Robust Stochastic Optimization Made Easy with RSOME

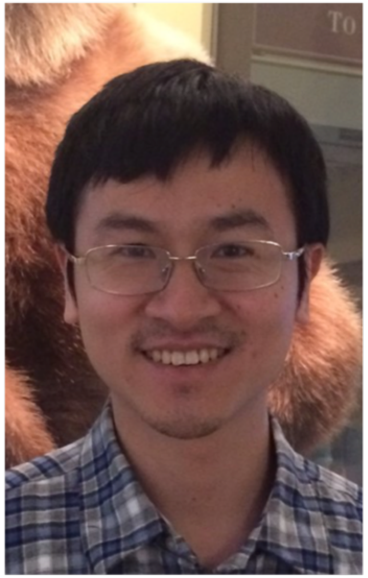
Melvyn Sim

Department of Analytics & Operations

NUS Business School

Collaborators

Zhi Chen



Peng Xiong

Robust Decision-Making

Model

Random variable: $\tilde{\boldsymbol{z}} \sim \mathbb{P}$

Ambiguity set: $\mathcal{F}, \mathbb{P} \in \mathcal{F}$

Decision *(here-and-now)* : $\boldsymbol{x} \in \mathcal{X}$

Output cost function: $f(\boldsymbol{x}, \tilde{\boldsymbol{z}})$

Decision Criterion:

$$\sup_{\mathbb{P} \in \mathcal{F}} \mathbb{E}_{\mathbb{P}} [f(\boldsymbol{x}, \tilde{\boldsymbol{z}})]$$

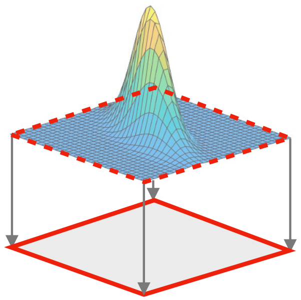
Solver:

$$\boldsymbol{x}^* = \arg\min_{\boldsymbol{x} \in \mathcal{X}} \sup_{\mathbb{P} \in \mathcal{F}} \mathbb{E}_{\mathbb{P}} [f(\boldsymbol{x}, \tilde{\boldsymbol{z}})]$$

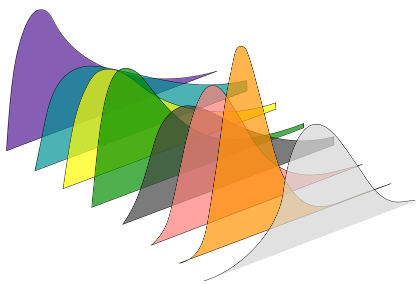
Ambiguity Set

- A family of distributions that share identical distributional information

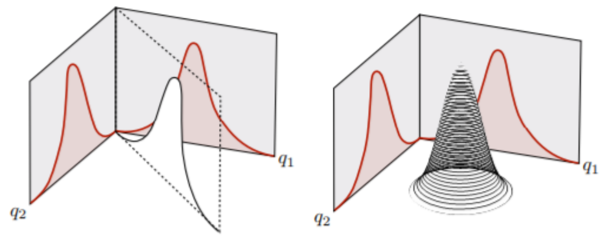
Support



Moments



Structural Information



Tractable Optimization Format

- Tractable optimization formats for deterministic optimization
  - E.g., LP, MIP, SOCP, etc
- Lack of tractable formats for optimization problems under uncertainty
- Goal is to develop a format for optimization under uncertainty that ensures tractability
  - Ability to incorporate state-of-the-art solvers

Content

- Scope of RSO
- Event-Wise Ambiguity Set
- Event-Wise Adaptation
- RSOME Toolbox
- Examples

RSO: Robust Stochastic Optimization

$$
\begin{aligned}
\min \quad & \sup_{\mathbb{P}\in\mathcal{F}} \mathbb{E}_{\mathbb{P}} \left[\boldsymbol{a}_0^\top(\tilde{s},\tilde{\boldsymbol{z}})\boldsymbol{w} + \boldsymbol{b}_0^\top(\tilde{s},\tilde{\boldsymbol{z}})\boldsymbol{x}(\tilde{s}) + \boldsymbol{c}_0^\top(\tilde{s})\boldsymbol{y}(\tilde{s},\tilde{\boldsymbol{z}}) + d_0(\tilde{s},\tilde{\boldsymbol{z}})\right] \\
\text{s.t}\ & \boldsymbol{a}_m^\top(s,\boldsymbol{z})\boldsymbol{w} + \boldsymbol{b}_m^\top(s,\boldsymbol{z})\boldsymbol{x}(s) + \boldsymbol{c}_m^\top(s)\boldsymbol{y}(s,\boldsymbol{z}) + d_m(s,\boldsymbol{z}) \le 0, && \forall \boldsymbol{z}\in\mathcal{Z}_s,\ \forall s\in[S],\ \forall m\in\mathcal{M}_1 \\
& \sup_{\mathbb{P}\in\mathcal{F}} \mathbb{E}_{\mathbb{P}} \left[\boldsymbol{a}_m^\top(\tilde{s},\tilde{\boldsymbol{z}})\boldsymbol{w} + \boldsymbol{b}_m^\top(\tilde{s},\tilde{\boldsymbol{z}})\boldsymbol{x}(\tilde{s}) + \boldsymbol{c}_m^\top(\tilde{s})\boldsymbol{y}(\tilde{s},\tilde{\boldsymbol{z}}) + d_m(\tilde{s},\tilde{\boldsymbol{z}})\right] \le 0, && \forall m\in\mathcal{M}_2 \\
& \left(\boldsymbol{w},\boldsymbol{x}(s),\boldsymbol{y}^0(s),\ldots,\boldsymbol{y}^{I_z}(s)\right)\in\mathcal{X}_s, && \forall s\in[S] \\
& x_j\in\mathcal{A}(\mathcal{C}_x^j), && \forall j\in[J_x] \\
& y_j\in\bar{\mathcal{A}}(\mathcal{C}_y^j,\mathcal{I}_y^j), && \forall j\in[J_y]
\end{aligned}
$$

RSO: Model Inputs

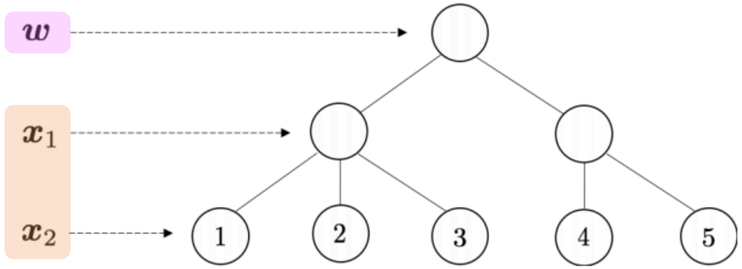
$$
\begin{array}{lll}
\min & \sup_{\mathbb{P}\in\mathcal{F}} \mathbb{E}_{\mathbb{P}}\left[\boldsymbol{a}_0^\top(\tilde{s},\tilde{\boldsymbol{z}})\boldsymbol{w} + \boldsymbol{b}_0^\top(\tilde{s},\tilde{\boldsymbol{z}})\boldsymbol{x}(\tilde{s}) + \boldsymbol{c}_0^\top(\tilde{s})\boldsymbol{y}(\tilde{s},\tilde{\boldsymbol{z}}) + d_0(\tilde{s},\tilde{\boldsymbol{z}})\right] & \\
\text{s.t} & \boldsymbol{a}_m^\top(s,\boldsymbol{z})\boldsymbol{w} + \boldsymbol{b}_m^\top(s,\boldsymbol{z})\boldsymbol{x}(s) + \boldsymbol{c}_m^\top(s)\boldsymbol{y}(s,\boldsymbol{z}) + d_m(s,\boldsymbol{z}) \leq 0, & \forall \boldsymbol{z}\in\mathcal{Z}_s,\ \forall s\in[S],\ \forall m\in\mathcal{M}_1 \\
& \sup_{\mathbb{P}\in\mathcal{F}} \mathbb{E}_{\mathbb{P}}\left[\boldsymbol{a}_m^\top(\tilde{s},\tilde{\boldsymbol{z}})\boldsymbol{w} + \boldsymbol{b}_m^\top(\tilde{s},\tilde{\boldsymbol{z}})\boldsymbol{x}(\tilde{s}) + \boldsymbol{c}_m^\top(\tilde{s})\boldsymbol{y}(\tilde{s},\tilde{\boldsymbol{z}}) + d_m(\tilde{s},\tilde{\boldsymbol{z}})\right] \leq 0, & \forall m\in\mathcal{M}_2 \\
& \left(\boldsymbol{w},\boldsymbol{x}(s),\boldsymbol{y}^0(s),\ldots,\boldsymbol{y}^{I_z}(s)\right)\in\mathcal{X}_s, & \forall s\in[S] \\
& x_j \in \mathcal{A}(\mathcal{C}_x^j), & \forall j\in[J_x] \\
& y_j \in \bar{\mathcal{A}}(\mathcal{C}_y^j,\mathcal{I}_y^j), & \forall j\in[J_y]
\end{array}
$$

$$
\left\{
\begin{array}{rcl}
\boldsymbol{a}_m(s,\boldsymbol{z}) & \triangleq & \boldsymbol{a}_{ms}^0 + \sum_{i\in[I_z]} \boldsymbol{a}_{ms}^i z_i \\
\boldsymbol{b}_m(s,\boldsymbol{z}) & \triangleq & \boldsymbol{b}_{ms}^0 + \sum_{i\in[I_z]} \boldsymbol{b}_{ms}^i z_i \\
\boldsymbol{c}_m(s) & \triangleq & \boldsymbol{c}_{ms} \\
d_m(s,\boldsymbol{z}) & \triangleq & d_{ms}^0 + \sum_{i\in[I_z]} d_{ms}^i z_i
\end{array}
\right.
$$

for given parameters $\boldsymbol{a}_{ms}^i\in\mathbb{R}^{J_w}, \boldsymbol{b}_{ms}^i\in\mathbb{R}^{J_x}, \boldsymbol{c}_{ms}\in\mathbb{R}^{J_y}, d_{ms}^i\in\mathbb{R} \quad \forall i\in[I_z]\cup\{0\},\ s\in[S]$

RSO: Decision Variables

$$
\begin{aligned}
\min \quad & \sup_{\mathbb{P}\in\mathcal{F}} \mathbb{E}_{\mathbb{P}} \left[\boldsymbol{a}_0^\top(\tilde{s},\tilde{\boldsymbol{z}})\boldsymbol{w} + \boldsymbol{b}_0^\top(\tilde{s},\tilde{\boldsymbol{z}})\boldsymbol{x}(\tilde{s}) + \boldsymbol{c}_0^\top(\tilde{s})\boldsymbol{y}(\tilde{s},\tilde{\boldsymbol{z}}) + d_0(\tilde{s},\tilde{\boldsymbol{z}})\right] & \\
\text{s.t}\ & \boldsymbol{a}_m^\top(s,\boldsymbol{z})\boldsymbol{w} + \boldsymbol{b}_m^\top(s,\boldsymbol{z})\boldsymbol{x}(s) + \boldsymbol{c}_m^\top(s)\boldsymbol{y}(s,\boldsymbol{z}) + d_m(s,\boldsymbol{z}) \le 0, & \forall \boldsymbol{z}\in\mathcal{Z}_s,\ \forall s\in[S],\ \forall m\in\mathcal{M}_1 \\
& \sup_{\mathbb{P}\in\mathcal{F}} \mathbb{E}_{\mathbb{P}} \left[\boldsymbol{a}_m^\top(\tilde{s},\tilde{\boldsymbol{z}})\boldsymbol{w} + \boldsymbol{b}_m^\top(\tilde{s},\tilde{\boldsymbol{z}})\boldsymbol{x}(\tilde{s}) + \boldsymbol{c}_m^\top(\tilde{s})\boldsymbol{y}(\tilde{s},\tilde{\boldsymbol{z}}) + d_m(\tilde{s},\tilde{\boldsymbol{z}})\right] \le 0, & \forall m\in\mathcal{M}_2 \\
& \left(\boldsymbol{w}, \boldsymbol{x}(s), \boldsymbol{y}^0(s),\dots,\boldsymbol{y}^{I_z}(s)\right) \in \mathcal{X}_s, & \forall s\in[S] \\
& x_j \in \mathcal{A}(\mathcal{C}_x^j), & \forall j\in[J_x] \\
& y_j \in \bar{\mathcal{A}}(\mathcal{C}_y^j, \mathcal{I}_y^j), & \forall j\in[J_y]
\end{aligned}
$$



RSO: A Unified Framework

$$
\begin{aligned}
\min \ & \sup_{\mathbb{P}\in\mathcal{F}} \mathbb{E}_{\mathbb{P}}\left[\boldsymbol{a}_0^\top(\tilde{s},\tilde{\boldsymbol{z}})\boldsymbol{w} + \boldsymbol{b}_0^\top(\tilde{s},\tilde{\boldsymbol{z}})\boldsymbol{x}(\tilde{s}) + \boldsymbol{c}_0^\top(\tilde{s})\boldsymbol{y}(\tilde{s},\tilde{\boldsymbol{z}}) + d_0(\tilde{s},\tilde{\boldsymbol{z}})\right] & \\
\text{s.t} \ & \boldsymbol{a}_m^\top(s,\boldsymbol{z})\boldsymbol{w} + \boldsymbol{b}_m^\top(s,\boldsymbol{z})\boldsymbol{x}(s) + \boldsymbol{c}_m^\top(s)\boldsymbol{y}(s,\boldsymbol{z}) + d_m(s,\boldsymbol{z}) \leq 0, & \forall \boldsymbol{z}\in\mathcal{Z}_s,\ \forall s\in[S],\ \forall m\in\mathcal{M}_1 \\
& \sup_{\mathbb{P}\in\mathcal{F}} \mathbb{E}_{\mathbb{P}}\left[\boldsymbol{a}_m^\top(\tilde{s},\tilde{\boldsymbol{z}})\boldsymbol{w} + \boldsymbol{b}_m^\top(\tilde{s},\tilde{\boldsymbol{z}})\boldsymbol{x}(\tilde{s}) + \boldsymbol{c}_m^\top(\tilde{s})\boldsymbol{y}(\tilde{s},\tilde{\boldsymbol{z}}) + d_m(\tilde{s},\tilde{\boldsymbol{z}})\right] \leq 0, & \forall m\in\mathcal{M}_2 \\
& \left(\boldsymbol{w},\boldsymbol{x}(s),\boldsymbol{y}^0(s),\ldots,\boldsymbol{y}^{I_z}(s)\right)\in\mathcal{X}_s, & \forall s\in[S] \\
& x_j \in \mathcal{A}(\mathcal{C}_x^j), & \forall j\in[J_x] \\
& y_j \in \bar{\mathcal{A}}(\mathcal{C}_y^j,\mathcal{I}_y^j), & \forall j\in[J_y]
\end{aligned}
$$

- RSO framework
  - ✓Ambiguity set

RSO: A Unified Framework

$$
\begin{aligned}
\min \quad & \sup_{\mathbb{P}\in\mathcal{F}} \mathbb{E}_{\mathbb{P}}\left[\boldsymbol{a}_0^\top(\tilde{s},\tilde{\boldsymbol{z}})\boldsymbol{w} + \boldsymbol{b}_0^\top(\tilde{s},\tilde{\boldsymbol{z}})\boldsymbol{x}(\tilde{s}) + \boldsymbol{c}_0^\top(\tilde{s})\boldsymbol{y}(\tilde{s},\tilde{\boldsymbol{z}}) + d_0(\tilde{s},\tilde{\boldsymbol{z}})\right] & \\
\text{s.t} \quad & \boldsymbol{a}_m^\top(s,\boldsymbol{z})\boldsymbol{w} + \boldsymbol{b}_m^\top(s,\boldsymbol{z})\boldsymbol{x}(s) + \boldsymbol{c}_m^\top(s)\boldsymbol{y}(s,\boldsymbol{z}) + d_m(s,\boldsymbol{z}) \le 0, & \forall \boldsymbol{z}\in\mathcal{Z}_s,\ \forall s\in[S],\ \forall m\in\mathcal{M}_1 \\
& \sup_{\mathbb{P}\in\mathcal{F}} \mathbb{E}_{\mathbb{P}}\left[\boldsymbol{a}_m^\top(\tilde{s},\tilde{\boldsymbol{z}})\boldsymbol{w} + \boldsymbol{b}_m^\top(\tilde{s},\tilde{\boldsymbol{z}})\boldsymbol{x}(\tilde{s}) + \boldsymbol{c}_m^\top(\tilde{s})\boldsymbol{y}(\tilde{s},\tilde{\boldsymbol{z}}) + d_m(\tilde{s},\tilde{\boldsymbol{z}})\right] \le 0, & \forall m\in\mathcal{M}_2 \\
& \left(\boldsymbol{w},\boldsymbol{x}(s),\boldsymbol{y}^0(s),\ldots,\boldsymbol{y}^{I_z}(s)\right)\in\mathcal{X}_s, & \forall s\in[S] \\
& x_j\in\mathcal{A}(\mathcal{C}_x^j), & \forall j\in[J_x] \\
& y_j\in\bar{\mathcal{A}}(\mathcal{C}_y^j,\mathcal{I}_y^j), & \forall j\in[J_y]
\end{aligned}
$$

- RSO framework
  - ✓Ambiguity set
  - ✓Optimization model

RSO: A Unified Framework

$$
\begin{array}{lll}
\min & \sup_{\mathbb{P}\in\mathcal{F}} \mathbb{E}_{\mathbb{P}}\left[\boldsymbol{a}_0^\top(\tilde{s},\tilde{\boldsymbol{z}})\boldsymbol{w} + \boldsymbol{b}_0^\top(\tilde{s},\tilde{\boldsymbol{z}})\boldsymbol{x}(\tilde{s}) + \boldsymbol{c}_0^\top(\tilde{s})\boldsymbol{y}(\tilde{s},\tilde{\boldsymbol{z}}) + d_0(\tilde{s},\tilde{\boldsymbol{z}})\right] & \\
\text{s.t} & \boldsymbol{a}_m^\top(s,\boldsymbol{z})\boldsymbol{w} + \boldsymbol{b}_m^\top(s,\boldsymbol{z})\boldsymbol{x}(s) + \boldsymbol{c}_m^\top(s)\boldsymbol{y}(s,\boldsymbol{z}) + d_m(s,\boldsymbol{z}) \leq 0, & \forall \boldsymbol{z}\in\mathcal{Z}_s,\ \forall s\in[S],\ \forall m\in\mathcal{M}_1 \\
& \sup_{\mathbb{P}\in\mathcal{F}} \mathbb{E}_{\mathbb{P}}\left[\boldsymbol{a}_m^\top(\tilde{s},\tilde{\boldsymbol{z}})\boldsymbol{w} + \boldsymbol{b}_m^\top(\tilde{s},\tilde{\boldsymbol{z}})\boldsymbol{x}(\tilde{s}) + \boldsymbol{c}_m^\top(\tilde{s})\boldsymbol{y}(\tilde{s},\tilde{\boldsymbol{z}}) + d_m(\tilde{s},\tilde{\boldsymbol{z}})\right] \leq 0, & \forall m\in\mathcal{M}_2 \\
& \left(\boldsymbol{w},\boldsymbol{x}(s),\boldsymbol{y}^0(s),\ldots,\boldsymbol{y}^{I_z}(s)\right) \in \mathcal{X}_s, & \forall s\in[S] \\
& x_j \in \mathcal{A}(\mathcal{C}_x^j), & \forall j\in[J_x] \\
& y_j \in \bar{\mathcal{A}}(\mathcal{C}_y^j,\mathcal{I}_y^j), & \forall j\in[J_y]
\end{array}
$$

- RSO framework
  - ✓Ambiguity set
  - ✓Optimization model
  - ✓Event-wise recourse adaptation

Content

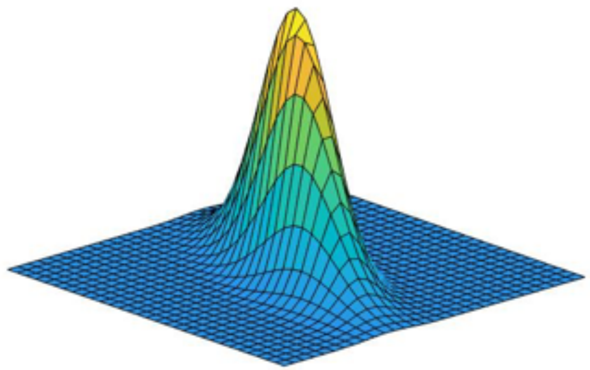
- Scope of RSO
- Event-Wise Ambiguity Set
- Event-Wise Adaptation
- RSOME Toolbox
- Examples

Event-Wise Ambiguity Set

$$\mathcal{F} = \left\{ \mathbb{P} \in \mathcal{P}_0\left(\mathbb{R}^{I_z} \times [S]\right) \;\middle|\; \begin{array}{ll} (\tilde{\boldsymbol{z}}, \tilde{s}) \sim \mathbb{P} & \\ \mathbb{E}_{\mathbb{P}}[\tilde{\boldsymbol{z}} \,|\, \tilde{s} \in \mathcal{E}_k] \in \mathcal{Q}_k & \forall k \in [K] \\ \mathbb{P}[\tilde{\boldsymbol{z}} \in \mathcal{Z}_s \,|\, \tilde{s} = s] = 1 & \forall s \in [S] \\ \mathbb{P}[\tilde{s} = s] = p_s & \forall s \in [S] \\ \text{for some } \boldsymbol{p} \in \mathcal{P} & \end{array} \right\}$$

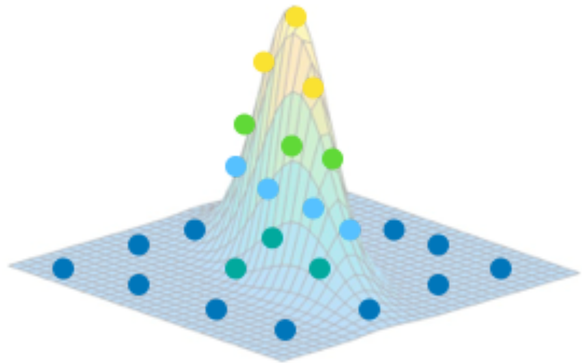
- Continuous random vector $\tilde{\boldsymbol{z}}$
- Discrete random scalar $\tilde{s}$

Distributional Information



Stochastic Programming

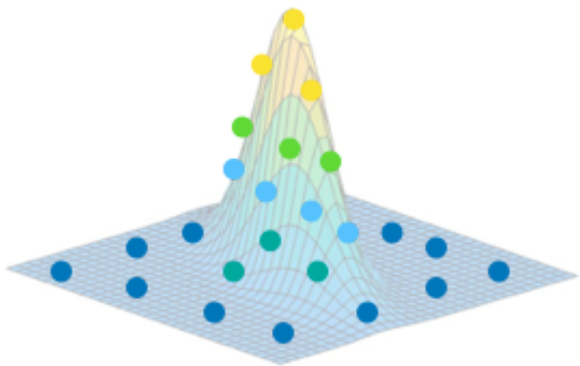
- Singleton
- Discretization
- Ambiguity set of discrete distributions



$$\mathcal{F} = \left\{ \mathbb{P} \in \mathcal{P}_0(\mathbb{R}^{I_z} \times [S]) \;\middle|\; \begin{array}{ll} (\tilde{\boldsymbol{z}}, \tilde{s}) \sim \mathbb{P} & \\ \mathbb{P}[\tilde{\boldsymbol{z}} = \hat{\boldsymbol{z}}_s \,|\, \tilde{s} = s] = 1 & \forall s \in [S] \\ \mathbb{P}[\tilde{s} = s] = p_s & \forall s \in [S] \end{array} \right\}$$

Stochastic Programming

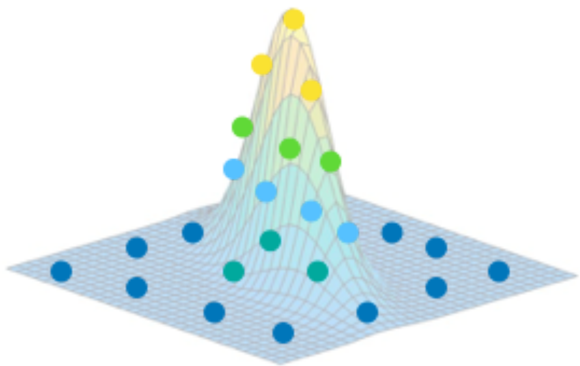
- Singleton
- Discretization
- Ambiguity set of discrete distributions
- Uncertain scenario probabilities



$$\mathcal{F} = \left\{ \mathbb{P} \in \mathcal{P}_0(\mathbb{R}^{I_z} \times [S]) \;\middle|\; \begin{array}{ll} (\tilde{\boldsymbol{z}}, \tilde{s}) \sim \mathbb{P} & \\ \mathbb{P}[\tilde{\boldsymbol{z}} = \hat{\boldsymbol{z}}_s \,|\, \tilde{s} = s] = 1 & \forall s \in [S] \\ \mathbb{P}[\tilde{s} = s] = p_s & \forall s \in [S] \\ \text{for some } \boldsymbol{p} \in \mathcal{P} & \end{array} \right\}$$

Stochastic Programming

- Singleton
- Discretization
- Ambiguity set of discrete distributions
- Uncertain scenario probabilities



$$\mathcal{F} = \left\{ \mathbb{P} \in \mathcal{P}_0(\mathbb{R}^{I_z} \times [S]) \;\middle|\; \begin{array}{ll} (\tilde{\boldsymbol{z}}, \tilde{s}) \sim \mathbb{P} & \\ \mathbb{P}[\tilde{\boldsymbol{z}} = \hat{\boldsymbol{z}}_s \,|\, \tilde{s} = s] = 1 & \forall s \in [S] \\ \mathbb{P}[\tilde{s} = s] = p_s & \forall s \in [S] \\ \text{for some } \boldsymbol{p} \in \mathcal{P} & \end{array} \right\}$$

Φ-divergences

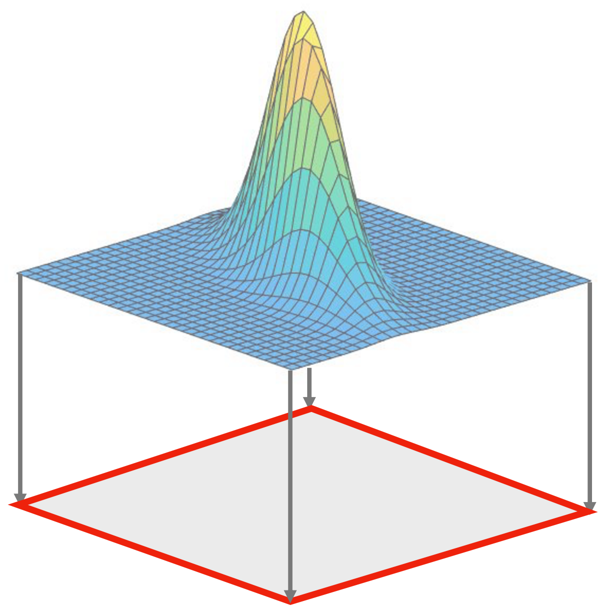
$$\mathcal{P} = \{\boldsymbol{p} \,|\, d_\phi(\boldsymbol{p}, \hat{\boldsymbol{p}}) \leq \theta\}$$

Robust Optimization

- Norm-based uncertainty sets

- Support information

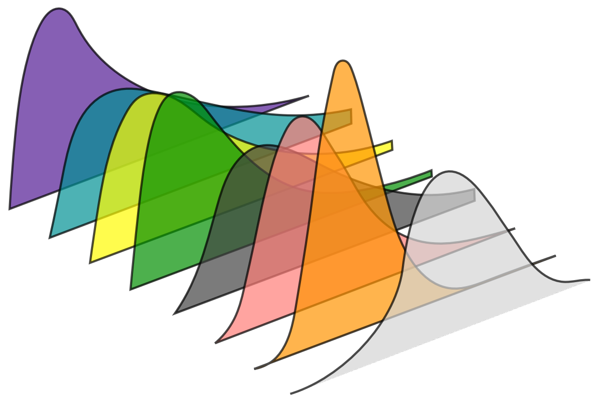
$$\mathcal{F} = \left\{ \mathbb{P} \in \mathcal{P}_0(\mathbb{R}^{I_z}) \;\middle|\; \begin{array}{l} \tilde{\boldsymbol{z}} \sim \mathbb{P} \\ \mathbb{P}[\tilde{\boldsymbol{z}} \in \mathcal{Z}] = 1 \end{array} \right\}$$



Ambiguity Sets with Generalized Moments

- Mean and dispersion

$$\mathcal{F} = \left\{ \mathbb{P} \in \mathcal{P}_0(\mathbb{R}^{I_u}) \;\middle|\; \begin{array}{l} \tilde{\boldsymbol{u}} \sim \mathbb{P} \\ \mathbb{E}_{\mathbb{P}}[\tilde{\boldsymbol{u}}] \in \mathcal{Q} \\ \mathbb{E}_{\mathbb{P}}[\boldsymbol{\phi}(\tilde{\boldsymbol{u}})] \leq \boldsymbol{\sigma} \\ \mathbb{P}[\tilde{\boldsymbol{u}} \in \mathcal{U}] = 1 \end{array} \right\}$$

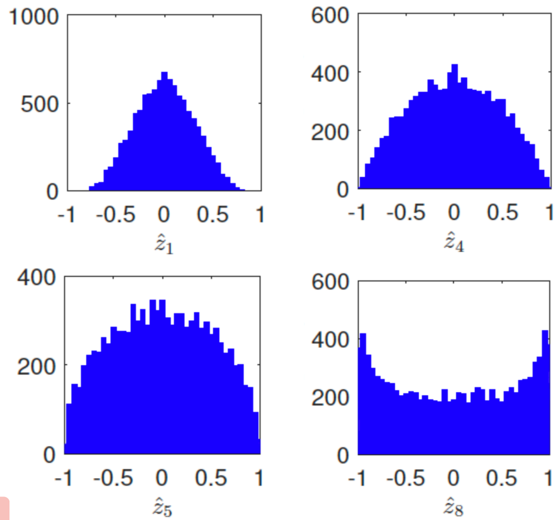


Ambiguity Sets with Generalized Moments

- Mean and dispersion

- Lifted expression:

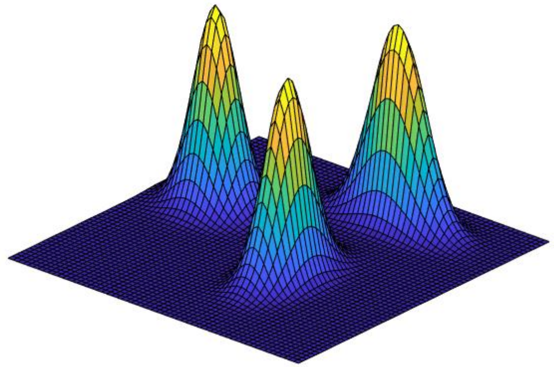
$$\mathcal{F} = \left\{ \mathbb{P} \in \mathcal{P}_0(\mathbb{R}^{I_u+I_v} \times \{1\}) \;\middle|\; \begin{array}{l} ((\tilde{\boldsymbol{u}}, \tilde{\boldsymbol{v}}), \tilde{s}) \sim \mathbb{P} \\ \mathbb{E}_{\mathbb{P}}[\tilde{\boldsymbol{u}} \,|\, \tilde{s} = 1] \in \mathcal{Q} \\ \mathbb{E}_{\mathbb{P}}[\tilde{\boldsymbol{v}} \,|\, \tilde{s} = 1] \leq \boldsymbol{\sigma} \\ \mathbb{P}[(\tilde{\boldsymbol{u}}, \tilde{\boldsymbol{v}}) \in \mathcal{Z} \,|\, \tilde{s} = 1] = 1 \\ \mathbb{P}[\tilde{s} = 1] = 1 \end{array} \right\}$$



$$\mathcal{Z} = \{(\boldsymbol{u}, \boldsymbol{v}) \mid \boldsymbol{u} \in \mathcal{U},\ \boldsymbol{v} \geq \boldsymbol{\phi}(\boldsymbol{u})\}$$

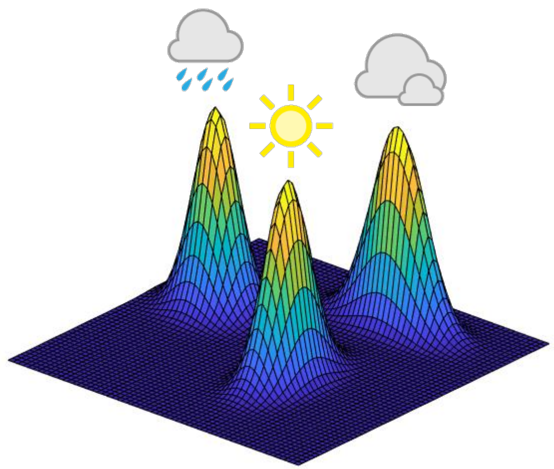
Mixture Distributions

- Multi-modality

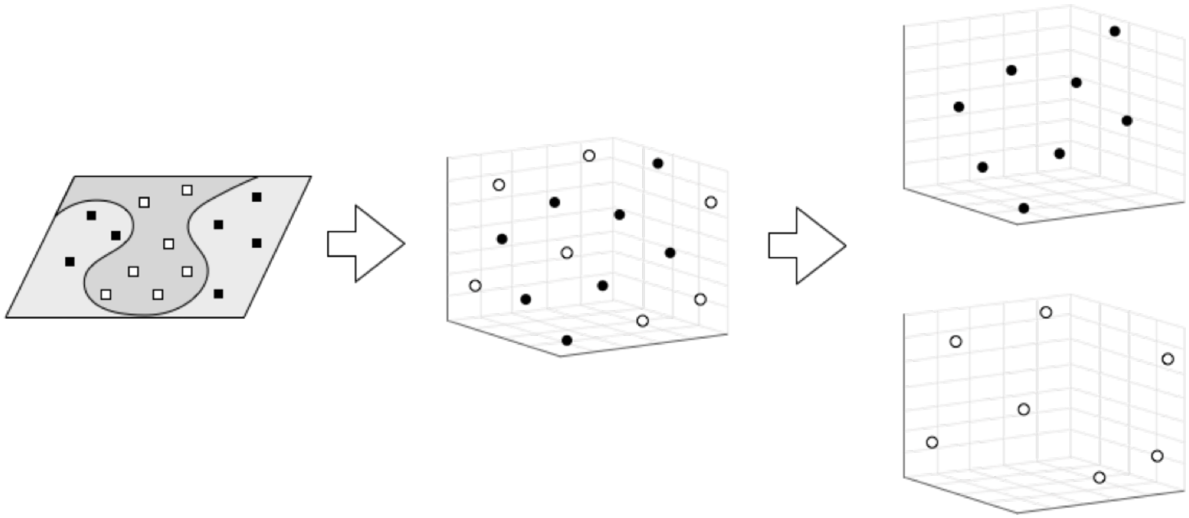


$$\mathcal{F} = \left\{ \mathbb{P} \in \mathcal{P}_0(\mathbb{R}^{I_u+I_v} \times [S]) \;\middle|\; \begin{array}{ll} ((\tilde{\boldsymbol{u}}, \tilde{\boldsymbol{v}}), \tilde{s}) \sim \mathbb{P} & \\ \mathbb{E}_{\mathbb{P}}[\tilde{\boldsymbol{u}} \,|\, \tilde{s} = s] = \boldsymbol{\mu}_s & \forall s \in [S] \\ \mathbb{E}_{\mathbb{P}}[\tilde{\boldsymbol{v}} \,|\, \tilde{s} = s] \leq \boldsymbol{\sigma}_s & \forall s \in [S] \\ \mathbb{P}[(\tilde{\boldsymbol{u}}, \tilde{\boldsymbol{v}}) \in \mathcal{Z}_s \,|\, \tilde{s} = s] = 1 & \forall s \in [S] \\ \mathbb{P}[\tilde{s} = s] = p_s & \forall s \in [S] \\ \text{for some } \boldsymbol{p} \in \mathcal{P},\ \boldsymbol{\mu}_s \in \mathcal{Q}_s & \forall s \in [S] \end{array} \right\}$$

Mixture Distributions

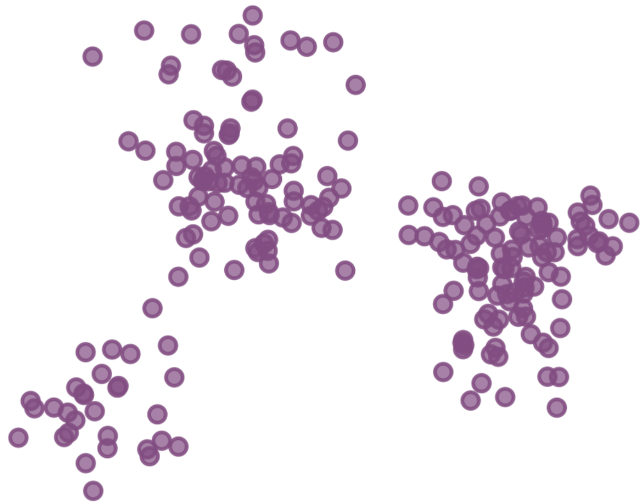


- Side information (Covariates)
  - E.g., weather versus taxi demand



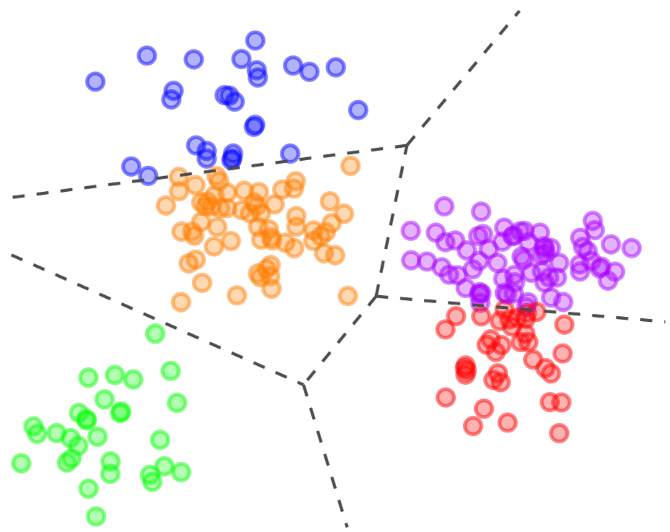
K-Means Clustering

- Cluster information

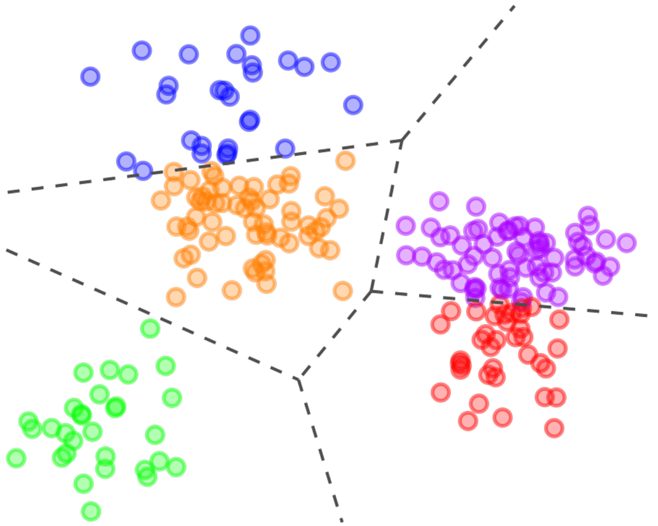


K-Means Clustering

- Cluster information



K-Means Clustering



- Cluster information

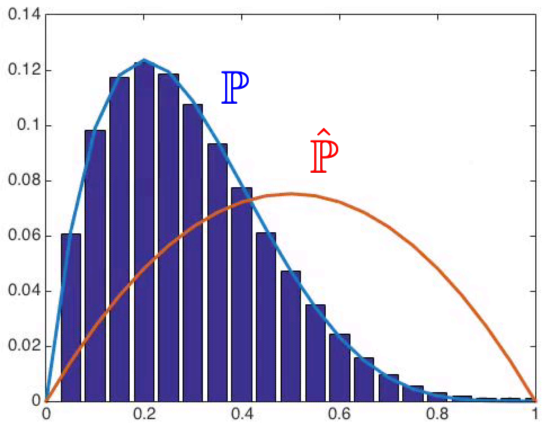
$$\mathcal{F} = \left\{ \mathbb{P} \in \mathcal{P}_0(\mathbb{R}^{I_u + I_v} \times [S]) \;\middle|\; \begin{array}{ll} ((\tilde{\boldsymbol{u}}, \tilde{\boldsymbol{v}}), \tilde{s}) \sim \mathbb{P} & \\ \mathbb{E}_{\mathbb{P}}[\tilde{\boldsymbol{u}} \,|\, \tilde{s} = s] \in \mathcal{Q}_s & \forall s \in [S] \\ \mathbb{E}_{\mathbb{P}}[\tilde{\boldsymbol{v}} \,|\, \tilde{s} = s] \leq \boldsymbol{\sigma}_s & \forall s \in [S] \\ \mathbb{P}[(\tilde{\boldsymbol{u}}, \tilde{\boldsymbol{v}}) \in \mathcal{Z}_s \,|\, \tilde{s} = s] = 1 & \forall s \in [S] \\ \mathbb{P}[\tilde{s} = s] = p_s & \forall s \in [S] \\ \text{for some } \boldsymbol{p} \in \mathcal{P}, \; \boldsymbol{\mu}_s \in \mathcal{Q}_s, \; \forall s \in [S] & \end{array} \right\}$$

Wasserstein Metric

Wasserstein Metric

$$d_W(\mathbb{P}, \hat{\mathbb{P}}) = \inf_{\mathbb{Q}} \mathbb{E}_{\mathbb{Q}}[\|\tilde{\boldsymbol{u}} - \tilde{\boldsymbol{u}}'\|]$$

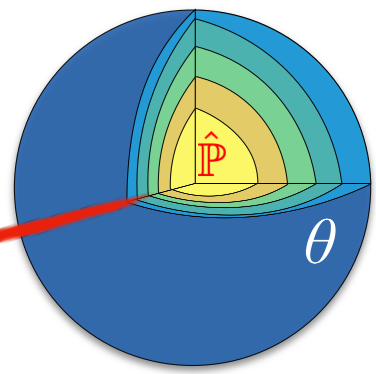
$$(\tilde{\boldsymbol{u}}, \tilde{\boldsymbol{u}}') \sim \mathbb{Q}, \ \Pi_{\tilde{\boldsymbol{u}}}\mathbb{Q} = \mathbb{P}, \ \Pi_{\tilde{\boldsymbol{u}}'}\mathbb{Q} = \hat{\mathbb{P}}$$



Wasserstein Ball

$$\mathcal{F}(\theta) = \{\mathbb{P} \mid d_W(\mathbb{P}, \hat{\mathbb{P}}) \leq \theta\}$$

$$\hat{\mathbb{P}} = \frac{1}{S}\sum_{s=1}^{S} \delta_{\hat{\boldsymbol{u}}_s}$$



Wasserstein Metric

- Wasserstein ambiguity set

$$\mathcal{G}(\theta) = \left\{ \mathbb{P} \in \mathcal{P}_0(\mathbb{R}^{I_u+1} \times [S]) \;\middle|\; \begin{array}{ll} ((\tilde{\boldsymbol{u}}, \tilde{v}), \tilde{s}) \sim \mathbb{P} & \\ \mathbb{E}_{\mathbb{P}}[\tilde{v} \,|\, \tilde{s} \in [S]\,] \leq \theta & \\ \mathbb{P}[(\tilde{\boldsymbol{u}}, \tilde{v}) \in \mathcal{Z}_s \,|\, \tilde{s} = s] = 1 & \forall s \in [S] \\ \mathbb{P}[\tilde{s} = s] = \frac{1}{S} & \forall s \in [S] \end{array} \right\}$$

Wasserstein Metric

- Wasserstein ambiguity set

$$\mathcal{G}(\theta) = \left\{ \mathbb{P} \in \mathcal{P}_0(\mathbb{R}^{I_u+1} \times [S]) \;\middle|\; \begin{array}{ll} ((\tilde{\boldsymbol{u}}, \tilde{v}), \tilde{s}) \sim \mathbb{P} & \\ \mathbb{E}_{\mathbb{P}}[\tilde{v} \,|\, \tilde{s} \in [S]] \leq \theta & \\ \mathbb{P}[(\tilde{\boldsymbol{u}}, \tilde{v}) \in \boldsymbol{\mathcal{Z}_s} \,|\, \tilde{s} = s] = 1 & \forall s \in [S] \\ \mathbb{P}[\tilde{s} = s] = \frac{1}{S} & \forall s \in [S] \end{array} \right\}$$

$$\mathcal{Z}_s = \left\{ (\boldsymbol{u}, v) \;\middle|\; \begin{array}{l} \boldsymbol{u} \in \mathcal{U} \\ v \geq \|\boldsymbol{u} - \hat{\boldsymbol{u}}_s\| \end{array} \right\} \quad \forall s \in [S]$$

Wasserstein Metric

- Wasserstein ambiguity set

$$\mathcal{G}(\theta) = \left\{ \mathbb{P} \in \mathcal{P}_0(\mathbb{R}^{I_u+1} \times [S]) \;\middle|\; \begin{array}{ll} ((\tilde{\boldsymbol{u}}, \tilde{v}), \tilde{s}) \sim \mathbb{P} & \\ \mathbb{E}_{\mathbb{P}}[\tilde{v} \,|\, \tilde{s} \in [S]] \leq \theta & \\ \mathbb{P}[(\tilde{\boldsymbol{u}}, \tilde{v}) \in \mathcal{Z}_s \,|\, \tilde{s} = s] = 1 & \forall s \in [S] \\ \mathbb{P}[\tilde{s} = s] = \frac{1}{S} & \forall s \in [S] \end{array} \right\}$$

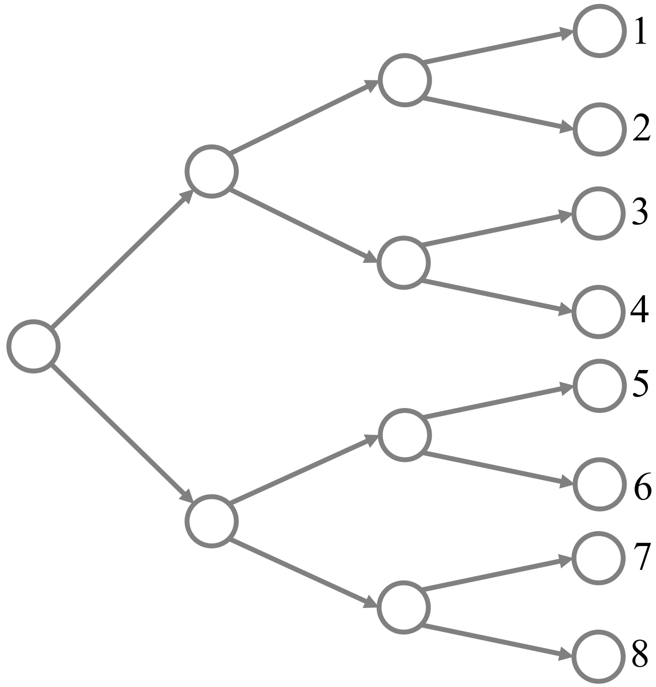
- Extra benefits
  - ✓Incorporation with other distributional information: generalized moments + Wasserstein metric
  - ✓Incorporation with probability uncertainties: Φ-divergence + Wasserstein metric

Content

- Scope of RSO
- Event-Wise Ambiguity Set
- Event-Wise Adaptation
- RSOME Toolbox
- Examples

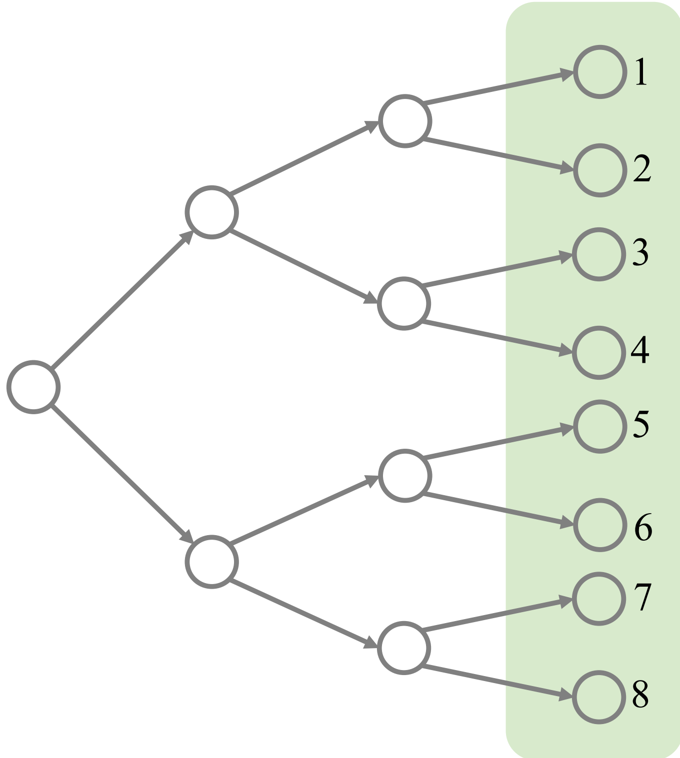
Event-Wise Recourse Adaptation

- An event consists a set of scenarios
- A set $\mathcal{C}$ of events
  - ✓Mutually exclusive
  - ✓Collectively exhaustive



Event-Wise Recourse Adaptation

- An event consists a set of scenarios
- A set $\mathcal{C}$ of events
  - ✓Mutually exclusive
  - ✓Collectively exhaustive

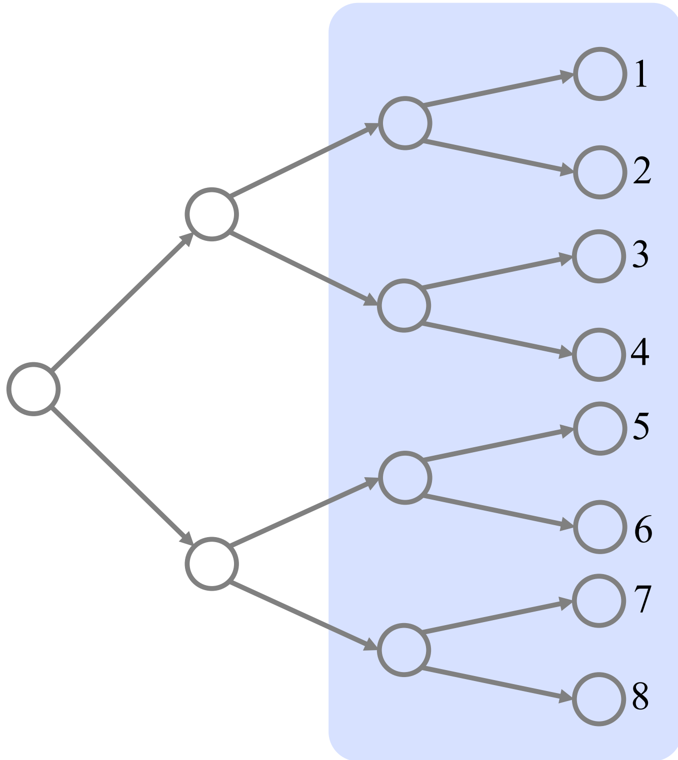


$$\mathcal{C} = \{\{1\}, \{2\}, \{3\}, \{4\}, \{5\}, \{6\}, \{7\}, \{8\}\}$$

Event-Wise Recourse Adaptation

- An event consists a set of scenarios
- A set $\mathcal{C}$ of events
  - ✓Mutually exclusive
  - ✓Collectively exhaustive

$$\mathcal{C} = \{\{1,2\},\{3,4\},\{5,6\},\{7,8\}\}$$



Event-Wise Recourse Adaptation

- Event-wise static adaptation

$$\mathcal{A}(\mathcal{C}) \triangleq \left\{ x : [S] \mapsto \mathbb{R} \;\middle|\; \begin{array}{l} x(s) = x^{\mathcal{E}}, \; \mathcal{E} = \mathcal{H}_{\mathcal{C}}(s) \\ \text{for some } x^{\mathcal{E}} \in \mathbb{R} \end{array} \right\}$$

Event-Wise Recourse Adaptation

- Event-wise static adaptation

$$\mathcal{A}(\mathcal{C}) \triangleq \left\{ x : [S] \mapsto \mathbb{R} \;\middle|\; \begin{array}{l} x(s) = x^{\mathcal{E}},\ \mathcal{E} = \mathcal{H}_{\mathcal{C}}(s) \\ \text{for some } x^{\mathcal{E}} \in \mathbb{R} \end{array} \right\}$$

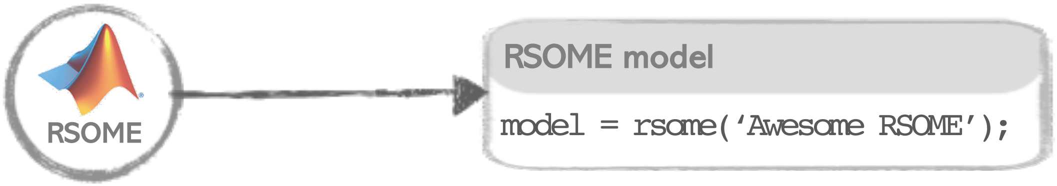
- Event-wise affine adaptation

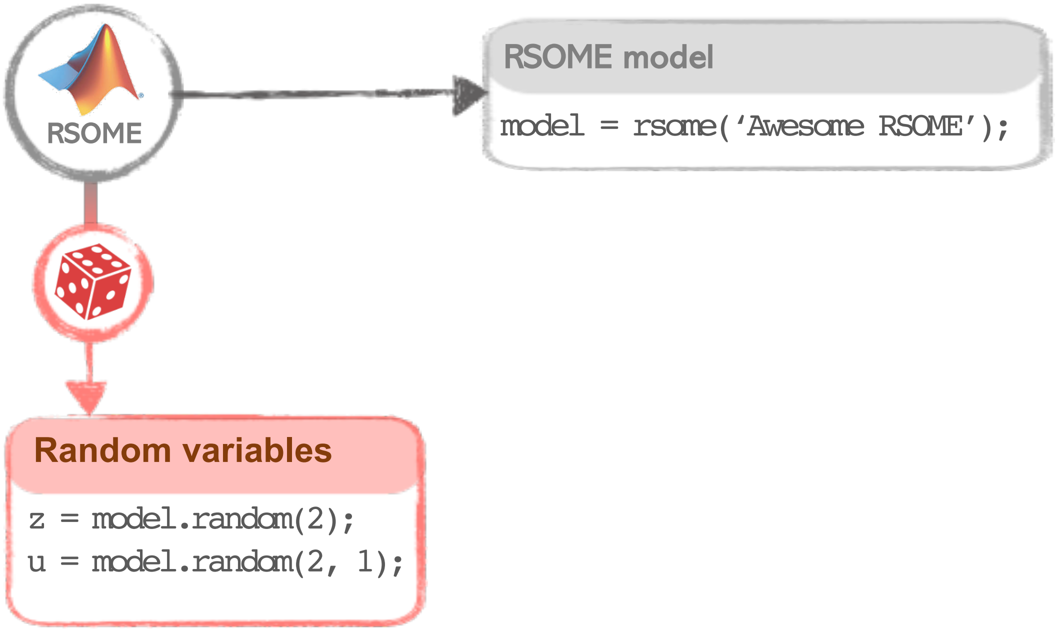
$$\bar{\mathcal{A}}(\mathcal{C}, \mathcal{I}) \triangleq \left\{ y : [S] \times \mathbb{R}^{I_z} \mapsto \mathbb{R} \;\middle|\; \begin{array}{l} y(s, \boldsymbol{z}) = y^0(s) + \sum_{i \in \mathcal{I}} y^i(s) z_i, \\ \text{for some } y^0, y^i \in \mathcal{A}(\mathcal{C}), i \in \mathcal{I} \end{array} \right\}$$

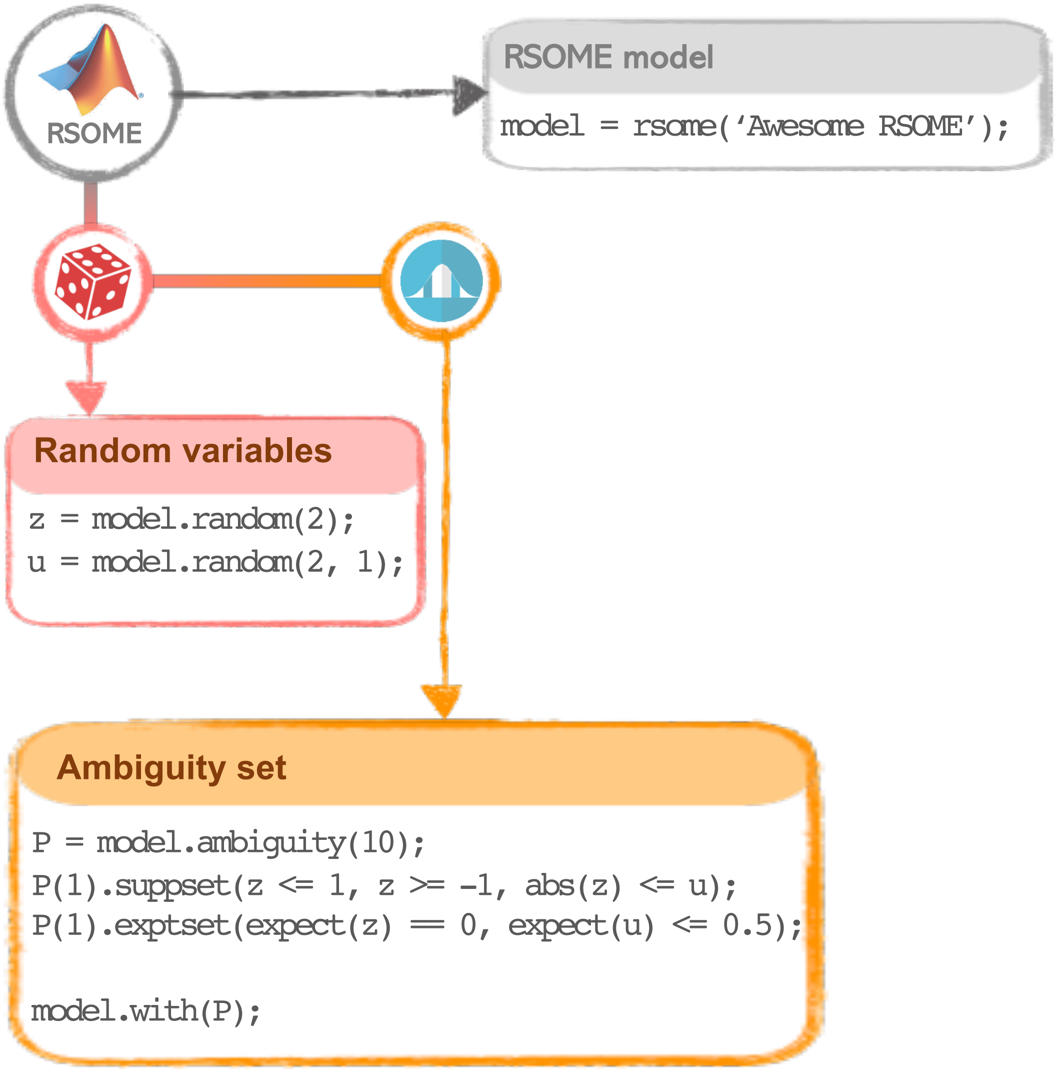
Content

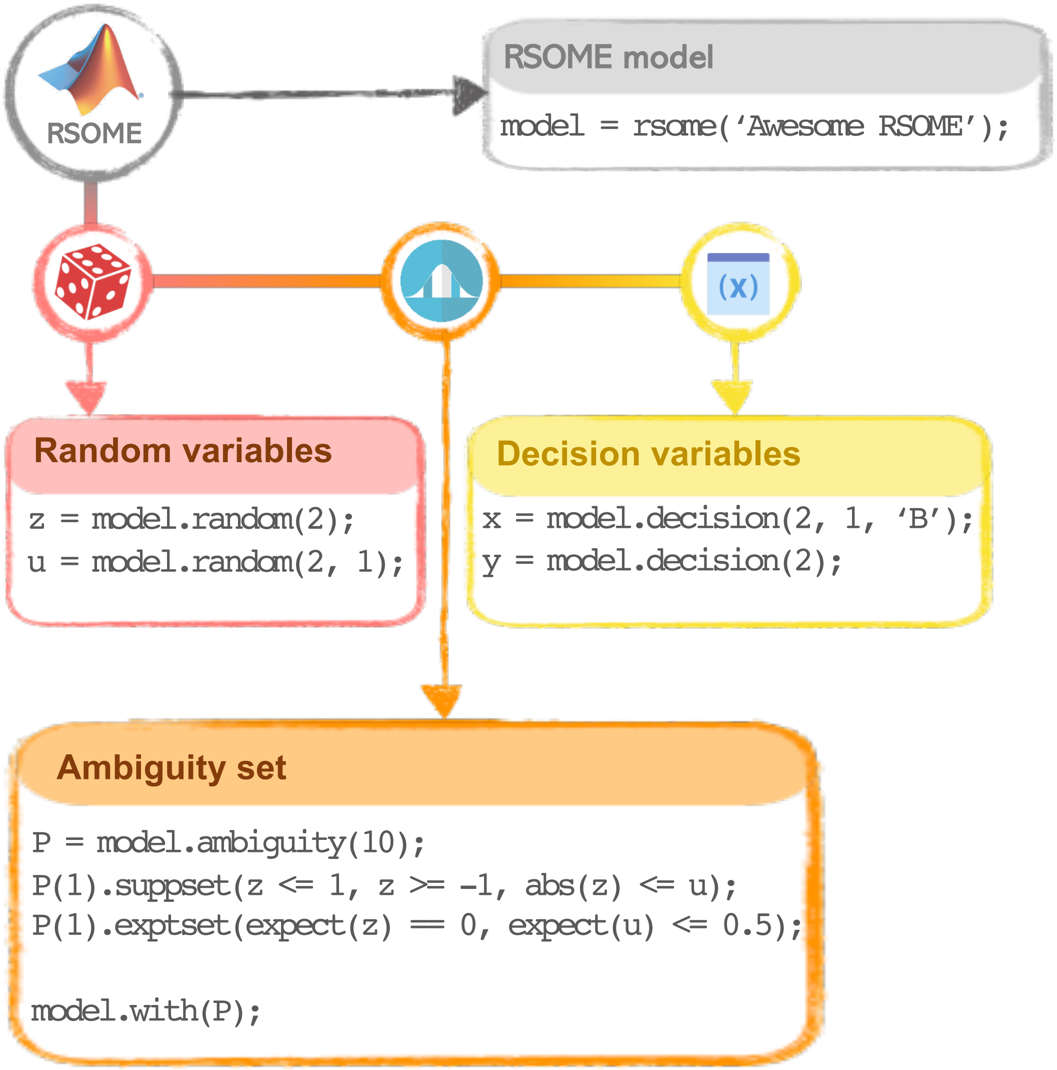
Robust Stochastic Optimization Made Easy

- Modeling toolbox (in development)
- OS
- Solvers

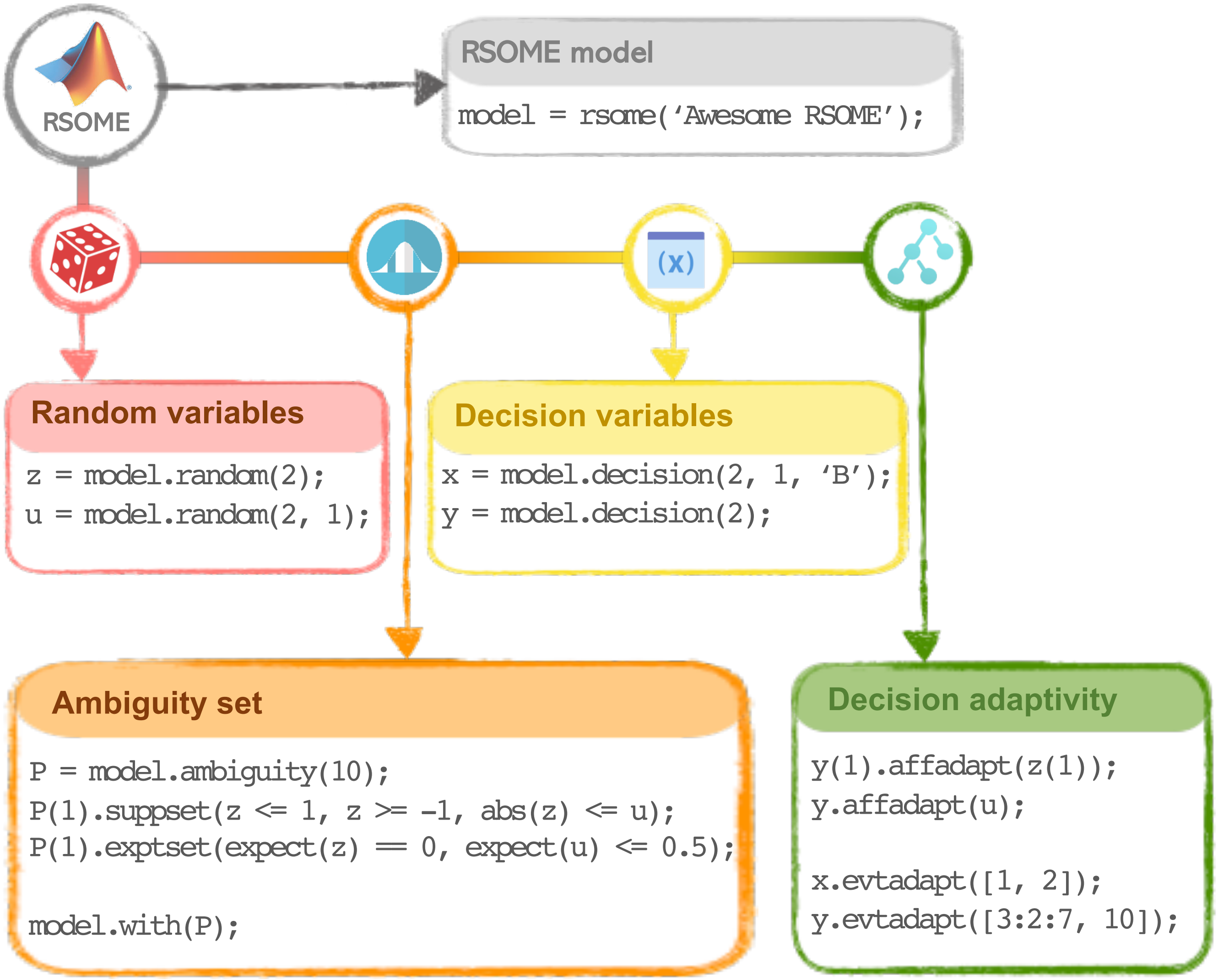

RSOME
RSOME model
model = rsome('Awesome RSOME');

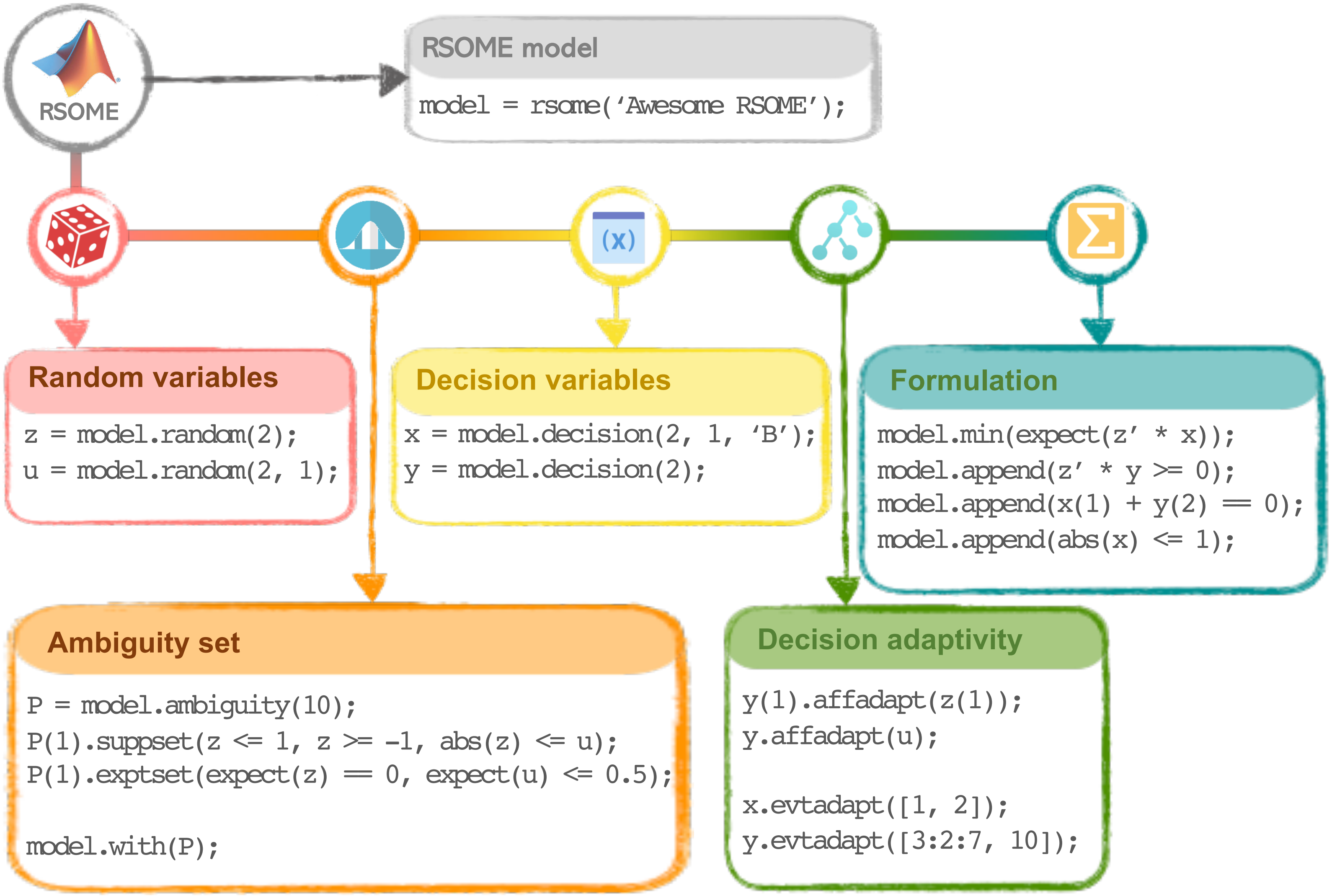

RSOME
RSOME model
model = rsome('Awesome RSOME');
Random variables
z = model.random(2);
u = model.random(2, 1);

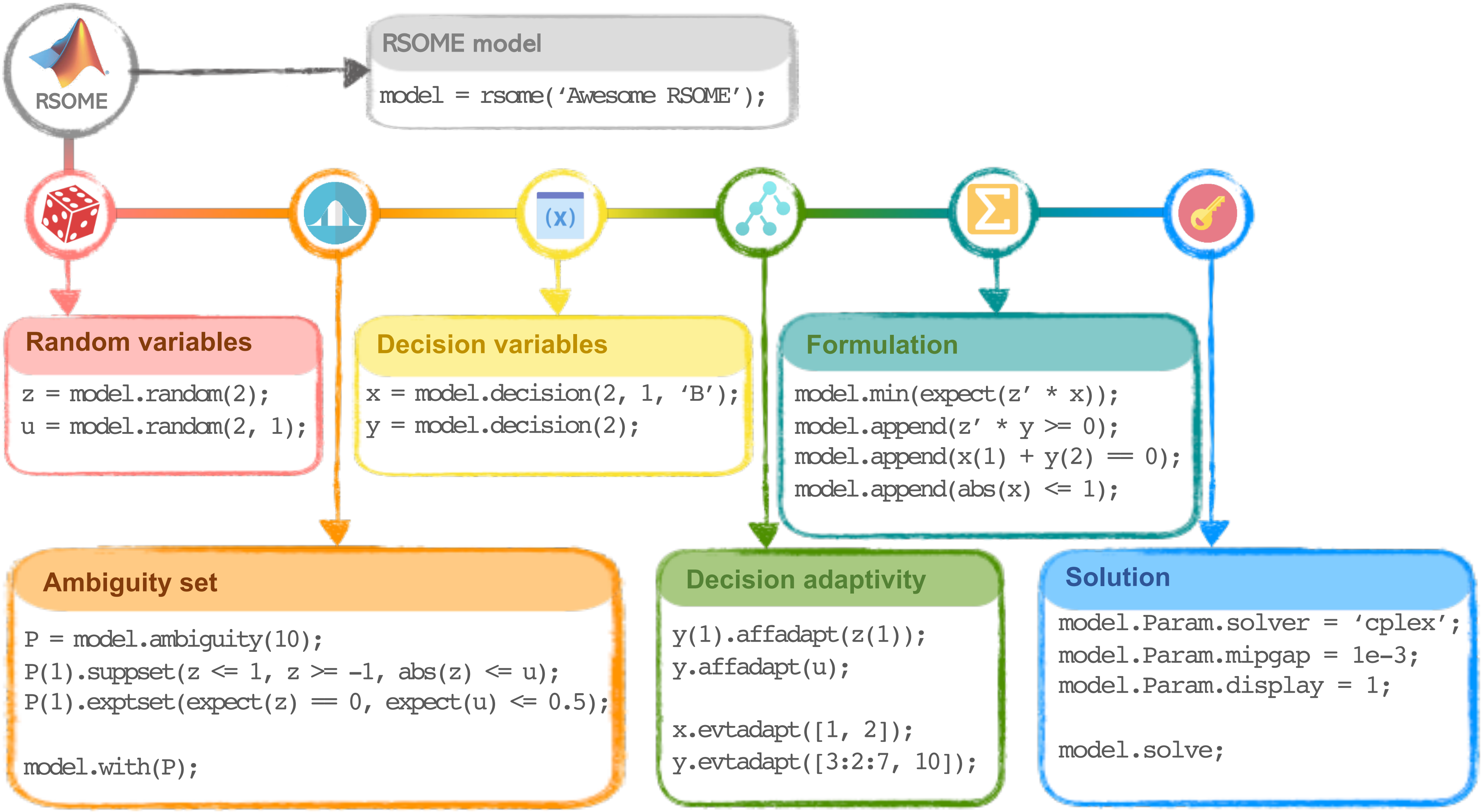

RSOME
RSOME model
model = rsome('Awesome RSOME');
Random variables
z = model.random(2);
u = model.random(2, 1);
Ambiguity set
P = model.ambiguity(10);
P(1).suppset(z <= 1, z >= -1, abs(z) <= u);
P(1).exptset(expect(z) == 0, expect(u) <= 0.5);
model.with(P);

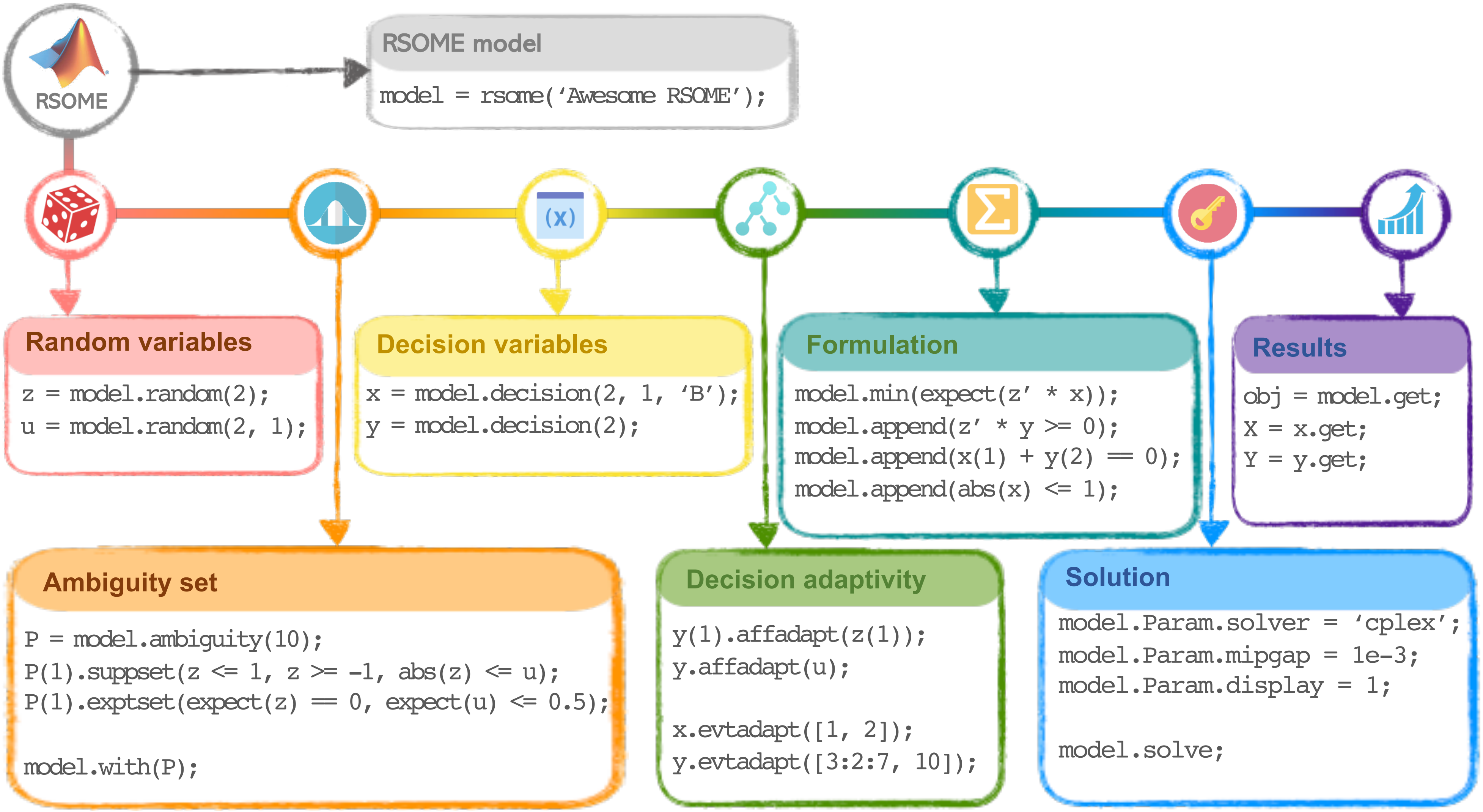


RSOME
RSOME model
model = rsome('Awesome RSOME');
Random variables
z = model.random(2);
u = model.random(2, 1);
Decision variables
x = model.decision(2, 1, 'B');
y = model.decision(2);
Ambiguity set
P = model.ambiguity(10);
P(1).suppset(z <= 1, z >= -1, abs(z) <= u);
P(1).exptset(expect(z) == 0, expect(u) <= 0.5);
model.with(P);


RSOME
RSOME model
model = rsome('Awesome RSOME');
Random variables
z = model.random(2);
u = model.random(2, 1);
Decision variables
x = model.decision(2, 1, 'B');
y = model.decision(2);
Ambiguity set
P = model.ambiguity(10);
P(1).suppset(z <= 1, z >= -1, abs(z) <= u);
P(1).exptset(expect(z) == 0, expect(u) <= 0.5);
model.with(P);
Decision adaptivity
y(1).affadapt(z(1));
y.affadapt(u);
x.evtadapt([1, 2]);
y.evtadapt([3:2:7, 10]);


RSOME
RSOME model
model = rsome('Awesome RSOME');
(x)
Random variables
z = model.random(2);
u = model.random(2, 1);
Decision variables
x = model.decision(2, 1, 'B');
y = model.decision(2);
Formulation
model.min(expect(z' * x));
model.append(z' * y >= 0);
model.append(x(1) + y(2) == 0);
model.append(abs(x) <= 1);
Ambiguity set
P = model.ambiguity(10);
P(1).suppset(z <= 1, z >= -1, abs(z) <= u);
P(1).exptset(expect(z) == 0, expect(u) <= 0.5);
model.with(P);
Decision adaptivity
y(1).affadapt(z(1));
y.affadapt(u);
x.evtadapt([1, 2]);
y.evtadapt([3:2:7, 10]);


RSOME
RSOME model
model = rsome('Awesome RSOME');
Random variables
z = model.random(2);
u = model.random(2, 1);
Decision variables
x = model.decision(2, 1, 'B');
y = model.decision(2);
Formulation
model.min(expect(z' * x));
model.append(z' * y >= 0);
model.append(x(1) + y(2) == 0);
model.append(abs(x) <= 1);
Ambiguity set
P = model.ambiguity(10);
P(1).suppset(z <= 1, z >= -1, abs(z) <= u);
P(1).exptset(expect(z) == 0, expect(u) <= 0.5);
model.with(P);
Decision adaptivity
y(1).affadapt(z(1));
y.affadapt(u);
x.evtadapt([1, 2]);
y.evtadapt([3:2:7, 10]);
Solution
model.Param.solver = 'cplex';
model.Param.mipgap = 1e-3;
model.Param.display = 1;
model.solve;



RSOME
RSOME model
model = rsome('Awesome RSOME');
Random variables
z = model.random(2);
u = model.random(2, 1);
Decision variables
x = model.decision(2, 1, 'B');
y = model.decision(2);
Formulation
model.min(expect(z' * x));
model.append(z' * y >= 0);
model.append(x(1) + y(2) == 0);
model.append(abs(x) <= 1);
Results
obj = model.get;
X = x.get;
Y = y.get;
Ambiguity set
P = model.ambiguity(10);
P(1).suppset(z <= 1, z >= -1, abs(z) <= u);
P(1).exptset(expect(z) == 0, expect(u) <= 0.5);
model.with(P);
Decision adaptivity
y(1).affadapt(z(1));
y.affadapt(u);
x.evtadapt([1, 2]);
y.evtadapt([3:2:7, 10]);
Solution
model.Param.solver = 'cplex';
model.Param.mipgap = 1e-3;
model.Param.display = 1;
model.solve;

Content

- Scope of RSOME
- Event-Wise Ambiguity Set
- Event-Wise Adaptation
- RSOME Toolbox
- Examples

Multi-Item Newsvendor Problem

- Order quantity w_i and random demand $\tilde{u}_i$

$$
\begin{aligned}
\min \quad & -\boldsymbol{p}^\top \boldsymbol{w} + \sup_{\mathbb{P} \in \mathcal{F}_W(\theta)} \mathbb{E}_{\mathbb{P}} \left[\sum_{i \in [I_u]} p_i (w_i - \tilde{u}_i)^+ \right] \\
\text{s.t.} \ & \boldsymbol{c}^\top \boldsymbol{w} = d, \ \boldsymbol{w} \geq \boldsymbol{0}
\end{aligned}
$$

Multi-Item Newsvendor Problem

- Wasserstein ambiguity set

$$
\mathcal{G}(\theta) = \left\{ \mathbb{P} \in \mathcal{P}_0(\mathbb{R}^{I_u+1} \times [S]) \;\middle|\; \begin{array}{ll} ((\tilde{\boldsymbol{u}}, \tilde{v}), \tilde{s}) \sim \mathbb{P} & \\ \mathbb{E}_{\mathbb{P}}[\tilde{v} \,|\, \tilde{s} \in [S]] \leq \theta & \\ \mathbb{P}[\tilde{\boldsymbol{u}} \in \mathcal{U},\ \tilde{v} \geq \|\tilde{\boldsymbol{u}} - \hat{\boldsymbol{u}}_s\| \,|\, \tilde{s} = s] = 1 & \forall s \in [S] \\ \mathbb{P}[\tilde{s} = s] = \frac{1}{S} & \forall s \in [S] \end{array} \right\}
$$

Multi-Item Newsvendor Problem

- Wasserstein ambiguity set

```
P = model.ambiguity(S);
P.exptset(expect(v) <= theta);
for s = 1:S
    P(s).suppset(u >= 0, u <= ubar, ...
                 norm(u - Us(:, s)) <= v);
end
pr = P.prob;
P.probset(pr == 1/S);
model.with(P);
```

$$\mathcal{G}(\theta) = \left\{ \mathbb{P} \in \mathcal{P}_0(\mathbb{R}^{I_u+1} \times [S]) \;\middle|\; \begin{array}{ll} ((\tilde{\boldsymbol{u}}, \tilde{v}), \tilde{s}) \sim \mathbb{P} & \\ \mathbb{E}_{\mathbb{P}}[\tilde{v} \,|\, \tilde{s} \in [S]] \leq \theta & \\ \mathbb{P}[\tilde{\boldsymbol{u}} \in \mathcal{U},\ \tilde{v} \geq \|\tilde{\boldsymbol{u}} - \hat{\boldsymbol{u}}_s\| \,|\, \tilde{s} = s] = 1 & \forall s \in [S] \\ \mathbb{P}[\tilde{s} = s] = \frac{1}{S} & \forall s \in [S] \end{array} \right\}$$

Multi-Item Newsvendor Problem

- Wasserstein ambiguity set

```
P = model.ambiguity(S);
P.exptset(expect(v) <= theta);
for s = 1:S
    P(s).suppset(u >= 0, u <= ubar, ...
                 norm(u - Us(:, s)) <= v);
end
pr = P.prob;
P.probset(pr == 1/S);
model.with(P);
```

$$\mathcal{G}(\theta) = \left\{ \mathbb{P} \in \mathcal{P}_0(\mathbb{R}^{I_u+1} \times [S]) \;\middle|\; \begin{array}{ll} ((\tilde{\boldsymbol{u}}, \tilde{v}), \tilde{s}) \sim \mathbb{P} & \\ \mathbb{E}_{\mathbb{P}}[\tilde{v} \,|\tilde{s} \in [S]] \leq \theta & \\ \mathbb{P}[\tilde{\boldsymbol{u}} \in \mathcal{U},\; \tilde{v} \geq \|\tilde{\boldsymbol{u}} - \hat{\boldsymbol{u}}_s\| \,|\tilde{s} = s] = 1 & \forall s \in [S] \\ \mathbb{P}[\tilde{s} = s] = \frac{1}{S} & \forall s \in [S] \end{array} \right\}$$

Multi-Item Newsvendor Problem

- Wasserstein ambiguity set

```
P = model.ambiguity(S);
P.exptset(expect(v) <= theta);
for s = 1:S
    P(s).suppset(u >= 0, u <= ubar, ...
                 norm(u - Us(:, s)) <= v);
end
pr = P.prob;
P.probset(pr == 1/S);
model.with(P);
```

$$\mathcal{G}(\theta) = \left\{ \mathbb{P} \in \mathcal{P}_0(\mathbb{R}^{I_u+1} \times [S]) \;\middle|\; \begin{array}{ll} ((\tilde{\boldsymbol{u}}, \tilde{v}), \tilde{s}) \sim \mathbb{P} & \\ \mathbb{E}_{\mathbb{P}}[\tilde{v} \,|\, \tilde{s} \in [S]] \leq \theta & \\ \mathbb{P}[\tilde{\boldsymbol{u}} \in \mathcal{U}, \;\; \tilde{v} \geq \|\tilde{\boldsymbol{u}} - \hat{\boldsymbol{u}}_s\| \,|\, \tilde{s} = s] = 1 & \forall s \in [S] \\ \mathbb{P}[\tilde{s} = s] = \frac{1}{S} & \forall s \in [S] \end{array} \right\}$$

Multi-Item Newsvendor Problem

- Wasserstein ambiguity set

```
P = model.ambiguity(S);
P.exptset(expect(v) <= theta);
for s = 1:S
    P(s).suppset(u >= 0, u <= ubar, ...
                 norm(u - Us(:, s)) <= v);
end
pr = P.prob;
P.probset(pr == 1/S);
model.with(P);
```

$$\mathcal{G}(\theta) = \left\{ \mathbb{P} \in \mathcal{P}_0(\mathbb{R}^{I_u+1} \times [S]) \;\middle|\; \begin{array}{ll} ((\tilde{\boldsymbol{u}}, \tilde{v}), \tilde{s}) \sim \mathbb{P} & \\ \mathbb{E}_{\mathbb{P}}[\tilde{v} \,|\, \tilde{s} \in [S]] \leq \theta & \\ \mathbb{P}[\tilde{\boldsymbol{u}} \in \mathcal{U}, \; \tilde{v} \geq \|\tilde{\boldsymbol{u}} - \hat{\boldsymbol{u}}_s\| \,|\, \tilde{s} = s] = 1 & \forall s \in [S] \\ \mathbb{P}[\tilde{s} = s] = \frac{1}{S} & \forall s \in [S] \end{array} \right\}$$

Multi-Item Newsvendor Problem

- Wasserstein ambiguity set

```
P = model.ambiguity(S);
P.exptset(expect(v) <= theta);
for s = 1:S
    P(s).suppset(u >= 0, u <= ubar, ...
                 norm(u - Us(:, s)) <= v);
end
pr = P.prob;
P.probset(pr == 1/S);
model.with(P);
```

$$\mathcal{G}(\theta) = \left\{ \mathbb{P} \in \mathcal{P}_0(\mathbb{R}^{I_u+1} \times [S]) \left| \begin{array}{ll} ((\tilde{\boldsymbol{u}}, \tilde{v}), \tilde{s}) \sim \mathbb{P} & \\ \mathbb{E}_{\mathbb{P}}[\tilde{v} \,|\, \tilde{s} \in [S]] \leq \theta & \\ \mathbb{P}[\tilde{\boldsymbol{u}} \in \mathcal{U}, \ \tilde{v} \geq \|\tilde{\boldsymbol{u}} - \hat{\boldsymbol{u}}_s\| \,|\, \tilde{s} = s] = 1 & \forall s \in [S] \\ \mathbb{P}[\tilde{s} = s] = \frac{1}{S} & \forall s \in [S] \end{array} \right. \right\}$$

Multi-Item Newsvendor Problem

- An upper bound via event-wise adaptation $\boldsymbol{y}(s, \boldsymbol{z})$, where $\boldsymbol{z} = (\boldsymbol{u}, v)$

$$
\begin{aligned}
\min \quad & -\boldsymbol{p}^\top \boldsymbol{w} + \sup_{\mathbb{P} \in \mathcal{F}_W(\theta)} \mathbb{E}_{\mathbb{P}} \left[\boldsymbol{p}^\top \boldsymbol{y}(s, \boldsymbol{z}) \right] \\
\text{s.t. } & \boldsymbol{c}^\top \boldsymbol{w} = d, \ \boldsymbol{w} \geq \boldsymbol{0} \\
& \boldsymbol{y}(s, \boldsymbol{z}) \geq \boldsymbol{0}, && \forall \boldsymbol{z} \in \mathcal{Z}_s, s \in [S] \\
& \boldsymbol{y}(s, \boldsymbol{z}) \geq \boldsymbol{w} - \boldsymbol{u}, && \forall \boldsymbol{z} \in \mathcal{Z}_s, s \in [S] \\
& y_i \in \bar{\mathcal{A}}(\mathcal{C}, \mathcal{I}), && \forall i \in [I_u]
\end{aligned}
$$

Multi-Item Newsvendor Problem

- Event-wise adaptation $y_i \in \bar{\mathcal{A}}(\mathcal{C}, \mathcal{I})$

```
w = decision(I);              % here-and-now decision w

y = decision(I);              % wait-and-see decision y
for s = 1:S
    y.evtadapt(s);           % each scenario is added as a new adaptive event of y
end
y.affadapt(u);               % y affinely depends on u
y.affadapt(v);               % y affinely depends on v
```

Multi-Item Newsvendor Problem

- Event-wise adaptation $y_i \in \bar{\mathcal{A}}(\mathcal{C}, \mathcal{I})$

```
w = decision(I);            % here-and-now decision w

y = decision(I);            % wait-and-see decision y
for s = 1:S
    y.evtadapt(s);         % each scenario is added as a new adaptive event of y
end
y.affadapt(u);              % y affinely depends on u
y.affadapt(v);              % y affinely depends on v
```

Multi-Item Newsvendor Problem

- Event-wise adaptation $y_i \in \bar{A}(\mathcal{C}, \mathcal{I})$

```
w = decision(I);               % here-and-now decision w

y = decision(I);               % wait-and-see decision y
for s = 1:S
    y.evtadapt(s);            % each scenario is added as a new adaptive event of y
end
y.affadapt(u);                 % y affinely depends on u
y.affadapt(v);                 % y affinely depends on v
```

Multi-Item Newsvendor Problem

- Model specification
  ✓An upper bound via event-wise adaptation $\boldsymbol{y}(s,\boldsymbol{z})$, where $\boldsymbol{z} = (\boldsymbol{u}, v)$

$$
\begin{aligned}
\min \quad & -\boldsymbol{p}^\top \boldsymbol{w} + \sup_{\mathbb{P} \in \mathcal{F}_W(\theta)} \mathbb{E}_{\mathbb{P}}\left[\boldsymbol{p}^\top \boldsymbol{y}(s,\boldsymbol{z})\right] \\
\text{s.t. } & \boldsymbol{c}^\top \boldsymbol{w} = d, \ \boldsymbol{w} \geq \boldsymbol{0} \\
& \boldsymbol{y}(s,\boldsymbol{z}) \geq \boldsymbol{0}, && \forall \boldsymbol{z} \in \mathcal{Z}_s, s \in [S] \\
& \boldsymbol{y}(s,\boldsymbol{z}) \geq \boldsymbol{w} - \boldsymbol{u}, && \forall \boldsymbol{z} \in \mathcal{Z}_s, s \in [S] \\
& y_i \in \bar{\mathcal{A}}(\mathcal{C}, \mathcal{I}), && \forall i \in [I_u]
\end{aligned}
$$

Multi-Item Newsvendor Problem

- Model specification
  - ✓An upper bound via event-wise adaptation $\boldsymbol{y}(s, \boldsymbol{z})$ where $\boldsymbol{z} = (\boldsymbol{u}, v)$

$$
\begin{aligned}
\min \quad & -\boldsymbol{p}^\top \boldsymbol{w} + \sup_{\mathbb{P} \in \mathcal{F}_W(\theta)} \mathbb{E}_{\mathbb{P}} \left[\boldsymbol{p}^\top \boldsymbol{y}(s, \boldsymbol{z}) \right] & \\
\text{s.t. } & \boldsymbol{c}^\top \boldsymbol{w} = d, \;\; \boldsymbol{w} \geq \boldsymbol{0} & \\
& \boldsymbol{y}(s, \boldsymbol{z}) \geq \boldsymbol{0}, & \forall \boldsymbol{z} \in \mathcal{Z}_s, s \in [S] \\
& \boldsymbol{y}(s, \boldsymbol{z}) \geq \boldsymbol{w} - \boldsymbol{u}, & \forall \boldsymbol{z} \in \mathcal{Z}_s, s \in [S] \\
& y_i \in \bar{\mathcal{A}}(\mathcal{C}, \mathcal{I}), & \forall i \in [I_u]
\end{aligned}
$$

```
model.min(-p'*w + expect(p'*y));

model.append(c' * w == d);
model.append(w >= 0);
model.append(y >= 0);
model.append(y >= w - z);
```

Comparison with Exact Solution

		θ					
		1	2	5	10	20	50
N	5	< 0.1	< 0.1	0.1	0.2	0.5	1.3
	10	< 0.1	< 0.1	0.2	0.3	1.0	1.7
	20	< 0.1	< 0.1	< 0.1	0.1	0.3	1.0
	50	< 0.1	< 0.1	< 0.1	0.1	0.3	0.6

Table 1: 90-th percentile optimality gaps (in %) of the scenario-wise linear decision rule approximation (5 items, 100 random instances).

Comparison with Exact Solution

		θ					
		1	2	5	10	20	50
N	5	< 0.1	0.1	0.2	0.3	0.6	0.3
	10	< 0.1	0.1	0.1	0.2	0.5	1.2
	20	< 0.1	0.1	0.1	0.2	0.6	1.6
	50	0.1	0.1	0.2	0.2	0.5	2.1

Table 1: 90-th percentile optimality gaps (in %) of the scenario-wise linear decision rule approximation (7 items, 100 random instances).

Experiment of Hanasusanto & Kuhn (2018)

Consider the second-stage problem of the form

$$f(\boldsymbol{u}) = \min\{\boldsymbol{e}^\top \boldsymbol{y} \mid \boldsymbol{y} \geq \boldsymbol{0}, \boldsymbol{y} \geq \boldsymbol{A}\boldsymbol{u} - \boldsymbol{b}\},$$

where $\boldsymbol{e}$ is a vector of ones. The problem does not have any here-and-now decision $\boldsymbol{w}$ and assumes that the random variable $\tilde{\boldsymbol{u}}$ resides in a box $\mathcal{U} = [0,1]^{I_u}$.

Consider the Wasserstein ambiguity set with a distance metric $\|\boldsymbol{u} - \boldsymbol{u}'\|_2^2$.

Two Approximation Methods

- Exact reformulation is a copositive program (COP approach) that can be approximated fairly well by a positive semidefinite program (Hanasusanto and Kuhn (2018)).

$$\boldsymbol{M} \succeq_{\mathcal{C}} \mathbf{0} \Longleftarrow \boldsymbol{M} : \boldsymbol{M} = \boldsymbol{M}_1 + \boldsymbol{M}_2, \boldsymbol{M}_1 \succeq \mathbf{0}, \boldsymbol{M}_2 = \boldsymbol{M}_2' \geq \mathbf{0}.$$

- Approximation using the event-wise affine adaptation is a second-order conic program.
- Performance based on randomly generated instances as in Hanasusanto and Kuhn (2018): solutions coincides for all instances!

Computation Times

		I_u									
		1		2		4		8		16	
S	5	0.3	< 0.1	0.3	< 0.1	0.3	< 0.1	0.5	< 0.1	2.3	< 0.1
	10	0.4	< 0.1	0.4	< 0.1	0.5	< 0.1	0.9	< 0.1	5.1	< 0.1
	20	0.6	< 0.1	0.7	< 0.1	0.8	< 0.1	1.8	< 0.1	11.6	< 0.1
	40	1.1	< 0.1	1.3	< 0.1	1.6	< 0.1	3.5	< 0.1	23.1	0.1
	80	2.3	< 0.1	2.5	< 0.1	3.2	< 0.1	7.8	0.1	51.1	0.1
	160	4.5	< 0.1	5.1	< 0.1	7.0	0.1	18.0	0.2	118.0	0.3
	320	9.2	0.1	10.8	0.1	15.5	0.2	45.4	0.3	281.5	0.6
	640	19.7	0.1	26.9	0.2	43.9	0.3	141.5	1.0	684.3	2.3

Table 1: Computation times (in seconds) of K_0-approximation (left) and event-wise affine adaptation (right).

Robust Stochastic Optimization Made Easy

RSOME toolbox

RSOME user guide

RSO paper