Name: (10 pts)

(please print neatly!)

Directions: Answer each of the following two (2) questions, making sure to read the instructions for each question as you proceed.

You may use the backs of the pages for scratch work or get scrap paper from me!

The last page is a bonus problem! Make sure you don't overlook it!

1. (20 pts) Solve the IVP

$$y'' + 2y' + y = 3e^{-x}, \quad y(0) = 1, \quad y'(0) = 1.$$

Hint: $(x^2e^{kx})' = kx^2e^{kx} + 2xe^{kx}$ and $(x^2e^{kx})'' = k^2x^2e^{kx} + 4kxe^{kx} + 2e^{kx}$ for all constants k.

SOLUTION: Hom! y"+24'+y=0 => (2+2(+1) => (r+1)(r+1)=0 => r=-1, r=-1 Ly cle-x + Coxe-x

Non-Hom: Y(x) = Ax2e-x ~> Y'= -Ax2e-x + ZAxe-x Y" = Ax2e x = 2Axe x = 2Axe + 2Ae

=> Y"+2V+Y= Ax2e-x-4Axe-x+2Ae-x + - 2Ax2e + 4Axe x + Ax2e x

IVP: • $y(0)=1 \Rightarrow C_1 = 1$. $-y' = -c_1e^{-x} + c_2e^{-x} + c_2e^{-x} + 3xe^{-x} = 1 = y'(0) = -c_1 + c_2$ $= > 1 + c_1 = c_2 = 2$

Ans: $y = e^{-x} + 2xe^{-x} + \frac{3}{2}x^2e^{-x}$

2.(a) (6 pts) True or False: $y_1(t) = e^{-2t}$ and $y_2 = e^{3t}$ are both solutions to the second-order homogeneous

Justify your claim!
$$y'' - y' - 6y = 0.$$

SOLUTION:

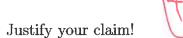
$$y'_1 - y'_1 - 6y_1 = 4e^{-2t} + 2e^{-2t} - 6e^{-2t}$$

 $= 0$

LUTION:

$$y_1' - y_1 - 6y_1 = 4e^{-2t} + 2e^{-2t} - 6e^{-2t}$$
 $\left(y_2'' - y_2' - 6y_2 = 9e^{-3t} - 3e^{-3t} - 6e^{-3t}\right)$
 $= 0$

(b) (4 pts) True or False: $y_1(t) = e^{-2t}$ and $y_2 = e^{3t}$ form a fundamental system of solutions for the second-order homogeneous ODE



SOLUTION:

iomogeneous ODE
$$TRUE \qquad y'' - y' - 6y = 0.$$

$$\underline{aim!}$$

$$W(e^{-2t}, e^{3t}) = det\begin{pmatrix} e^{-2t} & e^{3t} \\ -2e^{-2t} & 3e^{3t} \end{pmatrix}$$

(c) (10 pts) Without using undetermined coefficients, use parts (a) and (b) to find a particular solution of the non-homogeneous ODE

$$y'' - y' - 6y = e^{-3t}.$$

$$y_2 = e^{3t}$$

Simplify fully.

Solution:
$$g(t) = e^{-3t}$$
.

Simplify fully.

 $y_1 = e^{-2t}$
 $y_2 = e^{-3t}$
 $y_2 = e^{-3t}$
 $y_2 = e^{-3t}$
 $y_1 = e^{-2t}$
 $y_2 = e^{-3t}$
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 $y_2 = e^{-3t}$
 $y_2 = e^{-3t}$
 $y_1 = e^{-3t}$
 $y_2 = e^{-3t}$

$$= -e^{-2t} \left(\frac{e^{3t} \left(e^{-3t} \right)}{5e^{t}} dt + e^{3t} \left(\frac{e^{-2t} \left(e^{-5t} \right)}{5e^{t}} dt \right) \right)$$

$$e^{3t}$$
 $\int \frac{e^{-2t}(e^{-3t})}{5e^{t}} dt$

$$= -\frac{e^{-2t}}{5} \int e^{-t} dt + \frac{e^{3t}}{5} \int e^{-6t} dt$$

$$= \frac{e^{-3t}}{5} = \frac{1}{30} e^{-3t}$$

$$= \frac{5}{30}e^{-3t} = \frac{1}{6}e^{-3t}$$

Bonus: (5 pts) Use the method of undetermined coefficients to check the answer you got for problem 2(c). In other words: Use the method of undetermined coefficients to find a particular solution of the non-homogeneous ODE

 $y'' - y' - 6y = e^{-3t}.$

Hint: When using variation of parameters, some of the terms produced in the formula for Y(t) may have the form $(const)y_1$ or $(const)y_2$; to get that answer to match the one obtained using undetermined coefficients, you want to ignore any such terms! (make sure you know why!)

SOLUTION:

$$Y(t) = Ae^{-3t} Y'(t) = -3Ae^{-3t} Y''(t) = 9Ae^{-3t}$$

$$e^{-3t} = Y'' - Y' - 6Y$$

$$= 9Ae^{-3t} + 3Ae^{-3t} - 6Ae^{-3t}$$

$$= 6Ae^{-3t}$$

$$= 6A$$

$$\Rightarrow A = \frac{1}{6}$$
So $Y = \frac{1}{6}e^{-3t}$, as before.