MAC 2313, Section 12, Fall 2016 (Dr M-G)

Final

Wednesday, December 14, 2016

You are allowed to use a TI-30Xa (or any four-function calculator). No other calculator is allowed. You have 75 minutes. Present your solutions clearly *in ink*. Show all necessary steps in your method. Include enough comments or diagrams to convince me that you thoroughly understand. Begin each question (as opposed to part of question) on a fresh sheet of paper, use *one* side of the paper only, and ensure that your solutions are in the proper order at the end of the exam.

Each question is worth 10 points. Answer all four questions perfectly to obtain full credit.

1. (a) Show that
$$\int_0^1 \int_{\sqrt{y}}^1 e^{y/x^2}\, dx\, dy = \tfrac{1}{3}(e-1)$$
by first carefully sketching the region of integration and then using your diagram to reverse the order of integration.

(b) Let D be the subset of the positive quadrant in the x-y plane that is bounded by the lines $x = 0$, $y = x$ and the circle $x^2 + y^2 = 4$. Use a suitable change of coordinates to show that
$$\iint_D \{\sqrt{x^2+y^2} + y\}\, dx\, dy = \tfrac{2}{3}(\pi + 2\sqrt{2}).$$

2. The space curve C has vector equation $\mathbf{r} = \frac{1}{3}t^3\,\mathbf{i} + \frac{1}{2}t^2\,\mathbf{j} + t\,\mathbf{k}$, $0 \le t \le 2$.

(a) Find (i) the unit tangent vector $\mathbf{T}$, (ii) the principal unit normal vector $\mathbf{N}$, (iii) the curvature κ and (iv) the binormal unit vector $\mathbf{B}$ for C at the point $(\frac{1}{3}, \frac{1}{2}, 1)$.

(b) Show that the work $\int_C \mathbf{F}\cdot d\mathbf{r}$ done by the force $\mathbf{F} = y\,\mathbf{i} + 6x\,\mathbf{j} + 2z\,\mathbf{k}$ in moving a particle along C from $(0,0,0)$ to $(\frac{8}{3}, 2, 2)$ is 20.

3. Let C_1, C_2 and C_3 denote the straight-line segments from $(1,0,1)$ to $(3,-1,2)$, from $(3,-1,2)$ to $(5,1,3)$ and from $(5,1,3)$ to $(1,0,1)$, respectively; let $C = C_1 \cup C_2 \cup C_3$ be the resulting triangle; let S be the planar triangular surface that C encloses; and let $\mathbf{F} = z^2\,\mathbf{i} + x\,\mathbf{j} - 4y\,\mathbf{k}$.

(a) Parameterize S, and hence use Stokes' theorem to show that $\oint_C \mathbf{F}\cdot d\mathbf{r} = 9$.

(b) What is the area of S?

4. Let the vector field $\mathbf{F}$ be defined by
$$\mathbf{F} = y^2\mathbf{i} + z^2\mathbf{j} + x^2\mathbf{k}.$$
Let C be the ellipse on which the plane Π with equation $x + y + z = 5$ intersects the the circular cylinder with equation $x^2 + y^2 = 9$. Let S be the elliptical disk that lies in Π, is bounded by C and has positive upward orientation.

(a) Show that the upward flux of $\mathbf{F}$ through S is given by
$$\iint_S \mathbf{F}\cdot d\mathbf{S} = 306\pi.$$

(b) Show that the circulation of $\mathbf{F}$ around C, in the anticlockwise direction when viewed from above, is given by
$$\oint_C \mathbf{F}\cdot d\mathbf{r} = -90\pi.$$