# 1 Notation & Definitions

$$A : \Gamma(E) \to \Gamma(H) \tag{1}$$

## 1.1 Data Space

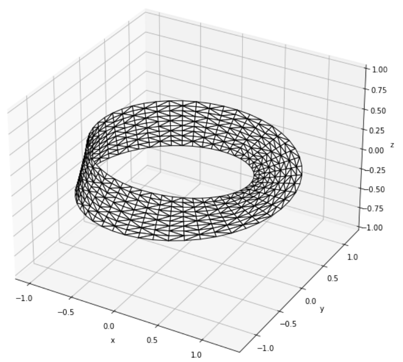


Figure 1: write up some words here

We use a fiber bundle model to represent the data, as proposed by Butler [2, 3].

One example of a fiber bundle is the mobius band shown in figure 1. A fiber bundle is a topological total space $E$ with an embedded fiber space $F$, a base space on which the fibers lie $K$ and the $\pi$ and $\sigma$ mappings between $E$ and $K$. The space of all $\sigma$ is $\Gamma$

$$\begin{tikzcd} F \arrow[r, hook] & E \arrow[d, bend right, "\pi"'] \\ & K \arrow[u, bend right, "\sigma"'] \end{tikzcd}$$

As illustrated by the mobius band example in figure 1, the vertical lines $F$ are the range of possible values embedded in $E$. The circle $K$ is a representation of the connectivity of the points in $E$. The function $\pi$ is the mapping from a point on a specific fiber $F_k | k \in K$ in $E$ to a location $k \in K$.? The section $\sigma$ is the mapping from locations $k$ on $K$ to points on $F_k$ in $E$. (Pull this back into more specific about fig, the general is making this more confusing.)

#### 1.1.1 Base Space $K$

One way to represent the topological space K is as a set composed of simplices, such as those shown in figure **??**. Simplices are a way of encoding the connectivity of each observation ($\sigma(k)$) to another:

**0-simplex** discrete observations (inventory records)

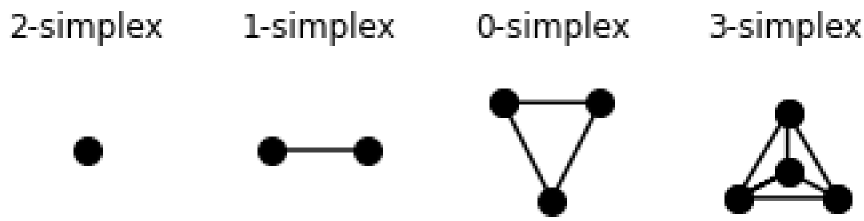


Figure 2: Simplices encode the connectivity of the data, from fully disconnected (0 simplex) observations to all observations are connected to at least 3 other observations. Higher order simplicies are outside the scope of this paper.

**1-simplex** 1D continuos data (timeseries)

**2-simplex** 2D continuos data (map)

**3-simplex** 3D continuos data (video)

In a locally trivial fiber bundle $E = F \times K$, it can be assumed that all $F_k$ for $k \in K$ are equal. A fiber bundle can be made locally trivial by approximating the total space E as a simplacial complex.

#### 1.1.2 Fiber Space $F$

The fibers encode the set of all possible values each observation can take. Spivak's formulation is that the fibers encode the union of the types of measurements[5]. For example, given the section $\Gamma(mobuisstrip) = \{\sigma_1 = \sin, \sigma_2 = \cos\}$: the fiber $F = \{float, float\}$.

#### 1.1.3 Subset & Streaming

$\Gamma(E)$ is the space of all points in $F$ returned by $\sigma$; therefore the points being visualized in a streaming or animation example can be considered a subset that lives on base space $U$ embedded in $K$ with the same fiber $\iota^*E$ and $\iota^*\sigma$.

$$\begin{tikzcd}
\iota^*E \arrow[r, hook] \arrow[d] & E \arrow[d, bend right] \\
U \arrow[r, hook, "\iota"] \arrow[u, dashed, bend right, "\iota^*\sigma"'] & K \arrow[u, dashed, bend right, "\sigma"']
\end{tikzcd}$$

### 1.2 Display Space

A physical display space can be thought of sets of $\mathbb{R}^7$ tuples, where

$$\mathbb{R}^7 = \{X, Y, Z, R, G, B, A\} \tag{2}$$

and the sets correspond to the sections on §, which is the topology of the output of the artist $A$. The space $H$ is a total space representing the predisplay space, with a fiber of $\mathbb{R}^7$ and a base space of §:

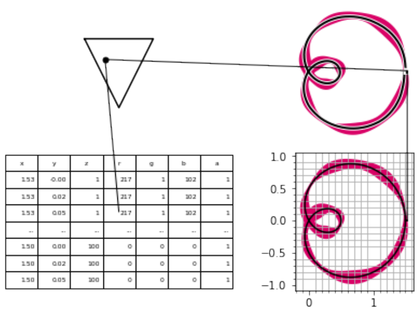


Figure 3

$$
\begin{tikzcd}
\mathbb{R}^7 \arrow[r, hook] & H \arrow[d, shift right, "\pi"'] \\
 & S \arrow[u, shift right, "\rho"']
\end{tikzcd}
$$

In the case of 2D screens, the predisplay space is a trivial fiber bundle $H = \mathbb{R}^7 \times S$. As illustrated in figure 3, a region on the screen defined by the corners $(x_1, y_1)$ and $(x_2, y_2)$ maps into a region on a 2-simplex in $S$ defined by $(\alpha_1, \beta_1)$ and $(\alpha_2, \beta_2)$. The function on the simplex $f$ returns the (R, G, B, A) value for that $(\alpha, \beta)$ pair. For a region,

$$\rho(S) = \int_{\alpha_1}^{\alpha_2} \int_{\beta_1}^{\beta_2} \int_{z_1}^{z_2} R, G, B, A$$

where the R,G,B,A values are derived from the how the data values are mapped to visual characteristics. The z component of the mapping to $\mathbb{R}^7$ is moved to the integration because this is a trivial space representing a 2D screen; $\rho$ varies depending on $H$.

## 1.3 Artist

$$A : \Gamma(E) \to \Gamma(H) \tag{3}$$

### 1.3.1 Screen to Data

$$
\begin{tikzcd}
E \arrow[d, shift right, "\pi"'] & H \arrow[d, shift right, "\pi"'] \\
K \arrow[u, shift right, "\sigma"'] & S \arrow[u, shift right, "\rho"'] \arrow[l, "\xi"']
\end{tikzcd}
$$

The pullback $\xi$ on $S \to K$ means that the values in $E$ can be directly mapped to a simplex in $S$, which means there's a mapping from screen space back to the values.

$$
\begin{tikzcd}
\xi E \arrow[rr, "\tau"] \arrow[dr, "\xi\sigma"'] & & H \arrow[dl] \\
 & S \arrow[ul, bend right] &
\end{tikzcd}
$$

### 1.3.2 Marks

Bertin describes a location on the plane as the signifying characteristic of a point, measurable length as the signifying characteristic of a line, and measurable size as the signifying characteristic of an area and that in display (pixel) space these are marks [1, 4].

$$H \underset{\rho(\xi^{-1}(J))}{\overset{}{\rightleftarrows}} S \underset{\xi^{-1}(J)}{\overset{\xi(s)}{\rightleftarrows}} J_k = \{j \in K | \exists \Gamma \text{ s.t. } \Gamma(0) = k \text{ and } \Gamma(1) = j\} \tag{4}$$

Each point $s$ in the display space $H$, the mark it belongs to can be found by mapping $s$ back to $K$ via the lookup on S described in section 1.2 then taking $\xi(s)$ back to a point on $k \in K$ which lies on the connected component $J \subset K$. To got back to the display space $H$ from the simplacial complex $J$ of the signifier implanted in the mark, the inverse image of $J \in S, \xi^{-1}(J)$ is pushed back to $S$, and then $\rho(\xi^{-1}(J))$ maps it into $R^7$.

#### 1.3.3 Visual Characteristics

Tau can preserves the measurement type properties (group scales)

#### 1.3.4 Visual Idioms: Equivalance class of artists

Two artists are equivalent when given data containers $\Gamma(E)$ of the same type, they output the same type of prerender $\Gamma(S)$:

$$\begin{array}{lcl} A_{\tau_2}: & & \Gamma(E) \longrightarrow \Gamma(H) \\ \downarrow\uparrow & & \\ A_{\tau_1}: & & \Gamma(E) \longrightarrow \Gamma(H) \end{array} \tag{5}$$