Neal's Funnel

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In this notebook we analyze Neal's Funnel (Neal, 2011) which defines a distribution that exemplifies the difficulties of sampling from some hierarchical models. Neal′s example has support for $y \in \mathbb{R}$ and $x \in \mathbb{R}^2$ with density

$$p(y, x) = \text{Normal}(y \mid 0, 3) \times \prod_{n=1}^{2} \text{normal}\left(x_n \mid 0, \exp\left(\frac{y}{2}\right)\right).$$

The Funnel

```
using Distributions, Plots

x = -2:0.01:2;
kernel(x, y) = logpdf(Normal(0, exp(y / 2)), x)
surface(x, x, kernel)
```

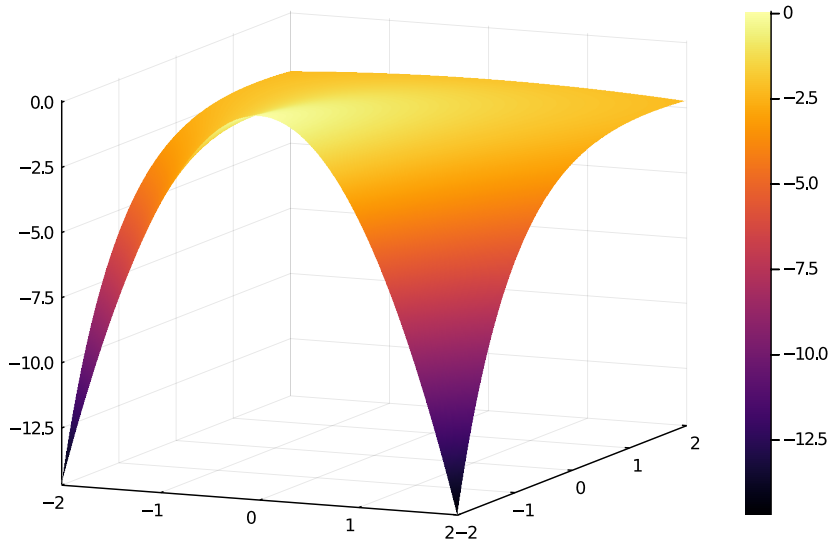

Figure: The Funnel

Reparameterization Trick

What if we reparameterize so that we can express y and x_n as standard normal distributions, by using a reparameterization trick[2]:

$$x^* \sim \text{Normal}(0, 0)$$

$$x = x^* \cdot \sigma_x + \mu_x$$

...this also works for multivariate distributions.

Applied to our example

We can provide the MCMC sampler a better-behaved posterior geometry to explore:

$$p(y^*, x^*) = \text{Normal}(y^* \mid 0, 0) \times \prod_{n=1}^{2} \text{Normal}(x_n^* \mid 0, 0)$$

$$y = 3y^*$$

$$x_n = \exp\left(\frac{y}{2}\right) x_n^*$$

The Funnel Tammed

Below there is is the Neal's Funnel reparameterized as standard normal density in 3-D.

```
kernel_reparameterized(x, y) = logpdf(Normal(), x)
surface(x, x,  kernel_reparameterized)
```

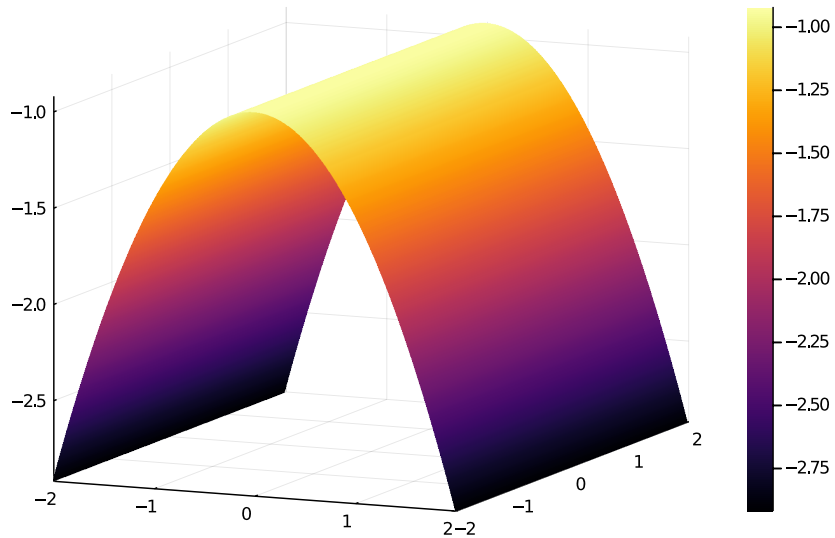

Figure: The Funnel Reparameterized

Environment

```
using InteractiveUtils
versioninfo()
```

```
Julia Version 1.6.0-rc1
Commit a58bdd9010* (2021-02-06 15:49 UTC)
Platform Info:
  OS: macOS (x86_64-apple-darwin20.3.0)
  CPU: Intel(R) Core(TM) i5-8500B CPU @ 3.00GHz
  WORD_SIZE: 64
  LIBM: libopenlibm
  LLVM: libLLVM-11.0.1 (ORCJIT, skylake)
Environment:
  JULIA_NUM_THREADS = 6
```

References

Neal, R. M. (2011). MCMC using Hamiltonian dynamics. In S. Brooks, A. Gelman, G. L. Jones, & X.-L. Meng (Eds.), Handbook of markov chain monte carlo.