Linear Algebra: Homework #2

Due on September 4, 2019

Professor MacArthur

Carson Storm

Problem 1 (1.4#1)

Compute the product using (a) the definition, as in Example 1, and (b) the row–vector rule for computing $A\boldsymbol{x}$. If aproduct is undefined, explain why.

$$\begin{bmatrix} -4 & 2 \\ 1 & 6 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ -2 \\ 7 \end{bmatrix}$$

Problem 2 (1.4#3)

Compute the product using (a) the definition, as in Example 1, and (b) the row–vector rule for computing $A\boldsymbol{x}$. If aproduct is undefined, explain why.

$$\begin{bmatrix} 6 & 5 \\ -4 & 4 \\ 7 & 6 \end{bmatrix} \begin{bmatrix} -2 \\ 3 \end{bmatrix}$$

Problem 3 (1.4#7)

Use the definition of $A\boldsymbol{x}$ to write the vector equation as a matrix equation.

$$\begin{bmatrix} 4 \\ -1 \\ 7 \\ -4 \end{bmatrix} x_1 + \begin{bmatrix} -5 \\ 3 \\ -5 \\ 1 \end{bmatrix} x_2 + \begin{bmatrix} 7 \\ -8 \\ 0 \\ 2 \end{bmatrix} x_3 = \begin{bmatrix} 6 \\ -8 \\ 0 \\ -7 \end{bmatrix}$$

Problem 4 (1.4#11)

Given A and $\boldsymbol{b}$, write the augmented matrix for the linear system that corresponds to the matrix equation $A\boldsymbol{x} = \boldsymbol{b}$. Then solve the system and write the solution as a vector.

$$A = \begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & 5 \\ -2 & -4 & -3 \end{bmatrix}, \boldsymbol{b} = \begin{bmatrix} -2 \\ 2 \\ 9 \end{bmatrix}$$

Problem 5 (1.4#15)

Let $A = \begin{bmatrix} 2 & -1 \\ -6 & 3 \end{bmatrix}$ and $\boldsymbol{b} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$. Show that the equation $A\boldsymbol{x} = \boldsymbol{b}$ does not have a solution for all possible $\boldsymbol{b}$, and describe the solution set of all $\boldsymbol{b}$ for which $A\boldsymbol{x} = \boldsymbol{b}$ *does* has a solution.

Problem 6 (1.4#19)

Can each vector in $\mathbb{R}^4$ be written as a linear combination of the columns of the matrix A? Do the columns of A span $\mathbb{R}^4$?

Problem 7 (1.4#21)

Let $\boldsymbol{v_1} = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}$, $\boldsymbol{v_2} = \begin{bmatrix} 0 \\ -1 \\ 0 \\ 1 \end{bmatrix}$, $\boldsymbol{v_3} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}$. Does $\{\boldsymbol{v_1}, \boldsymbol{v_2}, \boldsymbol{v_3}\}$ span $\mathbb{R}^3$? Why or why not?

Problem 8 (1.4#25)

Mark each statement as True or False. Justify your answer.

(a) The equation $A\boldsymbol{x} = b$ is referred to as the *vector equation*.

(b) A vector $\boldsymbol{b}$ is a linear combination of the columns of a matrix A if and only if the equation $A\boldsymbol{x} = \boldsymbol{b}$ has at least one solution.

(c) The equation $A\boldsymbol{x} = \boldsymbol{b}$ is consistent if the augmented matrix $\begin{bmatrix} A & \boldsymbol{b} \end{bmatrix}$ has a pivot position in every row.

(d) The first entry in the product $A\boldsymbol{x}$ is a sum of products.

(e) If the columns of an $m \times n$ matrix A span $\mathbb{R}^m$, then the equation $A\boldsymbol{x} = \boldsymbol{b}$ is consistent for each b in $\mathbb{R}^m$.

(f) If A is an $m \times n$ matrix and if the equation $A\boldsymbol{x} = \boldsymbol{b}$ is inconsistent for some $\boldsymbol{b}$ in $\mathbb{R}^m$, then A cannot have a pivot position in every row.

Problem 9 (1.5#1)

Determin if the system has a nontrival solution. Try to use as few row operations as possible.

$$\begin{aligned} 2x_1 - 5x_2 + 8x_3 &= 0 \\ -2x_2 - 7x_2 + x_3 &= 0 \\ 4x_1 + 2x_2 + 7x_3 &= 0 \end{aligned}$$

Problem 10 (1.5#3)

Determin if the system has a nontrival solution. Try to use as few row operations as possible.

$$\begin{aligned} -3x_1 + 5x_2 - 7x_3 &= 0 \\ -6x_1 + 7x_2 + x_3 &= 0 \end{aligned}$$

Problem 11 (1.5#7)

Describe all solutions of $A\boldsymbol{x} = \boldsymbol{0}$ in parametric form, where A is row equivalent to the given matrix.

$$\begin{bmatrix} 1 & 3 & -3 & 7 \\ 0 & 1 & -4 & 5 \end{bmatrix}$$

Problem 12 (1.5#11)

Describe all solutions of $A\boldsymbol{x} = \mathbf{0}$ in parametric form, where A is row equivalent to the given matrix.

$$\begin{bmatrix} 1 & -4 & -2 & 0 & 3 & -5 \\ 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Problem 13 (1.5#13)

Suppose the solution set of a certain linear system of equations can be described as $x_1 = 5 + 4x_3$, $x_2 = -2 - 7x_3$, with x_3 free. Use vectors to describe this set as a line in $\mathbb{R}^3$.

Problem 14 (1.5#15)

Follow the method of Example 3 to describe the solution of the following system in parametric form. Also, give a geometric description of the solution set and compare it to that in Exercise 5.

$$\begin{aligned} x_1 + 3x_2 + x_3 &= 1 \\ -4x_1 - 9x_2 + 2x_3 &= -1 \\ -3x_2 - 6x_3 &= -3 \end{aligned}$$

Problem 15 (1.5#19)

Find the parametric equation of the line through $\boldsymbol{a}$ and parallel to $\boldsymbol{b}$.

$$\boldsymbol{a} = \begin{bmatrix} -2 \\ 0 \end{bmatrix}, \boldsymbol{b} = \begin{bmatrix} -5 \\ 3 \end{bmatrix}$$

Problem 16 (1.5#25)

Prove the second part of Theorem 6: Let $\boldsymbol{w}$ be any solution of $A\boldsymbol{x} = \boldsymbol{b}$, and define $\boldsymbol{v_h} = \boldsymbol{w} - \boldsymbol{p}$. Show that $\boldsymbol{v_h}$ is a solution of $A\boldsymbol{x} = \mathbf{0}$. This shows that every solution of $A\boldsymbol{x} = \boldsymbol{b}$ has the form $\boldsymbol{w} = \boldsymbol{p} + \boldsymbol{v_h}$, with $\boldsymbol{p}$ a particular solution of $A\boldsymbol{x} = \boldsymbol{b}$ and $\boldsymbol{v_h}$ a solution of $A\boldsymbol{x} = 0$.

Problem 17 (1.5#27)

Suppose A is a 3×3 *zero* matrix (with all zero entries). Describe the solution set of the equation $A\boldsymbol{x} = \mathbf{0}$.

Problem 18 (1.5#39s)

Let A be an $m \times n$ matrix, and let $\boldsymbol{u}$ be a vector in $\mathbb{R}^n$ that satisfies the equation $A\boldsymbol{x} = \mathbf{0}$. Show that for any scalar c the vector $c\boldsymbol{u}$ also satisfies $A\boldsymbol{x} = \mathbf{0}$. [That is, show that $A(c\boldsymbol{u}) = \mathbf{0}$].