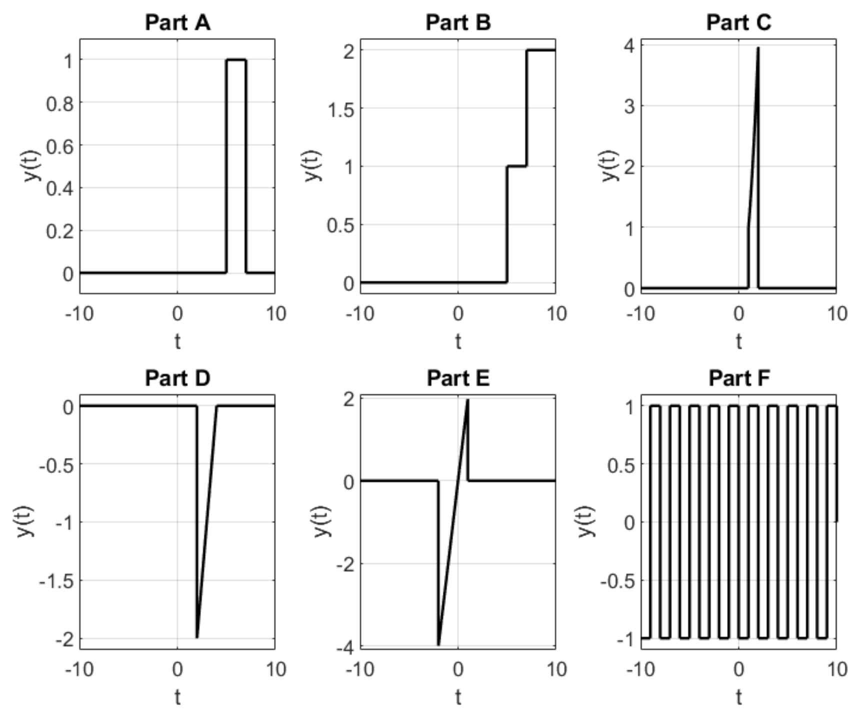
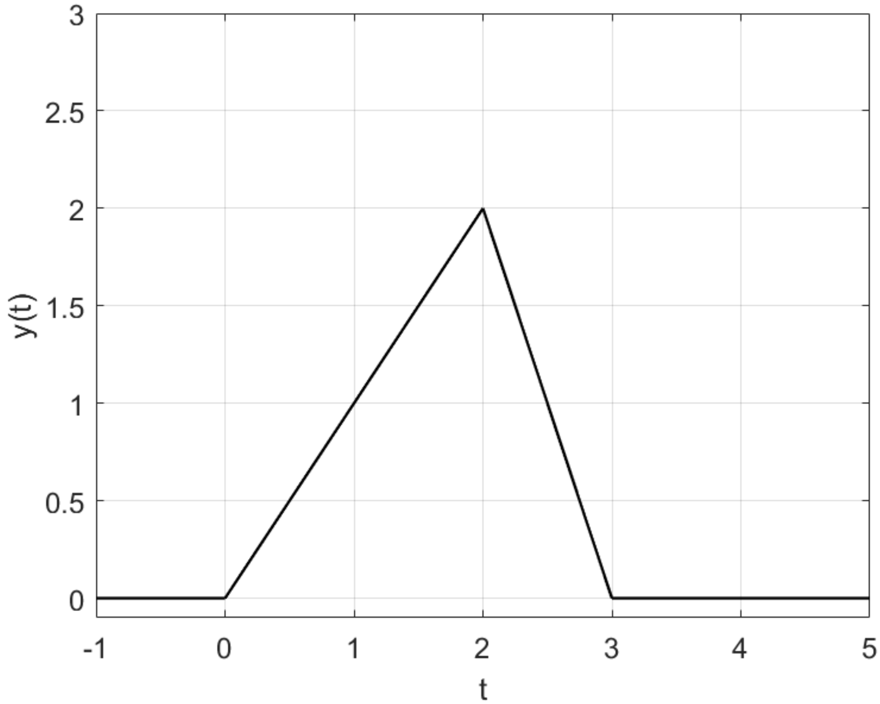
ECE 330: Signals & Systems - Problem Set

1) Sketch the following signals (check your work using MATLAB):
   a) $u(t-5) - u(t-7)$
   b) $u(t-5) + u(t-7)$
   c) $t^2[u(t-1) - u(t-2)]$
   d) $(t-4)[u(t-2) - u(t-4)]$
   e) $2t(u(t+2) - u(t-1))$
   f) **Bonus:** $\sum_{k=1}^{+\infty}(-1)^k(u(t-(k-1)) - u(t-k))$

   Solution:



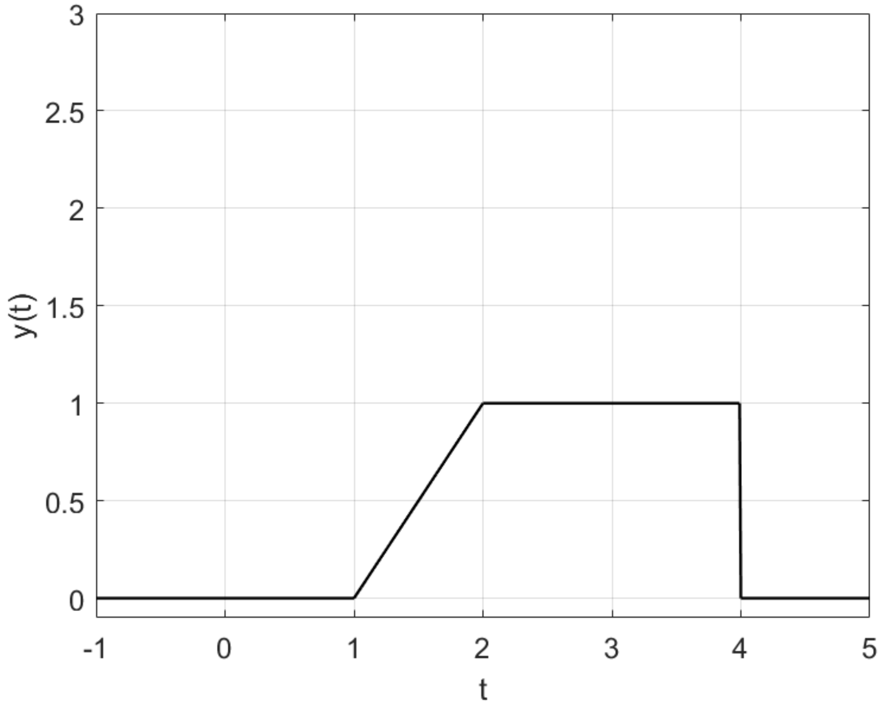
2) Give an expression for the signal, $y(t)$, shown in the figure using the unit step function.



Solution: The signal may be broken down into the sum of two separate signals: one for the rising portion and one for the decreasing portion. Both ramps are linear meaning the amplitude of the signal is proportional to t. Rising Ramp: $x_1(t) = t[u(t) - u(t-2)]$. Also, $u(t) - u(t-2)$ constrains the ramp from $0 \leq t \leq 2$. Then, for the decreasing ramp, it is again linear but now proportional by $-2t$ and occurs in the interval $2 \leq t \leq 3$. Thus, the decreasing ramp: $x_2(t) = -2(t-3)[u(t-2) - u(t-3)]$. Finally:

$$y_t(t) = y_1(t) + y_2(t) = tu(t) - 3(t-2)u(t-2) + 2(t-3)u(t-3)$$

3) Give an expression for the following signal shown in the figure using the step function.



Solution:

$$y(t) = (t-1)u(t-1) - (t-2)u(t-2) - u(t-4)$$

Each term in the solution serves a specific role in the visual characteristic of the signal. The first term, $(t-1)u(t-1)$ acts as the increasing ramp in the interval $1 \leq t \leq 2$.

Then, the following term $(t-2)u(t-2)$ cancels out the increasing ramp for $t > 2$. Lastly, $u(t-4)$ gives us the familiar rectangle ending at $t = 4$.

4) Simplify the following expressions containing the unit step:

a) $(t^3 + 3)\delta(t)$

b) $[sin(t^2 - \frac{\pi}{2})\delta(t)]$

c) $e^{-2t}\delta(t)$

d) $\frac{\omega^2+1}{\omega^2+9}\delta(\omega - 1)$

Solution:

a) $3\delta(t)$

b) $-\delta(t)$

c) $\delta(t)$

d) $\frac{1}{5}\delta(\omega - 1)$

5) Simplify the following unit impulse expressions:

a) $\int_{-\infty}^{\infty} \delta(t)e^{-j\omega t}dt$

b) $\int_{-\infty}^{\infty} \delta(t-2)cos(\frac{\pi t}{4})dt$

c) $\int_{-\infty}^{\infty} e^{-2(x-t)}\delta(2-t)dt$

Solution:

a) 1

b) 0

c) $e^{-2(x-2)}$

Exercises

Time Invariance

Recall that time invariance refers to a system where if the input is delayed by T seconds, the output is the same as before but delayed by T. In other words, a signal is time invariant if a delay between the input and output does not affect the signals output.

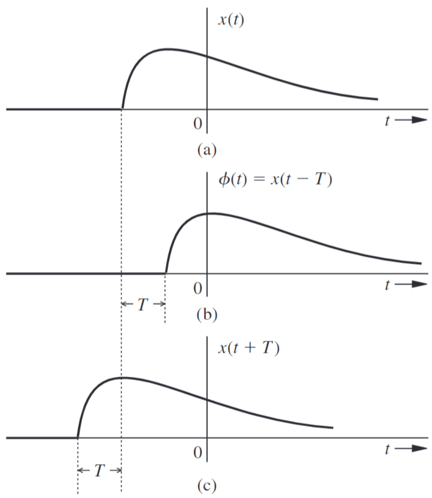


Figure 1: Time Delay Example

Answer the following questions related time invariance of systems. Determine if the following systems are time invariant. **Show your work**.

(Hint: If you suspect time variance, use an example to show the system fails to satisfy the time-invariance property).

a) $y(t) = x(t)u(t)$

b) $y(t) = \frac{d}{dt}x(t)$

c) $y(t) = sin(t)x(t-2)$

Solution:
a) This system is time variant. We can show this using a counter example. Let $x(t) = \delta(t+1)$. We see that $y(t) = 0$. Now, by applying a delay to our input, we have $x(t-2) = \delta(t-1)$ which means our new output is $y(t-2) = \delta(t-1)$ which does not equal our output without the delay.
b) This system is time invariant. Lets apply a delay to our input which gives us $y(t-T) = \frac{d}{d(t-T)}x(t-T) = \frac{d}{dt}x(t-T)$ which is just the output of the system to a

delayed input.
c) This system is time variant. If we apply a delay, we see that $y(t-T) = sin(t-T)x(t-2-T)$. If $sin(t)$ does not equal $sin(t-T)$ then the system is time variant.

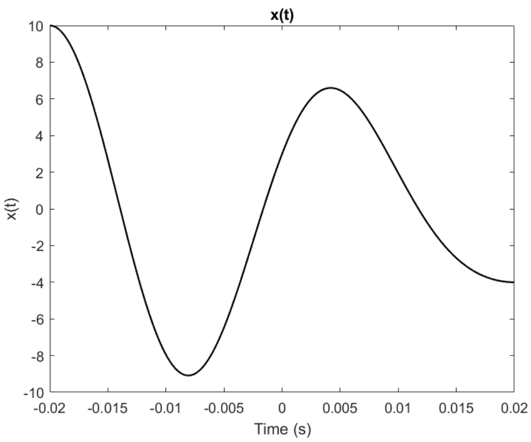
Sampling and Reconstruction

Let the T-periodic wave $s(t)$ be defined as

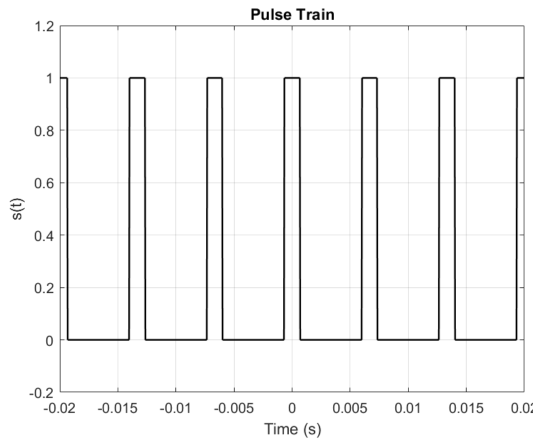
$$
s(t) = \begin{cases} 1, & |t| < T_0 \\ 0, & T_0 < |t| < T/2 \\ s(t+T), & \forall t \end{cases}
$$

Now, define the product of $s(t)$ and some arbitrary waveform $x(t)$ as $v(t) = x(t)s(t)$. We can treat this product as a continuous time sampling of $x(t)$ where portions of $x(t)$ associated with the zero parts of $x(t)$ are eliminated.

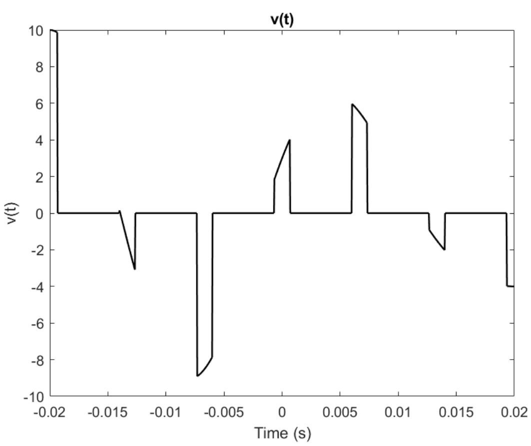
This is shown in the figure below.



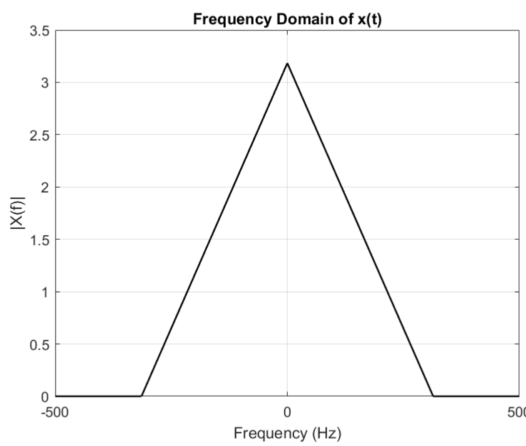
(a) $x(t)$ is some arbitrary signal.



(b) Pulse train representing our sampling.



(c) $v(t)$ representing our sampled signal.



(d) Frequency domain of $x(t)$. The nonzero portion ranges from $[-100\pi, 100\pi]$.

We will assume the Fourier transform of $x(t)$ is a simple triangular spectrum as depicted in the figure above. Let the nonzero band be defined between $-W < \Omega < W$ where $W = 100\pi$ rads/sec.

Begin by finding $S(\Omega)$, the Fourier transform representation of $s(t)$. Assume that $T = 1/150$ and $T_0 = 1/1500$.

Solution:

Use the multiplication in time-convolution in frequency property of the Fourier transform to find the expression for $V(\Omega)$. Sketch $V(\Omega)$ and answer the following questions.

What is the minimum T for which the portions of $V(\Omega)$ do not overlap?

Solution: $\frac{2\pi}{T} > 2W$

Now pass $v(t)$ through a low pass filter with impulse response

$$h(t) = (\frac{T}{2T_0})\frac{sin(100\pi t)}{\pi t}$$

to obtain $y(t) = v(t) * h(t)$. what is the output of the filter? **Hint: use the convolution in time-multiplication in frequency property.**

Solution: $y(t) = x(t)$