The Specification of Signal Track Query Language

1 Data Model

An **interval** is a DNA segment. Formally, an interval is a complex data type, which contains several fields as below:

- chr: The name of the chromosome on which the DNA segment is. Its type is string. It is a compulsory field.
- chrstart: The starting position of the DNA segment (inclusive). Its type is integer. It is a compulsory field. The first base in a chromosome is numbered 1.
- chrend: The ending position of the DNA segment (inclusive). Its type is integer. It is a compulsory field. It must be no less than the starting position (i.e., chrend ≥ chrstart), and cannot be larger than the length of the chromosome. If chrend is equal to chrstart, the interval is a single-position interval.
- value: The value associated to the DNA segment. Its type is float. It could be NULL. It is a compulsory field.
- strand: The strand on which the DNA segment is. Valid entries include "+", "-" or ".", meaning the positive strand, negative strand, do not know or do not care which strand, respectively. Its type is string. It is a optional field.
- other meta-data: other optional meta-data fields the DNA segment has. For example, a feature has a field called source showing the program that generated the feature. A DNA segment may has a name, such as a gene.

We define two value models to manipulate intervals' values: **each** and **total**:

- each: Each position within the interval is associated with the value.
- total: All positions within the interval collectively have the value. E-quivalently, each position has an equally partial amount of the value.

A **signal track** is a set of intervals. Formally, we define a track as a table such that

1. It is a unary relation, containing only one column of type interval.

2. Every tuple in it has the exact same interval fields. It is not allowed that one tuple has some field but another one does not have that field.

The intervals in the same track could be overlapping, and are not necessarily ordered.

2 Interval Location Comparison Relations

All eleven location comparison relations are given in Table 1.

Location Relations	Definition	Remark
I_1	I_1 .chr = I_2 .chr and (I_1 .chrend + 1 = I_2 .chrstart or	
is adjacent	I_1 .chrstart $-1 = I_2$.chrend)	It is symmetric.
to I_2	,	
I_1 coincides with I_2	I_1 .chr = I_2 .chr and I_1 .chrstart = I_2 .chrstart and	
	I_1 .chrend = I_2 .chrend	It is symmetric.
I_1 overlaps	I_1 .chr = I_2 .chr and I_1 .chrstart $\leq I_2$.chrend and	
with I_2	I_1 .chrend $\geq I_2$.chrstart	It is symmetric.
I_1	I_1 .chr = I_2 .chr and I_1 .chrstart $\leq I_2$.chrstart and	
contains I_2	I_1 .chrend $\geq I_2$.chrend	It is the inverse relation of is-within.
I_1 is	$I_1.\text{chr} = I_2.\text{chr} \text{ and } I_1.\text{chrstart} \ge I_2.\text{chrstart and}$	
within I_2	I_1 .chrend $\leq I_2$.chrend	It is the inverse relation of contains.
I_1	$I_1.\text{chr} = I_2.\text{chr}$ and $I_1.\text{chrstart} = I_2.\text{chrstart}$ and	
is prefix of	I_1 .chrend $\leq I_2$.chrend	
I_2		
-1	$I_1.\mathrm{chr} = I_2.\mathrm{chr}$ and $I_1.\mathrm{chr}$ start $\geq I_2.\mathrm{chr}$ start and	
is suffix of	I_1 .chrend = I_2 .chrend	
I_2		
I_1 precedes I_2	I_1 .chr = I_2 .chr and I_1 .chrend < I_2 .chrstart	
		It is the inverse relation of follows.
I_1 follows I_2	$I_1.\text{chr} = I_2.\text{chr} \text{ and } I_1.\text{chrstart} > I_2.\text{cheend}$	
		It is the inverse relation of precedes.
I_1	$I_1.\text{chr} = I_2.\text{chr} \text{ and } ($	L., , , , , ,
is upstream	$(I_2.\text{strand} = '+' \text{ and } I_1.\text{strand} = '+' \text{ and } I_1 \text{ precedes } I_2) \text{ or}$	Only when the two intervals have the same strand,
of I_2	$(I_2.\text{strand} = '+' \text{ and } I_1.\text{strand} = '.' \text{ and } I_1 \text{ precedes } I_2) \text{ or}$	it is the inverse relation of is-downstream-of;
	$(I_2.\text{strand} = '-' \text{ and } I_1.\text{strand} = '-' \text{ and } I_1 \text{ follows } I_2) \text{ or}$	otherwise,
	$(I_2.\text{strand} = \text{`-'} \text{ and } I_1.\text{strand} = \text{`.'} \text{ and } I_1 \text{ is follows } I_2))$	it is not the inverse relation of is-downstream-of.
I_1	$I_1.\text{chr} = I_2.\text{chr} \text{ and } ($	L., , , , , ,
is downstream	$(I_2.\text{strand} = '+' \text{ and } I_1.\text{strand} = '+' \text{ and } I_1 \text{ follows } I_2) \text{ or}$	Only when the two intervals have the same strand,
of I_2	$(I_2.\text{strand} = '+' \text{ and } I_1.\text{strand} = '.' \text{ and } I_1 \text{ follows } I_2) \text{ or}$	it is the inverse relation of is-upstream-of;
	$(I_2.\text{strand} = '-' \text{ and } I_1.\text{strand} = '-' \text{ and } I_1 \text{ precedes } I_2) \text{ or}$	otherwise,
	$(I_2.\text{strand} = '-' \text{ and } I_1.\text{strand} = '.' \text{ and } I_1 \text{ is precedes } I_2))$	it is not the inverse relation of is-upstream-of.

Table 1: Location Comparison Relations

3 Interval Operations

Length of an interval Given one interval I, its length, i.e., length(I), is defined as I.chrend -I.chrestart +1.

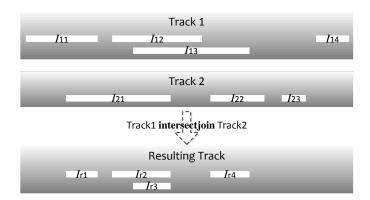
Distance between two intervals Given two intervals I_1 , I_2 , their distance,

i.e., distance(I_1 , I_2), is defined as below:

$$\operatorname{distance}(I_1,I_2) = \left\{ \begin{array}{ll} I_2.\operatorname{chrstart} - I_1.\operatorname{chrend} & \text{ if } I_1 \text{ precedes } I_2 \\ 0 & \text{ if } I_1 \text{ overlaps with } I_2 \\ I_1.\operatorname{chrstart} - I_2.\operatorname{chrend} & \text{ if } I_1 \text{ follows } I_2 \\ \operatorname{NaN} & \text{ if } I_1.\operatorname{chr} \neq I_2.\operatorname{chr} \end{array} \right.$$

4 Track Operations

Intersectjoin Given two tracks T_1 , T_2 , for each pair of overlapping intervals I_1 , I_2 , where I_1 is from T_1 and I_2 is from T_2 , we compute the common fragment of I_1 and I_2 . We denoted the operation of finding the common fragment of I_1 and I_2 by intersect (I_1, I_2) . Let $I_r = \text{intersect}(I_1, I_2)$.



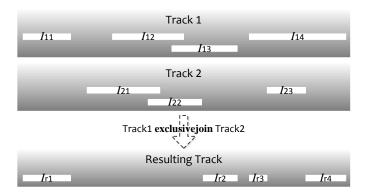
Assume that I_1 's value is v_1 and I_2 's value is v_2 . If we use each model, we can derive the value of I_r , v_r , by using any of the nine types of value-derivation in Table 2.

If we use total model, let v_{f1} (= $v_1 \times \frac{\text{length}(I_r)}{\text{length}(I_1)}$), v_{f2} (= $v_2 \times \frac{\text{length}(I_r)}{\text{length}(I_2)}$), we can derive the value of I_r , v_r , by using any of the nine types of value-derivation in Table 3.

 I_r can inherits all meta-data from I_1 . Specifically, if I_1 has $n \geq 0$ meta-data fields, $f_1, f_2, \ldots, f_n, I_r$ also has these n fields and $I_r.f_1 = I_1.f_1, I_r.f_2 = I_1.f_2, \ldots, I_r.f_n = I_1.f_n$.

Exclusivejoin Given two tracks T_1 , T_2 , for each interval I_1 in T_1 , we compute the fragment(s) of I_1 that are not covered by any interval in T_2 . Suppose that I_r is such one of the fragments of I_1 .

Here we have only one type of value-derivation, i.e., vd_left, which means that we derive I_r .value from the value of I_1 . Assume that I_1 's

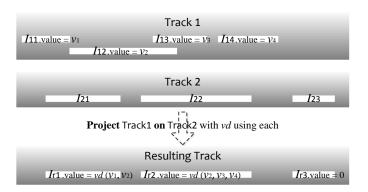


value is v_1 . If we use each model, I_r .value = v_1 . If we use total model, I_r .value = $v_1 \times \frac{\text{length}(I_r)}{\text{length}(I_1)}$.

 I_r can inherits all meta-data from I_1 .

Project-on Given two tracks T_1 , T_2 , for each interval I in T_2 , we generate an interval I_r that coincides with I, and derive a value for I_r from the values of the intervals in T_1 that overlaps with I_r (or I). It implies that we calculate a value for I_r based on its corresponding positions in T_1 . We propose five types of value-derivation.

If we use each model, we can derive the value of I_r by using any type of value-derivation in Table 4.

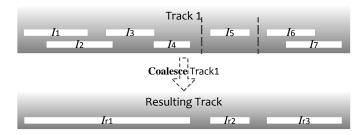


If we use total model, we can derive the value of I_r by using any type of value-derivation in Table 5.

 I_r can inherits all meta-data from I.

Coalesce Given a track T, we merge all overlapping and/or adjacent intervals. Consider a subset S of T such that the intervals in S span

continuous positions, we generate an interval I_r , whose starting position is the minimum of the starting positions of all intervals in S, and whose ending position is the maximum of the ending positions of all intervals in S. We can also derive a value for I_r . We have five types of value-derivation for coalesce as we have for project-on.



If we use each model, like project-on, for each position p within I_r , we first compute the value at p based on the values of the intervals in S that covers p, by using one type of value-derivation. Then, we compute the average of values across all positions of I_r .

If we use total model, how we derive I_r value is shown in Table 6.

The coalesce operation does not support reservation of meta-data.

Discretize Given a track T, we break down overlapping intervals using their starting and ending positions so that all resulting intervals are disjoint. Consider a subset S of T such that the intervals in S span continuous positions, we generate a list of adjacent intervals, each spanning from the starting position of some interval in S, to the ending position of some interval in S. Consider one interval in such a list, I_r . Here we also propose five types of value-derivation, each of which can be used to derive the value of I_r .



Suppose that the set \hat{S} comprises of all intervals in S that contains I_r . The derivation of I_r value under each different value model is shown in Table 7, Table 8, respectively.

The discretize operation does not support reservation of meta-data.

5 STQL Grammar Rules

```
QUERY := SELECT\_STAT FROM\_STAT (WHERE\_STAT)? (GROUPBY\_STAT)?
(ORDERBY_STAT)?
  FROM\_STAT := from FROM\_SOURCE
  FROM\_SOURCE := TRACK\ UNION\ TRACK\ |\ MULTIPLETRACK
  TRACK := RAW\_TRACK \mid TRANSFORM\_RES \mid OVERLAPJOIN\_RES
| SUBQUERY
  MULTIPLETRACK := TRACK (, TRACK)^*
  UNION := union all
  RAW_TRACK := Identifier (TRACKALIAS)?
  TRACKALIAS := Identifier
  TRANSFORM\_RES := TRANSFORM\_OP \mid LBracket TRANSFORM\_OP
RBracket TRACKALIAS
  TRANSFORM_OP := TRANSFORM (with VALUE_DER)?
  TRANSFROM := COALESCE TRACK | DISCRETIZE TRACK
  COALESCE := coalesce
  DISCRETIZE := discretize
  OVERLAPJOIN\_RES := OVERLAPJOIN\_OP \mid LBracket OVERLAPJOIN\_OP
RBracket TRACKALIAS
  OVERLAPJOIN_OP := OVERLAPJOIN (with (VALUE_DER_METADATA
| VALUE_DER | META_DATA))?
  OVERLAPJOIN := INTERSECTJOIN | EXCLUSIVEJOIN | PROJECT
  INTERSECTJOIN := TRACK intersection TRACK
  EXCLUSIVEJOIN := TRACK exclusivejoin TRACK
  PROJECT := project TRACK on (TRACK | CREATE_BINS)
  CREATE_BINS := generate bins with length Integer
  VALUE_DER_METADATA := VALUE_DER, META_DATA | META_DATA,
VALUE_DER
  VALUE_DER := VD_TYPE using VALUE_MODEL
  VD_TYPE := vd_sum | vd_diff | vd_product | vd_quotient | vd_avg |
vd_max | vd_min | vd_left | vd_right
  META\_DATA := metadata
  VALUE\_MODEL := VM\_TYPE model
  VM_TYPE := each \mid all
  SELECT\_STAT := select ((distinct)? FIELD (, FIELD)* | SELALLEXP)
  SELALLEXP := *
  FIELD := ARITH\_FUNC \mid AGG
```

```
ARITH\_FUNC := (MUL\_DIV \mid Number) ((+ \mid -) (MUL\_DIV \mid Num-
ber))?
   MUL_DIV := (ELEM \mid Number) ((* \mid /) (ELEM \mid Number))?
  ELEM := INTERVAL_ATTR | LBracket ARITH_FUNC RBracket
  INTERVAL\_ATTR := INTERVAL.ATTRNAME \mid TRACKNAME.ATTRNAME
  INTERVAL := TRACKNAME.interval
  TRACKNAME := Identifier \mid TRACKALIAS
  ATTRNAME := chr | chrstart | chrend | value | Identifier
   AGG := AGG_FUNC LBracket INTERVAL_ATTR RBracket | COUN-
T_ALL
   AGG\_FUNC := count \mid max \mid min \mid avg \mid sum
  COUNT_ALL := count LBracket SELALLEXP RBracket
   WHERE\_STAT := where OR\_PREDICATE
  OR\_PREDICATE := AND\_PREDICATE (or AND\_PREDICATE)?
  AND\_PREDICATE := NOT\_PREDICATE (and NOT\_PREDICATE)?
  NOT_PREDICATE := PREDICATE | not (PREDICATE | LBracket
OR_PREDICATE RBracket)
  PREDICATE := NUMERIC_COMP | LOCATION_COMP | PATTERN_MATCHING
  NUMERIC_COMP := (INTERVAL_ATTR | INTERVAL_LENGTH | IN-
TERVAL_DIS | Number) COMP_OP (INTERVAL_ATTR | INTERVAL_LENGTH
| INTERVAL_DIS | Number)
  INTERVAL_LENGTH := length LBracket (INTERVAL | CONS_INTERVAL)
RBracket
  INTERVAL_DIS := distance LBracket (INTERVAL | CONS_INTERVAL)
, (INTERVAL | CONS_INTERVAL) RBracket
  COMP_OP := < | = | ! = | > | <= | > =
  LOCATION_COMP := (INTERVAL | CONS_INTERVAL) LOC_COMP_OP
(INTERVAL | CONS_INTERVAL)
  LOC_COMP_OP := overlaps with | precedes | follows | matches | is prefix
of | is suffix of | coincides with | is within | contains | is upstream of | is
downstream of | is closest to
   CONS_INTERVAL := LeftSquareBracket CHR, CHRSTART, CHREND
(,STRAND)? RightSquareBracket
  CHR := Idendifier
  \mathbf{CHRSTART} := \mathbf{Integer}
  CHREND := Integer
  STRAND := + | -
  PATTERN\_MATCHING := (not)? like RegularExpression
  GROUPBY_STAT := group by INTERVAL_ATTR (, INTERVAL_ATTR)*
  ORDERBY\_STAT := order\ by\ INTERVAL\_ATTR\ (,\ INTERVAL\_ATTR)^*
  SUBQUERY := LBracket QUERY RBracket Identifier
```

6 STQL Query Examples

Q1 Find the locations of all intervals in a track T.

SELECT $T.\mathrm{interval.chr}, T.\mathrm{interval.chrstart}, T.\mathrm{interval.chrend}$ FROM T

Q2 Divide the whole genome into 100bp bins, and compute the value for each bin based on the values at corresponding positions in a track T using vd_sum and each model.

SELECT

FROM PROJECT T ON GENERATE BINS

WITH LENGTH 100 WITH vd_sum USING each model

Q3 Find all overlapping interval pairs, one from T_1 , the other from T_2 .

SELECT * FROM T_1 , T_2

WHERE T_1 .interval ovelaps with T_2 .interval

Q4 Find pairs of intervals such that (1) the first interval is from a track T_1 , covering a position p in chr1, and the second one is from T_2 ; (2) the first interval precedes the second one; (3) their distance is no larger than 10kbp.

SELECT *
FROM T_1, T_2

WHERE T1.interval contains [chr1, p, p] and T_1 .interval precedes T_2 .interval and distance $(T_1$.interval, T_2 .interval) <= 10000

Q5 Find the number of intervals in T with value larger than v for each chromosome.

SELECT T.interval.chr, count(*)

FROM T

WHERE T.interval.value > v GROUP BY T.interval.chr

Q6 Find the locations of all intervals in T_1 and T_2 .

SELECT *

FROM (SELECT T_1 .interval.chr, T_1 .interval.chrstart, T_1 .interval.chrend FROM T_1

UNION ALL

SELECT T_2 .interval.chr
, T_2 .interval.chr
start, T_2 .interval.chrend FROM T_2) t

value-derivation	$v_r =$	remark
vd_sum	$v_1 + v_2$	The sum of v_1 and v_2
vd_avg	$(v_1 + v_2)/2$	The average of v_1 and v_2
vd_diff	$v_1 - v_2$	The difference of v_1 and v_2
vd_product	$v_1 \times v_2$	The product of v_1 and v_2
vd_quotient	$v_1 \div v_2$	The quotient of v_1 and v_2
vd_max	$\max\{v_1, v_2\}$	The maximum of v_1 and v_2
vd_min	$\min\{v_1, v_2\}$	The minimum of v_1 and v_2
vd_left	v_1	I_1 .value
vd_right	v_2	I_2 .value

Table 2: Nine types of value-derivation for intersectjoin, under each model

value-derivation	$v_r =$	remark
vd_sum	$v_{f1} + v_{f2}$	The sum of v_{f1} and v_{f2}
vd_avg	$(v_{f1} + v_{f2})/2$	The average of v_{f1} and v_{f2}
vd_diff	$v_{f1} - v_{f2}$	The difference of v_{f1} and v_{f2}
vd_product	$v_{f1} \times v_{f2}$	The product of v_{f1} and v_{f2}
$vd_quotient$	$v_{f1} \div v_{f2}$	The quotient of v_{f1} and v_{f2}
vd_max	$\max\{v_{f1}, v_{f2}\}$	The maximum of v_{f1} and v_{f2}
vd_min	$\min\{v_{f1}, v_{f2}\}$	The minimum of v_{f1} and v_{f2}
vd_left	v_{f1}	The fractional share of I_1 .value
vd_right	v_{f2}	The fractional share of I_2 .value

Table 3: Nine types of value-derivation for intersectjoin, under total model

value-derivation	I_r .value =	remark
vd_sum	$\frac{\sum\limits_{p=I_r.\text{chrend}}\sum\limits_{\hat{I}\in\hat{S}}\hat{I}.\text{value}}{\sum\limits_{p=I_r.\text{chrstart}}\hat{S}=\{I' I'\in T_1\land I'\text{ covers }p\}}$ $\frac{\hat{S}=\{I' I'\in T_1\land I'\text{ covers }p\}}{\text{length}(I_r)}$	For each p within I_r , compute the sum of values of the intervals in T_1 that covers it. Compute the average value across I_r .
vd_avg	$\frac{I_r.\text{chrend}}{\sum\limits_{p=I_r.\text{chrstart}} \left(\frac{\hat{S}=\{I' I'\in T_1 \wedge I' \text{ covers } p\}}{\hat{S}=\{I' I'\in T_1 \wedge I' \text{ lovers } p\}}\right)}{\text{length}(I_r)}$	For each p within I_r , compute the average of values of the intervals in T_1 that covers it. Compute the average value across I_r .
vd_product	$\frac{\sum\limits_{p=I_r.\text{chrend}}^{I_r.\text{chrend}}\prod\limits_{\hat{I}\in\hat{S}}\hat{I}.\text{value}}{\sum\limits_{p=I_r.\text{chrstart}}^{\hat{I}\in\hat{I}}\hat{S}=\{I' I'\in T_1\land I'\text{ covers }p\}}$ $\frac{\hat{S}=\{I' I'\in T_1\land I'\text{ covers }p\}}{\text{length}(I_r)}$	For each p within I_r , compute the product of values of the intervals in T_1 that covers it. Compute the average value across I_r .
vd_max	$\frac{\sum\limits_{p=I_r.\text{chrend}}\sum\limits_{\hat{I}\in\hat{S}}\hat{I}.\text{value}}{\sum\limits_{p=I_r.\text{chrstart}}\hat{S}=\{I' I'\in T_1\land I'\text{ covers }p\}}$ $\frac{\hat{S}=\{I' I'\in T_1\land I'\text{ covers }p\}}{\text{length}(I_r)}$	For each p within I_r , compute the maximum of values of the intervals in T_1 that covers it. Compute the average value across I_r .
vd_min	$\frac{\sum\limits_{p=I_r.\text{chrend}}\sum\limits_{\hat{I}\in\hat{S}}\hat{I}.\text{value}}{\sum\limits_{p=I_r.\text{chrstart}}\hat{S}=\{I' I'\in T_1\land I'\text{ covers }p\}}$	For each p within I_r , compute the minimum of values of the intervals in T_1 that covers it. Compute the average value across I_r .

Table 4: Five types of value-derivation for project-on, under each model

value-derivation	I_r .value =	remark			
		The			
vd_sum	$\sum_{I' \in T_1 \wedge} \left(I'.\text{value} \times \frac{\text{length}(\text{intersect}(I_r, I'))}{\text{length}(I')} \right)$				
VG_Sum					
	I' ovelaps with I_r	values			
	$\sum_{I' \in T_1 \land} \left(I'.\text{value} \times \frac{\text{length}(\text{intersect}_{(I_r, I')})}{\text{length}_{(I')}} \right)$	The			
vd_avg	$\frac{I \text{ overlaps with } I_r}{ \{I' I' \in T_1 \land I' \text{ overlaps with } I_r\} }$	age			
		of the			
		values			
		The			
	$\left \prod_{\substack{I' \in T_1 \land \\ I' \text{ ovelaps with } I_r}} \left(I'.\text{value} \times \frac{\text{length}(\text{intersect}(I_r, I'))}{\text{length}(I')} \right) \right $				
vd_product					
	. ,	values			
		The			
,	$\max_{I' \in T_1 \land} \qquad \left(I'.\text{value} \times \frac{\text{length}(\text{intersect}(I_r, I'))}{\text{length}(I')} \right)$				
vd_max					
	I' ovelaps with I_r	of the			
		values			
vd_min		The mini-			
	$\min_{\substack{I' \in T_1 \land \\ I' \text{ ovelaps with } I_r}} \left(I'.\text{value} \times \frac{\text{length}(\text{intersect}(I_r, I'))}{\text{length}(I')} \right)$				
					values

Table 5: Five types of value-derivation for project-on, under total model

value-derivation	I_r .value =	remark
vd_sum	$\sum_{I_i \in S} I_i$.value	The sum of the values of the intervals in S
vd_avg	$\left(\sum_{I_i \in S} I_i.\text{value}\right)/ S $	The average of the values of the intervals in S
vd_product	$\prod_{I_i \in S} I_i. \text{value}$	The product of the values of the intervals in S
vd_max	$\max_{I_i \in S} I_i. \text{value}$	The maximum of the values of the intervals in S
vd_min	$\min_{I_i \in S} I_i. \text{value}$	The minimum of the values of the intervals in S

Table 6: Five types of value-derivation for coalesce, under total model

value-derivation I_r .value =		remark	
vd_sum	$\sum_{\hat{I} \in \hat{S}} \hat{I}.\text{value}$	The sum of the values the intervals in \hat{S}	
vd_avg	$\left(\sum_{\hat{I}\in\hat{S}}\hat{I}.\text{value}\right)/ \hat{S} $	The average of the values the intervals in \hat{S}	
vd_product	$\prod_{\hat{I} \in \hat{S}} \hat{I}.\text{value}$	The product of the values the intervals in \hat{S}	
vd_max	$\max_{\hat{I} \in \hat{S}} \hat{I}.\text{value}$	The maximum of the values the intervals in \hat{S}	
vd_min	$\min_{\hat{I} \in \hat{S}} \hat{I}.\text{value}$	The minimum of the values the intervals in \hat{S}	

Table 7: Five types of value-derivation for discretize, under each model $\,$

value-derivation	$I_r. \text{value} =$	remark
vd_sum	$\sum_{\hat{I} \in \hat{S}} \left(\hat{I}.\text{value} \times \frac{\text{length}(I_r)}{\text{length}(\hat{I})} \right)$	The sum of the values
vd_avg	$\left \left(\sum_{\hat{I} \in \hat{S}} \left(\hat{I}.\text{value} \times \frac{\text{length}(I_r)}{\text{length}(\hat{I})} \right) \right) / \hat{S} \right $	The average of the values
vd_product	$\prod_{\hat{I} \in \hat{S}} \left(\hat{I}.\text{value} \times \frac{\text{length}(I_r)}{\text{length}(\hat{I})} \right)$	The product of the values
vd_max	$\max_{\hat{I} \in \hat{S}} \left(\hat{I}.\text{value} \times \frac{\text{length}(I_r)}{\text{length}(\hat{I})} \right)$	The maximum of the values
vd_min	$\min_{\hat{I} \in \hat{S}} \left(\hat{I}.\text{value} \times \frac{\text{length}(\hat{I}_r)}{\text{length}(\hat{I})} \right)$	The minimum of the values

Table 8: Five types of value-derivation for discretize, under total model