

Rank Optimization

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1 Introduction

We devise here an analytical solution for the rank optimization for the approximation through random projections. This solution is then compared to the numerical approach.

2 Filling equation

$$u' = -\frac{P_s u^2}{P_s u^2 + P_n v^2} / s \quad (1)$$

$$v' = -\frac{P_n v^2}{P_s u^2 + P_n v^2} / n \quad (2)$$

we deduce

$$\frac{s}{P_s} \frac{u'}{u^2} - \frac{n}{P_n} \frac{v'}{v^2} = 0 \quad (3)$$

$$s u' + n v' = -1 \quad (4)$$

with $\alpha = \frac{s}{P_s}$ $\beta = \frac{n}{P_n}$
it follows

$$\frac{\alpha}{u} - \frac{\beta}{v} = k \quad (5)$$

$$s u + n v = l - t \quad (6)$$

Conditions à $t = 0$

$$l = s + n \quad (7)$$

$$k = \alpha - \beta \quad (8)$$

Returning to our system we have :

$$v = \frac{\beta u}{\alpha - k u} \quad (9)$$

$$s u + n \frac{\beta u}{\alpha - k u} = l - t \quad (10)$$

Hence

$$-k s u^2 + (s \alpha + n \beta + k(l - t)) u - \alpha(l - t) = 0 \quad (11)$$

giving the solution

$$u = \frac{(s \alpha + n \beta + k(l - t)) - \sqrt{(s \alpha + n \beta + k(l - t))^2 - 4 k s \alpha(l - t)}}{2 k s} \quad (12)$$

With μ being the ratio of signal retrieved,

$$t_{opt} = \mu \frac{(s - \mu) \alpha + n \beta}{k \mu - \alpha} + l \quad (13)$$