University of Victoria
Department of Computer Science
COMPUTER SCIENCE 355

Fall 2023 - ASSIGNMENT 1
(DUE Monday October 5 by 11:59PM on Brightspace).
Late assignments will not be accepted, so begin early.

Please upload a clear PDF of your solutions on Brightspace. The marker has the right to refuse to mark any assignment considered too messy or illegible. If any of the questions are unclear to you, or it is not obvious what is required, please ask in MS Teams or during office hours. You must show all the steps in your work to receive marks; the final solution only will not get the credit.

- Numbers in square brackets [ ] indicate the marks assigned to the question.

1. [6] Find the complement of the following expressions:
   a. $\bar{A}B + A\bar{B}$
   b. $(\bar{V}W + X)Y + \bar{Z}$
   c. $WX(\bar{Y}Z + Y\bar{Z}) + \bar{W}\bar{X}(\bar{Y} + Z)(Y + \bar{Z})$

a) $\bar{F} = \overline{\bar{A}B + A\bar{B}} = (A+\bar{B})\cdot(\bar{A}+B) = AB + \bar{A}\bar{B}$

b) $\bar{F} = \overline{(\bar{V}W + X)\cdot Y + \bar{Z}} = ((V+\bar{W})\cdot\bar{X} + \bar{Y})\cdot Z$

c) $\bar{F} = \overline{WX(\bar{Y}Z + Y\bar{Z}) + \bar{W}\bar{X}(\bar{Y}+Z)(Y+\bar{Z})}$

$= [\bar{W} + \bar{X} + (Y+\bar{Z})\cdot(\bar{Y}+Z)]\cdot[W + X + Y\bar{Z} + \bar{Y}Z]$

$= (\bar{W} + \bar{X} + YZ + \bar{Y}\bar{Z})\cdot(W + X + Y\bar{Z} + \bar{Y}Z)$

2. [2] Prove that the dual of the exclusive-OR is also its complement

$X \oplus Y = X\bar{Y} + \bar{X}Y$

$\text{Dual}(X \oplus Y) = \text{Dual}(X\bar{Y} + \bar{X}Y)$

$= (X+\bar{Y})\cdot(\bar{X}+Y)$

$= \overline{\overline{(X+\bar{Y})\cdot(\bar{X}+Y)}}$

$= \overline{\bar{X}\cdot Y + X\cdot\bar{Y}} = \overline{X \oplus Y}$

3. [6] Reduce the following Boolean expressions to the indicated number of literals:
   a. $\bar{X}\bar{Y} + XYZ + \bar{X}Y$ *to three literals*
   b. $X + Y(Z + \overline{X+Z})$ *to two literals*
   c. $\bar{W}X(\bar{Z} + \bar{Y}Z) + X(W + \bar{W}YZ)$ *to one literal*

a) $\bar{X}\bar{Y} + XYZ + \bar{X}Y = \bar{X} + XYZ = \bar{X} + YZ$ (Rule 22)

b) $X + Y(Z + \overline{X+Z}) = X + Y\cdot(Z + \bar{X}\cdot\bar{Z}) = X + YZ + \bar{X}Y\bar{Z}$

$= X + Y\bar{Z} + YZ = X + Y$

c) $\bar{W}X\cdot(\bar{Z} + \bar{Y}Z) + X(W + \bar{W}YZ)$

$= \bar{W}X\cdot(\bar{Z} + \bar{Y}) + X\cdot(W + YZ)$

$= \bar{W}X\bar{Z} + \bar{W}X\bar{Y} + XW + XYZ$

$= X\cdot(W + \bar{W}\bar{Z}) + \bar{W}X\bar{Y} + XYZ$

$= XW + X\bar{Z} + \bar{W}X\bar{Y} + XYZ$

$= X(W + \bar{W}\bar{Y}) + X(\bar{Z} + YZ)$

$= XW + X\bar{Y} + X\bar{Z} + XZ$

$= X(Z + \bar{Z} + W + \bar{Y}) = X\cdot(1 + W + \bar{Y}) = \underline{\underline{X}}$

4. [4] Simplify the following Boolean expressions to expressions containing a minimum number of literals. At every step indicate the Boolean Algebra rule used in the simplification.
   a. $F = (C + \bar{D} + \bar{C}\bar{D}).(\bar{B}C + B\bar{C} + BD).(\bar{C} + D)$

$$F = (C + \bar{D} + \bar{C}\bar{D})(\bar{B}C + B\bar{C} + BD)(\bar{C} + D) = (C + \bar{D})(\bar{B}C + B\bar{C} + BD)(\bar{C} + D) \quad 22$$
$$= (CD + \bar{C}\bar{D})(\bar{B}C + B\bar{C} + BD) \quad 11,8,1$$
$$= \bar{B}CD + B\bar{C}\bar{D} + BCD \quad 8,6$$
$$= CD + B\bar{C}\bar{D} \quad 7$$

   b. $F = C\bar{D}\bar{B} + C\bar{D} + B\bar{D}C + B\bar{C}A + \bar{B}D\bar{A}$

$$F = C\bar{D}\,\bar{B} + C\bar{D} + B\bar{D}C + B\bar{C}A + \bar{B}D\bar{A} = C\bar{D} + B\bar{C}A + \bar{B}D\bar{A} \quad 3,7,11$$

5. [2] Given that *A.B=0* and *A+B=1*, use algebraic manipulation to prove that

$$(A + C).(\bar{A} + B).(B + C) = B.C$$

$$(A+C)(\bar{A}+B)(B+C) = (AB+\bar{A}C+BC)\cdot(B+C)$$
$$= AB+\bar{A}C+BC \qquad (\text{Given } AB=0)$$
$$= 0 + C(\bar{A}+B) \qquad (A+B=1)$$
$$= C(\bar{A}+B)\cdot 1$$
$$= C(\bar{A}+B)\cdot(A+B)$$
$$= C(AB+\bar{A}B+B) = BC$$

6. [10] For the Boolean Functions E and F, as given in the following truth table:

X	Y	Z	E	F
0	0	0	0	1
0	0	1	1	0
0	1	0	1	1
0	1	1	0	0
1	0	0	1	1
1	0	1	0	0
1	1	0	1	0
1	1	1	0	1

a. List the minterms and maxterms of each function
b. List the minterms of $\bar{E}$ and $\bar{F}$
c. List the minterms of *E+F* and *E.F*
d. Express *E* and *F* in sum-of-minterms algebraic form
e. Simplify *E* and *F* to expressions with a minimum of literals

X \ YZ	00	01	11	10
0		1		1
1	1			1

E

X \ YZ	00	01	11	10
0	1			1
1	1		1	

F

a) $E = \sum m(1,2,4,6) = \prod M(0,3,5,7)$ | $F = \sum m(0,2,4,7) = \prod M(1,3,5,6)$

b) $\bar{E} = \sum m(0,3,5,7)$ $\qquad \bar{F} = \sum m(1,3,5,6)$

c) $E+F = \sum m(0,1,2,4,6,7)$ $\qquad E \cdot F = \sum m(2,4)$

d) $E = \bar{x}\bar{y}z + \bar{x}y\bar{z} + x\bar{y}\bar{z} + xy\bar{z}$

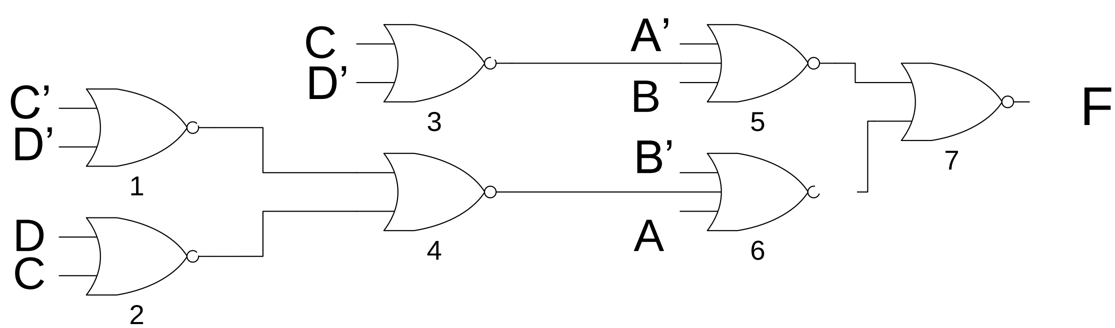
$F = \bar{x}\bar{y}\bar{z} + \bar{x}y\bar{z} + x\bar{y}\bar{z} + xyz$

e) $E = x\bar{z} + y\bar{z} + \bar{x}\bar{y}z$

$F = \bar{x}\bar{z} + \bar{y}\bar{z} + xyz$

7. [6] For the Boolean The following network was designed to realize the function:

$$F = \left(\bar{A} + B + \bar{C}D\right)\left[A + \bar{B} + \left(\bar{C} + \bar{D}\right)\left(C + D\right)\right]$$



a. Is this the correct circuit to realize the function? If so, explain why. If not, re-draw the circuit correctly and write out the function F and then simplify it.

Yes. Change the first NOR(7) to an AND, the second level NORs (5 and 6) to ORs the third level NORs (3, 4) to ANDs, the fourth lever NORs(1,2) to Ors. . . .Now the circuit exactly matches the expression.

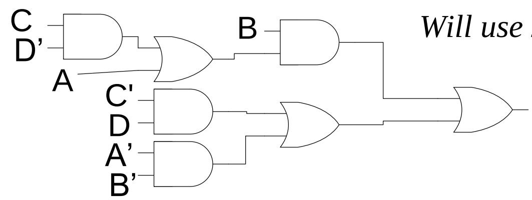
b. If A=B=0 and C=D=1 determine the value of output F (0 or 1)

F=1

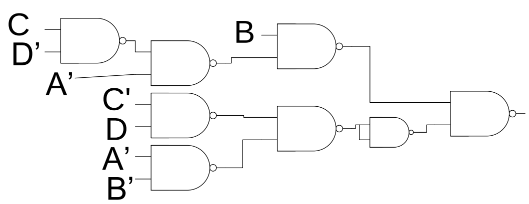
c. Redesign the network so that it realizes the same function but uses minimum number of 2-input NAND gates and 4 inverters.

F=(A'+B+C'D)(A+B'+C'D+CD') = C'D+(A'+B)(A+B'+CD')
= C'D+A'B'+AB+A'CD'+BCD' = C'D+ A'B'+AB+A'(B+B')CD'+BCD'
= C'D+ A'B'+AB+A'BCD'+A'B'CD'+BCD'= C'D+ A'B'+AB+BCD'
= C'D+ A'B'+B(A+CD')

All are 2-input gates except the outer OR. Will use 2 OR's to get this result.



To convert to NAND: Change the A input to A', insert an inverter between the two consecutive OR gates and then change all AND and OR gates to NAND gates. PROBLEM: We have 1 too many inverters (5 instead of 4) and there is an extra NAND gate!! We can use this NAND gate as an inverter. . . .just tie both inputs to the same signal.



8. [6] Optimize the following functions into (1) sum-of-products and (2) product-of-sums forms:
   a. $F(A,B,C,D)=\sum m(0,2,4,5,7,9,12,13,14)$

CD \ AB	00	01	11	10
00	1	1	1	
01		1	1	1
11		1		
10	1		1	

(1) $F = B\overline{C} + \overline{A}BD + \overline{A}\,\overline{B}\,\overline{D} + AB\overline{D} + A\overline{C}D$
(2) $F = \left(A + B + \overline{D}\right)\left(A + \overline{B} + \overline{C} + D\right)\left(\overline{A} + \overline{C} + \overline{D}\right)\left(\overline{A} + B + D\right)$

   b. $F(W,X,Y,Z)=\Pi M(1,2,4,5,7,8,10,15)$

YZ \ WX	00	01	11	10
00		0		0
01	0	0		
11		0	0	
10	0			0

(1) $F = \overline{W}\,\overline{X}\,\overline{Y}\,\overline{Z} + WX\overline{Y} + W\overline{Y}Z + \overline{X}YZ + XY\overline{Z}$

(2) $F = \left(W + \overline{X} + Y\right)\left(\overline{X} + \overline{Y} + \overline{Z}\right)\left(W + Y + \overline{Z}\right)\left(\overline{W} + X + Z\right)\left(X + \overline{Y} + Z\right)$

c. $F(A, B, C, D) = \sum m(0,1,7,9,13,14,15), \quad d = \sum m(5,6,10,12)$

***NOTE: *d* represents the don't care states and should be included as part of F.

CD \ AB	00	01	11	10
00	1		x	
01	1	x	1	1
11		1	1	
10		x	1	x

$\overline{C}D + BC$
$+ \overline{A}\,\overline{B}\,\overline{C}$

(1) $F = \overline{C}D + BC + \overline{A}\,\overline{B}\,\overline{C}$

(2) $F = \left(A + B + \overline{C}\right)\left(\overline{A} + B + \overline{C}\right)\left(\overline{B} + C + D\right)\left(\overline{A} + C + D\right)$

9. [6] Realize $F = B\left(C + \overline{C}A\right) + CA\left(\overline{B} + D\right) + C\left(\overline{B}A + D\right)$ as a circuit that uses a minimum number of 2-input NAND gates and inverters. *This should be done in three phases:*
   a. *Manipulate the equation (using Boolean Algebra and/or Karnaugh maps) to obtain a (possibly simplified) expression in which all operations require only two inputs*
   b. *Realize (i.e., draw) the circuit using only 2-input AND gates, OR gates and Inverters*
   c. *Redraw the circuit by changing AND and OR gates to NAND gates*

a) F = B(C+C'A)+CA(B'+D)+C(AB'+D)= BC+AB+AB'C+ACD+AB'C+CD
= BC+AB+AB'C+CD = BC+A(B+B'C)+CD = BC+AB+AC+CD