MAC 2313, Section 12, Spring 2016 (Dr M-G)

Fourth Test

Tuesday, December 06, 2016

You are allowed to use a TI-30Xa (or any 4-function calculator). No other calculator is allowed. You have 75 minutes. Present your solutions clearly *in ink*. Show all necessary steps in your method. Include enough comments or diagrams to convince me that you thoroughly understand. Begin each question (as opposed to part of question) on a fresh sheet of paper, use *one* side of the paper only, and ensure that your solutions are in the proper order at the end of the test.

Answer all four questions perfectly to obtain full credit.

1. Let C denote the ellipse with vector equation

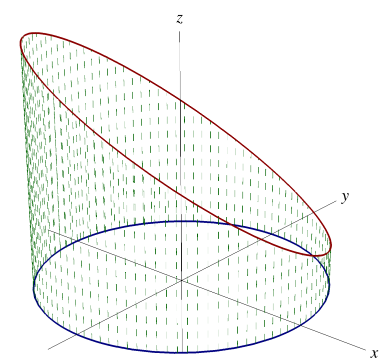
$$\mathbf{r} = 2\cos(t)\mathbf{i} + 2\sin(t)\mathbf{j} + \{2 - \cos(t) - \sin(t)\}\mathbf{k}, \quad 0 \le t \le 2\pi$$

(red curve in diagram). This is the closed curve in which the plane with equation $x + y + 2z = 4$ intersects the cylinder with equation $x^2 + y^2 = 4$. Let S_3 denote the planar elliptical disk with positive upward normal that is bounded by C. Let the vector field $\mathbf{F}$ be defined by $\mathbf{F} = z\,\mathbf{i} + x\,\mathbf{j} + y\,\mathbf{k}$.

(a) Calculate the circulation $\oint\limits_C \mathbf{F} \cdot \mathbf{dr}$ directly (as a line integral). [10]

(b) Calculate the flux $\iint\limits_{S_3} \nabla \times \mathbf{F} \cdot \mathbf{dS}$ directly (as a surface integral). [10]

Verify that your results agree with Stokes' theorem.



2. Let E be the volumetric region enclosed by the surface $S = S_1 \cup S_2 \cup S_3$, where S_1 is a circular disk of radius 2, centered at the origin and lying in the plane $z = 0$ (inside the blue circle in the diagram); S_2 is the part of the circular cylinder $x^2 + y^2 = 4$ that lies between the planes $z = 0$ and $x + y + 2z = 4$; and S_3 is the planar elliptical disk defined in Question 1. Let the vector field $\mathbf{F}$ be defined by $\mathbf{F} = x\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k}$.

(a) Calculate the volume integral $\iiint\limits_E \nabla \cdot \mathbf{F}\, dV$ directly. [10]

(b) Calculate the flux $\iint\limits_S \mathbf{F} \cdot \mathbf{dS}$ directly (as a surface integral). [10]

Verify that your results agree with the divergence theorem.

[Perfect score: $4 \times 10 = 40$]