Adversarial Search II: Alpha-Beta Pruning

CSE 415: Introduction to Artificial Intelligence
University of Washington
Winter, 2018

Alpha-Beta Pruning

Enhance minimax search with two extra values at each tree node that represent the interval in which the "solution" value must lie.

$[\alpha, \beta]$

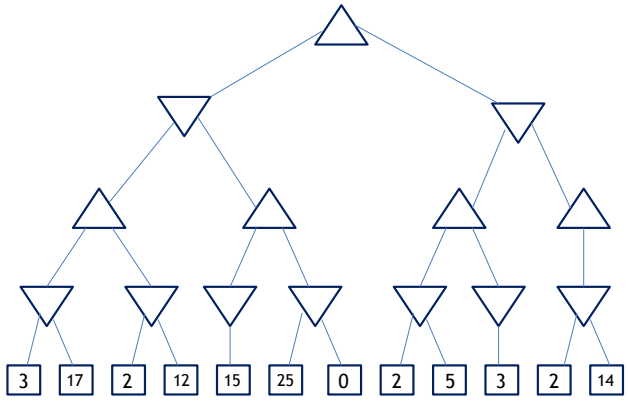
Initialize the root's to $[-\infty, \infty]$.
Update these at the current node, when possible.
If any node gets $\alpha \geq \beta$, then it is "finished", so "prune off" any of its children that remain.

Alpha-Beta Cutoffs

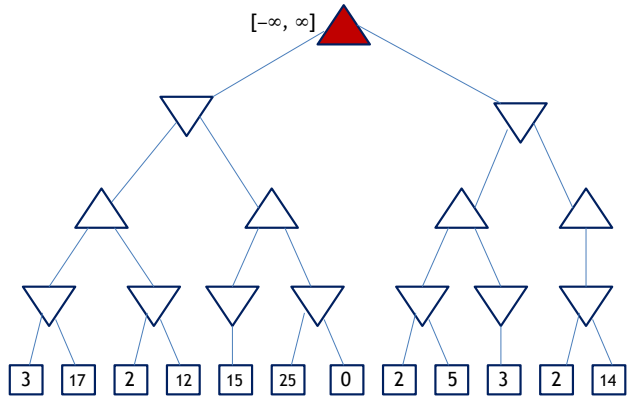
An alpha (beta) cutoff occurs at a Maximizing (minimizing) node when it is known that the maximizing (minimizing) player has a move that results in a value alpha (beta) and, subsequently, when an alternative to that move is explored, it is found that the alternative gives the opponent the option of moving to a lower (higher) valued position.

Any further exploration of the alternative can be canceled.

Alpha-Beta Pruning

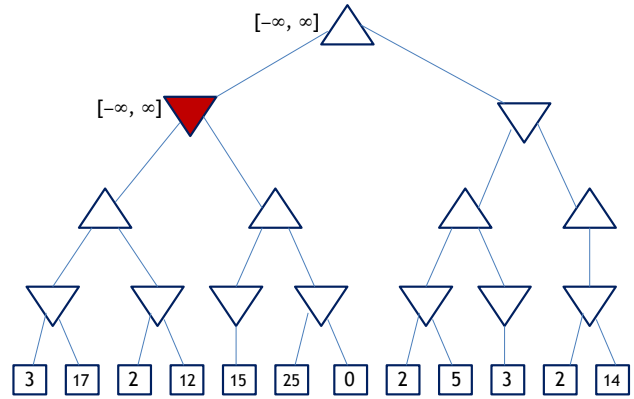


Alpha-Beta Pruning



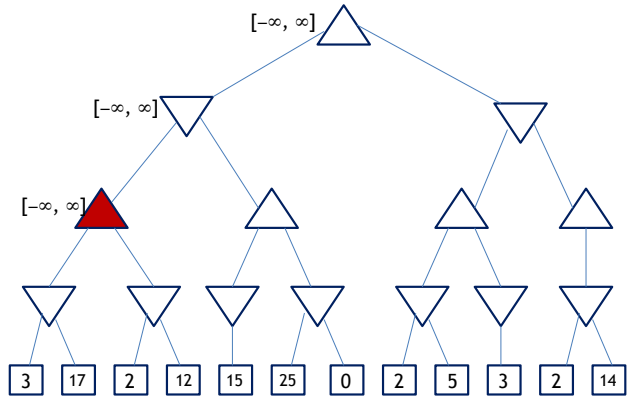
5

Alpha-Beta Pruning



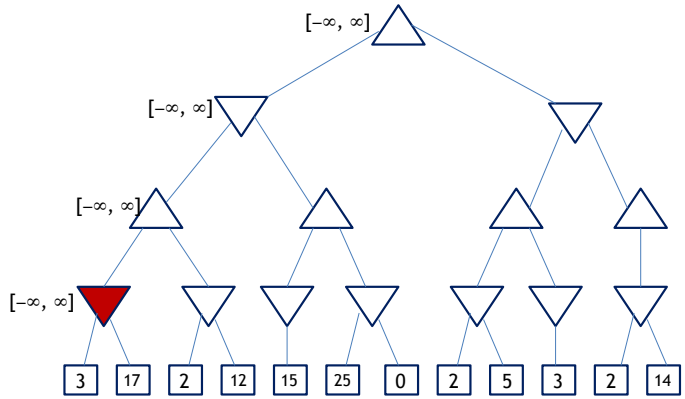
6

Alpha-Beta Pruning

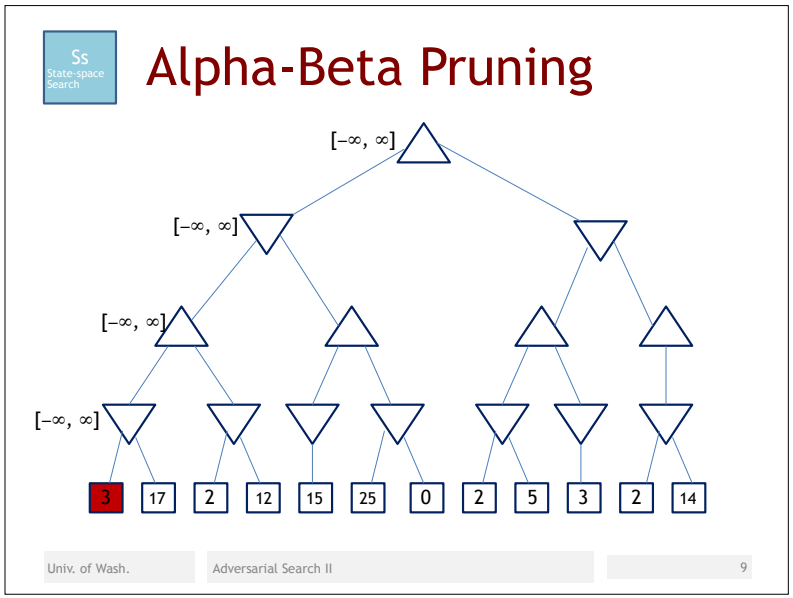


7

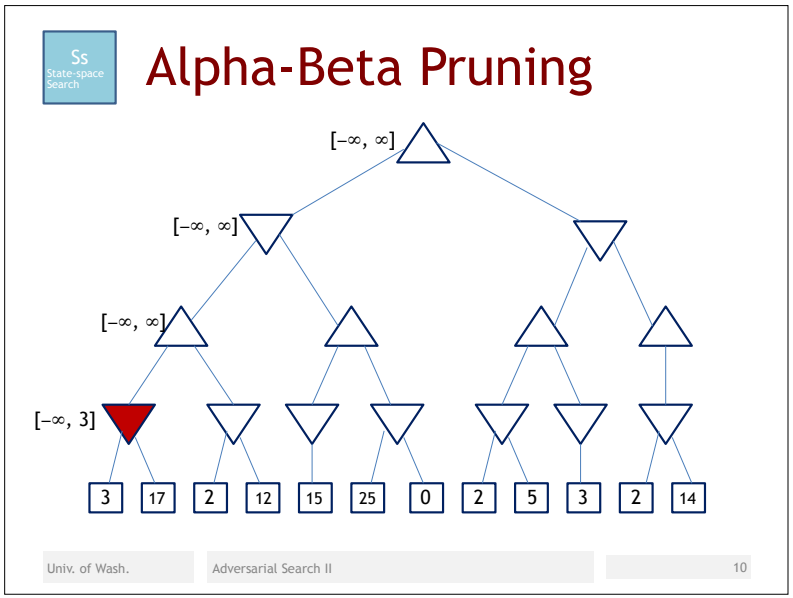
Alpha-Beta Pruning



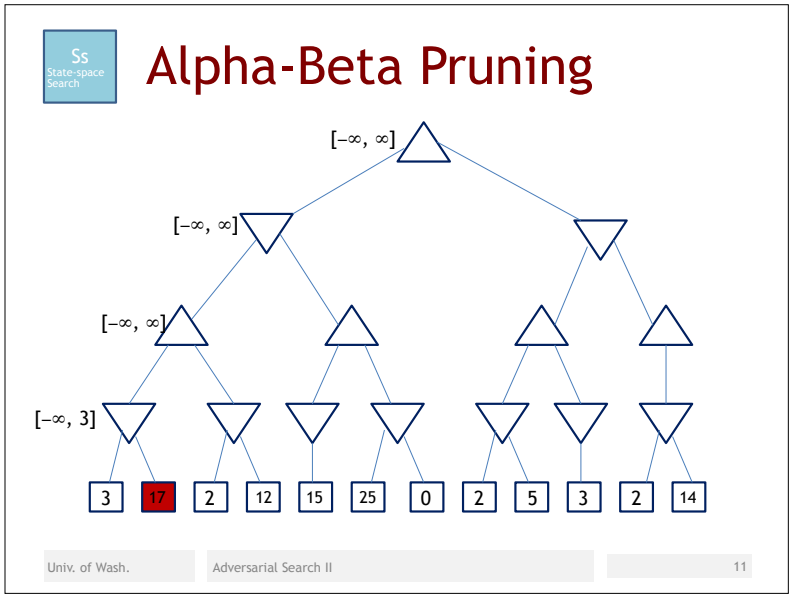
8


Ss
State-space Search
Alpha-Beta Pruning
[−∞, ∞]
[−∞, ∞]
[−∞, ∞]
[−∞, ∞]
3 17 2 12 15 25 0 2 5 3 2 14
Univ. of Wash.
Adversarial Search II
9

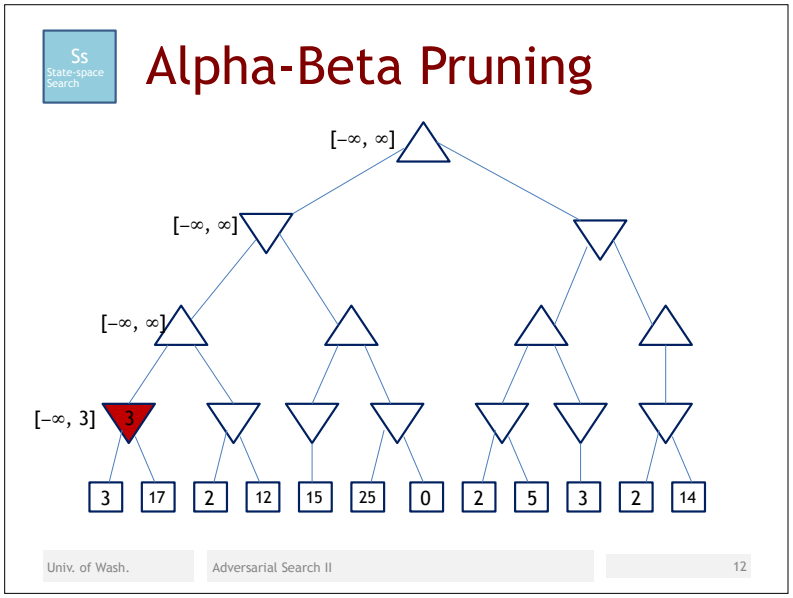
9


Ss
State-space Search
Alpha-Beta Pruning
[−∞, ∞]
[−∞, ∞]
[−∞, ∞]
[−∞, 3]
3 17 2 12 15 25 0 2 5 3 2 14
Univ. of Wash.
Adversarial Search II
10

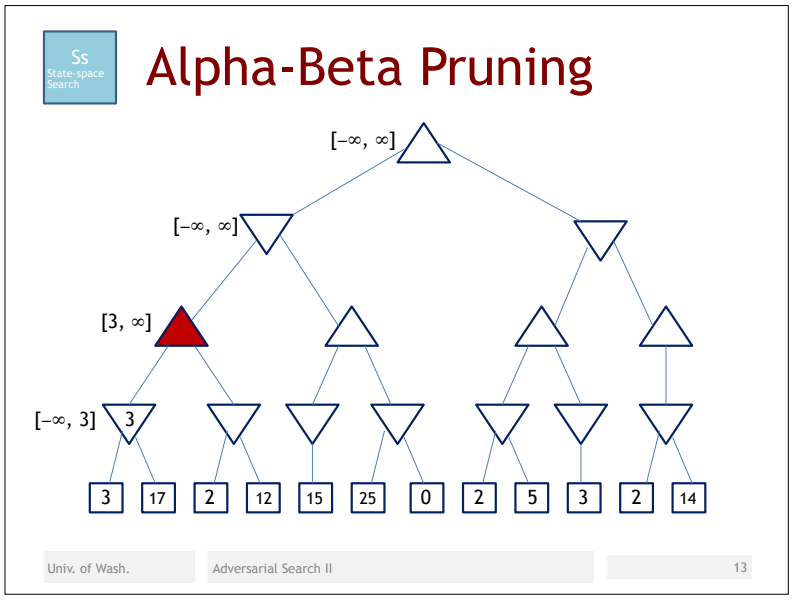
10


Ss
State-space Search
Alpha-Beta Pruning
[−∞, ∞]
[−∞, ∞]
[−∞, ∞]
[−∞, 3]
3 17 2 12 15 25 0 2 5 3 2 14
Univ. of Wash.
Adversarial Search II
11

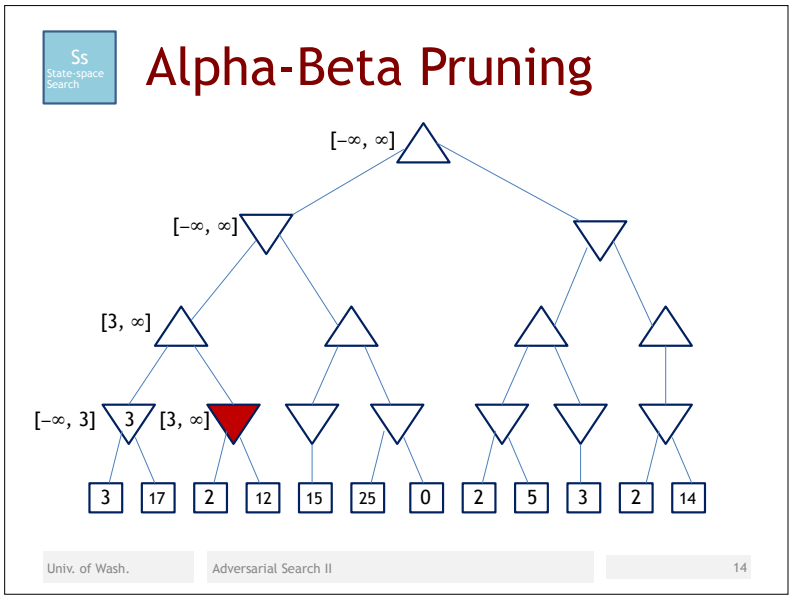
11


Ss
State-space Search
Alpha-Beta Pruning
[−∞, ∞]
[−∞, ∞]
[−∞, ∞]
[−∞, 3]
3 17 2 12 15 25 0 2 5 3 2 14
Univ. of Wash.
Adversarial Search II
12

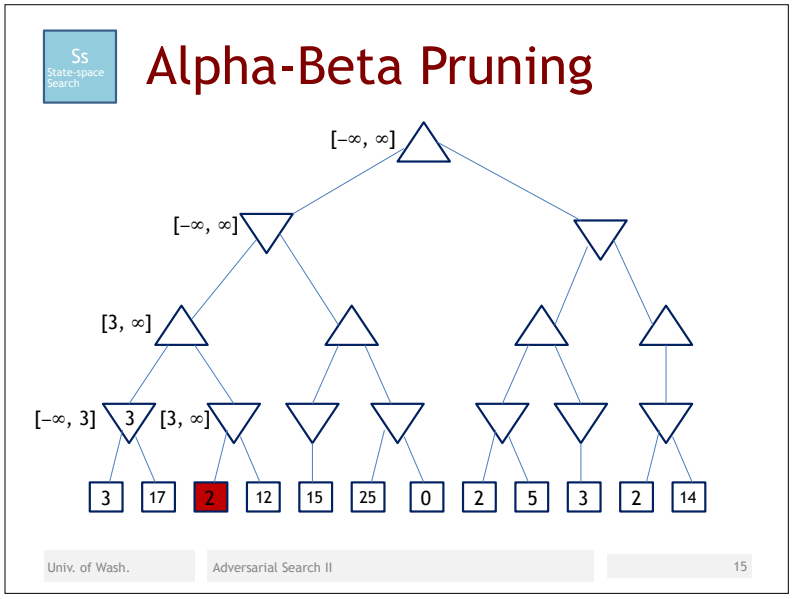
12


Ss
State-space Search
Alpha-Beta Pruning
[−∞, ∞]
[−∞, ∞]
[3, ∞]
[−∞, 3]
3
3 17 2 12 15 25 0 2 5 3 2 14
Univ. of Wash.
Adversarial Search II
13

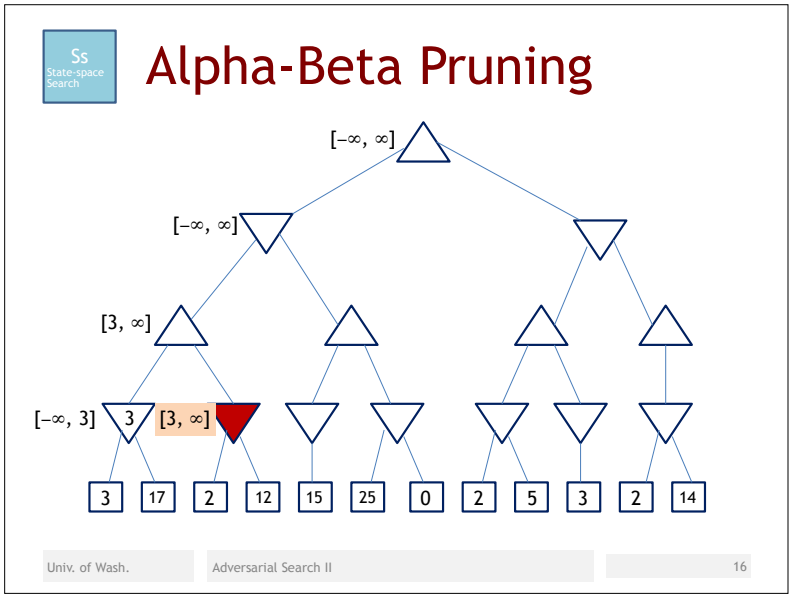
13


Ss
State-space Search
Alpha-Beta Pruning
[−∞, ∞]
[−∞, ∞]
[3, ∞]
[−∞, 3]
3
[3, ∞]
3 17 2 12 15 25 0 2 5 3 2 14
Univ. of Wash.
Adversarial Search II
14

14

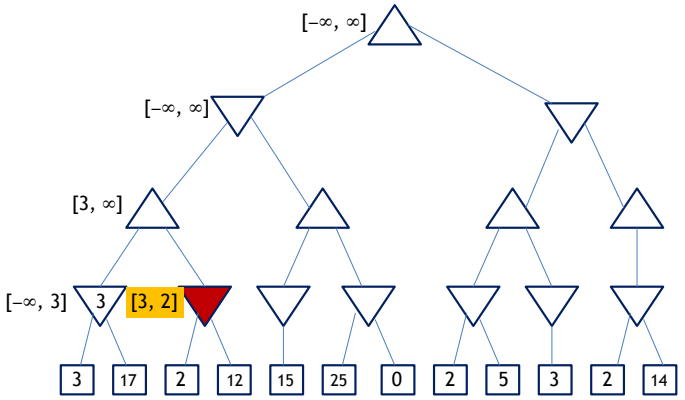

Ss
State-space Search
Alpha-Beta Pruning
[−∞, ∞]
[−∞, ∞]
[3, ∞]
[−∞, 3]
3
[3, ∞]
3 17 2 12 15 25 0 2 5 3 2 14
Univ. of Wash.
Adversarial Search II
15

15


Ss
State-space Search
Alpha-Beta Pruning
[−∞, ∞]
[−∞, ∞]
[3, ∞]
[−∞, 3]
3
[3, ∞]
3 17 2 12 15 25 0 2 5 3 2 14
Univ. of Wash.
Adversarial Search II
16

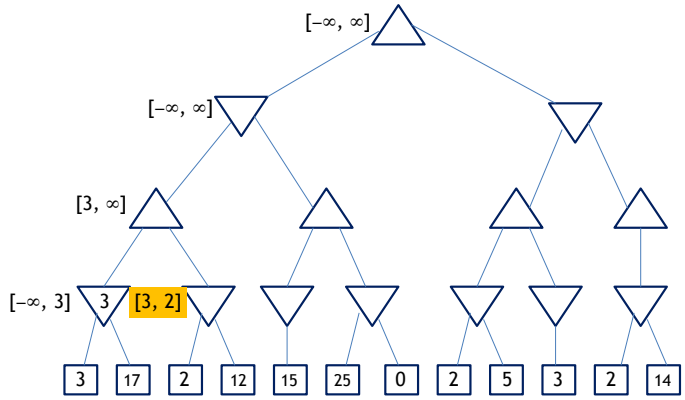
16

Alpha-Beta Pruning



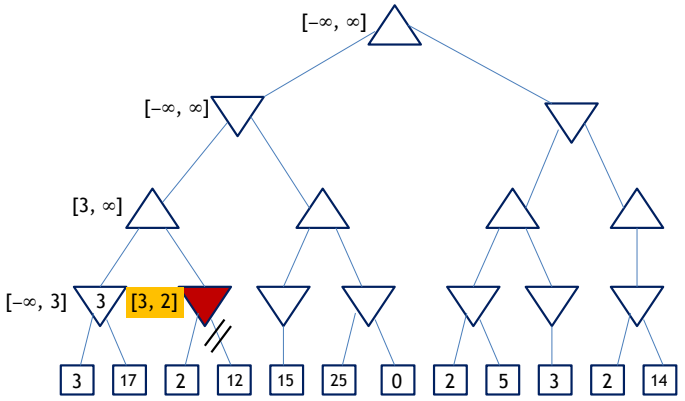
17

Alpha-Beta Pruning



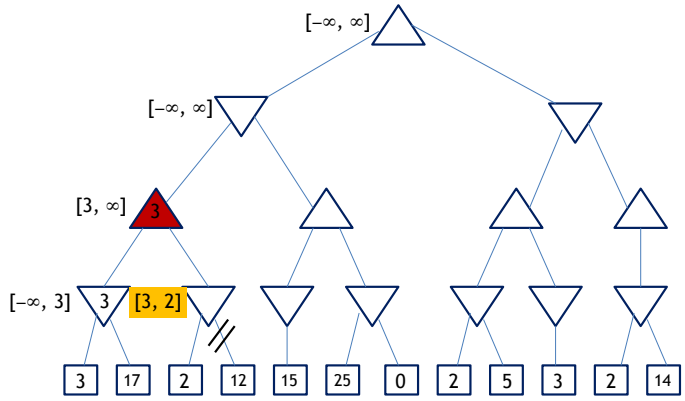
18

Alpha-Beta Pruning

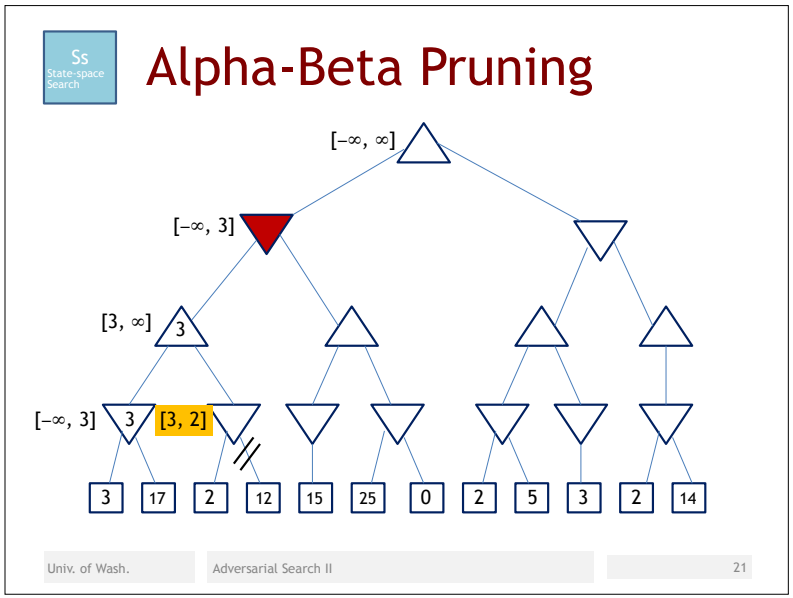


19

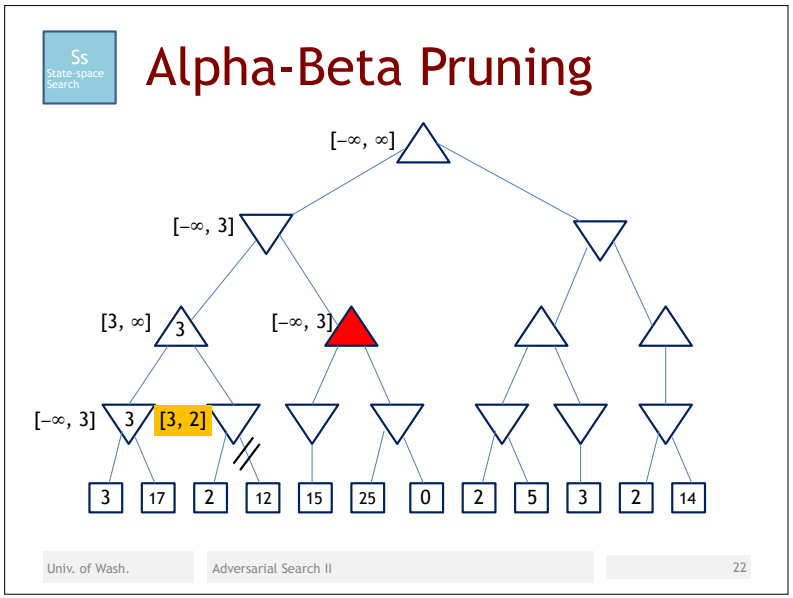
Alpha-Beta Pruning



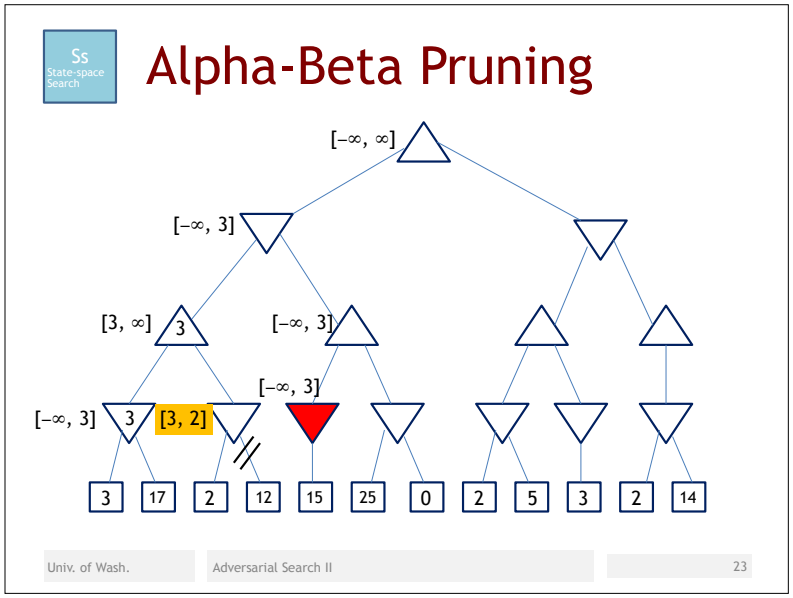
20


Alpha-Beta Pruning
[−∞, ∞]
[−∞, 3]
[3, ∞]
3
[−∞, 3]
3
[3, 2]
3 17 2 12 15 25 0 2 5 3 2 14
Univ. of Wash.
Adversarial Search II
21

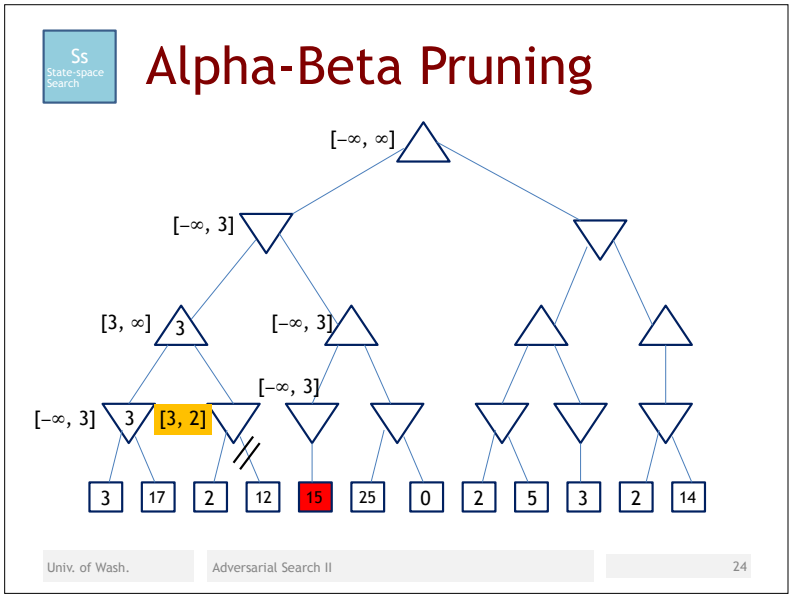
21


Alpha-Beta Pruning
[−∞, ∞]
[−∞, 3]
[3, ∞]
3
[−∞, 3]
[−∞, 3]
3
[3, 2]
3 17 2 12 15 25 0 2 5 3 2 14
Univ. of Wash.
Adversarial Search II
22

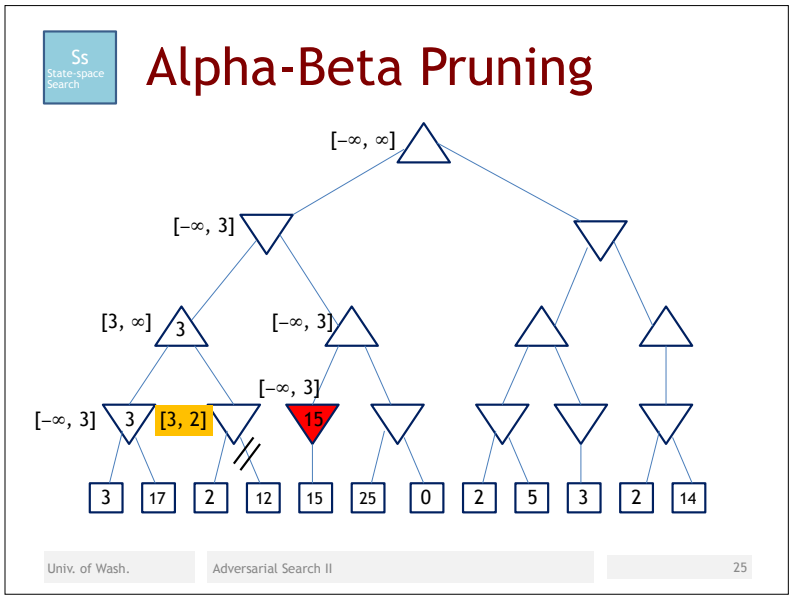
22


Alpha-Beta Pruning
[−∞, ∞]
[−∞, 3]
[3, ∞]
3
[−∞, 3]
[−∞, 3]
3
[3, 2]
3 17 2 12 15 25 0 2 5 3 2 14
Univ. of Wash.
Adversarial Search II
23

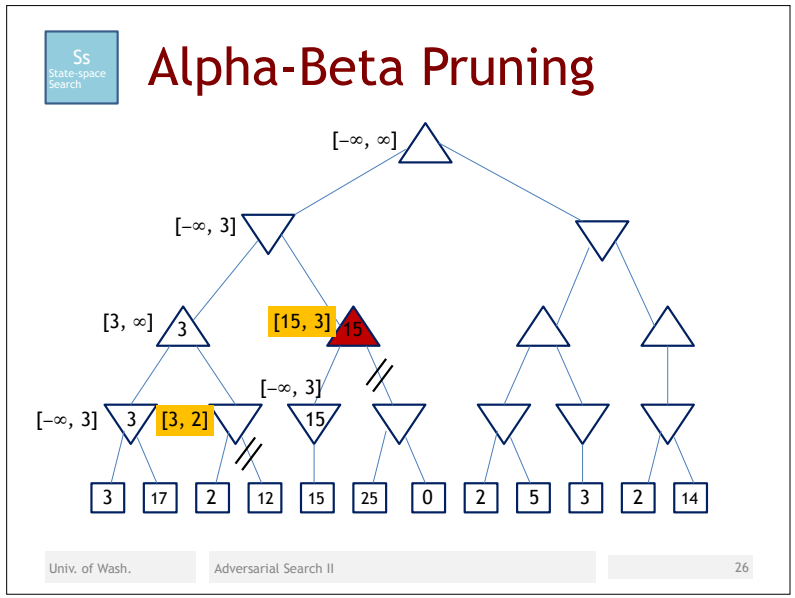
23


Alpha-Beta Pruning
[−∞, ∞]
[−∞, 3]
[3, ∞]
3
[−∞, 3]
[−∞, 3]
3
[3, 2]
3 17 2 12 15 25 0 2 5 3 2 14
Univ. of Wash.
Adversarial Search II
24

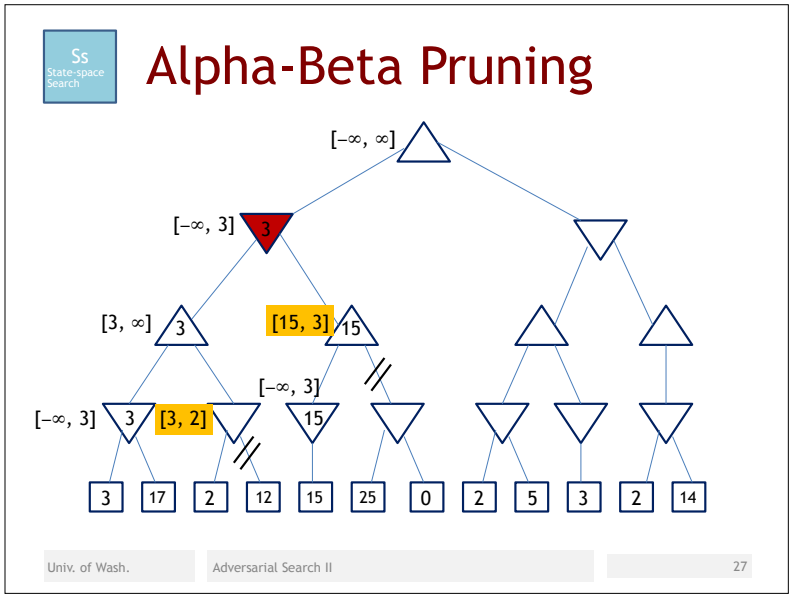
24


Ss
State-space Search
Alpha-Beta Pruning
[−∞, ∞]
[−∞, 3]
[3, ∞]
3
[−∞, 3]
[−∞, 3]
3
[3, 2]
15
3
17
2
12
15
25
0
2
5
3
2
14
Univ. of Wash.
Adversarial Search II
25

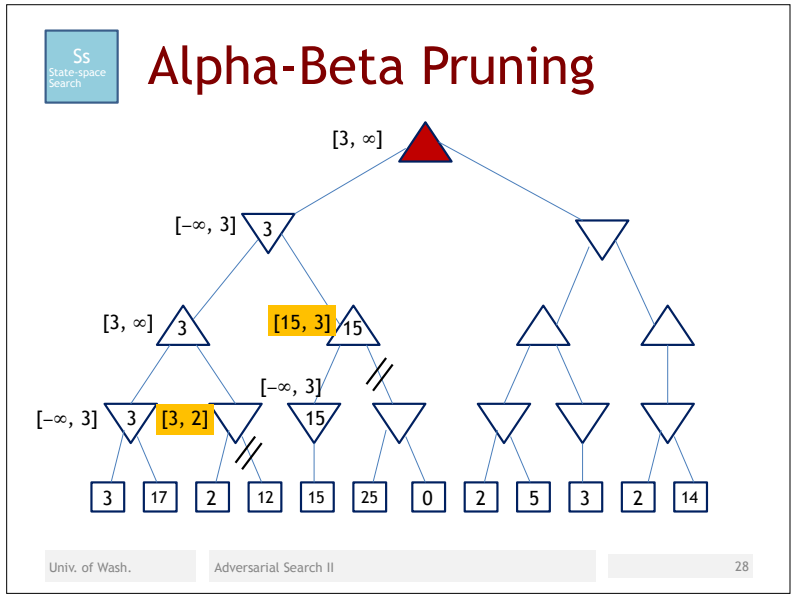
25


Ss
State-space Search
Alpha-Beta Pruning
[−∞, ∞]
[−∞, 3]
[3, ∞]
3
[15, 3]
15
[−∞, 3]
[−∞, 3]
3
[3, 2]
15
3
17
2
12
15
25
0
2
5
3
2
14
Univ. of Wash.
Adversarial Search II
26

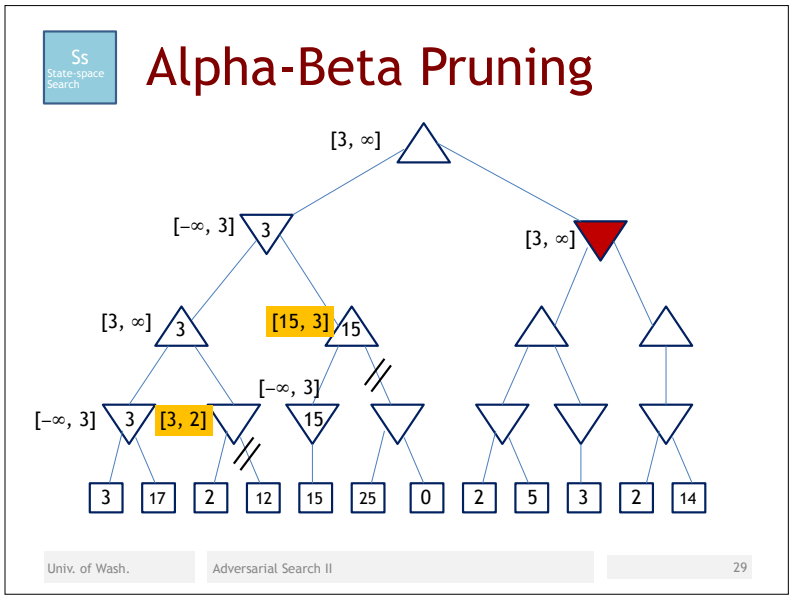
26


Ss
State-space Search
Alpha-Beta Pruning
[−∞, ∞]
[−∞, 3]
3
[3, ∞]
3
[15, 3]
15
[−∞, 3]
[−∞, 3]
3
[3, 2]
15
3
17
2
12
15
25
0
2
5
3
2
14
Univ. of Wash.
Adversarial Search II
27

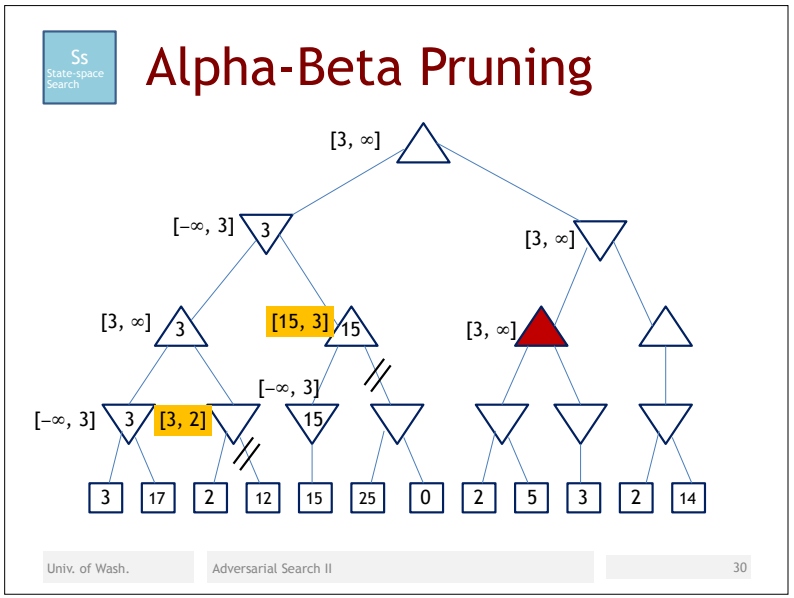
27


Ss
State-space Search
Alpha-Beta Pruning
[3, ∞]
[−∞, 3]
3
[3, ∞]
3
[15, 3]
15
[−∞, 3]
[−∞, 3]
3
[3, 2]
15
3
17
2
12
15
25
0
2
5
3
2
14
Univ. of Wash.
Adversarial Search II
28

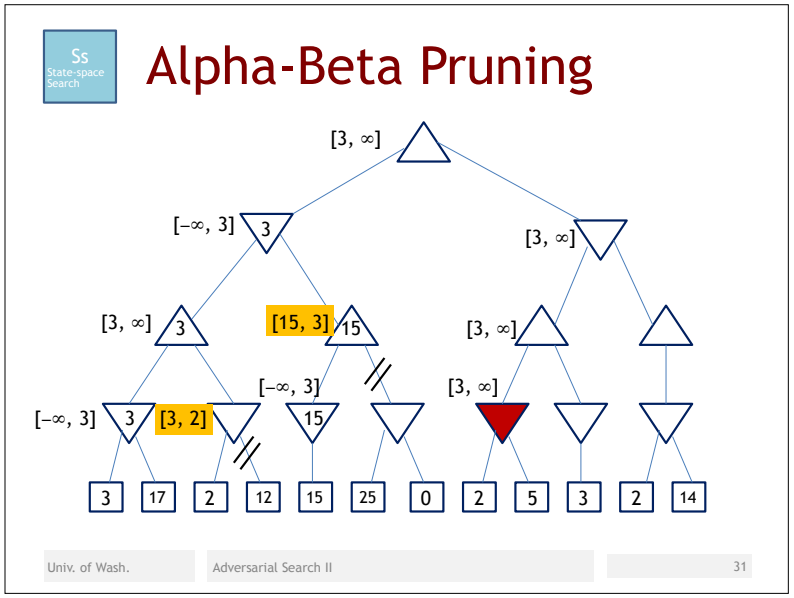
28


Ss
State-space Search
Alpha-Beta Pruning
[3, ∞]
[−∞, 3]
3
[3, ∞]
[3, ∞]
3
[15, 3]
15
[−∞, 3]
[−∞, 3]
3
[3, 2]
15
3
17
2
12
15
25
0
2
5
3
2
14
Univ. of Wash.
Adversarial Search II
29

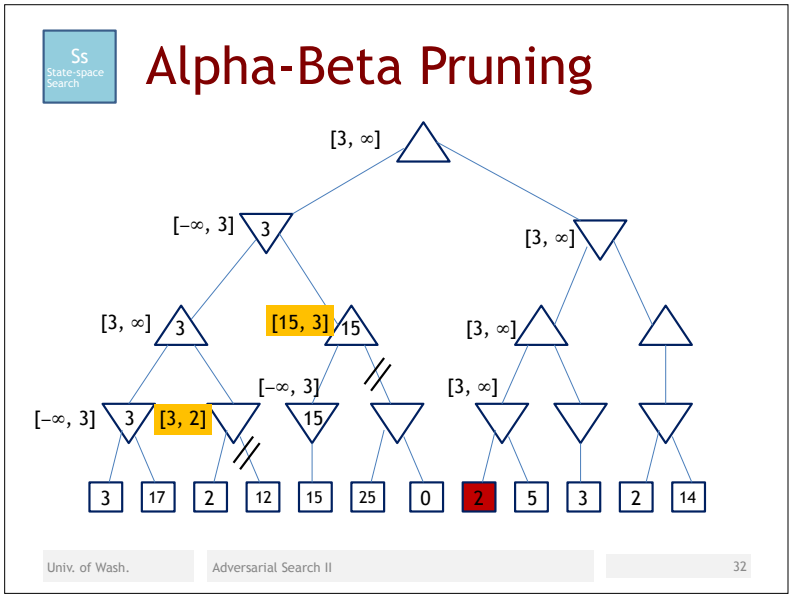
29


Ss
State-space Search
Alpha-Beta Pruning
[3, ∞]
[−∞, 3]
3
[3, ∞]
[3, ∞]
3
[15, 3]
15
[3, ∞]
[−∞, 3]
[−∞, 3]
3
[3, 2]
15
3
17
2
12
15
25
0
2
5
3
2
14
Univ. of Wash.
Adversarial Search II
30

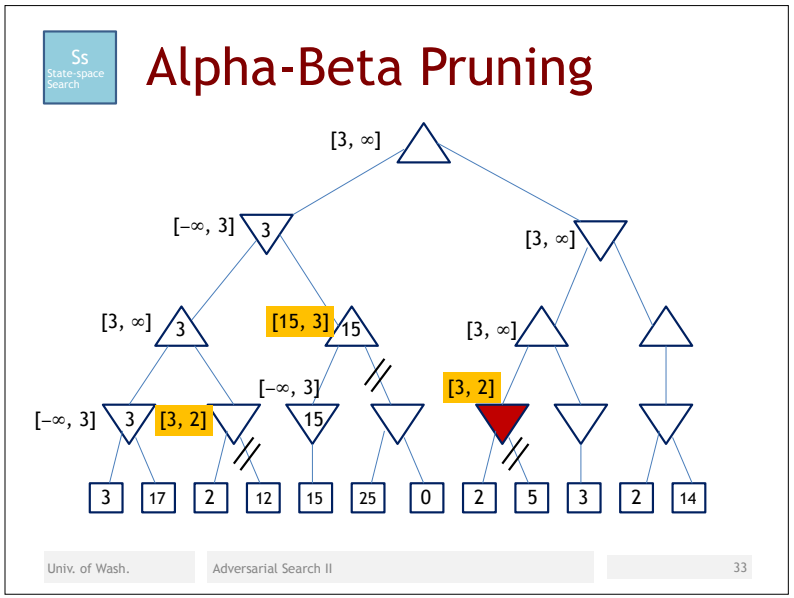
30


Ss
State-space Search
Alpha-Beta Pruning
[3, ∞]
[−∞, 3]
3
[3, ∞]
[3, ∞]
3
[15, 3]
15
[−∞, 3]
[3, ∞]
[−∞, 3]
3
[3, 2]
15
3
17
2
12
15
25
0
2
5
3
2
14
Univ. of Wash.
Adversarial Search II
31

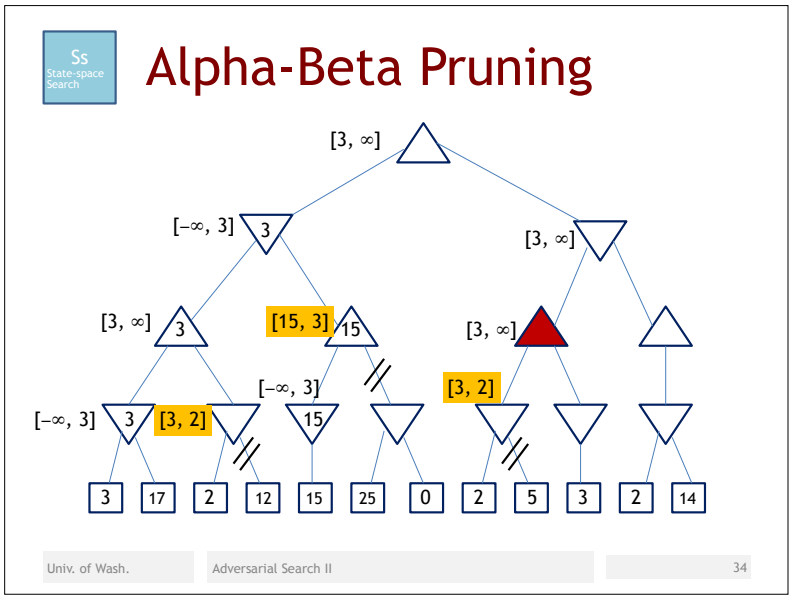
31


Ss
State-space Search
Alpha-Beta Pruning
[3, ∞]
[−∞, 3]
3
[3, ∞]
[3, ∞]
3
[15, 3]
15
[3, ∞]
[−∞, 3]
[−∞, 3]
3
[3, 2]
15
3
17
2
12
15
25
0
2
5
3
2
14
Univ. of Wash.
Adversarial Search II
32

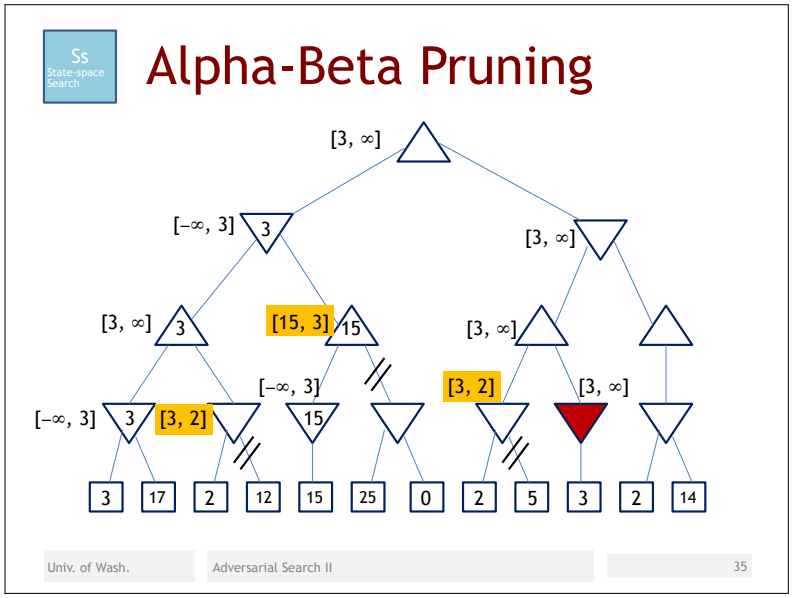
32


Ss
State-space Search
Alpha-Beta Pruning
[3, ∞]
[−∞, 3]
3
[3, ∞]
[3, ∞]
3
[15, 3]
15
[3, ∞]
[−∞, 3]
[3, 2]
[−∞, 3]
3
[3, 2]
15
3
17
2
12
15
25
0
2
5
3
2
14
Univ. of Wash.
Adversarial Search II
33

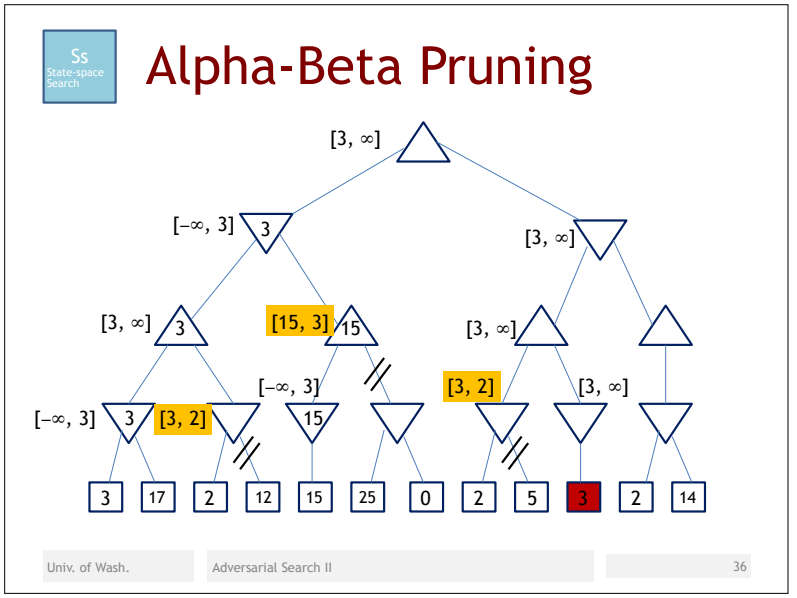
33


Ss
State-space Search
Alpha-Beta Pruning
[3, ∞]
[−∞, 3]
3
[3, ∞]
[3, ∞]
3
[15, 3]
15
[3, ∞]
[−∞, 3]
[3, 2]
[−∞, 3]
3
[3, 2]
15
3
17
2
12
15
25
0
2
5
3
2
14
Univ. of Wash.
Adversarial Search II
34

34


Ss
State-space Search
Alpha-Beta Pruning
[3, ∞]
[−∞, 3]
3
[3, ∞]
[3, ∞]
3
[15, 3]
15
[3, ∞]
[−∞, 3]
[3, 2]
[3, ∞]
[−∞, 3]
3
[3, 2]
15
3
17
2
12
15
25
0
2
5
3
2
14
Univ. of Wash.
Adversarial Search II
35

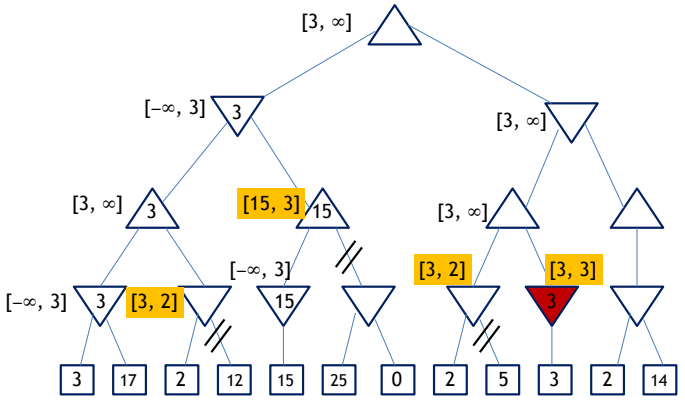
35


Ss
State-space Search
Alpha-Beta Pruning
[3, ∞]
[−∞, 3]
3
[3, ∞]
[3, ∞]
3
[15, 3]
15
[3, ∞]
[−∞, 3]
[3, 2]
[3, ∞]
[−∞, 3]
3
[3, 2]
15
3
17
2
12
15
25
0
2
5
3
2
14
Univ. of Wash.
Adversarial Search II
36

36

Ss State-space Search

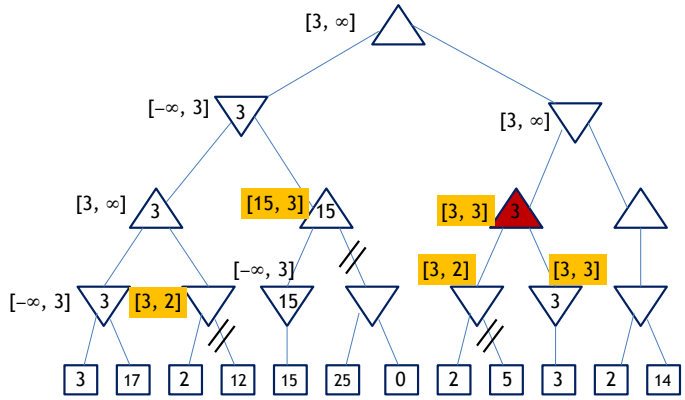
Alpha-Beta Pruning



37

Ss State-space Search

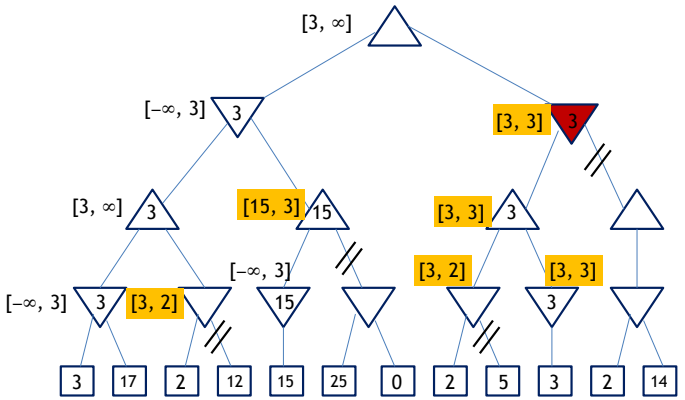
Alpha-Beta Pruning



38

Ss State-space Search

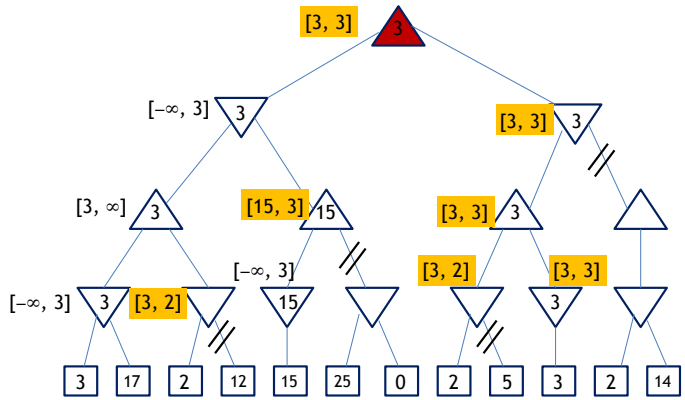
Alpha-Beta Pruning



39

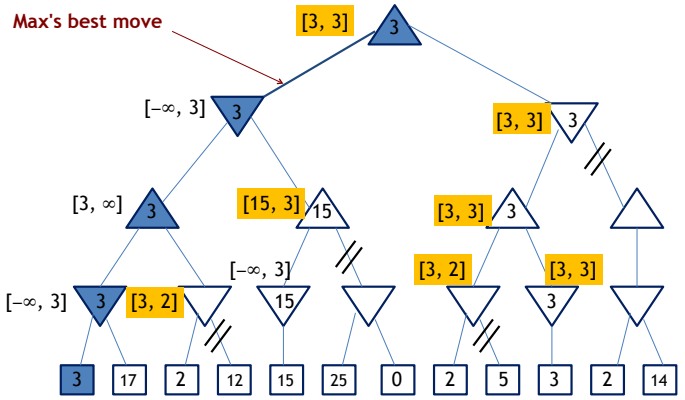
Ss State-space Search

Alpha-Beta Pruning



40

Alpha-Beta Pruning



41

Strategy to Increase the Number of Cutoffs

At each non-leaf level, perform a static evaluation of all successors of a node and order them best-first before doing the recursive calls. If the best move was first, the tendency should be to get cutoffs when exploring the remaining ones.

Or, use Iterative Deepening, with ply limits increasing from, say 1 to 15. Use results of the last iteration to order moves in the next iteration.

In games like chess, α-β pruning typically allows searching ahead 2 times as deep. It tends to reduce the effective branching factor from d to approx. sqrt(d).

42

Strategy to Increase the Number of Cutoffs

At each non-leaf level, perform a static evaluation of all successors of a node and order them best-first before doing the recursive calls. If the best move was first, the tendency should be to get cutoffs when exploring the remaining ones.

Or, use Iterative Deepening, with ply limits increasing from, say 1 to 15. Use results of the last iteration to order moves in the next iteration.

43