University Physics A(2) 2014

Worksheet #9: Entropy (1)

Name (名字):   Student number (学号):

Problems Show all working.

(1) [M&I.12.P.39] The interatomic spring stiffness for copper (Cu) is known to be 28 N/m. The mass of one mole of copper is 0.064 kg.

In the following, you will need to keep 5 significant figures (有效数字) in your calculations, so use these precise values for the constants:

$\hbar = 1.0546 \times 10^{-34}\, J \cdot s$ ("h-bar" = Planck's constant/2π)

$N_A = 6.0221 \times 10^{23}$ molecules/mole (Avogadro's number)

$k_B = 1.3807 \times 10^{-23}\, J/K$ (Boltzmann's constant).

(a) What is one quantum of energy for one oscillator in copper? (Remember that, in the Einstein model, the effective spring stiffness is 4 times the true interatomic spring stiffness.)

(b) The table below contains the number of ways to arrange a given number of quanta of energy in a particular block of copper. Fill in the blank spaces to complete the table, including calculating the temperature of the block. The energy E is measured relative to the ground state. Be sure to give the temperature to the nearest 0.1 K.

q	# ways	E, J	S, J/K	ΔE, J	ΔS, J/K	T, K
20	4.91 E26					
21	4.44 E27					
22	3.85 E28					

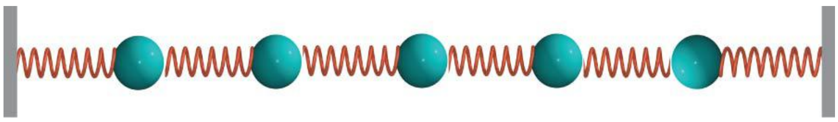
(c) There are 60 atoms in this object (it is a copper nanoparticle - 纳米粒子). What is the specific heat capacity per atom? Compare your answer to the high-temperature limit of $C = 3k_B$ =4.2 $\times 10^{-23}$ J/K/atom.

(2) [M&I.7.P.46] A nanoparticle consisting of four iron atoms (object 1) initially has 1 quantum of energy. It is brought into contact with a nanoparticle consisting of two iron atoms (object 2), which initially has 2 quanta of energy. The mass of one mole of iron is 56 grams, and the interatomic spring stiffness is 45 N/m.

(a) Using the Einstein model of a solid, calculate and plot $\ln \Omega_1$ vs q_1 (the number of quanta in object 1), $\ln \Omega_2$ vs q_1, and $\ln \Omega_{\text{total}}$ vs q_1 (put all three plots on the same graph). Show your work and explain briefly.

(b) Calculate the approximate temperature of the objects at equilibrium. State what assumptions or approximations you made.

(3) [M&I.P.47] The figure below shows a one-dimensional (1D) row of 5 microscopic objects, each of mass 4×10^{-26} kg, connected by bonds that can be modeled as springs with stiffness 15 N/m. These objects can only move along the *x* axis.



(a) Using the Einstein model, calculate the approximate entropy of this system for total energy of 0, 1, 2, 3, 4, and 5 quanta. (Hint: this is 1D, not 3D, so one object corresponds to how many oscillators?)

(b) Calculate the approximate temperature of the system when the total energy is 4 quanta.

(c) Calculate the approximate temperature of the system when the total energy is 4 quanta.

(d) If the temperature is raised very high, what is the approximate specific heat capacity per object? Compare with your result from part (c).