Linear Algebra: Homework #2

Due on September 4, 2019

Professor MacArthur

Carson Storm

Problem 1 (1.6#4)

Suppose an economy has four sectors, Agriculture (A), Energy (E), Manufacturing (M), and Transportation (T). Sector A sells 10% of its output to E and 25% to M and reatins the rese. Sectore E sells 30% of its output to A, 35% to M, and 25% to T and reatins the rest. Sector M sells 30% of its output to A, 15% to E, and 40% to T and reatins the rest. Sector T sells 20% of its output to A, 10% to E, and 30% to M and retains the rest.

a. Construct the exchange table for this economy.

b. [M] Find a set of equilibrium prices for the economy.

Problem 2 (1.6#7)

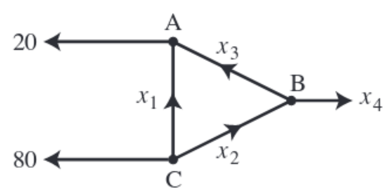
Alka-Seltzer contains sodium bicarbonate ($NaHCO_3$) and citric acid ($H_3C_6H_5O_7$). When a tablet is dissolved in water, the following reaction produces sodium citrate, water, and carbon dioxide (gas):

$$NaHCO_3 + H_3C_6H_5O_7 \rightarrow Na_3C_6H_5O_7 + H_2O + CO_2$$

Balance the chemical equaion using the vector equation approach.

Problem 3 (1.6#11)

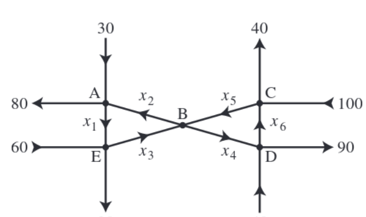
Find the general flow pattern of the network shown in the figure. Assuming that the flows are all nonnegative, what is the largest possible value for x_3?



Problem 4 (1.6#13)

a. Find the general flow pattern in the network shown in the figure.

b. Assuming that the flow must be in the directions indicated, find the minimum flows in the branches denoted by x_2, x_3, x_4 and x_5.



Problem 5 (1.7#1)

Determine if the vectors are linearly independent. Justify answer.

$$\begin{bmatrix}5\\0\\0\end{bmatrix}, \begin{bmatrix}7\\2\\-6\end{bmatrix}, \begin{bmatrix}9\\4\\-8\end{bmatrix}$$

Problem 6 (1.7#3)

Determine if the vectors are linearly independent. Justify answer.

$$\begin{bmatrix}1\\-3\end{bmatrix}, \begin{bmatrix}-3\\9\end{bmatrix}$$

Problem 7 (1.7#5)

Determine if the columns of the matrix form a linearly independent set. Justify your answer.

$$\begin{bmatrix}0 & -8 & 5\\3 & -7 & 4\\-1 & 5 & -4\\1 & -3 & 2\end{bmatrix}$$

Problem 8 (1.7#7)

Determine if the columns of the matrix form a linearly independent set. Justify your answer.

$$\begin{bmatrix}1 & 4 & -3 & 0\\-2 & -7 & 5 & 1\\-4 & -5 & 7 & 5\end{bmatrix}$$

Problem 9 (1.7#11)

Find the value(s) of h for which the vectors are linearly dependent. Justify your answer.

$$\begin{bmatrix}1\\-1\\4\end{bmatrix}, \begin{bmatrix}3\\-5\\7\end{bmatrix}, \begin{bmatrix}-1\\5\\h\end{bmatrix}$$

Problem 10 (1.7#13)

Find the value(s) of h for which the vectors are linearly dependent. Justify your answer.

$$\begin{bmatrix}1\\5\\-3\end{bmatrix}, \begin{bmatrix}-2\\-9\\6\end{bmatrix}, \begin{bmatrix}3\\h\\-9\end{bmatrix}$$

Problem 11 (1.7#15)

Determine by inspection whether the vectors are linearly independent. Justify your answer.

$$\begin{bmatrix}5\\1\end{bmatrix}, \begin{bmatrix}2\\8\end{bmatrix}, \begin{bmatrix}1\\3\end{bmatrix}, \begin{bmatrix}-1\\7\end{bmatrix}$$

Problem 12 (1.7#17)

Determine by inspection whether the vectors are linearly independent. Justify your answer.

$$\begin{bmatrix}3\\5\\-1\end{bmatrix}, \begin{bmatrix}0\\0\\0\end{bmatrix}, \begin{bmatrix}-6\\5\\4\end{bmatrix}$$

Problem 13 (1.7#19)

Determine by inspection whether the vectors are linearly independent. Justify your answer.

$$\begin{bmatrix}-8\\12\\-4\end{bmatrix}, \begin{bmatrix}2\\-3\\-1\end{bmatrix}$$

Problem 14 (1.7#21)

Mark each statment True or False. Justify each answer on the basis of a careful reading of the text.

1. The columns of a matrix A are linearly independent if the equation $A\boldsymbol{x} = 0$ has the trivial solution
2. If S is a linearly dependent set, then each vector is a linear combination of the other vectors in S.
3. The columns of any 4×5 matrix are linearly dependent.
4. If x and y are linearly independent, and if $\{x, y, z\}$ is linearly dependent, then z is in $Span\{x, y\}$

Problem 15 (1.7#23)

Describe the possible echelon forms of A, a 3×3 matrix with linearly independent columns. Use the notation of Example 1 in Section 1.2

Problem 16 (1.7#25)

Describe the possible echelon forms of A, a 2×2 matrix with linearly dependent columns. Use the notation of Example 1 in Section 1.2

Problem 17 (1.8#1)

Let $A = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$, and define $T : \mathbb{R}^2 \to R^2$ by $T(\boldsymbol{x}) = A\boldsymbol{x}$. Find the images under T of $\boldsymbol{u} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$, and $\boldsymbol{v} = \begin{bmatrix} a \\ b \end{bmatrix}$.

Problem 18 (1.8#2)

Let $A = \begin{bmatrix} .5 & 0 & 0 \\ 0 & .5 & 0 \\ 0 & 0 & .5 \end{bmatrix}, \boldsymbol{u} = \begin{bmatrix} 1 \\ 0 \\ -4 \end{bmatrix}$, and $\boldsymbol{v} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$. Define $T : \mathbb{R}^3 \to R^3$ by $T(\boldsymbol{x}) = A\boldsymbol{x}$. Find $T(\boldsymbol{u})$ and $T(\boldsymbol{v})$.

Problem 19 (1.8#3)

Find a vector $\boldsymbol{x}$ whose image under T is $\boldsymbol{b}$ and determine whether x is unique. $T(\boldsymbol{x}) = A\boldsymbol{x}$.

$$A = \begin{bmatrix} 1 & 0 & -2 \\ -2 & 1 & 6 \\ 3 & -2 & -5 \end{bmatrix}, \boldsymbol{b} = \begin{bmatrix} 1 \\ 9 \\ 3 \\ -6 \end{bmatrix}$$

Problem 20 (1.8#5)

Find a vector $\boldsymbol{x}$ whose image under T is $\boldsymbol{b}$ and determine whether x is unique. $T(\boldsymbol{x}) = A\boldsymbol{x}$.

$$A = \begin{bmatrix} 1 & -5 & -7 \\ -3 & 7 & 5 \end{bmatrix}, \boldsymbol{b} = \begin{bmatrix} -2 \\ -2 \end{bmatrix}$$

Problem 21 (1.8#7)

Let A be 6×5 matrix. What must a and b be in order to define $T : \mathbb{R}^a \to \mathbb{R}^b$ by $T(x) = A\boldsymbol{x}$?

Problem 22 (1.8#9)

Find all x in $\mathbb{R}^4$ that are mapped into the zero vector by the transofmration $\boldsymbol{x} \mapsto A\boldsymbol{x}$ for the given matrix A.

$$\begin{bmatrix} 1 & -4 & 7 & -5 \\ 0 & 1 & -4 & 3 \\ 2 & -6 & 6 & -4 \end{bmatrix}$$

Problem 23 (1.8#11)

Let $\boldsymbol{b} = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$, and let A be the matrix be the matrix in Exercise 9. Is $\boldsymbol{b}$ in the range of the linear transofmration $\boldsymbol{x} \mapsto A\boldsymbol{x}$? Why or why not?

Problem 24 (1.8#13)

Use a rectangular coordinate system to plot $\boldsymbol{u} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$, $\boldsymbol{v} = \begin{bmatrix} -2 \\ 4 \end{bmatrix}$, and their images under the given transformation T. Describe geometrically what T does to each vector x in $\mathbb{R}^2$.

$$T(\boldsymbol{x}) = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Problem 25 (1.8#16)

Use a rectangular coordinate system to plot $\boldsymbol{u} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$, $\boldsymbol{v} = \begin{bmatrix} -2 \\ 4 \end{bmatrix}$, and their images under the given transformation T. Describe geometrically what T does to each vector x in $\mathbb{R}^2$.

$$T(\boldsymbol{x}) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Problem 26 (1.8#30)

An *affine transofmration* $T : \mathbb{R}^n \rightarrow R^m$ has the form $T(\boldsymbol{x}) = A\boldsymbol{x} + \boldsymbol{b}$, with A an $m \times n$ matrix and $\boldsymbol{b}$ in $\mathbb{R}^m$. Show that T is *not* a linear transofmation when $\boldsymbol{b} \neq 0$

Problem 27 (1.8#33)

Show that the transformation T defined by $T(x_1, x_2) = (2x_1 - 3x_2, x_1 + 4, 5x_2)$ is not linear.