Linear Systems and Signals Notes

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Contents

Chapter 1

Signals and Systems

1.0.1 Definitions

Definition 1.0.1 (Signal Energy).

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

Definition 1.0.2 (Siganl Power).

$$P_x = lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt$$

Definition 1.0.3 (Time Shifting).

$$\phi(t+T) = x(t)$$
$$\phi(t) = x(t-T)$$

$x(t-T)$ represents $x(t)$ time shifted by T seconds.

1.1 Classification of Signals

I felt like 330 never really laid out the these basic ideas about siganls in a good way. For example, I remember Micheal and I didn't realize that anytime a signal is said to be from convolution, it is a LTI system.

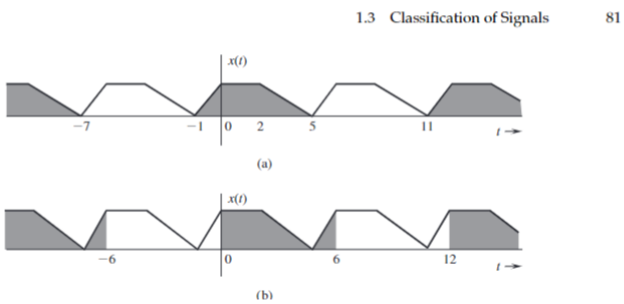
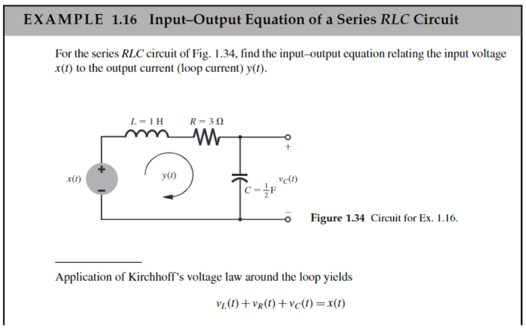


Figure 1.1: Example of a property

I like how the book goes over basic signal properites like time shifts, reversals, etc. instead of properites being introduced randomly throughout the semester (this is done well with fourier).



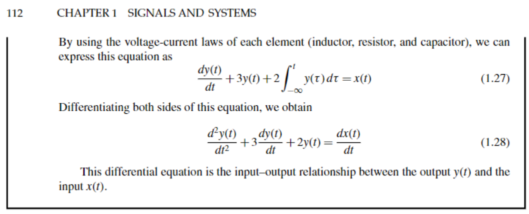
EXAMPLE 1.16 Input–Output Equation of a Series *RLC* Circuit

For the series *RLC* circuit of Fig. 1.34, find the input–output equation relating the input voltage $x(t)$ to the output current (loop current) $y(t)$.

Figure 1.34 Circuit for Ex. 1.16.

Application of Kirchhoff's voltage law around the loop yields

$$v_L(t) + v_R(t) + v_C(t) = x(t)$$



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By using the voltage-current laws of each element (inductor, resistor, and capacitor), we can express this equation as

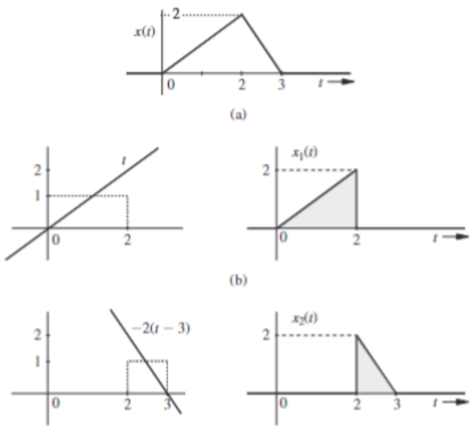
$$\frac{dy(t)}{dt} + 3y(t) + 2\int_{-\infty}^{t} y(\tau)\,d\tau = x(t) \qquad (1.27)$$

Differentiating both sides of this equation, we obtain

$$\frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} + 2y(t) = \frac{dx(t)}{dt} \qquad (1.28)$$

This differential equation is the input–output relationship between the output $y(t)$ and the input $x(t)$.

Example 1.1.1 (Unit Step Matlab Code). I think students are intially confused by the idea of using the unit step function to represent other signals such as a rectangle. Might be nice to have a little matlab script.



$$x_1(t) = t[u(t) - u(t-2)]$$

$$x_2(t) = -2(t-3)[u(t-2) - u(t-3)]$$

$$x(t) = x_1(t) + x_2(t)$$

Example 1.7 and drill 1.7, 1.8 are all good examples of using unit step.

1.1.1 The Exponential Function

We could wait to explain this when we do Laplace.

1.1.2 Even and Odd Functions

Useful but seems out of place/not needed considering how much content there is to cover.

1.2 Systems

Example 1.2.1.

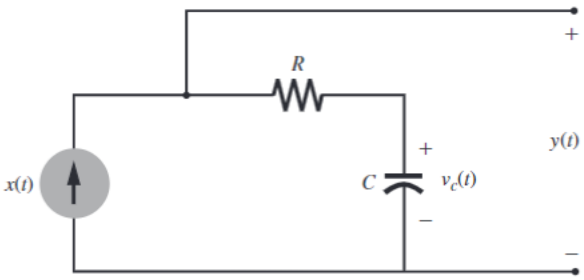
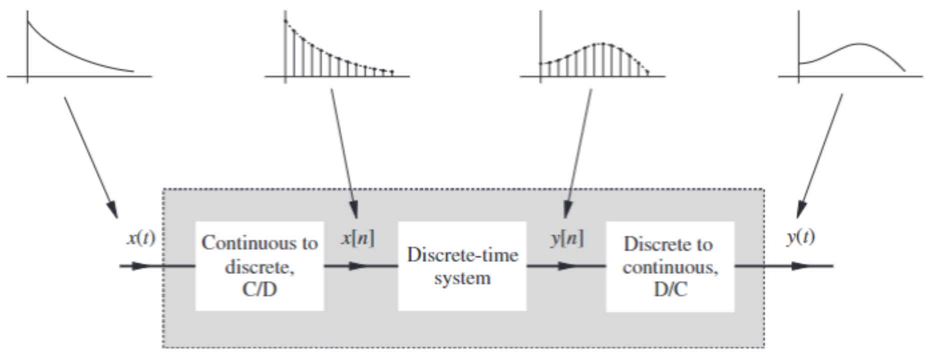


Figure 1.26 Example of a simple electrical system.

I think we should try to add a lot more circuit examples. Most students should have taken 230 by then or Physics 2.

1.3 Classification of Systems

Again, I think its a good idea to clearer go over all these system properties in a clear manner.



This is a good figure.

The end of the chapter has a lot of circuit examples that seem good.

Chapter 2

Time-Domain Analysis of Continuous-Time Systems

2.0.1 System Response to Internal Conditions

What is this? We never covered zero-input response in 330.

2.0.2 The Unit Impulse Response

The book goes right into describing systems as dif. eq's. Is this the approach we want to have as well? I know before we broke down systems into a few ways: Convolution, difference equations, dif. eq, and laplace?

I personally find a complete and full derivation of something such as convolution actually helps me in my understanding and intuition. When the math is abstracted it feels like theres gaps in my understanding and like I'm missing why something is something.

The below figure demonstrates a pretty good visualization for what convolution really does. I think this was never made clear (for me) in the videos.

I want to redo the convodemo from 203 with a better GUI and make a version for continuous convolution.

I never knew until now that convolution with the unit impulse is just the function $x(t)$ itself.

The book itself has lots of good matlab examples.

What is the thought process of discrete vs continuous for teaching order? Currently 330 splits it up in terms of "operation". It first goes over convolution and then difference and differential equations. The book groups everything in terms of continuous or discrete instead.

What is easier to intuitively understand first, discrete or continuous?

Again, I really like how they format this.

- Introduction
- Theory
- Example
- Matlab!!!

NOTE: Pg 189-190 has a good intuitive explanation of system response.

EXAMPLE 2.7 Using MATLAB to Find the Impulse Response

Determine the impulse response $h(t)$ for an LTIC system specified by the differential equation

$$(D^2+3D+2)y(t) = Dx(t)$$

This is a second-order system with $b_0 = 0$. First we find the zero-input component for initial conditions $y(0^-) = 0$, and $\dot{y}(0^-) = 1$. Since $P(D) = D$, the zero-input response is differentiated and the impulse response immediately follows as $h(t) = 0\delta(t) + [Dy_n(t)]u(t)$.

```
>> y_n = dsolve('D2y+3*Dy+2*y=0','y(0)=0','Dy(0)=1','t'); h = diff(y_n)
   h = 2/exp(2*t) - 1/exp(t)
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Therefore, $h(t) = (2e^{-2t} - e^{-t})u(t)$.

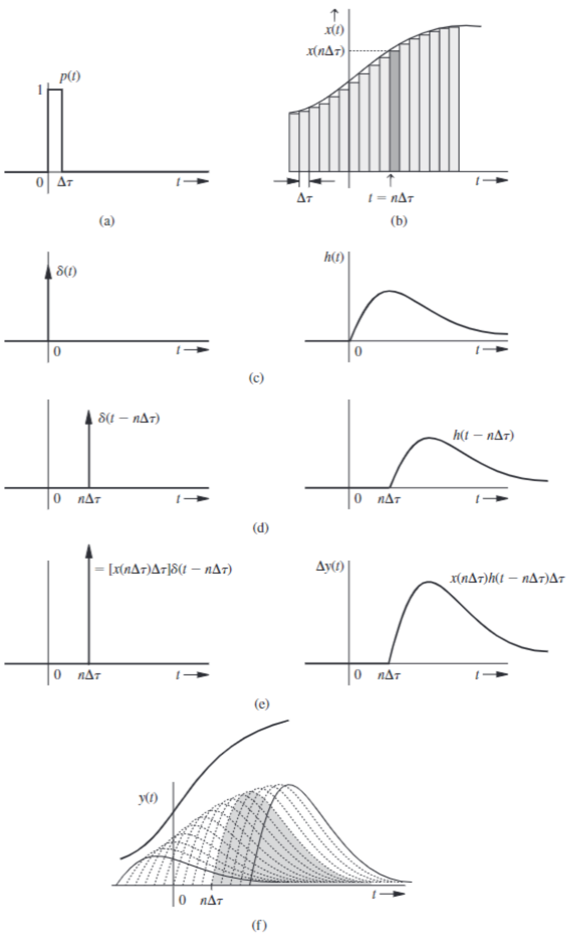


Figure 2.3 Finding the system response to an arbitrary input $x(t)$.

$$x(t) = \lim_{\Delta\tau \to 0} \sum_{\tau} x(n\Delta\tau)p(t - n\Delta\tau) = \lim_{\Delta\tau \to 0} \sum_{\tau} \left[\frac{x(n\Delta\tau)}{\Delta\tau}\right] p(t - n\Delta\tau)\Delta\tau$$

The term $[x(n\Delta\tau)/\Delta\tau]p(t - n\Delta\tau)$ represents a pulse $p(t - n\Delta\tau)$ with height $[x(n\Delta\tau)/\Delta\tau]$. As $\Delta\tau \to 0$, the height of this strip $\to \infty$, but its area remains $x(n\Delta\tau)$. Hence, this strip approaches an impulse $x(n\Delta\tau)\delta(t - n\Delta\tau)$ as $\Delta\tau \to 0$ (Fig. 2.3e). Therefore,

$$x(t) = \lim_{\Delta\tau \to 0} \sum_{\tau} x(n\Delta\tau)\delta(t - n\Delta\tau)\,\Delta\tau \tag{2.22}$$

To find the response for this input $x(t)$, we consider the input and the corresponding output pairs, as shown in Figs. 2.3c–2.3f and also shown by directed arrow notation as follows:

$$\begin{aligned}
\text{input} &\Longrightarrow \text{output} \\
\delta(t) &\Longrightarrow h(t) \\
\delta(t - n\Delta\tau) &\Longrightarrow h(t - n\Delta\tau) \\
[x(n\Delta\tau)\Delta\tau]\delta(t - n\Delta\tau) &\Longrightarrow [x(n\Delta\tau)\Delta\tau]h(t - n\Delta\tau) \\
\underbrace{\lim_{\Delta\tau \to 0} \sum_{\tau} x(n\Delta\tau)\delta(t - n\Delta\tau)\,\Delta\tau}_{x(t) \quad \text{[see Eq. (2.22)]}} &\Longrightarrow \underbrace{\lim_{\Delta\tau \to 0} \sum_{\tau} x(n\Delta\tau)h(t - n\Delta\tau)\Delta\tau}_{y(t)}
\end{aligned}$$

2.0.3 2.7-1 Op Amp Script Example

KCL equation at +v(t) node

$$\frac{x(t) - v(t)}{R_3} + \frac{y(t) - v(t)}{R_2} + \frac{0 - v(t)}{R_1} - C_2\frac{\mathrm{d}v}{\mathrm{d}t} = 0$$

KCL equation at inverting pin

$$\frac{v(t)}{R_1} + C_1\frac{\mathrm{d}y}{\mathrm{d}t}$$

Example 2.0.1.

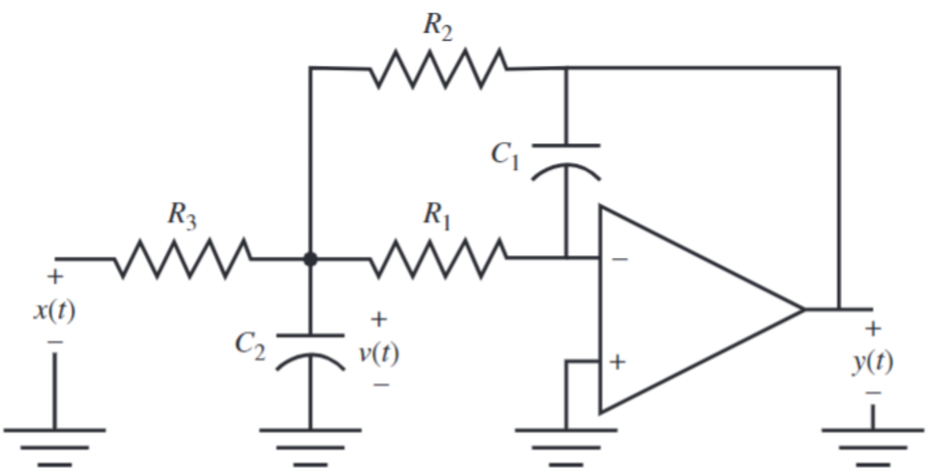


Figure 2.25 Operation-amplifier circuit.

Characteristic equation:

$$\lambda^2 + \frac{1}{C_2}\frac{1}{R_1 + R_2 + R_3}\lambda + \frac{1}{R_1 R_2 C_1 C_2} = (a_0\lambda^2 + a_1\lambda + a_2) = 0$$

Chapter 3

Additional Notes

- We should have videos or worksheets for the math behind some of the concepts such as partial fractions, integrals, etc.

- Try doing more circuit examples.

- Keep notation consistent throughout the course...

- I like how the book will introduce a concept, do examples, and then give ways to visualize or solve the problems in matlab. We should try doing that more in hw's or exercises and say hey! try this yourself and mess with it.

- What is the order idea: Discrete vs. Continuous, what first? Or both at the same time?

3.0.1 Peter's Notes

- Time invariance could be taught better with more examples. More graphical intuition.

- Stability was not made clear.

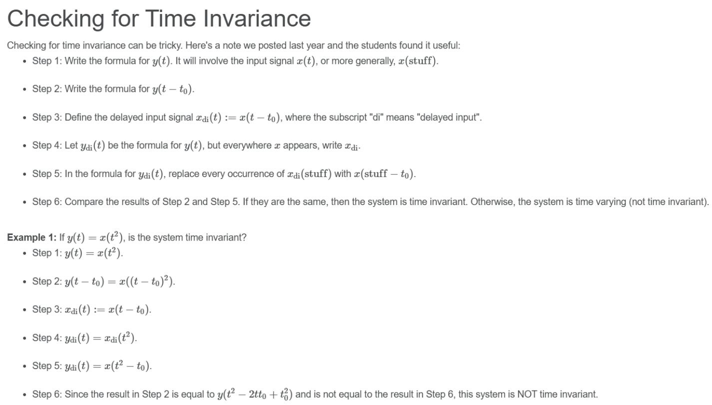
- Have videos or text with the required math for the class (i.e. differential equations, partial fractions).

3.0.2 Day by Day

Day1	Intro to Course + Basic Signals
Day2	Impulse Function, Unit Step, Impulse response
Day3	Systems + Properties
Day5	Discrete Time Convolution
Day6	Continuous Time Convolution
Day7	Differential Equations
Day8	Difference Equations

3.0.3 Student Questions/Comments

Example 3.0.1. I've heard from multiple people that time invariance (and properties in general) need to be exampled a lot better considering how important they are in the course.



Checking for Time Invariance

Checking for time invariance can be tricky. Here's a note we posted last year and the students found it useful:

- Step 1: Write the formula for $y(t)$. It will involve the input signal $x(t)$, or more generally, $x(\mathrm{stuff})$.
- Step 2: Write the formula for $y(t-t_0)$.
- Step 3: Define the delayed input signal $x_{\mathrm{di}}(t) := x(t-t_0)$, where the subscript "di" means "delayed input".
- Step 4: Let $y_{\mathrm{di}}(t)$ be the formula for $y(t)$, but everywhere x appears, write x_{di}.
- Step 5: In the formula for $y_{\mathrm{di}}(t)$, replace every occurrence of $x_{\mathrm{di}}(\mathrm{stuff})$ with $x(\mathrm{stuff}-t_0)$.
- Step 6: Compare the results of Step 2 and Step 5. If they are the same, then the system is time invariant. Otherwise, the system is time varying (not time invariant).

Example 1: If $y(t) = x(t^2)$, is the system time invariant?

- Step 1: $y(t) = x(t^2)$.
- Step 2: $y(t-t_0) = x((t-t_0)^2)$.
- Step 3: $x_{\mathrm{di}}(t) := x(t-t_0)$.
- Step 4: $y_{\mathrm{di}}(t) = x_{\mathrm{di}}(t^2)$.
- Step 5: $y_{\mathrm{di}}(t) = x(t^2-t_0)$.
- Step 6: Since the result in Step 2 is equal to $y(t^2-2tt_0+t_0^2)$ and is not equal to the result in Step 6, this system is NOT time invariant.

- How to know if something is BIBO stable? Considering it is the first question on the exam, it should be clear to students the steps.
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