A **sequence** is a function $\{a_n\}_{n=0}^{\infty}$ whose domain is the set of natural numbers (or some subset) $n = 0, 1, 2, 3, \ldots$. We list the elements of a sequence in order as

$$a_n = a_0, a_1, a_2, a_3, \ldots$$

Write the first six terms of the following sequences.

1. $\left\{\dfrac{n}{n+1}\right\}_{n=1}^{\infty}$

2. $\left\{\dfrac{(-1)^n(n+1)}{3^n}\right\}_{n=1}^{\infty}$

3. $\left\{\sqrt{n-3}\right\}_{n=3}^{\infty}$

4. $\left\{\cos\left(\frac{n\pi}{6}\right)\right\}_{n=0}^{\infty}$

Sequences can also be written **recursively**. That means that each term depends on the previous term. Determine the first six terms of each of the recursively defined sequences.

1. $a_1 = 4, a_n = 3 + a_{n-1}$.

2. $a_1 = 1, a_2 = -1, a_n = a_{n-2} + 3 \cdot a_{n-1}$.

3. $a_1 = 3, a_n = -2 \cdot a_{n-1}$.

Find a formula for the general term a_n of the sequence.

1. $\{2, 7, 12, 17, \ldots\}$

2. $\left\{\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots\right\}$

3. $\left\{\frac{3}{5}, -\frac{4}{25}, \frac{5}{125}, -\frac{6}{625}, \frac{7}{3125}, \ldots\right\}$

Determine the limit $\lim_{n\to\infty} a_n$ of the sequence, if it exists.

1. $\left\{\dfrac{n}{n+1}\right\}$

2. $\left\{\sqrt{n-3}\right\}$

3. $\left\{\cos\left(\frac{n\pi}{6}\right)\right\}$

4. $\left\{\dfrac{1}{2^n}\right\}$

A **series** is the sum of a sequence. A series can be the sum of a certain number of terms or it can be the infinite sum of the entire series. Notationally, we represent a series as

$$\sum_{n=1}^{N} a_n$$

The number below Σ tells us where to start our addition, the number above tells us where to stop.

Determine the following sums:

1. $\displaystyle\sum_{n=1}^{5} \frac{n}{n+1}$

2. $\displaystyle\sum_{n=4}^{10} n$

3. $\displaystyle\sum_{n=0}^{10} \frac{(-1)^n(n+1)}{3^n}$