

# Linear Algebra: Homework #3

Due on September 11, 2019

*Professor MacArthur*

Carson Storm

**Problem 1 (1.9#1)**

Assuming  $T$  is a linear transformation. Find the standard matrix of  $T$  where  $T : \mathbb{R}^2 \rightarrow \mathbb{R}^4, T(\mathbf{e}_1) = (3, 1, 3, 1)$  and  $T(\mathbf{e}_2) = (-5, 2, 0, 0)$ .

**Problem 2 (1.9#3)**

Assuming  $T$  is a linear transformation. Find the standard matrix of  $T$  where  $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  rotates points (about the origin) through  $3\pi/2$  radians (counterclockwise).

**Problem 3 (1.9#5)**

Assuming  $T$  is a linear transformation. Find the standard matrix of  $T$  where  $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  is a vertical shear transformation that maps  $\mathbf{e}_1$  into  $\mathbf{e}_1 - 2\mathbf{e}_2$  but leaves the vector  $\mathbf{e}_2$  unchanged.

**Problem 4 (1.9#10)**

Assuming  $T$  is a linear transformation. Find the standard matrix of  $T$  where  $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  first reflects points through the vertical  $x_2$ -axis and then rotates points  $\pi/2$  radians.

**Problem 5 (1.9#11)**

Assuming  $T$  is a linear transformation. Find the standard matrix of  $T$  where  $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  first reflects points through the  $x_1$ -axis and then reflects points through the  $x_2$ -axis. Show that  $T$  can also be described as a linear transformation that rotates points about the origin. What is the angle of that rotation?

**Problem 6 (1.9#15)**

Fill in the missing entries of the matrix, assuming that the equation holds for all values of the variables.

$$\begin{bmatrix} ? & ? & ? \\ ? & ? & ? \\ ? & ? & ? \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3x_1 - 2x_3 \\ 4x_1 \\ x_1 - x_2 + x_3 \end{bmatrix}$$

**Problem 7 (1.9#17)**

Show that  $T$  is a linear transformation by finding a matrix that implements the mapping. Note that  $x_1, x_2, \dots$  are not vectors but are entries in vectors.

$$T(x_1, x_2, x_3, x_4) = (0, x_1 + x_2, x_2 + x_3, x_3 + x_4)$$

**Problem 8 (1.9#20)**

Show that  $T$  is a linear transformation by finding a matrix that implements the mapping. Note that  $x_1, x_2, \dots$  are not vectors but are entries in vectors.

$$T(x_1, x_2, x_3, x_4) = 2x_1 + 3x_3 - 4x_4$$

**Problem 9 (1.9#21)**

Show that  $T$  is a linear transformation by finding a matrix that implements the mapping. Note that  $x_1, x_2, \dots$  are not vectors but are entries in vectors. Let  $T : \mathbb{R}^2 \rightarrow \mathbb{R}^5$  such that  $T(x_1, x_2) = (x_1 + x_2, 4x_1 + 5x_2)$ . Find  $\mathbf{x}$  such that  $T(\mathbf{x}) = (3, 8)$ .

**Problem 10 (1.9#25)**

Determine if the linear transformation in Exercise 17 is (a) one-to-one and (b) onto. Justify your answer.

**Problem 11 (2.1#1)****Problem 12 (2.1#2)****Problem 13 (2.1#3)****Problem 14 (2.1#5)****Problem 15 (2.1#7)****Problem 16 (2.1#9)****Problem 17 (2.1#13)****Problem 18 (2.1#15)****Problem 19 (2.1#21)****Problem 20 (2.1#27)****Problem 21 (2.2#1)****Problem 22 (2.2#5)**

**Problem 23** (2.2#7)

**Problem 24** (2.2#9)

**Problem 25** (2.2#13)

**Problem 26** (2.2#15)

**Problem 27** (2.2#17)

**Problem 28** (2.2#19)

**Problem 29** (2.2#21)

**Problem 30** (2.2#31)