Rank Optimization

Lionel Chiron

October 4, 2018

1 Introduction

We devise here an analytical solution for the rank optimization for the approximation through random projections. This solution is then compared to the numerical approach.

2 Filling equation

$$u' = -\frac{P_s u^2}{P_s u^2 + P_n v^2}/s \tag{1}$$

$$v' = -\frac{P_n v^2}{P_s u^2 + P_n v^2}/n \tag{2}$$

we deduce

$$\frac{s}{P_s}\frac{u'}{u^2} - \frac{n}{P_n}\frac{v'}{v^2} = 0 \tag{3}$$

$$su' + nv' = -1 \tag{4}$$

with $\alpha = \frac{s}{P_s}$ $\beta = \frac{n}{P_n}$
it follows

$$\frac{\alpha}{u} - \frac{\beta}{v} = k \tag{5}$$

$$su + nv = l - t \tag{6}$$

Conditions à $t = 0$

$$l = s + n \tag{7}$$

$$k = \alpha - \beta \tag{8}$$

Returning to our system we have :

$$v = \frac{\beta u}{\alpha - ku} \tag{9}$$

$$su + n\frac{\beta u}{\alpha - ku} = l - t \tag{10}$$

Hence

$$-ksu^2 + (s\alpha + n\beta + k(l-t))u - \alpha(l-t) = 0 \tag{11}$$

giving the solution

$$u = \frac{(s\alpha + n\beta + k(l-t)) - \sqrt{(s\alpha + n\beta + k(l-t))^2 - 4ks\alpha(l-t)}}{2ks} \tag{12}$$

With μ being one minus the ratio of signal retrieved and t_{opt} the optimal rank we have,

$$u(t_{opt}) = \mu \tag{13}$$

it gives

$$t_{opt} = \mu \frac{(s-\mu)\alpha + n\beta}{k\mu - \alpha} + l \tag{14}$$