Assignment 1

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Problem 1

Part 1:

$$
\begin{aligned}
\textsf{Property } p ::= \quad & \varepsilon \\
\mid \quad & np \\
\mid \quad & sp \\
\\
\textsf{Numerical Property } np ::= \quad & = n \\
\mid \quad & > n \\
\mid \quad & < n \\
\mid \quad & np \wedge np \\
\mid \quad & np \vee np \\
\mid \quad & (np) \\
\\
\textsf{String Property } sp ::= \quad & = s \\
\mid \quad & sp \vee sp \\
\\
\textsf{Schema } \tau ::= \quad & \textsf{number}\langle np \rangle \\
\mid \quad & \textsf{number}\langle \varepsilon \rangle \\
\mid \quad & \textsf{string}\langle sp \rangle \\
\mid \quad & \textsf{string}\langle \varepsilon \rangle \\
\mid \quad & \textsf{bool} \\
\mid \quad & [\tau^*] \\
\mid \quad & \{(s : \tau)^*\}
\end{aligned}
$$

Part 2:

$$\frac{}{\mathsf{false} \sim \mathsf{bool}} \ (\text{S-Bool-False}) \qquad \frac{}{\mathsf{true} \sim \mathsf{bool}} \ (\text{S-Bool-True}) \qquad \frac{}{n \sim \mathsf{number}\langle \varepsilon \rangle} \ (\text{S-Num})$$

$$\frac{n1 = n2}{n_1 \sim \mathsf{number}\langle = n_2 \rangle} \ (\text{S-Num-Equal}) \qquad \frac{n_1 < n_2}{n_1 \sim \mathsf{number}\langle < n_2 \rangle} \ (\text{S-Num-Less})$$

$$\frac{n_1 > n_2}{n_1 \sim \mathsf{number}\langle > n_2 \rangle} \ (\text{S-Num-Greater}) \qquad \frac{n \sim \mathsf{number}\langle p \rangle, n \sim \mathsf{number}\langle q \rangle}{n \sim \mathsf{number}\langle p \wedge q \rangle} \ (\text{S-Num-And})$$

$$\frac{n \sim \mathsf{number}\langle p \rangle \vee n \sim \mathsf{number}\langle q \rangle}{n \sim \mathsf{number}\langle p \vee q \rangle} \ (\text{S-Num-Or}) \qquad \frac{n \sim \mathsf{number}\langle p \rangle}{n \sim \mathsf{number}\langle (p) \rangle} \ (\text{S-Num-Brackets})$$

$$\frac{}{s \sim \mathsf{string}\langle \varepsilon \rangle} \ (\text{S-Str}) \qquad \frac{s_1 = s_2}{s_1 \sim \mathsf{string}\langle = s_2 \rangle} \ (\text{S-Str-Equal})$$

$$\frac{s \sim \mathsf{string}\langle p \rangle \vee s \sim \mathsf{string}\langle q \rangle}{s \sim \mathsf{string}\langle p \vee q \rangle} \ (\text{S-Str-Or}) \qquad \frac{\forall i = 0 \ldots |j| - 1 . a_i \sim \tau_i}{[a^*] \sim [\tau^*]} \ (\text{S-Array})$$

$$\frac{\forall s' \in s . j_{s'} \sim \tau_{s'}}{\{(s : j)^*\} \sim \{(s : \tau)^*\}} \ (\text{S-Dict})$$

Problem 2

Part 1:

$$\frac{s' \in s}{(.s'a, \{(s, j)^*\}) \mapsto (a, j_{s'})} \ (\text{E-Access}) \qquad \frac{i \in \{0 \ldots |j| - 1\}}{([i]a, [j^*]) \mapsto (a, j_i)} \ (\text{E-Index})$$

$$\frac{\forall i = 1 \ldots |[\{s, j\}^*]| - 1, s' \in s_i}{(|.s'a, [\{s, j\}^*]) \mapsto [j_{i,s'}] \text{ for } i = 1 \ldots |[\{s, j\}^*]|} \ (\text{E-Map-Access})$$

$$\frac{\forall l = 1 \ldots |[[j^*]^*]| - 1, i \in 0, \ldots, |j_l| - 1}{(|[i]a, [[j^*]^*]) \mapsto [j^*_{l,i}] \text{ for } l = 1 \ldots |[[j^*]]|} \ (\text{E-Map-Index})$$

Part 2:

$$\frac{a \sim \tau}{[i]a \sim [\tau]}$$

$$\forall \tau \quad \frac{}{\varepsilon \sim \tau} \text{(V-NON)} \qquad \frac{s' \in s, a \sim \tau_{j_{s'}}}{.s'a \sim \cancel{\{(s,j)^*\}} \{(s,\tau)^*, (s, \tau_{j_{s'}})\}} \text{(V-ACCESS)} \qquad \cancel{\frac{i \in \{0 \ldots |j|-1\}, a \sim \tau_{j_i}}{[i]a \sim [j^*]}} \text{(V-INDEX)}$$

$$\frac{\forall i = 1 \ldots |[\{s,j\}^*]| - 1, s' \in s_i, a \sim \tau_{j_{i,s'}}}{.|s'a \sim \cancel{[\{s,j\}^*]} [\{(s,\tau)^*, (s, \tau_{j_{s'}})\}]} \text{(V-MAP-ACCESS)}$$

$$\frac{\forall l = 1 \ldots |[[j^*]^*]| - 1, i \in 0, \ldots, |j_l| - 1, a \sim \tau_{\cancel{j_{l,i}}}}{|[i]a \sim \cancel{[[j^*]^*]} [[\tau]^*]^*} \text{(V-MAP-INDEX)}$$

Accessor safety: for all a, j, τ, if $a \sim \tau$ and $j \sim \tau$, then there exists a j' such that $(a, j) \overset{*}{\mapsto} \varepsilon, j'$.

Proof. Induction on the steps of derivation:

(1 step derivation)

trivial by case analysis of a.

(k step derivation, k > 1)

take 1 step like we did above then use the k-1 hypothesis.

□