MAC 2313, Section 12, Spring 2016 (Dr M-G)

Second Test

Tuesday, October 18, 2016

You are allowed to use a TI-30Xa (or any 4-function calculator). No other calculator is allowed. You have 75 minutes. Present your solutions clearly *in ink*. Show all necessary steps in your method. Include enough comments or diagrams to convince me that you thoroughly understand. Begin each question (as opposed to part of question) on a fresh sheet of paper, use *one* side of the paper only, and ensure that your solutions are in the proper order at the end of the test. You may assume that the Jacobian determinant is $J = \rho^2\sin(\phi)$ for a change of variables from Cartesian coordinates x, y, z to spherical polar coordinates ρ (distance from origin), ϕ (colatitude), θ (azimuth) in a triple integral and

$$J = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial y}{\partial u} \\ \frac{\partial x}{\partial v} & \frac{\partial y}{\partial v} \end{vmatrix}$$

for a general change of variables from x and y to u and v in a double integral. You may also assume the trigonometric identity $\cos(2\phi) = 1 - 2\sin^2(\phi)$.

Answer all four questions perfectly to obtain full credit.

1. A function f is defined on $(-\infty,\infty)\times(-\infty,\infty)$ by

$$f(x,y) = y^3 - 6xy + x^3.$$

(a) Find both critical points of f, and in each case determine whether the critical point is a local maximum, a local minimum or a saddle point.

(b) For $\mathbf{s} = 12\mathbf{i} - 5\mathbf{j}$, find the directional derivative $\frac{\partial f}{\partial s}$ at the point with position vector $\mathbf{r}_0 = 5\mathbf{i} - 3\mathbf{j}$. (The correct answer is an integer between 80 and 90.)

2. The exact value of

$$I_2 = \int_0^1 \int_x^{\sqrt[3]{x}} \sin(x/y)\,dy\,dx$$

is a number between $\frac{1}{7}$ and $\frac{1}{6}$. Calculate I_2 by first carefully sketching the region of integration and then using your diagram to reverse the order of integration.

3. Define

$$I_3 = \iint_T \{y - 2x\}\,dA$$

where

$$\{u(4\mathbf{i}+\mathbf{j}) + v(-\mathbf{i}+6\mathbf{j}) | 0 \le v \le u \le 1\}$$

is a triangle with vertices at $(0,0)$, $(4,1)$ and $(3,7)$.

(a) Do you expect $I_3 > 0$ or $I_3 < 0$? Justify your answer with a rough sketch.

(b) Calculate the exact value of I_3 by changing variables from x and y to u and v. (The absolute value of the correct answer is an integer between 7π and 8π.)

4. Use spherical polar coordinates to calculate the exact value of the triple integral

$$I_4 = \iiint_E \sqrt{x^2+y^2}\,dV$$

where E is the region bounded above by the sphere $x^2+y^2+z^2 = 16$ and below by the cone $z = \sqrt{x^2+y^2}$. (The correct answer satisfies $57 < I_4 < 58$.)

[Perfect score: $4 \times 10 = 40$]