How to determine whether $\sum_{n=1}^{\infty} a_n$ converges or diverges.

Throughout, let f be a function satisfying $f(n) = a_n$.

Question 1: *Can* my series converge?

ANSWER: Does $\lim_{n\to\infty} a_n$ exist *and* does $\lim_{n\to\infty} a_n = 0$?

- If *no*: You're done; $\sum_{n=1}^{\infty} a_n$ diverges.
- If *yes*: Your series *may* converge. **Go to Question 2**.

Question 2: Does my series have negative terms?

- If *no*: You have a positive series. **Go to Question 3**.
- If *yes*: Go to **Question 5**.

Question 3: Is my series a geometric series or a p-series?

- If *yes*: Use the info you know about **geometric series** and/or ***p*-series** and you're done.
- If *no*: Go to **Question 4**.

Question 4: If I squint at my series, does it kinda-sorta look like a geometric series or a p-series?

- If *yes*, use either **the comparison test** or **the limit comparison test**.
- If *no*:
  - Does my series have factorials and/or $(\text{constant})^n$?

    $\Longrightarrow$ **Use the Ratio Test!**
  - Does a_n have the form $a_n = (b_n)^n$ (a whole function to the nth power)?

    $\Longrightarrow$ **Use the Root Test!**
  - Does it look like I can find $\int_1^\infty f(x)\,dx$?

    $\Longrightarrow$ **(Try to) Use the Integral Test!** (f must be continuous, positive, and decreasing!)
  - If none of the ratio, root, or integral tests seem appropriate:

    $\Longrightarrow$ Ask whatever higher power you believe in for an intervention. (If you don't have a higher power, ask a friend to borrow theirs.)

    $\Longrightarrow$ Try looking at $\sum_{n=1}^{\infty} |a_n|$ directly by going back at **Question 3**.

Question 5: Is my series alternating? (i.e., is $a_n = (-1)^n b_n$ or $a_n = (-1)^{n+1} b_n$ where $\{b_n\}$ has all positive terms?)

- If *yes*: (Try to) Use the **Alternating Series Test!** (b_n must be decreasing and $\lim_{n\to\infty} b_n = 0$ must hold)
- If *no*:
  - Does my series have factorials and/or $(\text{constant})^n$?

    $\Longrightarrow$ **Use the Ratio Test!**
  - Does a_n have the form $a_n = (b_n)^n$ (a whole function to the nth power)?

    $\Longrightarrow$ **Use the Root Test!**
  - If neither the ratio nor root test seems applicable:

    $\Longrightarrow$ See **Question 4** about borrowing higher powers, etc.

    $\Longrightarrow$ Try looking at $\sum_{n=1}^{\infty} |a_n|$ directly by going back at **Question 3**.