

Topological Equivariant Artist Model

March 18, 2021

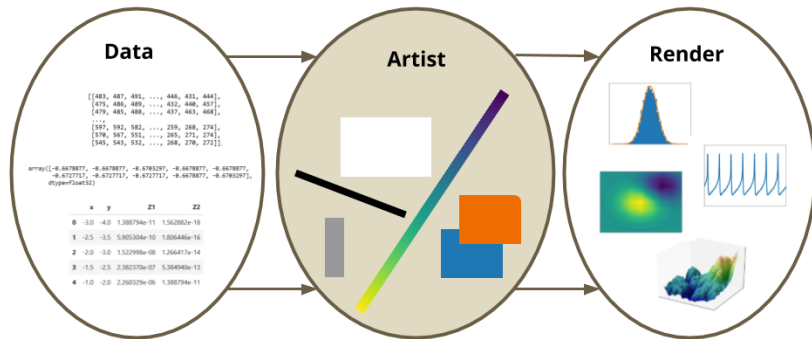
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Visualizations are structure preserving maps



The aim of this work is to rearchitecture Matplotlib to take advantage of developments in software design, data structures, and visualization to improve consistency, reusability, and discoverability, so domain specific tool developers can build structure preserving visualization tools.

Visualization component constraints

- equivariance** properties of data and visual encoding match
- continuity** connectivity of data and visual encoding match
- composibility** structure preserved by individual components is preserved in combined components

Tools are tuned to data continuity [15]

PassengerId	Survived	Age	Sex	Name	Embarked	Class	Fare	Survived
1	0	22	male	Mr. Owen Harris	C	3rd	5.12	0
2	1	38	female	Ms. Ellen O'Shea	C	3rd	7.25	1
3	0	26	male	Mr. William Henry	C	3rd	5.18	0
4	1	35	female	Ms. Louise Brock	C	3rd	7.92	1
5	0	35	male	Mr. John Bradley	C	3rd	5.17	0
6	1	35	female	Ms. Julia	C	3rd	7.12	1
7	0	35	male	Mr. James	C	3rd	5.15	0
8	1	35	female	Ms. Julia	C	3rd	7.12	1
9	0	35	male	Mr. James	C	3rd	5.15	0
10	1	35	female	Ms. Julia	C	3rd	7.12	1



Figure: Based on fig 2.5 in Munzner's VAD[1]

1 Tableau[2–4]

2 ggplot[5]

3 Vega[6],
Altair[7]

1 ImageJ[8],
ImagePlot[9]

2 Napari[10]

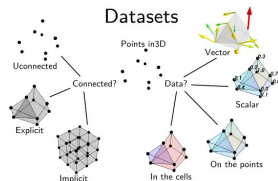


Figure: Data Representation, MayaVi 4.7.2 docs[11]

1 Gephi[12]

2 Graphviz[13]

3 Networkx[14]

Visualizations are tuned to data continuity[23]

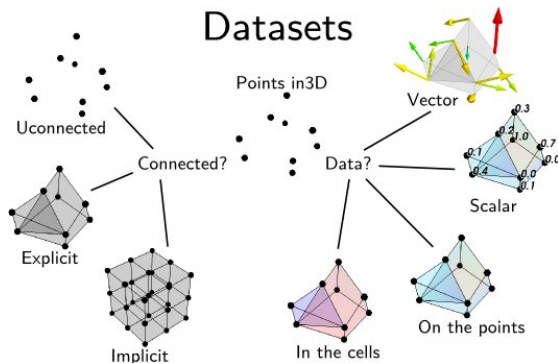
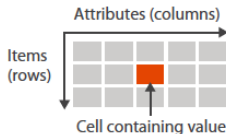


Figure: Data Representation, MayaVi 4.7.2 docs[11]

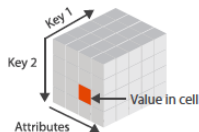
- 1 Matplotlib[16],
- 2 D3 [17]
- 3 VTK [18, 19], MayaVi[20], ParaView[21], Titan[22]

Structure is encoded in variables and continuity

→ Tables



→ *Multidimensional Table*



→ Geometry (Spatial)



Figure: Image is figure 2.8 in Munzner's Visualization Analysis and Design[1]

binding metadata are structural *keys* with associated *values* (Munzner [1])

continuity Fiber bundles can be a common data abstraction (Butler [24, 25])

variables Fibers can hold schema like encodings of variables (Spivak [26, 27])

Visualizations are (mostly) evaluated on equivariance

Expressiveness structure preserving mappings from data to graphic (Mackinlay [28])

Effectiveness design choices made in deference to perceptual saliency (Mackinlay [1, 29–31])

Naturalness easier to understand when properties match (Norman [32])

Graphical Integrity graphs show **only** the data (Tufte [33])

Models describe composition

language model APT, GoG: syntax, semantics, and grammar of graphics (Mackinlay, Wilkenson [28, 34, 35])

functional dependencies constrained maps between data and visual representation (Sugibuchi [36])

category theory the semiotics of visualization are commutative (Vickers [37])

algebraic process data (α) and viz (ω) transforms are symmetric (Kindlmann and Scheidegger [38])

D data

R representations

V visualizations

$$\begin{array}{ccccc} D & \xrightarrow{r_1} & R & \xrightarrow{\nu} & V \\ \alpha \downarrow & & & & \downarrow \omega \\ D & \xrightarrow{r_2} & R & \xrightarrow{\nu} & V \end{array}$$

Contributions

- Topological** topology preserving relationship between data and graphic via continuous maps
- Equivariant** property preservation from data component to visual representation as equivariant maps that carry a homomorphism of monoid actions
- Artist** functional oriented visualization tool architecture built on the mathematical model to demonstrate the utility of the model
- Model** prototype of the architecture built on Matplotlib's infrastructure to demonstrate the feasibility of the model

Topological Equivariant Artist Model

The Artist \mathcal{A} is a map from data \mathcal{E} to graphic \mathcal{H}

$$\mathcal{A} : \mathcal{E} \rightarrow \mathcal{H} \tag{1}$$

that carries a homomorphism of monoid actions

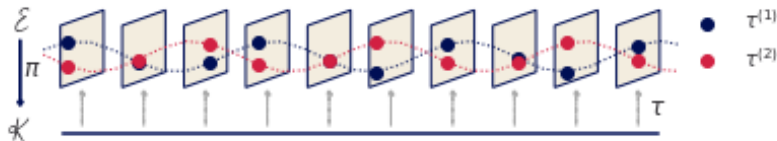
$$\varphi : M \rightarrow M' \tag{2}$$

such that artists are equivariant maps

$$\mathcal{A}(m \cdot r) = \varphi(m) \cdot \mathcal{A}(r) \tag{3}$$

with a deformation retraction from graphic to data space.

Data Bundle



A fiber bundle is a tuple (E, K, π, F) defined by the projection map π

$$F \hookrightarrow E \xrightarrow{\pi} K \quad (4)$$

where E is the total data space, F is the variable space, and K encodes the continuity.

Variables: Fiber

Given a space of all possible values \mathbb{U}

$$\begin{array}{ccc} \mathbb{U}_\sigma & \longrightarrow & \mathbb{U} \\ \pi_\sigma \downarrow & & \downarrow \pi \\ \mathcal{C} & \xrightarrow{\sigma} & \mathbf{DT} \end{array} \quad (5)$$

a fiber component is the restricted space $\mathbb{U}_{\sigma(c)}$.

$$F = \mathbb{U}_{\sigma(c)} = \mathbb{U}_T \quad (6)$$

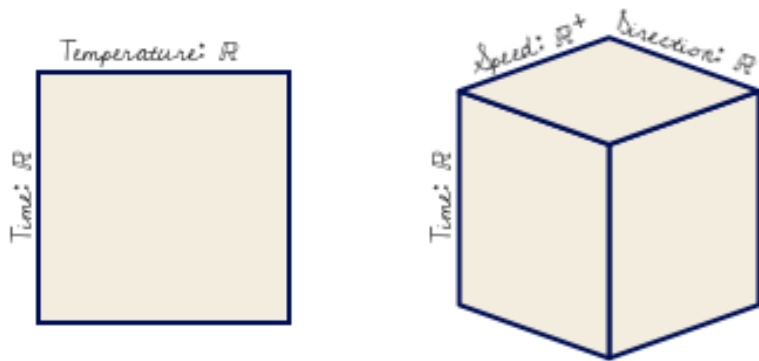
DT data types of the variables in the dataset

\mathbb{U} disjoint union of all values of type $T \in \mathbf{DT}$

\mathcal{C} variable names, $c \in \mathcal{C}$

\mathbb{U}_σ \mathbb{U} restricted to the data type of a named variable

Variable types are dimensions of the fiber



Figure

plane $F = \mathbb{R} \times \mathbb{R}$, (time, temperature)

cube $\mathbb{R} \times \mathbb{R}^+ \times \mathbb{R}$, (time, wind=(speed, direction))

Structure of Components: Monoid & Monoid Actions

A monoid M is a set with

associative binary operator $*$: $M \times M \rightarrow M$

identity element $e \in M$ such that $e * a = a * e = a$ for all $a \in M$.

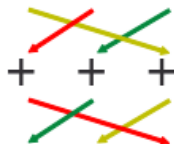
left monoid action

A set F with an action[39] $\bullet : M \times F \rightarrow F$ with the properties:

associativity for all $f, g \in M$ and $x \in F$, $f \bullet (g \bullet x) = (f * g) \bullet x$

identity for all $x \in F$, $e \in M$, $e \bullet x = x$

Monoid Actions: Permutation



Why monoids? partial orders

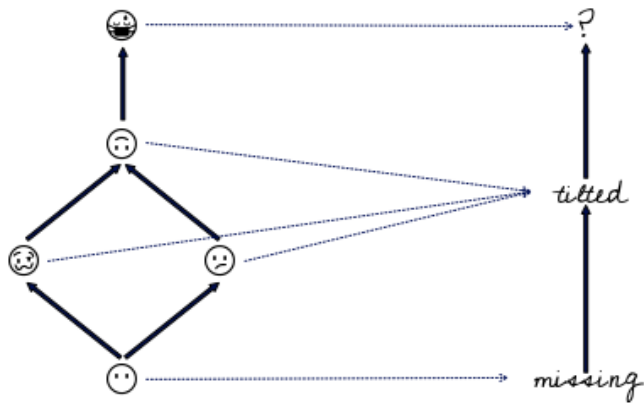
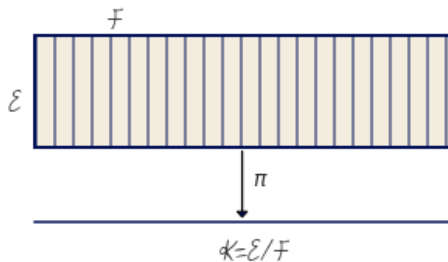


Figure: Inspired by definition 1.59 diagram in Spivak and Fong's An Invitation to Applied Category Theory [40]

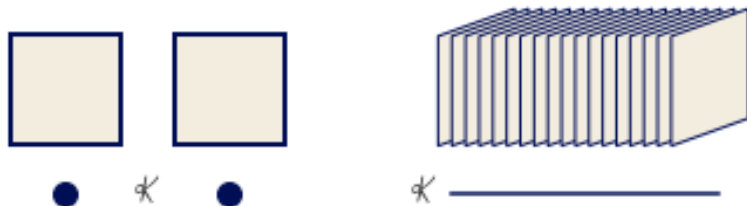
Data Continuity: Base space



where the total space can be decomposed into components

$$\pi: E_1 \oplus \dots \oplus E_i \oplus \dots \oplus E_n \rightarrow K \quad (7)$$

Data connectivity is encoded as the base space



Figure

points data is 0D discrete

line data is lies on the 1D continuous interval \mathcal{K}

For any fiber bundle, there exists a map

$$\begin{array}{ccc} F & \hookrightarrow & E \\ & & \pi \downarrow \uparrow \tau \\ & & K \end{array} \quad (8)$$

s.t. $\pi(\tau(k)) = k$. Set of all global sections is denoted $\Gamma(E)$.

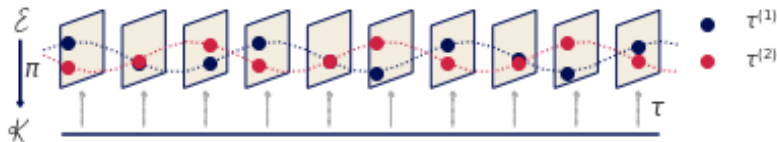
Record

Assuming a trivial fiber bundle $E = K \times F$, the section is

$$\tau(k) = (k, (g_{F_0}(k), \dots, g_{F_n}(k))) \quad (9)$$

where $g : K \rightarrow F$ is the index function into the fiber.

Sample dataset



- F is $\mathbb{R} \times \mathbb{R}$
- K is interval $[0, 1]$
- $\tau^{(1)}$ is a *sin* function
- $\tau^{(2)}$ is a *cos* function
- $\tau^{(1)}, \tau^{(2)} \in \Gamma(E)$

Restriction maps of a sheaf describe how local $\iota^*\tau$ can be glued into larger sections [41, 42].

$$\begin{array}{ccc} \iota^*E & \xhookrightarrow{\iota^*} & E \\ \pi \downarrow \uparrow \iota^*\tau & & \pi \downarrow \uparrow \tau \\ U & \xhookrightarrow{\iota} & K \end{array} \quad (10)$$

The inclusion map $\iota : U \rightarrow K$ pulls E over U such that the pulled back $\iota^*\tau$ only contains records over $U \subset K$.

Graphic Bundle

The graphics bundle is a tuple (H, S, π, D) defined by the projection map π

$$\begin{array}{ccc} D & \hookrightarrow & H \\ & & \pi \downarrow \uparrow \rho \\ & & S \end{array} \quad (11)$$

where ρ is the fully encoded graphic.

Example: 2D opaque image

The target display is $D = \mathbb{R}^5$ with elements

$$(x, y, r, g, b) \in D$$

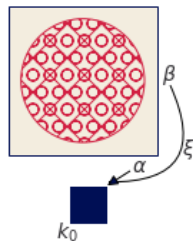
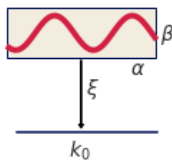
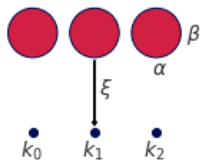
returned by ρ such that a graphic has color and 2D position.

Graphic Continuity

The surjective map $\xi : S \rightarrow K$

$$\begin{array}{ccc} E & & H \\ \pi \downarrow & & \pi \downarrow \\ K & \xleftarrow{\xi} & S \end{array} \quad (12)$$

goes from region $s \in S_k$ to its associated point k in data space.



Topological Equivariant Artist Model

The topological artist A is a monoid equivariant sheaf map

$$\begin{array}{ccccccc} E' & \xrightarrow{\nu} & V & \xleftarrow{\xi^*} & \xi^* V & \xrightarrow{Q} & H \\ & \searrow \pi & \downarrow \pi & & \downarrow \xi^* \pi & & \swarrow \pi \\ & & K & \xleftarrow{\xi} & S & & \end{array} \quad (13)$$

where the artist $A : \mathcal{O}(E) \rightarrow \mathcal{O}(H)$ takes as input $E' = \mathcal{J}^2(E)$.

The visual bundle is a tuple (V, K, π, P) defined by the projection map π

$$\begin{array}{ccc} P & \hookrightarrow & V \\ & & \downarrow \pi \\ & & K \end{array} \Bigg)^\mu \quad (14)$$

where μ is the visual variable encoding[43] of the data section τ .

Example: position and color

Given an artist with parameters $\{xpos, ypos, color\}$, a sample visual section μ could be $\{.5, .5, (255, 20, 147)\}$

Visual Channel Encoders

We define the visual transformers ν on components of the data bundle τ_i

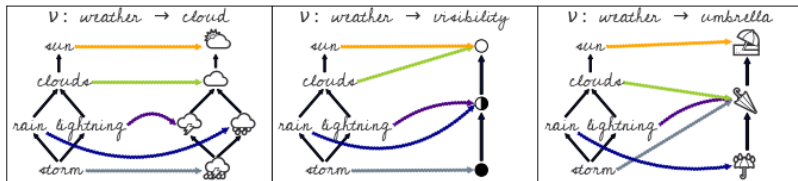
$$\{\nu_0, \dots, \nu_n\} : \{\tau_0, \dots, \tau_n\} \mapsto \{\mu_0, \dots, \mu_n\} \quad (15)$$

as the set of equivariant maps with the constraint

$$\nu_i(m_r(E_i)) = \varphi(m_r)(\nu_i(E_i)) \quad (16)$$

where $\varphi : M \rightarrow M'$ carries a homomorphism of monoid actions.

Example: Nominal Equivariance



Measurement Scale Groups

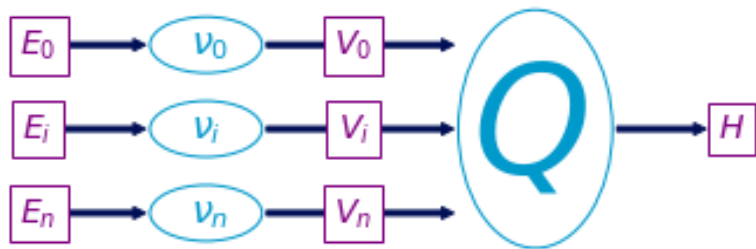
nominal	permutation	if $r_1 \neq r_2$ then $v(r_1) \neq v(r_2)$
ordinal	monotonic	if $r_1 \leq r_2$ then $v(r_1) \leq v(r_2)$
interval	translation	$v(x + c) = v(x) + c$
ratio	scaling	$v(xc) = v(x) * c$

Invalid v

Given $v_i(x) = .5$ and $t(x) = x + 2$,

$$v(t(r + 2)) \stackrel{?}{=} v(r) + 2$$
$$.5 \neq .5 + 2$$

Visualization Assembly Function



The glyph is the graphic generated by $Q(S_j)$ where the path connected components $J \subset K$ are defined

$$J = \{j \in K \text{ s. t. } \exists \gamma \text{ s.t. } \gamma(0) = k \text{ and } \gamma(1) = j\} \quad (17)$$

such that the path γ from k to j is a continuous function from the interval $[0,1]$ and S_j is the region

$$H \xrightleftharpoons[\rho(S_j)]{} S_j \xrightleftharpoons[\xi^{-1}(J)]{\xi(s)} J_k \quad (18)$$

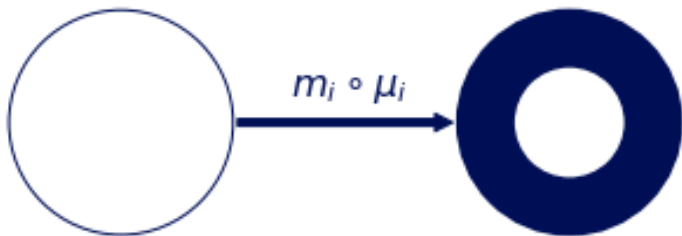
such that the glyph is differentiable, in keeping with Ziemkiewicz and Kosara's description of a glyph[44].

Visualization Equivariance

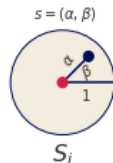
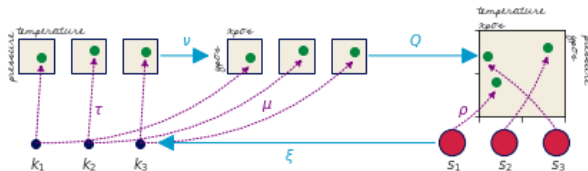
If Q is applied to μ, μ' that generate the same ρ

$$Q(\mu) = Q(\mu') \implies Q(m \circ \mu) = Q(m \circ \mu') \quad (19)$$

then the output of both sections acted on by the same monoid m must be the same.



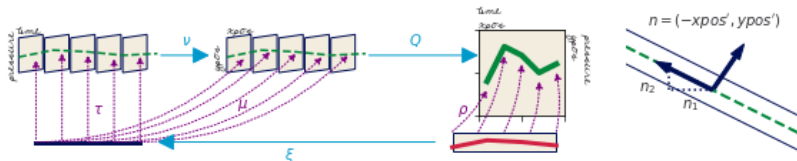
Scatter: $Q(xpos, ypos)(\alpha, \beta)$



$$x = size * \alpha \cos(\beta) + xpos$$

$$y = size * \alpha \sin(\beta) + ypos$$

Line: $Q(xpos, \hat{n}_1, ypos, \hat{n}_2)(\alpha, \beta)$

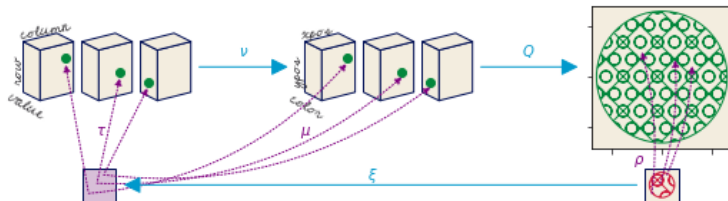


$$|n| = \sqrt{n_1^2 + n_2^2}, \quad \hat{n}_1 = \frac{n_1}{|n|}, \quad \hat{n}_2 = \frac{n_2}{|n|}$$

$$x = xpos(\xi(\alpha)) + width * \beta \hat{n}_1(\xi(\alpha))$$

$$y = ypos(\xi(\alpha)) + width * \beta \hat{n}_2(\xi(\alpha))$$

Image $Q(xpos, ypos, color)$

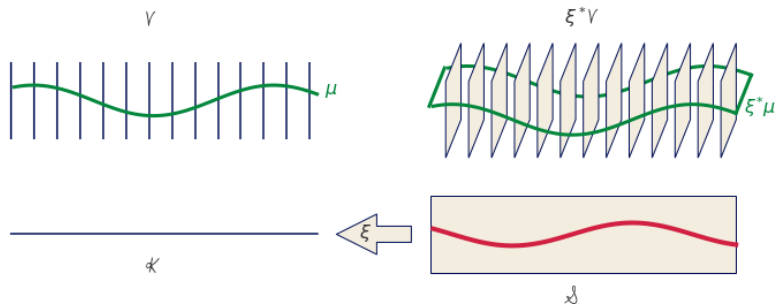


$$R = R(\xi(\alpha, \beta))$$

$$G = G(\xi(\alpha, \beta))$$

$$B = B(\xi(\alpha, \beta))$$

Assembly Function Factory



$$\hat{Q}(\mu(k))(s) := Q((\xi^*\mu)(s)) \quad (20)$$

such s can be factored out when $\xi^{-1}(k) = s$

Composition of artists

Given the family of artists $(E_i : i \in I)$ on the same image

$$+ := \sqcup_{i \in I} E_i \quad (21)$$

the $+$ operator defines a simple composition of artists.

When artists share a base space

$$K_2 \hookrightarrow K_1 \quad (22)$$

a composition operator can be defined such that the the artists can be considered to be acting on different components of the same section.

Equivalence class of artists

An approximation of the equivalence class of artists A'

$$A \in A' : A_1 \equiv A_2 \quad (23)$$

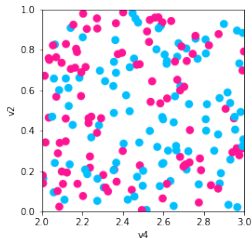
roughly treats two artists as equivalent if they

- act on the same visual bundle V
- have the same assembly function Q
- have the same continuity map ξ

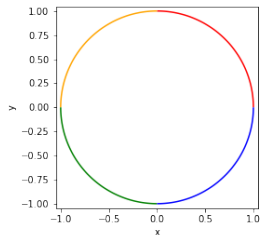
Artist

```
1 class ArtistClass(matplotlib.artist.Artist):
2     def __init__(self, data, transforms, *args, **kwargs):
3         # properties that are specific to the graphic
4         self.data = data
5         self.transforms = transforms
6         super().__init__(*args, **kwargs)
7
8     def assemble(self, **args):
9         # set the properties of the graphic
10
11    def draw(self, renderer):
12        # returns K, indexed on fiber then key
13        view = self.data.view(self.axes)
14        # visual channel encoding applied fiberwise
15        visual = {p: t['encoder'](view[t['name']])
16                  for p, t in self.transforms.items()}
17        self.assemble(**visual)
18        # pass configurations off to the renderer
19        super().draw(renderer)
```

Artists: Scatter & Line



```
1 fig, ax = plt.subplots()
2 artist = Point(data, transforms)
3 ax.add_artist(artist)
```



```
1 fig, ax = plt.subplots()
2 artist = Line(data, transforms)
3 ax.add_artist(artist)
```

Artists: Scatter & Line

```
1 class Point(mcollections.Collection):
2     def assemble(self, x, y, s, facecolors='C0' ):
3         # construct geometries of the circle glyphs in visual coordinates
4         self._paths = [mpath.Path.circle(center=(xi,yi), radius=si)
5                         for (xi, yi, si) in zip(x, y, s)]
6         # set attributes of glyphs, these are vectorized
7         # circles and facecolors are lists of the same size
8         self.set_facecolors(facecolors)
```

```
1 class Line(mcollections.LineCollection):
2     def assemble(self, x, y, color='C0'):
3         #assemble line marks as set of segments
4         segments = [np.vstack((vx, vy)).T for vx, vy in zip(x, y)]
5         self.set_segments(segments)
6         self.set_color(color)
```

Visual Transformations

```
1 cmap = color.Categorical({'true':'deeppink', 'false':'deepskyblue'})
2 transforms = {'x': {'name': 'v4', 'encoder': lambda x: x},
3               'y': {'name': 'v2', 'encoder': lambda x: x},
4               'facecolors': {'name': 'v3', 'encoder': cmap},
5               's': {'name': None,
6                     'encoder': lambda _: itertools.repeat(.02)}}
```

- `lambda x: x` is identity ν
- `{'name':None}` map into P without corresponding τ
- `color.Categorical` is custom ν

Custom Complex γ

```
1 class Categorical:
2     def __init__(self, mapping):
3         # check that the conversion is to valid colors
4         assert(mcolors.is_color_like(color) for color in mapping.values())
5         self._mapping = mapping
6
7     def __call__(self, value):
8         # convert value to a color
9         return [mcolors.to_rgba(self._mapping[v]) for v in values]
```

That we can test for action equivariance

```
1 def test_nominal(values, encoder):
2     m1 = list(zip(values, encoder(values)))
3     random.shuffle(values)
4     m2 = list(zip(values, encoder(values)))
5     assert sorted(m1) == sorted(m2)
```

Fiber Bundle

```
1  @dataclass
2  class FiberBundle:
3      """
4      Attributes
5      -----
6      K: {'tables': []}
7      F: {variable name: type}
8      """
9      K: dict
10     F: dict
```

Discrete Connectivity

```
1 class VertexSimplex: #maybe change name to something else
2     """Fiberbundle is consistent across all sections
3     """
4     FB = FiberBundle({'tables': ['vertex']},
5                       {'v1': float, 'v2': str, 'v3': float})
6
7     def __init__(self, sid = 45, size=1000, max_key=10**10):
8         # create random list of keys
9     def tau(self, k):
10         # e1 is sampled from F1, e2 from F2, etc...
11         return (k, (e1, e2, e3, e4))
```

1D Continuous Connectivity

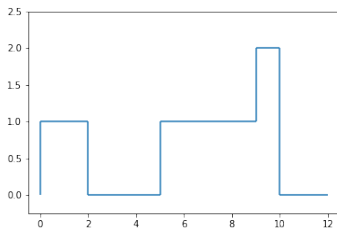
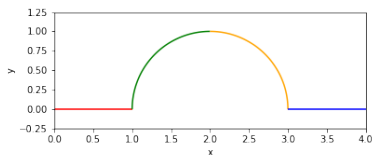
```
1 class EdgeSimplex:
2     FB = FiberBundle({'tables': ['vertex', 'edge']},
3                       {'x': float, 'y': float,
4                        'color': mtypes.Color()})
5     def __init__(self, num_edges=4, num_samples=1000):
6         self.keys = range(num_edge) #edge id
7         self.distances = np.linspace(0,1, num_samples)
8         # half generlized representation of arcs on a circle
9         self.angle_samples = np.linspace(0, 2*np.pi, len(self.keys)+1)
10    @staticmethod
11    def _color(edge):
12        return ['red', 'orange', 'green', 'blue'][edge%4]
13    @staticmethod
14    def _xy(edge, distances, start=0, end=2*np.pi):
15        # start and end are parameterizations b/c really there is
16        angles = (distances *(end-start)) + start
17        return np.cos(angles), np.sin(angles)
18    def tau(self, k): #will fix location on page on revision
19        x, y = self._xy(k, self.distances,
20                        self.angle_samples[k], self.angle_samples[k+1])
21        color = self._color(k)
22        return (k, (x, y, color))
```

```
1 def view(self, axes):
2     table = defaultdict(list)
3     for k in self.keys:
4         table['index'].append(k)
5         for (name, value) in zip(self.FB.fiber.keys(), self.tau(k)[1]):
6             table[name].append(value)
7     return table
```

VertexSimplex (name, value), value is scalar

EdgeSimplex (name, value), value is $[x_0, \dots, x_n]$

Same Artist, Different Data Configurations



```
simplex.GraphLine(FB, edge_table, vertex_table,  
                  connect=True)  
simplex.GraphLine(FB, edge_table, vertex_table,  
                  num_samples=2, connect=False)
```

Summary

- structure preserving maps from data to visual representation:
 - data and graphics have equivalent continuity
 - properties are equivariant under monoid actions
- fiber bundles with a schema are structure rich abstractions of
 - topologically complex heterogeneous data
 - target display spaces
- model can be iteratively integrated into existing Matplotlib architecture

Proposed Dissertation

- expansion of the mathematical framework to include worked out simple and complex addition
- formalization of definition of equivalence class A'
- implementation of artist with explicit ξ
- specification of interactive visualization
- mathematical formulation of a graphic with axes labeling
- implementation of new prototype artists that do not inherit from Matplotlib artists
- provisional mathematics and implementation of user level composite artists
- proof of concept domain specific user facing library

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Rendering: Define a Pixel

Given a pixel

$$p = [y_{top}, y_{bottom}, x_{right}, x_{left}] \quad (24)$$

the inverse map of the bounding box

$$S_p = \rho_{xy}^{-1}(p) \quad (25)$$

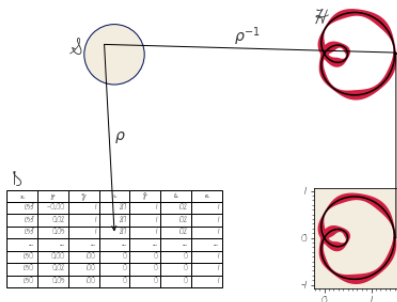
is a region $S_p \subset S$ such that

$$r_p = \iint_{S_p} \rho_r(s) ds^2 \quad (26)$$

$$g_p = \iint_{S_p} \rho_g(s) ds^2 \quad (27)$$

$$b_p = \iint_{S_p} \rho_b(s) ds^2 \quad (28)$$

yields the color of the pixel.



P Components

ν_i	μ_i	$\text{codomain}(\nu_i) \subset P_i$
position	x, y, z, theta, r	\mathbb{R}
size	linewidth, markersize	\mathbb{R}^+
shape	markerstyle	$\{f_0, \dots, f_n\}$
color	color, facecolor, markerfacecolor, edgecolor	\mathbb{R}^4
texture	hatch	\mathbb{N}^{10}
	linestyle	$(\mathbb{R}, \mathbb{R}^{+n, n\%2=0})$

GraphLine Data Model

```
1 class GraphLine:
2     def __init__(self, FB, edge_table, vertex_table, num_samples=1000,
3                 connect=False):
4         #set args as attributes and generate distance
5         if connect: # test connectivity if edges are continuous
6             assert edge_table.keys() == self.FB.F.keys()
7             assert is_continuous(vertex_table)
8
9     def tau(self, k):
10        # evaluates functions defined in edge table
11        return(k, (self.edges[c][k](self.distances)
12                  for c in self.FB.F.keys()))
13
14    def view(self, axes):
15        # walk the edge_vertex table to return the edge function
16        table = defaultdict(list)
17        for (i, (start, end)) in sorted(zip(self.ids, self.vertices),
18                                       key=lambda v:v[1][0]):
19            table['index'].append(i)
20            # same as view for line, returns nested list
21            for (name, value) in zip(self.FB.F.keys(), self.tau(i)[1]):
22                table[name].append(value)
23        return table
```