# Softmax Implementation for Linear Classifier

This project will teach you how to implement softmax activation and use PyTorch's automatic differentiation for training a linear classifier. You will:

1. **Implement softmax activation function** for multi-class classification with numerical stability
2. **Use PyTorch's automatic differentiation** instead of manual gradient computation
3. **Train a linear classifier** using default hyperparameters (lr=0.001, batch_size=32)
4. **Test the trained classifier on MNIST dataset** and achieve >90% accuracy
5. **Load pre-saved weights** and compare with the original linear classifier from 01_linear_feature.ipynb

**Important**: This notebook focuses on understanding softmax implementation and PyTorch's automatic differentiation. We will use PyTorch's cross-entropy loss function and optimizer for training.

***NOTE:*** When filling in the code, please REMOVE the `pass` statement. DO NOT remove the TODO coding highlight in your submission.

**Grading Criteria**:

- Test accuracy must be >90% (60% points)
- Code quality and implementation (10% points)
- Analysis and answers to questions (30% points)
- **Total: 5 points (50% of project score)**

In [ ]:

```
# # Google Colab Setup (Comment out for local computer running)
############################################################################
# Uncomment and set the path to your project folder in Google Drive
############################################################################
# from google.colab import drive
# drive.mount('/content/drive')

# FOLDERNAME = 'cpsc8430/assignments/project1/'
# assert FOLDERNAME is not None, "[!] Enter the foldername."

# # Now that we've mounted your Drive, this ensures that
# # the Python interpreter of the Colab VM can load
# # python files from within it.

# import sys
# sys.path.append('/content/drive/My Drive/{}'.format(FOLDERNAME))
# %cd /content/drive/My\ Drive/$FOLDERNAME
```

In [1]:

```
import torch
import torch.nn as nn
```

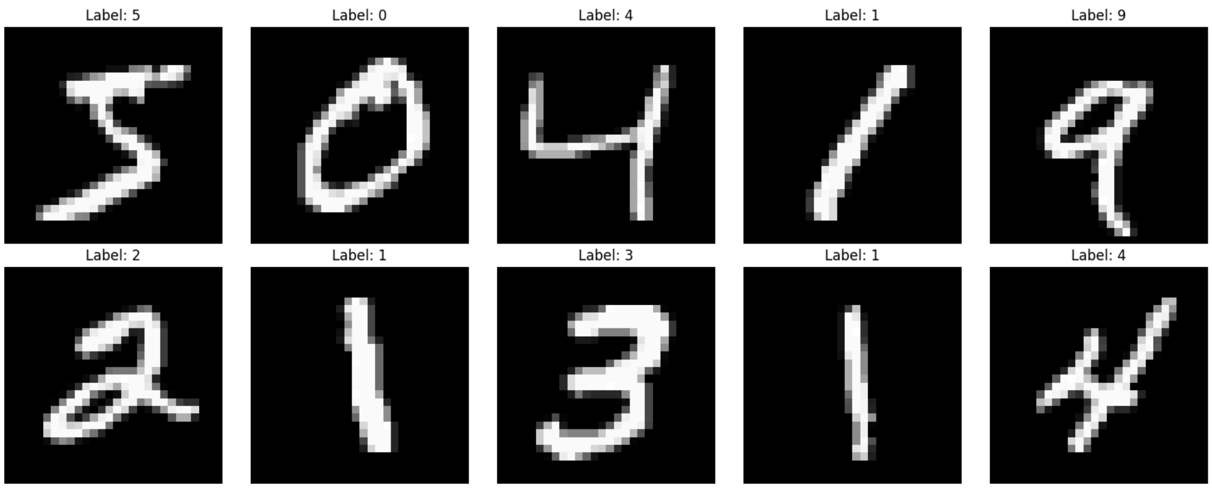
```
import torchvision
import matplotlib.pyplot as plt
import numpy as np
from PIL import Image
import os

# Set random seed for reproducibility
torch.manual_seed(42)
np.random.seed(42)
```

# Part 1: Load and Preprocess MNIST Dataset

First, let's load the MNIST dataset and prepare it for training.

In [2]:
```
# Load MNIST dataset
transform = torchvision.transforms.Compose([
    torchvision.transforms.ToTensor(),
    torchvision.transforms.Normalize((0.1307,), (0.3081,))  # MNIST mean and
])

train_dataset = torchvision.datasets.MNIST(
    root='cpsc8430/datasets/MNIST',
    train=True,
    download=True,
    transform=transform
)

test_dataset = torchvision.datasets.MNIST(
    root='cpsc8430/datasets/MNIST',
    train=False,
    download=True,
    transform=transform
)

print(f"Training set size: {len(train_dataset)}")
print(f"Test set size: {len(test_dataset)}")

# Visualize some training examples
plt.figure(figsize=(15, 6))
for i in range(10):
    plt.subplot(2, 5, i + 1)
    plt.imshow(train_dataset[i][0].squeeze(), cmap='gray')
    plt.title(f'Label: {train_dataset[i][1]}')
    plt.axis('off')
plt.tight_layout()
plt.show()
```

```
Training set size: 60000
Test set size: 10000
```




## Part 2: Implement Softmax Function

**TODO: Implement the softmax function from scratch**

The softmax function converts raw scores (logits) into probabilities:

$$\text{softmax}(x_i) = \frac{e^{x_i}}{\sum_{j=1}^{K} e^{x_j}}$$

**Important Hint for Numerical Stability**: To avoid numerical overflow when computing exponentials, subtract the maximum value from each input before applying exp:

$$\text{softmax}(x_i) = \frac{e^{x_i - \max_j x_j}}{\sum_{j=1}^{K} e^{x_j - \max_j x_j}}$$

This ensures that the largest exponent is 0, preventing overflow while maintaining the same mathematical result.

**Requirements**:

- Handle numerical stability (subtract max value before exp)
- Return probabilities that sum to 1
- Work with both 1D and 2D inputs

In [4]:
```
def softmax(x):
    """
    Compute softmax probabilities for input scores

    Args:
        x: Input scores tensor of shape (batch_size, num_classes) or (num_cl

    Returns:
        Softmax probabilities tensor of same shape
    """
    ##########################################################################
```

```
    # TODO: Implement softmax function
    # Hint: Use torch.exp() and torch.sum()
    # Hint: For numerical stability, subtract the max value before computing
    # Hint: The equation is: softmax(x_i) = exp(x_i - max_j x_j) / sum_j exp
    ############################################################################
    # *****START OF YOUR CODE (DO NOT DELETE/MODIFY THIS LINE)*****
    max_j = torch.max(x) # Get the max_j(x_j)
    exp_x = torch.exp(torch.add(x, (-1)*max_j))
    sum_exp_x = torch.sum(exp_x)

    return torch.div(exp_x, sum_exp_x)
    # *****END OF YOUR CODE (DO NOT DELETE/MODIFY THIS LINE)*****
    ############################################################################
    #                              END OF YOUR CODE
    ############################################################################


# Test your softmax implementation
test_scores = torch.tensor([[1.0, 2.0, 3.0], [4.0, 5.0, 6.0]])
test_probs = softmax(test_scores)
print(f"Test scores:\n{test_scores}")
print(f"Softmax probabilities:\n{test_probs}")
print(f"Probabilities sum to 1: {torch.allclose(torch.sum(test_probs, dim=1)

# Test 1D input as well
test_scores_1d = torch.tensor([1.0, 2.0, 3.0])
test_probs_1d = softmax(test_scores_1d)
print(f"\n1D test scores: {test_scores_1d}")
print(f"1D softmax probabilities: {test_probs_1d}")
print(f"1D probabilities sum to 1: {torch.allclose(torch.sum(test_probs_1d),
```

```
Test scores:
tensor([[1., 2., 3.],
        [4., 5., 6.]])
Softmax probabilities:
tensor([[0.0043, 0.0116, 0.0315],
        [0.0858, 0.2331, 0.6337]])
Probabilities sum to 1: False

1D test scores: tensor([1., 2., 3.])
1D softmax probabilities: tensor([0.0900, 0.2447, 0.6652])
1D probabilities sum to 1: True
```

# Part 3: Implement Linear Classifier Forward Pass

**TODO: Implement the forward pass of the linear classifier**

The forward pass computes: $f(x) = \text{softmax}(xW^T + b)$

In [13]:
```
def linear_classifier_forward(x, weights, bias):
    """
    Forward pass of linear classifier (returns logits, not probabilities)

    Args:
        x: Input features, shape (batch_size, input_features)
```

```
        weights: Weight matrix, shape (num_classes, input_features)
        bias: Bias vector, shape (num_classes,)

    Returns:
        Logits (raw output class scores), shape (batch_size, num_classes)
    """
    ##########################################################################
    # TODO: Implement forward pass
    # Hint: Compute linear transformation: x * weights^T + bias
    # Return the logits (do NOT apply softmax here)
    ##########################################################################
    # *****START OF YOUR CODE (DO NOT DELETE/MODIFY THIS LINE)*****
    return torch.add(torch.matmul(x, torch.transpose(weights, -2, 1)), bias)
    # *****END OF YOUR CODE (DO NOT DELETE/MODIFY THIS LINE)*****
    ##########################################################################
    #                               END OF YOUR CODE
    ##########################################################################




# Test your forward pass implementation
batch_size = 4
input_features = 784
num_classes = 10

test_x = torch.randn(batch_size, input_features)
test_weights = torch.randn(num_classes, input_features)
test_bias = torch.randn(num_classes)

test_output = linear_classifier_forward(test_x, test_weights, test_bias)
print(f"Input shape: {test_x.shape}")
print(f"Weights shape: {test_weights.shape}")
print(f"Bias shape: {test_bias.shape}")
print(f"Output shape: {test_output.shape}")
print(f"Output probabilities sum to 1: {torch.allclose(torch.sum(test_output
```

```
Input shape: torch.Size([4, 784])
Weights shape: torch.Size([10, 784])
Bias shape: torch.Size([10])
Output shape: torch.Size([4, 10])
Output probabilities sum to 1: False
```

## Part 4: Train Linear Classifier Using PyTorch

**TODO: Train the linear classifier using PyTorch's automatic differentiation**

Instead of implementing gradient descent manually, we'll use PyTorch's built-in optimizer and automatic differentiation. This will use the default learning rate of 0.001 and batch size of 32.

In [30]:
```
def train_linear_classifier(train_loader, num_epochs=10):
    """
    Train linear classifier using PyTorch's automatic differentiation
```

```
    Args:
        train_loader: DataLoader for training data
        num_epochs: Number of training epochs (default: 10)

    Returns:
        Trained weights, bias, and training history
    """
    # Initialize parameters
    input_features = 784  # 28x28 flattened
    num_classes = 10

    weights = None
    bias = None

    # Training history
    train_losses = []
    train_accuracies = []

    criterion = None
    optimizer = None


    ########################################################################
    # TODO: Initialize weights and bias, set requires_grad=True, and set up
    # Hint:
    #   - Use torch.randn() or torch.zeros() to initialize weights and bias
    #   - Use weights.requires_grad_(True) and bias.requires_grad_(True)
    #   - Use nn.CrossEntropyLoss() and torch.optim.SGD() with lr=0.001
    ########################################################################
    # *****START OF YOUR CODE (DO NOT DELETE/MODIFY THIS LINE)*****
    weights = torch.rand(num_classes, input_features).requires_grad_(True)
    bias = torch.rand(num_classes).requires_grad_(True)
    criterion = nn.CrossEntropyLoss()
    optimizer = torch.optim.SGD([weights, bias], lr=0.001, momentum=.9)

    # *****END OF YOUR CODE (DO NOT DELETE/MODIFY THIS LINE)*****
    ########################################################################
    #                               END OF YOUR CODE
    ########################################################################


    print(f"Starting training with {num_epochs} epochs, lr=0.001, batch_size

    for epoch in range(num_epochs):
        epoch_loss = 0.0
        correct = 0
        total = 0

        for batch_idx, (data, targets) in enumerate(train_loader):
            # Flatten input data
            data = data.view(-1, input_features)

            ####################################################################
            # TODO: Forward pass, compute loss, and perform backward pass ar
            # Hint:
```

```
            #   - Use your linear_classifier_forward function for the forwar
            #   - CrossEntropyLoss expects logits, not probabilities (comput
            #   - optimizer.zero_grad(), loss.backward(), optimizer.step()
            ################################################################
            # *****START OF YOUR CODE (DO NOT DELETE/MODIFY THIS LINE)*****

            predictions = linear_classifier_forward(data, weights, bias)
            loss = criterion(predictions, targets)
            optimizer.zero_grad()
            loss.backward()
            optimizer.step()

            # *****END OF YOUR CODE (DO NOT DELETE/MODIFY THIS LINE)*****
            ################################################################
            #                               END OF YOUR CODE
            ################################################################

            # Compute accuracy
            _, predicted = torch.max(predictions, 1)
            total += targets.size(0)
            correct += (predicted == targets).sum().item()
            epoch_loss += loss.item()

            if batch_idx % 100 == 0:
                print(f"Epoch {epoch+1}/{num_epochs}, Batch {batch_idx}, Los

        # Compute epoch statistics
        avg_loss = epoch_loss / len(train_loader)
        accuracy = 100 * correct / total

        train_losses.append(avg_loss)
        train_accuracies.append(accuracy)

        print(f"Epoch {epoch+1}/{num_epochs} - Loss: {avg_loss:.4f}, Accurac

    return weights, bias, train_losses, train_accuracies

# Create data loader with default batch size of 32
train_loader = torch.utils.data.DataLoader(train_dataset, batch_size=32, shu

# Test training function
print("Testing training function...")
weights, bias, losses, accuracies = train_linear_classifier(train_loader)
```

```
Testing training function...
Starting training with 10 epochs, lr=0.001, batch_size=32
Epoch 1/10, Batch 0, Loss: 12.4415
Epoch 1/10, Batch 100, Loss: 3.2843
Epoch 1/10, Batch 200, Loss: 0.9501
Epoch 1/10, Batch 300, Loss: 3.1565
Epoch 1/10, Batch 400, Loss: 1.7861
Epoch 1/10, Batch 500, Loss: 0.9610
Epoch 1/10, Batch 600, Loss: 0.4730
Epoch 1/10, Batch 700, Loss: 0.4359
Epoch 1/10, Batch 800, Loss: 0.5850
Epoch 1/10, Batch 900, Loss: 1.3495
Epoch 1/10, Batch 1000, Loss: 0.3172
Epoch 1/10, Batch 1100, Loss: 1.8061
Epoch 1/10, Batch 1200, Loss: 0.9012
Epoch 1/10, Batch 1300, Loss: 1.0479
Epoch 1/10, Batch 1400, Loss: 0.8743
Epoch 1/10, Batch 1500, Loss: 0.9093
Epoch 1/10, Batch 1600, Loss: 0.6413
Epoch 1/10, Batch 1700, Loss: 0.3850
Epoch 1/10, Batch 1800, Loss: 1.0981
Epoch 1/10 - Loss: 1.4792, Accuracy: 74.69%
Epoch 2/10, Batch 0, Loss: 0.8854
Epoch 2/10, Batch 100, Loss: 0.6879
Epoch 2/10, Batch 200, Loss: 0.2145
Epoch 2/10, Batch 300, Loss: 0.6000
Epoch 2/10, Batch 400, Loss: 0.5532
Epoch 2/10, Batch 500, Loss: 0.6550
Epoch 2/10, Batch 600, Loss: 0.6981
Epoch 2/10, Batch 700, Loss: 0.6675
Epoch 2/10, Batch 800, Loss: 0.2066
Epoch 2/10, Batch 900, Loss: 0.3138
Epoch 2/10, Batch 1000, Loss: 0.4688
Epoch 2/10, Batch 1100, Loss: 0.2440
Epoch 2/10, Batch 1200, Loss: 0.4842
Epoch 2/10, Batch 1300, Loss: 0.9978
Epoch 2/10, Batch 1400, Loss: 0.7010
Epoch 2/10, Batch 1500, Loss: 0.2916
Epoch 2/10, Batch 1600, Loss: 0.4465
Epoch 2/10, Batch 1700, Loss: 0.3272
Epoch 2/10, Batch 1800, Loss: 0.4109
Epoch 2/10 - Loss: 0.6394, Accuracy: 85.61%
Epoch 3/10, Batch 0, Loss: 0.2634
Epoch 3/10, Batch 100, Loss: 0.8189
Epoch 3/10, Batch 200, Loss: 0.4159
Epoch 3/10, Batch 300, Loss: 0.1367
Epoch 3/10, Batch 400, Loss: 0.6195
Epoch 3/10, Batch 500, Loss: 0.4370
Epoch 3/10, Batch 600, Loss: 0.3597
Epoch 3/10, Batch 700, Loss: 0.2153
Epoch 3/10, Batch 800, Loss: 0.3851
Epoch 3/10, Batch 900, Loss: 0.0634
Epoch 3/10, Batch 1000, Loss: 0.7177
Epoch 3/10, Batch 1100, Loss: 0.1801
Epoch 3/10, Batch 1200, Loss: 0.2924
Epoch 3/10, Batch 1300, Loss: 0.0322
```

```
Epoch 3/10, Batch 1400, Loss: 0.7142
Epoch 3/10, Batch 1500, Loss: 0.7064
Epoch 3/10, Batch 1600, Loss: 0.3128
Epoch 3/10, Batch 1700, Loss: 0.5808
Epoch 3/10, Batch 1800, Loss: 0.1392
Epoch 3/10 - Loss: 0.5311, Accuracy: 87.50%
Epoch 4/10, Batch 0, Loss: 0.5415
Epoch 4/10, Batch 100, Loss: 0.4175
Epoch 4/10, Batch 200, Loss: 0.2160
Epoch 4/10, Batch 300, Loss: 0.1849
Epoch 4/10, Batch 400, Loss: 0.3158
Epoch 4/10, Batch 500, Loss: 0.5108
Epoch 4/10, Batch 600, Loss: 0.1180
Epoch 4/10, Batch 700, Loss: 0.3329
Epoch 4/10, Batch 800, Loss: 0.2560
Epoch 4/10, Batch 900, Loss: 0.4034
Epoch 4/10, Batch 1000, Loss: 1.0943
Epoch 4/10, Batch 1100, Loss: 0.3325
Epoch 4/10, Batch 1200, Loss: 0.2246
Epoch 4/10, Batch 1300, Loss: 1.3358
Epoch 4/10, Batch 1400, Loss: 0.6801
Epoch 4/10, Batch 1500, Loss: 0.3333
Epoch 4/10, Batch 1600, Loss: 0.4160
Epoch 4/10, Batch 1700, Loss: 0.5832
Epoch 4/10, Batch 1800, Loss: 0.3350
Epoch 4/10 - Loss: 0.4765, Accuracy: 88.48%
Epoch 5/10, Batch 0, Loss: 0.5614
Epoch 5/10, Batch 100, Loss: 0.2980
Epoch 5/10, Batch 200, Loss: 0.5536
Epoch 5/10, Batch 300, Loss: 0.2882
Epoch 5/10, Batch 400, Loss: 0.4325
Epoch 5/10, Batch 500, Loss: 0.2940
Epoch 5/10, Batch 600, Loss: 0.2090
Epoch 5/10, Batch 700, Loss: 0.4702
Epoch 5/10, Batch 800, Loss: 0.3907
Epoch 5/10, Batch 900, Loss: 1.4988
Epoch 5/10, Batch 1000, Loss: 0.4017
Epoch 5/10, Batch 1100, Loss: 0.6380
Epoch 5/10, Batch 1200, Loss: 0.4785
Epoch 5/10, Batch 1300, Loss: 0.9969
Epoch 5/10, Batch 1400, Loss: 1.1298
Epoch 5/10, Batch 1500, Loss: 0.1826
Epoch 5/10, Batch 1600, Loss: 0.1854
Epoch 5/10, Batch 1700, Loss: 0.1882
Epoch 5/10, Batch 1800, Loss: 1.1886
Epoch 5/10 - Loss: 0.4411, Accuracy: 89.15%
Epoch 6/10, Batch 0, Loss: 0.6518
Epoch 6/10, Batch 100, Loss: 0.7630
Epoch 6/10, Batch 200, Loss: 0.6802
Epoch 6/10, Batch 300, Loss: 0.3795
Epoch 6/10, Batch 400, Loss: 0.2161
Epoch 6/10, Batch 500, Loss: 0.5714
Epoch 6/10, Batch 600, Loss: 0.2090
Epoch 6/10, Batch 700, Loss: 0.3637
Epoch 6/10, Batch 800, Loss: 0.1484
Epoch 6/10, Batch 900, Loss: 0.5043
```

```
Epoch 6/10, Batch 1000, Loss: 0.0679
Epoch 6/10, Batch 1100, Loss: 0.1424
Epoch 6/10, Batch 1200, Loss: 0.0786
Epoch 6/10, Batch 1300, Loss: 0.2907
Epoch 6/10, Batch 1400, Loss: 0.1984
Epoch 6/10, Batch 1500, Loss: 0.4914
Epoch 6/10, Batch 1600, Loss: 0.2324
Epoch 6/10, Batch 1700, Loss: 0.8605
Epoch 6/10, Batch 1800, Loss: 0.5752
Epoch 6/10 - Loss: 0.4163, Accuracy: 89.59%
Epoch 7/10, Batch 0, Loss: 0.1943
Epoch 7/10, Batch 100, Loss: 0.0553
Epoch 7/10, Batch 200, Loss: 0.2709
Epoch 7/10, Batch 300, Loss: 0.2796
Epoch 7/10, Batch 400, Loss: 0.6274
Epoch 7/10, Batch 500, Loss: 0.2606
Epoch 7/10, Batch 600, Loss: 0.0965
Epoch 7/10, Batch 700, Loss: 0.3887
Epoch 7/10, Batch 800, Loss: 0.2041
Epoch 7/10, Batch 900, Loss: 0.1280
Epoch 7/10, Batch 1000, Loss: 0.2686
Epoch 7/10, Batch 1100, Loss: 0.1036
Epoch 7/10, Batch 1200, Loss: 0.2804
Epoch 7/10, Batch 1300, Loss: 0.4824
Epoch 7/10, Batch 1400, Loss: 0.4010
Epoch 7/10, Batch 1500, Loss: 0.1711
Epoch 7/10, Batch 1600, Loss: 0.0581
Epoch 7/10, Batch 1700, Loss: 1.1092
Epoch 7/10, Batch 1800, Loss: 0.0930
Epoch 7/10 - Loss: 0.3972, Accuracy: 89.96%
Epoch 8/10, Batch 0, Loss: 1.1755
Epoch 8/10, Batch 100, Loss: 0.6855
Epoch 8/10, Batch 200, Loss: 0.2987
Epoch 8/10, Batch 300, Loss: 0.0648
Epoch 8/10, Batch 400, Loss: 0.4980
Epoch 8/10, Batch 500, Loss: 0.7719
Epoch 8/10, Batch 600, Loss: 0.5594
Epoch 8/10, Batch 700, Loss: 0.2720
Epoch 8/10, Batch 800, Loss: 0.2420
Epoch 8/10, Batch 900, Loss: 0.1266
Epoch 8/10, Batch 1000, Loss: 0.3864
Epoch 8/10, Batch 1100, Loss: 0.4126
Epoch 8/10, Batch 1200, Loss: 0.5604
Epoch 8/10, Batch 1300, Loss: 0.2868
Epoch 8/10, Batch 1400, Loss: 0.8295
Epoch 8/10, Batch 1500, Loss: 0.4256
Epoch 8/10, Batch 1600, Loss: 0.7177
Epoch 8/10, Batch 1700, Loss: 0.1713
Epoch 8/10, Batch 1800, Loss: 0.1773
Epoch 8/10 - Loss: 0.3821, Accuracy: 90.17%
Epoch 9/10, Batch 0, Loss: 0.4058
Epoch 9/10, Batch 100, Loss: 0.1199
Epoch 9/10, Batch 200, Loss: 0.4132
Epoch 9/10, Batch 300, Loss: 0.5403
Epoch 9/10, Batch 400, Loss: 0.3441
Epoch 9/10, Batch 500, Loss: 0.3502
```

```
Epoch 9/10, Batch 600, Loss: 0.8906
Epoch 9/10, Batch 700, Loss: 1.2111
Epoch 9/10, Batch 800, Loss: 0.2376
Epoch 9/10, Batch 900, Loss: 0.6784
Epoch 9/10, Batch 1000, Loss: 0.2072
Epoch 9/10, Batch 1100, Loss: 0.6299
Epoch 9/10, Batch 1200, Loss: 0.1653
Epoch 9/10, Batch 1300, Loss: 0.2968
Epoch 9/10, Batch 1400, Loss: 0.1013
Epoch 9/10, Batch 1500, Loss: 0.2695
Epoch 9/10, Batch 1600, Loss: 0.5796
Epoch 9/10, Batch 1700, Loss: 0.1824
Epoch 9/10, Batch 1800, Loss: 0.1072
Epoch 9/10 - Loss: 0.3693, Accuracy: 90.43%
Epoch 10/10, Batch 0, Loss: 0.4871
Epoch 10/10, Batch 100, Loss: 0.5844
Epoch 10/10, Batch 200, Loss: 0.7069
Epoch 10/10, Batch 300, Loss: 0.1279
Epoch 10/10, Batch 400, Loss: 0.5715
Epoch 10/10, Batch 500, Loss: 0.0900
Epoch 10/10, Batch 600, Loss: 0.2152
Epoch 10/10, Batch 700, Loss: 0.4432
Epoch 10/10, Batch 800, Loss: 1.1588
Epoch 10/10, Batch 900, Loss: 0.0873
Epoch 10/10, Batch 1000, Loss: 0.1554
Epoch 10/10, Batch 1100, Loss: 0.1839
Epoch 10/10, Batch 1200, Loss: 0.0611
Epoch 10/10, Batch 1300, Loss: 0.2619
Epoch 10/10, Batch 1400, Loss: 0.4704
Epoch 10/10, Batch 1500, Loss: 0.4738
Epoch 10/10, Batch 1600, Loss: 0.8374
Epoch 10/10, Batch 1700, Loss: 0.4826
Epoch 10/10, Batch 1800, Loss: 0.3284
Epoch 10/10 - Loss: 0.3590, Accuracy: 90.68%
```

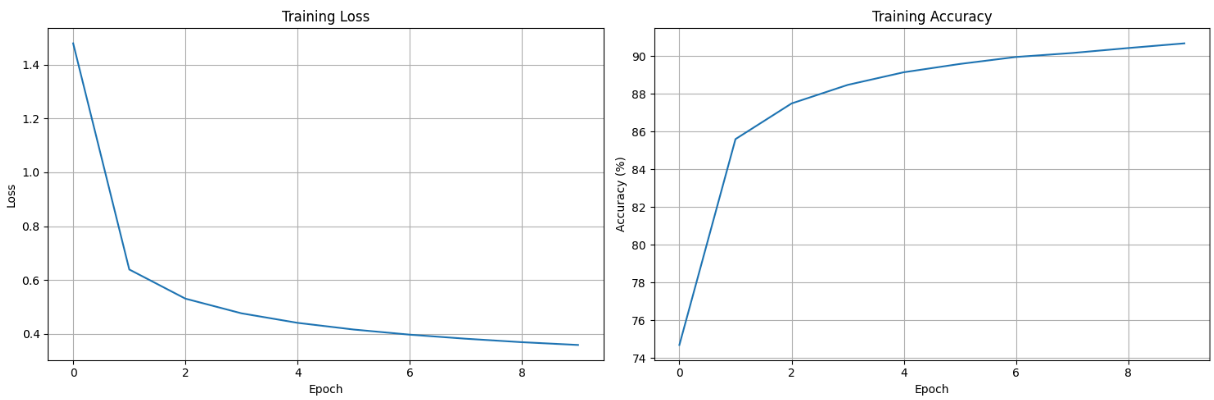
## Part 5: Visualize Training Progress

Plot the training curves to see how the model learns.

In [31]:

```python
# Plot training curves
plt.figure(figsize=(15, 5))

# Plot training loss
plt.subplot(1, 2, 1)
plt.plot(losses)
plt.title('Training Loss')
plt.xlabel('Epoch')
plt.ylabel('Loss')
plt.grid(True)

# Plot training accuracy
plt.subplot(1, 2, 2)
plt.plot(accuracies)
plt.title('Training Accuracy')
```

```
plt.xlabel('Epoch')
plt.ylabel('Accuracy (%)')
plt.grid(True)

plt.tight_layout()
plt.show()

print(f"Final training accuracy: {accuracies[-1]:.2f}%")
```




```
Final training accuracy: 90.68%
```

# Part 6: Visualize Learned Templates

**NEW SECTION: Visualize the learned weight templates for each digit class**

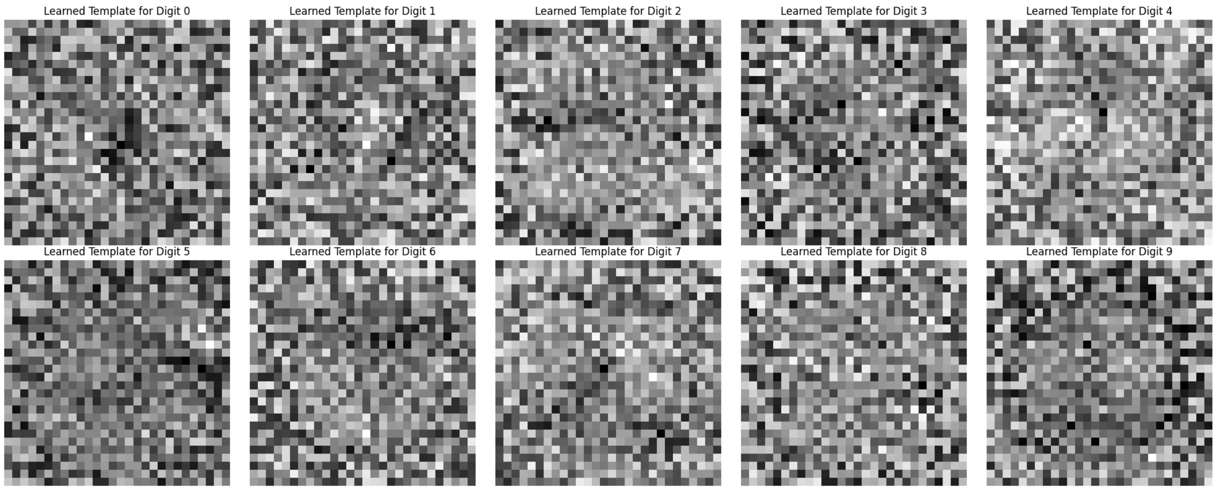
The weights of a linear classifier can be interpreted as learned templates for each class. Let's visualize what the model has learned.

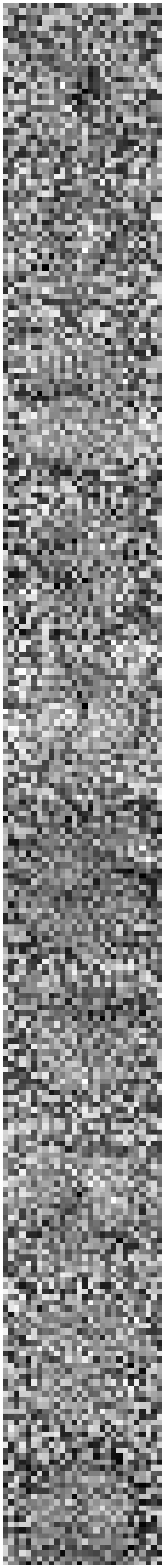
In [32]:
```
# Visualize learned templates
from PIL import Image

def visualize_learned_templates(weights, save_path='all_weights_combined_lea
    """
    Visualize the learned weight templates for each digit class

    Args:
        weights: Trained weight matrix, shape (num_classes, input_features)
        save_path: Path to save the combined visualization
    """
    # Reshape weights to 28x28 images
    num_classes, input_features = weights.shape
    assert input_features == 784, f"Expected 784 features, got {input_featur

    # Create a figure with 2x5 subplots for the 10 digits
    fig, axes = plt.subplots(2, 5, figsize=(20, 8))
    axes = axes.ravel()

    # Normalize weights for better visualization
    weights_normalized = weights.clone()
    for i in range(num_classes):
        # Normalize each class template to [0, 1] range
        w_min = weights[i].min()
```

```
        w_max = weights[i].max()
        if w_max > w_min:
            weights_normalized[i] = (weights[i] - w_min) / (w_max - w_min)

    # Plot each digit template
    for i in range(num_classes):
        # Reshape to 28x28
        template = weights_normalized[i].detach().view(28, 28).numpy()

        # Plot template
        axes[i].imshow(template, cmap='gray')
        axes[i].set_title(f'Learned Template for Digit {i}')
        axes[i].axis('off')

    plt.tight_layout()
    plt.show()

    # Create combined visualization (similar to 01_linear_feature.ipynb)
    # Create a 10x1 grid layout (single column)
    combined_img = np.zeros((28 * 10, 28))

    for i in range(num_classes):
        row = i  # Each digit gets its own row

        template = weights_normalized[i].detach().view(28, 28).numpy()
        combined_img[row*28:(row+1)*28, :] = template

    # Display combined image
    plt.figure(figsize=(12, 10))
    plt.imshow(combined_img, cmap='gray')
    plt.axis('off')
    plt.tight_layout()
    plt.show()

    # Save combined image directly as raw array
    combined_img_normalized = ((combined_img - combined_img.min()) / (combir
    img = Image.fromarray(combined_img_normalized, mode='L')
    img.save(save_path)

    print(f"✅ Learned templates visualization saved to: {save_path}")
    return combined_img

# Visualize the learned templates
print("Visualizing learned templates...")
learned_templates = visualize_learned_templates(weights)

# Also show individual templates with more detail
plt.figure(figsize=(20, 10))
for i in range(10):
    plt.subplot(2, 5, i + 1)
    template = weights[i].detach().view(28, 28).numpy()
    plt.imshow(template, cmap='gray')
    plt.title(f'Digit {i} Template (Raw Weights)')
    plt.colorbar()
    plt.axis('off')
```

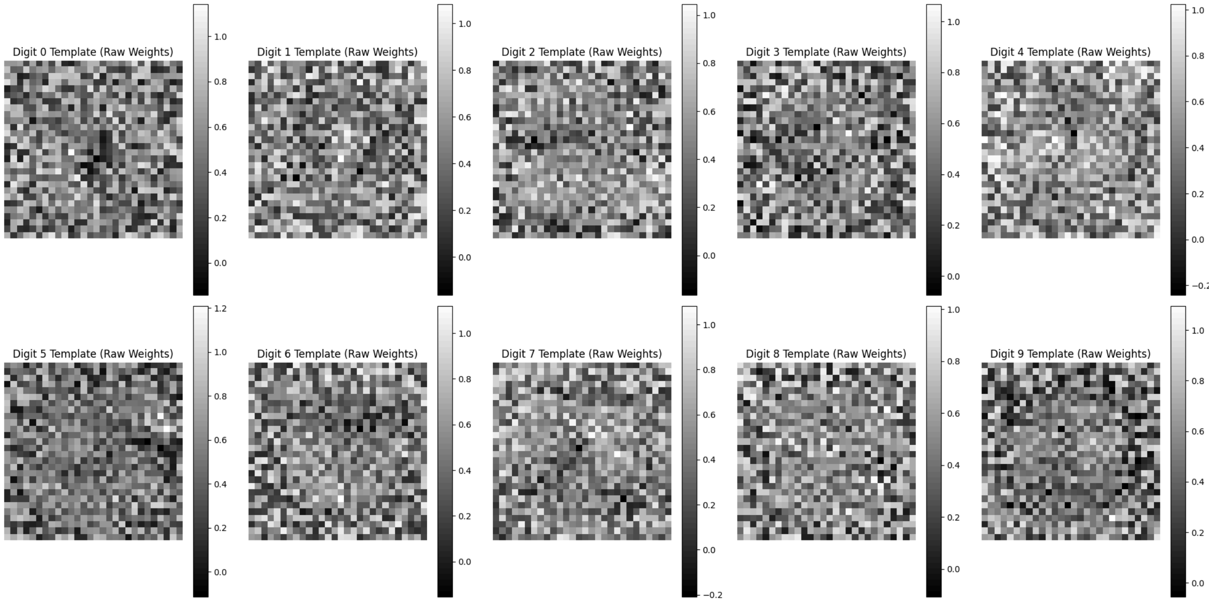
```python
plt.tight_layout()
plt.show()
```

```
Visualizing learned templates...
```





✅ Learned templates visualization saved to: all_weights_combined_learned.pn
g




# Part 7: Test on MNIST Test Set

Evaluate the trained model on the test set to get the final accuracy.

In [33]:
```
def evaluate_model(test_loader, weights, bias):
    """
    Evaluate the trained model on test set
    """
    correct = 0
    total = 0

    with torch.no_grad():
        for data, targets in test_loader:
            # Flatten input data
            data = data.view(-1, 784)

            # Forward pass
            predictions = linear_classifier_forward(data, weights, bias)

            # Get predictions
            _, predicted = torch.max(predictions, 1)

            # Update statistics

            total += targets.size(0)
            correct += (predicted == targets).sum().item()

    return 100 * correct / total

# Evaluate model
test_loader = torch.utils.data.DataLoader(test_dataset, batch_size=1000, shu
```

```
test_accuracy = evaluate_model(test_loader, weights, bias)

print(f"Test Accuracy: {test_accuracy:.2f}%")

# IMPORTANT: This accuracy should be greater than 90% for full credit
# assert test_accuracy > 90.0, f"Test accuracy {test_accuracy:.2f}% is below
# print(f"✅ Test accuracy {test_accuracy:.2f}% meets the requirement of >90

# Store the result for grading
test_accuracy_result = test_accuracy

# Test metadata for Gradescope auto-grading
# This cell will be automatically executed and evaluated
print(f"\n🎯 Final Test Result: {test_accuracy_result:.2f}%")
print(f"📊 Grading Status: {'✅ PASSED' if test_accuracy_result > 90.0 else
print(f"🎓 Points Earned: {60 if test_accuracy_result > 90.0 else 0}/60 for
```

```
Test Accuracy: 90.81%

🎯 Final Test Result: 90.81%
📊 Grading Status: ✅ PASSED
🎓 Points Earned: 60/60 for accuracy
```

## Part 8: Load Pre-saved Weights and Compare

**NEW SECTION: Load pre-saved weight images and apply them to the linear classifier**

In this section, we'll load the pre-saved weight images (like `all_digits_combined_learned.png`) and use them as weights for the linear classifier. We'll then compare the accuracy with the original implementation from 01_linear_feature.ipynb.

**Key Features:**

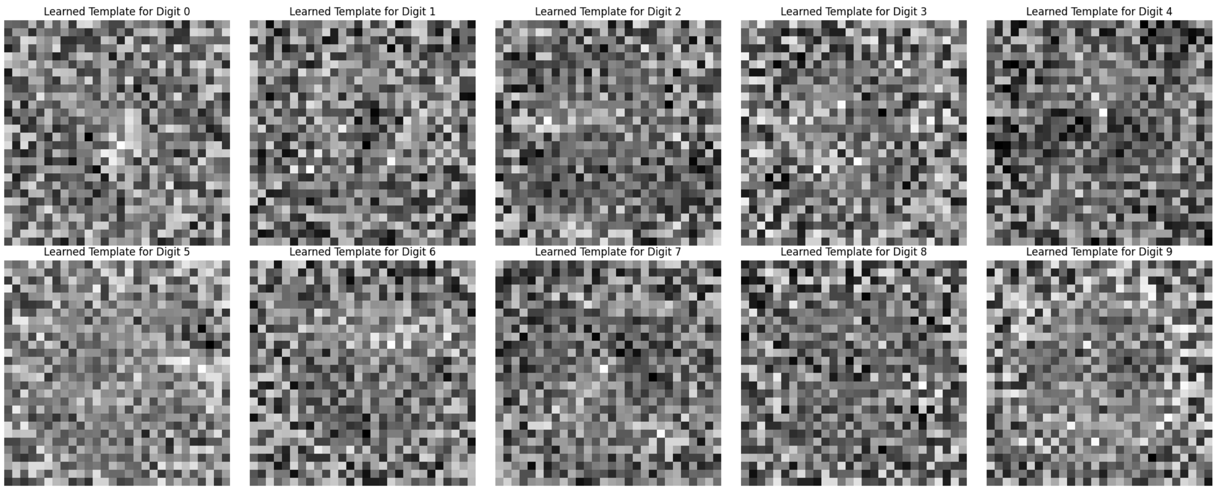
1. **Load images** using the same approach as 01_linear_feature.ipynb
2. **Apply eta adjustment** to modify the loaded images
3. **Create weight matrix** [10, 784] from normalized images
4. **Test accuracy** using the linear classifier evaluation function
5. **Compare results** across different eta values

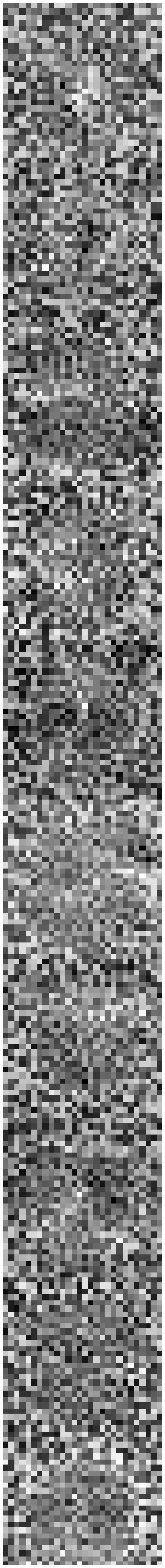
In [40]:
```
# Import the linear classifier functions from 01_linear_feature.ipynb
from cpsc8430.classifiers import (
    load_and_preprocess_image,
    create_weight_matrix,
    create_mnist_test_loader,
    evaluate_linear_classifier,
    create_random_bias
)

# Load weights from the saved image with different eta values
print("Loading weights from saved image...")
```

```
################################################################################
# TODO: Try different values of eta (e.g., 0.0, 0.25, 0.5, 0.75, 1.0) and ob
#       You can loop over several eta values and report the results for each
eta = 1
################################################################################


print(f"\n{'='*50}")
print(f"Testing with eta = {eta}")
print(f"{'='*50}")

try:
    # Load and preprocess image using eta (similar eta in 01_linear_feature.
    img_normalized = load_and_preprocess_image('all_weights_combined_learned

    # Create weight matrix [10, 784] from the normalized image
    weight_matrix = create_weight_matrix(img_normalized)

    # Create random bias
    bias = torch.randn(10)

    # Create test loader
    test_loader = create_mnist_test_loader()

    # Evaluation function
    def evaluate():
        correct = 0
        total = 0

        with torch.no_grad():
            for images, labels in test_loader:
                # Flatten the images
                images = images.view(-1, 784)

                # Forward pass: compute scores
                scores = torch.mm(images, weight_matrix.t()) + bias

                # Get predictions
                _, predicted = torch.max(scores, 1)

                # Update statistics
                total += labels.size(0)
                correct += (predicted == labels).sum().item()

        return 100 * correct / total

    # Test the classifier
    accuracy = evaluate()
    print(f"Test Accuracy: {accuracy:.2f}%")

    # Visualize the loaded weights with current eta
    print(f"Visualizing weights with eta={eta}...")
    visualize_learned_templates(weight_matrix, f'loaded_weights_eta_{eta:.2f

    # Show the effect of eta on a sample digit template
```

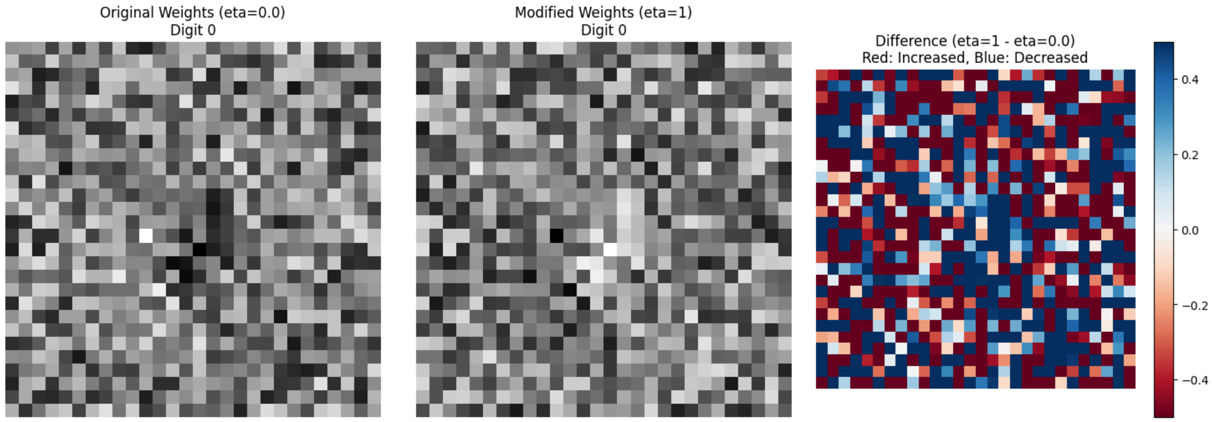
```python
    plt.figure(figsize=(15, 5))

    # For comparison, also load the original weights (eta=0.0)
    img_normalized_orig = load_and_preprocess_image('all_weights_combined_le
    original_weights = create_weight_matrix(img_normalized_orig)
    current_weights = weight_matrix

    # Show comparison for digit 0
    plt.subplot(1, 3, 1)
    template_orig = original_weights[0].detach().view(28, 28).numpy()
    plt.imshow(template_orig, cmap='gray')
    plt.title(f'Original Weights (eta=0.0)\nDigit 0')
    plt.axis('off')

    plt.subplot(1, 3, 2)
    template_current = current_weights[0].detach().view(28, 28).numpy()
    plt.imshow(template_current, cmap='gray')
    plt.title(f'Modified Weights (eta={eta})\nDigit 0')
    plt.axis('off')

    plt.subplot(1, 3, 3)
    difference = template_current - template_orig
    plt.imshow(difference, cmap='RdBu', vmin=-0.5, vmax=0.5)
    plt.title(f'Difference (eta={eta} - eta=0.0)\nRed: Increased, Blue: Decr
    plt.colorbar()
    plt.axis('off')

    plt.tight_layout()
    plt.show()

    print(f"✅ Eta={eta} analysis completed")

except Exception as e:
    print(f"❌ Error with eta={eta}: {e}")

print(f"\n{'='*60}")
```

```
Loading weights from saved image...

==================================================
Testing with eta = 1
==================================================
Test Accuracy: 0.11%
Visualizing weights with eta=1...
```


Learned Template for Digit 0
Learned Template for Digit 1
Learned Template for Digit 2
Learned Template for Digit 3
Learned Template for Digit 4
Learned Template for Digit 5
Learned Template for Digit 6
Learned Template for Digit 7
Learned Template for Digit 8
Learned Template for Digit 9



✅ Learned templates visualization saved to: loaded_weights_eta_1.00.png




✅ Eta=1 analysis completed

============================================================

# Inline Questions

## Question 1

What is the purpose of the softmax function in multi-class classification? How does it help with gradient descent?

*Your Answer:*

The softmax functions works to create a probability distribution that is differentiable. This key aspect of the softmax function is what enables GD to work in the first place.

## Question 2

What is the significance of the cross-entropy loss function in classification problems? Why is it preferred over mean squared error?

*Your Answer:*

Cross entropy loss focuses on how much the scores diverge from the target since it relies on probablistic calculations. This results in a steeper optimization when compared to MSE (which is not divergent by nature nor reflects probability, thus regressive problems work better than classification). This means that classifications are more likely to fall into local minima and vanishing gradients if MSE is used.

## Question 3

**Analyze the learned templates visualization**

Look at the learned weight templates for each digit class. What patterns do you observe? How do these templates relate to the actual digit shapes? Why might some templates look clearer than others? What does this tell us about how the linear classifier learns to distinguish between different digit classes?

*Your Answer:*

You can see the general shape of the numbers. Some are more crumby than others, but all have the general curves. I think some look different than others because people write those numbers very differently. The classifier seems to learn the edges of the shapes and even highlight the differences rather than the exact shape. Structured, linear data (with linear separation) fairs better than more complex variations which we see with certain digits (like `4` and even `1`).

## Question 4

**Weight loading and comparison**

When you load the pre-saved weights, 'all_weights_combined_learned.png' , from the image file, how does the testing accuracy compare to the direct trained weights (Part 7)? What is the major reason to cause differences in performance?

*Your Answer:*

The testing accuracy is not as good as the direct trained weights. At `eta = 0`, the accuracy is ~65%. While the direct trained weights gives us 90%> accuracy. The major reason I see is with the bias vector. Rather than used trained, tuned biases, a random vector of biases is created in its stead. Weights and biases go hand-in-hand during training.

## Question 5

**Effect of Different Eta Values on Testing Accuracy**

In Part 8, try different values of $\eta$ from the set $\{0, 0.25, 0.5, 0.75, 1\}$ when loading the pre-saved weights from the image file. For each $\eta$, report the testing accuracy you obtain.

*Your Answer:*

- Testing accuracy for $\eta = 0$: 65.29%
- Testing accuracy for $\eta = 0.25$: 66.64%
- Testing accuracy for $\eta = 0.5$: 9.8%
- Testing accuracy for $\eta = 0.75$: 0.11%
- Testing accuracy for $\eta = 1$: 0.11%

How does the trend of testing accuracy as $\eta$ changes compare to what you observed in 01_linear_feature.ipynb? Why do you think the trend is different in this notebook?

In the 01_linear_feature.ipynb we see the opposite trend as the grayscale changes. This is likely because the weights from the learned png are learned such that negative squares (darker) are associated with divergence and positive (lighter) with convergence.