Topological Equivariant Artist Model for Visualization Library Architecture

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Abstract—The abstract goes here.

Index Terms—

1 Introduction

This paper uses methods from topology and category theory to develop a model of the transformation from data to graphical representation. This model provides a language to specify how data is structured and how this structure is carried through in the visualization, and serves as the basis for a functional approach to implementing visualization library components. Topology allows us to describe the structure of the data and graphics in a generalizable, scalable, and trackable way. Category theory provides a framework for separating the transformations implemented by visualization libraries from the various stages of visualization and therefore can be used to describe the constraints imposed on the library components [1], [2]. Well constrained modular components are inherently functional [3], and a functional framework yields a library implementation that is likely to be shorter, clearer, and more suitd to distributed, concurrent, and on demand tasks [4]. Using this functional approach, this paper contributes a practical framework for decoupling data processing from visualization generation in a way that allows for modular visualization components that are applicable to a variety of data sets in different formats. is it OK that this is something reviewer 4 wrote

2 Related Work

This work aims to develop a model for describing visualization transformations that can be translated into visualization library architecture. In doing so, we describe how visualization libraries attempt this goal and build on work that formally describes what properties of data should be preserved. We restrict the properties of data that should be preserved to

continuity how elements in a dataset are connect to each other, e.g. discrete rows in a table, networked nodes, pixels in an image, points on a line

equivariance functions on data that have an equivalent effect on the graphical representation, e.g. rotating a matrix has a matching rotation of the image, translating the points on a line has a matching visual shift in the line plot

2.1 Continuity

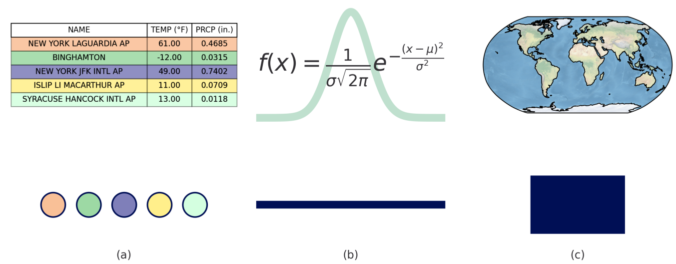

Fig. 1: Continuity is how elements in a data set are connected to each other, which is distinct from how the data is structured. The rows in (a) are discrete, therefore they have discrete continuity as illustrated by the discrete dots. The gaussian in (b) is a 1D continuous function, therefore the continuity of the elements of the gaussian can be represented as a line on an interval (0,1). In (c), every element of the globe is connected to its nearest neighbors, which yields a 2D continuous continuity as illustrated by the square.

Continuity is

We care about continuity b/c

We express continuity using fiber bundles, which are ... [5], [6]

$$F \hookrightarrow E \xrightarrow{\pi} K \tag{1}$$

2.2 Equivariance

What is it?

$$\begin{tikzcd} \text{data} \arrow[r] \arrow[d, "\text{function}"] & \text{representation} \arrow[r] & \text{visual stimulus} \arrow[d, "\text{visual equivalent to fu}"] \\ \text{data} \arrow[r] & \text{representation} \arrow[r] & \text{visual stimulus} \end{tikzcd}$$

3 Modeling Visualization Stages: Fiber Bundles

The Artist $\mathcal{A}$ is a transformation from

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Fig. 2: Continuity is implicit in choice of visualization rather than explicitely in choice of data container. The line plots in (b) are generated by a 2D table (a). Structurally this table can be identical to the 2D matrix (a) that generates the image in (c).

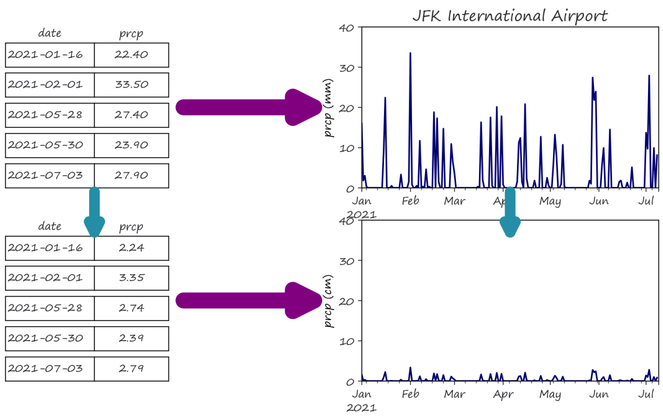


Fig. 3: Equivariance is that a transformation on the data has a corresponding transformation in the graphical representation. For example, in this figure the data is scaled by a factor 10. Equivalently the line plot is scaled by factor of 10, resulting in a shrunken line plot. Either a transformation on the data side can induce a transformation on the visual side, or a transformation on the visual side indicates that there is also a transformation on the data side.

3.1 Data Bundle E

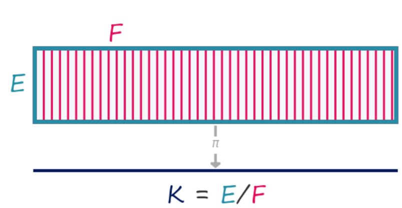


Fig. 4: this is gonna be replaced w/ more concrete

We use topology to model

The The continuity of the data is encoded in the base space K.

The properties of the variables are encoded in the fiber space F. The fiber is a topological space

Spivak provides notation for describing the set of all possible values U_σ of a single column with name c and type T. This set of values is the fiber "F" $F = U_\sigma(c)$. When data is multivariate, the fiber F is the cartesian product of each columns fiber space (5)

3.1.0.1 Structured Data: Section τ:

$$\begin{tikzcd} F \arrow[r, hook] & E \arrow[d, "\pi"', shift right] \\ & K \arrow[u, bend right, "\tau"'] \end{tikzcd} \tag{2}$$

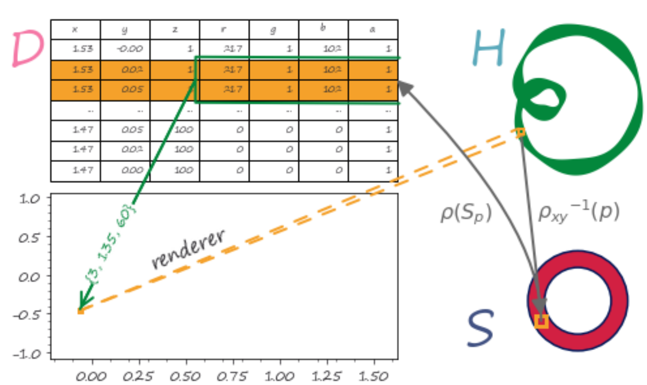
3.1.0.2 Structure: Continuity and Equivariant Actions:

$$\begin{tikzcd} F_i \arrow[d, "m_j"'] \arrow[dr, "m_j \circ m_k"] & \\ F_i \arrow[r, "m_k"'] & F_i \end{tikzcd} \tag{3}$$

3.1.1 *Continuity of Data: sheaf $\mathcal{O}$*

Fig. 5

3.2 Graphic Bundle H



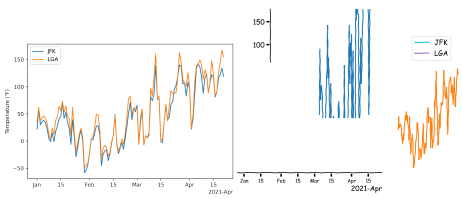
$$\begin{tikzcd} D \arrow[r, hook] & H \arrow[d, "\pi"', shift right] \\ & S \arrow[u, bend right, "\rho"'] \end{tikzcd} \tag{4}$$

3.2.0.1 Continuity: Base Space S:
3.2.0.2 Equivariance: Fiber D:
3.2.0.3 Structured Data: Visual ρ:

3.3 Artist

$$A : \mathcal{O}(E) \rightarrow \mathcal{O}(H) \tag{5}$$

4 Union of Artists



(a) Artists with shared μ_i renderered correctly (b) Artists without shared μ_i

Fig. 6: Simulation results for the network.

5 Construction and formal properties of Artists

$$\begin{tikzcd} E \arrow[r, "\nu"] \arrow[dr, "\pi"'] & V \arrow[d, "\pi"] & \xi^*V \arrow[l, "\xi^*"'] \arrow[r, "Q"] \arrow[d, "\xi^*\pi"'] & H \arrow[dl, "\pi"] \\ & K & S \arrow[l, "\xi"'] & \end{tikzcd} \tag{6}$$

5.1 Graphic to Data: ξ

$$\begin{tikzcd} E \arrow[d, "\pi"'] & H \arrow[d, "\pi"'] \\ K & S \arrow[l, "\xi"'] \end{tikzcd} \tag{7}$$

5.2 Visual Bundle V

$$\begin{tikzcd} P \arrow[r, hook] & V \arrow[d, "\pi"'] \\ & K \arrow[u, bend right, "\mu"'] \end{tikzcd} \tag{8}$$

5.3 Data to Representation: ν

$$\{\nu_0, \ldots, \nu_n\} : \{\tau_0, \ldots, \tau_n\} \mapsto \{\mu_0, \ldots, \mu_n\} \tag{9}$$

We enforce the equivariance constraint

$$\begin{tikzcd} E_i \arrow[r, "\nu_i"] \arrow[d, "m_r"'] & V_i \arrow[d, "m_v"] \\ E_i \arrow[r, "\nu_i"] & V_i \end{tikzcd} \tag{10}$$

5.3.1 *Representation to Visual: Q*

6 Case Study

We implement the **arrows** in fig:api

1

7 Discussion

7.1 Limitations

7.2 future work

8 Conclusion

The conclusion goes here.

Appendix A Rendering: ρ

Appendix B Manufacturing $\hat{Q} \leftarrow Q$

$$\begin{tikzcd} E \arrow[r, "\nu"] \arrow[dr, "\pi"'] & V \arrow[d, "\pi"'] & \xi^*V \arrow[l, "\xi^*"'] \arrow[r, "Q"] \arrow[d, "\xi^*\pi"'] & H \arrow[dl, "\pi"] \\ & K \arrow[u, bend right, "\mu"'] & S \arrow[l, "\xi"'] \arrow[u, bend right, "\xi^*\mu"'] & \end{tikzcd} \tag{11}$$

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Hannah Aizenman Biography text here.

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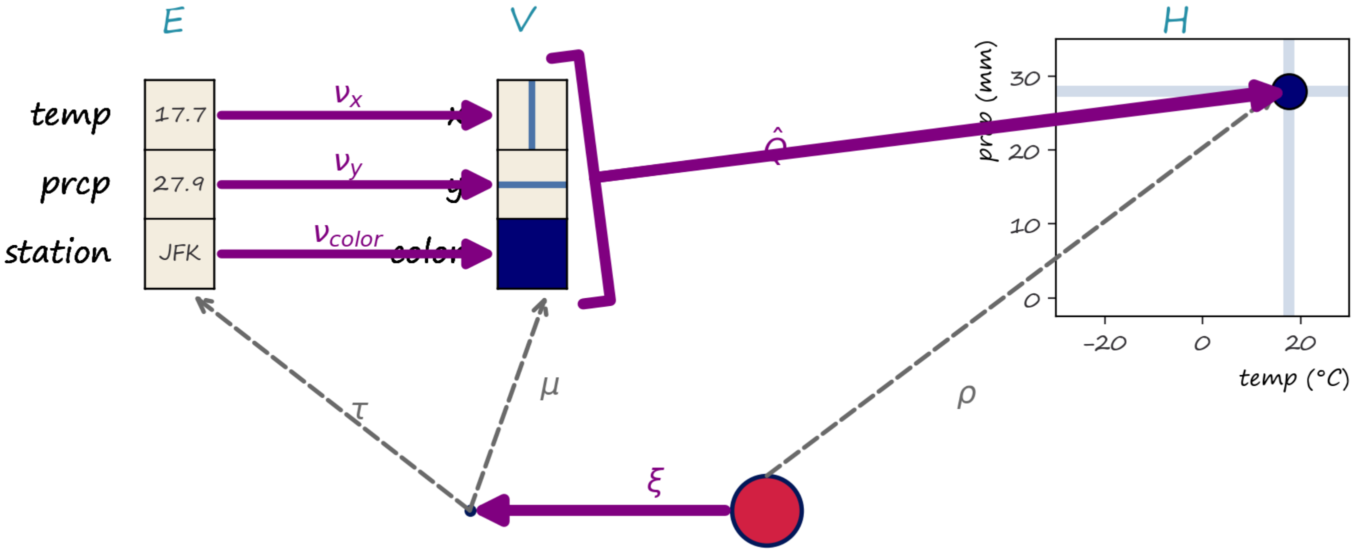
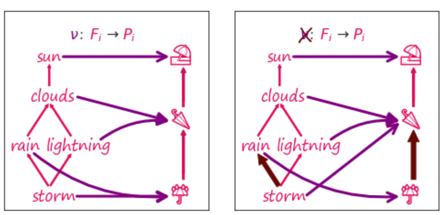
Michael Grossberg Biography text here.





(a) Artists with shared μ_i renderered correctly

(b) Artists without shared μ_i

Fig. 7: Simulation results for the network.

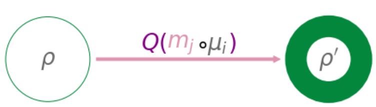


Fig. 8: rework this as a commutative box w/ the r in E row associated w/ this qhat(k)

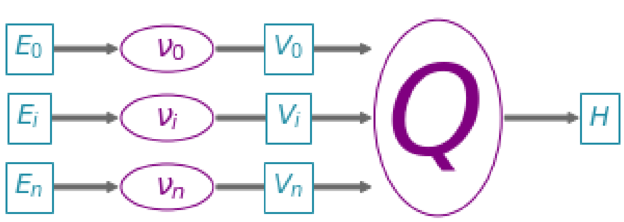


Fig. 9: add in xi!