A Theory for Coloring Bivariate Statistical Maps

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Consideration of some practical uses of statistical bivariate maps—for example, display of association between variables—leads to principles for making effective use of color to represent data values. Effective color schemes for bivariate maps are viewed as continuous transformations from color models to the unit square with appropriate restrictions involving hue, saturation, and brightness. Several schemes, including those used by the U.S. Census Bureau, are criticized on the basis of this theory.

KEY WORDS: Bivariate color maps; Two-trait choropleth maps; Statistical graphics; Statistics in cartography; Color perception models.

1. INTRODUCTION

About six years ago the United States Bureau of the Census (Census) introduced color schemes that allow two variables—for example, educational attainment and family income—to be represented on the same map. Such bivariate maps have the potential to reveal relationships between variables more effectively than a side-by-side comparison of the two corresponding univariate maps. But it is not clear that Census bivariate maps fully achieve this purpose. Two recent papers in *The American Statistician* (Fienberg 1979, Wainer and Francolini 1980) have complained that these maps are hard to interpret. In spite of apparent difficulties with the implementation, the idea of using color to solve a problem of representation in four dimensions (two spatial dimensions—longitude and latitude—plus two statistical variables) is an important one for descriptive statistics and for exploratory data analysis. It is an idea with possible applications far beyond the attractive presentation of Census data, and with a future made especially promising by current rapid developments in color computer graphics.

This paper discusses some of the attributes (e.g., clear display of association between variables) that would be desirable for bivariate maps and gives a theoretical framework for selecting color schemes targeted to produce particular attributes. Thus, it becomes possible to predict which schemes are most worthy of being tested in practice.

The papers of Fienberg (1979) and of Wainer and Francolini (1980) contain relevant historical and bibliographical information not repeated here and color reproductions of Census bivariate maps. The Appendix gives a very brief description of two color models basic to our development and of associated color codes used for precision throughout the paper.

2. BASIC PRINCIPLES IN A UNIVARIATE SETTING

Some of the issues we need to consider for bivariate maps can be approached more simply in the univariate case. We begin with these.

Statistical maps may be continuous or discrete on either of two levels. First, the *statistical* variable may be continuous or it may be discrete (or grouped). Second, in the *cartographic* sense, an isopleth or "contour" map can be used for a variable—for example, temperature—with a known (or smoothed) value at each point on the map. Alternatively, a choropleth or "patch" map may be used for data available by discrete reporting unit—for example, median family income by census tract. We restrict attention to the discrete option in both instances, only remarking that almost everything in this paper has a direct continuous analog, and that continuous representations are often better (Tukey 1979, Sibert 1980).

On choropleth maps, levels of a statistical variable are often represented and distinguished by a selection of colors or patterns. These visual codes are called degrees of a retinal variable. In many cases of practical interest, such as the U.S. divided into counties or a city divided into census tracts, patches become too small for effective use of patterns. We consider only color retinal variables here.

We now illustrate two principles that seem important in selecting colors to represent levels of a statistical variable.

Principle I. (Order). If the levels of a statistical variable are ordered, then the colors chosen to represent them should be perceived as preserving the order.

For example, suppose the levels of the statistical variable, L_1 = less than \$5000, L_2 = \$5000 to \$9999, L_3 = \$10,000 to \$14,999, and L_4 = \$15,000 and above are to be represented, respectively, by the degrees D_i of the retinal variable $R = [D_1, D_2, D_3, D_4]$. Then some reasonable choices for R are

R_a = [*yellow, orange, red, purple*] =
[$2pa$, $5pa$, $8pa$, $11pa$],
R_b = [*white, pale pink, pink, red*] =
[a, $8ea$, $8la$, $8pa$],

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R_c = [*white, light grey, dark grey, black*] = [a, e, l, p],

R_d = [*white, pale yellow tint, orange tint, red*] = [a, $2ea$, $5la$, $8pa$],

since each of these consists of an ordered progression in hue, in saturation, in brightness, or in a combination of these perceptual aspects of color. (See the Appendix, especially Figs. 4 and 5.) A poor choice of retinal variable would be

R_e = [*yellow, pale pink, pink, red*] = [$2pa$, $8ea$, $8la$, $8pa$],

because of the gap between the lowest degree and the other three. A disastrously counter-intuitive choice would be

R_f = [*white, dark grey, black, light grey*] = [a, l, p, e]

(Wainer and Francolini 1980, p. 83).

A second principle focuses attention on some of the difficulties in using color to represent quantitative information. The signal one intends to send can be distorted by noise in the form of inexact color reproduction, of variability in color perception even among people not considered color-blind, of spurious effects produced by various light sources and by optical illusions, and so on (Birren 1961, Gerritsen 1975).

Principle II. (Separation). Important differences in the levels of a statistical variable should be represented by colors clearly perceived as different.

If, as in the examples above, there are only four levels, then we would suppose that all differences are important and would want to choose four colors spaced quite comfortably apart in one of the color models. R_d might be preferable to R_b, since in R_d more visual cues separate each pair of colors. Although

R_g = [*very pale pink*, . . . , *pink*] = [$8ca$, $8ea$, $8ga$, $8ia$]

adheres to Principle I, it has very poor separation.[1] On the other hand, the extreme separation in

R_h = [*yellow, red, blue, green*] = [$1pa$, $7pa$, $13pa$, $19pa$]

defeats our ability to perceive order (Principle I).

Other desirable properties could be listed, but Principles I and II illustrate the idea of using the perceptual aspects of color in the effective display of statistical information. Moreover, these two principles are crucial ones in producing useful color schemes for bivariate maps.

[1] If the statistical variable is to be displayed at many levels, say 12, we are approaching continuity. Here differences between neighboring levels are less important, and the retinal variable [$8ca$, $8ea$, . . . , $8pa$, . . . , $8pl$] might be a reasonable choice, especially if the map is to be printed using only black and red inks.

3. BIVARIATE OVERLAY SCHEMES

A bivariate scheme is an array of colors [C_{ij}]. A region of the map where the ith level of the first variable and the jth level of the second coincide will be colored C_{ij}. Suppose, for definiteness, that each variable has four levels so that the scheme is a square array of 16 elements. As in the univariate case the problem is to choose the colors wisely. At the least we would hope to select 16 mutually distinguishable colors (Principle II) such that progressions in any direction make sense visually (Principle I).

One systematic way to choose the C_{ij} is to "cross" two univariate retinal variables. In Figure 1 each C_{ij} is obtained by subtractive mixing of colors D_i and E_j. The result is called an *overlay* scheme because a bivariate map in such a scheme can be made by superimposing two transparent marginal maps. Figure 2 shows the result of combining R_e (yellow and reds) with a similar retinal variable in yellow and blues. The subtractive combination of D_4 = *blue* and E_1 = *yellow* is C_{41} = *green*, and so on. This is the Census scheme reproduced in Fienberg (1979).

In view of the criticism of R_e in the last section, we should not be surprised that the Census scheme is less than perfect. The illogical gap between the lowest degree and the higher degrees of each component has been propagated to the bivariate scheme, breaking it into two visually distinct subsets demarked by the heavy line in Figure 2. Another problem is the bewildering similarity of the nine violets C_{ij}, i, j = 2, 3, 4. A strength of this scheme is that the four colors at its corners are clearly distinct from each other, an attribute noted by Census (1976) and exploited by Fabsitz and Feinlieb (1979) among others.

Table 1 shows three overlay schemes based on components that satisfy Principle I. For example, Scheme 1

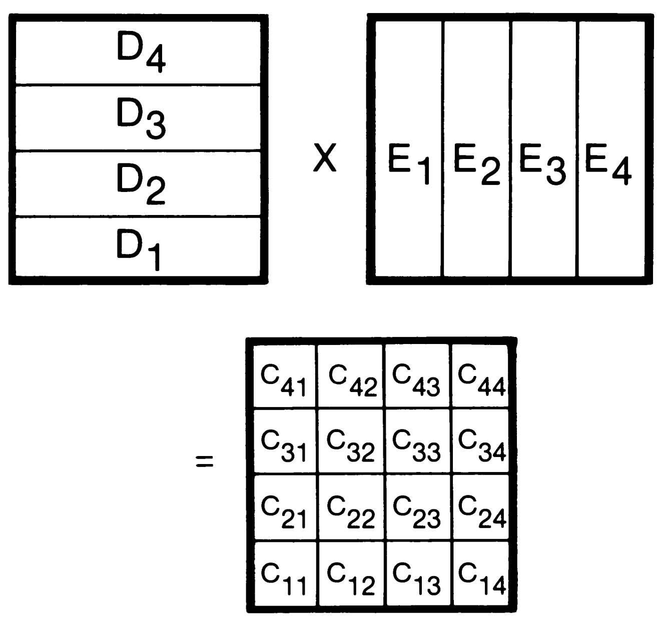

Figure 1. Crossing Two Retinal Variables To Form an Overlay Scheme

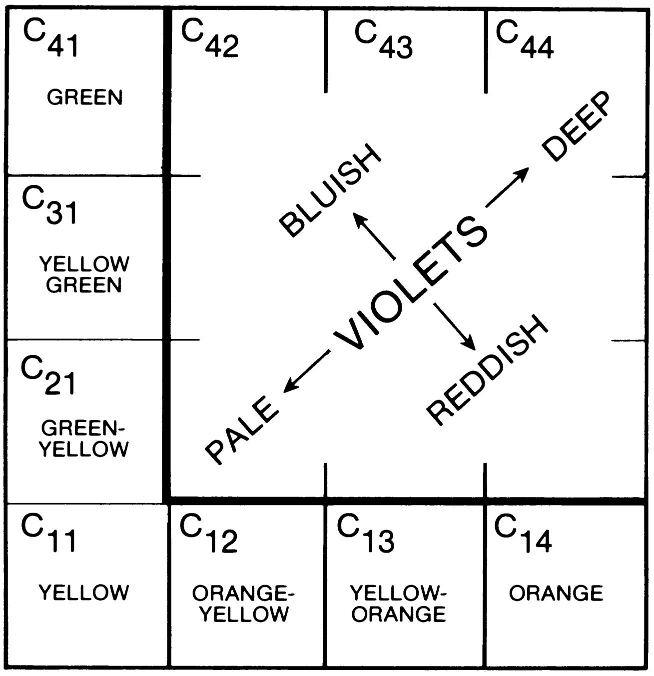

Figure 2. A Census Scheme

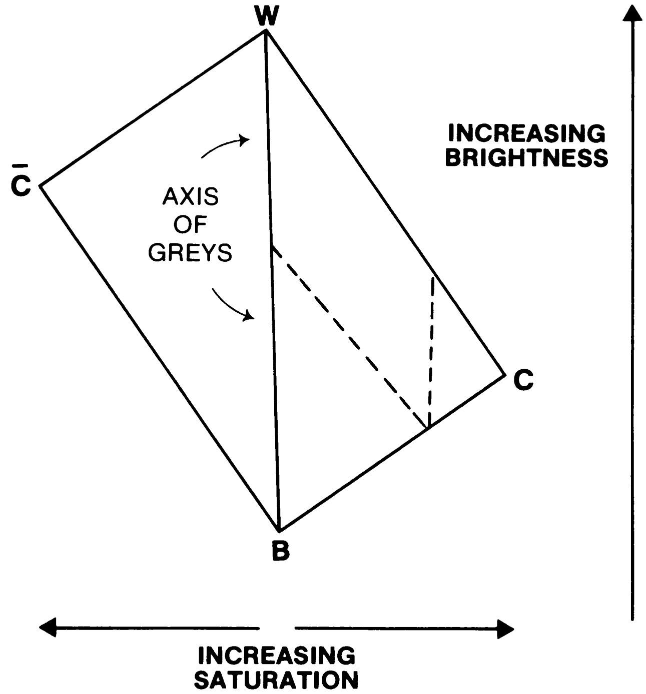

Figure 3. Section of Cube Model Containing Zero-Saturation Line (Axis of Greys)

crosses [000 *white*, 030, 060, 090 *magenta*] with [000 *white*, 003, 006, 009 *cyan*] as suggested by Wainer (1978). For an overlay scheme the color codes for D_i and E_j combine to give C_{ij}—for example, 090 *magenta* superimposed upon 009 *cyan* gives 099 *blue*.

Notice the location of the 16 color points within the cube model in each instance. The colors for Scheme 1 are lattice points on one face (area = 1.0) of the cube. Those in Scheme 2 lie in a parallelogram (area = 1.4) that slices through the cube at vertices *white*, *black*, and two complementary hues, here chosen to be C = 990 *orange* and $\bar{C}$ = 009 *cyan*. (See Figure 3.) Elements of Scheme 3 come from the much smaller parallelogram (area = .3) indicated by dotted lines in Figure 3. Of these three examples, Scheme 2 has the best theoretical color separation and Scheme 3 the worst. We shall see that Scheme 2 has other desirable attributes as well. Scheme 1 has no better color separation than do the nine similar colors in the Census scheme tested, with unfavorable results, by Wainer and Francolini (1980).[2]

Table 1. Elements C_{ij} of Schemes 1, 2, and 3 Designated by Hickethier Codes and Some Color Names

		j			
i	*(Scheme)*	*1*	*2*	*3*	*4*
4	(1)	009 cyan	039	069	099 blue
	(2)	009 cyan	339 cyan shade	669	999 black
	(3)	444 med. grey	554	774	994 brown
3	(1)	006	036	066	096
	(2)	006	336	666 dk grey	996 dk brown
	(3)	333 lt. grey	443	663 or tone	883
2	(1)	003 cyan tint	003 blue tint	063	093
	(2)	003 cyan tint	333 lt. grey	663 or tone	993 burnt or
	(3)	111	221	441	661
1	(1)	000 white	030 mag tint	060	090 magenta
	(2)	000 white	330 or tint	660	990 orange
	(3)	000 white	110	330	550 or tint

4. ADDITIONAL PRINCIPLES FOR BIVARIATE MAPS

Before going on to consider other possible schemes, let us think about how we might want to use a bivariate map in practice. The analogy with a bivariate scatterplot comes immediately to mind. We can use a scatterplot to assess the degree and type of association between two variables and to locate extreme cases. Information that involves only one of the variables is easier to get from a univariate plot, but we might well use a scatterplot to see the conditional distribution of one variable at a given level of the other. With bivariate maps the issues are much the same, but information from the two geodetic dimensions is introduced as well.

By way of illustration, imagine a bivariate map of a metropolitan area displaying data on income (scheme rows) and education (columns), and two corresponding univariate maps. The two univariate maps could be

[2] This scheme can be obtained from Scheme 1 by adding code 900 *yellow* to each of the codes for elements with i = 1 or j = 1. These seven colors are thus projected to the opposite cube face while the other nine remain unchanged. The Census scheme used in Map 7 for each SMSA of the *Urban Atlas* (Census 1975) appears to arise from lattice points in a rectangular subset of the original cube face, and thus would have poorer separation than Scheme 1.

used to see whether we agree with such statements as these:

1. Low income tracts occur mainly in the center of the city.

2. Tracts with highly educated populations are mainly in the suburban areas with an especially heavy concentration in the northern suburbs.

We might try to use the bivariate map to provide quick answers to questions such as the following. (Whether any of these questions can be answered easily by comparing the two univariate maps depends on the number and size of the tracts and on the intricacy of the interplay between the variables.)

3. Are all of the districts with both low income and low education near the center of the city?

4. Are there any high-education, low-income tracts? If so, are these isolated, scattered tracts, or clusters of tracts?

5. What is the range of educational levels among tracts with the most highly paid people?

6. Is there a positive association between the two variables? What spatial patterns are formed by exceptional cases?

Questions such as 3 and 4 can be answered by looking for a color located at a specific corner of the scheme. Moderately good color separation, so that corner colors stand out, is the main requirement here. An easy answer to 5 would require that the column of the scheme corresponding to high education be visually distinct from other columns and that the colors in that column form a useful visual sequence. To answer 6 we need to be able to tell which tracts and how many tracts have colors that occur on or near the principal diagonal of the scheme, and to tell which colors come from above the diagonal and which come from below. These considerations lead us to formulate two additional desirable properties for bivariate maps.

Principle III. (Rows and Columns). If preservation of univariate information or display of conditional distributions is a goal, then the levels of the component variables should not interact to obscure one another.

Principle IV. (Diagonal). If display of positive association is a goal, scheme elements should resolve themselves visually into three classes: those on or near the principal diagonal,[3] those above it, and those below.

Fienberg (1979, p. 176) suggests looking at diagonals and guesses that the goals involved in Principles III and IV may conflict, making it difficult to achieve both at the same time. Since a bivariate map seems to be the only descriptive technique available for displaying associations among all variables, statistical and spatial, priority will often be given to Principle IV. Of course Principle IV is not applicable unless both statistical variables are at least ordinal.

The Census scheme displayed in Figure 2 violates both Principle III and Principle IV. In row 4, for example, we find green, blue-violet and purple; and each of these colors is closely related to colors in other rows. The diagonal contains yellow and three of nine closely related violets. In the next section we discuss schemes that do better.

5. GENERAL COLOR SCHEMES

A three-dimensional color model M contains all possible colors arranged systematically. To select the colors for a 4×4 scheme so that Principle I is satisfied, we employ a bicontinuous transformation from some subset U of M onto the unit square, and then discretize the result by selecting the 16 colors that fall at the lattice points $((i - 1)/3, (j - 1)/3), i, j = 1, 2, 3, 4$ of the square. The continuity of the transformation preserves the systematic arrangement of the colors in M. (A little reflection will reveal that the transformation ought to be reasonably smooth also.)

Principle II is satisfied provided U is large enough. Since neither the dimensions of the statistical variables (e.g., years and dollars) nor those of the retinal variables are commensurate, the area of U can be only a very rough guide to appropriate color separation. But clearly, if $U \supset V$, then a scheme derived from U will excel one derived from V in this respect. We have already given the example that Scheme 1 is better than Scheme 3.

Principle III is satisfied if vertical lines in the unit square are images of points in M with constant hue, constant brightness, or constant saturation; and similarly for horizontal lines. None of the examples given so far satisfies Principle III.

In order to satisfy Principle IV, the principal diagonal of the square must be the image of points in M that form a class visually separate from other scheme colors. Points of zero saturation—white, greys, and black—seem to be an especially good choice, but points of maximum saturation or of constant hue might also serve as diagonal elements. In Scheme 2 the diagonal has zero saturation with blues above the diagonal increasing in saturation toward the upper left corner, and with oranges below the diagonal increasing in saturation toward the lower right corner. Scheme 1 has diagonal elements of constant hue.

A general scheme produced by choosing a transformation that will produce desirable attributes is not necessarily an overlay scheme. With older technologies, the ability to make a bivariate map by overlaying one univariate map upon another was a great convenience, but with the most recent computer graphics techniques, color maps in any desired scheme—univariate or bivariate—can be composed directly on a monitor or on photographic film. Thus, it is of practical

[3] If a negative association is anticipated, one of the variables can be reversed in order (e.g., from "Percent below poverty level" to "Percent above poverty level") or the scheme can be recast to call attention to the other diagonal.

Table 2. Elements C_{ij} of Scheme 4 Designated by Color Codes (Ostwald/Hickethier) and Names

i	j: 1	2	3	4
4	20*na*/509 saturated blue-green	22*ne*/939 leafgreen shade	24*ni*/968 dark olive green shade	*n*/999 black
3	18*ia*/014 cyan tint	20*ie*/527 blue-green tone	*i*/666 dark grey	12*ni*/698 dark purple shade
2	15*ea*/001 pale (sky) blue tint	*e*/222 light grey	8*ie*/572 red tone (rose)	10*ne*/393 magenta shade
1	*a*/000 white	3*ea*/110 pale orange tint (peach)	5*ia*/430 red-orange tint	8*na*/590 saturated red

as well as theoretical interest to consider promising examples of nonoverlay schemes.

The advantages of Scheme 2 are its good color separation, its logical arrangement of colors, and its zero-saturation diagonal. Scheme 4 in Table 2 retains the latter two advantages and improves color separation by taking colors from a *curved* slice through the Ostwald model to capture a variety of hues. (Note the dotted line in Figure 5.) Cool colors lie above the diagonal and warm colors below. Scheme 4 is not an overlay scheme, but marginal maps using retinal variables $[C_{11}, \ldots, C_{14}]$ and $[C_{11}, \ldots, C_{41}]$ could be compared with a bivariate map in this scheme without confusion.

Scheme 5 in Table 3 uses part of the surface of the Ostwald solid so that its rows (corresponding to "latitude" lines of the solid) are of constant saturation and brightness, and its columns (from "longitude" lines) are of constant hue. Thus Principle III is satisfied. For variety, five levels of each variable are indicated in this

Table 3. Elements C_{ij} of Scheme 5 Designated By Ostwald Codes and Some Color Names

i	j: 1 red	2 orange	3 yellow	4 leafgreen	5 seagreen
5 pale tint	8*ca* pale pink	5*ca* pale orange	2*ca* pale yellow	23*ca* pale leafgreen	20*ca* pale seagreen
4 medium tint	8*ia*	5*ia*	2*ia*	23*ia*	20*ia*
3 pure hue	8*pa* vivid red	5*pa* vivid orange	2*pa* vivid yellow	23*pa* vivid leafgreen	20*pa* vivid seagreen
2 medium shade	8*pe*	5*pe*	2*pe*	23*pe*	20*pe*
1 deep shade	8*pi* maroon	5*pi* brown	2*pi* olive	34*pi* deep leafgreen	20*pi* deep seagreen

Table 4. Characteristics of Various Schemes

Scheme	Principle: I (Order)	II (Separation)	III (Rows/Col's)	IV (Diagonal)
Census	No	—	No	No
1 (Mag./Cyan)	Yes	1.0	No	Weak
2 (Or./Cyan)	Yes	1.4	No	Yes
3 (Or./Grey)	Yes	0.3	No	No
4 (Twist)	Yes	>2	No	Yes
5 (Hues/Brt.)	Yes	>2	Yes	No

NOTE: Numerical ratings for Principle II are approximate areas of sections of the Hickethier cube model from which scheme colors are derived, except for the Census scheme, which involves a discontinuity. Higher numbers indicate better separation.

example. Schemes 4 and 5 can also be represented in terms of the cube model, but less intuitively. Table 4 summarizes the characteristics of Schemes 1 through 5.

We have discussed only square schemes so far, but for data more naturally plotted in polar coordinates (e.g., wind direction and velocity) or in triangular coordinates (e.g., population percentages of whites, blacks, and others), we could make schemes in these shapes. One circular scheme comes easily from an "equatorial" section of the Ostwald solid and one triangular scheme can be based on the planar section of the cube with vertices 900 *yellow*, 090 *magenta*, and 009 *cyan*. In these cases the ideas behind Principles III and IV need to be reexpressed.

Since color is a three-dimensional quantity, the question arises whether trivariate maps can be made. A fundamental limitation here is that the color solids are not the cartesian products of their perceptually important components—hue, saturation, and brightness. For example, at minimum saturation (grey scale), distinctions among hues are meaningless; and at minimum brightness (black), hue and saturation are degenerate. The practical possibilities for trivariate color maps seem limited.

6. CONCLUSION

Empirical testing will be necessary to determine which schemes for coloring bivariate maps are most effective. We have seen, however, that theoretical considerations involving the perceptually important aspects of color can be brought to bear in predicting which schemes will be successful in making maps to serve various purposes. Allocation of experimental resources for testing map schemes should be made on the basis of these predictions.

Specifically, we have seen that schemes similar to Schemes 2 or 4 should be effective in displaying bivariate relationships and those similar to Scheme 5 are useful in displaying univariate or conditional information. Scheme 3 and the published Census schemes have various disadvantages that should be avoided.

The particular examples of schemes discussed here

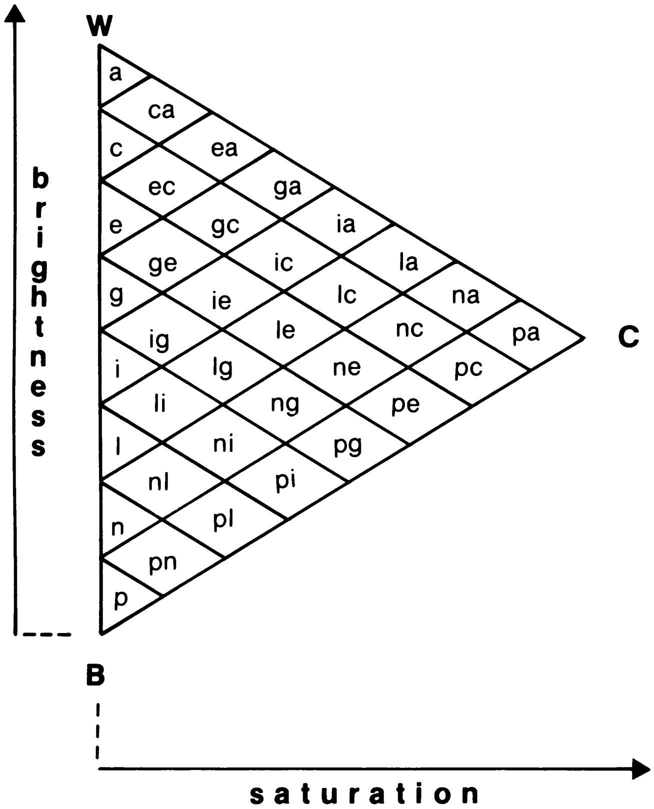

Figure 4. Ostwald Triangle With Color Codes

were chosen to illustrate the ideas of this paper and no claim is made that any of them is optimal. Furthermore, each map-making situation is unique, with its own constraints as to type of data, technology of reproduction, style, and so on. Circumstances may require attention to principles in addition to those we have discussed here. The approach used here is general enough, however, that it ought to be worthy of consideration in all but the most unusual of circumstances.

ADDENDUM. During Winter and Spring 1981 I have worked with Albert Yen at Lawrence Berkeley Laboratory to produce several computer-generated examples of maps in Schemes 1, 2, 4, and 5. Although no formal testing has been done, I am willing to venture from our experience so far that actual maps look very much as the theory predicts they should.

Maps were constructed in Schemes 2 and 4 for several pairs of statistical variables with different degrees of association. In each series, as Kendall's tau decreases from one to zero, maps range from somber unsaturated patterns to ones including more and more regions with saturated colors. For zero tau, the distinction between uncorrelated principal components (with remaining *spatial* correlation) and random data (with none) is striking. Scatterplots do not show this distinction. With Scheme 4 a "Christmas Effect" (mainly reds and greens) warns of negatively associated data. I understand that two-color reproductions of Scheme 4 maps, provided by Lawrence Berkeley Laboratory, will appear in a forthcoming article by Gentleman.

7. APPENDIX: COLOR MODELS AND CODES

Normal perception of visible light involves three aspects—hue, saturation, and brightness. Roughly, hue corresponds to average wavelength, saturation is inverse to variability of wavelengths, and brightness is proportional to wave amplitude. Here we describe briefly two of the many three-dimensional color models, in which all possible colors are represented as points arranged according to perceptual continua in these three aspects.

The Ostwald solid (Ostwald 1969) is based on triangles—each with vertices at black B, white W, and a pure (maximum saturation) hue C—and a color circle. (See Figures 4 and 5.) Ostwald color codes combine a hue number from the circle with a pair of letters giving the position in the appropriate triangle. For example, 8*ea*, 8*ie*, and 8*pi* are the codes for a pink (red tint), a rose (red tone), and a maroon (red shade), respectively. Greys, which lie on the line WB common to all such triangles, are given by a single letter.

The Hickethier solid (c.f. Küppers 1972) is the unit cube, in which the coordinates of each point (y, m, c), $0 \le y, m, c \le 1$ give the respective fractions of subtractive primary components—yellow, magenta, and

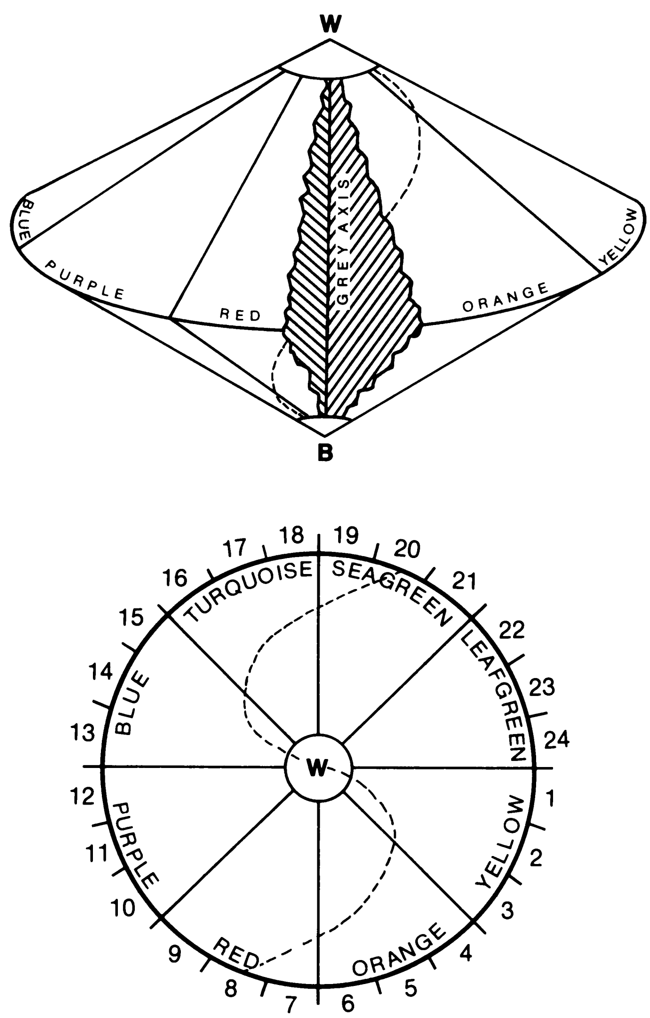

Figure 5. Side and Top Views of Ostwald Model

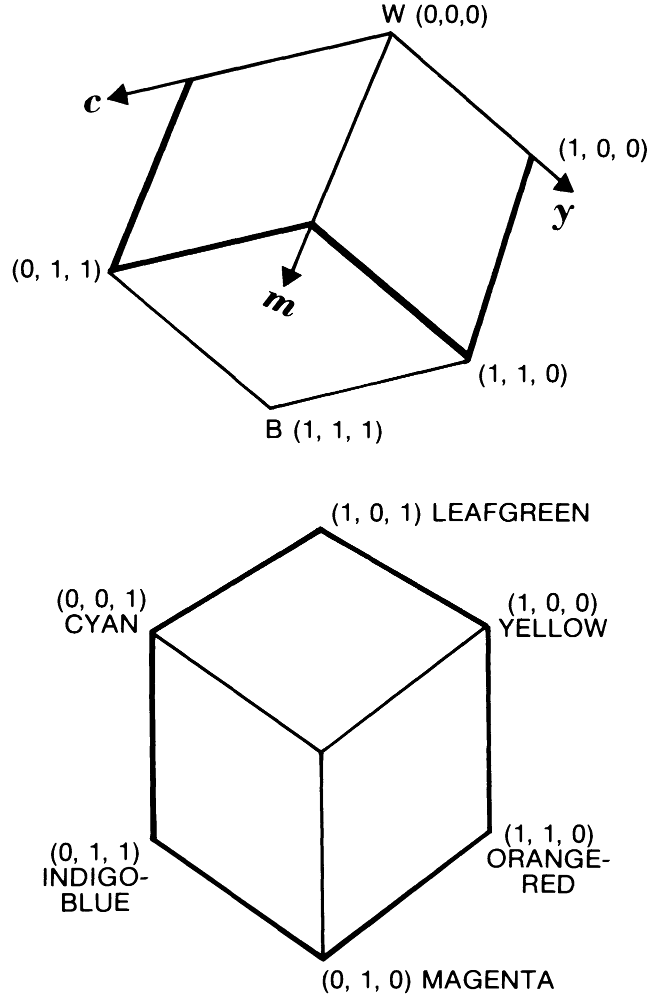

Figure 6. Two Views of Hickethier Cube Model

cyan—filtered out of white light to produce the color at that point. (See Figure 6.) It is customary to consider 10^3 uniformly spaced lattice points within the cube, represented by codes 000 through 999, where e.g., 316 is the code for (3/9, 1/9, 6/9) and (0, 1/2, 2/3) is coded either 046 or 056. (The difference is perceptually trivial.) This model can be used to predict the result of combining colors of different hues. For example, superimposed 900 *yellow* and 009 *cyan* filters pass light of color 900 + 009 = 909 *leafgreen*. These codes refer to the subtractive character of colors. Thus, the subtractive combining of colors is represented by a kind of addition, without "carrying," of their codes.

For many purposes these two color models are equivalent: a bicontinuous one-to-one transformation exists between them, linking points of identical color in each. Given a choice, we use the model that illustrates an idea most easily and intuitively.

For more extensive discussions of color models, see for example, Gerritsen (1975), Itten (1979), Küppers (1972), and Ostwald (1969).

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* Contains elementary discussions of color theory and models.

† Has color reproductions of Census-style bivariate maps.