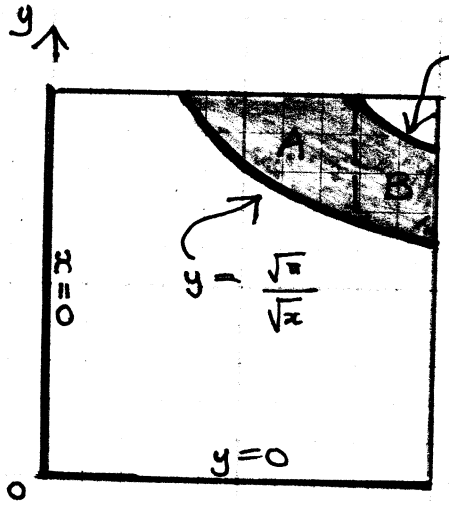
Sea level is where $z = 0 \Rightarrow \sin(xy^2) = 0$



$\Rightarrow xy^2 =$ a multiple of π

But $0 \le x \le 2$, $0 \le y \le 2$

$\Rightarrow 0 \le xy^2 \le 2.2^2 = 8$

$\Rightarrow 0 \le xy^2 < 3\pi$

So the only multiples we have to consider are 0, 1 and 2.

$xy^2 = 0 \Rightarrow x = 0$ or $y = 0$

$xy^2 = \pi \Rightarrow y = \sqrt{\pi}/\sqrt{x}$

$xy^2 = 2\pi \Rightarrow y = \sqrt{2\pi}/\sqrt{x}$

So the sea level contour is as shown (not to scale, but good enough) We are interested only in the "wet" portion of this contour. The south bank hits $y = 2$ where $x\,2^2 = \pi$ or $x = \pi/4$; the north bank hits where $x\,2^2 = 2\pi$ or $x = \pi/2$. So area of water =

area A + area B

$$= \int_{\pi/4}^{\pi/2} \left(2 - \frac{\sqrt{\pi}}{\sqrt{x}}\right) dx + \int_{\pi/2}^{2} \left(\frac{\sqrt{2\pi}}{\sqrt{x}} - \frac{\sqrt{\pi}}{\sqrt{x}}\right) dx$$

$$= 2\left(\frac{\pi}{2} - \frac{\pi}{4}\right) - \sqrt{\pi}\int_{\pi/4}^{\pi/2} x^{-1/2}\,dx + \left(\sqrt{2\pi} - \sqrt{\pi}\right)\int_{\pi/2}^{2} x^{-1/2}\,dx$$

$$= \frac{\pi}{2} - \sqrt{\pi}\; 2x^{1/2}\Big|_{\pi/4}^{\pi/2} + \left(\sqrt{2}-1\right)\sqrt{\pi}\; 2x^{1/2}\Big|_{\pi/2}^{2}$$

$$= \frac{\pi}{2} - \sqrt{\pi}\left(\sqrt{2}\sqrt{\pi} - \sqrt{\pi}\right) + \left(\sqrt{2}-1\right)\sqrt{\pi}\left(2\sqrt{2} - \sqrt{2}\sqrt{\pi}\right)$$

$$= 2\sqrt{2}\left(\sqrt{2}-1\right)\sqrt{\pi} - \pi/2 \approx 0.506$$