad 2 \quad n-2 \quad 3$

$n-3 \qquad n-4$

$1 \quad n-2 \quad 2 \quad n-1 \quad 3$

$n \qquad n-3$

$n-1 \qquad n-4$

$n-2 \qquad n-5$

$n-13$

$0 \; (n-13 \quad 13 \quad n-12 \quad 12 \quad n-11 \quad 11 \quad n-10 \quad 10 \quad n-9 \quad 9 \quad n-8 \quad 8$

$n-13 \qquad -5 \quad -5 \quad -4 \quad -3 \quad -4$

$n-7 \quad 7 \; (n-6 \quad 6 \quad n-5 \quad 5 \quad n-4 \quad 4 \quad n-3 \quad 3 \quad n-2 \quad 2 \quad n-1) \quad 1 \quad n) (27$

$n-8 \qquad n-7$

$n-1 \quad 2 \quad n-2 \quad 3 \quad n-3 \quad 4 \quad n-4 \quad 5 \quad n-5 \quad 6 \quad n-6$

$-5 \quad -3 \quad -4 \quad -5 \quad -6 \quad -4 \quad -1$

$4 \quad n-1 \quad 2 \quad n-2 \quad 3 \quad n-3 \quad 1 \quad n$

$\;\; n \quad\;\; n-1 \;\; n-2 \quad\; n-3 \;\; n-4 \qquad\quad n-6 \qquad\qquad\qquad\qquad\qquad\qquad\qquad n-13$

$0 \; (n \;\; 1 \;\; n-1 \;\; 2 \;\; n-2 \;\; 3 \;\; n-3 \;\; 4 \;\; n-4 \;\; 5 \;\; n-5 \;\; 6 \;\; n-6) \; (7$

$n-6 \qquad\qquad\qquad\qquad\qquad -7 \qquad\qquad\qquad -2 \qquad\quad n-13$

$0 \; (n-6 \;\; 6 \;\; n-5 \;\; 5 \;\; n-4 \;\; 4 \; (n-3 \;\; 3 \;\; n-2 \;\; 2 \;\; n-1) \;\; 1 \;\; n) \; (13$

$n-6 \qquad\qquad\qquad\qquad n-6 \;\; n-5 \;\; n-3 \;\; n-2 \;\; n-3 \;\; n-2$

$0 \; (n-6 \;\; 6 \;\; n-5 \;\; 5 \;\; n-4 \;\; 4 \;\; n-3 \;\; 3 \;\; n-2 \;\; 1 \;\; n-1 \;\; 2 \;\; n \;\; 13$

$-3 \;\; -4 \;\; -2 \;\; -3 \;\; -4 \;\; -5$

$n \;\; 3 \;\; n-1 \;\; 1 \;\; n-2 \;\; 2 \;\; n-3$

$n-4 \;\; n-2 \;\; n-3 \;\; n-4 \;\; n-5 \;\; n-3$

$n-3 \;\; 1 \;\; n-1 \;\; 2 \;\; n-2 \;\; 3 \;\; n$

$n-3$

$n-4$

$n-5$

$n-3$ $n-4$ $n-5$ $n-3$ $n-4$ $n-5$

n 3 $n-1$ 4

$n-13$

27

$n-3$

n 3

$n-1$ 2

$n-2$ 1

$n-5$ $n-4$ $n-3$ $n-5$ $n-4$ $n-3$

n 5 $n-1$ — 2 $n-3$ 1 $n-2$

$n-4$

n 4

$n-1$ 3

$n-2$ 2

$n-3$ 1

$n-4$ $n-5$ $n-3$ $n-4$

n 4 $n-1$ 2 $n-2$ $n-3$

$n-4$ $n-5$ $n-6$ $n-7$ $n-4$ $n-5$ $n-6$ $n-7$

n 4 $n-1$ 5 $n-2$ 2 $n-3$ 3 $n-4$

$n-5$

n 5

$n-1$ 4

$n-2$ 3

$n-3$ 2

$n-4$ 1

$n-5 \quad n-4 \quad n-3 \quad n-5 \quad n-4 \quad n-3$

$n \quad 5 \quad n-1 \quad - \quad 2 \quad n-3 \quad 1 \quad n-2$

$n \quad n-1 \quad n-2 \quad n-3 \quad n-4 \quad n-5 \quad \boxed{n-6} \quad \boxed{n-7} \quad n-9 \quad n-10$

$0 \quad n \quad 1 \quad n-1 \quad 2 \quad n-2 \quad 3 \quad n-3 \quad 4 \quad n-4 \quad 5 \quad n-5$

$K = n-2$

$n \quad n-1 \quad n-2 \quad n-3 \quad n-4 \quad n-5 \qquad n-2i-1$

$0 \; (n \; (1 \;\; n-1 \;\; 2) \;\; n-2 \;\; 3 \;\; n-3 \; \cdots \qquad n-i)(i+1 \; \cdots \; 2i+1)$

$n-i \qquad\qquad\qquad\qquad n-2i+1$

$0 \; (n-i \quad \ldots \qquad n \mid (2i+1 \; \cdots \; i+1)$

$n-2 \quad n-3 \quad n-2 \quad n-3$

$n \;\; 2 \;\; n-1 \;\; 1 \;\; n-2$

$i = 4$

$n-4 \qquad n-3 \qquad n-2$

$0 \; (n-4 \;\; 4 \;\; n-3 \;\; 3 \;\; n-2 \; (1 \;\; n-1 \;\; 2) \; n \;) \; (2i+1 \; \ldots \; i+1$

$k = n - 4$

$n \quad n-1 \quad n-2 \quad n-3 \quad n-4 \quad n-5 \quad n-6 \quad n-7 \quad n-8 \quad n-k$

$0 \quad n \quad 1 \quad (n-1 \quad (2 \quad n-2 \quad 3 \quad n-3 \quad 4) \quad n-4 \quad 5 \quad n-5 \quad 6 \quad n-6 \quad 7, n-7)$

$2 \to 4$, $n-5$

$2 \to 2$, $n-6$

$(8 \quad \ldots \quad 14) \equiv$

$n = 2 + c \cdot 13$

PROP: Dado i, existe r t.q. existe solução p/ (n, k), onde $n = c \cdot r$, $c \geq 1$ e $k = n - i$

$K = n-3$

$n \quad n-1 \quad n-2 \qquad n-3 \quad n-4 \qquad n-5 \quad n-6 \qquad\qquad n-10$

$0 \; (n \quad 1 \quad n-1 \quad 2 \quad n-2 \quad 3 \quad n-3 \quad 4 \quad n-4 \quad 5 \quad n-5 \qquad n-i \,)$

$c \rightarrow n-c$

$n-1 \qquad n-2 \quad n-3 \qquad n-4 \qquad n-5 \qquad n-6 \quad n-7 \qquad n-8 \quad n-9 \qquad n-10$

$0 \quad n-1 \qquad n-2 \quad 2 \,)(\, n-3 \quad 3 \quad n-4 \,)(\, 4 \quad n-5 \quad 5 \,)$

$n-3 \qquad\qquad n-4$

$n-1 \; (\; 2 \qquad\qquad 1 \,)(\, n-3$

$K = n - 1$

$n \quad n-1 \quad n-2 \quad n-3 \quad n-6$

$0 \; (n \;\; 1 \;\; n-1) \; (2 \;\; n-2 \;\; 3) \; (n-3 \;\; 4 \;\; n-4) \; (5 \;\; n-5 \;\; 6)$

$n-1 \quad n-2 \quad n-1 \quad n-3 \quad n-4 \quad n-6$

$0 \; (n-1 \;\; 1 \;\; n) \; (3 \;\; n-2) \; (2) \; (n-4 \;\; 4 \;\; n-3) \; (6) \; n-5 \;\; 5)$

$n-2 \quad n-1 \quad n-2 \quad n-7$

$0 \; (n-2 \;\; 3 \;\; n \;\; 1 \;\; n-1) \; (6 \;\; n-3 \;\; 4 \;\; n-4 \;\; 2)$

$0 \quad n \quad 1 \quad n-1 \quad 2 \quad n-2 \quad 3 \quad n-3 \quad 4 \quad n-4 \quad \overset{n-9}{} \quad 5$

$0 \quad 9 \quad 1 \quad 8 \quad 2 \quad 7 \quad 3 \quad 6 \quad 4 \quad 5$

$0 \quad 6 \quad \overset{1}{5} \quad 1 \quad 4 \quad 8 \quad 3 \quad 9 \quad 7 \quad 2$

$0 \quad n-3 \quad n-2 \qquad\qquad n-7$

n $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad n-9$

$0\ (n\ \ 1\ \ n-1\ \ 2\ \ n-2\ \ 3\ \ n-3\ \ 4\ \ n-4)(5 \qquad\qquad 9)$

$n-4 \qquad\qquad n-4 \qquad n-1 \quad n-9$

$0\ (n-4\ \ 4\ \ n-3\ \ 3\ \ n-2\ \ (2\ \ n-1\ \ 1)\ n)(9 \qquad\qquad 5)$

$n-4 \qquad\qquad n-3 \qquad n-2 \quad n-9$

$0\ (n-4\ \ 4\ \ n-3\ \ 3\ \ n-2\ \ (1\ \ n-1\ \ 2)\ n)\ (9 \qquad\qquad 5$

$\qquad\qquad\qquad\qquad\qquad 1 \qquad 3 \qquad 2$

$n\ \ \ n-1\ \ \ n-2 \rightsquigarrow n-3\ \ \ n-4\ \ \ n-5$

$n-1\ \ n-2\ \ \ n-3\ \ n-4\ \ \ n-5\ \ n-6\ \ \ n-7\ \ n-8$

$n\ \ 1\ \ n-1\ \ 2\ \ n-2\ \ 3\ \ n-3\ \ 4\ \ n-9$

$n-4\ \ n-5\ \ \ n-3\ \ n-4\ \ \ n-5\ \ n-6\ \ \ n-4\ \ n-5$

$n\ \ 4\ \ n-1\ \ 2\ \ n-2\ \ 3\ \ n-3\ \ 1\ \ n-4$

$n-1 \quad n-2 \quad n-3 \quad n-4 \quad n-5 \quad n-6 \quad n-7 \quad n-7$

$\Big(n \quad 1 \quad n-1 \quad 2 \quad n-2 \quad 3 \quad n-3 \quad 4 \Big) \; n-4 \qquad n-8 \Big)$

$n-11$

$n-5 \quad 6$

9

$n-1 \qquad n-3 \qquad n-4 \; n-3 \qquad n-4 \qquad n-3 \; 4 \; n-4 \qquad n-10 \qquad n-11 \qquad n-19$

$\underbrace{n\Big(1 \quad n-2 \quad 2 \quad n-1 \quad 3 \quad \cdots \quad 5\Big)}\Big(n-5\Big)\Big(6 \qquad n-9\Big) \cdots \qquad \overbrace{10-14}^{9}$

$n \quad n-1 \quad n-2$

$\cdots$

$n-3 \quad n-4 \quad n-5$

$n-5 \qquad n-10$

$\underbrace{n\Big(5 \qquad 1\Big)} \; \underbrace{\Big(n-9 \quad \cdots \quad n-5\Big)}_{9} \cdots \qquad 14-10$

0

$n-14$

n $\qquad$ $n-9$

$0\,(n \quad 1 \quad n-1 \quad 2 \quad n-2 \quad 3 \quad n-3 \quad 4 \quad n-4)\,(5 \qquad\qquad 9)$

$n-4 \qquad\qquad n-4 \qquad n-1 \qquad n-9$

$0\,(n-4 \quad 4 \quad n-3 \quad 3 \quad n-2 \quad (2 \quad n-1 \quad 1)\,n)\,(9 \qquad\qquad 5)$

$n-4 \qquad\qquad n-3 \qquad\qquad n-2 \qquad n-9$

$0\,(n-4 \quad 4 \quad n-3 \quad 3 \quad n-2 \quad (1 \quad n-1 \quad 2)\,n)\,(9 \qquad\qquad 5$

$n-1 \qquad n-4$

$n \quad 1 \quad n-1 \quad 2 \quad n-2 \quad 3 \quad n-3 \quad 4 \quad n-4$

$n-2$

$n-3$

$$0 \quad n \left(1 \quad \overset{n-2}{(n-1} \quad 2 \quad \overset{n-5}{n-2)} \right) \left(3 \quad n-3 \quad \overset{n-8}{4} \right) \cdots \quad 5) \overset{n-10}{(n-5} \quad \cdots \quad \overset{n-19}{n-9)}$$

$$(10 \quad \cdots \quad \overset{n-28}{14)}$$

$$(n-14 \quad n-18)$$

$$\overset{n-37}{(19} \quad 23)$$

$$0 \quad \overset{n}{n} \quad \overset{n-5}{(5} \quad \cdots \quad \overset{n-10}{1)} \quad (n-9 \quad \cdots \quad \overset{n-19}{n-5)} \quad (14 \quad \cdots \quad \overset{n-28}{10)} \quad (n-18 \quad \cdots \quad n-4)$$

$$19$$

$$0 \quad n \left(\overset{n-3}{1} \quad (n-2 \quad 2 \quad \overset{n-5}{n-1)} \right) \left(4 \quad n-3 \quad \overset{n-7}{3)} \quad n-4 \quad \cdots \quad \overset{n-14}{7)} \right) (n-7 \quad \cdots$$

$$0 \quad n \quad (1 \quad n-1 \quad 2)(n-2 \quad 3 \quad n-3)\,4 \cdots 5 \quad n-5 \quad \cdots \quad n-9)$$

$$n-7$$

$$0 \quad n \quad (1 \quad n-1 \quad 2) \quad n-2 \quad 3 \quad n-3 \quad 4 \cdots 5 \quad n-5 \quad \cdots \quad n-9)$$

$$n-2 \qquad n-4 \qquad n-6 \qquad n-$$

$$\underbrace{0 \quad (n \quad (2 \quad n-1 \quad 1)(n-3 \quad 3 \quad n-2) \quad 4 \quad \cdots \quad 6 \quad (n-6))}_{13}$$

0 10 1 9 2 8 3 7 4 6 5

7 2 7 6 5 3 5 4 1 6

0 7 9 2 8 3 6 1 5 4 10

$0 \; (n^{n} \; 1^{n-1} \; n-1^{n-2})^{n-3} \; (2^{n-4} \; n-2^{n-5} \; 3)^{n-6} \; (n-3 \; 4 \; n-4)^{5\,n-9} \cdots (\quad)$

$0 \; (n-1^{n-1} \; 1^{n-2} \; n^{n-1})^{n-3} \; (3^{n-5} \; n-2^{n-4} \; 2)^{n-6} \; (n-4 \; 4 \; n-3) \cdots (\quad)$

$0 \; (n-4^{n-4} \; 4^{n-8} \; n-3^{n-7} \; 3^{n-6} \; n-2^{n-5} \; (2^{n-4} \; n-1^{n-3} \; 1^{n-2}) \; n^{n-1} \;)^{n-9} \; (9 \quad 5)$

$0 \; (n-4^{n-4} \; 4^{n-8} \; n-3^{n-7} \; 3^{n-6} \; n-2^{n-5} \; (1^{n-3} \; n-1^{n-2} \; 2^{n-3}) \; n^{n-2} \;)^{n-9} \; (9 \quad 5)$

$n = 3n' + 2$

$n - 25 = 6$

6 5 4 3 2 1

0 6 (1 5 2 4 3)

10+2

3 1 2 3 4 5

0 (3 4 2 5 1) 6

12 11 10 5

0 12 1 (11 2 10) (3 9) (4 8 5 7 6)

9 10 5 8

0 (9 3 10 2 11) 1 (6 7 5 8 4) 12

−12 −11

$\boxed{+10 \quad +9}$

$n = 17 \quad K = 15$

$-17 \quad -16 \quad -15$

$+14 \quad +13 \quad +12$

14 9 6 3 2 1

0 17 1 (16 2 15) (3 14 4) (13 5 12) (6 11 7) (10 8 9)

14 14 13 12 11 10

12 7 8 12 14 13 14 9 4 5 11 13 10 11

0 (12 5 13) 1 (15 2 16) (7 11 6) 17 (4 14 3)

6 1 2

(9 8 10)

(9 8 10) (4 14 3) 17

$K = n - 1$ ✓

13 11

(3 14 4) 17 (6 11 7)

Blocos

± resolver $K = n - 2$

$K = \frac{n}{2}$

((16 2 15) 1 (13 5 12))

$n=14$
$k=14$

0 14 1 13 2 12 3 11 4 10 5 9 6 8 7

$n=14$
$k=7$

0 7 1 6 2 5 3 4 11 12 10 13 9 14 8

$n=7$
$k=7$

$n=6$
$k=6$

+ 8

7 6 5 4 3 2 1 7 1 2 3 4 5 6

0 7 1 6 2 5 3 (4 11) 12 10 13 9 14 8

0 7 13 9 2 5 10 12 8 14 6 1 4 3 11

7 6 4 7 3 5 2 4 6 8 5 3 1 8

$n=18$
$k=9$

0 9 1 8 2 7 3 6 4 5 14 15 13 16 12 17 11 18 10

0 6 1 5 2 4 3

8 14 9 13 10 12 11

$n \qquad 0 \; \dots \; n$

Ex: $n = 4$ $\qquad P = 4\ 0\ 3\ 2\ 1$

$D(P) = 4\ 3\ 1\ 1$

Ex: $n = 9$ $\qquad P = 0\ 9\ 1\ 8\ 2\ 7\ 3\ 6\ 4\ 5 \rightsquigarrow \ell = 0$

$D(P) = \cancel{9}\ \cancel{8}\ 7\ 6\ 5\ 4\ 3\ 2\ 1$

$\underbrace{\cancel{8}}_{K}\ \underbrace{7\ 6}_{K}$

$\lfloor \frac{n}{2} \rfloor \geq \ell = 1$

2

DEF: Dados n e ℓ com $\ell \leq \lfloor \frac{n}{2} \rfloor$, uma (n,ℓ)-permutação é uma permutação de $\{0, \dots, n\}$ (começando em 0) t.q. $D(P) = \{1, \dots, n-\ell\}$ e os números $\{n-2\ell, \dots, n-\ell\}$ aparecem duas vezes.

Conj: Existe (n,ℓ)-permutação $\forall n, \ell$ com $\ell \leq \lfloor \frac{n}{2} \rfloor$.

Já "resolvemos" para $\ell = 0, 1, 2$

DEF: DADOS n E ℓ COM $\ell \leq \lfloor \frac{n}{2} \rfloor$, UMA (n,ℓ)-PERMUTAÇÃO É UMA PERMUTAÇÃO DE $\{0, \ldots, n\}$ T.Q. $D(p) \subseteq \{1, \ldots, n-\ell\}$ E OS NÚMEROS $\{\underbrace{n-2\ell}, \ldots, \underbrace{n-\ell}\}$ APARECEM DUAS VEZES.

CONJ 1: EXISTE (n,ℓ)-PERMUTAÇÃO $\forall n, \ell$ COM $\ell \leq \lfloor \frac{n}{2} \rfloor$.

SÓ "RESOLVEMOS" PARA $\ell = 0, 1, 2$

CONJ 2: DADO ℓ, EXISTE $n_0 = n_0(\ell)$ T.Q. EXISTE (n,ℓ)-PERMUTAÇÃO $\forall n \geq n_0$.

CONJ 3: DADO ℓ, EXISTE $r = r(\ell)$ E $n_0 = n_0(\ell)$ T.Q. EXISTE (n,ℓ)-PERMUTAÇÃO COM $n = n_0 + m \cdot r_0$ $\forall m \in \mathbb{N}$

$n_0 = 7$

$r_0 = 5$

$n = 7, 12, 17, \ldots$

$n = 10$

$$P_0^{10} = 0\,(10\ 1\ 9)(2\ 8\ 3)(7\ 4\ 6)\,5 \qquad (\ell=0)$$

$\cancel{10}$ 9 8 7 6 5 4 3 2 $\cancel{1}$

$$0\,(9\ 1\ 10)(3\ 8\ 2)(6\ 4\ 7)\,5$$

9 8 9 7 5 6 4 2 3 2

$$P_0^{12} = 0\,(12\ 1\ 11)(2\ 10\ 3)(9\ 4\ 8)(5\ 7\ 6)$$

12 9 6 3

$$P_1^{12} = 0\,(11\ 1\ 12)(3\ 10\ 2)(8\ 4\ 9)(6\ 7\ 5)$$

11 9 6 3

n $n-1$ $n-2$ $n-3$ $n-4$

$$\boxed{P_0^{18}} = 0\,(18\ 1\ 17\ 2\ 16\ 3\ 15\ 4\ 14)(5\ 13\ 6\ 12\ 7\ 11\ 8\ 10\ 9)$$

18 $n-2$ 14 $n-4$ 9

$$0\,(14\ 4\ 15\ 3\ 16\ 2\ 17\ 1\ 18)(9\ 10\ 8\ 11\ 7\ 12\ 6\ 13\ 5)$$

14 14 17 9

16 1 17 2 18

15 16

-18	$-18,$
$+14$	-17
-14	$+16$
-17	$+15$

$P_0^{10} = 0 \quad 10 \quad (1 \quad 9 \quad 2)(8 \quad 3 \quad 7)(4 \quad 6 \quad 5)$

10 9 8 7 6 5 4 3 2 1

$P_2^{10} \quad 0 \, (7 \; 3 \; 8)(5 \; 6 \; 4) \; 10 \, (2 \; 9 \; 1)$

7 3 6 8

$P_0^{22} = 0 \quad 22 \, (1 \quad 21 \quad 2)(20 \quad 3 \quad 19)(4 \quad 18 \quad 5)(17 \quad 6 \quad 16)(7 \quad 15 \quad 8)(14 \quad 9 \quad 13)(10 \; 12 \; 11)$

n+1 18

22 21 18 15 12 9 6 3

$0 \, (19 \; 3 \; 20)(5 \; 18 \; 4) \; 22 \, (2 \; 21 \; 1) \quad (13 \quad 9 \quad 14)(11 \; 12 \; 10)(16 \; 6 \; 1)(8 \; 15 \; 7)$

19 15 18 20 12 3 6 9

P_2^{10+12r}

P_0^n $\quad 0 \quad n \quad 1 \quad (n-1 \quad 2 \quad n-2 \quad 3 \quad n-3)$

$\underline{n \quad n-1 \quad n-2}$

-2

$0 \quad 7 \quad 1 \quad (6 \quad 2 \quad 5 \quad 3 \quad 4)$

$7 \quad 6 \quad 5$

-6

-5

$k \geq 3$

$7+5k$

$+4$

P_3

$+3$

$+2$

$n-5$

$n-4$

$n-3$

$n-3$

$n-4 \qquad n-5$

$= 0 \quad 22 \quad 1 \quad (21 \quad 2 \quad 20 \quad 3 \quad 19) \quad (4 \quad 18 \quad 5 \quad 17 \quad 6) \quad (16 \quad 7 \quad 15 \quad 8 \quad 14) \quad (9 \quad 13 \quad 10 \quad 12 \quad 11)$

$n \quad n-1 \quad n-2 \qquad\qquad 15$

$n-3$

$n-4$

$n-5$

2

$-$

Graceful Tree Conjecture

1. Relação com decomposição:

2. Nosso problema é rotular os vértices de um caminho com $n+1$ vértices com $\{0, \dots, n\}$ de forma que os rótulos das arestas são o multiconjunto

$$\{1, \dots, n-\ell\} \cup \{n-2\ell, \dots, n-\ell\}$$

3. Será que podemos generalizar nosso problema p/ árvores?

Conj. Seja T uma árvore com $n+1$ vértices. Queremos rotular os vértices de T t.q. os rótulos das arestas são o multiconjunto

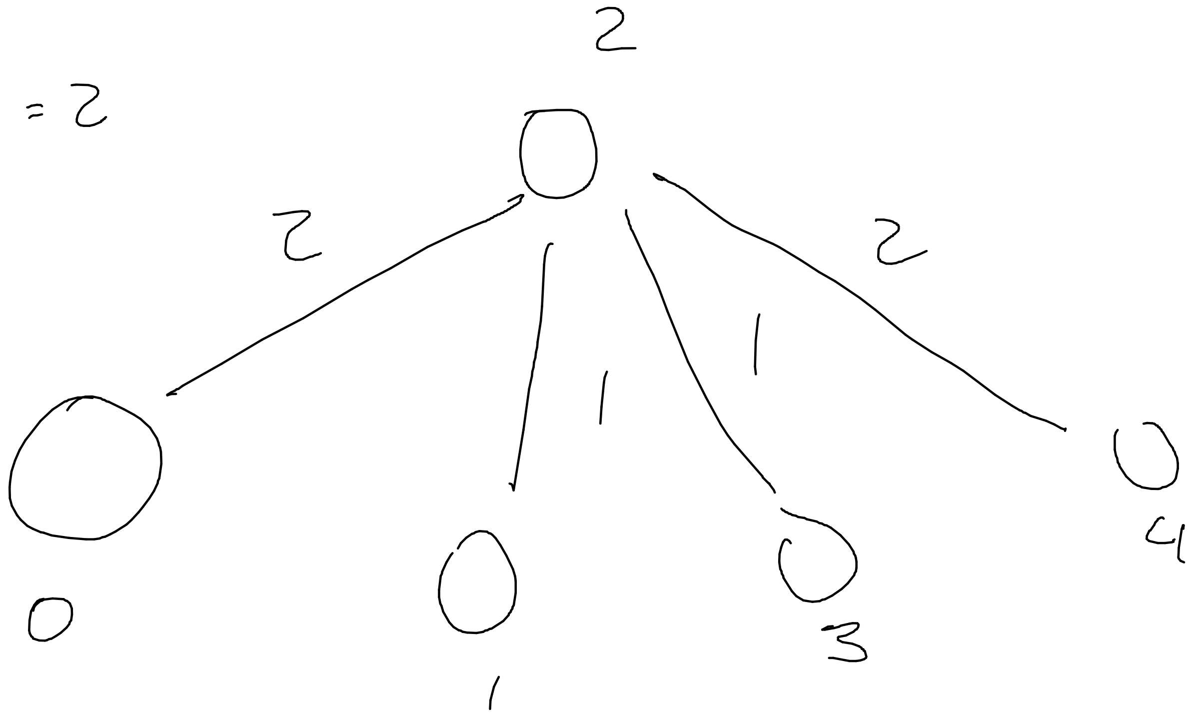
$$\{1, \dots, n-\ell\} \cup \{n-2\ell, \dots, n-\ell\}$$

CONJ. SEJA T UMA ÁRVORE COM $n+1$ VÉRTICES. QUEREMOS ROTULAR OS VÉRTICES DE T T.Q. OS RÓTULOS DAS ARESTAS SÃO O MULTICONJUNTO

$$\{1, \dots, n-\ell\} \cup \underline{\{n-2\ell, \dots, n-\ell\}}$$

$\mathrm{diam}(P_{n+1}) = n$

$\mathrm{diam}(S_{n+1}) = 2$

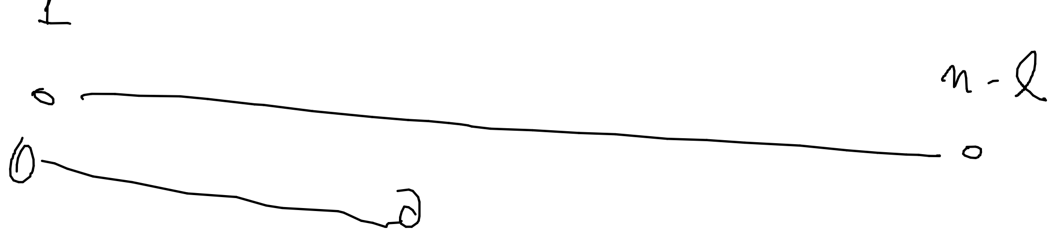


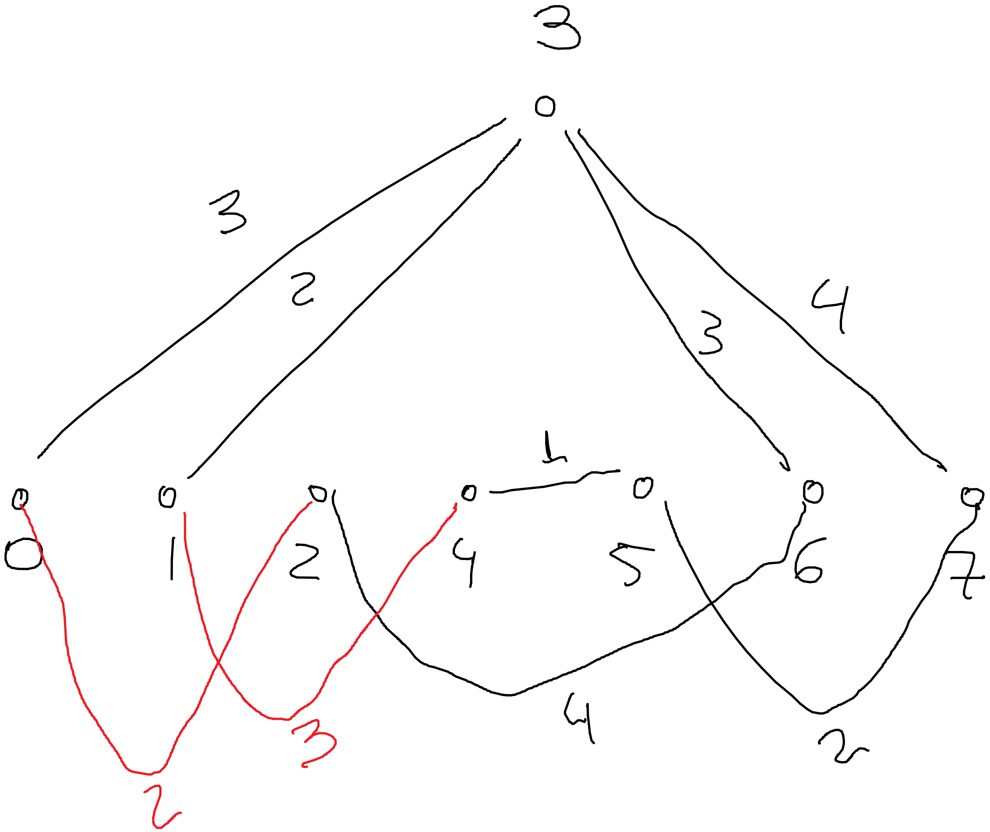
$n = 4$

$\ell = 0$

$\ell = 1$ X

$\{1,2,3\} \cup \{1\}$





$n = 7$

$l = 3$

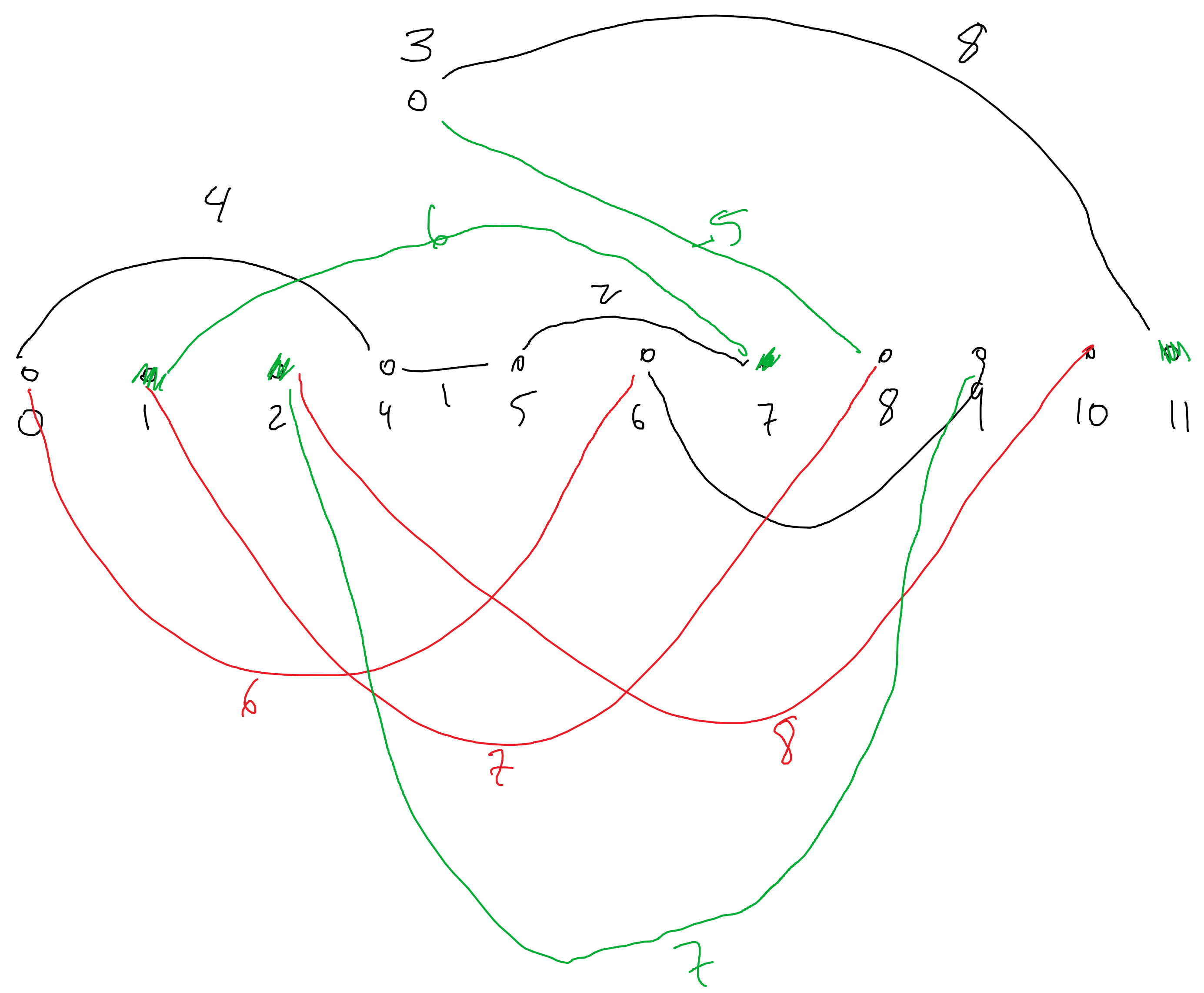
1 2 3 4

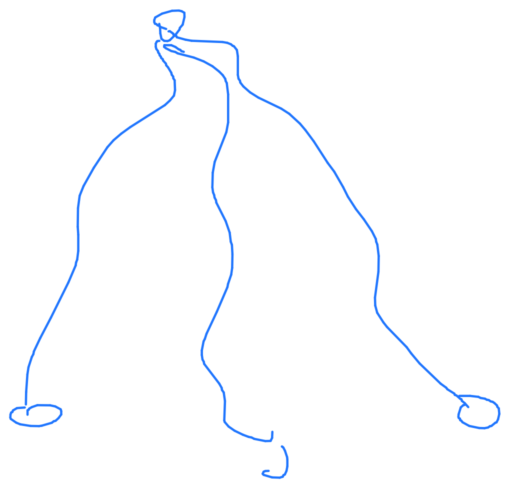
2 3 4

1 2 3 4

1 2 3

6 7 8





11 3 8 1 7 5 4 0 6 9 2 10

8 5 7 6 2 1 4 6 3 7 8

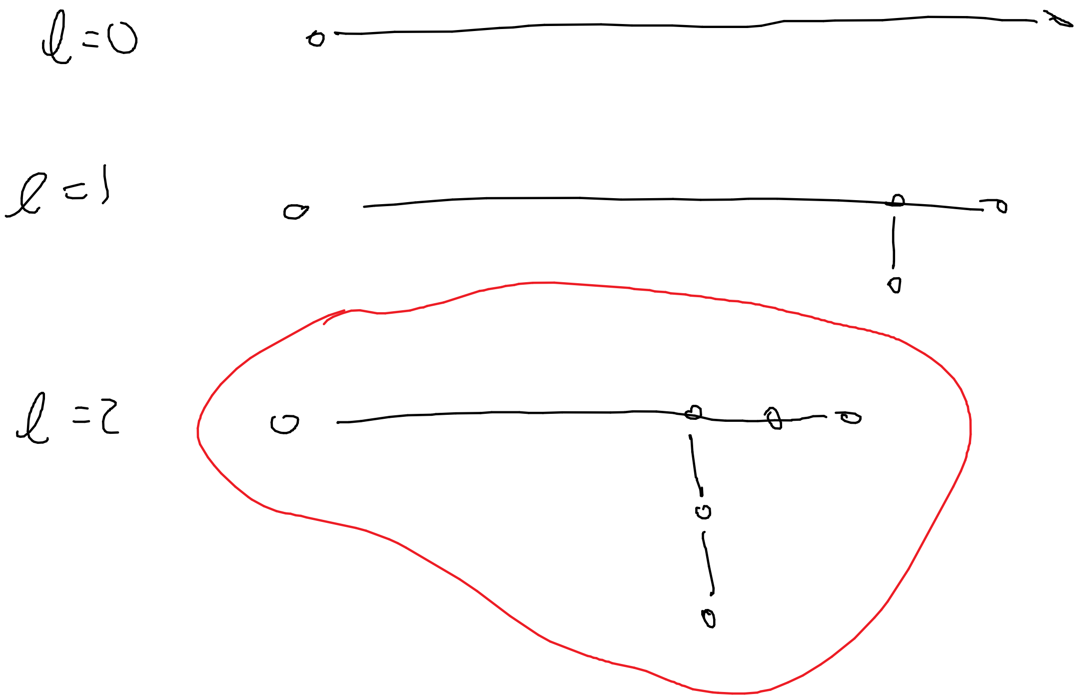
0 8 3 10 4 6 7 11 5 2 9 1

$n+1$ VÉRTICES

↳ MÚLTIPLO DE 3

EX: $n = 3$

ℓ



$n = 10$

$\ell = 3$

$0 \quad n-\ell \quad 1 \quad n-\ell-1 \quad \ldots$

7 6 5 4 7 6 5 2 6 3

0 7 1 6 10 3 9 4 2 8 5

5 4 3 2

8 9 10

4 3 2 1

7 6 5

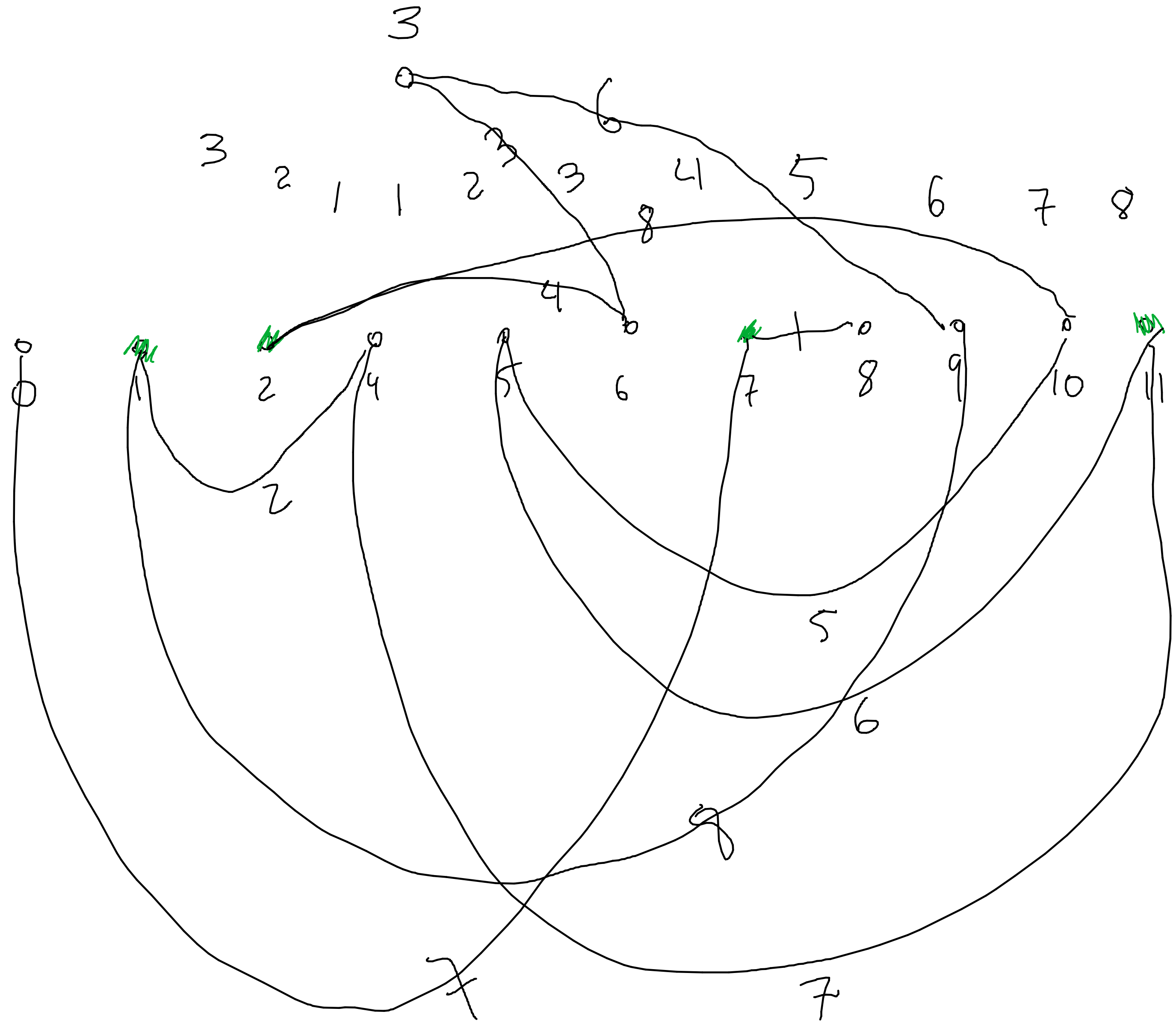
7 6 5 4 3 2 1

0 7 1 8 2 5 3 4

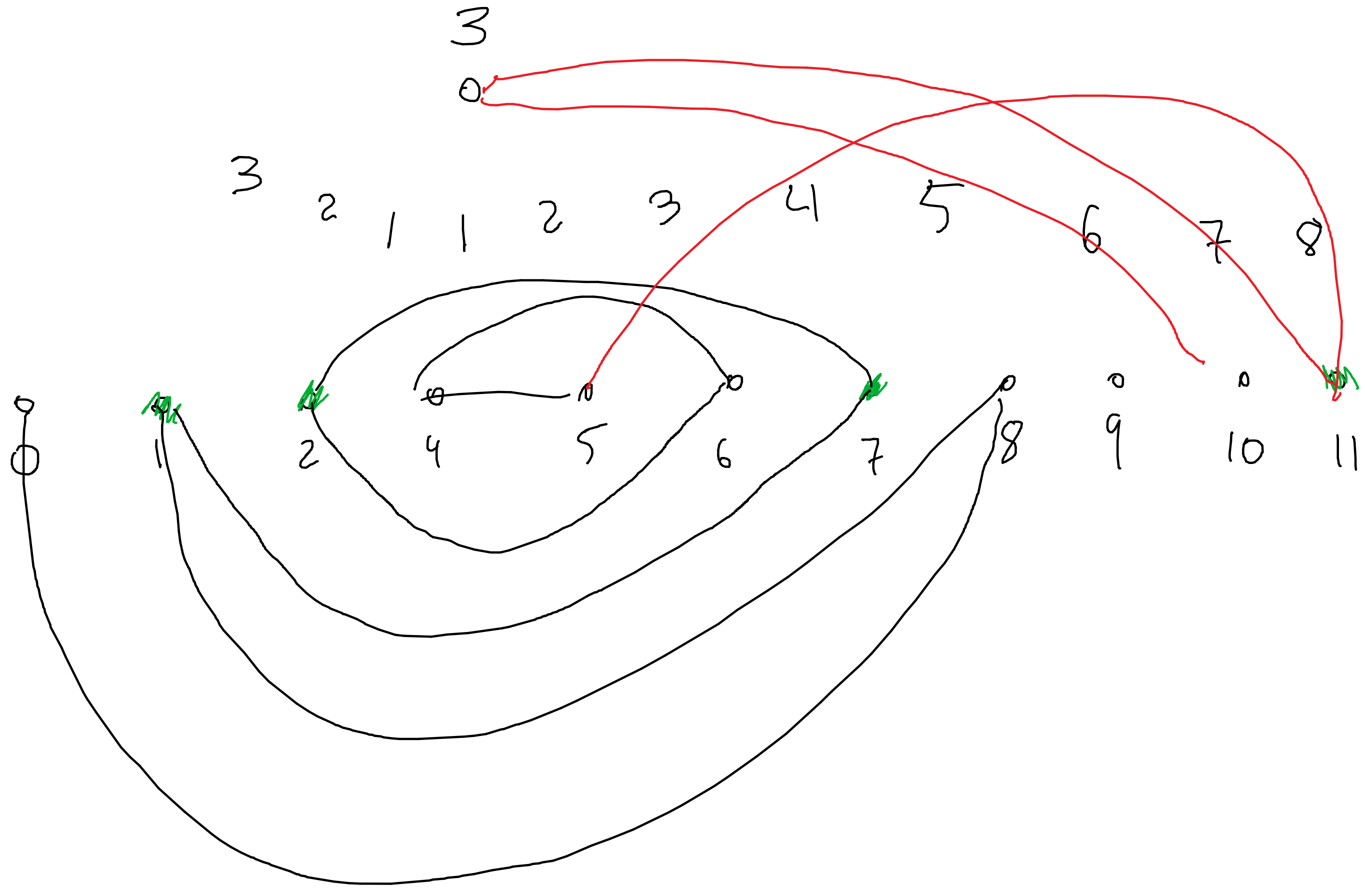
7 6

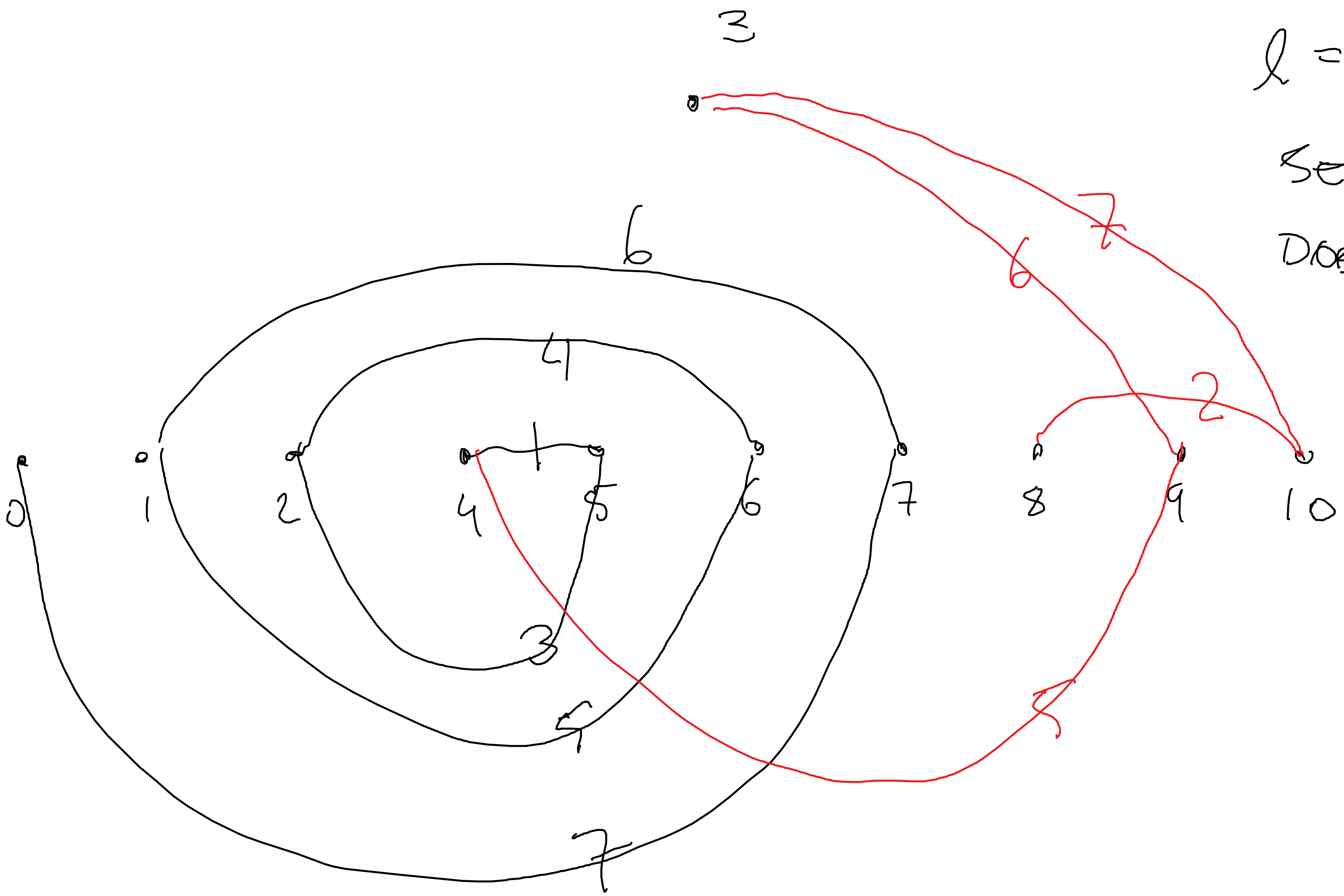
5 5

6 9 10



7, 8


3
0
3 2 1 1 2 3 4 5 6 7 8
0 1 2 4 5 6 7 8 9 10 11



$\ell = 3$

SEM 8, 9, 10

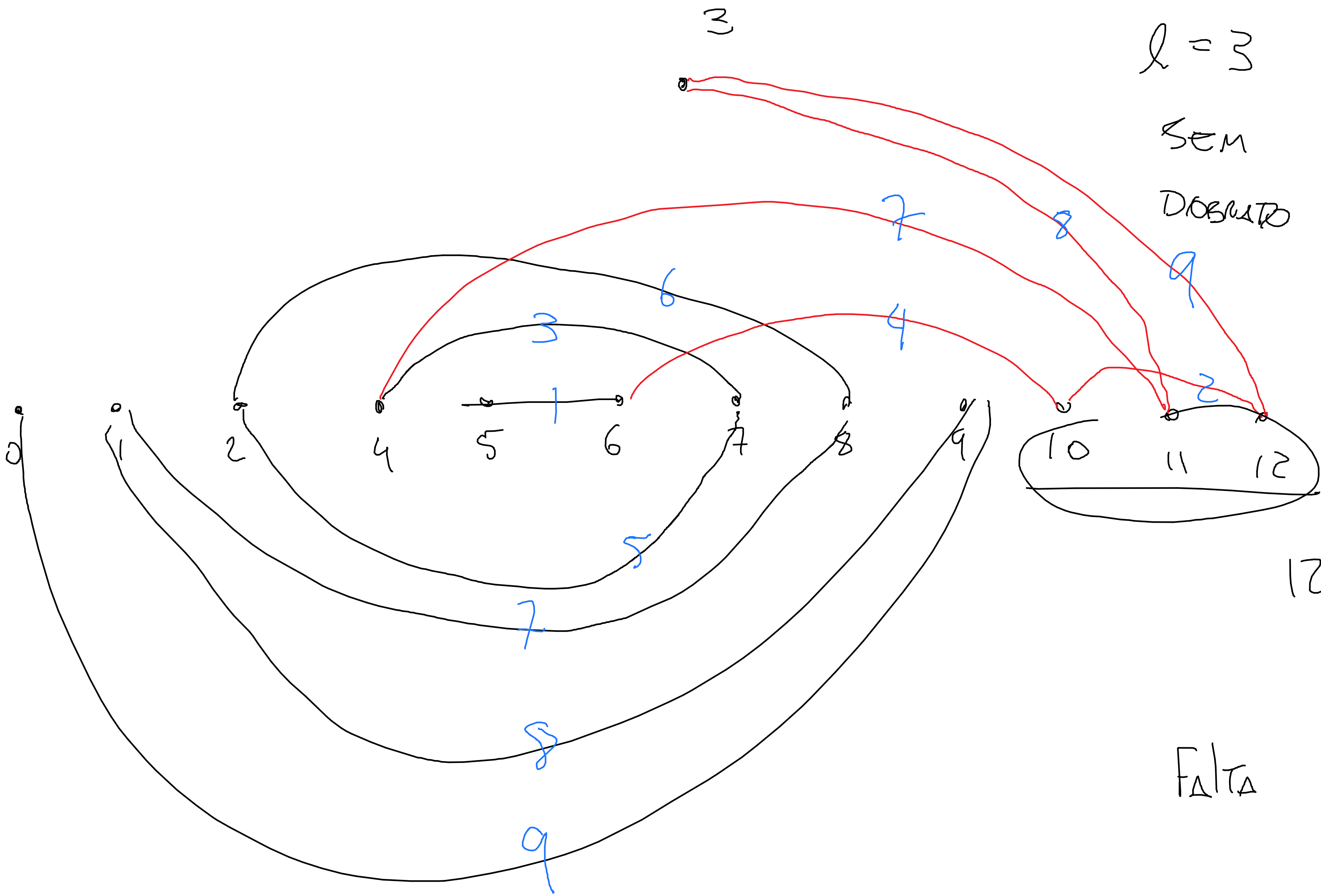
DOBRADO 5, 6, 7

FALTA 2, 5, 6, 7

$l = 3$

SEM 10, 11, 12

DOBRADO 9, 8, 7



9 8

12 3 11

Falta

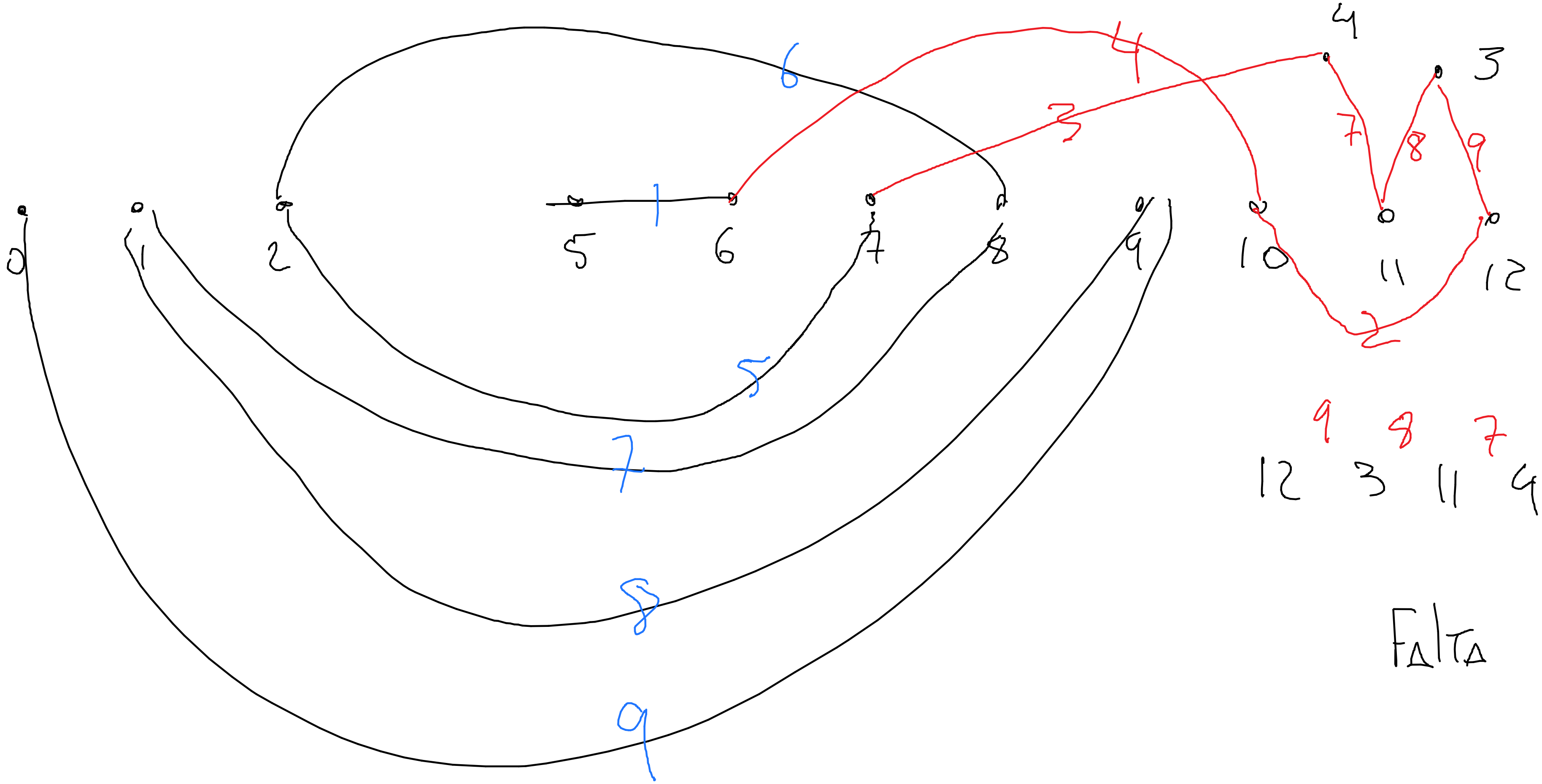
0 12 1 11 2 10 3 9 4 8 5 7 6

0 9 1 8 2 7 4 11 3 12 10 6 5

$\ell = 3$

SEM 10, 11, 12

DOBRADO 9, 8, 7



9 8 7 6

12 3 11 4 10

Falta 4, 3

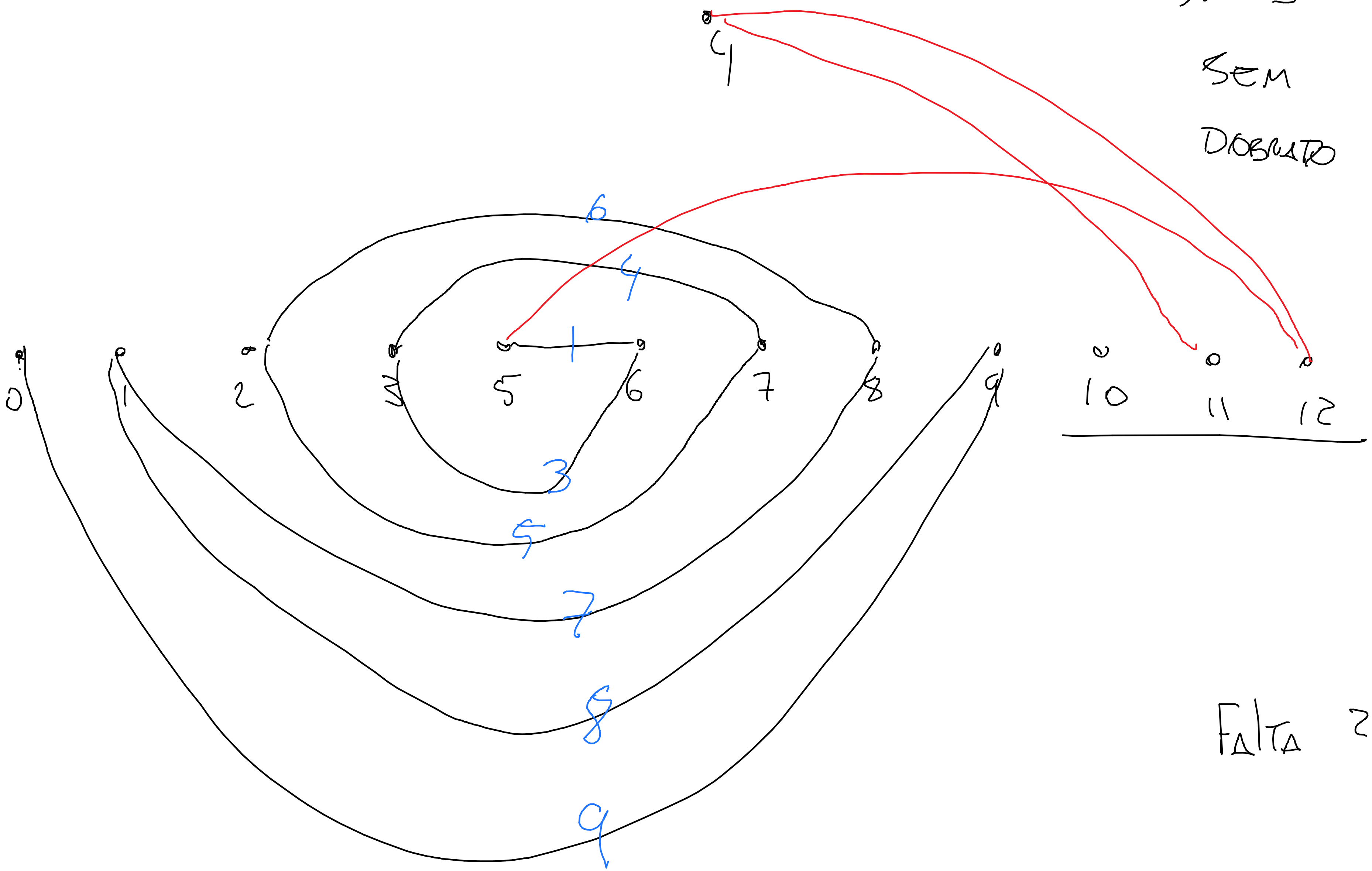
0 12 1 11 2 10 3 9 4 8 5 7 6

0 9 1 8 2 7 4 11 3 12 10 6 5

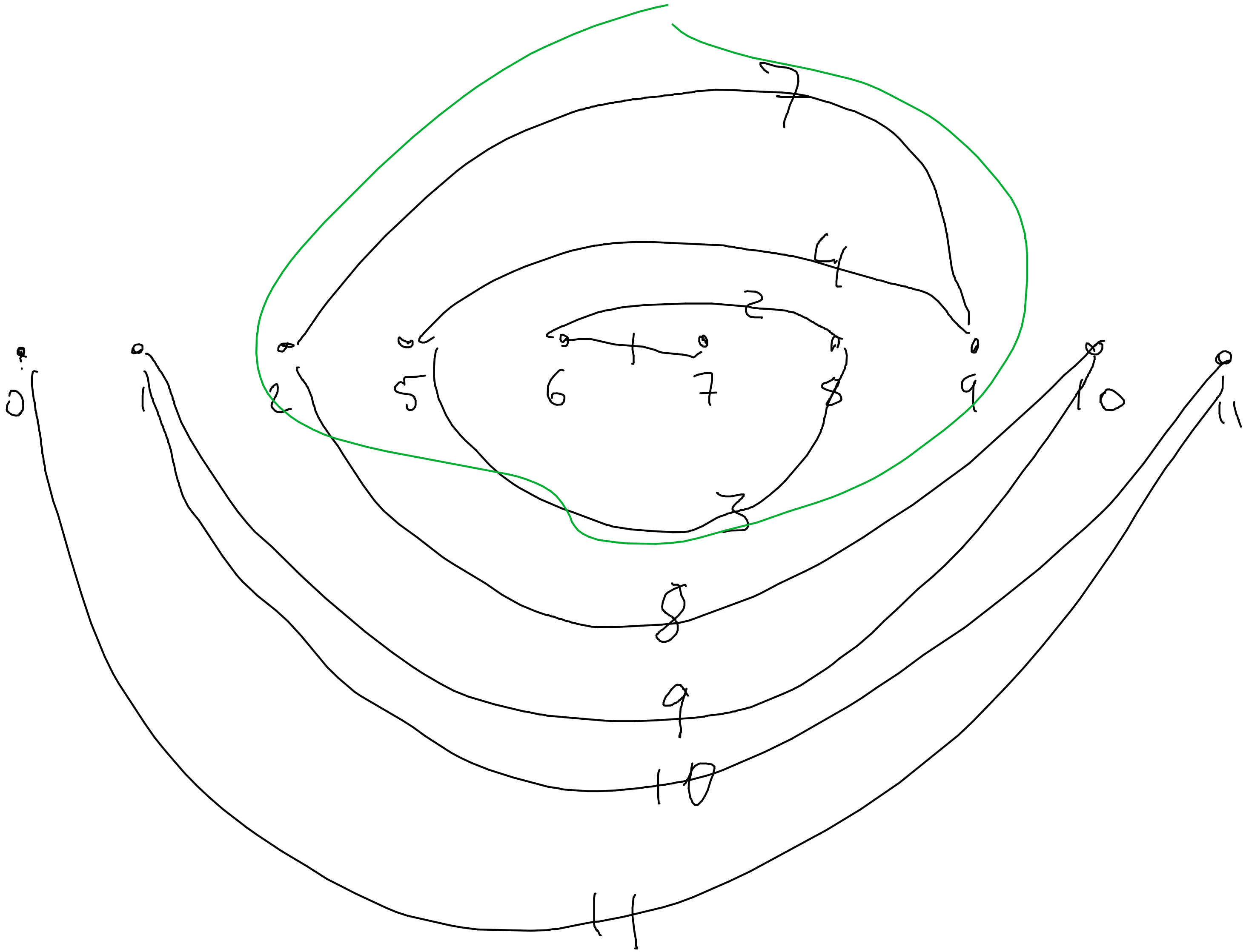
$\ell = 3$

SEM 10, 11, 12

DOBRADO 9, 8, 7



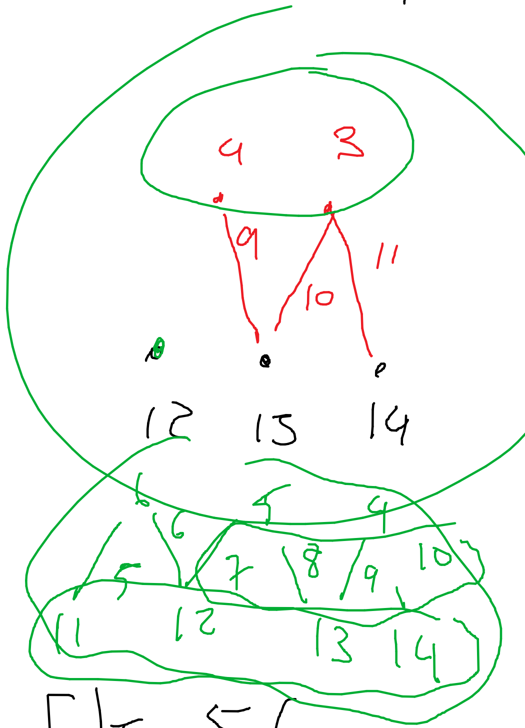
FALTA 2, 7, 8, 9



$l = 3$

SEM 14, 13, 12

DOBRADO 9, 10, 11

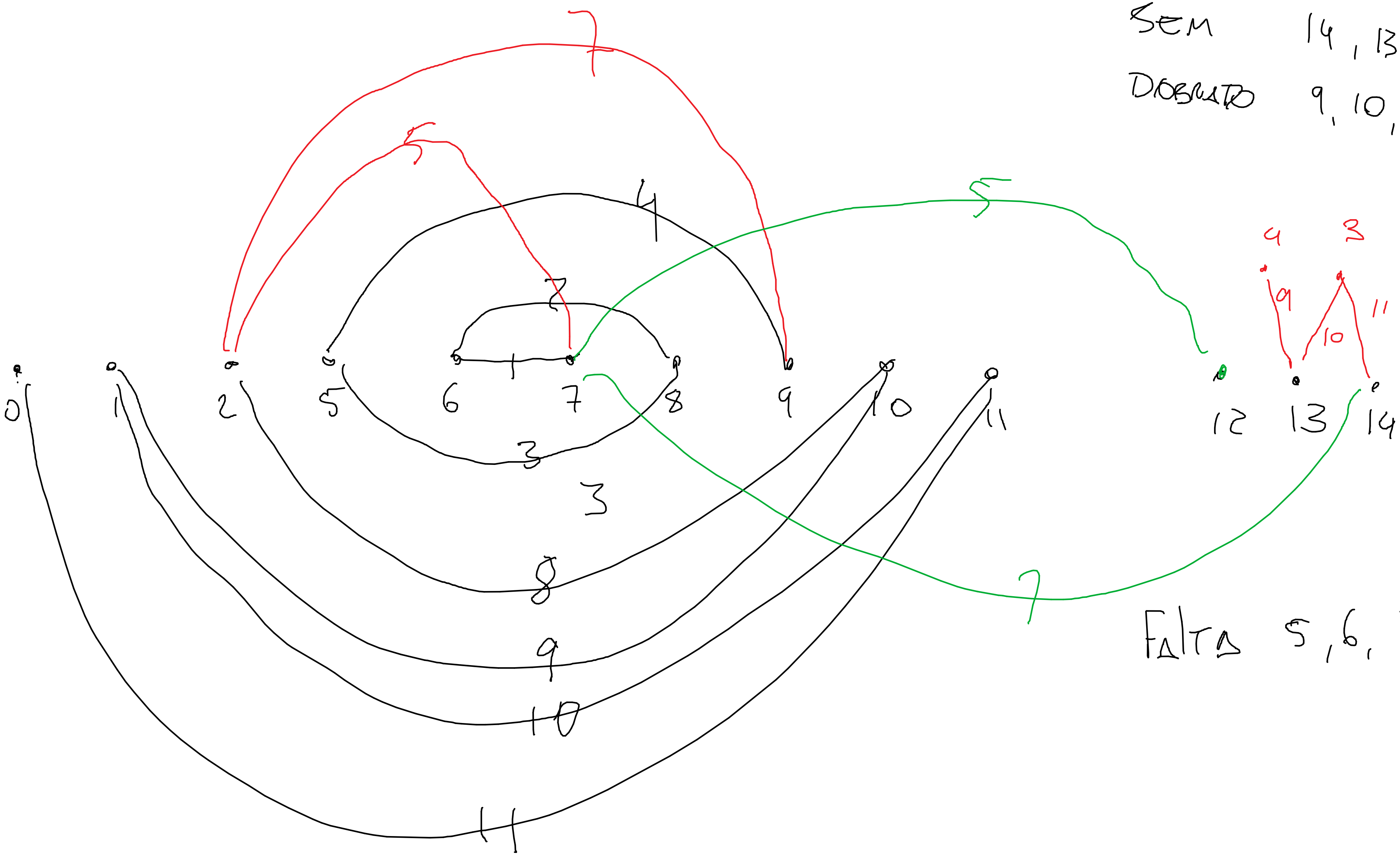


Falta 5 6

$\ell = 3$

SEM 14, 13, 12

DOBRADO 9, 10, 11

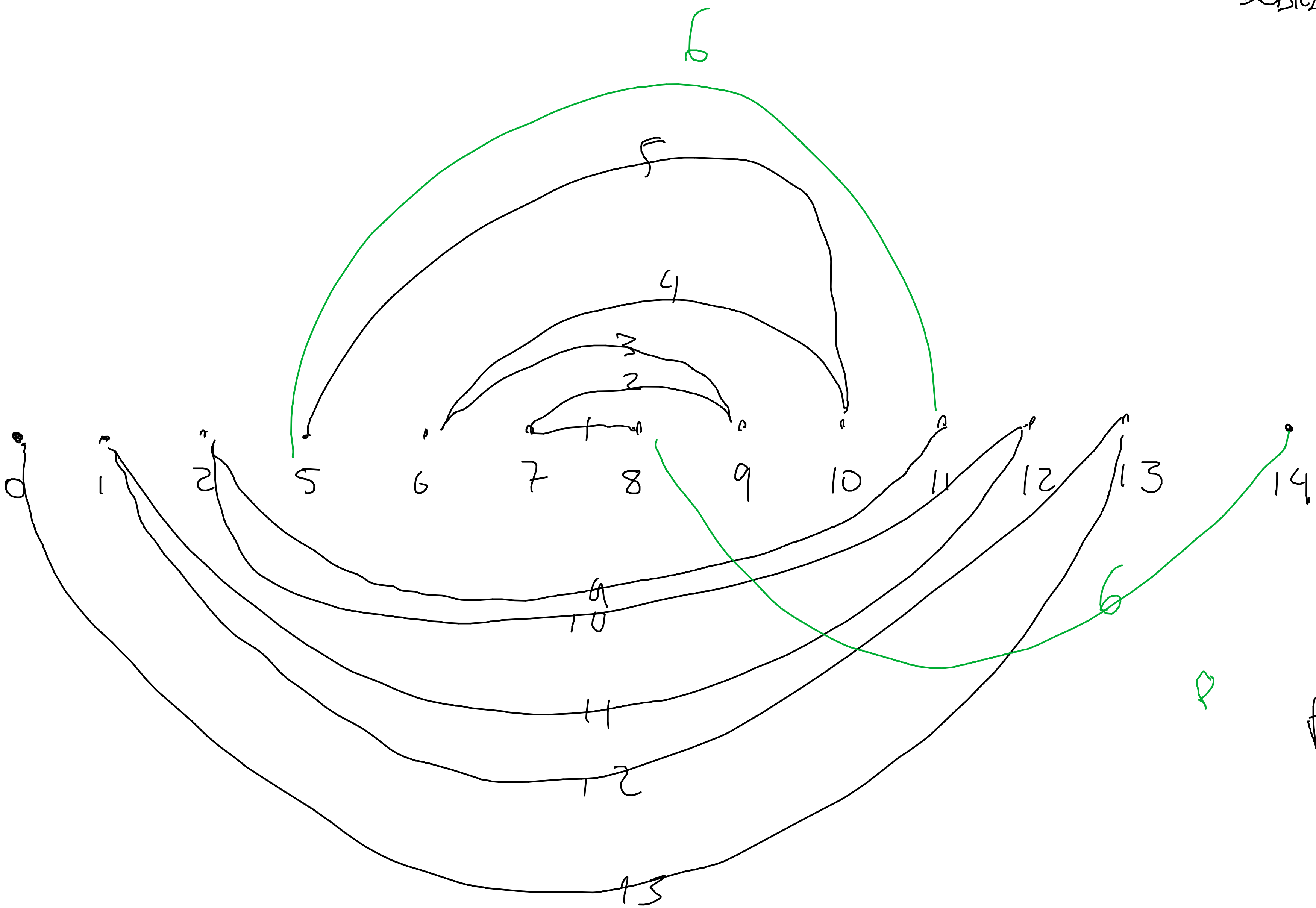


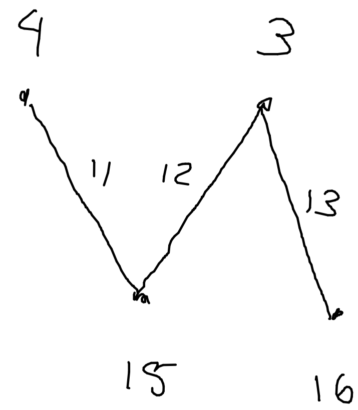
FALTA 5, 6, 7

$\ell = 3$

SEM 14, 15, 16

DOBRA 11 12 13





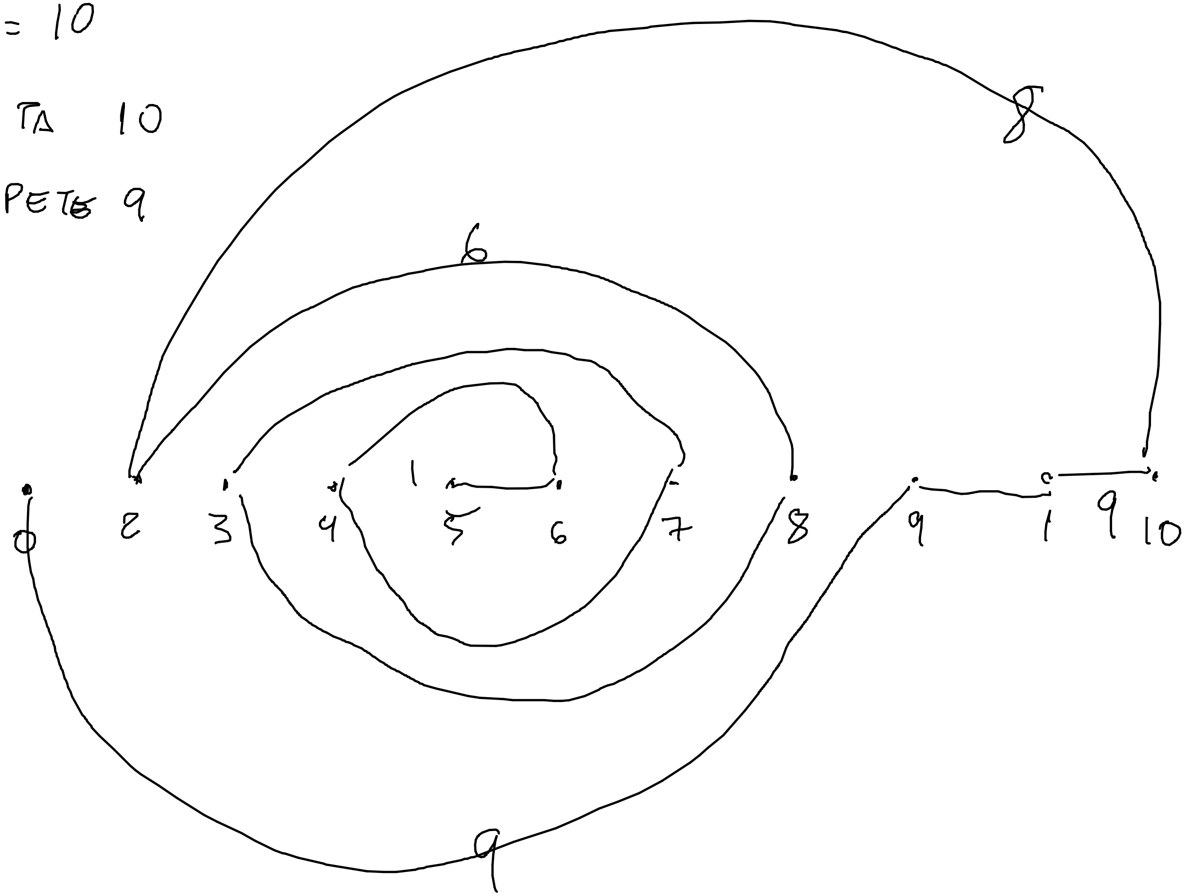
FALTA 6 7 8

$\ell = 1$

$n = 10$

EVITA 10

REPETE 9



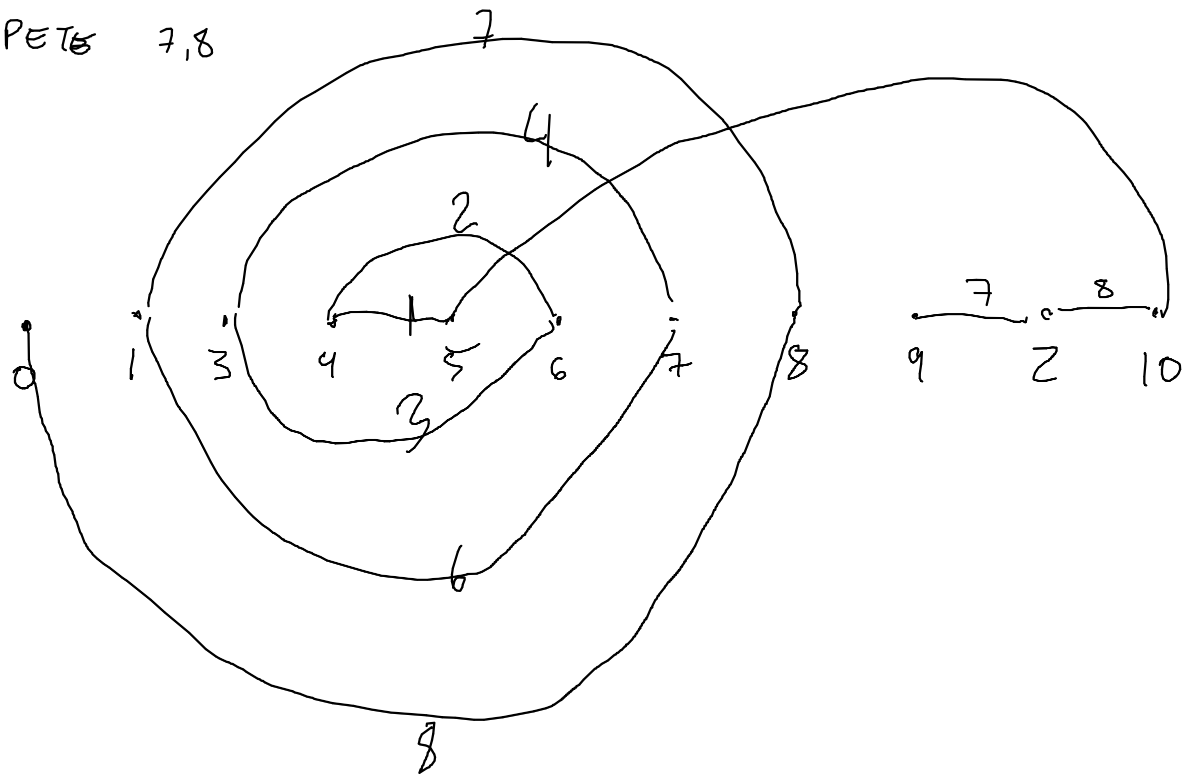
FALTA 7, 8

$\ell = 2$

$n = 10$

EVITA 9, 10

REPETE 7, 8



FALTA 5

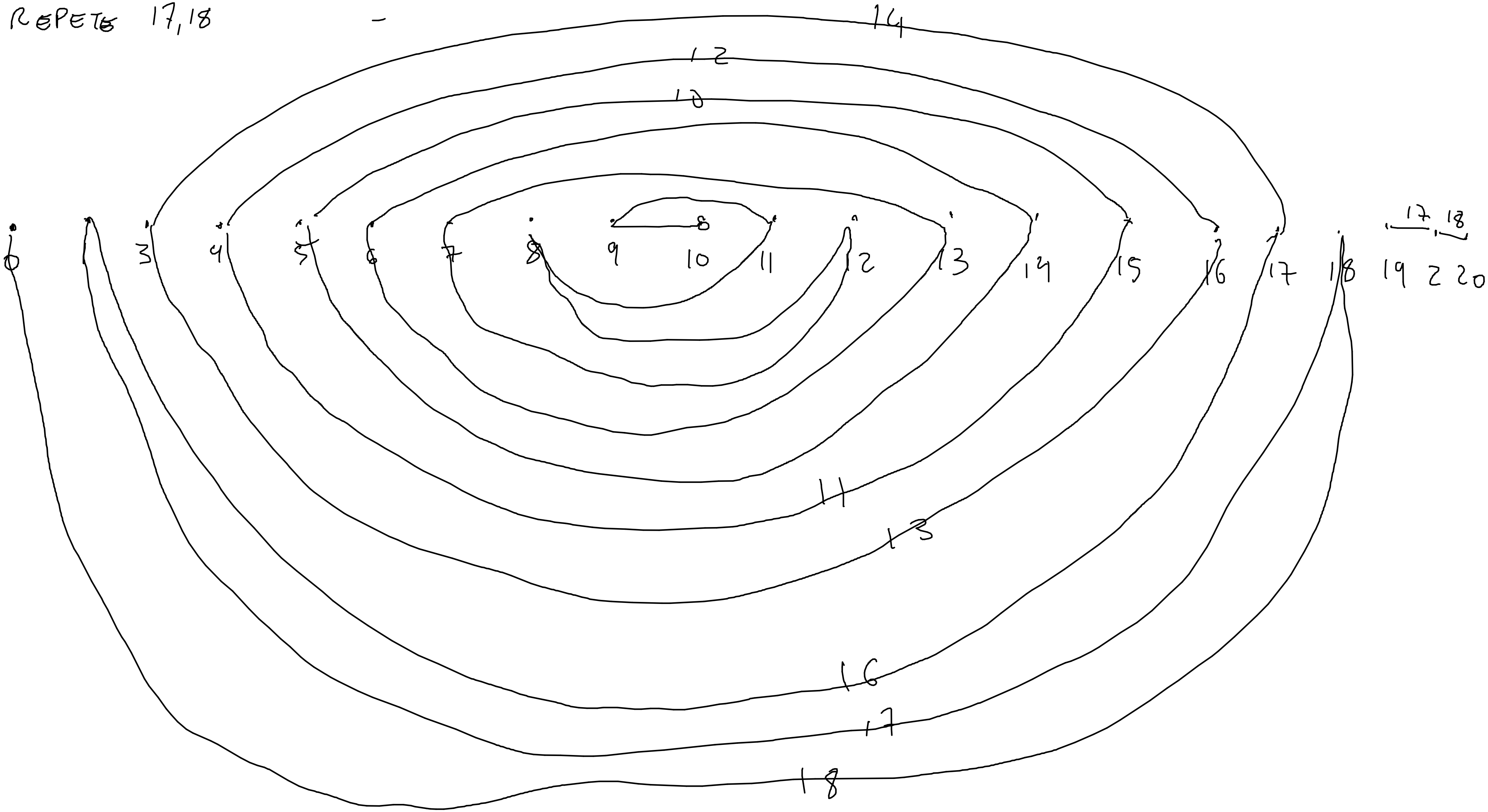
$l = 2$

FALTA 15

$n = 20$

EVITA 19, 20

REPETE 17, 18



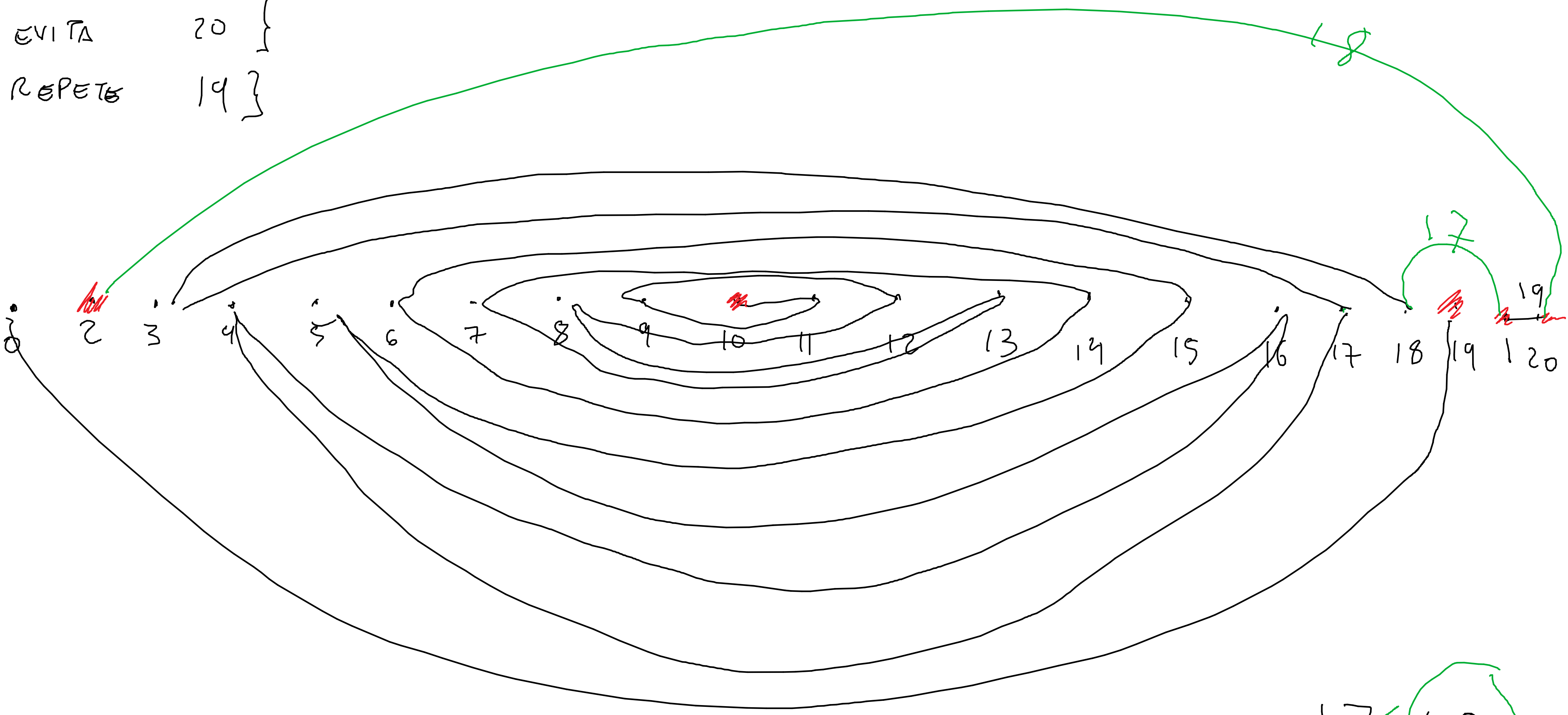
$\ell = 1$

$n = 20$

EVITA 20

REPETE 19

Falta 15



~~17~~, 18

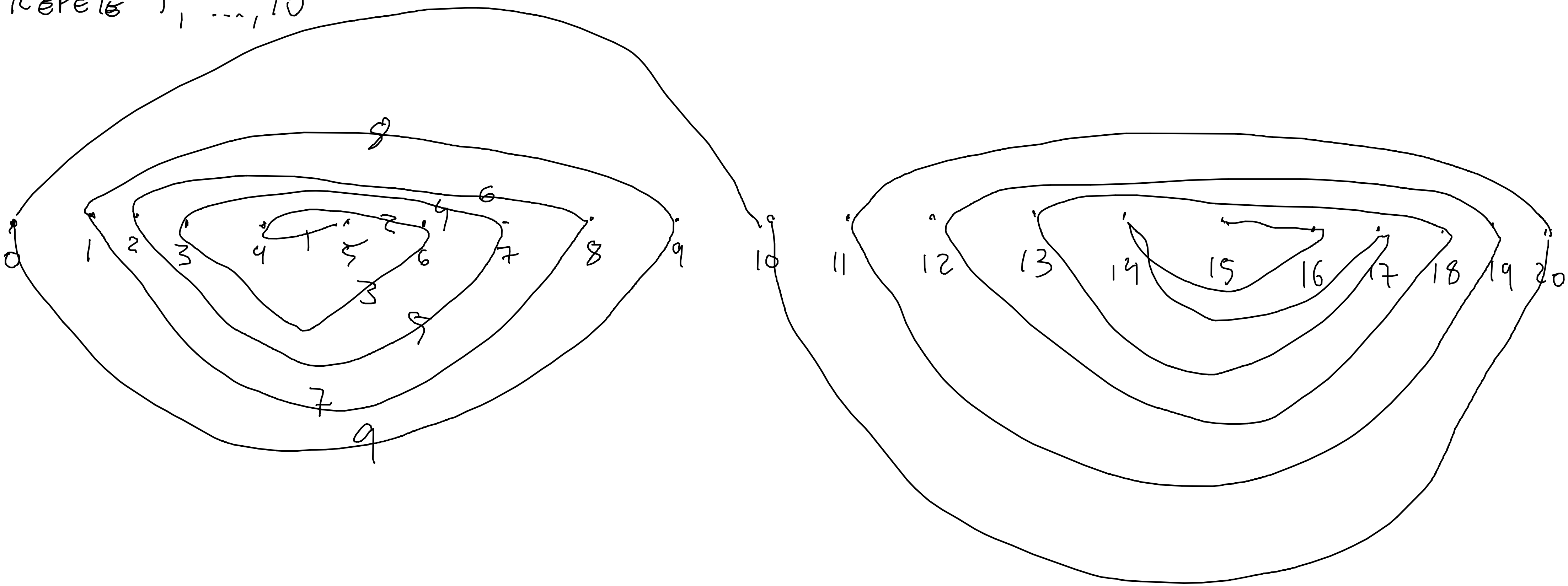
$\ell = 10$

FALTA 15

$n = 20$

EVITA 11, ..., 20

REPETE 1, ..., 10



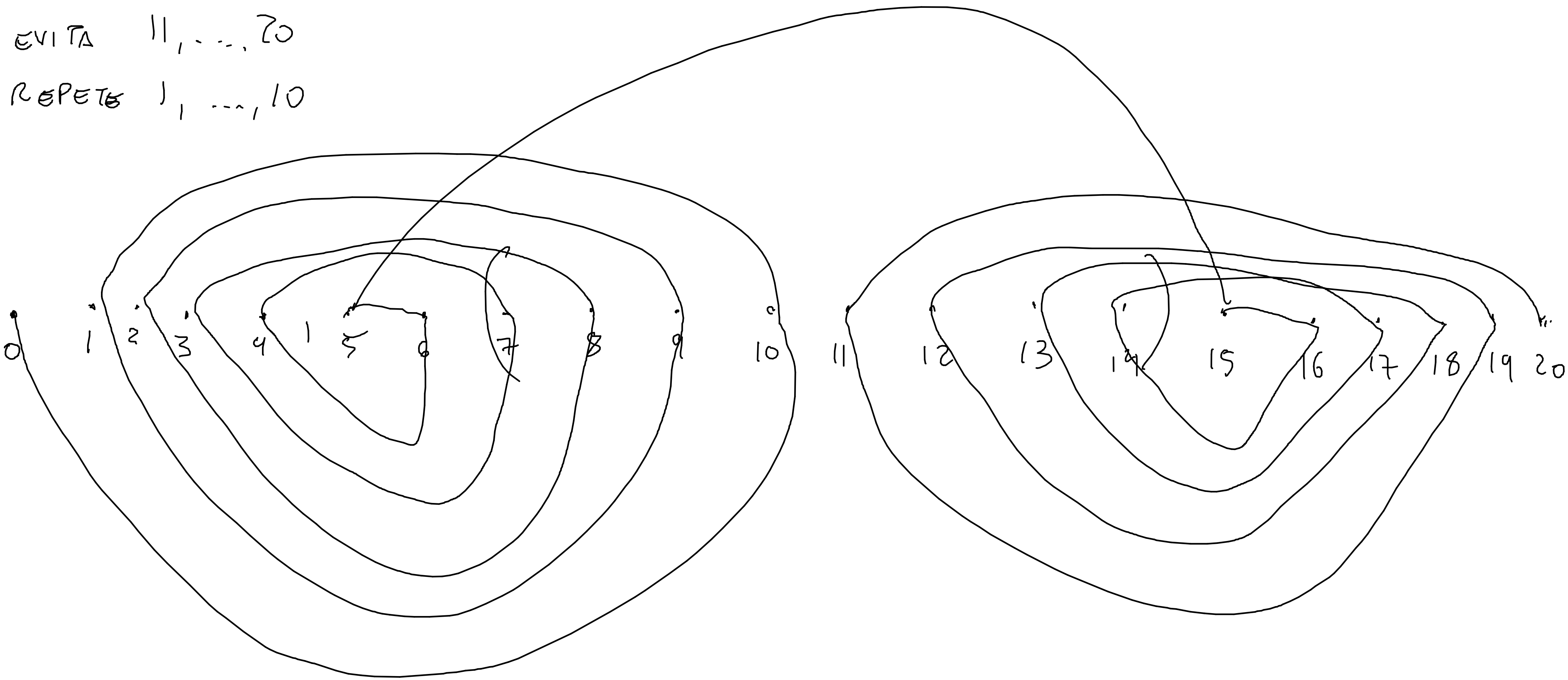
$\ell = 10$

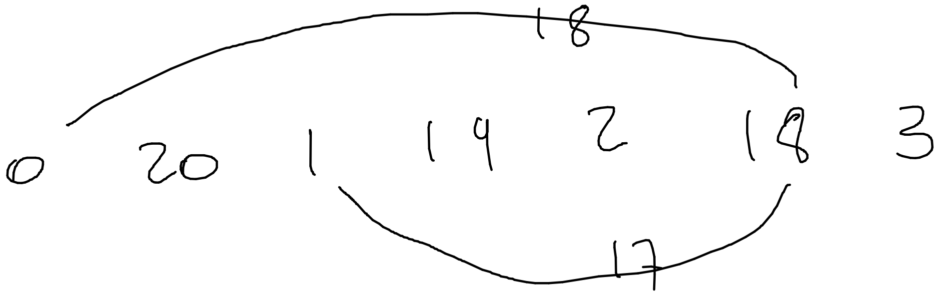
$n = 20$

EVITA $11, \ldots, 20$

REPETE $1, \ldots, 10$

FALTA 15





$\ell = 3$

$n = 20$

EVITA 20 19 18

REPETE 17 16 15

17

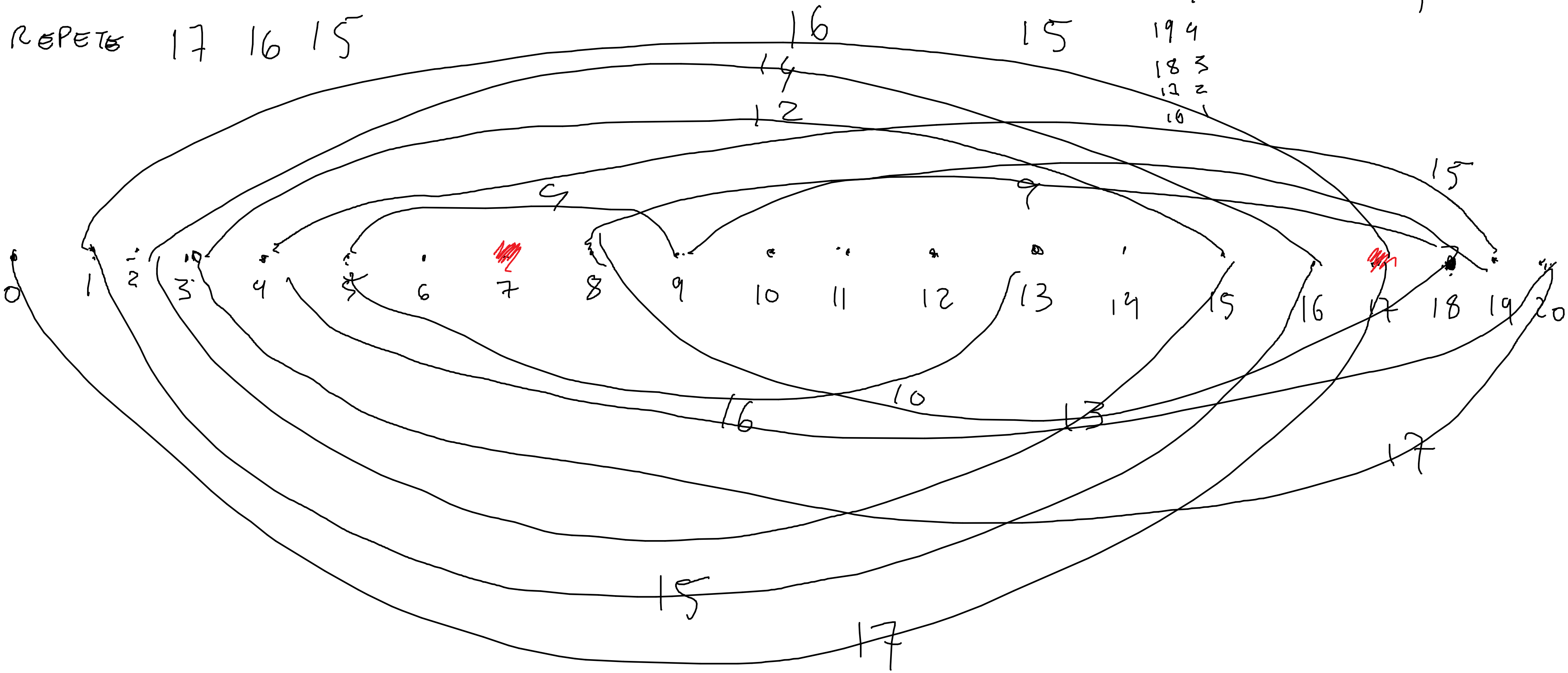
20 3
19 2
18 1

16

20 4
19 3
18 2
17 1

FALTA 15

20 5 — 1
19 4
18 3
17 2
16 1



12

$\ell = 2$

$n = 20$

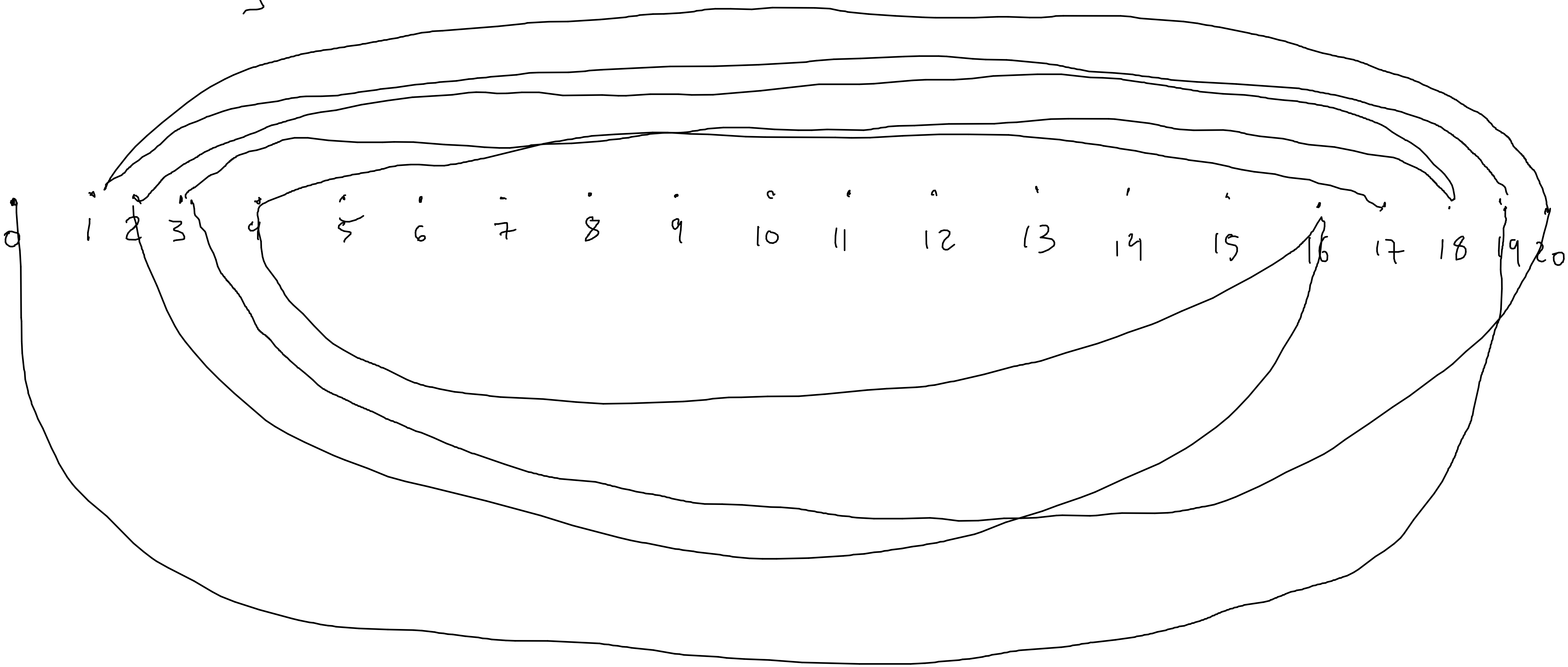
EVITA 19, 20

REPETE 17, 18

FALTA 15

$0\,(20\;\;1\;\;19)\,(2\;\;18\;\;3)$

$0\,(19\;\;1\;\;20)\,(3\;\;18\;\;2)$

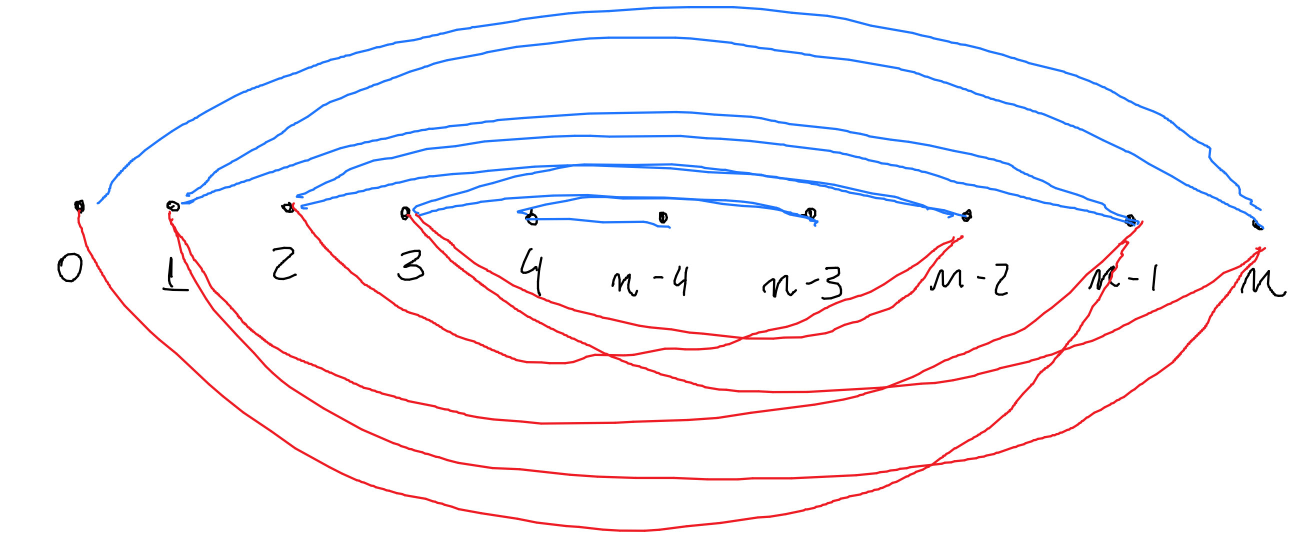


n

$$0\ (n\ \ 1\ \ n-1)^{n-3}\ (2\ \ n-2\ \ 3)^{n-6}\ (n-3\ \ 4\ \ n-4)^{n-9}\ \cdots \qquad n\equiv 0 \pmod 3$$

$n-1$

$$0\ (n-1\ \ 1\ \ n)^{n-3}\ (3\ \ n-2\ \ 2)^{n-6}\ (n-4\ \ 4\ \ n-3)^{n-9}$$



$$0\ (n^{n}\quad 1^{n-1}\quad n-1\quad 2)^{n-4}\ (n-2\quad 3\quad n-3\quad 4)^{n-8}\ (n-4\quad 5\quad n-5\quad 6)$$

$$0\ (2^{2}\quad n-1\quad 1\quad n)^{n-4}\ (4\quad n-3\quad 3\quad n-2)^{n-8}\ (6\quad n-5\quad 5\quad n-4)$$

$f(x) = n - x + 1 \quad : \quad f(n) = 1,\ f(1) = n$

$|f(x) - f(y)| = |n - x + 1 - (n - y + 1)|$

f

$$0\ (n-1^{n-1}\quad 2^{n-3}\quad n^{n-2}\quad 1^{n-1})\ (n-3\quad 4\quad n-2\quad 3) \quad =$$

$$0\ (n-1^{n-1}\quad 1\quad n)^{n-3}\ (3\quad n-2\quad 2)^{n-6}\ (n-4\quad 4\quad n-3)^{n-9}$$

$$\underbrace{\begin{pmatrix} 0 & \tilde{n} & \hat{1} & \hat{n-1} & \tilde{2} & \overline{n-2} & \tilde{3} & n-3 \end{pmatrix}}\ \underbrace{\begin{pmatrix} \end{pmatrix}}$$

n

$$0\begin{pmatrix} \overset{2}{2} & n-1 & 1 & n \end{pmatrix}\begin{pmatrix} \overset{n-4}{4} & n-3 & 3 & n-2 \end{pmatrix}\begin{pmatrix} \overset{n-8}{6} & n-5 & 5 & n-4 \end{pmatrix}$$

$f(x) = n - x + 1 \quad : \quad f(n) = 1 \,,\ f(1) = n$

$|f(x) - f(y)| = |n - x + 1 - (n - y + 1)|$

$$0 \xrightarrow{f} 0\begin{pmatrix} \overset{n-1}{n-1} & \overset{n-3}{2} & \overset{n-2}{n} & \overset{n-1}{1} \end{pmatrix}\begin{pmatrix} n-3 & 4 & n-2 & 3 \end{pmatrix} =$$

$$0\begin{pmatrix} \overset{n-1}{n-1} & 1 & n \end{pmatrix}^{n-3}\begin{pmatrix} 3 & n-2 & 2 \end{pmatrix}^{n-6}\begin{pmatrix} n-4 & 4 & n-3 \end{pmatrix}^{n-9}$$

$$0 \ \overset{n}{(} n \quad 1 \quad n-1 \quad 2 \quad n-2 \,\overset{n-5}{)(}\, 3 \quad n-3 \quad 4 \quad n-4 \quad 5 \,\overset{n-10}{)(}\, n-5 \quad 6 \quad n-6 \quad 7 \quad n-7)$$

$$0 \ \overset{n-2}{(} n-2 \quad 2 \quad n-1 \quad 1 \quad n \,\overset{n-5}{)(}\, 5 \quad n-4 \quad 4 \quad n-3 \quad 3 \,\overset{n-10}{)(}\, n-7 \quad 7 \quad n-6 \quad 6 \quad n-5)$$

$$f(x) = n - x + 1 \quad : \quad \underline{f(n) = 1, \ f(1) = n}$$

$$0 \ \overset{3}{(} 3 \quad n-1 \quad 2 \quad n \quad 1)$$

$$0 \ \overset{n-1}{(} n-1 \quad 1 \quad n \,\overset{n-3}{)(}\, 3 \quad n-2 \quad 2 \,\overset{n-6}{)(}\, n-4 \quad 4 \quad n-3 \overset{n-9}{)}$$

$\boxed{n-1}$ $n-2$ $n-3$ $n-4$ $n-9$

$0\ (n\ (1\ \ n-1\ \ 2)\ \ n-2\ \ 3\ \ n-3\ \ 4\ \ n-4)\ (5\ \dots\ 9)$

$n-4$ $n-3$ $n-2$ $n-9$

$0\ (n-4\ \ 4\ \ n-3\ \ 3\ \ n-2\ \ 1\ \ n-1\ \ 2\ \ n)\ (9\ \dots\ 5)$

$n-6$ $n-5$ $n-2$ $n-1$

$n-4$ $n-6$ $n-5$ $n-4$ $n-3$ $n-6$ $n-5$ $n-4$ $n-3$ $n-$

$0\ (n-4\ \ 2\ \ n-3\ \ 1\ \ n-2\ \ 4\ \ n-1\ \ 3\ \ n)$

$n-3$ $n-8$ $n-7$ $n-6$ $n-5$ $n-4$ $n-3$ $n-6$ $n-10$ $n-11$ $n-12$ $n-3$ $n-14$

$0\ (n-3\ \ 5\ \ n-2\ \ 4\ \ n-1\ \ 3\ \ n)\ (6\ \ n-4\ \ 7\ \ n-5\ \ 8\ \ n-6$

$1\ \dots\ K\ \ K+1\ \dots\ K+r\ \ K+r+1\ \dots\ 2K+r$

$n-1$ $n-3$ $n-6$ $n-9$

$0\ (n-1\ \ 1\ \ n)\ (3\ \ n-2\ \ 2)\ (n-4\ \ 4\ \ n-3)$

$\ell = 2 \cdot L = 6$

0 n 1 $n-1$ 2 $n-2$ 3 $n-3$ 4 $n-4$ 5 $n-5$ 6 n-6

$n-7$ $n-6$ $n-7$ $n-6$

$n-6$

0 $n-6$ $n-5$ $n-4$ $n-3$ 4 $n-2$ 5 $n-1$ 6 n

-11 5 6
-10 4 5 6
-9 3 4 5 ~~6~~
-8 2 3 4 ~~5~~ 6
-7 1 2 3 4 ~~5~~ 6
-6 1 2 3 4 5 6

n 6 $n-1$ 7 $n-2$

1 ... 5

$n-10$ $n-11$ $n-10$ $n-11$

$n-5$ $n-6$ $n-7$ $n-8$ 2 $n-9$ 1 $n-10$

EXCLUI	DOBRA
n	$n-6$
$n-1$	$n-7$
$n-2$	$n-8$
$n-3$	$n-9$
$n-4$	$n-10$
$n-5$	$n-11$

$n-5 \quad n-6$

$0 \quad \underbrace{(n \quad 1 \quad n-1 \quad 2 \quad n-2 \quad 3 \quad n-3 \quad 4}\ (n-4 \quad)($

$n \equiv 0 \bmod 2$

$n-2i \qquad\qquad\qquad n-2j-1$

$i \quad (n-i \quad \ldots \quad n-j) \quad j+1$

$n-j-i \qquad\qquad\qquad n-i-j-1$

$i \quad (n-j \qquad\qquad n-i) \quad j+1$

$\ell = 3$

$0 \; (n \quad 1 \quad (n-1 \quad (2 \quad \overset{n-4}{n-2}) \quad 3) \quad n-3 \quad 4 \quad n-4 \quad 5 \quad)$

$0 \; (n-5 \quad \overset{n-4}{5} \quad n-4 \quad 4 \quad n-3 \quad 3 \quad n-2 \quad 2 \quad n-1 \quad 1 \quad n)$

$n-3 \quad n-4 \quad n-5 \quad n-6 \quad n-7 \quad n-8 \quad n-9$

$0 \quad n-3 \quad 1 \quad n-4 \quad 2 \quad n-5 \quad 3 \quad n-6 \quad 4 \quad n-7 \quad 5 \quad 6 \quad 7 \quad \ldots$

$n-x-1 \qquad n-x-i-1$

$n-i \qquad x \qquad n-i-1$

$2(n-i)-x-1$

$n-i-x-1 \qquad n-x-i$

$\frac{n}{2} \qquad \frac{n}{2}+1 \qquad \frac{n}{2}+2$

Exclui	DOBRA
n	$n-3$
$n-1$	$n-4$
$n-2$	$n-5$

n	$n-1$	$n-2$
3	2	1
4	3	2
5	4	3

$n-i-x \qquad n-i-1-x$

$x < n-i,\ n-i-1$

$n-i \qquad x \qquad n-i-1$

$n-i \qquad \underline{2(n-i)-x-1} \qquad n-i-1$

$i+1 \qquad i$

$n-i \qquad x \qquad n-i-1$

$i+1$

$i \qquad i+1$

$n-i \qquad n \qquad n-i-1$

$$2(n-i)-x-1 = n$$

$$2n-2i-x-1 = n$$

$$2i+x+1 = n$$

$$i = \frac{n-x-1}{2}$$

$$n-i-(i+1) = i+1$$

$$n-2 = 3i$$

$n-x-1 \qquad n-x-i-1$

$n-i \qquad x \qquad n-i-1$

$2(n-i)-x-1$

$n-i-x-1 \qquad n-x-i$

$n = 14$ $\quad n-2 = 3i$ $\quad i = 4$

0 14 1 13 2 12 3 11 4 10 5 9 6 8 7

5 4 5 4

0 5 1 13 2 12 3 11 4 10 14 9 6 8 7

4 5

$x < n-i,\ n-i-1$

$i \quad i-1$

$n-i \quad x \quad n-i-1$

$n-2i$

$i+1$

$i-1 \quad i$

$\underbrace{n-i \quad n-1}$ $\quad n-i-1$

$n-i-i = n-2i$

$n-2i = i+1$

$n-1 = 3i \qquad i = \frac{n-1}{3}$

$n-1$

$n-1$ $\quad$ $n-3$ $\quad$ $n-4$

$0 \quad n-1 \quad 2 \quad n-2 \quad 3 \quad \cdots \quad \frac{n}{2}$

$0 \; (n-1 \quad 1 \quad n) \; (3 \quad n-2 \quad 2)$

$0 \; (n-2 \quad 2 \quad n-1 \quad 1 \quad n) \; (5 \qquad 3)$

$n-1 \quad n-2 \quad n-3 \quad n-4 \quad n-5$

$0 \quad n-1 \quad 1 \quad n-2 \quad 2 \quad n-3$

$n-2 \quad n-3 \quad n-4 \quad n-5 \quad n-6$

$0 \quad n-2 \quad 1 \quad n-3 \quad 2 \quad n-4$

$n-2 \quad n-3$

$0 \quad n-2 \quad 1 \quad n-3 \quad 2 \quad n \quad 3$

Exclumos

n

$n-2$

$n-2$

$n-1$

$1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 13 \quad 2 \quad 3 \quad 4$

$0 \quad 1 \quad 3 \quad 6 \quad 10 \quad 15 \quad 2 \quad 4 \quad 7 \quad 11$

$2 \quad 4\,5 \quad 7\,8\,9 \quad 11\,12\,13\,14$

$m-2 \quad m-5 \quad m-6 \quad m-7 \quad m-8 \quad m-13 \quad m-13$

$0 \; (m-2 \quad 1 \quad m-3) \; (2 \quad m-4 \quad 3) \; (m-5 \quad 4 \quad m-6) \; (5 \quad m-7 \quad 6) \; m-7$

$m-9 \quad m-$

$m-2 \quad m-3 \quad m-7 \quad m-8 \quad m-9$

$m \quad m-4 \quad m-5$

$m \quad m-1$

$m-1 \quad m-6$

$m-3 \quad m-5 \quad m-8$

$0 \; (m-3 \quad 1 \quad m-2) \; (3 \quad m-4 \quad 2) \; (m-6 \quad 4 \quad m-5) \; (6 \quad m-7 \quad 5)$

$\frac{m}{3} \quad \frac{m}{3}-1 \quad \frac{m}{3}-2$

$0 \quad \frac{m}{3} \quad 1 \quad \frac{m}{3}-1 \quad 2 \quad \ldots$

$n-1 \quad n-2 \quad n-3 \quad n-4 \quad n-5$

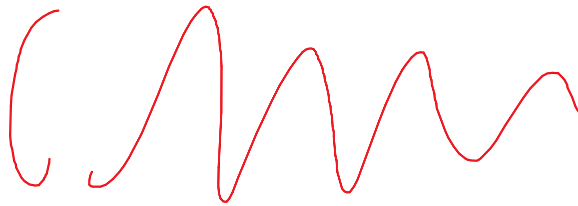
$0 \quad n-1 \quad 1 \quad n-2 \quad 2 \quad n-3 \quad 3 \quad n-4 \quad 4 \quad n-5 \quad 5 \quad n-6 \quad 6$

$n-1 \quad n-2 \quad n-3 \quad n-4 \quad n-2 \quad n-3 \quad n-6$

$0 \quad n-1 \quad 1 \quad n-2 \quad 2 \quad n \quad 3 \quad n-3 \quad 4 \quad n-4$

$0 \quad n+1 \quad 1 \quad n \quad 2 \quad n-1 \quad 3$

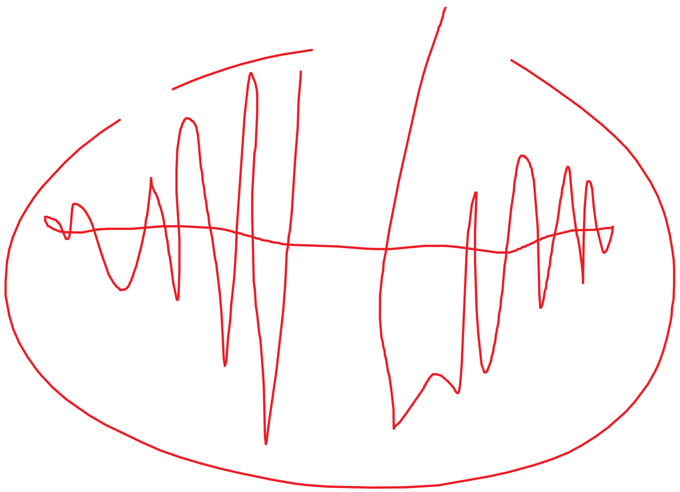
$0\ \underbrace{(n \qquad n-i)\ (i+1 \qquad j)}$



0 7 1 6 2 5 3 4

  4 3 4 3 4 3

0 4 1 5 2 6 3



7 6 5 4 3 2 1 7 1 2 3 4 5 6

0 7 1 6 2 5 3 4 11 12 10 13 9 14 8

$n-5 \quad n-6 \quad n-7 \quad n-8$

$0 \quad n'+7 \quad 1 \quad n'+6 \quad 2 \quad n'+5$

$0 \; (n \;\; 1 \;\; n-1 \;\; 2 \;\; n-2) \; (3 \;\; n-3 \;\; 4 \;\; n-4 \;\; 5) \; (n-5$

$n-10$

$n-3 \quad n-4 \quad n-5 \quad n-6 \quad n-4 \quad n-5 \quad n-3 \quad n-4 \quad n-5 \quad n-6$

$0 \; (n-3 \;\; 1 \;\; n-4 \; (2 \;\; n-2) \; (3 \;\; n \;\; 4 \;\; n-1 \;\; 5)) \; (n-5 \; \ldots$

$\cancel{n-9} \quad \cancel{n-8} \quad \cancel{n-7}$

$n = n' + 14$

$n-10$

$0 \; (\qquad 2) \; (n-8$

n- n- n- n- n-1

○ (n-3 1 n-4 (2 n-2) (3 n 4 n-1 5

Exclui

n

n-1

n-2

2

Dobrar

n-3

n-4

n-5

○

n-3 (n,3) (n-1,2) (n-2,1) (n-3,0)

n-4 (n,4) (n-1,3) (n-2,2) (n-3,1) (n-4,0)

n-5 (n,5) (n-4,1)

2x

n-6

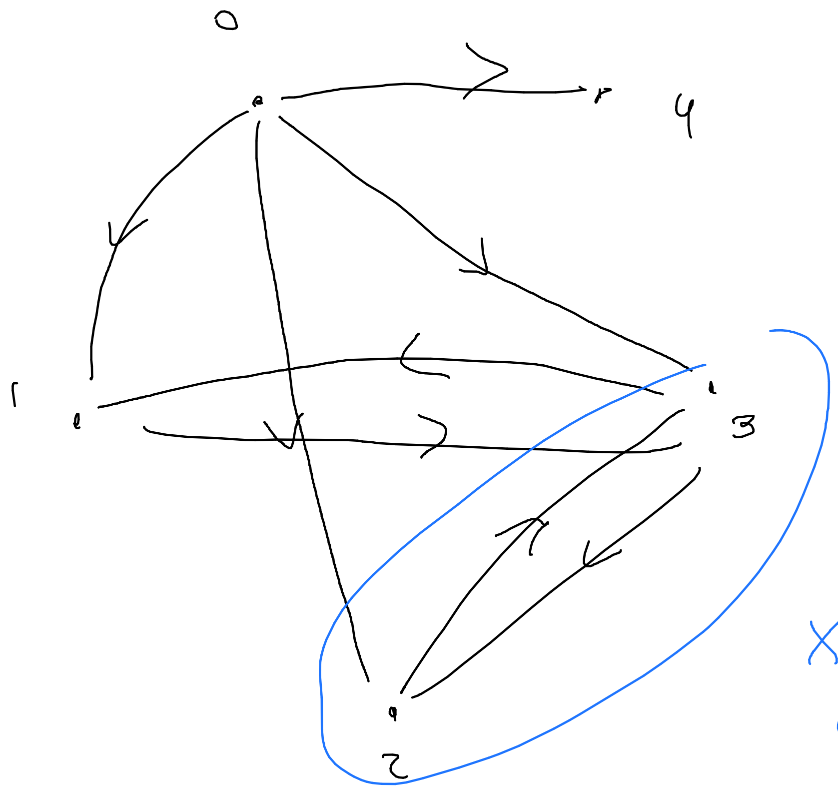
n-7

n-8

n-9

1x

$n = 4$



$(\vee \vee) \wedge (\vee \vee) \wedge (\vee \vee)$

$(x_{2,3} \vee x_{3,2})$

$x_{2,3} + x_{3,2} \leq 1$

0	0
1	0
0	1
1	1

F

$(\overline{x_{2,3}} \vee \overline{x_{3,2}})$

SE TENHO VARIÁVEIS $x_1, \dots, x_n$ E GOSTARIA QUE NO MÁXIMO k DELAS SEJAM SATISFEITAS

$$\bigwedge_{S \subseteq \binom{[n]}{k+1}} \left(\bigvee_{i \in S} \overline{x_i} \right)$$

$$\begin{array}{ccc} x_1 & x_2 & x_3 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{array}$$

T (primeiras quatro linhas)

F (últimas quatro linhas)

EX: $k = 1$

$$\bigwedge_{(i,j) \in \binom{[n]}{2}} (\overline{x_i} \vee \overline{x_j})$$

$$(\overline{x_1} \vee \overline{x_2} \vee \overline{x_3})$$

$$(\overline{x_1} \vee \overline{x_2}) \wedge (\overline{x_1} \vee \overline{x_3}) \wedge (\overline{x_2} \vee \overline{x_3})$$

$$\bigwedge_{(a,b) \in \binom{[n]}{k}}$$

$[n] = \{1, \dots, n\}$

SUBCONJUNTOS DE $[n]$ COM k ELEMENTOS

$$(\overline{x_1} \vee \overline{x_2} \vee \overline{x_3})$$

u TEM GRAU ≥ 1 : $\left(\bigvee_{e \in \delta(u)} \overset{0}{x_e}\right)$ $\Bigg\}$ 0 $\Big\{ \forall u \neq 0$

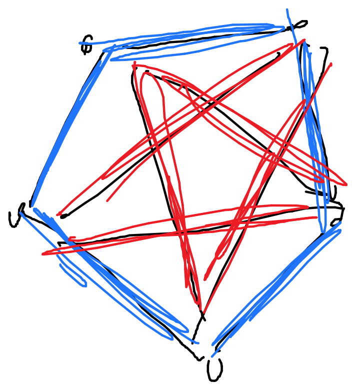
u TEM GRAU ≤ 1 : $\bigwedge_{\{e,f\} \subseteq \binom{\delta(u)}{2}} \left(\underset{\substack{1\\0}}{\bar{x}_e} \vee \underset{\substack{1\\0}}{\bar{x}_f}\right)$ $\quad | \quad |$

u TEM GRAU ≤ 2 $\bigwedge_{\{e,f,g\} \subseteq \binom{\delta(u)}{3}} \left(\bar{x}_e \vee \bar{x}_f \vee \bar{x}_g\right)$ $\Big\{ \forall u \neq 0$

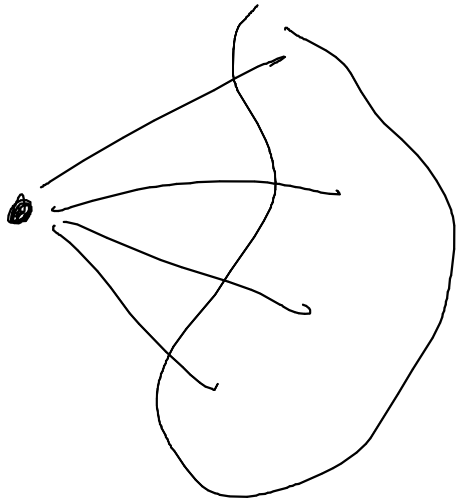
PARA $H \subseteq G$ SER CONEXO, PRECIO $\forall u$ UM CAMINHO $P_{0u} \subseteq H$

$\forall u, \forall e$, $\underbrace{y_{u,e} \leq}_{\{0,1\}} x_e$

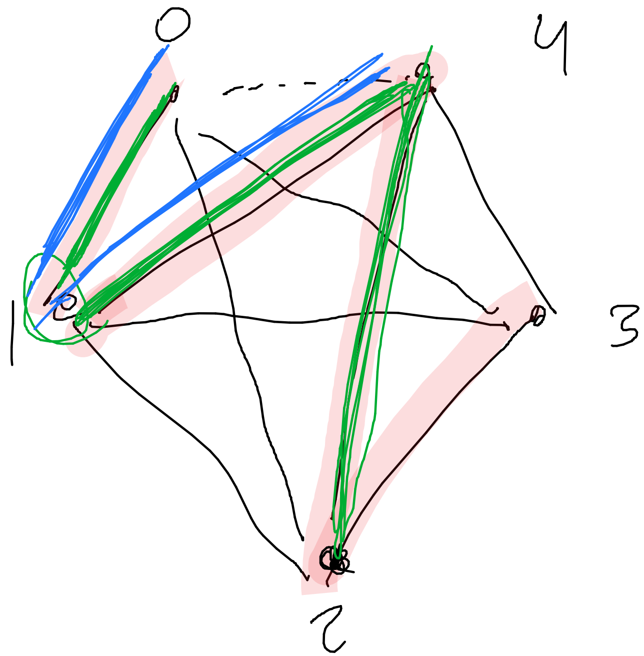
K_5



É possível color



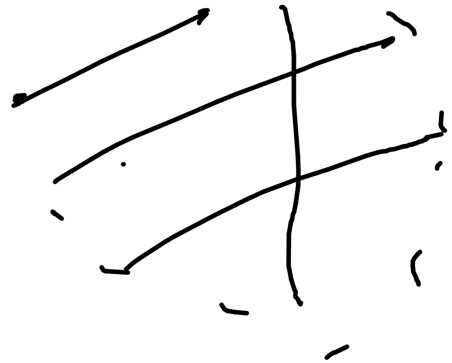
$n = 4 \qquad l = 1$



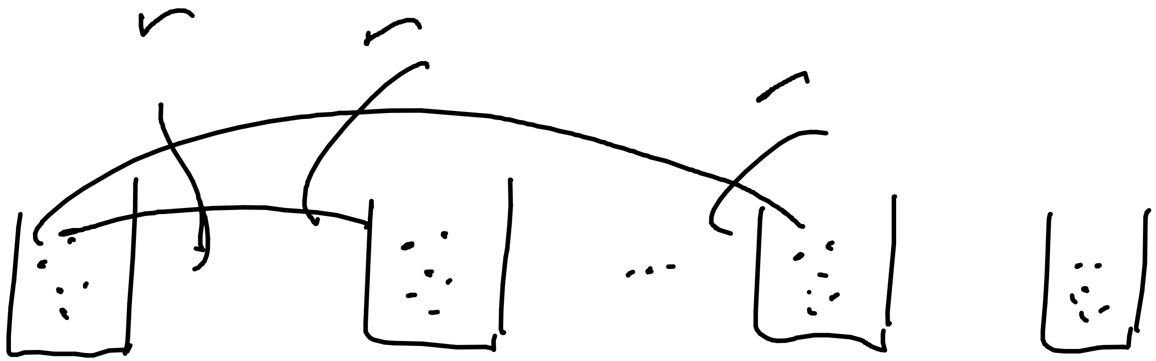
G É r-REGULAR COM n VÉRTICES

0 1 1 0 0 1

$e(G) = \frac{n \cdot r}{2}$



$n \cdot r$ PONTOS



1

n

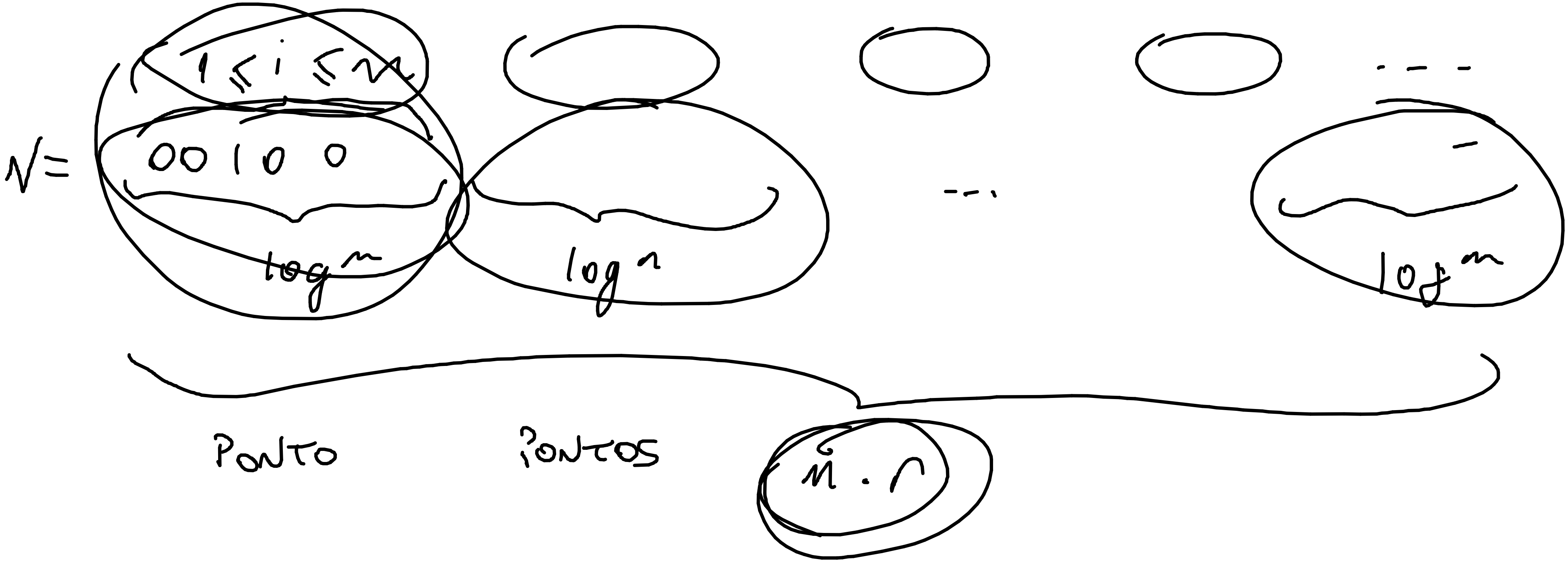
$n \cdot r$

2

$\log\left(\left(\frac{n \cdot r}{2}\right)!\right)$

$\frac{(n \cdot r)!}{\left(\frac{n \cdot r}{2}\right)!}$

$$\boxed{|V| = n \cdot r \cdot \lceil \log_2 n \rceil} = O(n \log n) \leq O(n^2)$$



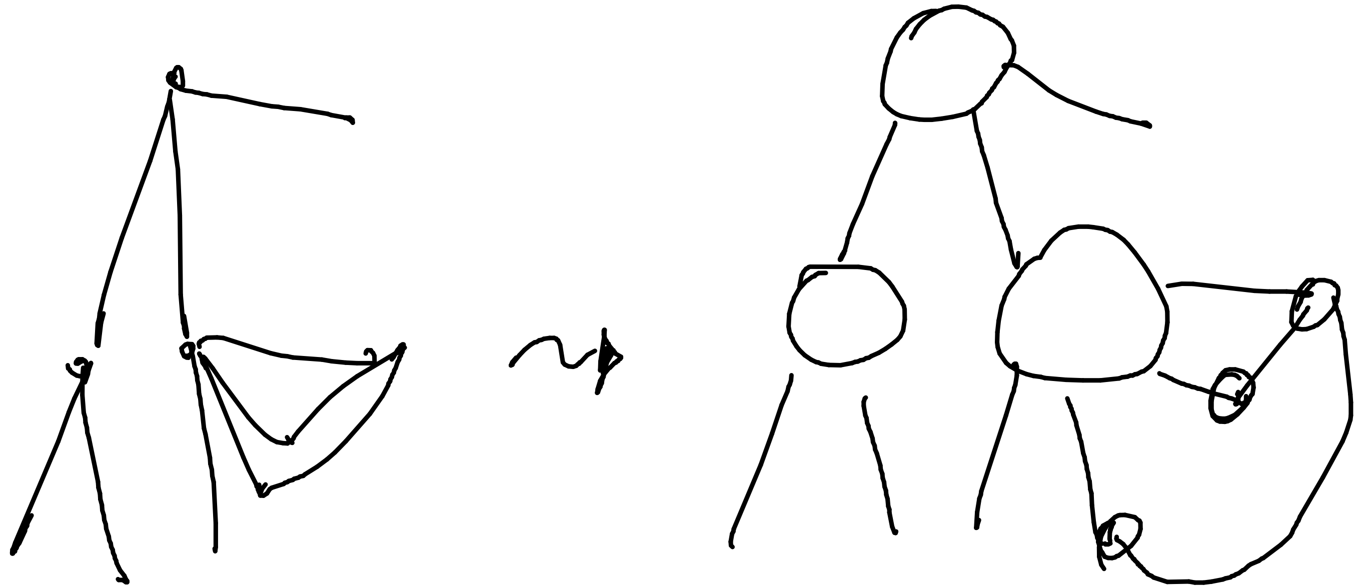
12 14 24

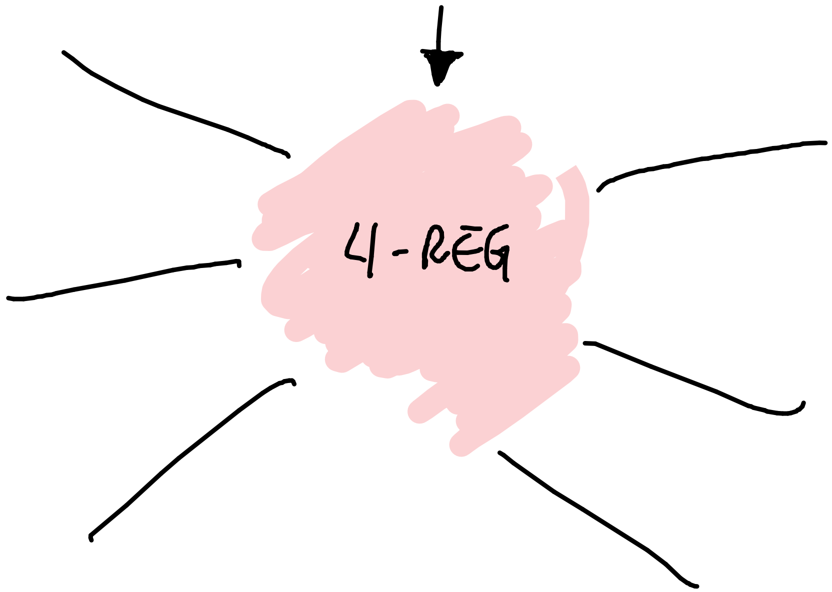
0 1 0 0 0 1

13 23 34

$\binom{4}{2}$

H




4-REG

GRAFOS 5-REGULAR $d(x) = 5 \quad \forall x \in V(G)$

EMPARELHAMENTO PERFEITO:

1) $M \subseteq E(G)$ T.q. $d_M(x) = 1 \quad \forall x \in V(G)$

OU SEJA UM SUBGRAFO 1-REGULAR GERADOR

2) $M \subseteq E(G)$ É UM CONJUNTO COM $\frac{n}{2}$ ARESTAS INDEPENDENTES

$$\underset{\{x,y\}}{e} \cap \underset{\{u,v\}}{f} = \emptyset$$

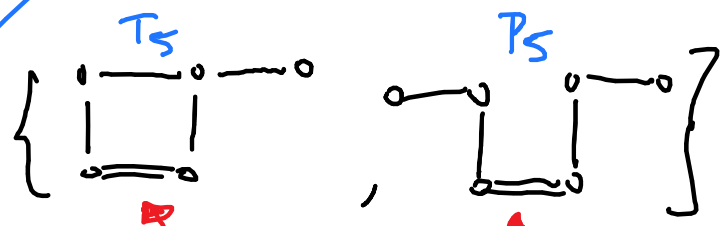
DEF: UMA P_5-DECOMPOSIÇÃO DE G É UMA FAMÍLIA $\mathcal{D} = \{H_1, \ldots, H_k\}$ DE SUBGRAFOS DE G T.q.

1. $H_i \approx P_5$
2. $E(H_i) \cap E(H_j) = \emptyset$
3. $E(G) = \bigcup_i E(H_i)$

$P_5 =$ 

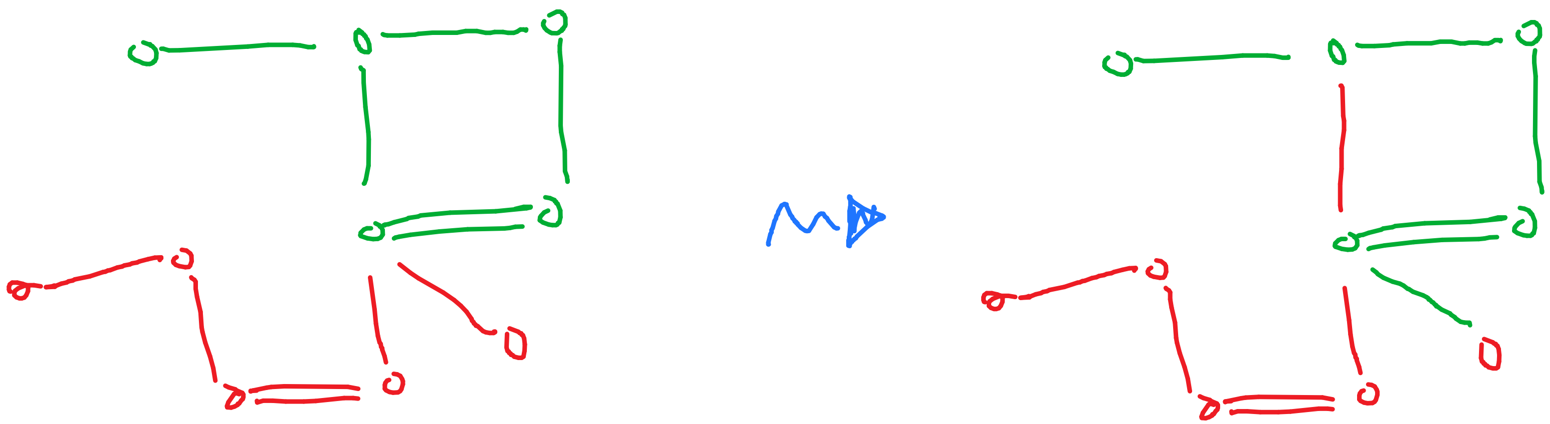
Conjectura: Se G é 5-regular e contém um emp. perf., então G admite uma P_5-decomposição

É possível encontrar é uma



-decom

Arestas do emp. perfeito



INSTÂNCIAS : DADO n , COMPUTA $N = \binom{n}{2}$

DIGAMOS 20 $\longrightarrow$ 190

VETOR BINÁRIO DE COMP. N

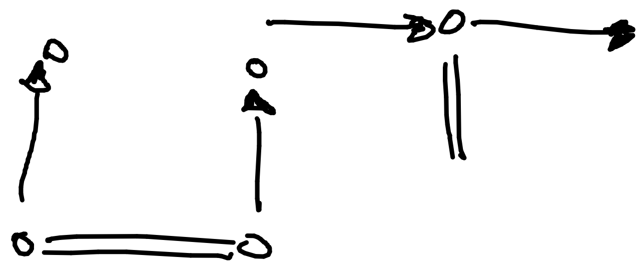
P_5 –

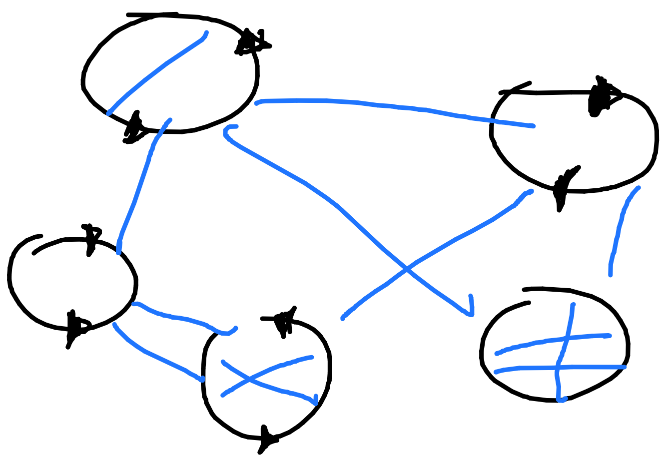
$P_2 \ldots P_K$ – DEC.

TEOREMA: SEJA G UM GRAFO 3-REGULAR.
ENTÃO G ADMITE P_3-DECOMP. SE E SÓ SE G POSSUI EMP. PERFEITO

PROVA: $G' = G - M$ É 2-REGULAR

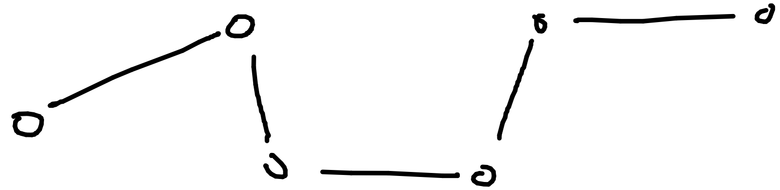
EMP. PERF.





DEF: UMA P_5-~~DECOMPOSIÇÃO~~ EMPACOTAMENTO DE G É UMA FAMÍLIA $\mathcal{D} = \{H_1, \dots, H_k\}$ DE SUBGRAFOS DE G T.Q.

1. $H_i \approx P_5$
2. $E(H_i) \cap E(H_j) = \emptyset$
3. ~~$E(G) = \bigcup_i E(H_i)$~~

$P_5 =$ 

G = GRAFO QUALQUER

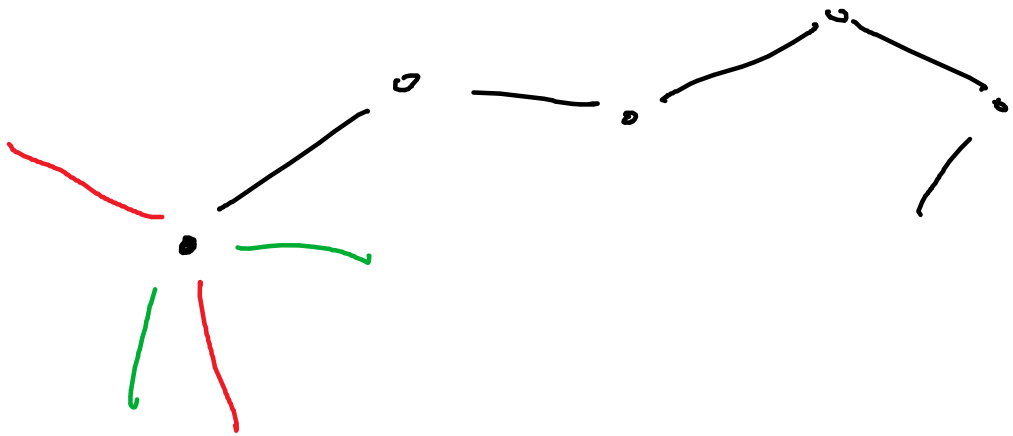
1. MINIMIZAR O TAMANHO DO MAIOR P_5-EMPACOTAMENTO $< \frac{m}{2}$
2. MINIMIZAR O NÚMERO DE VÉRTICES COM GRAU $\neq 5$ $= 0$

PARA CADA VTX x, DEFEITO $(x) = |5 - d(x)|$. QUEREMOS MINIMIZAR

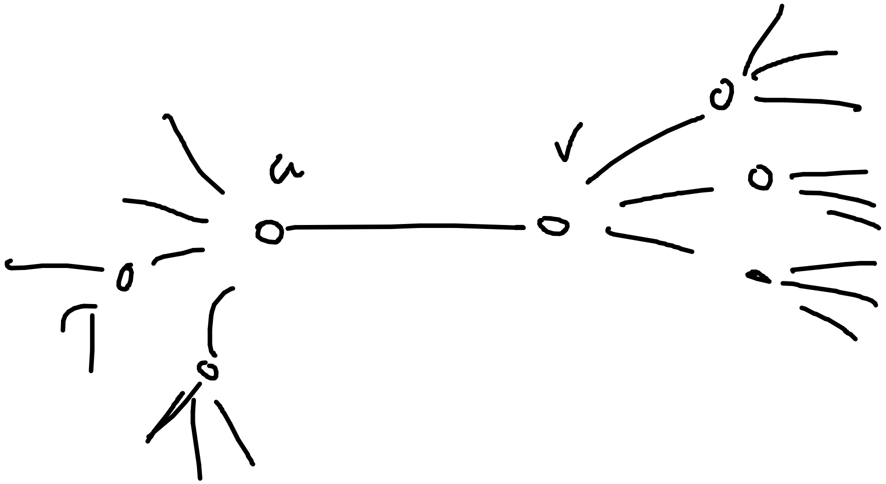
$$|E(G)| = \frac{1}{2}\sum d(v) = \frac{5n}{2}$$

$$\frac{5n}{2} = |E(G)| = \sum_{P \in D} |E(P)| = 5|D|$$

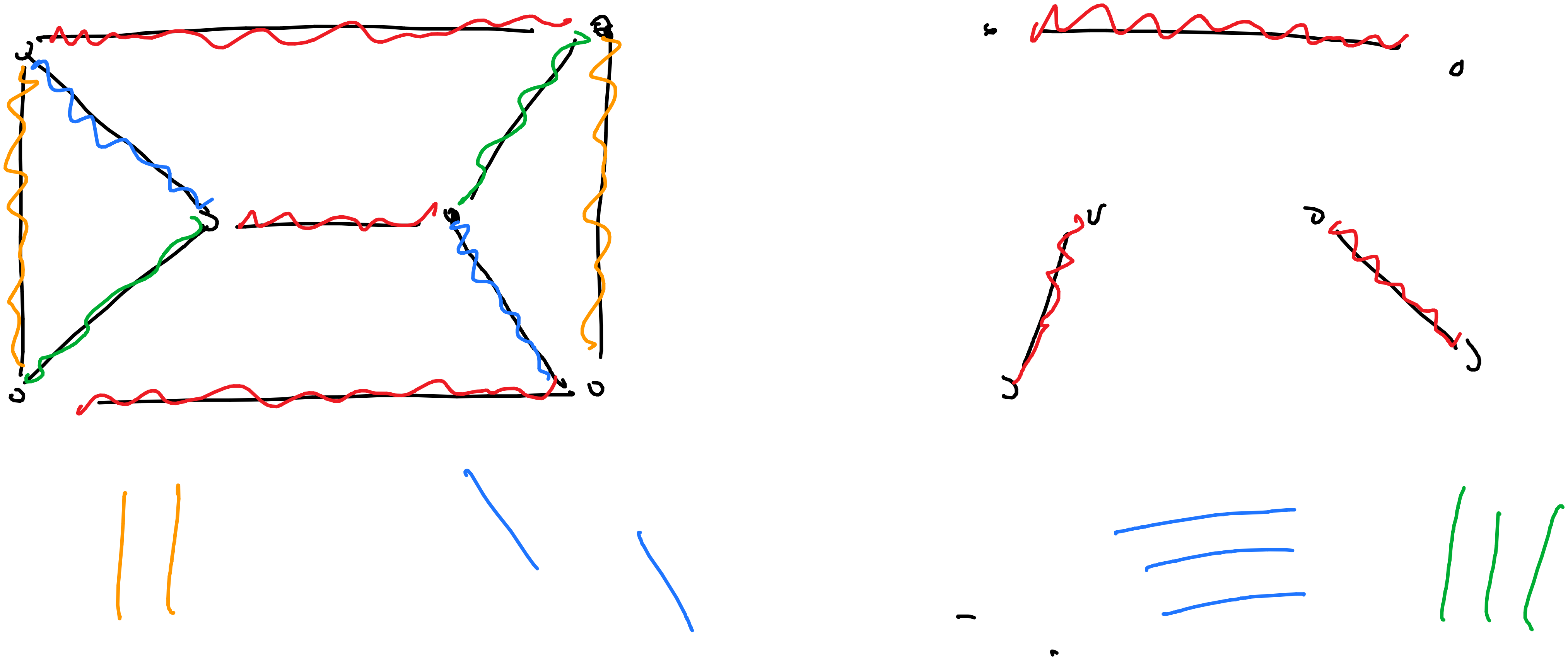
$$|D| = \frac{n}{2}$$



FOR EACH $e \in E(G)$:



EMPARELHAMENTO



Coloração própria = coloração na qual cada cor induz um emparelhamento

= coloração na qual cada cor induz uma floresta na qual cada componente é um caminho de comprimento 1

Linear arboricity = coloração na qual cada cor induz uma floresta na qual cada componente é um caminho ~~de comprimento 1~~

Coloração Própria

$$\chi'(G) \leqslant \Delta(G) + 1$$

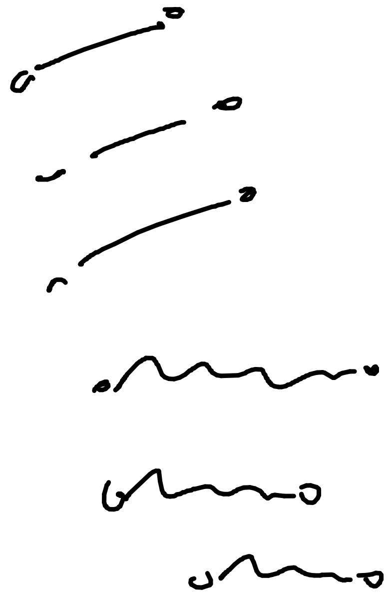
Teo. Vizing.

Conjectura:

$$\chi'_{la}(G) \leqslant \frac{\Delta(G)}{2} + 1$$

$\parallel$

Menor número de cores em uma coloração na qual cada cor induz uma floresta linear



PROPOSTA :

→ TESTAR A CONJECTURA PARA GRAFOS COM ATÉ 11 VTXS

→ ESCREVER UM P.L.I. :

USAR CALLBACK

- VARIÁVEL $x_{e,c} = \begin{cases} 1 & \text{SE } e \text{ ESTÁ COLORIDA COM } c \\ 0 & \text{CASO CONTRÁRIO} \end{cases}$ $\forall e \in E(G)$, $\forall c \in \mathcal{C}$

  $\mathcal{C}$ → CONJ. DE CORES

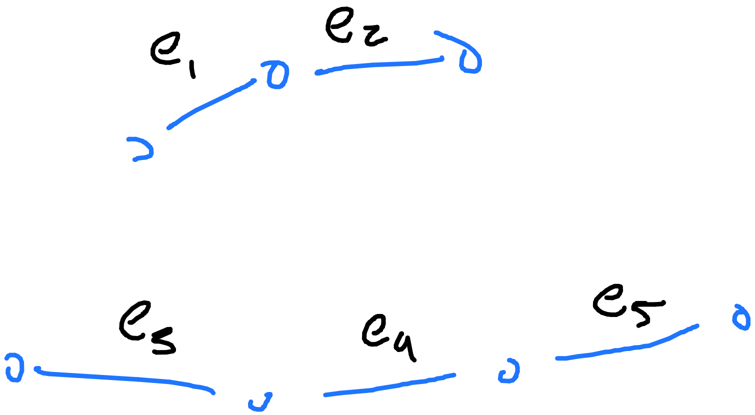
- CADA ARESTA RECEBE EXATAMENTE UMA COR : $\sum_{c \in \mathcal{C}} x_{e,c} = 1 \quad \forall e \in E(G)$

COL. PRÓPRIA : $d_c(u) \leq 1$
↳ GRAU DE u NA COR c

ARB. LINEAR : $d_c(u) \leq 2$
↳ GRAU DE u NA COR c

⋮

Callback



$$\sum_{e \in S} x_{e,c} \leqslant 4 \quad \forall c \in \mathcal{C} \text{ e } \forall S \subseteq E_5(G)$$

S não é um caminho

$$x_{e_1,1} + \cdots + x_{e_5,1} \leqslant 4$$

$$x_{e_1,2} + \cdots + x_{e_5,2} \leqslant 4$$

$$\vdots$$

EXERCÍCIO:

- ESCREVER AS RESTRIÇÕES $\left(\text{EX: } \sum_{c \in \mathcal{C}} x_{e,c} = 1 \quad \forall e \in E(G)\right)$
- ESSAS RESTRIÇÕES SÃO SUFICIENTES?
  OU SEJA, UMA SOLUÇÃO QUE RESPEITE TODAS AS RESTRIÇÕES SATISFAZ NOSSO PROBLEMA?
- QUAIS OS RESTRIÇÕES VÃO PRO CALLBACK?
- PROGRAMAR (ORGANIZADO)
- TESTAR EM TODOS OS GRAFOS
- FAZER FUNCIONAR O GUROBI