§13.4, #42*

From

$$\boldsymbol{\rho} \;=\; t\mathbf{i} + \cos^2(t)\mathbf{j} + \sin^2(t)\mathbf{k}$$

we obtain

$$\mathbf{v} \;=\; \dot{\boldsymbol{\rho}} \;=\; \mathbf{i} + 2\cos(t)\{-\sin(t)\}\mathbf{j} + 2\sin(t)\cos(t)\mathbf{k} \;=\; = \;\mathbf{i} - \sin(2t)\mathbf{j} + \sin(2t)\mathbf{k}$$

implying

$$v \;=\; |\mathbf{v}| \;=\; \sqrt{1 + \{-\sin(2t)\}^2 + \{-\sin(2t)\}^2} \;=\; \sqrt{1 + 2\sin^2(2t)}$$

and hence

$$a_T \;=\; \frac{dv}{dt} \;=\; \tfrac{1}{2}\{1 + 2\sin^2(2t)\}^{-1/2}\{2 \cdot 2\sin(2t)\cdot\cos(2t)\cdot 2\} \;=\; \frac{2\sin(4t)}{\sqrt{1+2\sin^2(2t)}}$$

with

$$\mathbf{T} \;=\; \widehat{\mathbf{v}} \;=\; \frac{\mathbf{v}}{v} \;=\; \frac{1}{\sqrt{1+2\sin^2(2t)}}\{\mathbf{i} - \sin(2t)\mathbf{j} + \sin(2t)\mathbf{k}\}$$

and

$$\mathbf{a} \;=\; \dot{\mathbf{v}} \;=\; \ddot{\boldsymbol{\rho}} \;=\; 0\mathbf{i} - 2\cos(2t)\mathbf{j} + 2\cos(2t)\mathbf{k}$$

Now—and no earlier—we can follow our star and set $t = \frac{1}{6}\pi \Longrightarrow \sin(2t) = \sin\left(\frac{1}{3}\pi\right) = \frac{1}{2}\sqrt{3}$, $\cos(2t) = \cos\left(\frac{1}{3}\pi\right) = \frac{1}{2}$ and $\sin(4t) = \sin\left(\frac{2}{3}\pi\right) = \frac{1}{2}\sqrt{3}$. We obtain

$$v \;=\; \sqrt{\tfrac{5}{2}}, \qquad a_T \;=\; \sqrt{\tfrac{6}{5}}$$

with

$$\mathbf{T} \;=\; \sqrt{\tfrac{2}{5}}\,\mathbf{i} - \sqrt{\tfrac{3}{10}}\,\mathbf{j} + \sqrt{\tfrac{3}{10}}\,\mathbf{k} \;=\; \tfrac{1}{\sqrt{10}}\{2\mathbf{i} - \sqrt{3}\mathbf{j} + \sqrt{3}\mathbf{k}\}, \qquad \mathbf{a} \;=\; 0\mathbf{i} - \mathbf{j} + \mathbf{k}.$$

But we know that also

$$\mathbf{a} \;=\; a_T\mathbf{T} + a_N\mathbf{N} + 0\,\mathbf{B}.$$

So

$$a_N\mathbf{N} \;=\; \mathbf{a} - a_T\mathbf{T} \;=\; 0\,\mathbf{i} - \mathbf{j} + \mathbf{k} - \sqrt{\tfrac{6}{5}}\left(\sqrt{\tfrac{2}{5}}\,\mathbf{i} - \sqrt{\tfrac{3}{10}}\,\mathbf{j} + \sqrt{\tfrac{3}{10}}\,\mathbf{k}\right) \;=\; \tfrac{2}{5}\{-\sqrt{3}\mathbf{i} - \mathbf{j} + \mathbf{k}\},$$

implying

$$\mathbf{N} \;=\; \widehat{a_N\mathbf{N}} \;=\; \tfrac{1}{\sqrt{5}}\{-\sqrt{3}\mathbf{i} - \mathbf{j} + \mathbf{k}\}$$

and

$$a_N \;=\; |a_N\mathbf{N}| \;=\; \tfrac{2}{5}|-\sqrt{3}\mathbf{i} - \mathbf{j} + \mathbf{k}| \;=\; \tfrac{2}{5}\sqrt{5} \;=\; \tfrac{2}{\sqrt{5}}$$

or, if you prefer, $a_N = \mathbf{a}.\mathbf{N} = \frac{1}{\sqrt{5}}\{0\cdot(-\sqrt{3}) + (-1)(-1) + 1.1\} = \frac{2}{\sqrt{5}}$ (because $\mathbf{T}.\mathbf{N} = 0$ and $\mathbf{N}.\mathbf{N} = 1$. Also

$$\mathbf{B} \;=\; \mathbf{T}\times\mathbf{N} \;=\; \tfrac{1}{\sqrt{10}\sqrt{5}}\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -\sqrt{3} & \sqrt{3} \\ -\sqrt{3} & -1 & 1 \end{vmatrix} \;=\; \frac{0\mathbf{i} - \mathbf{j} - \mathbf{k}}{\sqrt{2}}$$

and

$$\kappa \;=\; a_N/v^2 \;=\; \tfrac{4}{5\sqrt{5}}.$$