Tensor_Data_Processing

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1 Data Manipulation

- The basic data structure used in deep learning is the *n*-dimensional array, which is also called a *tensor*.
 - A rank 0 tensor corresponds to a *number*.
 - A rank 1 tensor corresponds to a vector.
 - A rank 2 tensor corresponds to a matrix.
 - Tensors of higher rank do not have special names.
- The class Tensor in PyTorch is similar to the class ndarray in Numpy. However, it also enables GPU acceleration and automatic differentiation.
- Together, these properties make the Tensor class very useful for deep learning.

```
[2]: print(torch.__version__)
```

2.1.0+cu121

2 Vector

- We will denote vectors by boldface lower-case letters (such as \mathbf{x} , \mathbf{y} , and \mathbf{z}).
- Operations between matrices and vectors will behave as if the vector were a column matrix:

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix},$$

where x_1, \dots, x_n are elements of the vector.

```
[3]: # A vector with 4 integers in the range [0,3]

# Unless otherwise specified, a new tensor is stored in main memory, enabling

□ CPU-based computation

x = torch.arange(4)

print(type(x))

print(x)
```

```
<class 'torch.Tensor'>
tensor([0, 1, 2, 3])
```

[4]: # Acessing the i-th element: x[i]. Indices start at zero. print(x[3])

tensor(3)

```
[5]: # Vector shape (dimensionality)
print(len(x))
print(x.size())
print(x.shape)
print(type(x.size()))
```

4
torch.Size([4])
torch.Size([4])
<class 'torch.Size'>

3 Matrices

- We will denote matrices by boldface capital letters (such as X, Y, and Z).
- We will let $\mathbf{A} \in \mathbb{R}^{m \times n}$ denote that matrix \mathbf{A} is composed of m rows and n columns of real numbers.
- We can represent a matrix $\mathbf{A} \in \mathbb{R}^{m \times n}$ by a table:

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix},$$

where the element in the i^{th} row and j^{th} column is given by a_{ij} .

- For any matrix $\mathbf{A} \in \mathbb{R}^{m \times n}$, the shape of \mathbf{A} is (m, n).
- A matrix is called a *square matrix* if the number of rows is the same as the number of columns.

```
[6]: # Reshape function: changes the shape of a tensor without changing the number
    of elements or their values
    A = torch.arange(20)
    print(A)
    B = A.reshape(5, 4)
    print(B)
    print(B[2, 3])
    print(B[2][3])
```

tensor([0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19])

```
tensor([[ 0, 1, 2, 3],
            [4, 5, 6, 7],
            [8, 9, 10, 11],
            [12, 13, 14, 15],
            [16, 17, 18, 19]])
    tensor(11)
    tensor(11)
[7]: # Reshaping convention
    A = torch.arange(20)
    print(A)
    # From vector to matrix
    c = 0
    B = torch.zeros((5, 4), dtype=int)
    for i in range(5):
        for j in range(4):
            B[i, j] = A[c]
            c += 1
    print(B)
    # From matrix to vector
    c = 0
    C = torch.zeros(20, dtype=int)
    for i in range(5):
        for j in range(4):
            C[c] = B[i, j]
            c += 1
    print(C)
    tensor([ 0, 1,
                    2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17,
            18, 19])
    tensor([[ 0, 1, 2, 3],
            [4, 5, 6, 7],
            [8, 9, 10, 11],
            [12, 13, 14, 15],
            [16, 17, 18, 19]])
    tensor([ 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17,
            18, 19])
[8]: # Matrix shape
    print(len(B))
    print(B.size())
    print(B.shape)
    torch.Size([5, 4])
    torch.Size([5, 4])
```

```
[9]: # Use -1 for a dimension that can be automatically inferred
     A1 = torch.arange(20).reshape(5, -1);
     A2 = torch.arange(20).reshape(-1, 4);
     print(A1 == A2)
     tensor([[True, True, True, True],
             [True, True, True, True],
             [True, True, True, True],
             [True, True, True, True],
             [True, True, True, True]])
[10]: # Transpose
     B1 = B.T
     print(B1)
     B2 = B1.transpose(1, 0)
     print(B2)
     tensor([[ 0, 4, 8, 12, 16],
             [1, 5, 9, 13, 17],
             [2, 6, 10, 14, 18],
             [ 3, 7, 11, 15, 19]])
     tensor([[ 0, 1, 2, 3],
             [4, 5, 6, 7],
             [8, 9, 10, 11],
             [12, 13, 14, 15],
             [16, 17, 18, 19]])
        Tensors
[11]: # A rank 3 tensor
     X = torch.arange(24).reshape(2, 3, -1)
     print(X)
     tensor([[[ 0, 1, 2, 3],
              [4, 5, 6, 7],
              [8, 9, 10, 11]],
             [[12, 13, 14, 15],
              [16, 17, 18, 19],
              [20, 21, 22, 23]])
         Commonly-used Tensor Constructors
[12]: print(torch.ones((2, 3, 4))) # Initialize with ones
```

tensor([[[1., 1., 1., 1.],

[1., 1., 1., 1.],

```
[1., 1., 1., 1.]],
             [[1., 1., 1., 1.],
              [1., 1., 1., 1.],
              [1., 1., 1., 1.]])
[13]: print(torch.zeros(2, 3)) # Initialize with zeros
     tensor([[0., 0., 0.],
             [0., 0., 0.]])
[14]: print(torch.randn(3, 4)) # Initialize with samples from a Gaussian distribution
       \hookrightarrowwith mean 0 and standard deviation 1
     tensor([[-0.5730, 0.0458, -1.0057, -0.2085],
             [0.3169, -0.1660, -0.0406, 0.0657],
             [0.0972, -0.1748, -1.0358, -0.7907]])
[15]: print(torch.tensor([[2, 1, 4, 3], [1, 2, 3, 4]])) # Initialize from Python lists
     tensor([[2, 1, 4, 3],
             [1, 2, 3, 4]])
         Common Tensor Operators
[16]: A = torch.arange(20, dtype=torch.float32).reshape(5, 4)
      B = A.clone() # Assign a copy of `A` to `B` by allocating new memory
      print(A)
      print(A + B)
      print(A * B)
      print(A + 2)
     tensor([[ 0., 1., 2., 3.],
             [4., 5., 6., 7.],
             [8., 9., 10., 11.],
             [12., 13., 14., 15.],
             [16., 17., 18., 19.]])
     tensor([[ 0., 2., 4., 6.],
             [8., 10., 12., 14.],
             [16., 18., 20., 22.],
             [24., 26., 28., 30.],
             [32., 34., 36., 38.]])
     tensor([[ 0., 1., 4.,
                                9.],
             [ 16., 25., 36., 49.],
             [ 64., 81., 100., 121.],
             [144., 169., 196., 225.],
             [256., 289., 324., 361.]])
```

```
tensor([[ 2., 3., 4., 5.],
             [6., 7., 8., 9.],
             [10., 11., 12., 13.],
             [14., 15., 16., 17.],
             [18., 19., 20., 21.]])
[17]: # Summations (analogously for A.mean())
      print(A.sum())
      print(A.sum(dim=0))
      print(A.sum(dim=1))
     tensor(190.)
     tensor([40., 45., 50., 55.])
     tensor([ 6., 22., 38., 54., 70.])
[18]: # Functions are applied elementwise
      print(torch.exp(A))
      print(A**2)
      print(torch.pow(A, 2))
      print(torch.cos(A))
     tensor([[1.0000e+00, 2.7183e+00, 7.3891e+00, 2.0086e+01],
             [5.4598e+01, 1.4841e+02, 4.0343e+02, 1.0966e+03],
             [2.9810e+03, 8.1031e+03, 2.2026e+04, 5.9874e+04],
             [1.6275e+05, 4.4241e+05, 1.2026e+06, 3.2690e+06],
             [8.8861e+06, 2.4155e+07, 6.5660e+07, 1.7848e+08]])
     tensor([[ 0.,
                      1.,
                            4.,
                                  9.],
             [ 16., 25., 36., 49.],
             [ 64., 81., 100., 121.],
             [144., 169., 196., 225.],
             [256., 289., 324., 361.]])
     tensor([[ 0., 1., 4.,
             [ 16., 25., 36., 49.],
             [ 64., 81., 100., 121.],
             [144., 169., 196., 225.],
             [256., 289., 324., 361.]])
     tensor([[ 1.0000, 0.5403, -0.4161, -0.9900],
             [-0.6536, 0.2837, 0.9602, 0.7539],
             [-0.1455, -0.9111, -0.8391, 0.0044],
             [0.8439, 0.9074, 0.1367, -0.7597],
             [-0.9577, -0.2752, 0.6603, 0.9887]])
[19]: # Concatenation
      print(torch.cat((A, B), dim=0))
      print(torch.cat((A, B), dim=1))
     tensor([[ 0., 1., 2., 3.],
             [4., 5., 6., 7.],
```

```
[ 8., 9., 10., 11.],
    [12., 13., 14., 15.],
    [16., 17., 18., 19.],
    [ 0., 1., 2., 3.],
    [ 4., 5., 6., 7.],
    [ 8., 9., 10., 11.],
    [12., 13., 14., 15.],
    [16., 17., 18., 19.]])

tensor([[ 0., 1., 2., 3., 0., 1., 2., 3.],
    [ 4., 5., 6., 7., 4., 5., 6., 7.],
    [ 8., 9., 10., 11., 8., 9., 10., 11.],
    [12., 13., 14., 15., 12., 13., 14., 15.],
    [16., 17., 18., 19., 16., 17., 18., 19.]])
```

7 Dot Product

• Given two vectors $\mathbf{x}, \mathbf{y} \in \mathbb{R}^d$, we will let $\mathbf{x}^T \mathbf{y}$ denote their *dot product*. The dot product is the sum of the products of corresponding elements:

$$\mathbf{x}^T\mathbf{y} = \sum_{i=1}^d x_i y_i = x_1 y_1 + x_2 y_2 + \ldots + x_d y_d$$

```
[20]: x = torch.arange(4, dtype=torch.float32)
y = torch.ones(4, dtype=torch.float32)
print(x)
print(y)
print(x.dot(y))

tensor([0., 1., 2., 3.])
tensor([1., 1., 1., 1.])
tensor(6.)
```

- Dot products are useful in many different ways.
- Given two vectors $\mathbf{x}, \mathbf{y} \in \mathbb{R}^d$, if the elements of \mathbf{y} are non-negative and sum to one (that is, $\sum_{i=1}^d y_i = y_1 + y_2 + ... + y_d = 1$), then the dot product $\mathbf{x}^T \mathbf{y}$ is the weighted average of the elements in \mathbf{x} by the elements in \mathbf{y}
- Given two vectors $\mathbf{x}, \mathbf{y} \in \mathbb{R}^d$ of *length* one (that is, $\|\mathbf{x}\|_2 = \|\mathbf{y}\|_2 = 1$, as detailed below), the dot product $\mathbf{x}^T \mathbf{y}$ is the cosine of the angle between them.

8 Matrix-Vector Multiplication

• Let $\mathbf{A} \in \mathbb{R}^{m \times n}$ and $\mathbf{x} \in \mathbb{R}^n$. Let us write \mathbf{A} in terms of its row vectors:

$$\mathbf{A} = \begin{bmatrix} \mathbf{a}_1^T \\ \mathbf{a}_2^T \\ \vdots \\ \mathbf{a}_m^T \end{bmatrix},$$

where each $\mathbf{a}_i^T \in \mathbb{R}^n$ is a row vector representing the i^{th} row of the matrix \mathbf{A} .

• Ax is the column vector of length m whose i^{th} element is $\mathbf{a}_i^T \mathbf{x}$:

$$\mathbf{A}\mathbf{x} = egin{bmatrix} \mathbf{a}_1^T \ \mathbf{a}_2^T \ dots \ \mathbf{a}_m^T \end{bmatrix} \mathbf{x} = egin{bmatrix} \mathbf{a}_1^T \mathbf{x} \ \mathbf{a}_2^T \mathbf{x} \ dots \ \mathbf{a}_m^T \mathbf{x} \end{bmatrix}.$$

• Multiplication by $\mathbf{A} \in \mathbb{R}^{m \times n}$ maps vectors from \mathbb{R}^n to \mathbb{R}^m .

```
[21]: # Matrix-vector multiplication
print(A)
print(x)
print(torch.mv(A, x))
```

9 Matrix Multiplication

• Consider two matrices $\mathbf{A} \in \mathbb{R}^{n \times k}$ and $\mathbf{B} \in \mathbb{R}^{k \times m}$:

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1k} \\ a_{21} & a_{22} & \cdots & a_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nk} \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1m} \\ b_{21} & b_{22} & \cdots & b_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ b_{k1} & b_{k2} & \cdots & b_{km} \end{bmatrix}.$$

• Denote by $\mathbf{a}_i^T \in \mathbb{R}^k$ the row vector corresponding to the i^{th} row of \mathbf{A} , and by $\mathbf{b}_j \in \mathbb{R}^k$ the column vector corresponding to the j^{th} column of \mathbf{B} , so that

$$\mathbf{A} = egin{bmatrix} \mathbf{a}_1^T \ \mathbf{a}_2^T \ dots \ \mathbf{a}_n^T \end{bmatrix}, \quad \mathbf{B} = egin{bmatrix} \mathbf{b}_1 & \mathbf{b}_2 & \cdots & \mathbf{b}_m \end{bmatrix}.$$

• $\mathbf{AB} \in \mathbb{R}^{n \times m}$ is a matrix given by:

$$\mathbf{A}\mathbf{B} = \begin{bmatrix} \mathbf{a}_1^T \\ \mathbf{a}_2^T \\ \vdots \\ \mathbf{a}_n^T \end{bmatrix} \begin{bmatrix} \mathbf{b}_1 & \mathbf{b}_2 & \cdots & \mathbf{b}_m \end{bmatrix} = \begin{bmatrix} \mathbf{a}_1^T \mathbf{b}_1 & \mathbf{a}_1^T \mathbf{b}_2 & \cdots & \mathbf{a}_1^T \mathbf{b}_m \\ \mathbf{a}_2^T \mathbf{b}_1 & \mathbf{a}_2^T \mathbf{b}_2 & \cdots & \mathbf{a}_2^T \mathbf{b}_m \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{a}_n^T \mathbf{b}_1 & \mathbf{a}_n^T \mathbf{b}_2 & \cdots & \mathbf{a}_n^T \mathbf{b}_m \end{bmatrix}.$$

```
[22]: # Matrix(-matrix) multiplication
      print(A)
      B = torch.arange(12, dtype=torch.float32).reshape(4,3)
      print(B)
      C = torch.mm(A, B)
      print(C)
     tensor([[ 0., 1., 2., 3.],
             [4., 5., 6., 7.],
             [8., 9., 10., 11.],
             [12., 13., 14., 15.],
             [16., 17., 18., 19.]])
     tensor([[ 0., 1., 2.],
             [3., 4., 5.],
             [6., 7., 8.],
             [ 9., 10., 11.]])
     tensor([[ 42., 48., 54.],
             [114., 136., 158.],
```

10 Norms

- Let $\mathbf{x}, \mathbf{y} \in \mathbb{R}^d$ be vectors and $\alpha \in \mathbb{R}$ be a scalar. A *norm* is a function f that maps a vector to a scalar. A *norm* must satisfy the following properties:
 - 1. $f(\mathbf{x}) \geq 0$.
 - 2. $f(\mathbf{x} + \mathbf{y}) \le f(\mathbf{x}) + f(\mathbf{y})$.

[186., 224., 262.], [258., 312., 366.], [330., 400., 470.]])

- 3. $f(\alpha \mathbf{x}) = |\alpha| f(\mathbf{x})$.
- 4. $\mathbf{x} = \mathbf{0} \Leftrightarrow f(\mathbf{x}) = 0$, where **0** denotes the zero vector.
- The L_2 norm $\|\cdot\|_2$ gives the square root of the sum of the squares of the elements of a vector:

$$\|\mathbf{x}\|_2 = \sqrt{\sum_{i=1}^d x_i^2} = \sqrt{x_1^2 + x_2^2 + \ldots + x_d^2},$$

which is the length of the vector. Because of the importance of this norm, the subscript 2 is often omitted, so that $\|\mathbf{x}\| = \|\mathbf{x}\|_2$.

• The L_1 norm $\|\cdot\|_1$ gives the sum of the absolute values of the elements of a vector:

$$\|\mathbf{x}\|_1 = \sum_{i=1}^d |x_i| = |x_1| + |x_2| + \ldots + |x_d|.$$

• The L_i distance between vectors $\mathbf{x}, \mathbf{y} \in \mathbb{R}^d$ is defined through the L_i norm as $\|\mathbf{x} - \mathbf{y}\|_i$.

11 Broadcasting Mechanism

- Mathematically, elementwise operations between tensors require them to have the same shape.
- Conveniently, under certain conditions, PyTorch can interpret and perform elementwise operations between tensors of different shapes using the *broadcasting mechanism*:
 - First, one or both tensors are expanded by copying elements in order to create two tensors with the same shape.
 - Second, the elementwise operation is carried on the resulting tensors.
- We typically broadcast across a dimension where an array has length 1, such as in the following example.

```
[23]: print('Tensors:')
      a = torch.arange(3).reshape((3, 1))
      b = torch.arange(2).reshape((1, 2))
      print(a)
      print(b)
      print('Sum with broadcasting:')
      print(a + b)
      print('Expanded tensors:')
      a expanded = torch.cat((a, a), dim=1)
      b expanded = torch.cat((b, b, b), dim=0)
      print(a expanded)
      print(b_expanded)
      print('Sum with expansion:')
      print(a_expanded + b_expanded)
     Tensors:
     tensor([[0],
              [1],
              [2]])
     tensor([[0, 1]])
     Sum with broadcasting:
     tensor([[0, 1],
              [1, 2],
              [2, 3]])
     Expanded tensors:
     tensor([[0, 0],
              [1, 1],
              [2, 2]])
     tensor([[0, 1],
              [0, 1],
              [0, 1]
     Sum with expansion:
     tensor([[0, 1],
              [1, 2],
```

```
[2, 3]])
```

```
[24]: # Another example
      A = torch.arange(20, dtype=torch.float32).reshape(5, 4)
      print(A)
      B1 = A.sum(dim=1);
      B2 = A.sum(dim=1, keepdims=True);
      print(B1)
      print(B2)
      print(B1.size())
      print(B2.size())
     tensor([[ 0., 1., 2., 3.],
             [4., 5., 6., 7.],
             [8., 9., 10., 11.],
             [12., 13., 14., 15.],
             [16., 17., 18., 19.]])
     tensor([ 6., 22., 38., 54., 70.])
     tensor([[ 6.],
             [22.],
             [38.],
             [54.],
             [70.11)
     torch.Size([5])
     torch.Size([5, 1])
[25]: A/B2 # A/B1 does not work
[25]: tensor([[0.0000, 0.1667, 0.3333, 0.5000],
              [0.1818, 0.2273, 0.2727, 0.3182],
              [0.2105, 0.2368, 0.2632, 0.2895],
              [0.2222, 0.2407, 0.2593, 0.2778],
              [0.2286, 0.2429, 0.2571, 0.2714]])
```

12 Tensor Indexing and Slicing

- Elements in a tensor can be accessed by indices that start at zero
- The range i:j includes all the elements from index i up to index j, including the element at i but excluding the element at j (just like the function range)
- As in Python lists, a negative index can be used to access elements starting from the last element along a dimension (for example, -1 denotes the index of last element along a dimension)

```
[26]: # Indexing and slicing an array
A = torch.arange(5)
print(A)
print(A[2:5])
```

```
print(A[-2])
      print(A[:3]) # A[0:3]
      print(A[2:]) # A[2:5]
      print(A[:]) # A[0:5]
     tensor([0, 1, 2, 3, 4])
     tensor([2, 3, 4])
     tensor(3)
     tensor([0, 1, 2])
     tensor([2, 3, 4])
     tensor([0, 1, 2, 3, 4])
[27]: # Indexing and slicing a matrix
      X = torch.arange(20, dtype=torch.float32).reshape(5, 4)
      print(X)
      print(X[1:3, :])
      print(X[1:3, :2])
      print(X[-1, :])
      print(X[-3:-1, :])
     tensor([[ 0., 1., 2., 3.],
             [4., 5., 6., 7.],
             [8., 9., 10., 11.],
             [12., 13., 14., 15.],
             [16., 17., 18., 19.]])
     tensor([[ 4., 5., 6., 7.],
             [8., 9., 10., 11.]])
     tensor([[4., 5.],
             [8., 9.]])
     tensor([16., 17., 18., 19.])
     tensor([[ 8., 9., 10., 11.],
             [12., 13., 14., 15.]])
```

13 Saving Memory

- Running operations can cause new memory to be allocated to store the results.
- We do not want to allocate memory unnecessarily all the time.
 - In machine learning, we might have hundreds of megabytes of parameters (or more!)
- Where possible, we want to perform operations *in-place*.

```
[28]: # In-place example
X = torch.arange(20, dtype=torch.float32).reshape(5, 4)
Y = 10*X
print(id(X), id(Y))
Y = Y + X # not in-place
print(id(X), id(Y))
Y += X # in-place
```

```
print(id(X), id(Y))
      Y[:] = Y + X \# in-place, overrides the content of the tensor Y with the tensor
       \hookrightarrow Y + X
      print(id(X), id(Y))
     136511199190432 136511199184272
     136511199190432 136511199190992
     136511199190432 136511199190992
     136511199190432 136511199190992
[29]: # Warning: the assignment operator `=` does not copy data, it just assigns names
      A = torch.arange(20, dtype=torch.float32).reshape(5, 4)
      B = A
      B[0, :] = -1
      print(A)
      # The method `clone` creates a copy of a tensor
      A = torch.arange(20, dtype=torch.float32).reshape(5, 4)
      B = A.clone()
      B[0, :] = -1
      print(A)
     tensor([[-1., -1., -1., -1.],
             [4., 5., 6., 7.],
             [8., 9., 10., 11.],
             [12., 13., 14., 15.],
             [16., 17., 18., 19.]])
     tensor([[ 0., 1., 2., 3.],
             [4., 5., 6., 7.],
             [8., 9., 10., 11.],
             [12., 13., 14., 15.],
             [16., 17., 18., 19.]])
     14 Converting Tensors to Other Objects
[30]: # Converting to a NumPy array, or vice-versa
      A = X.numpy()
      print(type(A))
      B = torch.tensor(A)
      print(type(B))
```

<class 'numpy.ndarray'>
<class 'torch.Tensor'>

a = torch.tensor([3.5])

print(a)

[31]: # Converting a size-1 tensor to a Python scalar

```
print(a.item())
print(float(a))
print(int(a))

tensor([3.5000])
3.5
3.5
```

15 Recommended reading

3

• Dive into Deep Learning: Chapters 1, 2.1, 2.2, and 2.3.

16 [Storing this notebook as a pdf]