Have:

$$\frac{1}{\left(1-\frac{y}{5}\right)\left(1-\frac{y}{9}\right)y}$$

Want:

$$\frac{A}{1-\frac{y}{5}} + \frac{B}{1-\frac{y}{9}} + \frac{C}{y}$$

Set them equal:

$$\frac{1}{\left(1-\frac{y}{5}\right)\left(1-\frac{y}{9}\right)y} = \frac{A}{1-\frac{y}{5}} + \frac{B}{1-\frac{y}{9}} + \frac{C}{y}.$$

Multiply by den. of LHS:

$$1 = Ay\left(1-\frac{y}{9}\right) + By\left(1-\frac{y}{5}\right) + C\left(1-\frac{y}{5}\right)\left(1-\frac{y}{9}\right).$$

Note:

- when $y=0$: $1 = 0 + 0 + C(1-0)(1-0) \Rightarrow \boxed{C=1}$

  $y=5$: $1 = 5A\left(1-\frac{5}{9}\right) + 0 + 0 \Rightarrow 1 = 5A\left(\frac{4}{9}\right)$

  $\Rightarrow 1 = \frac{20A}{9}$

  $\Rightarrow \boxed{A = \frac{9}{20}}$

  $y=9$: $1 = 0 + 9B\left(1-\frac{9}{5}\right) + 0 \Rightarrow 1 = 9B\left(\frac{-4}{5}\right)$

  $\Rightarrow 1 = \frac{-36}{5}B$

  $\Rightarrow \boxed{B = \frac{-5}{36}}$

Note: By plugging in A, B, C, you get the right decomp. In my answers, I also did some simplification!