# Topological Equivariant Artist Model

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Advisor: Dr. Michael Grossberg
Committee: Dr. Robert Haralick, Dr. Lev Manovich, Dr. Huy Vo
External Member: Dr. Marcus Hanwell

# Visualizations are structure preserving maps

| DATE | LATITUDE | LONGITUDE | TAVG |
|---|---|---|---|
| 2021-01-01 | 42.1 | -77.1 | 27.86 |
| 2021-01-01 | 40.6386 | -73.7622 | 35.96 |
| 2021-01-01 | 42.7 | -75.5 | 30.02 |
| 2021-01-01 | 43.0078 | -73.6511 | 27.68 |
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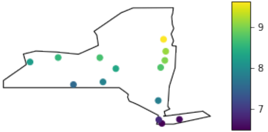

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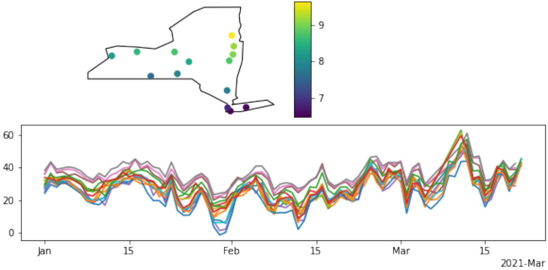

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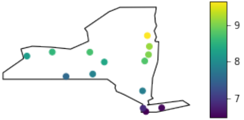


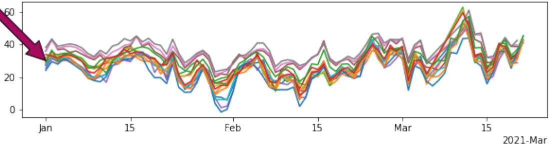

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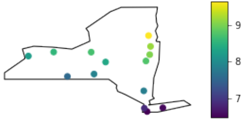


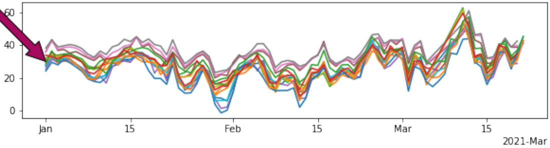


equivariance: properties of data and visual encoding match

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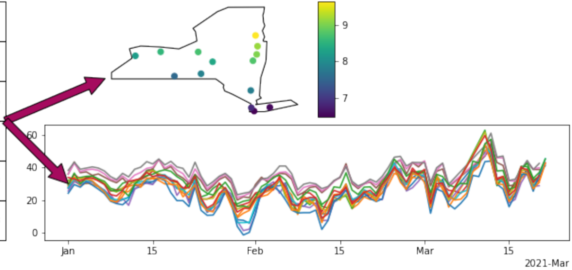


equivalence properties of data and visual encoding match

continuity connectivity of data and visual encoding match

# Domain specific libraries assume data structure[HA06]

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Vega[SWH14]
Altair[Van+18]
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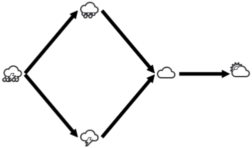

Gephi[BHJ09]
Graphviz[Ell+02]
Networkx[HSS08]

# General purpose libraries can't[TM04]

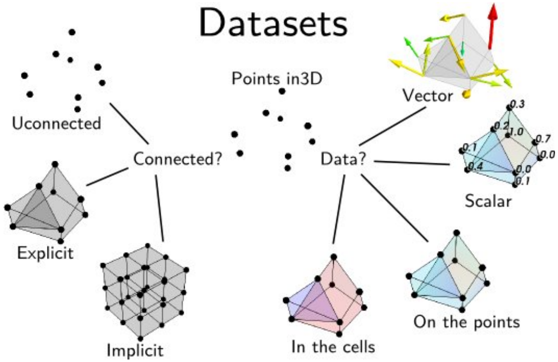


Figure: Data Representation, MayaVi 4.7.2 docs[Dat]

1. Matplotlib[Hun07] →Seaborn[Wt20], xarray [HH17]
2. D3 [BOH11]
3. VTK [Han+15; Gev+12](MayaVi[RV11])→ Titan[BJ09], ParaView[AGL05]

# Best practices in visualization design

Expressiveness structure preserving mappings[Mac86]

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Expressiveness structure preserving mappings[Mac86]

Graphical Integrity graphs show **only** the data[Tuf01]

Naturalness easier to understand when properties match[Nor93]

# Contributions

| | |
|---|---|
| Topological | continuity |
| Equivariant | monoid action |
| Artist | Matplotlib $artist : data \to graphic$ |
| Model | |

# Topological Equivariant Artist Model

An artist $\mathscr{A}$ is an equivariant map

$$\mathscr{A} : \mathscr{E} \to \mathscr{H}$$

from data $\mathscr{E}$ space to graphic $\mathscr{H}$ space.

# Model data as a fiber bundle [BB92; BP89]

A fiber bundle is a tuple $(E, K, \pi, F)$ defined by the map $\pi$

$$F \hookrightarrow E \xrightarrow{\pi} K$$

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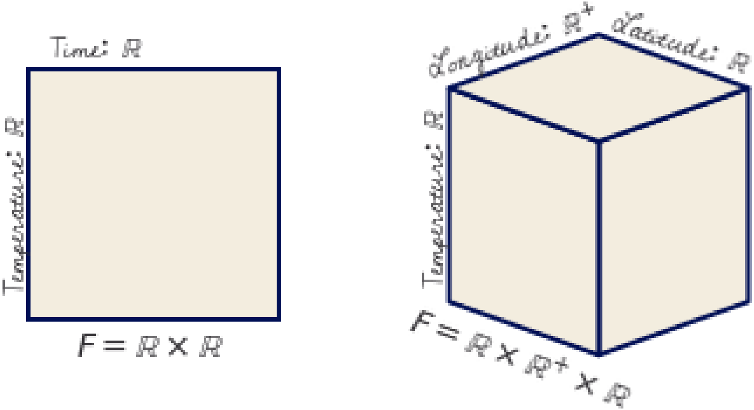
total space $E$ topology

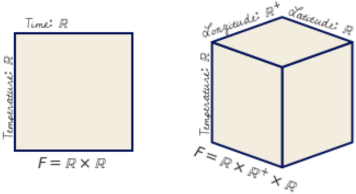
fiber space $F$ schema

base space $K$ continuity

# Encode variable types in a schema like fiber [Spi10; Spi]



# Monoids are the structure of the components of $F$




$$F = F_0 \times \ldots \times F_i \times \ldots \times F_n$$

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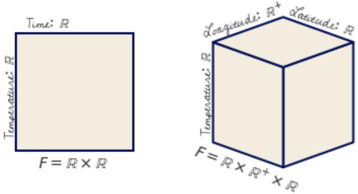


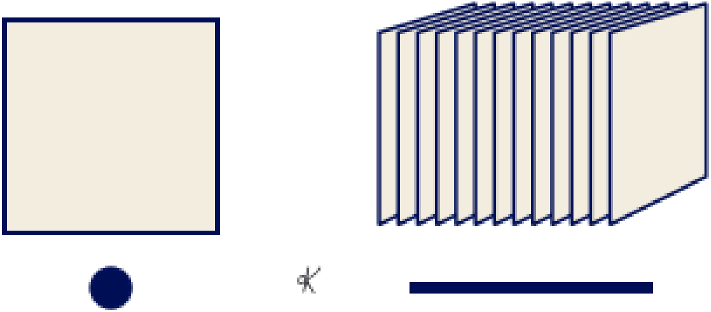

$$F = F_0 \times \ldots \times F_i \times \ldots \times F_n$$

Monoid actions $M_i$ (e.g. rotation, partial ordering) define the structure on $F_i$

$$\bullet : M_i \times F_i \to F_i$$

where $\bullet$ is associative and has an identity action.

# $K$ is an indexing space



# $K$ is the space of *keys* into data in $E$[Mun14]

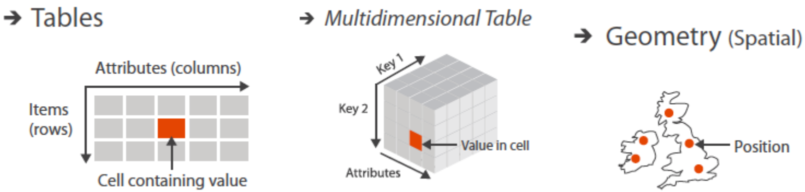
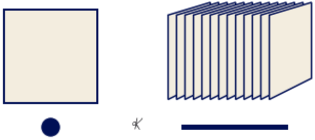



Figure: Figure 2.8 in Munzner's Visualization Analysis and Design[Mun14]

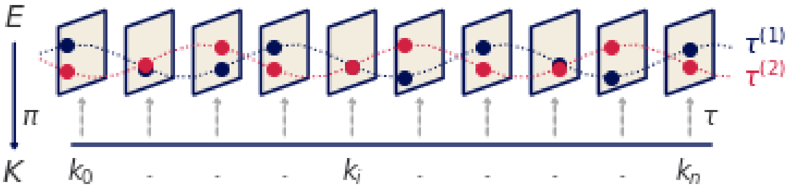


# Data are sections $\tau$ on $E$

For any fiber bundle, there exists a map

$$\begin{tikzcd} F \arrow[r, hook] & E \arrow[d, "\pi"'] \\ & K \arrow[u, bend right, "\tau"'] \end{tikzcd}$$

s.t. $\pi(\tau(k)) = k$. $\Gamma(E)$ is the set of all global sections.



# Graphic Bundle $(H, S, \pi, D)$

$$
\begin{tikzcd}
F \arrow[r, hook] & E \arrow[d, "\pi"'] \\
& K \arrow[u, bend right, "\tau"']
\end{tikzcd}
\qquad
\begin{tikzcd}
D \arrow[r, hook] & H \arrow[d, "\pi"'] \\
& S \arrow[u, bend right, "\rho"']
\end{tikzcd}
$$

# Graphic Bundle $(H, S, \pi, D)$

Continuity is preserved via the many $s$ to one $k$ map $\xi : S \to K$

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$D$ is a proxy for the target display, for example $(x, y, r, g, b) \in D$

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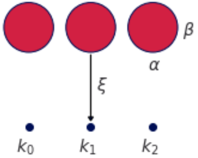

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$$\begin{tikzcd}
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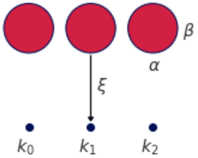


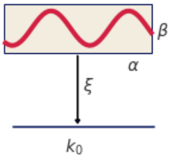

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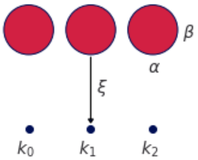


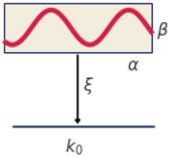


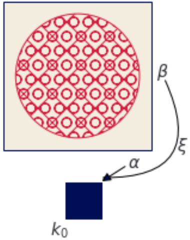

# Visual bundle $(V, K, \pi, P)$

$$\mathcal{A} : \mathcal{E} \to \mathcal{H}$$

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$$\begin{tikzcd}
E' \arrow[r, "\nu"] \arrow[dr, "\pi"'] & V \arrow[d, "\pi"] & \xi^* V \arrow[l, "\xi^*"'] \arrow[r, "Q"] \arrow[d, "\xi^*\pi"'] & H \arrow[dl, "\pi"] \\
 & K & S \arrow[l, "\xi"'] &
\end{tikzcd}$$

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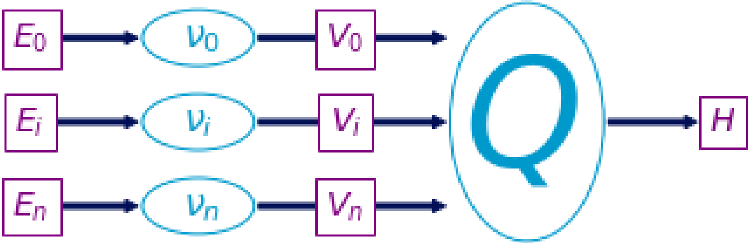
$$A : \mathcal{O}(E) \to \mathcal{O}(H)$$

# Visualization Assembly Function



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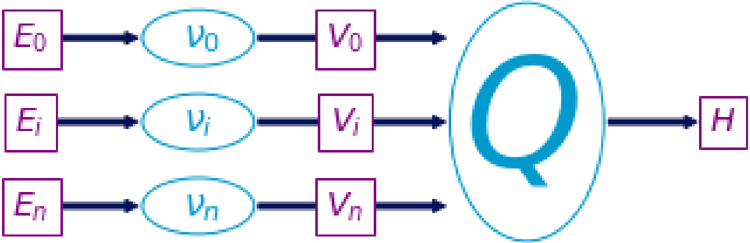


$$\{\nu_0, \ldots, \nu_n\} : \{\tau_0, \ldots, \tau_n\} \mapsto \{\mu_0, \ldots, \mu_n\}$$

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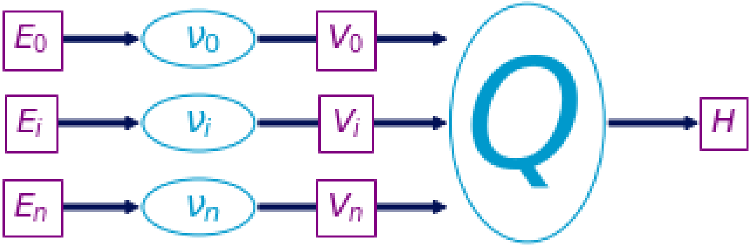


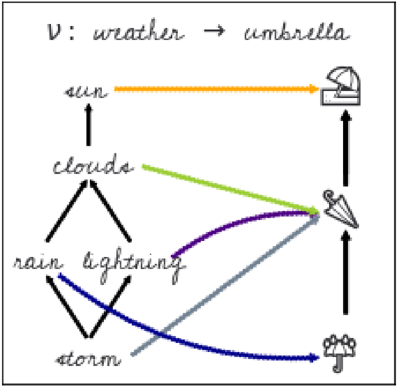

$$\{\nu_0, \dots, \nu_n\} : \{\tau_0, \dots, \tau_n\} \mapsto \{\mu_0, \dots, \mu_n\}$$

$$Q = \nu \circ \tau$$

# Group Equivariance: Stevens' Scales [Ste46]

| scale | group | constraint |
|---|---|---|
| nominal | permutation | if $r_1 \neq r_2$ then $\nu(r_1) \neq \nu(r_2)$ |
| ordinal | monotonic | if $r_1 \leqslant r_2$ then $\nu(r_1) \leqslant \nu(r_2)$ |
| interval | translation | $\nu(x + c) = \nu(x) + c$ |
| ratio | scaling | $\nu(xc) = \nu(x) * c$ |

# Monoid Equivariance: Partial Orders



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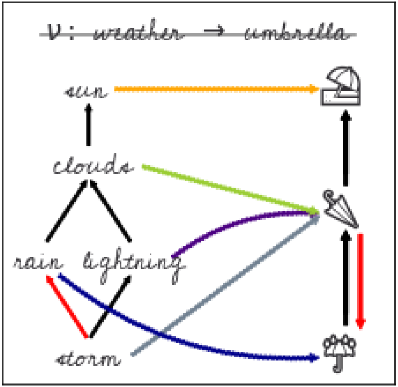

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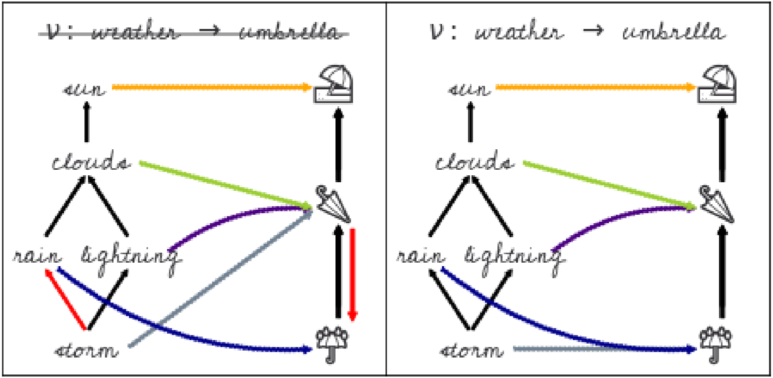
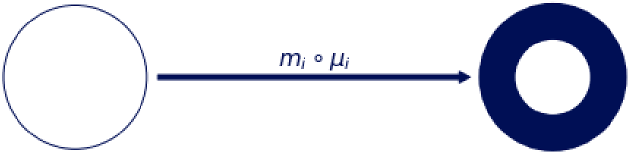
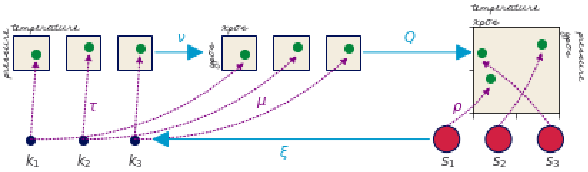


# Visualization Equivariance




$$Q(\mu) = Q(\mu') \implies Q(m \circ \mu) = Q(m \circ \mu')$$

# Scatter: $Q(xpos, ypos)(\alpha, \beta)$



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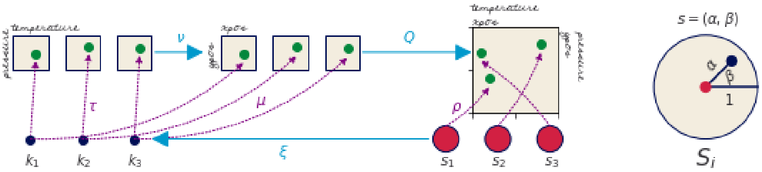
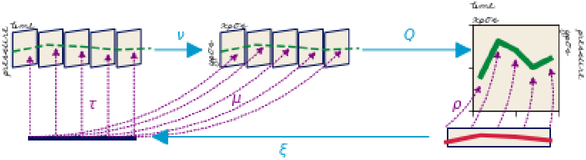


# Line: $Q(xpos, \hat{n}_1, ypos, \hat{n}_2)(\alpha, \beta)$



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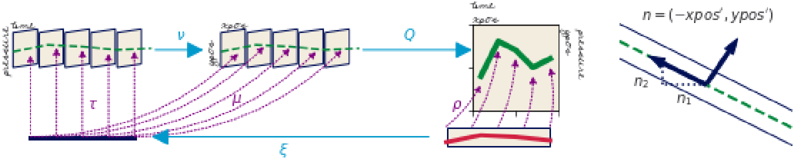
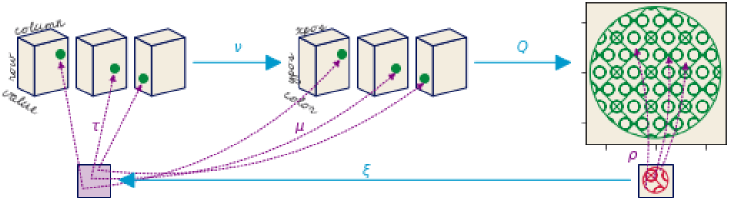
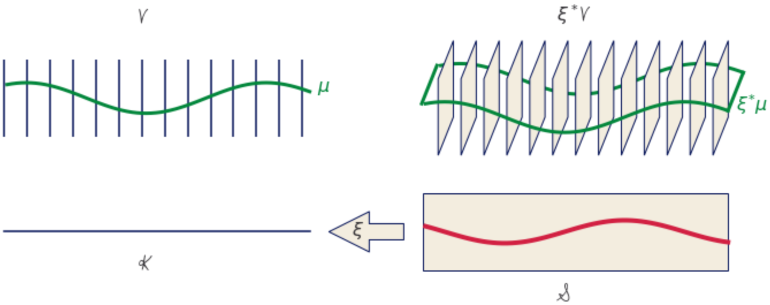


# Image $Q(xpos, ypos, color)$

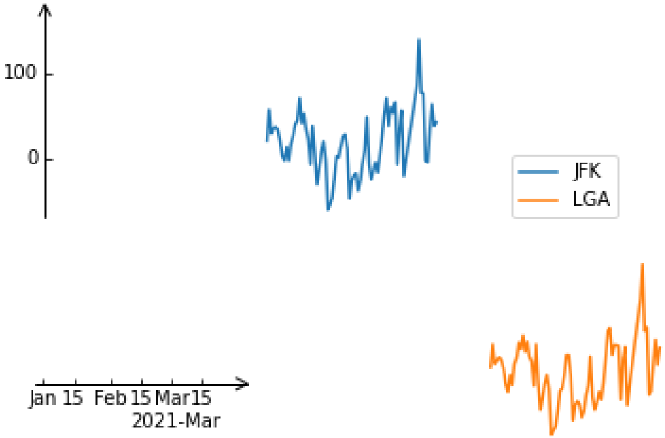


# Build $Q$ over $K$: $\hat{Q}$




$$\hat{Q}(\mu(k))(s) \coloneqq Q((\xi^*\mu)(s))$$

# Composition of artists $+ :\coloneqq \sqcup E_i$



# TEAM driven rearchitecture of Matplotlib

- complex visualizations

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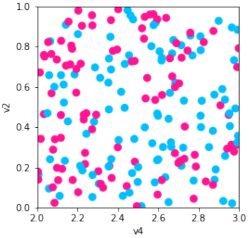
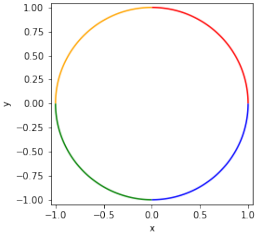
- complex visualizations
- structure preserving maps from data to visual
    - data and graphics have equivalent continuity
    - properties are equivariant under monoid actions

# TEAM driven rearchitecture of Matplotlib

- complex visualizations
- structure preserving maps from data to visual
  - data and graphics have equivalent continuity
  - properties are equivariant under monoid actions
- fiber bundles are an abstraction
  - topologically complex heterogenous data
  - target display spaces

# How do we make things?




```
fig, ax = plt.subplots()
artist = Point(data, transforms)
ax.add_artist(artist)
```

```
fig, ax = plt.subplots()
artist = Line(data, transforms)
ax.add_artist(artist)
```

# $\nu$

```
cmap =  color.Categorical({'true':'deeppink', 'false':'deepskyblue'})
transforms = {'x': {'name': 'v4', 'encoder': lambda x: x},
                'y': {'name': 'v2', 'encoder': lambda x: x},
                'facecolors': {'name':'v3', 'encoder': cmap},
                's':{'name': None ,
                    'encoder': lambda _: itertools.repeat(.02)}}
```

- `lambda x: x` is identity $\nu$
- `{'name':None}` map into $P$ without corresponding $\tau$
- `color.Categorical` is custom $\nu$

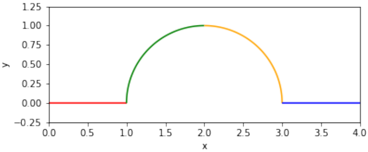
```
class ArtistClass(matplotlib.artist.Artist):
    def __init__(self, E, V, *args, **kwargs):
        # set properties that are specific to the artist
        # stash the input E and V
        super().__init__(*args, **kwargs)

    def qhat(self, **args):
        # set the properties of the graphic

    def draw(self, renderer):
        # returns tau, indexed on fiber then key
        tau = self.E.view(self.axes)
        # visual channel encoding applied fiberwise
        visual = {p_i: nu_i(tau_i)
                  for p_i, nu_i, tau)i
                  in zip(self.V, tau)}
        self.qhat(**visual)
        # pass configurations off to the renderer
        super().draw(renderer)
```

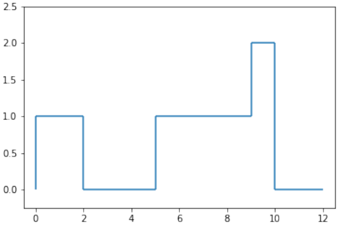
```python
class Point(mcollections.Collection):
    def assemble(self, x, y, s, facecolors='C0' ):
        # construct geometries of the circle glyphs in visual coordinates
        # set attributes of glyphs

class Line(mcollections.LineCollection):
    def assemble(self, x, y, color='C0'):
        # assemble line marks as set of segments
```

# Continuity

```
class PointData:
    # Fiberbundle is consistent across all sections
    FB = FiberBundle({'tables': ['vertex']},
        {'v1': float, 'v2': str, 'v3': float})
    def tau(self, k):
        return # tau evaluated at one point k

class LineData:
    FB = FiberBundle({'tables': ['edge']},
                {'x' : float, 'y':  float, 'color':mtypes.Color()})
    def tau(self, k):
        return # tau evaluated on interval k
```

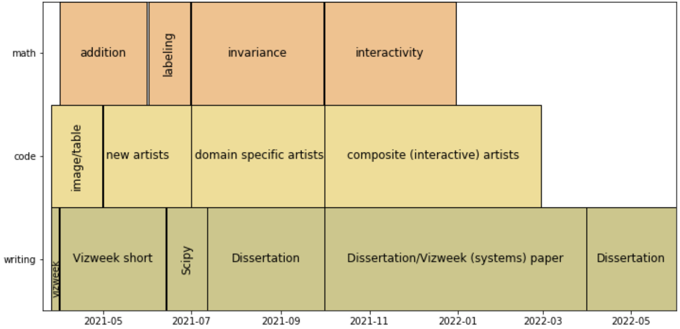
# Same Artist, Different $E$



```
LineData(FB, edge_table, vertex_table, connect=True)
LineData(FB, edge_table, vertex_table, num_samples=2, connect=False)
```

# Proposed Work



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# Mathematical Models of Vizualization

algebraic process data and viz transforms are symmetric [KS14]

$$\begin{array}{ccccc} E & \xrightarrow{\nu} & V & \xrightarrow{Q} & H \\ {\scriptstyle m}\downarrow & & & & \downarrow{\scriptstyle \varphi(m)} \\ E & \xrightarrow[\nu]{} & V & \xrightarrow[Q]{} & H \end{array}$$

language APT and GoG: syntax, semantics, and grammar [Mac86; Mac87; Wil05]

functional dependencies relationship between components [SSS09]

category theory *understanding* $=$ *read* $\circ$ *render* [VFR13]

# Fiber is all possible values a variable can be [Spi10; Spi]

Given a space of all possible values $\mathbb{U}$

$$
\begin{tikzcd}
\mathbb{U}_\sigma \arrow[r] \arrow[d, "\pi_\sigma"'] & \mathbb{U} \arrow[d, "\pi"] \\
C \arrow[r, "\sigma"'] & \mathbf{DT}
\end{tikzcd}
$$

a fiber component is the restricted space $\mathbb{U}_{\sigma(c)}$.

$$F = \mathbb{U}_{\sigma(c)} = \mathbb{U}_T$$

**DT** data types of the variables in the dataset

$\mathbb{U}$ disjoint union of all values of type $T \in \mathbf{DT}$

$C$ variable names, $c \in C$

$\mathbb{U}_\sigma$ $\mathbb{U}$ restricted to the data type of a named variable

# Monoid actions

A monoid $M$ is a set with

associative binary operator $* : M \times M \to M$

identity element $e \in M$ such that $e * a = a * e = a$ for all $a \in M$.

## left monoid action

A set $F$ with an action[nLa21] $\bullet : M \times F \to F$ with the properties:

**associativity** for all $f, g \in M$ and $x \in F$, $f \bullet (g \bullet x) = (f * g) \bullet x$

**identity** for all $x \in F, e \in M$, $e \bullet x = x$

# Keeping track of sections with sheafs

Restriction maps of a sheaf describe how local $\iota^*\tau$ can be glued into larger sections [Ghr14; Ghr18]

$$\begin{tikzcd}
\iota^* E \arrow[r, hook, "\iota^*"] \arrow[d, shift right, "\pi"'] & E \arrow[d, shift right, "\pi"'] \\
U \arrow[r, hook, "\iota"] \arrow[u, shift right, bend right, "\iota^*\tau"'] & K \arrow[u, shift right, bend right, "\tau"']
\end{tikzcd}$$

The inclusion map $\iota : U \to K$ pulls $E$ over $U$ such that the pulled back $\iota^*\tau$ only contains records over $U \subset K$.

# Rendering: Define a Pixel

Given a pixel

$$p = \left[y_{top}, y_{bottom}, x_{right}, x_{left}\right]$$

the inverse map of the bounding box

$$S_p = \rho_{xy}{}^{-1}(p)$$

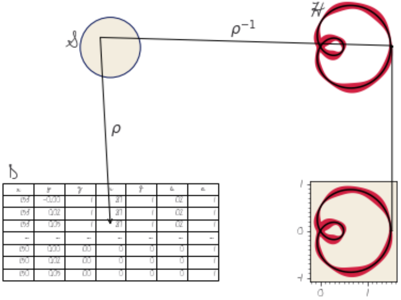
is a region $S_p \subset S$ such that

$$r_p = \iint_{S_p} \rho_r(s)\, ds^2 \quad (1)$$

$$g_p = \iint_{S_p} \rho_g(s)\, ds^2 \quad (2)$$

$$b_p = \iint_{S_p} \rho_b(s)\, ds^2 \quad (3)$$

yields the color of the pixel.



# $\mathcal{A} : \mathcal{E} \to \mathcal{E}$

The topological artist is a sheaf map

$$A : \mathcal{O}(E) \to \mathcal{O}(H)$$

that carries homomorphism of monoid actions $\varphi : M \to M'$ [Ceg19]

$$A(m \cdot r) = \varphi(m) \cdot A(r)$$

# Visual Channel Encoders

We define the visual transformers $\nu$ on components of the data bundle $\tau_i$

$$\{\nu_0, \ldots, \nu_n\} : \{\tau_0, \ldots, \tau_n\} \mapsto \{\mu_0, \ldots, \mu_n\}$$

as the set of equivariant maps with the constraint

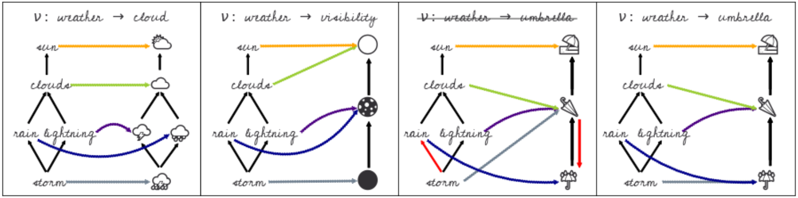
$$\nu_i(m_r(E_i)) = \varphi(m_r)(\nu_i(E_i))$$

where $\varphi : M \to M'$ carries a homomorphism of monoid actions.

# $P$ Components

| $\nu_i$ | $\mu_i$ | $codomain(\nu_i) \subset P_i$ |
|---|---|---|
| position | x, y, z, theta, r | $\mathbb{R}$ |
| size | linewidth, markersize | $\mathbb{R}^+$ |
| shape | markerstyle | $\{f_0, \ldots, f_n\}$ |
| color | color, facecolor, markerfacecolor, edgecolor | $\mathbb{R}^4$ |
| texture | hatch | $\mathbb{N}^{10}$ |
| | linestyle | $(\mathbb{R}, \mathbb{R}^{+n, n\%2=0})$ |

# Monoid Equivariance: Partial Orders



# Glyph

The glyph is the graphic generated by $Q(S_j)$ where the path connected components $J \subset K$ are defined

$$J = \{j \in K \text{ s. t. } \exists\gamma \text{ s.t. } \gamma(0) = k \text{ and } \gamma(1) = j\}$$

such that the path $\gamma$ from $k$ to $j$ is a continuous function from the interval [0,1] and $S_j$ is the region

$$H \underset{\rho(S_j)}{\rightleftarrows} S_j \underset{\xi^{-1}(J)}{\overset{\xi(s)}{\rightleftarrows}} J_k$$

such that the glyph is differentiable, in keeping with Ziemkiewicz and Kosara's description of a glyph[ZK09].

# Artist Equivalance class

When artists share a base space

$$K_2 \hookrightarrow K_1$$

a composition operator can be defined such that the the artists can be considered to be acting on different components of the same section.

# Complex ν

```python
class Categorical:
    def __init__(self, mapping):
        # check that the conversion is to valid colors
        assert(mcolors.is_color_like(color) for color in mapping.values())
        self._mapping = mapping

    def __call__(self, value):
        # convert value to a color
        return [mcolors.to_rgba(self._mapping[v]) for v in values]
```

That we can test for action equivariance

```python
def test_nominal(values, encoder):
    m1 = list(zip(values, encoder(values)))
    random.shuffle(values)
    m2 = list(zip(values, encoder(values)))
    assert sorted(m1) == sorted(m2)
```

# Artist

```python
class ArtistClass(matplotlib.artist.Artist):
    def __init__(self, data, transforms, *args, **kwargs):
        # properties that are specific to the graphic
        self.data = data
        self.transforms = transforms
        super().__init__(*args, **kwargs)

    def assemble(self, **args):
        # set the properties of the graphic

    def draw(self, renderer):
        # returns K, indexed on fiber then key
        view = self.data.view(self.axes)
        # visual channel encoding applied fiberwise
        visual = {p: t['encoder'](view[t['name']])
                    for p, t in self.transforms.items()}
        self.assemble(**visual)
        # pass configurations off to the renderer
        super().draw(renderer)
```

# Artists: Scatter & Line

```python
class Point(mcollections.Collection):
    def assemble(self, x, y, s, facecolors='C0' ):
        # construct geometries of the circle glyphs in visual coordinates
        self._paths = [mpath.Path.circle(center=(xi,yi), radius=si)
                        for (xi, yi, si) in zip(x, y, s)]
        # set attributes of glyphs, these are vectorized
        # circles and facecolors are lists of the same size
        self.set_facecolors(facecolors)

class Line(mcollections.LineCollection):
    def assemble(self, x, y, color='C0'):
        #assemble line marks as set of segments
        segments = [np.vstack((vx, vy)).T for vx, vy in zip(x, y)]
        self.set_segments(segments)
        self.set_color(color)
```

# View

```python
def view(self, axes):
    table = defaultdict(list)
    for k in self.keys:
        table['index'].append(k)
        for (name, value) in zip(self.FB.fiber.keys(), self.tau(k)[1]):
            table[name].append(value)
    return table
```

`VertexSimplex` (name, value), value is scaler

`EdgeSimplex` (name, value), value is [x0, ..., xn]

# Fiber Bundle

```
@dataclass
class FiberBundle:
"""
Attributes
----------
K: {'tables': []}
F: {variable name: type}
"""
    K: dict
    F: dict
```

# GraphLine Data Model

```
class GraphLine:
    def __init__(self, FB, edge_table, vertex_table, num_samples=1000,
                        connect=False):
        #set args as attributes and generate distance
        if connect: # test connectivity if edges are continuous
            assert edge_table.keys() == self.FB.F.keys()
            assert is_continuous(vertex_table)

    def tau(self, k):
        # evaluates functions defined in edge table
        return(k, (self.edges[c][k](self.distances)
                        for c in self.FB.F.keys()))

    def view(self, axes):
        # walk the edge_vertex table to return the edge function
        table = defaultdict(list)
        for (i, (start, end)) in sorted(zip(self.ids, self.vertices),
                                            key=lambda v:v[1][0]):
            table['index'].append(i)
            # same as view for line, returns nested list
            for (name, value) in zip(self.FB.F.keys(), self.tau(i)[1]):
                table[name].append(value)
        return table
```