Topological Artist Model

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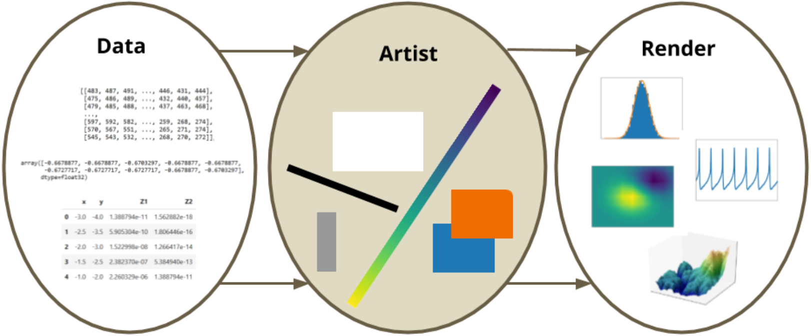
What do visualization libraries do?



Why are we doing this?

1. visualization software design patterns are tuned to data structures[16]
2. visualization types tuned to data structures[35]
3. need to build a tool that supports domain specific structures and algorithms
4. tool needs to be general purpose enough to support lots of domains
5. tools needs to support 2D, 3D, static, dynamic, and interactive viz

Everything is a table

DATA: longitude, latitude = *map*(*source*("World"))
TRANS: bd = *max*(birth-death, 0)
COORD: *project.mercator*()
ELEMENT: *point*(*position*(lon*lat), *size*(bd), *color*(*color.red*))
ELEMENT: *polygon*(*position*(longitude*latitude))

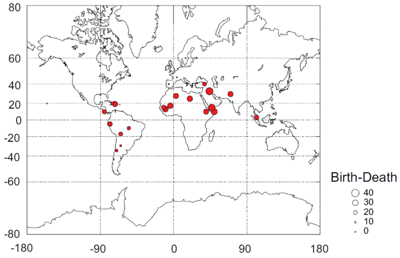


Figure 1.5 *Excess birth (vs. death) rates in selected countries*

Figure: Figure 1.5 in Wilkenson's Grammar of Graphics[40]

1. ggplot[39], protovis[4], D3 [5], vega[27], altair[37]
2. tends to be declarative[17]
3. users describe how to compose visual elements [41]

TAM: build the visual elements

Everything is an image

1. ImageJ[28], ImagePlot[32], Napari[29]
2. build plugins into existing system where image is primary input

TAM: more general than tables and images

Everything is ?

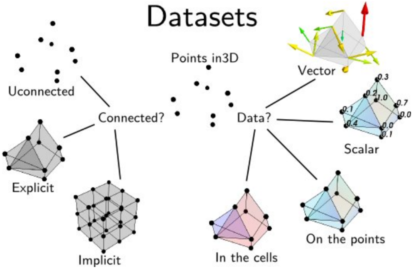


Figure: Image is from the Data Representation chapter of the MayaVi 4.7.2 documentation.[12]

1. Matplotlib[18], VTK [14, 15], MayaVi[26], ParaView[3], Titan[6]
2. each visualization type API is tuned to the data structure

TAM: common data abstraction interface between data and viz

Variables & Topology

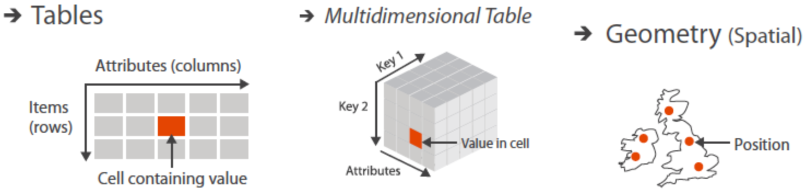


Figure: Image is figure 2.8 in Munzner's Visualization Analysis and Design[24]

	metadata are *keys* with associated *values* (Munzner [24])
topology	Fiber bundles can be a common data abstraction (Butler [7, 8])
variables	Fiber as a database schema (Spivak [30, 31])

Tam: variables are *values* and *keys* locate them in topology

Property Matching

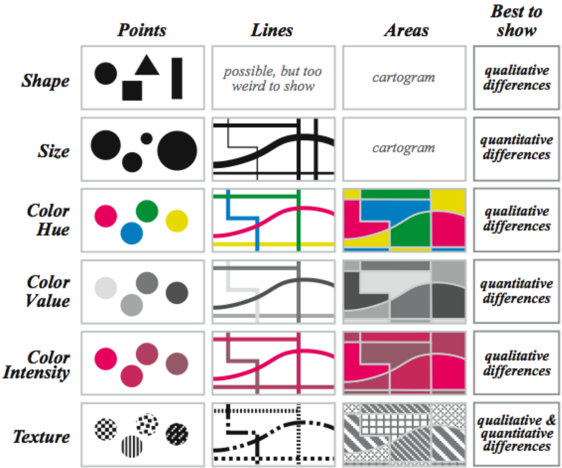


Figure: This tabular form of Bertin's retinal variables is from Understanding Graphics [23] who reproduced it from Krygier and Wood's *Making Maps: A Visual Guide to Map Design for GIS*[20]

Evaluating visualizations

Expressiveness structure preserving mappings from data to graphic (Mackinlay [21])

Effectiveness design choices made in deference to perceptual saliency [**(**]Mackinlay [9–11, 24])

Naturalness easier to understand when properties match (Norman [25])

Graphical Integrity graphs show **only** the data (Tufte [36])

Mathematical Frameworks for evaluating visualization

1. APT: visualization has syntax and semantics like a language (Mackinlay [21, 22])
2. Visualization has functional dependencies that can be represented as graphs (Sugibuchi [33])
3. the semiotics of visualization are commutative in a category theory framework (Vickers [38])

Tam: Framework for building visualizations

Visualization is commutative maps

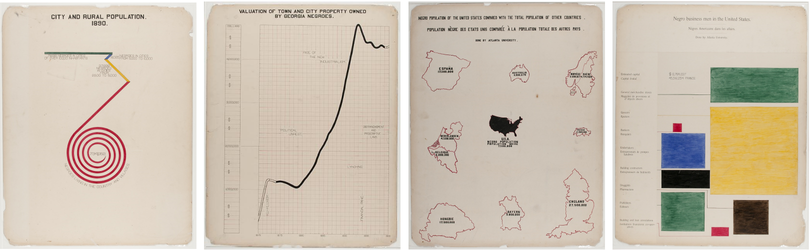
data (α) and viz (ω) symmetries (Kindlmann and Scheidegger [19])

$v \circ r_2 \circ \alpha = \omega \circ v \circ r_1$

$$\begin{tikzcd} D \arrow[r, "r_1"] \arrow[d, "\alpha"'] & R \arrow[r, "\nu"] & V \arrow[d, "\omega"] \\ D \arrow[r, "r_2"'] & R \arrow[r, "\nu"'] & V \end{tikzcd}$$

Tam: Add topology and make it functional

So why?



(a) (b) (c) (d)

Figure: Du Bois' data portraits from the Prints and Photographs collection of the Library of Congress [1, 2, 13, 34]

Framework for building any valid visualization

Contributions

1. a formal description of the topology preserving relationship between data and graphic via continuous maps
2. a formal description of the property preservation from data component to visual representation as equivariant maps that carry a homomorphism of monoid actions
3. abstraction of data structure using fiber bundles with schema like fibers to encode components and topology
4. algebraic sum operator such that more complex visualizations can be built from simple ones
5. a functional oriented visualization tool architecture built on the mathematical model to demonstrate the utility of the model
6. a prototype of the architecture built on Matplotlib's infrastructure to demonstrate the feasibility of the model

Topological Artist Model

$$\mathscr{A} : \mathscr{E} \to \mathscr{H} \tag{1}$$

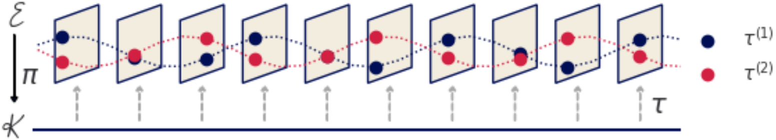
which is a map from data E to graphic H that carries a homomorphism of monoid actions

$$\varphi : M \to M' \tag{2}$$

such that artists are equivariant maps

$$\mathscr{A}(m \cdot r) = \varphi(m) \cdot \mathscr{A}(r) \tag{3}$$

Data Bundle



a fiber bundle is a tuple (E, K, π, F) defined by the projection map π

$$F \hookrightarrow E \xrightarrow{\pi} K \tag{4}$$

Variables: Fiber

$$F = \mathbb{U}_{\sigma(c)} = \mathbb{U}_T \tag{5}$$

where $\mathbb{U}_{\sigma(c)}$ is

$$\begin{tikzcd} \mathbb{U}_\sigma \arrow[r] \arrow[d, "\pi_\sigma"'] & \mathbb{U} \arrow[d, "\pi"] \\ C \arrow[r, "\sigma"'] & \mathbf{DT} \end{tikzcd} \tag{6}$$

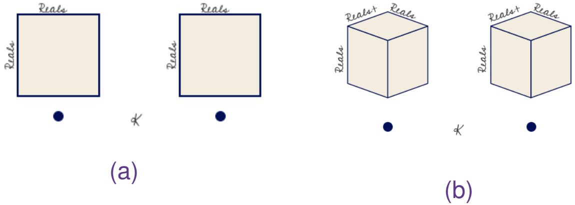
DT data types of the variables in the dataset

$\mathbb{U}$ disjoint union of all values of type $T \in \mathbf{DT}$

C variable names

$\mathbb{U}_\sigma$ $\mathbb{U}$ restricted to the data type of a named variable

Sample Fiber



Figure

6a $F = \mathbb{R} \times \mathbb{R}$, for example (time, temperature)

6b $\mathbb{R} \times \mathbb{R}^{+} \times \mathbb{R}$, for example (time, wind=(speed, direction))

Fiber Components

We can decouple decouple F into components F_i

$$F = F_0 \times \ldots \times F_i \times \ldots \times F_n \tag{7}$$

Structure of Components: Monoids

A monoid M is a set with

associative binary operator $* : M \times M \to M$

identity element $e \in M$ such that $e * a = a * e = a$ for all $a \in M$.

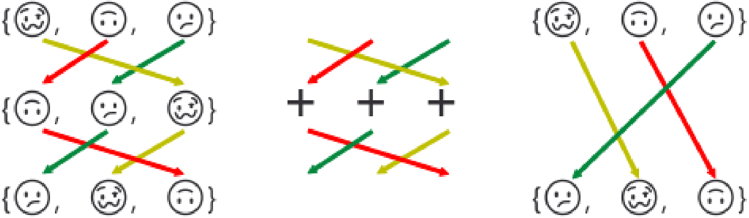
Monoid Actions

A left monoid action of M_i is a set F_i with an action $\bullet : M \times F_i \to F_i$ with the properties:

associativity for all $f, g \in M_i$ and $x \in F_i$, $f \bullet (g \bullet x) = (f * g) \bullet x$

identity for all $x \in F_i, e \in M_i$, $e \bullet x = x$

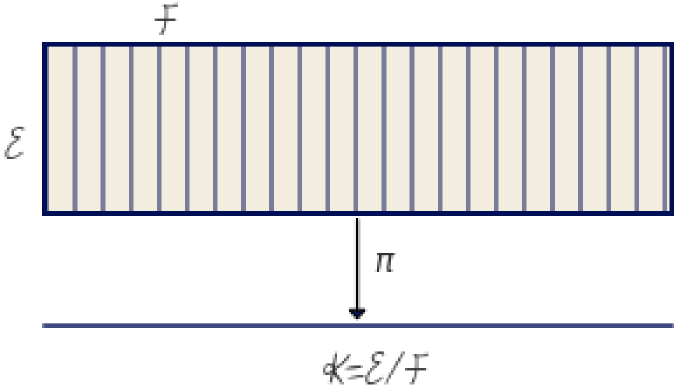
Monoid Composition: Permutation



Monoid Composition: Partial Order

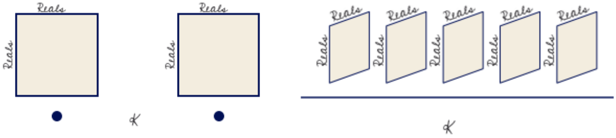
MAKE THIS Hasse Diagram (borrow from theories of composablity)

Continuity: base space



Figure

Sample: Base Space



Values: Section

For any fiber bundle, there exists a map

$$\begin{tikzcd} F \arrow[r, hook] & E \arrow[d, "\pi"'] \\ & K \arrow[u, bend right, "\tau"'] \end{tikzcd} \tag{9}$$

such that $\pi(\tau(k)) = k$.
The set of all global sections is denoted as $\Gamma(E)$.

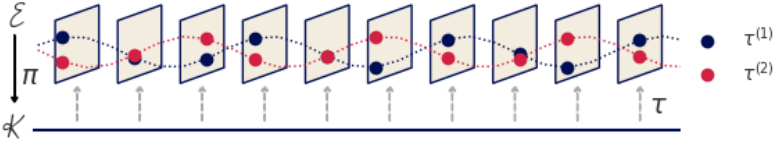
Record

Assuming a trivial fiber bundle $E = K \times F$, the section is

$$\tau(k) = (k, (g_{F_0}(k), \ldots, g_{F_n}(k))) \tag{10}$$

where $g : K \to F$ is the index function into the fiber.

Sample dataset



Fiber Bundle

The graphics bundle is a tuple (H, S, π, D) defined by the projection map π

$$D \hookrightarrow H \xrightarrow{\pi} S \tag{11}$$

Continuity: Base space

The surjective map $\xi : S \to K$

$$\begin{tikzcd} E \arrow[d, "\pi"'] & H \arrow[d, "\pi"'] \\ K & S \arrow[l, "\xi"'] \end{tikzcd} \tag{12}$$

goes from region $s \in S_k$ to its associated point s.

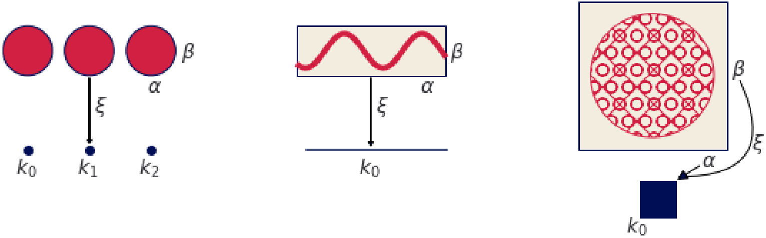
Sample Display Target: Fiber

Assume a 2D opaque image $D = \mathbb{R}^5$ with elements

$$(x, y, r, g, b) \in D \tag{13}$$

such that a rendered graphic only has 2D position and color.

Continuity Retraction Maps



Rendering: Define a Pixel

Given a pixel

$$p = \left[y_{top}, y_{bottom}, x_{right}, x_{left}\right] \tag{14}$$

the inverse map of the bounding box

$$S_p = {\rho_{xy}}^{-1}(p) \tag{15}$$

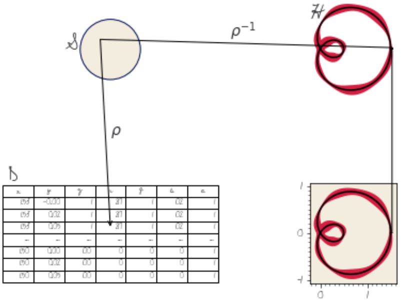
is a region $S_p \subset S$ such that

$$r_p = \iint_{S_p} \rho_r(s) ds^2 \tag{16}$$

$$g_p = \iint_{S_p} \rho_g(s) ds^2 \tag{17}$$

$$b_p = \iint_{S_p} \rho_b(s) ds^2 \tag{18}$$

yields the color of the pixel.



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