§13.3, #50

From

$$\boldsymbol{\rho} \;=\; t\mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k}$$

we obtain

$$\dot{\boldsymbol{\rho}} \;=\; \mathbf{i} + 2t\mathbf{j} + 3t^2\mathbf{k}$$

implying

$$\mathbf{T} \;=\; \widehat{\dot{\boldsymbol{\rho}}} \;=\; \frac{1}{\sqrt{1+4t^2+9t^4}}\{\mathbf{i} + 2t\mathbf{j} + 3t^2\mathbf{k}\}$$

and hence

$$\begin{aligned}\dot{\mathbf{T}} &= \frac{d}{dt}\left\{\frac{1}{\sqrt{1+4t^2+9t^4}}\right\}\{\mathbf{i} + 2t\mathbf{j} + 3t^2\mathbf{k}\} + \frac{1}{\sqrt{1+4t^2+9t^4}}\frac{d}{dt}\left\{\mathbf{i} + 2t\mathbf{j} + 3t^2\mathbf{k}\right\} \\ &= -\tfrac{1}{2}\{1+4t^2+9t^4\}^{-3/2}\{0 + 4\cdot 2t + 9\cdot 4t^3\}\{\mathbf{i} + 2t\mathbf{j} + 3t^2\mathbf{k}\} \;+ \\ &\qquad\qquad \frac{1}{\sqrt{1+4t^2+9t^4}}\left\{0\mathbf{i} + 2\mathbf{j} + 6t\mathbf{k}\right\}\end{aligned}$$

so that for $t = 1$ we obtain

$$\dot{\mathbf{T}} \;=\; \frac{1}{7\sqrt{14}}\{-11\mathbf{i} - 8\mathbf{j} + 9\mathbf{k}\}$$

So for $t = 1$ we obtain

$$\mathbf{T} \;=\; \frac{1}{\sqrt{14}}\{\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}\}, \qquad \mathbf{N} \;=\; \widehat{\dot{\mathbf{T}}} \;=\; \frac{1}{\sqrt{266}}\{-11\mathbf{i} - 8\mathbf{j} + 9\mathbf{k}\}$$

implying

$$\mathbf{B} \;=\; \mathbf{T}\times\mathbf{N} \;=\; \frac{1}{\sqrt{14}\sqrt{266}}\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 2 & 3 \\ -11 & -8 & 9 \end{vmatrix} \;=\; \frac{1}{\sqrt{14}\sqrt{266}}\{42\mathbf{i} - 42\mathbf{j} + 14\mathbf{k}\} \;=\; \frac{3\mathbf{i} - 3\mathbf{j} + \mathbf{k}}{\sqrt{19}}$$

So a normal vector to the osculating plane is $3\mathbf{i} - 3\mathbf{j} + \mathbf{k}$. Since it passes through the point with position vector $\mathbf{i}+\mathbf{j}+\mathbf{k}$, its equation is therefore $3x - 3y + z = (3\mathbf{i} - 3\mathbf{j} + \mathbf{k})\cdot(\mathbf{i}+\mathbf{j}+\mathbf{k}) = 1$. Likewise, a normal vector to the normal plane is $\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$. So its equation is $x + 2y + 3z = (\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})\cdot(\mathbf{i} + \mathbf{j} + \mathbf{k}) = 6$.