MAC 2313, Section 12, Spring 2016 (Dr M-G)

First Test

Thursday, September 22, 2016

You are allowed to use a TI-30Xa (or any 4-function calculator). No other calculator is allowed. You have 75 minutes. Present your solutions clearly *in ink*. Show all necessary steps in your method. Include enough comments or diagrams to convince me that you thoroughly understand. Begin each question (as opposed to part of question) on a fresh sheet of paper, use *one* side of the paper only, and ensure that your solutions are in the proper order at the end of the test. You may assume that $\boldsymbol{\rho}_1 = x_1\mathbf{i} + y_1\mathbf{j} + z_1\mathbf{k}, \boldsymbol{\rho}_2 = x_2\mathbf{i} + y_2\mathbf{j} + z_2\mathbf{k}$ implies

$$\boldsymbol{\rho}_1 \times \boldsymbol{\rho}_2 \;=\; (y_1\,z_2 - y_2\,z_1)\,\mathbf{i} + (x_2\,z_1 - x_1\,z_2)\;\mathbf{j} + (x_1\,y_2 - y_1\,x_2)\,\mathbf{k} \;\equiv\; \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix}.$$

Also, if and where appropriate, an output from one solution may be used as an input for another. (There is no point to recalculating what you already know!)

Answer all three questions perfectly to obtain full credit.

1. Write $\mathbf{s} = -3\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$ as the sum of vectors $\mathbf{p}$ and $\mathbf{h}$, where $\mathbf{p}$ is parallel to $\mathbf{q} = -2\mathbf{i} - \mathbf{j} + 2\mathbf{k}$ and $\mathbf{h}$ is perpendicular to $\mathbf{q}$. Verify that $\mathbf{h} \cdot \mathbf{p} = 0$. [10]

2. Points A, B and C with position vectors $\mathbf{a} = \mathbf{i} + \mathbf{j} + 3\mathbf{k}$, $\mathbf{b} = 2\mathbf{i} + 3\mathbf{j} - \mathbf{k}$ and $\mathbf{c} = 3\mathbf{i} + 5\mathbf{j} + 4\mathbf{k}$, respectively, are vertices of a parallelogram whose fourth vertex is D.

- **(a)** Find the displacement of B relative to A (that is, the vector $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$).
- **(b)** Find the displacement of C relative to A.
- **(c)** Hence (that is, using vector methods) find the exact area of the parallelogram.
- **(d)** Find the angle at vertex A (that is, the angle between AB and AC). [10]

- **(e)** What is $\mathbf{d}$, the position vector of the fourth vertex, D?
- **(f)** Find an equation for the plane Π in which the parallelogram $ABCD$ lies.
- **(g)** Obtain parametric equations for the line through C and D. [10]

3. The lines L_1 and L_2 are known to be skew. L_1 has parametric equations $x = 9 + 2s, y = -1 - s, z = 4 + 2s$. L_2 has parametric equations $x = 2 + 2t, y = -3 - 3t, z = 1 - 2t$. Find the (shortest) distance between the two lines. [10]

[Perfect score: $4 \times 10 = 40$]