



Let the flow be in the positive  $x$  direction. Then the velocity of the water is  $\underline{w} = \frac{7}{2} \underline{i}$ . Let  $\underline{b}$  be the velocity of the boat relative to the water, and  $\underline{a}$  the actual velocity relative to the land. Then

$$\underline{b} + \underline{w} = \underline{a}$$

Let  $\underline{a}$  have direction cosines  $\cos \alpha$ ,  $\cos \beta$  and  $\cos \gamma$  (cosines of angles with  $x$ ,  $y$  and  $z$  axes, respectively). Then from the diagram, specifically triangle ONM, we have

$$\cos(\beta) = \frac{3}{\sqrt{2^2+3^2}}, \quad \cos(\pi-\alpha) = \frac{2}{\sqrt{2^2+3^2}}, \quad \cos(\gamma) = 0$$

$$\Rightarrow \hat{\underline{a}} = \cos \alpha \underline{i} + \cos \beta \underline{j} = \frac{-2\underline{i} + 3\underline{j}}{\sqrt{13}}$$

(because  $\cos \alpha = -\cos(\pi-\alpha)$ ). Also,  $b = |\underline{b}| = 13 \text{ (km/h)}$

$$\text{But } \underline{b} = \underline{a} - \underline{w} = a \hat{\underline{a}} - \frac{7}{2} \underline{i} = \left( \frac{-2a}{\sqrt{13}} - \frac{7}{2} \right) \underline{i} + \frac{3a}{\sqrt{13}} \underline{j}$$

$$\text{So } b = \sqrt{\left( \frac{-2a}{\sqrt{13}} - \frac{7}{2} \right)^2 + \left( \frac{3a}{\sqrt{13}} \right)^2} = 13 \Rightarrow a^2 + \frac{14a}{\sqrt{13}} - \frac{627}{4} = 0$$

$$\Rightarrow a = \frac{1}{2} \left( \sqrt{\frac{8347}{13}} - \frac{14}{\sqrt{13}} \right) \approx 10.728. \text{ So time taken is}$$

$$\frac{OM}{a} = \frac{\sqrt{13}}{a} = \frac{26}{\sqrt{8347-14}} \approx 0.33608 \text{ hr} \approx 20.165 \text{ mins}$$

$$\text{Also, if } \theta \text{ is the angle DOP then } \cos \theta = \hat{\underline{b}} \cdot \underline{i} = \frac{\underline{b} \cdot \underline{i}}{b} = \frac{\underline{b} \cdot \underline{i}}{13} = \frac{\frac{-2a}{\sqrt{13}} - \frac{7}{2}}{13} = \frac{-63 - 2\sqrt{8347}}{338} \approx -0.72699 \Rightarrow \theta \approx 136.6^\circ$$