Linear Algebra: Homework #3

Due on September 11, 2019

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Problem 1 (1.9#1)

Assuming T is a linear transformation. Find the standard matrix of T where $T : \mathbb{R}^2 \to \mathbb{R}^4, T(\boldsymbol{e_1}) = (3, 1, 3, 1)$ and $T(\boldsymbol{e_2}) = (-5, 2, 0, 0)$.

Problem 2 (1.9#3)

Assuming T is a linear transformation. Find the standard matrix of T where $T : \mathbb{R}^2 \to \mathbb{R}^2$ rotates poing (about the origin) through $3\pi/2$ radians (counterclockwise).

Problem 3 (1.9#5)

Assuming T is a linear transformation. Find the standard matrix of T where $T : \mathbb{R}^2 \to \mathbb{R}^2$ is a vertical shear transformation that maps $\boldsymbol{e_1}$ into $\boldsymbol{e_1} - 2\boldsymbol{e_2}$ but leaves the vector $\boldsymbol{e_2}$ unchanged.

Problem 4 (1.9#10)

Assuming T is a linear transformation. Find the standard matrix of T where $T : \mathbb{R}^2 \to \mathbb{R}^2$ first reflects points through the vertical x_2-axis and then rotates points $\pi/2$ radians.

Problem 5 (1.9#11)

Assuming T is a linear transformation. Find the standard matrix of T where $T : \mathbb{R}^2 \to \mathbb{R}^2$ first reflects points through the x_1-axis and then reflects points through the x_2-axis. Show that T can also be describd as a linear transformation that roates points about the origin. What is the angle of that rotation?

Problem 6 (1.9#15)

Fill in the missing entries of the matrix, assuming that the equation holds for all values of the variables.

$$\begin{bmatrix} ? & ? & ? \\ ? & ? & ? \\ ? & ? & ? \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{Bmatrix} 3x_1 - 2x_3 \\ 4x_1 \\ x_1 - x_2 + x_3 \end{Bmatrix}$$

Problem 7 (1.9#17)

Show that T is a linear transformation by finiding a matrix that implments the mapping. Note that $x_1, x_2, \ldots$ are not vectors but are entries in vectors.

$$T(x_1, x_2, x_3, x_4) = (0, x_1 + x_2, x_2 + x_3, x_3 + x_4)$$

Problem 8 (1.9#20)

Show that T is a linear transformation by finiding a matrix that implments the mapping. Note that $x_1, x_2, \ldots$ are not vectors but are entries in vectors.

$$T(x_1, x_2, x_3, x_4) = 2x_1 + 3x_3 - 4x_4$$

Problem 9 (1.9#21)

Show that T is a linear transformation by finiding a matrix that implments the mapping. Note that $x_1, x_2, \ldots$ are not vectors but are entries in vectors. Let $T : \mathbb{R}^2 \to \mathbb{R}^5$ such that $T(x_1, x_2) = (x_1 + x_2, 4x_1 + 5x_2)$. Find $\boldsymbol{x}$ such that $T(\boldsymbol{x}) = (3, 8)$.

Problem 10 (1.9#25)

Determine if the linear transformation in Exercise 17 is (a) one-to-one and (b) onto. Justfiy your answer.

Problem 11 (2.1#1)

Compute each matrix sum or product if it defined. If an expression is undefined, explain why.

$$A = \begin{bmatrix} 2 & 0 & -1 \\ 4 & -5 & 2 \end{bmatrix}, B = \begin{bmatrix} 7 & -5 & 1 \\ 1 & -4 & -3 \end{bmatrix}, C = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}, D = \begin{bmatrix} 3 & 5 \\ -1 & 4 \end{bmatrix}, E = \begin{bmatrix} -5 \\ 3 \end{bmatrix}$$

Find $-2A, B - 2A, AC$ and CD.

Problem 12 (2.1#2)

Compute each matrix sum or product if it defined. If an expression is undefined, explain why.

$$A = \begin{bmatrix} 2 & 0 & -1 \\ 4 & -5 & 2 \end{bmatrix}, B = \begin{bmatrix} 7 & -5 & 1 \\ 1 & -4 & -3 \end{bmatrix}, C = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}, D = \begin{bmatrix} 3 & 5 \\ -1 & 4 \end{bmatrix}, E = \begin{bmatrix} -5 \\ 3 \end{bmatrix}$$

Find $-A + 2B, 3C - E, CB$ and EB.

Problem 13 (2.1#3)

Let $A = \begin{bmatrix} 4 & -1 \\ 5 & -2 \end{bmatrix}$. Compute $3I_2 - A$ and $(3I_2)A$.

Problem 14 (2.1#5)

Compute the product AB in two ways: (a) by the definition, where $A\boldsymbol{b_1}$ and $A\boldsymbol{b_2}$ are computed separately, and (b) by the row-column rule for computing AB.

Problem 15 (2.1#7)

If a matrix A is 5×3 and the product AB is 5×7, what is the size of B.

Problem 16 (2.1#9)

Problem 17 (2.1#13)

Problem 18 (2.1#15)

Problem 19 (2.1#21)

Problem 20 (2.1#27)

Problem 21 (2.2#1)

Problem 22 (2.2#5)

Problem 23 (2.2#7)

Problem 24 (2.2#9)

Problem 25 (2.2#13)

Problem 26 (2.2#15)

Problem 27 (2.2#17)

Problem 28 (2.2#19)

Problem 29 (2.2#21)

Problem 30 (2.2#31)