Make any stupid plot you want

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Abstract

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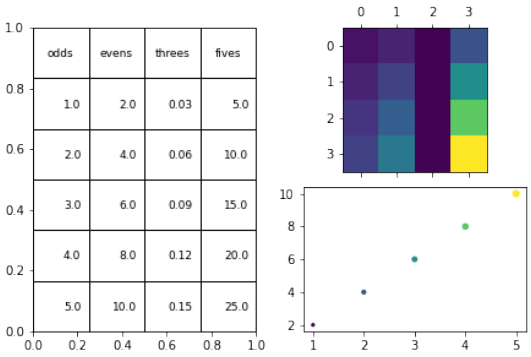


Figure 1: Implicit in visualization is the assumption that these three representations of data are equivalent, specifically that the measurements within a variable and relations of the measurements of each variable are preserved.

1 Introduction

1.1 Thesis statement

We define a visualization as a transform from data to graphic that preserves the topology of the data and the properties of the measurement type. In fig 1, we implicitly assume that the translation from table to heatmap has preserved the order of observations (the rows) and that the perceptually uniform sequential colormap has been applied such that the ordering relation on floats matches the ordering on the colormap (darker colors map to larger numbers). We also make this assumption about color in the scatter map, and that the translation to size and position on screen also respect the ordering on floats. In this work, we propose to mathematically describe the transform of data to visual space such that we can make explicit the implicit topology and types visualizations preserve. We then propose a new architecture for the Python visualization library Matplotlib [6] based on these descriptions because the Matplolib artist layer is analogous to the transforms.

1.2 What is a viz

? Acquired codes of meaning

2 Not all data are tables

Tables, images (Lev), graphs (network X)

3 Notation & Definitions

In this section we introduce a mathematical description of the visualization pipeline where artist A functions transform data of type $\Gamma(E)$ to an intermediate representation in prerendered display space of type $\Gamma(H)$:

$$A : \Gamma(E) \to \Gamma(H) \tag{1}$$

$$A : \sigma \to \rho \tag{2}$$

- A is the function that converts an instance of data $\Gamma(E)$ to an instance of a visual representation $\Gamma(H)$
- E is a locally trivial fiber bundle over K representing data space.
- K is a triangulizable space encoding the connectivity of the observations in the data.
- H is a fiber bundle over S representing visual space
- S is a simplacial complex of triangles encoding the connectivity of the visualization of $\Gamma(E)$
- $\sigma : K \to E$ is the data being visualized
- $\rho : S \to H$ is the render map

When E is a trivial fiber bundle $E = F \times K$, it can be assumed that all fibers F_k over $k \in K$ are equal. Fiber bundles are product spaces of toplological spaces, which are a set of points with a set of neighborhoods for each point[5, 9].

3.1 Data Model

We use a fiber bundle model to represent the data, as proposed by Butler [2, 3]. A fiber bundle is a structure (E, K, π, F) consisting of topological spaces E, K, F and the map from total space to base space:

$$\begin{array}{ccc} F & \hookrightarrow & E \\ & & \swarrow \pi \\ & & K \end{array} \tag{3}$$

where there is a bijection from F to every fiber F_k over point $k \in K$ in E and the function $\pi : E \to K$ is the map into the K quotient space of E. Every point in the base space $k \in K$ has a local open set neighborhood U [5, 9]

$$\begin{array}{ccc} \pi^{-1}(U) & -\varphi \rightarrow & U \times F \\ \downarrow \pi & \swarrow_{\text{proj}_U} & \\ U & & \end{array} \tag{4}$$

such that $\varphi : \pi^{-1}(U) \to U \times F$ is a homeomorphism where π and proj_U both map to U and the fiber over k $F_k = \pi^{-1}(k \in K)$ is homomorphic to the fiber F.

The section σ is the mapping $\sigma : K \to E$

$$\begin{array}{ccc} F & \hookrightarrow & E \\ & & \swarrow \pi \quad \nwarrow \sigma \\ & & K \end{array} \tag{5}$$

such that it is the right inverse of π

$$\pi(\sigma(k)) = k \text{ for all } k \in K \tag{6}$$

In a locally trivial fiber bundle, $\sigma = K \times E$ [5, 9]:

$$\sigma(k) = (k, g(k)) \tag{7}$$

where the domain of $g(k)$ is F_k. The space of sections over $U \subset K$ is called a sheaf and the space of all possible sections σ of E is $\Gamma(E)$. All datasets $\sigma \in \Gamma(E)$ have the same variables F and connectivity K but can have different values such that $\sigma_i \neq \sigma_j$.

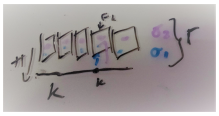
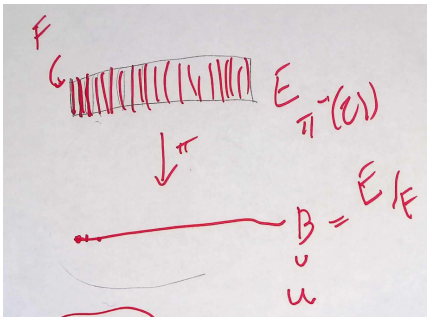


Figure 2: write up some words here

As illustrated by figure 2, the vertical lines F are the range of possible temperature values embedded in the total space E. The base space K of the fiber bundle describes the connectivity of the points in E; in figure 2 the connectivity of the timeseries is encoded in the line representation of K.

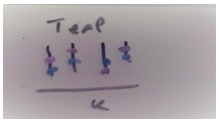
3.1.1 Base Space K

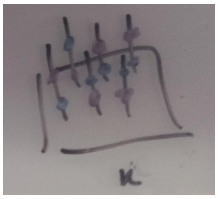


K is the quotient space of E, meaning it is the set of equivalence classes of elements p in of E defined via the map $\pi : E \to K$ that sends each point $p \in E$ to its equivalence class in $[p] \in K$ [8]. As shown in figure **??**, the fibers F divide E into smaller spaces consisisting of F and an open set neighborhood around F. This subdivision is projected down to the toplology τ_k

$$\tau_K = \{U \subseteqq K : \{p \in E : [p] \in U\} \in \tau_E\} \tag{8}$$

where $[p] \in U$ is the point $k \in K$ with an open set around it that has an open preimage in E under the surjective map $\pi : p \to [p]$.





We use K to encode the connectivity of the points p. In figure **??**, there is only one data field in p, temperature, but the points p are connected differently. In a timeseries, the temperature p at time t is dependent on the value at p_{t-1} and p_{t+1} is dependent on the value in p_t; this connectivity is expressed as a one dimensional K where K is the number line. In the case of the map, every point p is dependent on its nearest neighbors on the plane, and one way to express this is by encoding K as a plane. K does not know the time or latitude or longitude of the point - those are metadata variables potentially encoded in p because they are ways of describing the connectivity rather than the connectivity itself. The mapping $sigma : K \to E$ provides the binding between the key on K and the value p in E [7].

3.1.2 Fiber Space F

Spivak models the fiber space F as schema and the data as sheafs (localized σ functions) on the schema [10]. He defines the type specification

$$\pi : U \to DT \tag{9}$$

where DT is the set of data types (as identified by their names) and U is the disjoint set of all possible objects x of all types in DT. This means that for each type $T \in DT$,

the preimage $\pi^{-1}(T) \subset U$ is the domain of T, and $x \in \pi^1(T) \subset U$ is an object of type T. Spivak then defines a schema (C, σ) of type π, where π is the universe of all types, such that

$$\sigma : C \to DT \tag{10}$$

where C is the finite set of names of data fields in E. The set of all values restricted to the datatypes in DT is U_σ

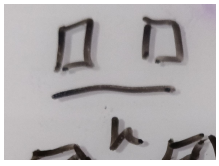
$$\begin{array}{ccc} U_\sigma & \longrightarrow & U \\ \downarrow{\scriptstyle \pi_\sigma} & & \downarrow{\scriptstyle \pi} \\ C & -\sigma \to & DT \end{array} \tag{11}$$

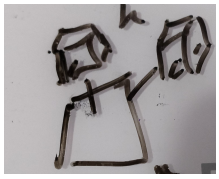
The pullback $U_\sigma \coloneqq \sigma^{-1}(U)$ restricts U to the datatypes of the fields in C such that U_σ is the fiber product $U \times_{DT} C$, and the pullback $\pi_\sigma : U_\sigma \to C$ specifies the domain bundle U_σ over C induced by σ. This domain bundle backs the fiber F in the data total space E

$$F = \prod_{i \in I} U_{\sigma_i} = \tag{12}$$

where F is the cartesian product of all sets in the disjoint union U_σ

The record function is the sigma function





The fibers F are a topological space embedded in E on which lie the set of all possible values. For example, if F is the interval $[0, 1]$, then $g(k)$ from equation **??** returns a single

measurement x in the interval F:

$$g(k) = x, \text{ where } 0 \leq x \leq 1 \tag{13}$$

The fiber in figure **??** is the space of possible temperature values in ∘ celsius, ranging from [start, end], similar to the interval F in equation 13. F can be any number of dimensions, for example in figure **??** time is encoded as a second dimension. Given:

- interval of all possible temperature values $[T_{min}, T_{max}]$
- interval of all possible time values $[t_{min}, t_{max}]$

then F is the cross product $F = [T_{min}, T_{max}] \times [t_{min}, t_{max}]$, and $g(k)$ listed in equation **??** is:

$$g(k) = (x_0, x_1) \text{ where } x_0 \in [T_{min}, T_{max}], x_1 \in [t_{min}, t_{max}] \tag{14}$$

When E is trivial, then we can decompose E so that each E

3.1.3 Subset

$\Gamma(E)$ is the space of all points in F returned by σ; therefore the points being visualized in a streaming or animation example can be considered a subset that lives on base space $U \subset K$ with the same fiber F

$$\begin{array}{ccc} \iota^* E & \hookrightarrow & E \\ \downarrow{\scriptstyle \iota^*\sigma} & & \downarrow{\scriptstyle \sigma} \\ U & \xrightarrow{\iota} & K \end{array} \tag{15}$$

where $\iota^* E$ and $\iota^* \sigma$ are E and σ restricted to points $k \in U \subset K$.

3.2 Prerender Space

A physical display space can be thought of sets of $\mathbb{R}^7$ tuples, where

$$\mathbb{R}^7 = \{X, Y, Z, R, G, B, A\} \tag{16}$$

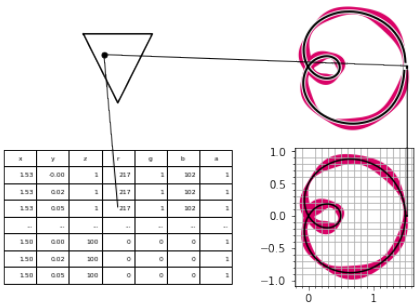


Figure 3

and the sets correspond to the sections on §, which is the topology of the output of the artist A. The space H is a total space representing the predisplay space, with a fiber of $\mathbb{R}^7$ and a base space of §:

$$\begin{array}{ccc} \mathbb{R}^7 & \hookrightarrow & H \\ & & \pi \swarrow \quad \nwarrow \rho \\ & & S \end{array} \tag{17}$$

In the case of 2D screens, the predisplay space is a trivial fiber bundle $H = \mathbb{R}^7 \times S$. As illustrated in figure 3, a region on the screen defined by the corners (x_1, y_1) and (x_2, y_2) maps into a region on a 2-simplex in S defined by (α_1, β_1) and (α_2, β_2). The function on the simplex f returns the (R, G, B, A) value for that (α, β) pair. For a region,

$$\rho(S) = \int_{\alpha_1}^{\alpha_2} \int_{\beta_1}^{\beta_2} \int_{z_1}^{z_2} R, G, B, A$$

where the R,G,B,A values are derived from the how the data values are mapped to visual characteristics. The z component of the mapping to $\mathbb{R}^7$ is moved to the integration because this is a trivial space representing a 2D screen; ρ varies depending on H.

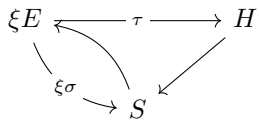
3.3 Artist

$$A : \Gamma(E) \to \Gamma(H) \tag{18}$$

3.3.1 Screen to Data

$$\begin{array}{ccc} E & & H \\ \scriptstyle{\pi}\downarrow & & \scriptstyle{\pi}\downarrow \\ K & \xleftarrow{\xi} & S \end{array} \tag{19}$$

The pullback ξ on $S \to K$ means that the values in E can be directly mapped to a simplex in S, which means there's a mapping from screen space back to the values.



(20)

3.3.2 Marks

Bertin describes a location on the plane as the signifying characteristic of a point, measurable length as the signifying characteristic of a line, and measurable size as the signifying characteristic of an area and that in display (pixel) space these are marks [1, 4].

$$H \underset{\rho(\xi^{-1}(J))}{\overset{}{\rightleftarrows}} S \overset{\xi(s)}{\underset{\xi^{-1}(J)}{\rightleftarrows}} J_k = \{j \in K | \exists \Gamma \text{ s.t. } \Gamma(0) = k \text{ and } \Gamma(1) = j\} \tag{21}$$

Each point s in the display space H, the mark it belongs to can be found by mapping s back to K via the lookup on S described in section 3.2 then taking $\xi(s)$ back to a point on $k \in K$ which lies on the connected component $J \subset K$. To got back to the display space H from the simplacial complex J of the signifier implanted in the mark, the inverse image of $J \in S, \xi^{-1}(J)$ is pushed back to S, and then $\rho(\xi^{-1}(J))$ maps it into R^7.

3.3.3 Channels

Acts on different parts of F, types means measurement groups , can be broken out so Tau can preserves the measurement type properties (group scales)

Tau is fully flexible and can do whatever; knows about fiber & neighborhood of fiber. Can in theory approximate hatching/dashing/etc can be approximated w/ functions and neighborhood of k.

3.3.4 Visual Idioms: Equivalance class of artists

Two artists are equivalent when given data containers $\Gamma(E)$ of the same type, they output the same type of prerender $\Gamma(S)$:

$$\begin{array}{lll} A_{\tau_2}: & \Gamma(E) \longrightarrow \Gamma(H) \\ \downarrow\uparrow & & \\ A_{\tau_1}: & \Gamma(E) \longrightarrow \Gamma(H) \end{array} \tag{22}$$

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