1 Introduction

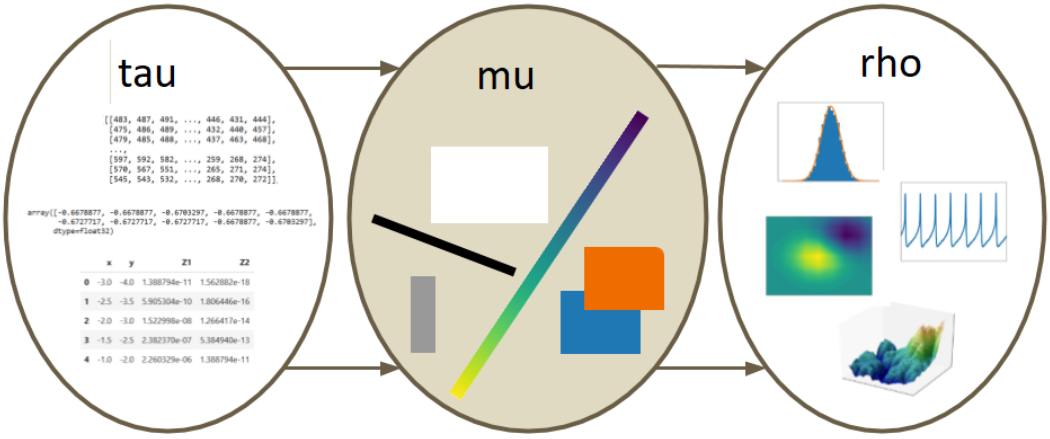


Figure 1: Visualization is equivariant maps between data and visual encoding of the variables and assembly of those encodings into a graphic. will replace w/ overarching figure w/ same structure

This work is motivated by a need for a visualization library that developers could build complex, domain specific tools tuned to the semantics and structure carried in domain specific data. The core architecture also needs to be robust to the big data needs of many visualization practitioners, and therefore support distributed and streaming data needs. To support both exploratory and confirmatory visualization[84], this tool needs to support 2D and 3D, static, dynamic and interactive visualizations. Specifically, this work was driven by a rearchiture the Python visualization library Matplotlib[41] to meet modern data visualization needs. We aim to take advantage of developments in software design, data structures, and visualization to improve the consistency, composibility, and discoverability of the API.

This work presents a mathematical description of how data is transformed into graphic representations, as shown in figure 1. While many researchers have identified and described important aspects of visualization, they have specialized in such different ways as to not provide a model general enough to natively support the full range of data and visualization types many general purpose modern visualization tools may need to support. In this work, we present a framework for understanding visualization as equivariant maps between topological spaces. Using this mathematical formalism, we can interpret and extend prior work and also develop new tools. We validate our model by using it to re-design artist and data access layer of Matplotlib, a general purpose visualization tool.

2 Background

Visual algorithms that display information are designed such that structure of data is assumed according to Tory and Möller [82]. Specifically they note that discrete and continuous data and their attributes form a discipline independent design space [61]. This idea can be seen in modern general purpose visualization architecture, where many libraries are either defined around specific data structures or make implicit assumptions about the data or the

types of visualizations they will support. In proposing a new architecture, we contrast the trade offs libraries make, describe different types of data continuity, and discuss metrics by which a visualization library is traditionally evaluated.

2.1 Tools

The library driving this rearchitecture, Matplotlib, aims to be flexible enough that developers can build domain specific libraries on top of it. To preserve this flexibility, Matplotlib enforces minimal constraints on what sorts of data the user is allowed to input and supports very many visual algorithms, from primative marks to computationally complex visualizations. Instead of enforcing constraints at the API level, Matplotlib carries implicit assumptions about data continuity in how each function interfaces with the input data. This has lead to poor API consistency and brittle code as every visual algorithm has a very different point of view on how the data is structured.

A commonly cited alternative is the family of tools built on top of Wilkenson's Grammar of Graphics (GoG) [90], including ggplot[88], protovis[13] and D3 [14], vega[70] and altair[85]. This framework is very popular in the data visualization community because it lends itself to a declarative interface [37] which allows end users to describe the visualization they are trying to create. It is also the wrong abstraction layer since the grammer exposes specific ways to compose parts together to build a chart, but not ways to build new parts[92].

A different class of user facing tools are those that support images, such as ImageJ[71]. These tools mostly have some support for visualizing non image components of a complex data set, but mostly in service to the image being visualized. These tools are ill suited for general purpose library developers as the architecture is to build plugins into the existing system [93] rather than domain specific tools that are built on top. Even the digital humanities oriented macro ImagePlot[79], which supports some non-image aggregate reporting charts, is still built around image data as the primary input.

There are also visualization tools designed around scientifically complex data that support explicit description of the data, such as vtk[31, 35] and its derivatives such as MayaVi[65] and extensions such as ParaView[4]. These libraries, generally speaking, are architectured more as graphics libraries than visualization libraries, meaning they lack a clear distinction between the graphic construction and the rendering of that graphic. This is very close to the existing Matplotlib architecture, where the data, visual encoding, and rendering are jumbled in ways that make it hard to figure out what the code is doing. We are proposing a functional framework instead in large part to clearly encapsulate these separate responsibilities of a visualization tool. In turn, this should make it easier to reuse components in the building of new tools.

Add somehow: Software Design Patterns for Information Visualization (heer, agrawala)

2.2 Data

As mentioned, one of the drivers of this work was to facilitate building libraries that could natively support domain specific data containers. Fiber bundles are one such way model to model containers, as Butler proposes because they encode the continuity of the data separately from the types of variables and are flexible enough to support discrete and ND continuous datasets [16, 17]. Since Butler's model lacks a robust way of describing the variables, we fold in Spivak's Simplacial formulation of databases [74, 75] so that we can encode a schema like description of the data in the fiber bundle.

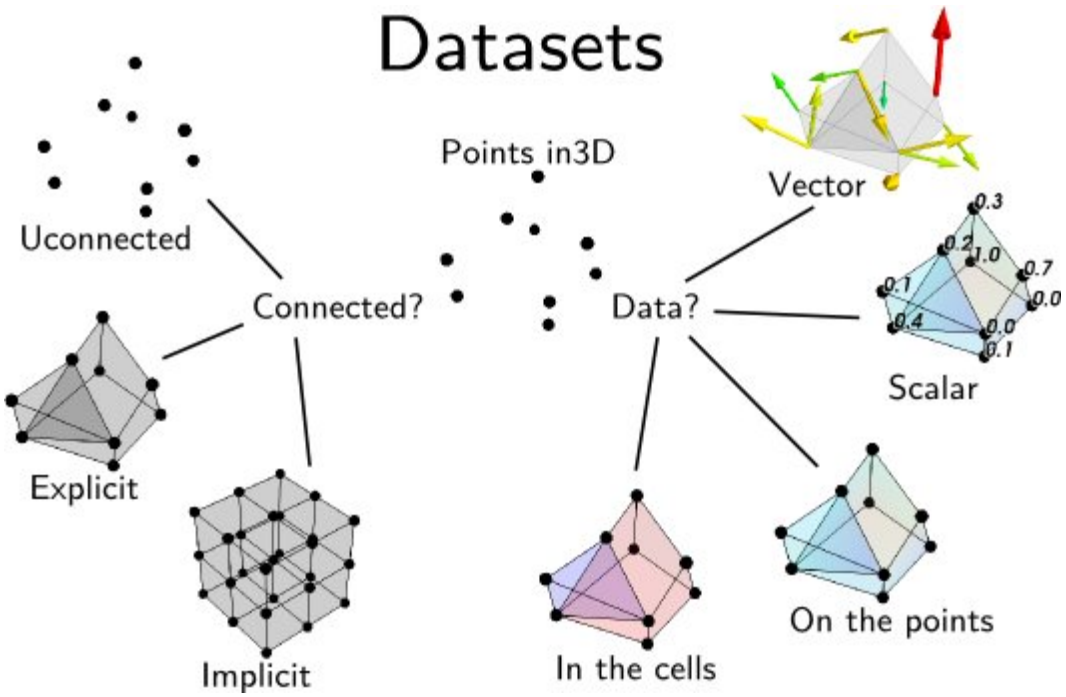


Figure 2: One way to describe data is by the connectivity of the points in the dataset. A database for example is often discrete unconnected points, while an image is an implicitely connected 2D grid. This image is from the Data Representation chapter of the MayaVi 4.7.2 documentation.[27]

As shown in figure 2, there are many types of connectivity. A database typically consists of unconnected records, while an image is an implicit 2D grid and a network is some sort of explicitly connected graph. In this work we will refer to the points of the dataset as *records* to indicate that a point can be a vector of heterogenous elements. Each *component* of the record is a single object, such as a temperature measurement, a color value, or an image. We also generalize *component* to mean all objects in the dataset of a given type, such as all temperatures or colors or images. The way in which these records are connected is the *connectivity*, *continuity*, or more generally *topology*.

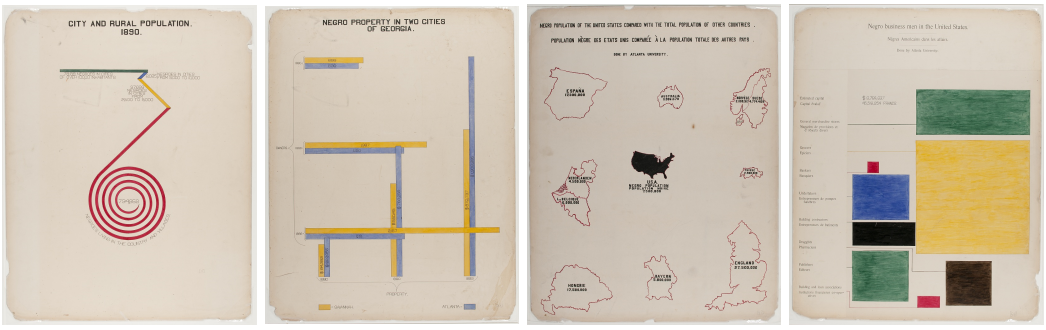
definitions
records points, observations, entries
components variables, attributes, fields
connectivity how the records are connected to each other

Often this topology has metadata associated with it, describing for example when or where the measurement was taken. Building on the idea of metadata as *keys* and their associated *value* proposed by Munzner [56], we propose that information rich metadata are part of the components and instead the values are keyed on coordinate free structural ids. In contrast to Munzner's model where the semantic meaning of the key is tightly coupled

to the position of the value in the dataset, our model allows for renaming all the metadata, for example changing the coordinate systems or time resolution, without imposing new semantics on the underlying structure.

2.3 Visualization

A visualization tool produces a graphical design and an image rendered based on that design, as described by Mackinlay [51]. He defines the graphical design as the set of encoding relations from data to visual representation[50], and the design rendered in an idealized abstract space is what throughout this paper we will refer to as a graphic. In addition to the graphic representations, Byrne et al. describe how visualizations have figurative representations that have meaning due to their similarity in shape to external concepts [18]. Mackinlay proposes that a visualization tool's expressiveness is a measure of how much of the structure of the data the tool encodes, while the tool's effectiveness describes how much design choices are made in deference to perceptual saliency [23–25, 57].



(a) (b) (c) (d)

Figure 3: Du Bois' data portraits[28] of post reconstruction Black American life exemplify that the fundemental characteristics of data visualization is that the visual elements vary in proportion to the source data. In figure 3a, the length of each segment maps to population; in figure 3b, the bar charts are intersected to show the number of property owners and how much they own in a given year; in figure 3c the countries are scaled to population size; and figure 3d is a treemap where the area of the rectangle is representative of the number of businesses in each field. The images here are from the Prints and Photographs collection of the Library of Congress [1, 2, 80, 81]

We propose that the Du Bois in figure 3 are representative of the expressivity a core architecture should allow a downstream library to express. This is because while the Du Bois figures are not the common scatter, bar, or line plot [30], they conform to the constraint that a visualization is a mapping from data to visuals. In figure 3c, Du Bois combines a graphical representation where glyph size varies by population with a figurative representation of those glyphs as the countries the data is from, which means that the semantic and numerical properties of the data are preserved in the graph. The visual representations in the Du Bois figures are in proportion to the quantitative data being represented, so the a chart is faithful according to Tufte's Lie Principal[83]. The properties of the representation match the properties of the information being represented, so they are fairly understandable according to Norman's Naturalness Principal[60]. As with the Du Bois data portraits, it is

fundemental that any architecture tool we build ensures that the data properties match the visual properties.

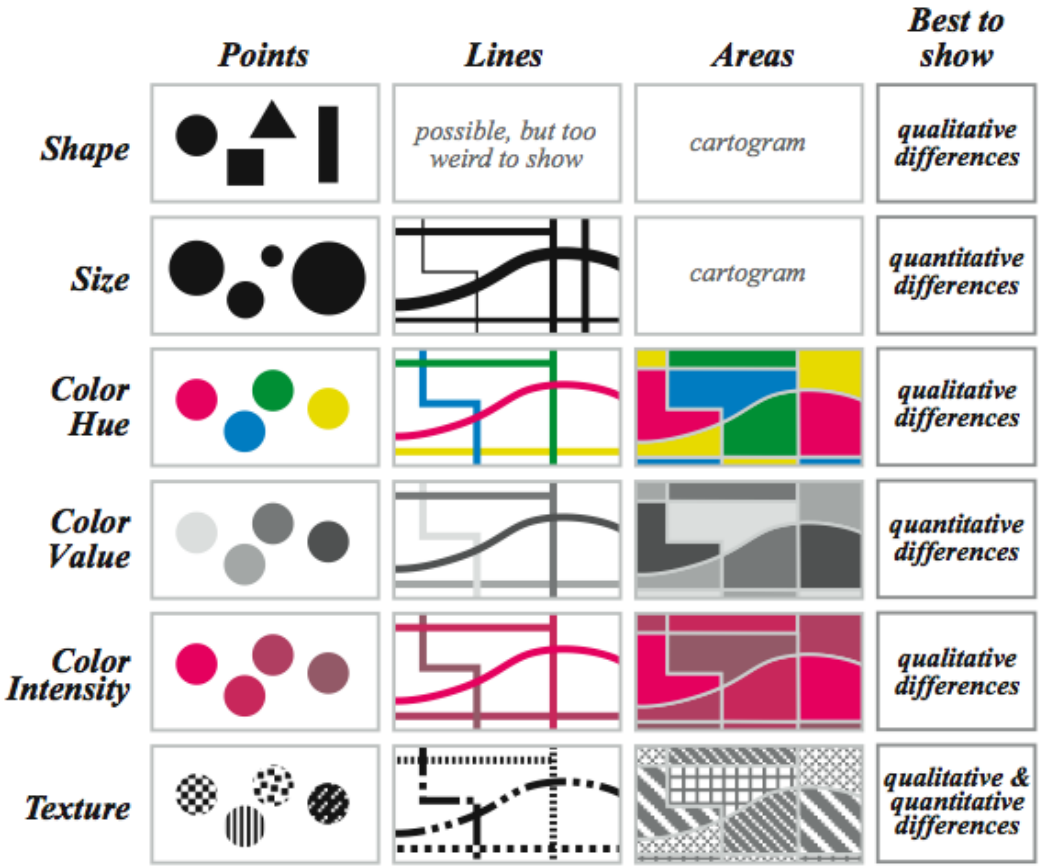


Figure 4: Retinal variables are a codification of how position, size, shape, color and texture are used to illustrate variations in the components of a visualization. The best to show column describes which types of information can be expressed in the corresponding visual encoding. This tabular form of Bertin's retinal variables is from Understanding Graphics [52] who reproduced it from *Making Maps: A Visual Guide to Map Design for GIS* [45]

The visual properties were first codified by Bertin[10], who also described the types of data that paired with the visual properties shown in figure 4. It is at this encoding level that Mackinlay formalized expressiveness as a homomorphic mapping which preserves some binary operator from one domain to another [51]. A functional dependency framework for evaluating these equivariances was proposed by Sugibuchi et al [**sugibuchiFramwork2009**], and an algbraic basis for visualization design and evaluation was proposed by Kindlmann and Scheideggar[44]. These visual encodings are composited into the point, line, and area graphical marks, as shown in figure 4 correspond. Marks can be generalized to glyphs, which are graphical objects that convey one or more attributes of the data entity mapped to it[57, 86]. As retinal variables and marks are the building blocks of most visualizations,

they must be fully expressible in a framework for building visualization tools. By building visualization concepts into the core architecture, developers can incorporate assessments of the visualization, such as as quality metrics[11] or invariance [44] of visualizations with respect to graphical encoding choices.

2.4 Contribution

should I mention the categorical framework here too? This work presents a mathematical model of the transformation from data to graphic representation and a proof of concept implementation. Specifically, this work contributes

1. a functional oriented visualization tool architecture
2. topology-preserving maps from data to graphic
3. monoidal action equivariant maps from component to visual variable
4. algebraic sum such that more complex visualizations can be built from simple ones
5. prototype built on Matplotlib's infrastructure

In contrast to mathematical models of visualization that aim to evaluate visualization design, we propose a topological framework for building tools to build visualizations. We defer judgement of expressivity and effectiveness to developers building domain specific tools, but provide them the framework to do so.

3 Topological Artist Model

As discussed in the introduction, visualization is generally defined as structure preserving maps from data to graphic representation. In order to formalize this statement, we describe the connectivity of the records using topology and define the structure on the components in terms of the monoid actions on the component types. By formalizing structure in this way, we can evaluate the extent to which a visualization preserves the structure of the data it is representing and build structure preserving visualization tools. We introduce the notion of an artist $\mathscr{A}$ as a structure preserving map from data $\mathscr{E}$ to $\mathscr{H}$

$$\mathscr{A} : \mathscr{E} \to \mathscr{H} \tag{1}$$

We model the data $\mathscr{E}$, graphic $\mathscr{H}$, and intermediate visual encoding $\mathscr{V}$ stages of visualization as topological structures that encapsulate types of variables and continuity; by doing so we can develop implementations that keep track of both in ways that let us distribute computation while still allowing assembly and dynamic update of the graphic. To explain which structure the artist is preserving, we first describe how we model data(3.1), graphics(3.2), and intermediate visual characteristics (3.3) as fiber bundles. We then discuss the equivariant maps between data and visual characteristics (3.3.2) and visual characteristics and graphics (3.3.3) that make up the artist. xi should maybe be moved down to artist but is more readable in graphics

3.1 Data Space E

Building on Butler's proposal of using fiber bundles as a common data representation format for visualization data[16, 17], a fiber bundle is a tuple (E, K, π, F) defined by the projection map π

$$F \hookrightarrow E \xrightarrow{\pi} K \tag{2}$$

that binds the components of the data in F to the continuity represented in K. The fiber bundle models the properties of data component types F (3.1.1), the continuity of records K (3.1.3), the collections of records τ (3.1.4), and the space E of all possible datasets with these components and continuity.

By definition fiber bundles are locally trivial[47, 73], meaning that over a localized neighborhood we can dispense with extra structure on E and focus on the components and continuity. We use fiber bundles as the data model because they are inclusive enough to express all the types of data described in section 2.2.

3.1.1 Variables: Fiber Space F

To formalize the structure of the data components, we use notation introduced by Spivak [75] that binds the components of the fiber to variable names and types. Spivak constructs a set $\mathbb{U}$ that is the disjoint union of all possible objects of types $\{T_0, \ldots, T_n\} \in \mathbf{DT}$, where $\mathbf{DT}$ are the data types of the variables in the dataset. He then defines the single variable set $\mathbb{U}_\sigma$

$$\begin{array}{ccc} \mathbb{U}_\sigma & \longrightarrow & \mathbb{U} \\ {\scriptstyle \pi_\sigma}\downarrow & & \downarrow{\scriptstyle \pi} \\ C & \xrightarrow[\sigma]{} & \mathbf{DT} \end{array} \tag{3}$$

which is $\mathbb{U}$ restricted to objects of type T bound to variable name c. Given σ, the fiber for a one variable dataset is

$$F = \mathbb{U}_{\sigma(c)} = \mathbb{U}_T \tag{4}$$

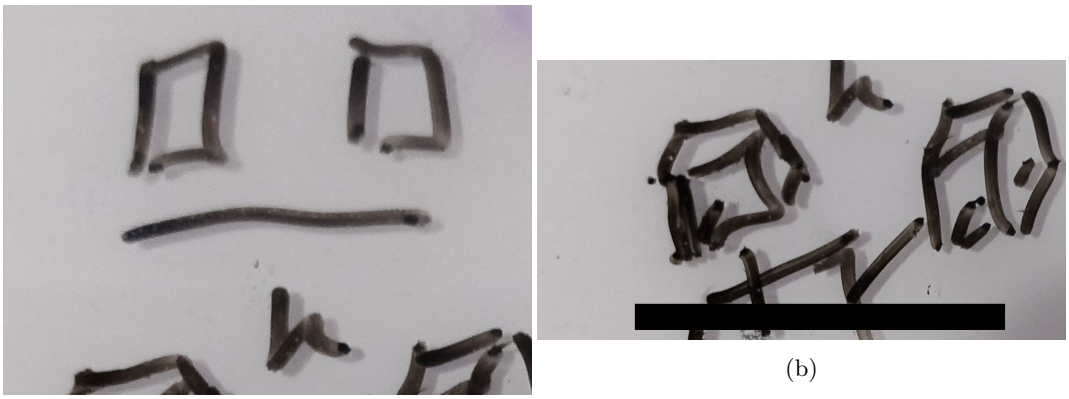
where σ is the schema binding variable name c to its datatype T. A dataset with multiple variables has a fiber that is the cartesian cross product of $\mathbb{U}_\sigma$ applied to all the columns:

$$F = \mathbb{U}_{T_1} \times \ldots \mathbb{U}_{T_i} \ldots \times \mathbb{U}_{T_n} \tag{5}$$

which is equivalent to

$$F = F_0 \times \ldots \times F_i \times \ldots \times F_n \tag{6}$$

which allows us to decouple F into components F_i.



(a) (b)

Figure 5: These two datasets have the same base space K but figure 5a has fiber $F = \mathbb{R} \times \mathbb{R}$ which is (time, temperature) while figure 5b has fiber $\mathbb{R}^+ \times \mathbb{R}^2$ which is (time, wind=(speed, direction))

For example, the data in figure 5a is a pair of times and °K temperature measurements taken at those times. Time is a positive number of type `datetime` which can be resolved to floats $\mathbb{U}_{\texttt{datetime}} = \mathbb{R}$. Temperature values are real positive numbers $\mathbb{U}_{\texttt{float}} = \mathbb{R}^+$. The fiber is

$$\mathbb{U} = \mathbb{R} \times \mathbb{R}^+ \tag{7}$$

where the first component F_0 is the set of values specified by ($c_0 = \mathit{time}, T_0 = \texttt{datetime}, \mathbb{U}_\sigma = \mathbb{R}$) and F_1 is specified by ($c_1 = \mathit{temperature}, T_1 = \texttt{float}, \mathbb{U}_\sigma = \mathbb{R}^+$) and is the set of values $\mathbb{U}_\sigma = \mathbb{R}^+$. In figure 5b, temperature is replaced with wind. This wind variable is of type `wind` and has two components speed and direction $\{(s, d) \in \mathbb{R}^2 \mid 0 \leq s,\, 0 \leq d \leq 360\}$. Therefore, the fiber is

$$F = \mathbb{R}^+ \times \mathbb{R}^2 \tag{8}$$

such that F_1 is specified by ($c_1 = \mathit{wind}, T_1 = \texttt{wind}, \mathbb{U}_\sigma = \mathbb{R}^2$). As illustrated in figure **??**, Spivak's framework provides a consistent way to describe potentially complex components of the input data.

3.1.2 Measurement Scales: Monoid Actions

Implementing expressive visual encodings requires formally describing the structure on the components of the fiber, which we define as the action of a monoid on the component. While structure on a set of values is often described algebraically as operations or through the actions of a group, for example Steven's scales [77], we generalize to monoids to support more component types. Monoids are also commonly found in functional programming because they specify compositions of transformations [78, 94].

A monoid [55] M is a set with an associative binary operator $* : M \times M \to M$. A monoid has an identity element $e \in M$ such that $e * a = a * e = a$ for all $a \in M$. As defined on a component of F, a left monoid action [3, 72] of M_i is a set F_i with an action

$\bullet : M \times F_i \to F_i$ with the properties:

$$\textbf{associativity} \text{ for all } f, g \in M_i \text{ and } x \in F_i,\ f \bullet (g \bullet x) = (f * g) \bullet x$$
$$\textbf{identity} \text{ for all } x \in F_i, e \in M_i,\ e \bullet x = x$$

As with the fiber F the total monoid space M is the cartesian product

$$M = M_0 \times \ldots \times M_i \times \ldots \times \ldots M_n \tag{9}$$

of each monoid M_i on F_i. The monoid is also added to the specification of the fiber $(c_i, T_i, \mathbb{U}_\sigma M_i)$

Steven's described the measurement scales[46, 77] in terms of the monoid actions on the measurements: nominal data is permutable, ordinal data is monotonic, interval data is translatable, and ratio data is scalable [87]. For example, given the interval scale fiber component ($c = temperature$, $T = \texttt{float}$, $\mathbb{U}_\sigma = \mathbb{R}$):

- monoid operator addition $* = +$
- monoid operations: $f : x \mapsto x + 1$, $g : x \mapsto x + 2$
- monoid action operator composition $\bullet = \circ$

then the translation monoid actions on temperature satisfy the condition

$$\begin{array}{ccc} \mathbb{R} & & \\ {\scriptstyle x+1^\circ}\downarrow & \searrow^{(x+1^\circ)\circ(x+2^\circ)} & \\ \mathbb{R} & \xrightarrow[x+2^\circ]{} & \mathbb{R} \end{array} \tag{10}$$

where 1° and 2° are valid distances between two temperatures x.

While many component types will be one of the measurement scale types, we generalize to monoids specifically for the case of partially ordered set. Given a set $W = \{mist, drizzle, rain\}$, then the map $f : W \to W$ defined by

1. $f(rain) = drizzle$,
2. $f(drizzle) = mist$
3. $f(mist) = mist$

is order preserving such that $mist \leq drizzle \leq rain$ but has no inverse since $drizzle$ and $mist$ go to the same value $mist$. Therefore order preserving maps do not form a group, and instead we generalize to monoids to support partial order component types. Defining the monoid actions on the components serves as the basis for identifying the invariance[44] that must be preserved in the visual representation of the component.

3.1.3 Continuity: Base Space K

The base space K is way to express the connectivity of the records. This is assumed in the choice of visualization, for example a line plot implies 1D continuous data and a scatter plot implies discrete records, but an explicit representation allows for verifying that the topology of the graphic representation is equivalent to the topology of the data.

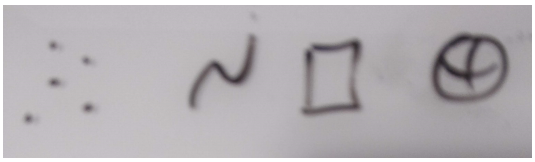


Figure 6: The topological base space K encodes the connectivity of the data space, for example if the data is independent points or a map or on a sphere

As illustrated in figure 6, K is akin to an indexing space into E that describes the structure of E. K can have any number of dimensions and can be continuous or discrete.

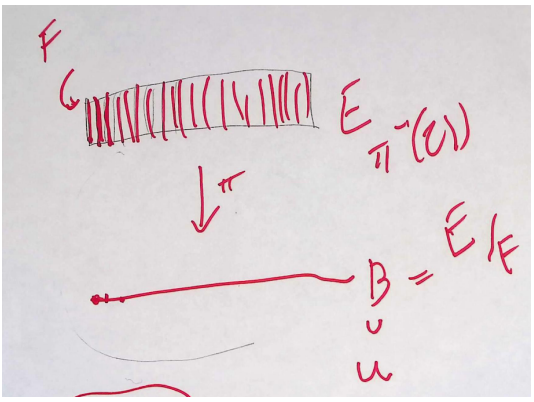


Figure 7: The base space E is divided into fiber segments F. The base space K acts as an index into the records in the fibers. this figure might be good all the way up top to lay out the components of fb

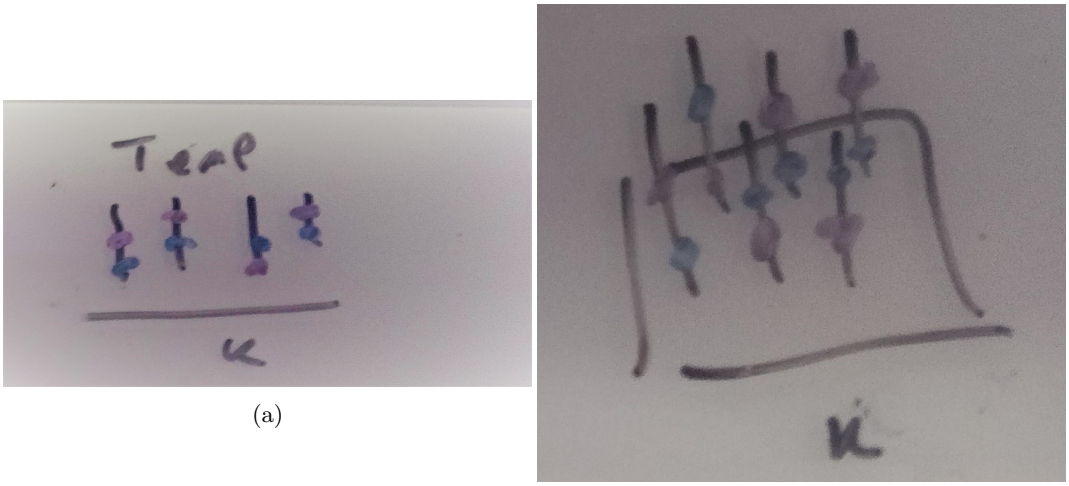
Formally K is the quotient space [64] of E meaning it is the finest space[6] such that every $k \in K$ has a corresponding fiber F_k[64]. In figure 7, E is a rectangle divided by vertical fibers F, so the minimal K for which there is always a mapping $\pi : E \to K$ is the line.

As with fibers and monoids, we can decompose the total space into components $\pi : E_i \to K$ where

$$\pi : E_1 \oplus \ldots \oplus E_i \oplus \ldots \oplus E_n \to K \tag{11}$$

which is a decomposition of F. The K remains the same because the connectivity of records does not change just because there are fewer elements in each record.

The datasets in figure 8 have the same fiber of (temperature, time). In figure 8a the fibers lie over discrete K such that the records in the datasets in the fiber bundles are discrete. The same fiber in figure 8b lies over a continuous interval Ksuch that the records are samples from a continuous function defined on K. By encoding this continuity in the model as K the data model now explicitly carries information about its structure such that the implicit assumptions of the visualization algorithms are now explicit. This in turn allows for building algorithms that can work with distributed or streaming data, since it has a common data access interface with a promise that the data exists.



(a) (b)

Figure 8: These two datasets have the same (time, temperature) fiber. In figure **??** the total space E is discrete over points $k \in K$, meaning the records in the fiber are also discrete. In figure **??** E lies over the continuous interval K, meaning the records in the fiber are sampled from a continuous space. revamp figure: F=Plane, k1 = dots, k2=line

3.1.4 Data: Sections τ

A set of records is a section $\tau : K \to E$ of the fiber bundle. For example, in the special case of a table [75], K is a set of row ids, F is the columns, and the section τ returns the record at a given key in K. In general, for any fiber bundle, there is always a map

$$\begin{array}{ccc} F & \hookrightarrow & E \\ & & \pi \downarrow \uparrow \tau \\ & & K \end{array} \tag{12}$$

such that $\pi(\tau(k)) = k$. There can be many sections τ and the space of all global sections is $\Gamma(E)$. Assuming a trivial fiber bundle $E = K \times F$, the section is

$$\tau(k) = (k, (g_{F_0}(k), \ldots, g_{F_n}(k))) \tag{13}$$

where $g : K \to F$ is the index function into the fiber. This formulation of the section also holds on locally trivial sections of a non-trivial fiber bundle. Because we can decompose the bundle and the fiber, we can formulate τ as

$$\tau = (\tau_0, \ldots, \tau_i, \ldots, \tau_n) \tag{14}$$

where each section τ_i is a variable or set of variables.

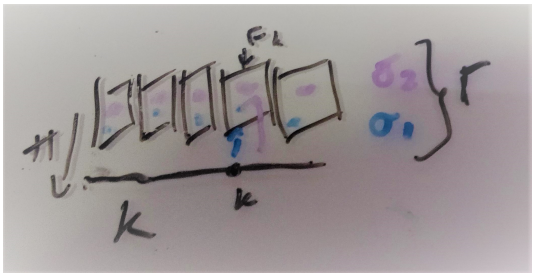


Figure 9: Fiber (time, temperature) with an interval K basespace. The sections τ_i and τ_j are constrained such that the time variable must be monotonic, which means each section is a timeseries of temperature values. They are included in the global set of sections $\tau_1, \tau_2 \in \Gamma(E)$

In the example in figure 9, the fiber is (*time*, *temperature*) as described in figure 5 and the base space is the interval K. The section τ_i resolves to a series of monotonically increasing in time records of (time, temperature) values. Section τ_j returns a different timeseries of (time, temperature) values. Both sections are included in the global set of sections $\tau_i, \tau_j \in \Gamma(E)$.

The section can be any instance of a data container, for example a numpy array[36], a pandas series or dataframe[67], or an xarray[40]. A univariate numpy array that stores an image is a section of a fiber bundle where K is a 2D continuous plane and the F is $(\mathbb{R}^3, \mathbb{R}, \mathbb{R})$ where $\mathbb{R}^3$ is color, and the other two components are the x and y positions of the sampled data in the image. A series could store the values of τ_i and a second series could be $dsection_j$. We could also fatten the fiber to hold two temperature series, such that a section would be an instance of a dataframe with a time column and two temperature columns. While the series and dataframe explicitly have a time index column, they are components in our model and the index is always assumed to be random keys. An instance of an xarray would also be a τ for example if the data is a temperature field than the K could be a continuous volume and the components would be the temperature and the time, latitude, and longitude the measurements were sampled at. As with the dataframe, the semantic index labels are considered components and the indicies are instead assumed to be random. A section can also be an instance of a distributed data container, such as a dask array [69]. As with the other containers, K and F are defined in terms of the index and dtypes of the components of the array. The section can remain curried until render since the model is functional, in keeping with dasks compute graph architecture. Our model provides a common interface to these widely used data containers without sacrificing the semantic structure embedded in each container.

3.1.5 Sheaf and Stalk

To support working with a subset of data, we can take the sheaf $\mathcal{O}(E)$ as input into the artist. E restricted over a small enough neighborhood $U \subset K$ is a locally trivial bundle over U[47]. The sheaf $O(E)$ is the localized section of fibers $\iota^*\tau : U \to \iota^* E$

$$\begin{array}{ccc} \iota^* E & \xleftarrow{\iota^*} & E \\ {\scriptstyle\pi}\downarrow\uparrow{\scriptstyle\iota^*\tau} & & {\scriptstyle\pi}\downarrow\uparrow{\scriptstyle\tau} \\ U & \xleftarrow{\iota} & K \end{array} \tag{15}$$

pulled back over the neighborhood U via the inclusion map $\iota : U \to K$. The localized section is the germ $\xi^*\tau$. The neighborhood of points k_i surrounding the point k the sheaf lies over is the stalk $\mathcal{F}_b$ [73, 76].

While E is only the fiber F_k over a specific k, the stalk includes the local behavior of the section at k which can include information about nearby records. In practice, often the only information needed from the stalk is some finite number n of derivatives, which is captured by the jet bundle $\mathcal{J}^n$ [43, 58] with $\mathcal{J}^0(E) = E$. In this work, we at most need $\mathcal{J}^2(E)$ which is the value at τ and its first and second derivatives. The sheaf facilitates interactions such as zooming or updating the graphic based on zooming data; the jet ensures that the artist only needs a single record on k to render a piece of a graphic. This allows for the creation of highly concurrent and possibly distributed artists.

3.2 Graphic: H

We introduce a graphic bundle to hold the essential information necessary to render a graphical design constructed by the artist. As with the data, we can represent the target graphic as a section ρ of a bundle (H, S, π, D). The graphic bundle H consists of a base S(3.2.1) that is a thickened form of K a fiber D(3.2.2) that is an idealized display space, and sections ρ(3.2.3) that encode a graphic where the visual characteristics are fully specified.

3.2.1 Idealized Display D

To fully specify the visual characteristics of the image, we construct a fiber D that is an infinite resolution version of the target space. Typically H is trivial and therefore sections can be thought of as mappings into D.

In this work, we assume a 2D opaque image $D = \mathbb{R}^5$ with elements

$$(x,\, y,\, r,\, g,\, b) \in D \tag{16}$$

such that a rendered graphic only consists of 2D position and color. To support overplotting and transparency, the fiber could be $D = \mathbb{R}^7$ such that $(x, y, z, r, g, b, a) \in D$ specifies the target display. By abstracting the target display space as D, the model can support different targets, such as a 2D screen or 3D printer.

3.2.2 Continuity of the Graphic S

Since we propose that data topology K is by some measure preserved in the graphic, we introduce a graphic topology $S$to encode the carried over structure. These topologies are not always identical because the underlying topology S of a graphic may need more dimensions than the data topology KIn a typical 2D display (ignoring depth), visible elements need to have 2D extent so that for example a scatter marker has an area greater than 0 or a line is not infinitely thin.

Since K with fewer than two dimensions need to be thickened, we define the mapping from graphic S to data K

$$\begin{array}{ccc} E & & H \\ {\scriptstyle\pi}\downarrow & & {\scriptstyle\pi}\downarrow \\ K & \xleftarrow{\ \xi\ } & S \end{array} \tag{17}$$

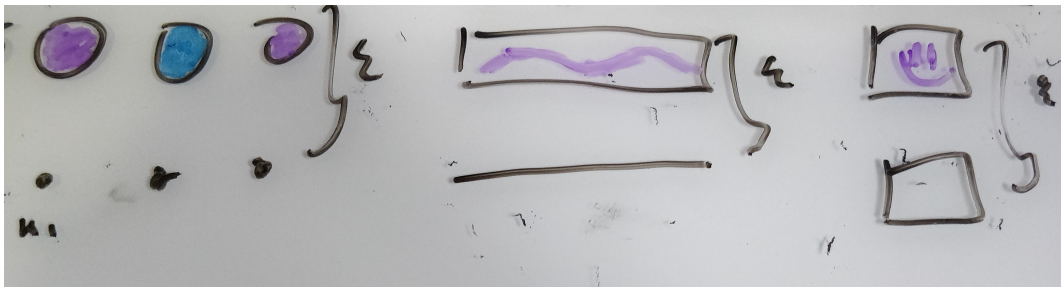


Figure 10: The scatter and line graphic base spaces have one more dimension of continuity than K so that S can encode physical aspects of the glyph, such as shape (a circle) or thickness. The heatmap has the same continuity in the graphic S as in the data K. add α, β coordinates to figures

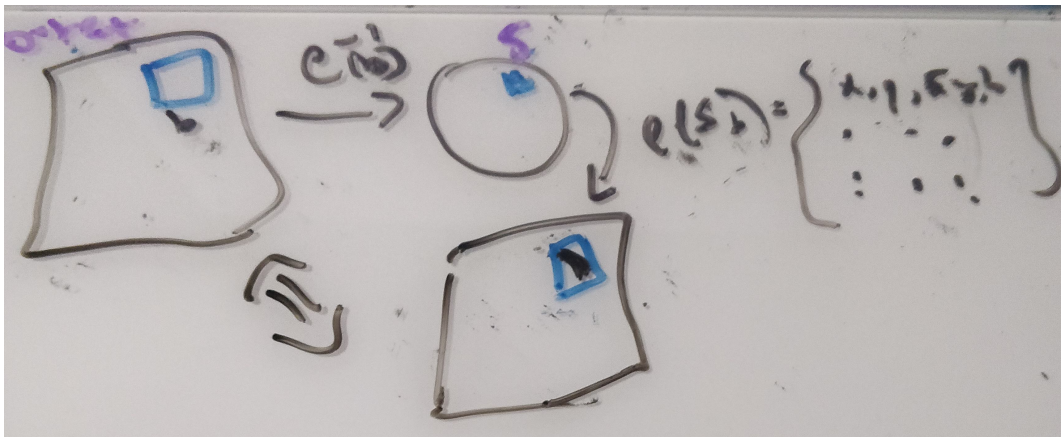


Figure 11: To render a graphic, a pixel p is selected in the display space, which is defined in the same coordinates as the x and y components in DThe inverse mapping $\rho_{xy}{}^{(}p)$ returns a region $S_p \subset S$. $\rho(S_p)$ returns the list of elements $(x, y, r, g, b) \in D$ that lie over S_p. The integral over the (r, g, b) elements is the color of the pixel.

as the deformation retraction [68] $\xi : S \to K$ that goes from a region $s \in S_k$ to its associated point s, such that when $\xi(s) = k$, $\xi^* \tau(s) = \tau(s)$. While dimensions can be added to S, it retains the same continuity as K.

When K is discrete points and the graphic is a scatter plot, each point $k \in K$ corresponds to a 2D disk S_k as shown in figure 10. In the case of 1D continuous data and a line plot, the region β over a point α_i specifies the thickness of the line in S for the corresponding τ on k. The heatmap has the same continuity in data space and graphic space such that no extra dimensions are needed in S. The mapping function ξ provides a way to identify the part of the visual transformation that is specific to the the connectivity of the data rather than the values; for example it is common to flip a matrix when displaying an image.

3.2.3 Renderering ρ

This section describes how we go from a graphical design to a rendered image, where the graphical design is the section $\rho : S \to H$. It is sufficient to sketch out how an arbitrary pixel would be renderer, where a pixel p in a real display corresponds to a region S_p in the idealized display. To determine the color of the pixel, we aggregate the color values over the region via integration.

For a 2D screen, the pixel is defined as a region $p = [y_{top}, y_{bottom}, x_{right}, x_{left}]$ of the rendered graphic. Since the x and y in p are in the same coordinate system as the x and y components of D the inverse map of the bounding box $S_p = {\rho_{xy}}^{-1}(p)$ is a region $S_p \subset S$. To compute the color, we integrate on S_p

$$r_p = \iint_{S_p} \rho_r(s) ds^2 \tag{18}$$

$$g_p = \iint_{S_p} \rho_g(s) ds^2 \tag{19}$$

$$b_p = \iint_{S_p} \rho_b(s) ds^2 \tag{20}$$

As shown in figure 11, a pixel p in the output space is selected and inverse mapped into the corresponding region $S_p \subset S$. This triggers a lookup of the ρ over the region S_p, which yields the set of elements in D that specify the (r, g, b) values corresponding to the region p. The color of the pixel is then obtained by taking the integral of $\rho_{rgb}(S_p)$. In general, ρ is an abstraction of rendering specifications, for example PDF[12], SVG[63], or an openGL scene graph[22] or rendering engines such as cairo[20] and AGG[5]. Implementation of ρ is out of scope for this work.

3.3 Artist

The artist is the function that converts data into graphics; its name is taken from the analogous part of Matplotlib[42] that builds visual elements to pass off to the renderer. The artist A is a mapping from E padded with data from $E' = \mathcal{J}(E)$ to a graphic that is a section $\rho \in \Gamma(H)$

$$\begin{array}{ccccccc} E' & \xrightarrow{\nu} & V & \xleftarrow{\xi^*} & \xi^*V & \xrightarrow{Q} & H \\ & \searrow^{\pi} & \downarrow{\scriptstyle \pi} & & \downarrow{\scriptstyle \xi^*\pi} & \swarrow_{\pi} & \\ & & K & \xleftarrow{\xi} & S & & \end{array} \tag{21}$$

which due to using the jet is point wise such that the input can be $\tau(k)$. The encoders $\nu : E' \to V$ converts the data components to visual components(3.2.2). The continuity map $\xi : SK$ then pulls back the visual bundle V over S(3.3.2). Then the assembly function $Q : \xi^*V \to H$ composites the fiber components of ξ^*V into a graphic in H(3.3.3). This functional decomposition of the visualization artists allows us to specify what are the responsibilities of each function in a fully constrained way. In turn, this allows for building reusable components at each stage of the transformation.

3.3.1 Visual Fiber Bundle V

We introduce a visual bundle V to store the visual representations the artist needs to composite into a graphic. The visual bundle $(V,\ K,\ \pi,\ P)$ has section $\mu : V \to K$ that resolves to a visual variable in the fiber P. The visual bundle V is the latent space of possible parameters of a visualization type, such as a scatter or line plot. We define P in terms of the parameters of a visualization libraries compositing functions; for example table 1 is a sample of the fiber space for Matplotlib [41].

$\boldsymbol{\nu_i}$	$\boldsymbol{\mu_i}$	$\boldsymbol{codomain(\nu_i)}$
position	x, y, z, theta, r	$\mathbb{R}$
size	linewidth, markersize	$\mathbb{R}^+$
shape	markerstyle	$\{f_0, \ldots, f_n\}$
color	color, facecolor, markerfacecolor, edgecolor	$\mathbb{R}^4$
texture	hatch	$\mathbb{N}^{10}$
	linestyle	$(\mathbb{R}, \mathbb{R}^{+^{n, n\%2=0}})$

Table 1: Some possible components of the fiber P for a visualization function implemented in Matplotlib

A section $\mu$is a tuple of visual values that specifies the visual characteristics of a part of the graphic. For example, given a fiber of $\{xpos, ypos, color\}$ one possible section could be $\{.5, .5, (255, 20, 147)\}$. The $codomain(\nu_i)$ determines the monoid actions on μ_i. These fiber components are implicit in the library, by making them explicit as components of the fiber we can build consistent definitions and expectations of how these parameters behave.

3.3.2 Visual Encoders ν

As introduced in section **??**, there are many ways to encode data visually. We define the visual transformers ν as the set of independent conversion functions

$$\{\nu_0, \ldots, \nu_n\} : \{\tau_0, \ldots, \tau_n\} \mapsto \{\mu_0, \ldots, \mu_n\} \tag{22}$$

where $\nu_i : \tau_i \mapsto \mu_i$ is an equivariant map such that there is a monoid homomorphism from F_i to $vfiber_i$. As mentioned in section 3.1.2, we choose monoid actions as the basis for equivariance because they define the structure on the fiber components. A validly constructed ν is one where the diagram of the monoid transform m

$$\begin{array}{ccc} E_i & \xrightarrow{\nu_i} & V_i \\ {\scriptstyle m_x}\downarrow & & \downarrow{\scriptstyle m_v} \\ E_i & \xrightarrow{\nu_i} & V_i \end{array} \tag{23}$$

```
[2]: nu = {'confused':'😕', 'woozy':'🥴', 'shruggy':'🙃'}

[3]: nu.keys()

[3]: dict_keys(['confused', 'woozy', 'shruggy'])

[4]: nu.values()

[4]: dict_values(['😕', '🥴', '🙃'])

14]: values

14]: ['woozy', 'shruggy', 'confused']

15]: [nu[v] for v in values]

15]: ['🥴', '🙃', '😕']
```

Figure 12: In this artis, ν maps the strings to the emojis. For ν to be equivariant, a shuffle in the words should have an equivalent shuffle in the emojies, and a shuffle in the emojies should have an equivalent shuffle in the words.

commutes such that $\nu_i(m_x(E_i)) = m_v(\nu_i(E_i))$. This equivariance constraint yields guidance on what makes for an invalid transform. For example, the conversion $\nu_i(x) = .5$ does not commute under translation monoid action $t(x) = x + 2$

$$\nu(t(x+2)) \stackrel{?}{=} \nu(x) + \nu(2) \tag{24}$$

$$.5 \neq .5 + .5 \tag{25}$$

On the other hand figure 12 illustrates a valid ν mapping from **Strings** to symbols. The group action on these sets is permutation, so shuffling the words must have an equivalent shuffle of the symbols they are mapped to. To preserve ordinal and partial order monoid actions, ν must be a monotonic function such that given $x_1, x_2 \in E_i$, if $delement_1 \leq x_2$ then $\nu(x_1) \leq \nu(x_2)$. For interval scale data, ν is equivariant under translation monoid actions if $\nu(x + c) = \nu(x) + \nu(c)$. For ratio data, there must be equivalent scaling $\nu(xc) = \nu(x)\nu(c)$. These constraints can be embedded into our artist such that the ν functions are equivariant; they also provide guidance on constructing new equivariant ν functions.

3.3.3 Graphic Assembler Q

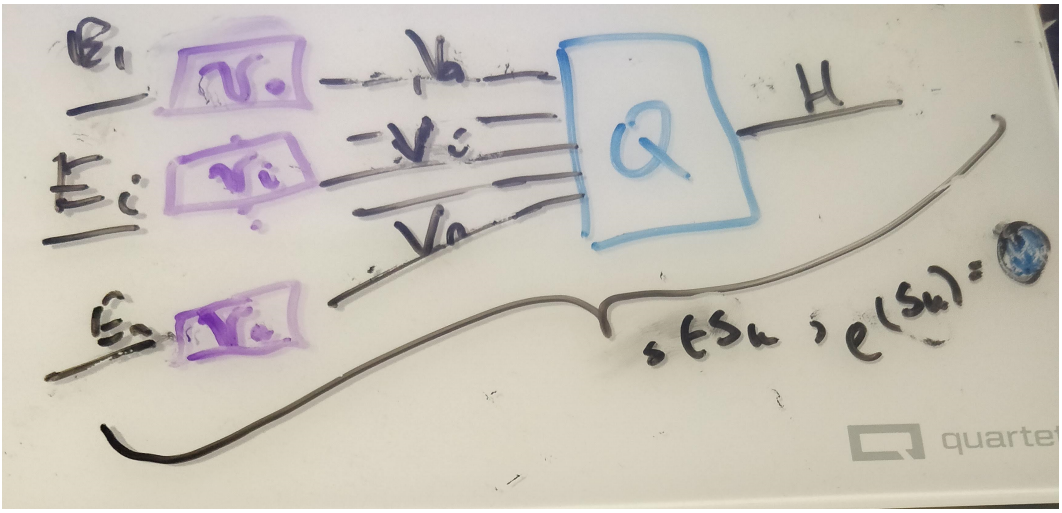


Figure 13: ν functions convert data τ_i to visual characteristics μ_i, then Q assembles μ_i into a graphic ρ such that there is a map ξ preserving the continuity of the data. ρ applied to a region of connected components S_j generates a part of a graphic, for example the point graphical mark.

As shown in figure 13, the assembly function Q combines the fiber F_i wise ν transforms into a graphic in H. Together, ν and Q are a map-reduce operation: map the data into their visual encodings, reduce the encodings into a graphic. As with ν the constraint on Q is that for every monoid action on the input μ there is a corresponding monoid action on the output ρ.

Since we define the equivariant map as $Q : \mu \mapsto \rho$, we define an action on the subset of graphics $Q(\Gamma(V)) \in \Gamma(H)$ that Q can generate. We then define the constraint on Qsuch

that if Q is applied to μ, μ' that generate the same ρ then the output of both sections acted on by the same monoid m must be the same.

Lets call the visual encodings $\Gamma(V) = X$ and the graphic $Q(\Gamma(V)) = Y$. If for all monoids $m \in M$ and for all $\mu, \mu' \in X$, the output is equivalent

$$Q(\mu) = Q(\mu') \implies Q(m \circ \mu) = Q(m \circ \mu') \tag{26}$$

then a group action on Y can be defined as $m \circ \rho = \rho'$. The transformed graphic ρ' is equivariant to a transform on the visual bundle $\rho' = Q(m \circ \mu)$ on a section that $\mu \in Q^{-1}(\rho)$ that must be part of generating ρ.

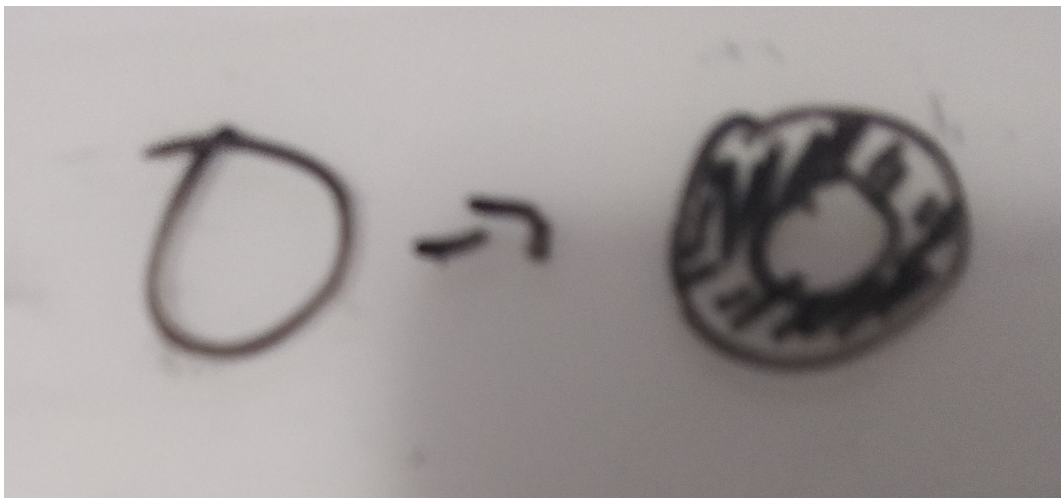


Figure 14: These two glyphs are generated by the same Q function, but differ in the value of the edge thickness parameter μ_i. A valid Q is one where a shift in μ_i is reflected in the glyph generated by ρ.

The glyph in figure 14 has the following characterstics Pspecified by (*xpos*, *ypos*, *color*, *thickness*) such that one section is $\mu = (0, 0, 0, 1)$ and $Q(\mu) = \rho$ generates a piece of the thin hollow circle. The equivariance constraint on $Q$is that the action $m = (e, e, e, x + 2)$, where e is identity, applied to μ such that $\mu' = (e, e, e, 3)$ has an equivalent action on ρthat causes $Q(\mu')$ to be equivalent to the thicker circle in figure 14.

We can describe a mark [9, 21] as Q with input of all the regions s that map back to a set of connected components $J \subset K$:

$$J = \{j \in K \text{ s. t. } \exists \gamma \text{ s.t. } \gamma(0) = k \text{ and } \gamma(1) = j\} \tag{27}$$

where the path[26] γ from k to j is a continuous function from the interval [0,1]. We define the mark as the graphic generated by $Q(S_j)$

$$H \underset{\rho(S_j)}{\overset{}{\rightleftarrows}} S_j \overset{\xi(s)}{\underset{\xi^{-1}(J)}{\rightleftarrows}} J_k \tag{28}$$

such that for every mark there is at least one corresponding section on K. We define a glyph as a composite of multiple Q in keeping with Munzner's definition of the glyph as an object with internal structure arising from multiple marks[57].

3.3.4 Assembly Q

In this section we formulate the minimal Q that will generate distinguishable graphical marks: non-overlapping scatter points, a non-infinitely thin line, and a heatmap.

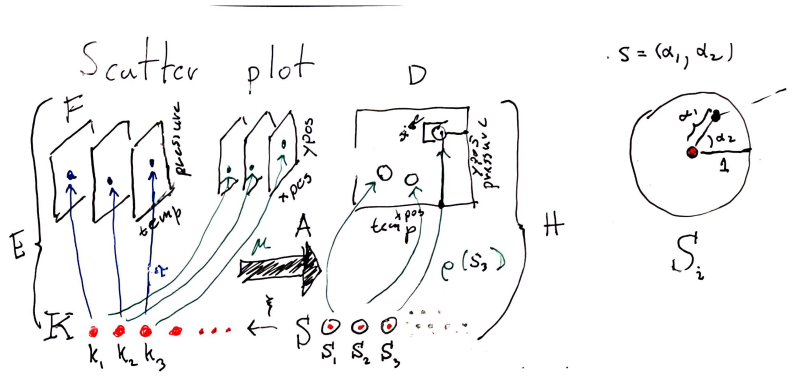


Figure 15: The data is discrete points (temperature, time). Via ν these are converted to (xpos, ypos) and pulled over discrete S. These values are then used to parameterize ρwhich returns a color based on the parameters (xpos,ypos) and position α, β on S_k that ρ is evaluated on.

The scatter plot in figure **??** can be defined as $Q(xpos, ypos)(\alpha, \beta)$ where color $\rho_{RGB} = (0, 0, 0)$ is defined as part of Q and $s = (\alpha, \beta)$ defines the region on S. The position of this swatch of color can be computed relative to the location on the disc S_k as shown in figure 15:

$$x = size \bullet \alpha \bullet \cos(\beta) + xpos \tag{29}$$

$$y = size \bullet \alpha \bullet \sin(\beta) + ypos \tag{30}$$

such that $\rho(s) = (x, y, 0, 0, 0)$ colors the point (x,y) black.

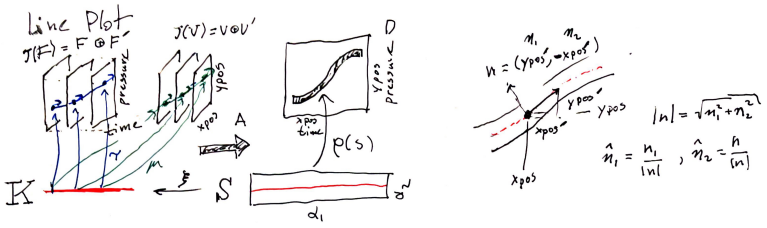


Figure 16: The line fiber ($time$, $temp$) is thickend with the derivative ($time'$, $temperature'$ because that information will be necessary to figure out the tangent to the point to draw a thick line. This is because the line needs to be pushed perpendicular to the tangent of (xpos, ypos). this is gonna move once this gets regenerated w/ labels The data is converted to visual characteristics (xpos, ypos). The α coordinates on S specifies the position of the line, the β coordinate specifies thickness.

The line plot $Q(xpos, \hat{n_1}, ypos, \hat{n_2})(\alpha, \beta)$ shown in fig 15 exemplifies the need for the jet discussed in section **??**. The line needs to know the tangent of the data to draw an envelope above and below each (xpos,ypos) such that the line appears to have a thickness. The magnitude of the thickness is

$$|n| = \sqrt{{n_1}^2 + {n_2}^2} \tag{31}$$

such that the normal is

$$\hat{n_1} = \frac{n_1}{|n|},\ \hat{n_2} = \frac{n_2}{|n|} \tag{32}$$

which yields components of ρ

$$x = xpos(\xi(\alpha)) + \beta\hat{(}n_1)(\xi(\alpha)) \tag{33}$$

$$y = ypos(\xi(\alpha)) + \beta\hat{(}n_2)(\xi(\alpha)) \tag{34}$$

where (x,y) look up the position $\xi(\alpha)$ on the data and then apply thickness β at that location.

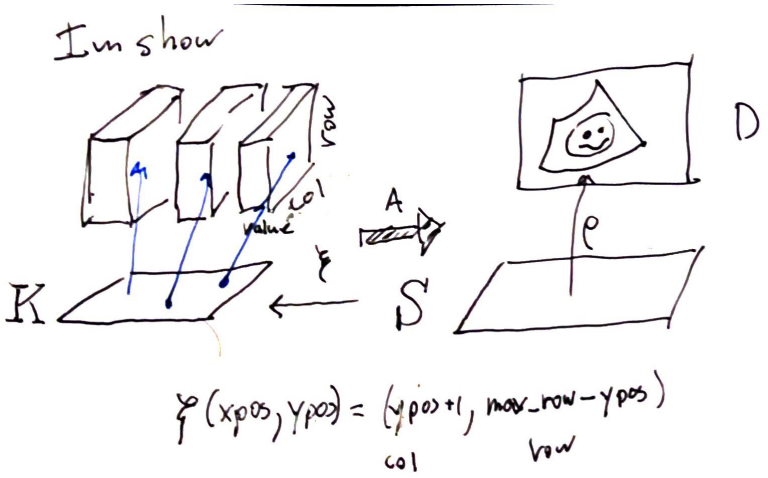


Figure 17: The only visual parameter a image requires is color since ξ encodes the mapping between position in data and position in graphic.

The image $Q(color)$ in figure 17 is a direct lookup $\xi : S \to K$ such that

$$R = R(\xi(\alpha, \beta)) \tag{35}$$

$$G = G(\xi(\alpha, \beta)) \tag{36}$$

$$B = B(\xi(\alpha, \beta)) \tag{37}$$

where ξ may do some translating to a convention expected by Q for example reorientng the array such that the first row in the data is at the bottom of the graphic.

3.3.5 Assembly factory $\hat{Q}$

The graphic base space S is not accessible in many architectures, including Matplotlib, because the rendering is tightly interlaced with the graphical design; instead we can construct a factory function $\hat{Q}$ over K that can build a Q. As shown in eq 21, Q is a bundle map $Q : \xi^* V \to H$ where $\xi^* V$ and H are both bundles over S.

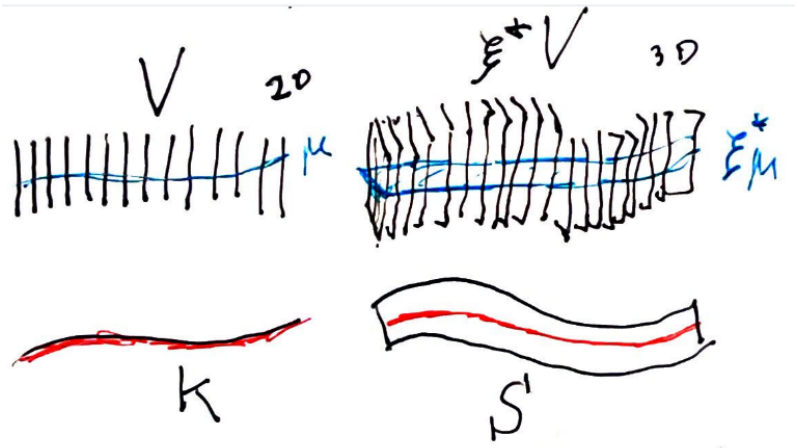


Figure 18: The pullback of the visual bundle $\xi^* V$ is the replication of a μ over all points s that map back to a single k. Because the μ is the same, we can constract a $\hat{Q}$ on μ over k that will fabricate the Q for the equivalent set of s associated to that k

The preimage of the continuity map $\xi^{-1}(k) \subset S$ is such that many graphic continuity points $s \in S_K$ go to one data continuity point k; therefore, by definition the pull back $\xi^* V \mid_{\xi^{-1}(k)} = \xi^{-1}(k) \times P$ copies the visual fiber Vover the preimage in $\xi^{-1}(k)$. This is illustrated in figure 18, where the 1D fiber over K is copied repeatedly to become the 2D fiber with identical components over S. Given a section $\xi^* \mu$ pulled back from μ on $\pi : V \to K$ and a point $s \in \xi^{-1}(k)$ in the preimage of k the pulled back section $\xi^* \mu(s) = \xi^*(\mu(k))$ is the image $(k, \mu(k)) \mapsto (s, \xi^* \mu(s))$. This means that $\xi^* \mu$ is identical for all s where $\xi(s) = k$, which is illustrated in figure 18 as each dot on $P$is equivalent to the line intersection $P^* \mu$.

Given the equivalence between μ and $\xi^* \mu$ defined above, the reliance on S can be factored out. When Q maps visual sections $Q : \Gamma(\xi^* V) \to \Gamma(H)$, if we restrict Q input to $\xi^* \mu$then $\rho(s) \coloneqq Q(\xi^* \mu)(s)$. Since $\xi^* \mu(s) = \xi^*(\mu(k))$ and $\xi^* \mu$ is constant on $\xi^{-1}(k)$, we can define a Q factory function $\hat{Q}(\mu(k))(s) \coloneqq Q((\xi^* \mu)(s))$ where $\xi^{-1}(k) = k$.

Factoring out s $\hat{Q}(\mu(k)) = Q(\xi^* \mu)$ generates a curried Q. In fact, $\hat{Q}$ is a map from visua space to graphic space $\hat{Q} : \Gamma(V) \to \Gamma(H)$ locally over k such that $\hat{Q} : \Gamma(V_k) \to \Gamma(H \mid_{\xi^{-1}(k)})$. This allows us to construct a $\hat{Q}$ that only depends on K, such that for each $\mu(k)$ there is part of $\rho \mid_{\xi^{-1}(k)}$. The construction of $\hat{Q}$ allows us to retain the functional map reduce benefits of Q without having to majorly restructure the existing rendering pipeline.

3.3.6 Composition of Artists: +

To build graphics that are the composites of multiple artists, we define a simple addition operator that is the disjoint union of fiber bundles E. For example, in figure 19 the scatter

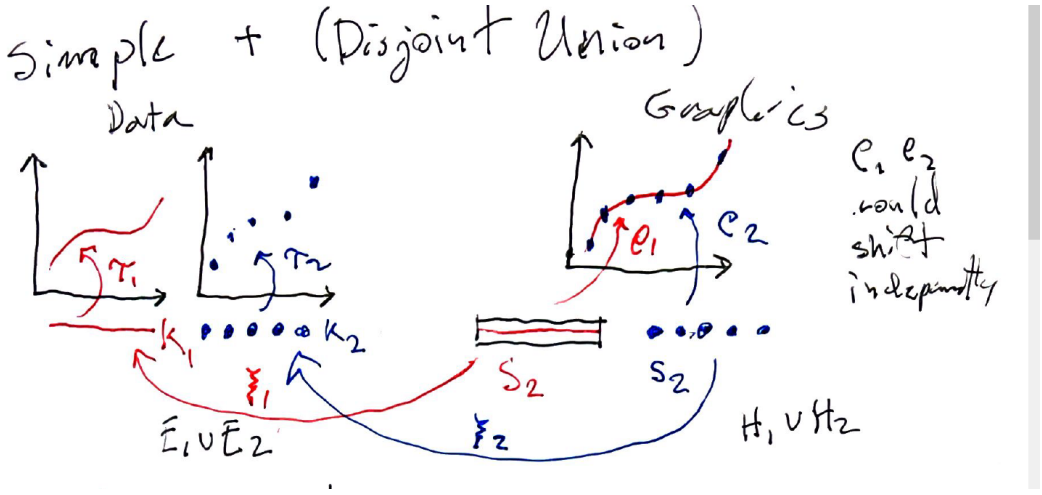


Figure 19: τ_1 and τ_2 are distinct datasets passed through artists A_1 and A_2 to generate graphics ρ_1 and ρ_2. These graphics happen to be rendered to the same image, but otherwise have no intrinsinc link.

plot E_1 and the line plot E_2 have different Kthat are mapped to separate S. To fully display both graphics, the composite graphic $A_1 + A_2$ needs to include all records on both K_1 and K_2, which are the sections on the disjoint union $K_1 \sqcup K_2$. This in turn yields disjoint graphics $S_1 \sqcup S_2$ rendered to the same image. Constraints can be placed on the disjoint union such as that the fiber components need to have the same νposition encodings or that the position μneed to be in a specified range. There are situations where $K_2 \hookrightarrow K_1$ that underpin more complex, especially interactive, visualizations; these cases require defining a more complex addition operator that is out of scope for this work.

3.3.7 Equivalance class of artists A'

It is impractical to implement an artist for every single graphic; instead we implement the equivalence class of artists $\{A \in A' : A_1 \equiv A_2\}$ which is essentially an artist where the ν functions have not yet been input. Equivalent artists have the same fiber bundle V and same assembly function Q but act on different sections μ. To further simplify implementation, we identify a minimal P associated with each A' that defines what visual characteristics of the graphic must originate in the data such that the graphic is identifiable as a given chart type.

Figure 20: Each of these graphics is generated by a different artist Awhich is the equivalance class of scatter plots A'this is gonna be a whole bunch of scatter plots

For example, a scatter plot of red circles is the output of one artist, a scatter plot of green squares the output of another, as shown in figure **??**. These two artists are equivalent since their only difference is in the literal visual encodings (color, shape). Shape and color could also be defined in Q but the position must come from the fiber $P = (xpos, ypos)$ since fundementally a scatter plot is the plotting of one position against another[30]. We also use

this criteria to identify derivative types, for example the bubble chart[83] is a type of scatter where by definition the glyph size is mapped from the data. The criteria for equivalence class membership serves as the basis for evaluating invariance[44].

3.4 Making the fiber bundle computable

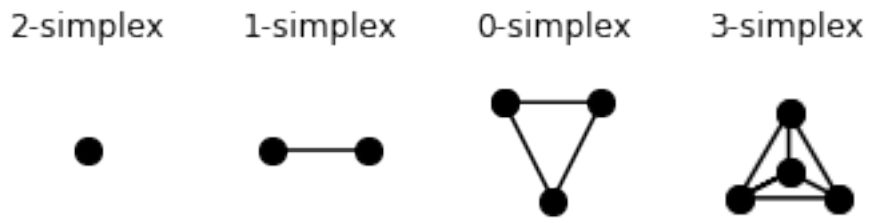


Figure 21: Simplices can encode the connectivity of the data, from fully disconnected (0 simplex) records to all records are connected to at least 3 others

One way of expressing the connectivity of records in a dataset is to implement K as a simplacial complex, which is a set of simplicies such as those shown in figure 21. The advantage of triangulization is that it is general enough to work for more complex topology based visualization methods [38] while also providing a consistent interface of vertices, edges, and faces for ξ to map into. When triangulated, the simplices encode the continuity in the data

simplex	**continuity**	τ
vertex	discrete	$\tau(k)$
edge	1D	$\tau(k,\ j)$
face	2D	$\tau(k,\ j,\ )$

Table 2

such that each section is bound to a simplex $k \in K$. As shown in table 2, in a 1D continuous spaces each τ lies distance j along edge k, while in a 2D continuous space each τ lies at coordinate j,on the face k. This is directly analogous to indexing to express connectivity in N-D arrays, while also natively supporting graphs and trees as they are simplacial complices of nodes and edges. Path connected components are then sections where edges or faces meet.

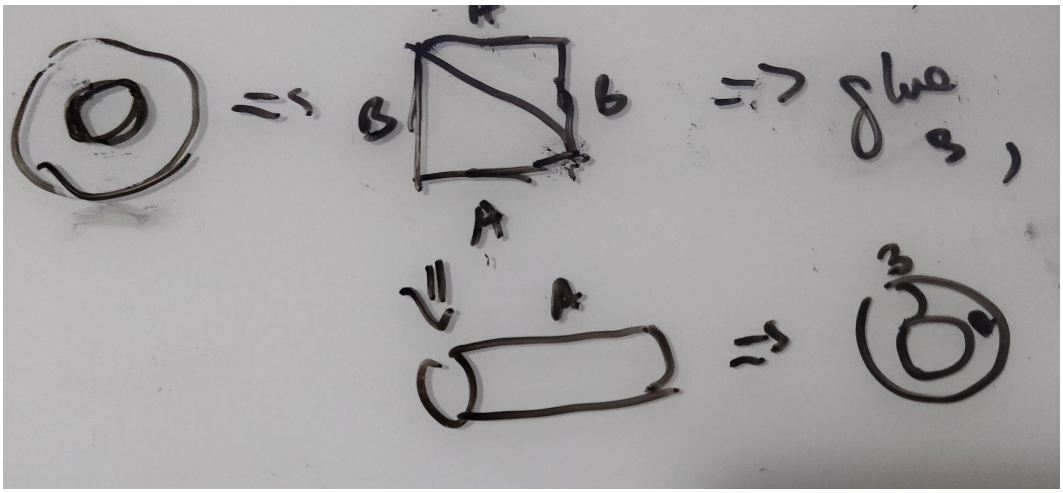


Figure 22: The torus $E$is unraveled into a simplacial complex of 2 faces K. Transition functions are defined on the edges of Ksuch that surface can be glued back into the torus. add cross sections a and b to ring and color same as edges in complex

An advantage of fiber bundles is the ability to encode data continuity that is more complex than points, a line or a plane. For example, the tori in figure 22 can be unraveled into a simplacial complex of two triangles with labeled edges. Transition functions δ are defined on the edges such that a can be glued to a' and b to b' to reconstruct the torus. This simplacial complex is then used as the base space encoding the continuity of data that lies in the torus. A constraint on the transition functions is that the monoid actions on the fibers on the edges of E are commutative $M * F \mapsto \delta(MF) = M * \delta(F)$

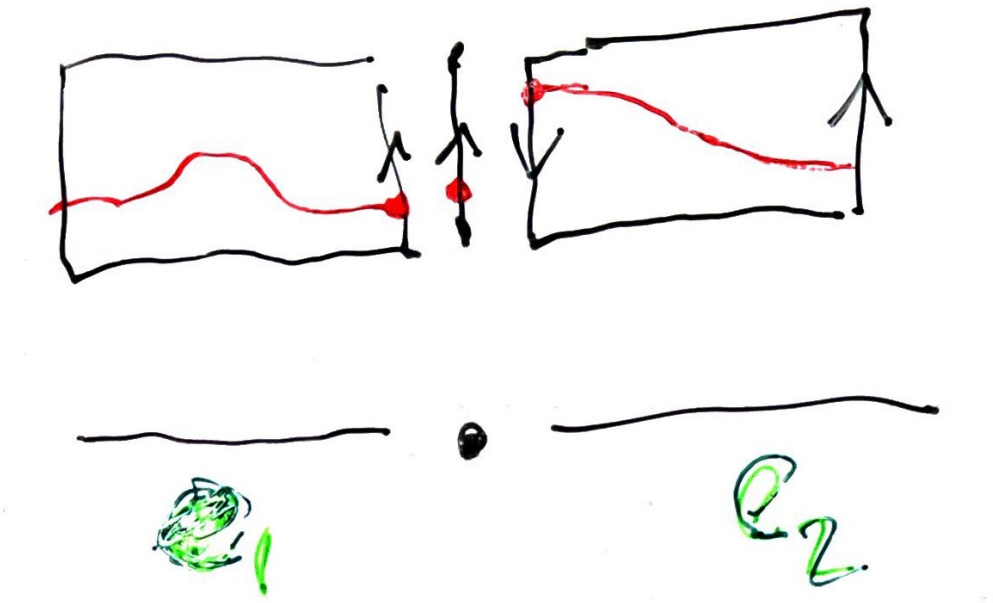


Figure 23: Many non-trivial spaces can be made locally trivial by dividing Einto locally trivial subspaces and defining transition functions between the edges on $K$for how to glue the two subspaces such that the $\tau$are continuous.

Another advantages of triangulization is that it provides a way to encode non-trivial structures such as the mobius strip[54]. As shown in figure 23, one way of making the mobius strip trivial is to seperate it into two spaces E_1 and E_2 and then define transition functions that specify that the edges of E_1 need to be reversed to line up with E_2 such that the sections along the edges meet. As with the torus, the transition functions must preserve monoid commutativity. While we generally do not deal with complex or non-trivial bundles, the ability to implement them native to the model allows for a great deal of expressivity in encoding data. This is especially important when developing a tool that needs to be general enough to support any number of very domain specific types of data.

4 Prototype Implementation: Matplottoy

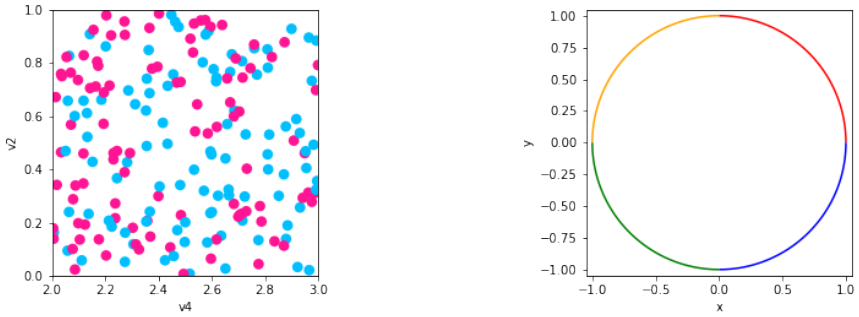


Figure 24: Scatter plot and line plot implemented using prototype artists and data models, building on Matplotlib rendering.

To prototype our model, we implemented the artist classes for the scatter and line plots shown in figure 24 because they differ in every attribute: different visual channels ν that composite to different marks Q with different continuities $\xi\dot{\text{W}}$e make use of the Matplotlib figure and axes artists [41, 42] so that we can initially focus on the data to graphic transformations.

To generate the images in figure 24, we instantiate `fig, ax` artists that will contain the new `Point, Line` primitive objects we implemented based on our topology model.

```
fig, ax = plt.subplots()
artist = Point(data, transforms)
ax.add_artist(artist)
```

```
fig, ax = plt.subplots()
artist = Line(data, transforms)
ax.add_artist(artist)
```

We then add the A'=`Point` and A'=`Line` artists that construct the scatter and line graphics. The arguments to the artist are the data E=`data` that is to be plotted and the aesthetic configuration ν=`transforms`. We implement the artists as equivalence classes A' because it would be impractical to implement a new artist for every aesthetic setting, such as one artist for red lines and another for green.

4.1 Artist Class A'

The artist is the piece of the matplotlib architecture that constructs an internal representation of the graphic that the render then uses to draw the graphic. In the prototype artist, `transform` is a dictionary of the form `{parameter:(variable, encoder)}` where parameter is a component in P, variable is a component in F, and the ν encoders are passed in as functions or callable objects. The data bundle E is passed in as a `data` object. By binding data and transforms to A' inside `__init__`, the `draw` method is a fully specified artist A.

```
class ArtistClass(matplotlib.artist.Artist):
    def __init__(self, data, transforms, *args, **kwargs):
        # properties that are specific to the graphic but not the channels
        self.data = data
        self.transforms = transforms
        super().__init__(*args, **kwargs)

    def assemble(self, visual):
        # set the properties of the graphic

    def draw(self, renderer, *args, **kwargs):
        # returns K, indexed on fiber then key
        view = self.data.view()
        # visual channel encoding applied fiberwise
        visual = {p: encoder(view.get(f, None)) for
                     p, (f, encoder) in self.transforms.items()}
        self.assemble(visual)
        # pass configurations off to the renderer
        super().draw(renderer, *args, **kwargs)
```

The data is fetched in section τ via a `view` method on the data because the input to the artist is a section on E. The return view object has a `get` method to support querying for components that are not in F which we exploit to support parameters in the visual fiber that are not bound to fiber components in F. The ν functions are then applied to the data to generate the μ=`visual` input to Q. An explicit ξ is not implemented since that would mean copying a single μon $k$to all the associated s, as illustrated in figure 10, and that is unnecessary overhead for these scatter and line plots. In $\hat{Q}$=`assemble` the artist generates instructions for the render by setting the attributes that are related to the graphic. These

are the settings that would have to be serialized in order to recreate a static version of the graphic. Although `assemble` could be implemented outside the class such that it returns an object the artist could then parse to set attributes, the attributes are directly set here to reduce indirection. The ν functions could be evaluated in this function to avoid passing over Ktwice but are not done so here to demonstrate the seperability of ν and $\hat{Q}$The last step in the artist function is handing itself off to the renderer.

The `Point` artist builds on `collection` artists because collections are optimized to efficiently draw a sequence of primitive point and area marks. In this prototype, the scatter marker shape is fixed as a circle, and the only visual fiber components are x and y position, size, and the facecolor of the marker.

```
class Point(mcollections.Collection):
    def __init__(self, data, transforms, *args, **kwargs):
        super().__init__(*args, **kwargs)
        self.data = data
        self.transforms = transforms

    def assemble(self, visual):
        # construct geometries of the circle marks in visual coordinates
        self._paths = [mpath.Path.circle(center=(x,y), radius=s)
                       for (x, y, s) in zip(visual['x'],visual['y'], visual['s'])]
        # set attributes of marks, these are vectorized
        # circles and facecolors are lists of the same size
        self.set_facecolors(visual['facecolors'])

    def draw(self, renderer, *args, **kwargs):
        # query data for a vertex table K
        view = self.data.view()
        visual = {p: encoder(view.get(f, None)) for
                     p, (f, encoder) in self.transforms.items()}
        self.assemble(visual)
        # call the renderer that will draw based on properties
        super().draw(renderer, *args, **kwargs)
```

The `view` method repackages the data as a fiber component indexed table of vertices, as described in section 3.4; even though the `view` is fiber indexed, each vertex at an index $k$has corresponding values in section $\tau(k_i)$ such that all the data on one vertex maps to one marker. To ensure the integrity of the section, `view` must be atomic, meaning that the values cannot change after the method is called in draw until a new call in draw. This table is converted to a table of visual variables. It is then passed into `assemble`, where it is used to individually construct the vector path of each circular marker with center `(x,y)` and size `x` and set the colors of each circle. Since `view` returns a τ all these operations could be applied on a section on one k or a subset of K.

The only difference between the `Point` and `Line` objects is in the `view` and `assemble` function because line has different continuity from scatter and is represented by a different type of graphical mark.

```
class Line(mcollections.LineCollection):
    def assemble(self, visual):
        #assemble line marks as set of segments
        segments = [np.vstack((vx, vy)).T for vx, vy
                    in zip(visual['x'], visual['y'])]
        self.set_segments(segments)
        self.set_color(visual['color'])

    def draw(self, renderer, *args, **kwargs):
        # query data source for edge table
        view = self.data.view()
        visual = {p: encoder(view.get(f, None)) for
                     p, (f, encoder) in self.transforms.items()}
        self.assemble(visual)
        super().draw(renderer, *args, **kwargs)
```

In the `Line` artist, `view` returns a table of edges. Each edge consists of (x,y) points sampled along the line defined by the edge and information such as the color of the edge. As with `Point`, the data is then converted into visual variables. In `assemble`, this visual representation is composed into a set of line segments and then the colors of each line segment are set. The colors are guaranteed to correspond to the correct segment because of the atomicity constraint on view.

4.2 Encoders ν

As mentioned above, the encoding dictionary is specified by the visual fiber component, the corresponding data fiber component, and the mapping function. The visual parameter serves as the dictionary key because the visual representation is constructed from the encoding applied to the data $\mu = \nu \circ \tau$. For the scatter plot, the mappings for the visual fiber components $P = (x, y, facecolors, s)$ are defined as

```
cmap =  color.Categorical({'true':'deeppink', 'false':'deepskyblue'})
transforms = {'y': ('v1', lambda x: x),
              'x': ('v3', lambda x: x),
             'facecolors': ('v2', cmap),
             's':(None ,lambda _: itertools.repeat(.02))}
```

where the position *(x,y)* ν transformers are identity functions. The size s transformer is not acting on a component of F, instead it is a ν that returns a constant value. While size could be embedded inside the `assembly` function, it is added to the transformers to illustrate user configured visual parameters that could either be constant or mapped to a component in F. The identity and constant ν are explicitly implemented here to demonstrate their implicit role in the visual pipeline, but they could be optimized away. More complex encoders can be implemented as callable classes, such as

```
class Categorical:
    def __init__(self, mapping):
        # check that the conversion is to valid colors
        assert(mcolors.is_color_like(color) for color in mapping.values())
        self._mapping = mapping

    def __call__(self, value):
        # convert value to a color
        return [mcolors.to_rgba(self._mapping[v]) for v in values]
```

where `__init__` can validate that the output of the ν is a valid element of the P component the ν function is targeting. Creating a callable class also provides a simple way to swap out the specific (data, value) mapping without having to reimplement the validation or conversion logic.

A test for equivariance can be implemented trivially such that it is independent of data or encoder.

```
def test_nominal(values, encoder):
    m1 = list(zip(values, encoder(values)))
    random.shuffle(values)
    m2 = list(zip(values, encoder(values)))
    assert sorted(m1) == sorted(m2)
```

In this example, `is_nominal` checks for equivariance of permutation group actions by applying the encoder to a set of values, shuffling values, and checking that (value, encoding) pairs remain the same. This equivariance test can be implemented as part of the artist or encoder, but for minimal overhead, the equivariant it is implemented as part of the library tests.

4.3 Data E

The data input into the will often be a wrapper class around an existing data structure, but must meet the following criteria:

1. specify the fiber components F and connectivity K

2. have a that returns an atomic object that encapsulates τ

3. the view object must have that returns a fiber component

To support specifying the fiber bundle, we define an optional `FiberBundle` class

```
class FiberBundle:
    def __init__(self, base, fiber):
        """
        base:  {'tables': ['vertex', 'edge', 'face']}
        fiber: {'component name': {'type':, 'monoid':,  'range':}}
        """
        self.base = base
        self.fiber = fiber
```

that asks the user to specify how $K$is triangulated and the attributes of F. The `assembly` functions expect tables that match the continuity of the graphic; scatter expects a vertex table because it is discontinuous, line expects an edge table because it is 1D continuous. The fiber informs appropriate choice of ν therefore it is a dictionary of attributes of the fiber components. I've basically stripped this out of the artists above so should I just ditch this section?

To generate the scatter plot in figure 24, we fully specify a dataset with random keys and values in a section chosen at random form the corresponding fiber component. The fiberbundle `FB` is a class level attribute since all instances of `code`VertexSimplex come from the same fiberbundle.

```
class VertexSimplex: #maybe change name to something else
    """Fiberbundle is consistent across all sections
    """
    FB = FiberBundle({'tables': ['vertex']},
            {'v1': {'type': float,'monoid':'interval','range': [0,1]},
             'v2': {'type': str, 'monoid':'nominal', 'range':['true', 'false']},
             'v3': {'type': float, 'monoid':'interval', 'range':[2,3]}})

    def __init__(self, sid = 45, size=1000, max_key=10**10):
        # create random list of keys
    def tau(self, k):
        # e1 is sampled from F1, e2 from F2, etc...
        return (k, (e1, e2, e3, e4))

    def view(self):
        table = defaultdict(list)
```

```
        for k in self.keys:
            table['index'] = k
            # on each iteration, add one (name, value) pair per component
            for (name, value) in zip(self.FB.fiber.keys(), self.tau(k)[1]):
                table[name].append(value)
        return table
```

The view method returns a dictionary where the key is a fiber component name and the value is a list of values in the fiber component. The table is built one call to `tau` at a time, guaranteeing that all the fiber component values are over the same k. Table has a `get` method as it is a method on Python dictionaries. In contrast, the line in `EdgeSimplex` is defined as the functions `_color,_xy` on each edge.

```
class EdgeSimplex:
    # assign a class level FB attribute
    def __init__(self, num_edges=4, num_samples=1000):
        self.keys = range(num_edge) #edge id
        # distance along edge
        self.distances = np.linspace(0,1, num_samples)
        # half generlized representation of arcs on a circle
        self.angle_samples = np.linspace(0, 2*np.pi, len(self.keys)+1)

    @staticmethod
    def _color(edge):
        colors = ['red','orange', 'green','blue']
        return colors[edge%len(colors)]

    @staticmethod
    def _xy(edge, distances, start=0, end=2*np.pi):
        # start and end are parameterizations b/c really there is
        angles = (distances *(end-start)) + start
        return np.cos(angles), np.sin(angles)

    def tau(self, k): #will fix location on page on revision
        x, y = self._xy(k, self.distances,
                        self.angle_samples[k], self.angle_samples[k+1])
        color = self._color(k)
        return (k, (x, y, color))

    def view(self, simplex):
        table = defaultdict(list)
```

```
        for k in self.keys:
            table['index'].append(k)
            # (name, value) pair, value is [x0, ..., xn] for x, y
            for (name, value) in zip(self.FB.fiber.keys(), self.tau(k, simplex)[1]):
                table[name].append(value)
```

Unlike scatter, the line `tau` method returns the functions on the edge evaluated on the interval [0,1]. By default these means each `tau` returns a list of 1000 x and y points and the associated color. As with scatter, `view` builds a table by calling `tau` for each kUnlike scatter, the line table is a list where each item contains a list of points. This bookkeeping of which data is on an edge is used by the `assembly` functions to bind segments to their visual properties.

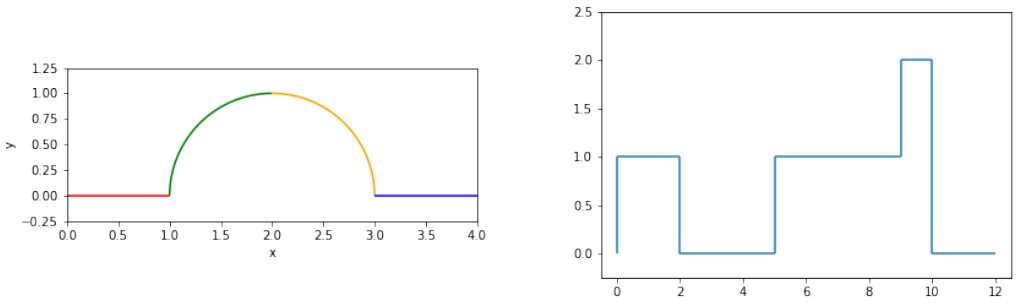


Figure 25: Continuous and discontinuous lines as defined by different data models, but generated with the same A'=`artist`

The graphics in figure 25 are made using the `Line` artist and the `Graphline` data source

```
class GraphLine:
    def __init__(self, FB, edge_table, vertex_table, num_samples=1000, connect=False):
        #s set args as attributes and generate distance
        if connect: # test connectivity if edges are continuous
            assert edge_table.keys() == self.FB.F.keys()
            assert is_continuous(vertex_table)

    def tau(self, k, simplex='edge'):
        # evaluates functions defined in edge table
        return(k, (self.edges[c][k](self.distances) for c in self.FB.F.keys()))

    def view(self, simplex='edge'):
        """walk the edge_vertex table to return the edge function
```

```
        """
        table = defaultdict(list)
        #sort since intervals lie along number line and are ordered pair neighbors
        for (i, (start, end)) in sorted(zip(self.ids, self.vertices), key=lambda v:v[1][0]):
            table['index'].append(i)
            # same as view for line, returns nested list
            for (name, value) in zip(self.FB.F.keys(), self.tau(i, simplex)[1]):
                table[name].append(value)
        return table
```

where if told that the data is connected, the data source will check for that connectivity by constructing an adjacency matrix. The multicolored line is a connected graph of edges with each edge function evaluated on 1000 samples

```
simplex.GraphLine(FB, edge_table, vertex_table, connect=True)
```

while the stair chart is discontinuous and only needs to be evaluated at the edges of the interval

```
simplex.GraphLine(FB, edge_table, vertex_table, num_samples=2, connect=False)
```

such that one advantage of this model is it helps differentiate graphics that have different artists from graphics that have the same artist but make different assumptions about the source data.

4.4 Case Study: Penguins

For this case study, we use the Palmer Penguins dataset[32, 39] since it is multivariate and has a varying number of penguins. We use a version of the data packaged as a pandas dataframe[59, 67] since that is a very commonly used Python labled data structure. The wrapper is very thin since here there is explicitly only one section.

```
class DataFrameSection:
    def __init__(self, dataframe):
        self._tau = dataframe.iloc
        self._view = dataframe
    def view(self):
        return self._view
```

The pandas indexer is a key valued set of discrete vertices, so there is no need to repackage for triangulation. As with the previous examples, there is no need to implement an explicit `get` method since the `dataframe` object has a get method.

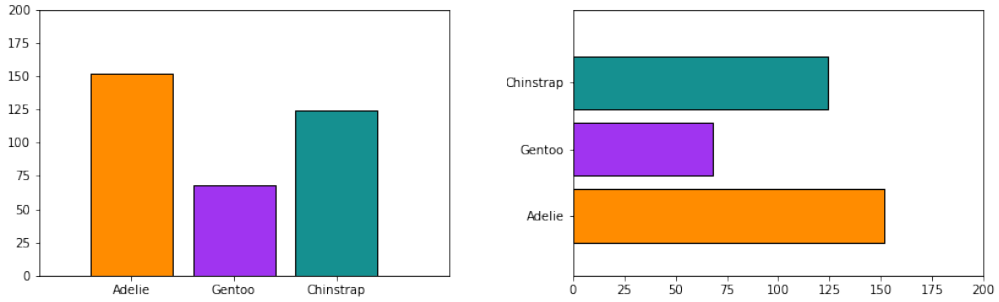


Figure 26: Frequency of Penguin types visualized as discrete bars.

The bar charts in figure 26 are generated with a `Bar` artist. The have the same required P components of (position, length). In of `Bar` an additional parameter is set, `orientation` which only applies holistically to the graphic and never to individual data parameters. Explicitly differentiate between parameters in V and ones that are only in $\hat{Q}$ is another way this model allows for cleaner separation of roles in the code.

```
class Bar(mcollections.Collection):
    def __init__(self, data, transforms, *args, **kwargs):
        # parameter of the graphic
        self.orientation = kwargs.pop('orientation', 'v')

        super().__init__(*args, **kwargs)
        self.data = data
        self.transforms = transforms

    @staticmethod
    def _make_bars(orientation, position, width, floor, length):
        if orientation in {'vertical', 'v'}:
            xval, xoff, yval, yoff = position, width, floor, length
        elif orientation in {'horizontal', 'h'}:
            xval, xoff, yval, yoff = floor, length, position, width
        return [[(x, y), (x, y+yo), (x+xo, y+yo), (x+xo, y), (x, y)]
                for (x, xo, y, yo) in zip(xval, xoff, yval, yoff)]


    def assemble(self, visual):
        #set some defaults
        visual['width'] = visual.get('width', itertools.repeat(0.8))
        visual['floor'] = visual.get('floor', itertools.repeat(0))
```

```
        visual['facecolors'] = visual.get('facecolors', 'C0')
        #build bar glyphs based on graphic parameter
        verts = self._make_bars(self.orientation, visual['position'],
                    visual['width'], visual['floor'], visual['length'])
        self._paths = [mpath.Path(xy, closed=True) for xy in verts]
        self.set_edgecolors('k')
        self.set_facecolors(visual['facecolors'])

    def draw(self, renderer, *args, **kwargs):
        view = self.data.view()
        visual = utils.convert_transforms(view, self.transforms)
        self.assemble(visual)
        super().draw(renderer, *args, **kwargs)
        return
```

The `draw` method identical to the ones above, but here the visual transformations are factored out into a separate function. The `assemble` function sets some defaults, constructs bars, and sets their edge color to black. The `_make_bars` function is somewhat factored out because this is an operation that may be used by other bar making functions that may not be able to make use of bars assemble or draw.

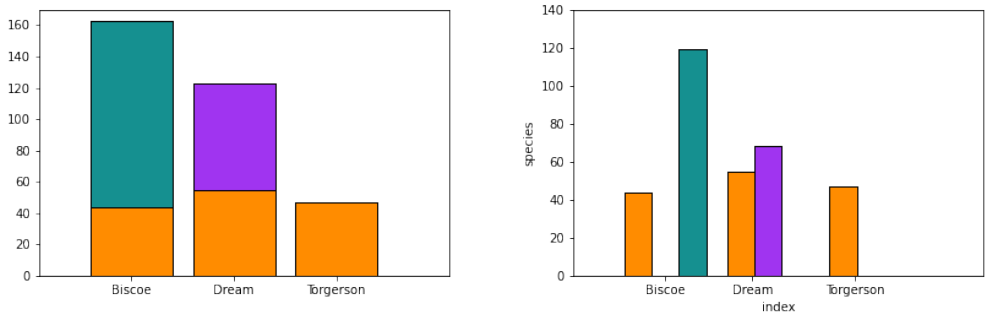


Figure 27: Penguin count disaggregated by island and species

For example, the `MultiBar` artist that makes figure 27 reuses `_make_bars` but does not reuse the assemble function because the composition of elements forces fundemental differences in glyph construction. As demonstrated in the `init`, the composite bar chart has orientation and whether it is stacked or not. While the stacked bar chart and the grouped bar chart could be seperate artists, as demonstrated they share so much overlapping code that it is far less redundant to implement them together. looking at the mess that is this code, I'm a) not convinced these should be combined b) no longer convinced this provides anything over just bar if it isn't rewritten to use bar more

```
class MultiBar(mcollections.Collection):
    def __init__(self, data, transforms, *args, **kwargs):
        #set the orientation of the graphic
        self.orientation = kwargs.pop('orientation', 'v')
        # set how the bar glyphs are put together to create the graphic
        self.stacked = kwargs.pop('stacked', False)
        # rest is same as other artist __init__s

        #this needs to be factored out but just want to finish now
        self.width = kwargs.pop('width', .8)

    def assemble(self, visual, view):
        (groups, gencoder) = self.transforms['length']
        ngroups = len(np.atleast_1d(groups))
        visual['floor'] = visual.get('floor', np.empty(len(view[groups[0]])))
        visual['facecolors'] = visual.get('facecolors', 'C0')
        # make equal width stacked columns
        if 'width' not in visual and self.stacked:
            visual['width'] = itertools.repeat(self.width)

        # make equal with groups
        if not self.stacked:
            visual['width'] = itertools.repeat(self.width/ngroups)
            offset = (np.arange(ngroups) /ngroups) * self.width
        else:
            offset = itertools.repeat(0)

        # make the bars and arrange them
        verts = []
        for group, off in zip(groups, offset):
            verts.extend(Bar._make_bars(self.orientation, visual['position'] + off,
                         visual['width'], visual['floor'], view[group]))
            if self.stacked: #add stacked bar to previous bar
                visual['floor'] += view[group]

        # convert lengths after all calculations are made and reorient if needed
        # here or in transform machinery?
        if self.orientation in {'v', 'vertical'}:
            tverts = [[(x, gencoder(y)) for (x, y) in vert]
```

```
                            for vert in verts]
        elif self.orientation in {'h', 'horizontal'}:
            tverts = [[(gencoder(x), y) for (x, y) in vert]
                            for vert in verts]
        self._paths = [mpath.Path(xy, closed=True) for xy in tverts]
        #flatted columns of colors to match list of bars
        self.set_facecolor(list(itertools.chain.from_iterable(visual['facecolors'])))
        self.set_edgecolors('k')

    def draw(self, renderer, *args, **kwargs):
        view = self.data.view()
        #exclude converting the group visual length, special cased in assemble
        visual = utils.convert_transforms(view, self.transforms, exclude=['length'])
        # pass in view because nu is not distributable so may need to apply it
        # after visual assembly
        self.assemble(visual, view)
        super().draw(renderer, *args, **kwargs)
        return
```

In the `__draw__`, a utility function is used for conversions, but the length transforms are held until after assembly because the length is computed by adding the current length to the previous and many transforms are not distributable such that $\nu(x_0 + x_1 + x_2) = \nu(x_0) + \nu(x_1) + \nu(x_2)$. Inside `assemble`, the glyphs are either shifted vertically (`stacked`) or horizontally (`grouped`) such that the positions are recorded and added to with the next group. This function allows multiple columns to be mapped to a visual parameter, but it must be equal numbers of columns

```
{'position': ('island', lambda x: {'Biscoe':0, 'Dream':1, 'Torgersen':2}[x]),
 'length':(['Adelie', 'Chinstrap', 'Gentoo'], lambda x: x),
 'facecolors': (['Adelie_s', 'Chinstrap_s', 'Gentoo_s'],
        color.Categorical({'Adelie':'#FF8C00',
                           'Gentoo':'#159090',
                           'Chinstrap':'#A034F0'}))}
```

such as in this example where for each column contributing to a segment of the bar there is a corresponding column of colors for this segment. The reason the multibar can work with such a trasnformer is because it is relying on the data model to do most of the bookkeeping of which values get mapped to which bars. This also yields a much simpler function call to the artist

```
fig, ax = plt.subplots()
artist = bar.MultiBar(table, trans, orientation='h', stacked=True)
ax.add_artist(artist)
```

where `trans` is the same dictionary for both stacked and grouped version, as is the `DataFrameSection` object `table`. The only difference between the two versions is the `stacked` flag, and the only difference between figures 26 is the `orientation` argument. By decomposing the architecture into data, visual encoding, and assembly steps, we are able to build components that are more flexible and also more self contained than the existing code base.

This API may want to be redesigned such that there's a way to clearly couple the columns when doing multindex broadcasting

5 Discussion

This work contributes a mathematical description of the transformation from data to visual representation. Combining Butler's fiber bundle model of data with Spivaks formalism of data schemas provides a way of decoupling topology from variability such that the model can support a very large variety of datasets, including discrete relational tables, multivariate high resolution spatio temporal datasets, and complex networks. Modeling the graphic as a fiber bundle provides a way to separate the target display space from the topology of the graphic. By decomposing the mapping from data to visual representation as encoding ν, assembly Q, and a mapping between data and graphic topologies $\xi$and formalizing what equivariance each stage needs to preserved, this work derives constraints that visualization library authors could embed in their code to guarantee visualizations that are equivariant transforms of the input data.

This work generalizes previous research constraining visual encodings from data components to graphic components as equivariant maps to components that are N-dimensional. Furthermore, it precisely defines the glyph as the visual element constructed from data on a simplex in a simplacial complex where the simplex is discrete or continuous. This is a restatement of for example Bertin's definition of a line that encapsulates all points on the continuous line. By modeling the data topology along with its variablity, this work also provides a generalization of topology preservation as a deformation retraction from graphic space to data space. By using a functional paradigm, we can deconstruct the graphic to the glyph associated with each data point or even to pieces of a glyph; therefore the renderer has full flexibility in how to to generate the image, while the graphic to data topology maps provide a way to keep track of which part of the image belongs to which data point.

The toy prototype built using this model validates that is usable for a general purpose visualization tool since it can be iteratively integrated into the existing architecture rather than starting from scratch/ Factoring out glyph formation into assembly functions allows for much more clarity in how the glyphs differ. This prototype demonstrates that this framework can generate the fundemental marks, point (scatter plot), line (line chart), and area (bar chart). Furthermore, the grouped and stacked bar examples demonstrate that this model supports composition of glyphs into more complex graphics. These composite examples also rely on the fiber bundles section base book keeping to keep track of which components contribute to the attributes of the glyph. Implementing this example

using a Pandas dataframe demonstrates the ease of incorporating existing widely used data containers rather than requiring users to conform to one stands.

5.1 Limitations

So far this model has only been worked out for a single data set tied to a primitive mark, but it should be extensible to compositing datasets and complex glyphs. The examples and prototype have so far only been implemented for the static 2D case, but nothing in the math limits to 2D and expansion to the animated case should be possible because the model is formalized in terms of the sheaf. While this model supports equivariance of figurative glyphs generated form parameters of the data[7, 19], it currently does not have a way to evaluate the semantic accuracy of the figurative representation. This model also does not currently factor in effectiveness, but potentially effectiveness criteria could be incorporated into a scheme for assigning encoders and assembling glyphs. Also, even though the model is designed to be backend independent, it has only really been tested against the AGG backend. It is especially unknown how this framework interfaces with high performance rendering libraries such as openGL[22]. Because this model has been limited to the graphic design space, it does not address the critical task of laying out the graphics in the image

This model and the associated prototype is deeply tied to Matplotlib's existing architecture. While the model is expected to generalize to other libraries, such as those built on Mackinlay's APT framework, this has not been worked through. In particular, Mackinlay's formulation of graphics as a language with semantic and syntax lends itself a declarative interface[49], which Heer and Bostock use to develop a domain specific visualization language that they argue makes it simpler for designers to construct graphics without sacrificing expressivity [37]. Similarly, the model presented in this work formulates visualization as equivariant maps from data space to visual space, and is designed such that developers can build software libraries with data and graphic topologies tuned to specific domains.

5.2 Future Work

While the model and prototype demonstrate that generation of simple marks from the data, there is a lot of work left to develop a model that underpins a minimally viable library. Foremost is implementing a data object that encodes data with a 2D continous topology and an artist that can consume data with a 2D topology to visualize the image[33, 34, 82] and also encoding a separate heatmap[48, 91] artist that consumes 1D discrete data. A second important proof of concept artist is a boxplot[89] because it is a graphic that assumes computation on the data side and the glyph is built from semantically defined components and a list of outliers.

The model supports simple composition of glyphs by overlaying glyphs at the same position, but more work is needed to define an operator where the fiber bundles have shared $S_2 \hookrightarrow S_1$ such that fibers could be pulled back over the subset. This complex operator is necessary for building semantically meaningful components such as interactive visualizations that update on shared S, such as brush-linked views[8, 15] and verifying constraints in multiple view systems [62]. The models simple addition supports axes as standalone artists with overlapping visual position encoding, but the complex operator would allow for keeping labels consistent with text when for example the horizontal and vertical positions of the data are changed. In summary, the proposed scope of work for the dissertation is

- expansion of the mathematical framework to include complex addition

- mathematical formulation of a graphic with axes labeling
- implementation of data with 2D continuous topology and a compatible artist
- provisional mathematics and implementation of user level composite artists
- proof of concept domain specific user facing library

Additionally, implementing the complex addition operator would allow for developing a mathematical formalism and prototype of how interactivity would work in this model. Other potential tasks for future work is implementing a data object for a non-trivial fiber bundle and exploiting the models section level formalism to build distributed data source models and concurrent artists. This could be pushed further to integrate with topological[38] and functional [66] data analysis methods. While this paper formulates visualization in terms of monoidal action homomorphisms between fiberbundles, the model lends itself to a categorical formulation[29, 53] that could be further explored.

6 Conclusion

Despite the alternative formalism to Mackinlay's APT framework, it is indebted to APT for providing a guideline for issues the model needs to address. As with APT, this works aims to develop a framework on which to build visualization tools. Unlike APT, our model is geared towards balancing the needs of the the scientific and information visualization communities; therefore we present a framework that can describe both how the components of the data are encoded and how the continuity is preserved in the graphic. We hope that this framework can distill the constraints on the data to graphic mapping such that it can be used to improve the architecture of a heavily used visualization tool and allow developers to more easily build domain specific tools.

Yes, need to figure out how to wrap urls cleanly in bib

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