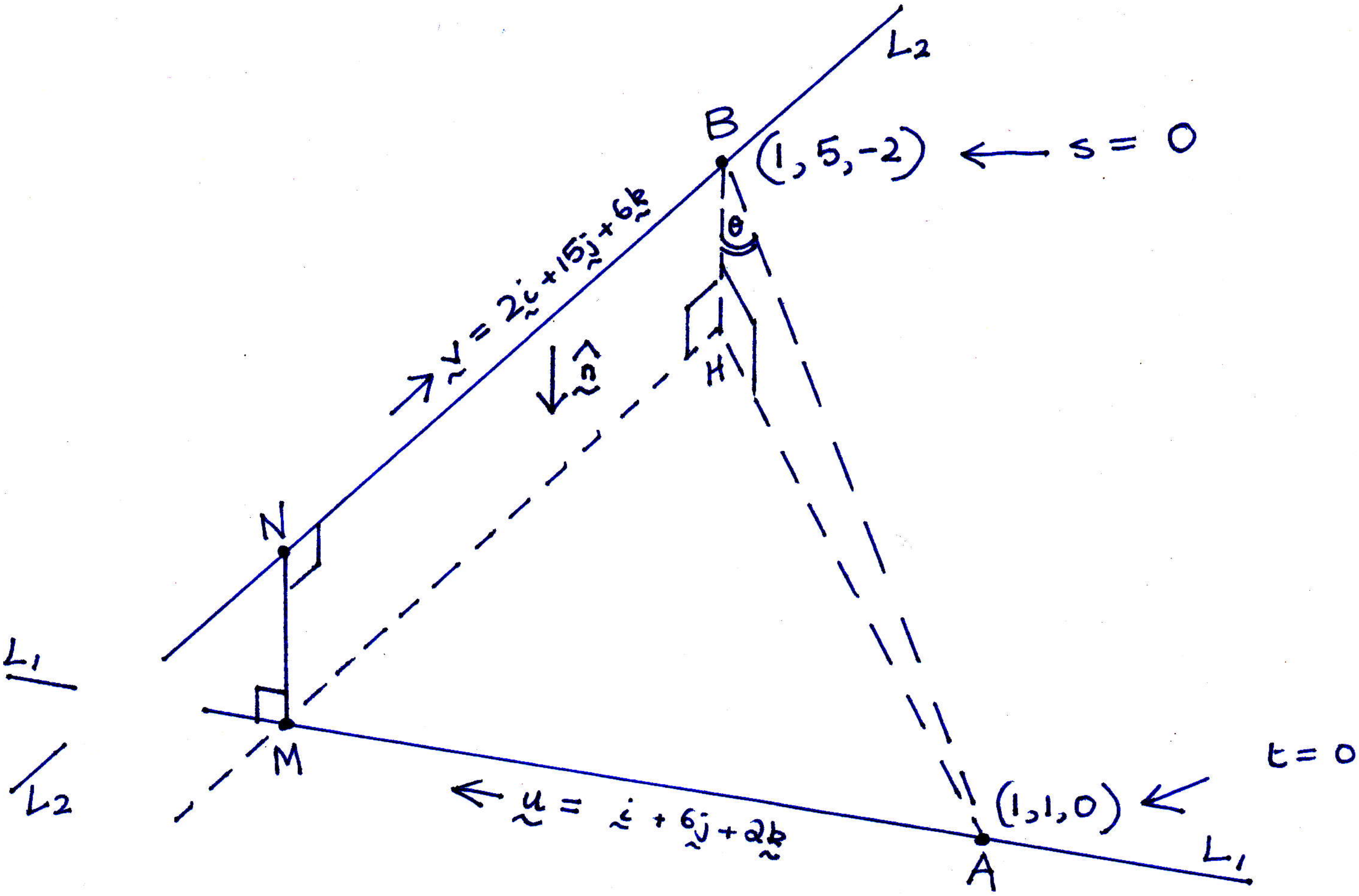

$$NM = BH = BA\cos(\theta) = \frac{BA\,BH\cos(\theta)}{BH} = \frac{\overrightarrow{BA}\cdot\overrightarrow{BH}}{BH}$$

$$= \overrightarrow{BA}\cdot\left(\frac{\overrightarrow{BH}}{BH}\right) = \overrightarrow{BA}\cdot\widehat{\overrightarrow{BH}} = \overrightarrow{BA}\cdot\hat{\mathbf{n}}$$

But $\mathbf{u}\times\mathbf{v} = 6\mathbf{i} - 2\mathbf{j} + 3\mathbf{k} \Rightarrow \hat{\mathbf{n}} = \dfrac{6\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}}{\sqrt{6^2 + (-2)^2 + 3^2}}$

$$= \frac{1}{7}\{6\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}\}$$

Also $\mathbf{a} = \mathbf{i} + \mathbf{j} + 0\mathbf{k}$, $\mathbf{b} = \mathbf{i} + 5\mathbf{j} - 2\mathbf{k} \Rightarrow \overrightarrow{BA} = \mathbf{a} - \mathbf{b}$

$$= 0\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}$$

So $NM = (0\mathbf{i} - 4\mathbf{j} + 2\mathbf{k})\cdot\dfrac{(6\mathbf{i} - 2\mathbf{j} + 3\mathbf{k})}{7} = \dfrac{0\cdot 6 + (-4)(-2) + 2\cdot 3}{7}$

$= \dfrac{14}{7} = 2 =$ distance between lines

Now $\overrightarrow{OH} = \overrightarrow{OB} + \overrightarrow{BH} = \mathbf{b} + 2\hat{\mathbf{n}}$

$$= \mathbf{i} + 5\mathbf{j} - 2\mathbf{k} + \frac{12}{7}\mathbf{i} - \frac{4}{7}\mathbf{j} + \frac{6}{7}\mathbf{k} = \frac{1}{7}\{19\mathbf{i} + 31\mathbf{j} - 8\mathbf{k}\}$$

So HM has equation

$$\underset{\sim}{\rho} = \overrightarrow{OH} + s\underset{\sim}{v} = \frac{19}{7}\underset{\sim}{i} + \frac{31}{7}\underset{\sim}{j} - \frac{8}{7}\underset{\sim}{k} + s(2\underset{\sim}{i} + 15\underset{\sim}{j} + 6\underset{\sim}{k})$$

$$= \left(\frac{19}{7} + 2s\right)\underset{\sim}{i} + \left(\frac{31}{7} + 15s\right)\underset{\sim}{j} + \left(\frac{-8}{7} + 6s\right)\underset{\sim}{k}$$

L_1 has equation $\underset{\sim}{\rho} = (1+t)\underset{\sim}{i} + (1+6t)\underset{\sim}{j} + 2t\underset{\sim}{k}$

They meet where $\frac{19}{7} + 2s = 1+t$, $\frac{31}{7} + 15s = 1+6t$, $-\frac{8}{7} + 6s = 2t$

or $s = \frac{16}{7}$, $t = \frac{44}{7}$ (check that all three equations are satisfied).

That is, they meet at M with position vector

$$\overrightarrow{OM} = \frac{51}{7}\underset{\sim}{i} + \frac{271}{7}\underset{\sim}{j} + \frac{88}{7}\underset{\sim}{k} \quad \text{(from either equation)}$$

Because L_2 has equation $\underset{\sim}{\rho} = (1+2s)\underset{\sim}{i} + (5+15s)\underset{\sim}{j} + (-2+6s)\underset{\sim}{k}$ and $BN = HM$, we can find the coordinates of N by setting $s = \frac{16}{7}$ in the equation for L_2. We obtain

$$\overrightarrow{ON} = \overrightarrow{OB} + \overrightarrow{BN} = \left(1 + 2\cdot\frac{16}{7}\right)\underset{\sim}{i} + \left(5 + 15\cdot\frac{16}{7}\right)\underset{\sim}{j} + \left(-2 + 6\cdot\frac{16}{7}\right)\underset{\sim}{k}$$

$$= \frac{39}{7}\underset{\sim}{i} + \frac{275}{7}\underset{\sim}{j} + \frac{82}{7}\underset{\sim}{k}$$

So the points of closest approach are M with coordinates $\frac{1}{7}(51, 271, 88)$ on L_1 and N with coordinates $\frac{1}{7}(39, 275, 82)$ on L_2. (By subtraction we verify that

$$\overrightarrow{NM} = \overrightarrow{OM} - \overrightarrow{ON}$$

$$= \frac{12}{7}\underset{\sim}{i} - \frac{4}{7}\underset{\sim}{j} + \frac{6}{7}\underset{\sim}{k}$$

$$= 2\left\{\frac{6}{7}\underset{\sim}{i} - \frac{2}{7}\underset{\sim}{j} + \frac{3}{7}\underset{\sim}{k}\right\} = 2\hat{n})$$