MAC 2313, Section 12, Spring 2016 (Dr M-G)

Third Test

Thursday, November 10, 2016

You are allowed to use a TI-30Xa (or any 4-function calculator). No other calculator is allowed. You have 75 minutes. Present your solutions clearly *in ink*. Show all necessary steps in your method. Include enough comments or diagrams to convince me that you thoroughly understand. Begin each question (as opposed to part of question) on a fresh sheet of paper, use *one* side of the paper only, and ensure that your solutions are in the proper order at the end of the test.

Answer all four questions perfectly to obtain full credit.

1. For the three-dimensional motion defined by $\mathbf{r} = \frac{1}{3}t^3\,\mathbf{i} + \frac{1}{\sqrt{2}}t^2\,\mathbf{j} + t\,\mathbf{k}$, find exactly

(a) the unit tangent vector $\mathbf{T}$ [3]

(b) the principal unit normal vector $\mathbf{N}$ [3]

(c) the curvature κ and [2]

(d) the binormal vector $\mathbf{B}$ [2]

at the moment when $t = 1$. You should check that your answers for (**a**), (**b**) and (**d**) are mutually orthogonal. [10]

Hint: The correct expression for v^2 is a perfect square (so there are no square roots in the correct expression for v).

2. Find the exact value of the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ for

$$\mathbf{F} \;=\; z\,\mathbf{i} + y(x^2 + z^2)\,\mathbf{j} - x\,\mathbf{k}$$

where C is defined by $\mathbf{r} = \sin(t)\,\mathbf{i} + \sqrt{1+t}\,\mathbf{j} + \cos(t)\,\mathbf{k}$ for $0 \le t \le 2\pi$. [10]

Hint: The correct answer lies between 9 and 10.

3. **(a)** Verify that $\mathbf{F} = (y + ze^x)\mathbf{i} + x\,\mathbf{j} + e^x\,\mathbf{k}$ is a conservative (or irrotational or potential or path-independent) vector field, that is, ϕ exists such that $\mathbf{F} = \nabla\phi$.

(b) Recover the potential $\phi = \phi(x, y, z)$ by integrating along a suitable contour from $(0, 0, 0)$ to (X, Y, Z) and then substituting x, y and z for X, Y and Z.

You should check that your resultant $\nabla\phi$ indeed equals $\mathbf{F}$. [10]

4. Calculate the *upward* flux of the vector field

$$\mathbf{F} \;=\; y\mathbf{i} + z^2\mathbf{j} + x\mathbf{k}$$

through the planar triangular surface whose vertices are at $(0, 1, 1)$, $(2, 3, -1)$ and $(1, 1, -1)$. [10]

Hint: The correct answer is an integer.

[Perfect score: $4 \times 10 = 40$]