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ФАКУЛЬТЕТ «Информатика и системы управления» (ИУ)

КАФЕДРА «Информационная безопасность» (ИУ8)

ЛАБОРАТОРНАЯ РАБОТА №1
ПО ДИСЦИПЛИНЕ
«ТЕОРИЯ ИГР И ИССЛЕДОВАНИЕ ОПЕРАЦИЙ»
НА ТЕМУ:

АНАЛИТИЧЕСКИЙ И ЧИСЛЕННЫЙ (БРАУНА-РОБИНСОН) МЕТОДЫ РЕШЕНИЯ АНТАГОНИСТИЧЕСКОЙ ИГРЫ В СМЕШАННЫХ СТРАТЕГИЯХ

ВАРИАНТ 5

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Цель работы - изучить аналитический (обратной матрицы) и численный (Брауна-Робинсона) методы нахождения смешанных стратегий в антагонистической игре двух лиц в нормальной форме.

Постановка задачи и методические указания

Найдите цену игры и оптимальные стратегии обоих игроков методами обратной матрицы и Брауна-Робинсон. Сравните полученные результаты.

Отчет должен содержать: титульный лист; цель работы; постановку задачи; решение матричной игры аналитическим методом; этапы решения матричной игры в смешанных стратегиях численным методом Брауна-Робинсон за обоих игроков (в виде таблицы и графиков) до уровня погрешности $\varepsilon \leq 0,1$; сравнительную оценку погрешностей, полученных аналитически и приближенным решением.

Ход работы

```
In [1]: from IPython.display import display, Math, Latex
        pretty_print_default(False)
```

Зададим матрицу игры:

```
In [2]: C = matrix(SR, 3, 3, [8, 12, 10, 1, 6, 19, 17, 11, 11])
        display(Math(latex(C)))
```

$$\begin{pmatrix} 8 & 12 & 10 \\ 1 & 6 & 19 \\ 17 & 11 & 11 \end{pmatrix}$$

Стратегии игрока A:

```
In [3]: x = C.rows()
        Math(latex(table(x)))
```

Out[3]:

$$(8,\ 12,\ 10) \quad (1,\ 6,\ 19) \quad (17,\ 11,\ 11)$$

Стратегии игрока B:

In [4]:
```
y = C.columns()
Math(latex(table(y)))
```

Out[4]:

$$(8,\ 1,\ 17) \quad (12,\ 6,\ 11) \quad (10,\ 19,\ 11)$$

Аналитический метод

Цена игры ищется по формуле $v = \frac{1}{u \cdot C^{-1} \cdot u^T}$:

In [5]:
```
u = matrix(SR, 1, C.nrows(), [1] * C.nrows())
v = (1/(u * C^(-1) * u.transpose()))
Math(latex(v))
```

Out[5]:

$$\left(\frac{845}{76} \right)$$

Оптимальные стратегии игрока A можно найти по формуле $x^* = \frac{u \cdot C^{-1}}{u \cdot C^{-1} \cdot u^T}$:

In [6]:
```
x_star = v * (u * C^(-1))
Math(latex(x_star))
```

Out[6]:

$$\left(\frac{39}{76} \quad \frac{3}{38} \quad \frac{31}{76} \right)$$

Оптимальные стратегии игрока B можно найти по формуле $y^* = \frac{C^{-1} \cdot u^T}{u \cdot C^{-1} \cdot u^T}$:

In [7]:
```
y_star = v * (C^(-1) * u.transpose()).transpose()
Math(latex(y_star))
```

Out[7]:

$$\left(\frac{3}{152} \quad \frac{11}{19} \quad \frac{61}{152} \right)$$

Метод Брауна-Робинсон

Верхняя и нижняя оценки игр:

$$\overline{v}[k] = \max_{i \in A} \sum_{j \in B} c_{ij} \tilde{y}_j[k] = \sum_{j \in B} c_{i[k+1]j} \tilde{y}_j[k]$$

$$\underline{v}[k] = \max_{i \in A} \sum_{i \in A} c_{ij} \tilde{x}_j[k] = \sum_{i \in A} c_{i[k+1]j} \tilde{x}_j[k]$$

Для оценки игры имеем:

$$\max_k \frac{1}{k} \underline{v}[k] \leq v \leq \min_k \frac{1}{k} \overline{v}[k]$$

Оценка погрешности:

$$\varepsilon[k] = \min_k \frac{1}{k} \overline{v}[k] - \max_k \frac{1}{k} \underline{v}[k]$$

In [8]:
```
def BrownRobinson(C, eps):
    x = C.rows()
    y = C.columns()

    ki, xk, yk, ei = 1, 0, 0, eps + 1

    winA = [0] * C.nrows()
    lossB = [0] * C.ncols()

    vxk, vyk = dict(), dict()
    v, vx, vy = [], [], []
```

```
for i in range(C.nrows()):
    vxk[i], vyk[i] = 0, 0

rows = [[
    '$k$',
    'Выбор игрока $A$',
    'Выбор игрока $B$',
    'Выигрыш $A$',
    'Проигрыш $B$',
    '$\\overline{v}^{k}$',
    '$\\underline{v}^{k}$',
    '$\\varepsilon$',
    '$v$'
]]
while ei > eps:
    winA = [xi + yi for xi, yi in zip(winA, y[yk])]
    lossB = [xi + yi for xi, yi in zip(lossB, x[xk])]

    maxX, minY = max(winA), min(lossB)

    vxk[xk] += 1
    vyk[yk] += 1
    vx.append(maxX / ki)
    vy.append(minY / ki)
    ei = min(vx) - max(vy)
    v.append((min(vx) + max(vy)) / 2)
    rows.append([ki, xk + 1, yk + 1, winA, lossB, vx[-1], vy[-1], ei, v[-1

    countX = winA.count(maxX)
    countY = lossB.count(minY)
```

```
            xk = winA.index(maxX)
            yk = lossB.index(minY)

            if countX > 1 and countY > 1:
                pass
            elif countX == 1 and countY > 1:
                if lossB[xk] == minY:
                    yk = xk
            elif countY == 1 and countX > 1:
                if winA[yk] == maxX:
                    xk = yk
            ki += 1
        x_strategies = matrix(1, 3, [i / (ki - 1) for i in vxk.values()])
        y_strategies = matrix([i / (ki - 1) for i in vyk.values()])
        return v[-1], x_strategies, y_strategies, rows

cost, x_strategies, y_strategies, rows = BrownRobinson(C, 1/10)
display(Math(latex(table(rows))))
```

k	Выбор A	Выбор B	Выигрыш A	Проигрыш B	$\overline{v}^k$	$\underline{v}^k$	ε	v
1	1	1	$[8, 1, 17]$	$[8, 12, 10]$	17	8	9	$\frac{25}{2}$
2	3	1	$[16, 2, 34]$	$[25, 23, 21]$	17	$\frac{21}{2}$	$\frac{13}{2}$	$\frac{55}{4}$
3	3	3	$[26, 21, 45]$	$[42, 34, 32]$	15	$\frac{32}{3}$	$\frac{13}{3}$	$\frac{77}{6}$
4	3	3	$[36, 40, 56]$	$[59, 45, 43]$	14	$\frac{43}{4}$	$\frac{13}{4}$	$\frac{99}{8}$
5	3	3	$[46, 59, 67]$	$[76, 56, 54]$	$\frac{67}{5}$	$\frac{54}{5}$	$\frac{13}{5}$	$\frac{121}{10}$
6	3	3	$[56, 78, 78]$	$[93, 67, 65]$	13	$\frac{65}{6}$	$\frac{13}{6}$	$\frac{143}{12}$
7	3	3	$[66, 97, 89]$	$[110, 78, 76]$	$\frac{97}{7}$	$\frac{76}{7}$	$\frac{15}{7}$	$\frac{167}{14}$
8	2	3	$[76, 116, 100]$	$[111, 84, 95]$	$\frac{29}{2}$	$\frac{21}{2}$	$\frac{15}{7}$	$\frac{167}{14}$
9	2	2	$[88, 122, 111]$	$[112, 90, 114]$	$\frac{122}{9}$	10	$\frac{15}{7}$	$\frac{167}{14}$
10	2	2	$[100, 128, 122]$	$[113, 96, 133]$	$\frac{64}{5}$	$\frac{48}{5}$	$\frac{68}{35}$	$\frac{414}{35}$
11	2	2	$[112, 134, 133]$	$[114, 102, 152]$	$\frac{134}{11}$	$\frac{102}{11}$	$\frac{102}{77}$	$\frac{887}{77}$
12	2	2	$[124, 140, 144]$	$[115, 108, 171]$	12	9	$\frac{8}{7}$	$\frac{80}{7}$

13	3	2	$[136, 146, 155]$	$[132, 119, 182]$	$\frac{155}{13}$	$\frac{119}{13}$	$\frac{97}{91}$	$\frac{2073}{182}$
14	3	2	$[148, 152, 166]$	$[149, 130, 193]$	$\frac{83}{7}$	$\frac{65}{7}$	1	$\frac{159}{14}$
15	3	2	$[160, 158, 177]$	$[166, 141, 204]$	$\frac{59}{5}$	$\frac{47}{5}$	$\frac{33}{35}$	$\frac{793}{70}$
16	3	2	$[172, 164, 188]$	$[183, 152, 215]$	$\frac{47}{4}$	$\frac{19}{2}$	$\frac{25}{28}$	$\frac{633}{56}$
17	3	2	$[184, 170, 199]$	$[200, 163, 226]$	$\frac{199}{17}$	$\frac{163}{17}$	$\frac{101}{119}$	$\frac{2685}{238}$
18	3	2	$[196, 176, 210]$	$[217, 174, 237]$	$\frac{35}{3}$	$\frac{29}{3}$	$\frac{17}{21}$	$\frac{473}{42}$
19	3	2	$[208, 182, 221]$	$[234, 185, 248]$	$\frac{221}{19}$	$\frac{185}{19}$	$\frac{103}{133}$	$\frac{2991}{266}$
20	3	2	$[220, 188, 232]$	$[251, 196, 259]$	$\frac{58}{5}$	$\frac{49}{5}$	$\frac{26}{35}$	$\frac{393}{35}$
21	3	2	$[232, 194, 243]$	$[268, 207, 270]$	$\frac{81}{7}$	$\frac{69}{7}$	$\frac{5}{7}$	$\frac{157}{14}$
22	3	2	$[244, 200, 254]$	$[285, 218, 281]$	$\frac{127}{11}$	$\frac{109}{11}$	$\frac{53}{77}$	$\frac{1725}{154}$
23	3	2	$[256, 206, 265]$	$[302, 229, 292]$	$\frac{265}{23}$	$\frac{229}{23}$	$\frac{107}{161}$	$\frac{3603}{322}$
24	3	2	$[268, 212, 276]$	$[319, 240, 303]$	$\frac{23}{2}$	10	$\frac{9}{14}$	$\frac{313}{28}$
25	3	2	$[280, 218, 287]$	$[336, 251, 314]$	$\frac{287}{25}$	$\frac{251}{25}$	$\frac{109}{175}$	$\frac{3909}{350}$
26	3	2	$[292, 224, 298]$	$[353, 262, 325]$	$\frac{149}{13}$	$\frac{131}{13}$	$\frac{55}{91}$	$\frac{2031}{182}$
27	3	2	$[304, 230, 309]$	$[370, 273, 336]$	$\frac{103}{9}$	$\frac{91}{9}$	$\frac{37}{63}$	$\frac{1405}{126}$
28	3	2	$[316, 236, 320]$	$[387, 284, 347]$	$\frac{80}{7}$	$\frac{71}{7}$	$\frac{4}{7}$	$\frac{78}{7}$
29	3	2	$[328, 242, 331]$	$[404, 295, 358]$	$\frac{331}{29}$	$\frac{295}{29}$	$\frac{113}{203}$	$\frac{4521}{406}$
30	3	2	$[340, 248, 342]$	$[421, 306, 369]$	$\frac{57}{5}$	$\frac{51}{5}$	$\frac{19}{35}$	$\frac{779}{70}$
31	3	2	$[352, 254, 353]$	$[438, 317, 380]$	$\frac{353}{31}$	$\frac{317}{31}$	$\frac{115}{217}$	$\frac{4827}{434}$
32	3	2	$[364, 260, 364]$	$[455, 328, 391]$	$\frac{91}{8}$	$\frac{41}{4}$	$\frac{29}{56}$	$\frac{1245}{112}$
33	1	2	$[376, 266, 375]$	$[463, 340, 401]$	$\frac{376}{33}$	$\frac{340}{33}$	$\frac{29}{56}$	$\frac{1245}{112}$
34	1	2	$[388, 272, 386]$	$[471, 352, 411]$	$\frac{194}{17}$	$\frac{176}{17}$	$\frac{29}{56}$	$\frac{1245}{112}$
35	1	2	$[400, 278, 397]$	$[479, 364, 421]$	$\frac{80}{7}$	$\frac{52}{5}$	$\frac{29}{56}$	$\frac{1245}{112}$
36	1	2	$[412, 284, 408]$	$[487, 376, 431]$	$\frac{103}{9}$	$\frac{94}{9}$	$\frac{29}{56}$	$\frac{1245}{112}$
37	1	2	$[424, 290, 419]$	$[495, 388, 441]$	$\frac{424}{37}$	$\frac{388}{37}$	$\frac{29}{56}$	$\frac{1245}{112}$
38	1	2	$[436, 296, 430]$	$[503, 400, 451]$	$\frac{218}{19}$	$\frac{200}{19}$	$\frac{29}{56}$	$\frac{1245}{112}$
39	1	2	$[448, 302, 441]$	$[511, 412, 461]$	$\frac{448}{39}$	$\frac{412}{39}$	$\frac{29}{56}$	$\frac{1245}{112}$
40	1	2	$[460, 308, 452]$	$[519, 424, 471]$	$\frac{23}{2}$	$\frac{53}{5}$	$\frac{29}{56}$	$\frac{1245}{112}$
41	1	2	$[472, 314, 463]$	$[527, 436, 481]$	$\frac{472}{41}$	$\frac{436}{41}$	$\frac{29}{56}$	$\frac{1245}{112}$
42	1	2	$[484, 320, 474]$	$[535, 448, 491]$	$\frac{242}{21}$	$\frac{32}{3}$	$\frac{29}{56}$	$\frac{1245}{112}$
43	1	2	$[496, 326, 485]$	$[543, 460, 501]$	$\frac{496}{43}$	$\frac{460}{43}$	$\frac{29}{56}$	$\frac{1245}{112}$

44	1	2	$[508, 332, 496]$	$[551, 472, 511]$	$\frac{127}{11}$	$\frac{118}{11}$	$\frac{29}{56}$	$\frac{1245}{112}$
45	1	2	$[520, 338, 507]$	$[559, 484, 521]$	$\frac{104}{9}$	$\frac{484}{45}$	$\frac{29}{56}$	$\frac{1245}{112}$
46	1	2	$[532, 344, 518]$	$[567, 496, 531]$	$\frac{266}{23}$	$\frac{248}{23}$	$\frac{29}{56}$	$\frac{1245}{112}$
47	1	2	$[544, 350, 529]$	$[575, 508, 541]$	$\frac{544}{47}$	$\frac{508}{47}$	$\frac{29}{56}$	$\frac{1245}{112}$
48	1	2	$[556, 356, 540]$	$[583, 520, 551]$	$\frac{139}{12}$	$\frac{65}{6}$	$\frac{29}{56}$	$\frac{1245}{112}$
49	1	2	$[568, 362, 551]$	$[591, 532, 561]$	$\frac{568}{49}$	$\frac{76}{7}$	$\frac{29}{56}$	$\frac{1245}{112}$
50	1	2	$[580, 368, 562]$	$[599, 544, 571]$	$\frac{58}{5}$	$\frac{272}{25}$	$\frac{99}{200}$	$\frac{4451}{400}$
51	1	2	$[592, 374, 573]$	$[607, 556, 581]$	$\frac{592}{51}$	$\frac{556}{51}$	$\frac{193}{408}$	$\frac{9089}{816}$
52	1	2	$[604, 380, 584]$	$[615, 568, 591]$	$\frac{151}{13}$	$\frac{142}{13}$	$\frac{47}{104}$	$\frac{2319}{208}$
53	1	2	$[616, 386, 595]$	$[623, 580, 601]$	$\frac{616}{53}$	$\frac{580}{53}$	$\frac{183}{424}$	$\frac{9463}{848}$
54	1	2	$[628, 392, 606]$	$[631, 592, 611]$	$\frac{314}{27}$	$\frac{296}{27}$	$\frac{89}{216}$	$\frac{4825}{432}$
55	1	2	$[640, 398, 617]$	$[639, 604, 621]$	$\frac{128}{11}$	$\frac{604}{55}$	$\frac{173}{440}$	$\frac{9837}{880}$
56	1	2	$[652, 404, 628]$	$[647, 616, 631]$	$\frac{163}{14}$	11	$\frac{3}{8}$	$\frac{179}{16}$
57	1	2	$[664, 410, 639]$	$[655, 628, 641]$	$\frac{664}{57}$	$\frac{628}{57}$	$\frac{163}{456}$	$\frac{10211}{912}$
58	1	2	$[676, 416, 650]$	$[663, 640, 651]$	$\frac{338}{29}$	$\frac{320}{29}$	$\frac{79}{232}$	$\frac{5199}{464}$
59	1	2	$[688, 422, 661]$	$[671, 652, 661]$	$\frac{688}{59}$	$\frac{652}{59}$	$\frac{153}{472}$	$\frac{10585}{944}$
60	1	2	$[700, 428, 672]$	$[679, 664, 671]$	$\frac{35}{3}$	$\frac{166}{15}$	$\frac{37}{120}$	$\frac{2693}{240}$
61	1	2	$[712, 434, 683]$	$[687, 676, 681]$	$\frac{712}{61}$	$\frac{676}{61}$	$\frac{143}{488}$	$\frac{10959}{976}$
62	1	2	$[724, 440, 694]$	$[695, 688, 691]$	$\frac{362}{31}$	$\frac{344}{31}$	$\frac{69}{248}$	$\frac{5573}{496}$
63	1	2	$[736, 446, 705]$	$[703, 700, 701]$	$\frac{736}{63}$	$\frac{100}{9}$	$\frac{19}{72}$	$\frac{1619}{144}$
64	1	2	$[748, 452, 716]$	$[711, 712, 711]$	$\frac{187}{16}$	$\frac{711}{64}$	$\frac{19}{72}$	$\frac{1619}{144}$
65	1	1	$[756, 453, 733]$	$[719, 724, 721]$	$\frac{756}{65}$	$\frac{719}{65}$	$\frac{19}{72}$	$\frac{1619}{144}$
66	1	1	$[764, 454, 750]$	$[727, 736, 731]$	$\frac{382}{33}$	$\frac{727}{66}$	$\frac{19}{72}$	$\frac{1619}{144}$
67	1	1	$[772, 455, 767]$	$[735, 748, 741]$	$\frac{772}{67}$	$\frac{735}{67}$	$\frac{19}{72}$	$\frac{1619}{144}$
68	1	1	$[780, 456, 784]$	$[743, 760, 751]$	$\frac{196}{17}$	$\frac{743}{68}$	$\frac{19}{72}$	$\frac{1619}{144}$
69	3	1	$[788, 457, 801]$	$[760, 771, 762]$	$\frac{267}{23}$	$\frac{760}{69}$	$\frac{19}{72}$	$\frac{1619}{144}$
70	3	1	$[796, 458, 818]$	$[777, 782, 773]$	$\frac{409}{35}$	$\frac{773}{70}$	$\frac{19}{72}$	$\frac{1619}{144}$
71	3	3	$[806, 477, 829]$	$[794, 793, 784]$	$\frac{829}{71}$	$\frac{784}{71}$	$\frac{19}{72}$	$\frac{1619}{144}$
72	3	3	$[816, 496, 840]$	$[811, 804, 795]$	$\frac{35}{3}$	$\frac{265}{24}$	$\frac{19}{72}$	$\frac{1619}{144}$
73	3	3	$[826, 515, 851]$	$[828, 815, 806]$	$\frac{851}{73}$	$\frac{806}{73}$	$\frac{19}{72}$	$\frac{1619}{144}$
74	3	3	$[836, 534, 862]$	$[845, 826, 817]$	$\frac{431}{37}$	$\frac{817}{74}$	$\frac{19}{72}$	$\frac{1619}{144}$

75	3	3	[846, 553, 873]	[862, 837, 828]	$\frac{291}{25}$	$\frac{276}{25}$	$\frac{19}{72}$	$\frac{1619}{144}$
76	3	3	[856, 572, 884]	[879, 848, 839]	$\frac{221}{19}$	$\frac{839}{76}$	$\frac{19}{72}$	$\frac{1619}{144}$
77	3	3	[866, 591, 895]	[896, 859, 850]	$\frac{895}{77}$	$\frac{850}{77}$	$\frac{19}{72}$	$\frac{1619}{144}$
78	3	3	[876, 610, 906]	[913, 870, 861]	$\frac{151}{13}$	$\frac{287}{26}$	$\frac{19}{72}$	$\frac{1619}{144}$
79	3	3	[886, 629, 917]	[930, 881, 872]	$\frac{917}{79}$	$\frac{872}{79}$	$\frac{19}{72}$	$\frac{1619}{144}$
80	3	3	[896, 648, 928]	[947, 892, 883]	$\frac{58}{5}$	$\frac{883}{80}$	$\frac{19}{72}$	$\frac{1619}{144}$
81	3	3	[906, 667, 939]	[964, 903, 894]	$\frac{313}{27}$	$\frac{298}{27}$	$\frac{19}{72}$	$\frac{1619}{144}$
82	3	3	[916, 686, 950]	[981, 914, 905]	$\frac{475}{41}$	$\frac{905}{82}$	$\frac{19}{72}$	$\frac{1619}{144}$
83	3	3	[926, 705, 961]	[998, 925, 916]	$\frac{961}{83}$	$\frac{916}{83}$	$\frac{19}{72}$	$\frac{1619}{144}$
84	3	3	[936, 724, 972]	[1015, 936, 927]	$\frac{81}{7}$	$\frac{309}{28}$	$\frac{19}{72}$	$\frac{1619}{144}$
85	3	3	[946, 743, 983]	[1032, 947, 938]	$\frac{983}{85}$	$\frac{938}{85}$	$\frac{19}{72}$	$\frac{1619}{144}$
86	3	3	[956, 762, 994]	[1049, 958, 949]	$\frac{497}{43}$	$\frac{949}{86}$	$\frac{19}{72}$	$\frac{1619}{144}$
87	3	3	[966, 781, 1005]	[1066, 969, 960]	$\frac{335}{29}$	$\frac{320}{29}$	$\frac{19}{72}$	$\frac{1619}{144}$
88	3	3	[976, 800, 1016]	[1083, 980, 971]	$\frac{127}{11}$	$\frac{971}{88}$	$\frac{19}{72}$	$\frac{1619}{144}$
89	3	3	[986, 819, 1027]	[1100, 991, 982]	$\frac{1027}{89}$	$\frac{982}{89}$	$\frac{19}{72}$	$\frac{1619}{144}$
90	3	3	[996, 838, 1038]	[1117, 1002, 993]	$\frac{173}{15}$	$\frac{331}{30}$	$\frac{19}{72}$	$\frac{1619}{144}$
91	3	3	[1006, 857, 1049]	[1134, 1013, 1004]	$\frac{1049}{91}$	$\frac{1004}{91}$	$\frac{19}{72}$	$\frac{1619}{144}$
92	3	3	[1016, 876, 1060]	[1151, 1024, 1015]	$\frac{265}{23}$	$\frac{1015}{92}$	$\frac{19}{72}$	$\frac{1619}{144}$
93	3	3	[1026, 895, 1071]	[1168, 1035, 1026]	$\frac{357}{31}$	$\frac{342}{31}$	$\frac{19}{72}$	$\frac{1619}{144}$
94	3	3	[1036, 914, 1082]	[1185, 1046, 1037]	$\frac{541}{47}$	$\frac{1037}{94}$	$\frac{19}{72}$	$\frac{1619}{144}$
95	3	3	[1046, 933, 1093]	[1202, 1057, 1048]	$\frac{1093}{95}$	$\frac{1048}{95}$	$\frac{19}{72}$	$\frac{1619}{144}$
96	3	3	[1056, 952, 1104]	[1219, 1068, 1059]	$\frac{23}{2}$	$\frac{353}{32}$	$\frac{19}{72}$	$\frac{1619}{144}$
97	3	3	[1066, 971, 1115]	[1236, 1079, 1070]	$\frac{1115}{97}$	$\frac{1070}{97}$	$\frac{19}{72}$	$\frac{1619}{144}$
98	3	3	[1076, 990, 1126]	[1253, 1090, 1081]	$\frac{563}{49}$	$\frac{1081}{98}$	$\frac{19}{72}$	$\frac{1619}{144}$
99	3	3	[1086, 1009, 1137]	[1270, 1101, 1092]	$\frac{379}{33}$	$\frac{364}{33}$	$\frac{19}{72}$	$\frac{1619}{144}$
100	3	3	[1096, 1028, 1148]	[1287, 1112, 1103]	$\frac{287}{25}$	$\frac{1103}{100}$	$\frac{19}{72}$	$\frac{1619}{144}$
101	3	3	[1106, 1047, 1159]	[1304, 1123, 1114]	$\frac{1159}{101}$	$\frac{1114}{101}$	$\frac{19}{72}$	$\frac{1619}{144}$
102	3	3	[1116, 1066, 1170]	[1321, 1134, 1125]	$\frac{195}{17}$	$\frac{375}{34}$	$\frac{19}{72}$	$\frac{1619}{144}$
103	3	3	[1126, 1085, 1181]	[1338, 1145, 1136]	$\frac{1181}{103}$	$\frac{1136}{103}$	$\frac{19}{72}$	$\frac{1619}{144}$
104	3	3	[1136, 1104, 1192]	[1355, 1156, 1147]	$\frac{149}{13}$	$\frac{1147}{104}$	$\frac{19}{72}$	$\frac{1619}{144}$
105	3	3	[1146, 1123, 1203]	[1372, 1167, 1158]	$\frac{401}{35}$	$\frac{386}{35}$	$\frac{19}{72}$	$\frac{1619}{144}$

106	3	3	[1156, 1142, 1214]	[1389, 1178, 1169]	$\frac{607}{53}$	$\frac{1169}{106}$	$\frac{19}{72}$	$\frac{1619}{144}$
107	3	3	[1166, 1161, 1225]	[1406, 1189, 1180]	$\frac{1225}{107}$	$\frac{1180}{107}$	$\frac{19}{72}$	$\frac{1619}{144}$
108	3	3	[1176, 1180, 1236]	[1423, 1200, 1191]	$\frac{103}{9}$	$\frac{397}{36}$	$\frac{19}{72}$	$\frac{1619}{144}$
109	3	3	[1186, 1199, 1247]	[1440, 1211, 1202]	$\frac{1247}{109}$	$\frac{1202}{109}$	$\frac{19}{72}$	$\frac{1619}{144}$
110	3	3	[1196, 1218, 1258]	[1457, 1222, 1213]	$\frac{629}{55}$	$\frac{1213}{110}$	$\frac{19}{72}$	$\frac{1619}{144}$
111	3	3	[1206, 1237, 1269]	[1474, 1233, 1224]	$\frac{423}{37}$	$\frac{408}{37}$	$\frac{19}{72}$	$\frac{1619}{144}$
112	3	3	[1216, 1256, 1280]	[1491, 1244, 1235]	$\frac{80}{7}$	$\frac{1235}{112}$	$\frac{19}{72}$	$\frac{1619}{144}$
113	3	3	[1226, 1275, 1291]	[1508, 1255, 1246]	$\frac{1291}{113}$	$\frac{1246}{113}$	$\frac{19}{72}$	$\frac{1619}{144}$
114	3	3	[1236, 1294, 1302]	[1525, 1266, 1257]	$\frac{217}{19}$	$\frac{419}{38}$	$\frac{19}{72}$	$\frac{1619}{144}$
115	3	3	[1246, 1313, 1313]	[1542, 1277, 1268]	$\frac{1313}{115}$	$\frac{1268}{115}$	$\frac{19}{72}$	$\frac{1619}{144}$
116	3	3	[1256, 1332, 1324]	[1559, 1288, 1279]	$\frac{333}{29}$	$\frac{1279}{116}$	$\frac{19}{72}$	$\frac{1619}{144}$
117	2	3	[1266, 1351, 1335]	[1560, 1294, 1298]	$\frac{1351}{117}$	$\frac{1294}{117}$	$\frac{19}{72}$	$\frac{1619}{144}$
118	2	2	[1278, 1357, 1346]	[1561, 1300, 1317]	$\frac{23}{2}$	$\frac{650}{59}$	$\frac{19}{72}$	$\frac{1619}{144}$
119	2	2	[1290, 1363, 1357]	[1562, 1306, 1336]	$\frac{1363}{119}$	$\frac{1306}{119}$	$\frac{19}{72}$	$\frac{1619}{144}$
120	2	2	[1302, 1369, 1368]	[1563, 1312, 1355]	$\frac{1369}{120}$	$\frac{164}{15}$	$\frac{19}{72}$	$\frac{1619}{144}$
121	2	2	[1314, 1375, 1379]	[1564, 1318, 1374]	$\frac{1379}{121}$	$\frac{1318}{121}$	$\frac{19}{72}$	$\frac{1619}{144}$
122	3	2	[1326, 1381, 1390]	[1581, 1329, 1385]	$\frac{695}{61}$	$\frac{1329}{122}$	$\frac{19}{72}$	$\frac{1619}{144}$
123	3	2	[1338, 1387, 1401]	[1598, 1340, 1396]	$\frac{467}{41}$	$\frac{1340}{123}$	$\frac{19}{72}$	$\frac{1619}{144}$
124	3	2	[1350, 1393, 1412]	[1615, 1351, 1407]	$\frac{353}{31}$	$\frac{1351}{124}$	$\frac{19}{72}$	$\frac{1619}{144}$
125	3	2	[1362, 1399, 1423]	[1632, 1362, 1418]	$\frac{1423}{125}$	$\frac{1362}{125}$	$\frac{19}{72}$	$\frac{1619}{144}$
126	3	2	[1374, 1405, 1434]	[1649, 1373, 1429]	$\frac{239}{21}$	$\frac{1373}{126}$	$\frac{19}{72}$	$\frac{1619}{144}$
127	3	2	[1386, 1411, 1445]	[1666, 1384, 1440]	$\frac{1445}{127}$	$\frac{1384}{127}$	$\frac{19}{72}$	$\frac{1619}{144}$
128	3	2	[1398, 1417, 1456]	[1683, 1395, 1451]	$\frac{91}{8}$	$\frac{1395}{128}$	$\frac{19}{72}$	$\frac{1619}{144}$
129	3	2	[1410, 1423, 1467]	[1700, 1406, 1462]	$\frac{489}{43}$	$\frac{1406}{129}$	$\frac{101}{387}$	$\frac{8701}{774}$
130	3	2	[1422, 1429, 1478]	[1717, 1417, 1473]	$\frac{739}{65}$	$\frac{109}{10}$	$\frac{151}{585}$	$\frac{13151}{1170}$
131	3	2	[1434, 1435, 1489]	[1734, 1428, 1484]	$\frac{1489}{131}$	$\frac{1428}{131}$	$\frac{301}{1179}$	$\frac{26501}{2358}$
132	3	2	[1446, 1441, 1500]	[1751, 1439, 1495]	$\frac{125}{11}$	$\frac{1439}{132}$	$\frac{25}{99}$	$\frac{2225}{198}$
133	3	2	[1458, 1447, 1511]	[1768, 1450, 1506]	$\frac{1511}{133}$	$\frac{1450}{133}$	$\frac{299}{1197}$	$\frac{26899}{2394}$
134	3	2	[1470, 1453, 1522]	[1785, 1461, 1517]	$\frac{761}{67}$	$\frac{1461}{134}$	$\frac{149}{603}$	$\frac{13549}{1206}$
135	3	2	[1482, 1459, 1533]	[1802, 1472, 1528]	$\frac{511}{45}$	$\frac{1472}{135}$	$\frac{11}{45}$	$\frac{337}{30}$
136	3	2	[1494, 1465, 1544]	[1819, 1483, 1539]	$\frac{193}{17}$	$\frac{1483}{136}$	$\frac{37}{153}$	$\frac{3437}{306}$

137	3	2	[1506, 1471, 1555]	[1836, 1494, 1550]	$\frac{1555}{137}$	$\frac{1494}{137}$	$\frac{295}{1233}$	$\frac{27695}{2466}$
138	3	2	[1518, 1477, 1566]	[1853, 1505, 1561]	$\frac{261}{23}$	$\frac{1505}{138}$	$\frac{49}{207}$	$\frac{4649}{414}$
139	3	2	[1530, 1483, 1577]	[1870, 1516, 1572]	$\frac{1577}{139}$	$\frac{1516}{139}$	$\frac{293}{1251}$	$\frac{28093}{2502}$
140	3	2	[1542, 1489, 1588]	[1887, 1527, 1583]	$\frac{397}{35}$	$\frac{1527}{140}$	$\frac{73}{315}$	$\frac{7073}{630}$
141	3	2	[1554, 1495, 1599]	[1904, 1538, 1594]	$\frac{533}{47}$	$\frac{1538}{141}$	$\frac{97}{423}$	$\frac{9497}{846}$
142	3	2	[1566, 1501, 1610]	[1921, 1549, 1605]	$\frac{805}{71}$	$\frac{1549}{142}$	$\frac{145}{639}$	$\frac{14345}{1278}$
143	3	2	[1578, 1507, 1621]	[1938, 1560, 1616]	$\frac{1621}{143}$	$\frac{120}{11}$	$\frac{289}{1287}$	$\frac{28889}{2574}$
144	3	2	[1590, 1513, 1632]	[1955, 1571, 1627]	$\frac{34}{3}$	$\frac{1571}{144}$	$\frac{2}{9}$	$\frac{101}{9}$
145	3	2	[1602, 1519, 1643]	[1972, 1582, 1638]	$\frac{1643}{145}$	$\frac{1582}{145}$	$\frac{287}{1305}$	$\frac{29287}{2610}$
146	3	2	[1614, 1525, 1654]	[1989, 1593, 1649]	$\frac{827}{73}$	$\frac{1593}{146}$	$\frac{143}{657}$	$\frac{14743}{1314}$
147	3	2	[1626, 1531, 1665]	[2006, 1604, 1660]	$\frac{555}{49}$	$\frac{1604}{147}$	$\frac{95}{441}$	$\frac{9895}{882}$
148	3	2	[1638, 1537, 1676]	[2023, 1615, 1671]	$\frac{419}{37}$	$\frac{1615}{148}$	$\frac{71}{333}$	$\frac{7471}{666}$
149	3	2	[1650, 1543, 1687]	[2040, 1626, 1682]	$\frac{1687}{149}$	$\frac{1626}{149}$	$\frac{283}{1341}$	$\frac{30083}{2682}$
150	3	2	[1662, 1549, 1698]	[2057, 1637, 1693]	$\frac{283}{25}$	$\frac{1637}{150}$	$\frac{47}{225}$	$\frac{5047}{450}$
151	3	2	[1674, 1555, 1709]	[2074, 1648, 1704]	$\frac{1709}{151}$	$\frac{1648}{151}$	$\frac{281}{1359}$	$\frac{30481}{2718}$
152	3	2	[1686, 1561, 1720]	[2091, 1659, 1715]	$\frac{215}{19}$	$\frac{1659}{152}$	$\frac{35}{171}$	$\frac{3835}{342}$
153	3	2	[1698, 1567, 1731]	[2108, 1670, 1726]	$\frac{577}{51}$	$\frac{1670}{153}$	$\frac{31}{153}$	$\frac{3431}{306}$
154	3	2	[1710, 1573, 1742]	[2125, 1681, 1737]	$\frac{871}{77}$	$\frac{1681}{154}$	$\frac{139}{693}$	$\frac{15539}{1386}$
155	3	2	[1722, 1579, 1753]	[2142, 1692, 1748]	$\frac{1753}{155}$	$\frac{1692}{155}$	$\frac{277}{1395}$	$\frac{31277}{2790}$
156	3	2	[1734, 1585, 1764]	[2159, 1703, 1759]	$\frac{147}{13}$	$\frac{131}{12}$	$\frac{23}{117}$	$\frac{2623}{234}$
157	3	2	[1746, 1591, 1775]	[2176, 1714, 1770]	$\frac{1775}{157}$	$\frac{1714}{157}$	$\frac{275}{1413}$	$\frac{31675}{2826}$
158	3	2	[1758, 1597, 1786]	[2193, 1725, 1781]	$\frac{893}{79}$	$\frac{1725}{158}$	$\frac{137}{711}$	$\frac{15937}{1422}$
159	3	2	[1770, 1603, 1797]	[2210, 1736, 1792]	$\frac{599}{53}$	$\frac{1736}{159}$	$\frac{91}{477}$	$\frac{10691}{954}$
160	3	2	[1782, 1609, 1808]	[2227, 1747, 1803]	$\frac{113}{10}$	$\frac{1747}{160}$	$\frac{17}{90}$	$\frac{2017}{180}$
161	3	2	[1794, 1615, 1819]	[2244, 1758, 1814]	$\frac{1819}{161}$	$\frac{1758}{161}$	$\frac{271}{1449}$	$\frac{32471}{2898}$
162	3	2	[1806, 1621, 1830]	[2261, 1769, 1825]	$\frac{305}{27}$	$\frac{1769}{162}$	$\frac{5}{27}$	$\frac{605}{54}$
163	3	2	[1818, 1627, 1841]	[2278, 1780, 1836]	$\frac{1841}{163}$	$\frac{1780}{163}$	$\frac{269}{1467}$	$\frac{32869}{2934}$
164	3	2	[1830, 1633, 1852]	[2295, 1791, 1847]	$\frac{463}{41}$	$\frac{1791}{164}$	$\frac{67}{369}$	$\frac{8267}{738}$
165	3	2	[1842, 1639, 1863]	[2312, 1802, 1858]	$\frac{621}{55}$	$\frac{1802}{165}$	$\frac{89}{495}$	$\frac{11089}{990}$
166	3	2	[1854, 1645, 1874]	[2329, 1813, 1869]	$\frac{937}{83}$	$\frac{1813}{166}$	$\frac{133}{747}$	$\frac{16733}{1494}$
167	3	2	[1866, 1651, 1885]	[2346, 1824, 1880]	$\frac{1885}{167}$	$\frac{1824}{167}$	$\frac{265}{1503}$	$\frac{33665}{3006}$

168	3	2	[1878, 1657, 1896]	[2363, 1835, 1891]	$\frac{79}{7}$	$\frac{1835}{168}$	$\frac{11}{63}$	$\frac{1411}{126}$
169	3	2	[1890, 1663, 1907]	[2380, 1846, 1902]	$\frac{1907}{169}$	$\frac{142}{13}$	$\frac{263}{1521}$	$\frac{34063}{3042}$
170	3	2	[1902, 1669, 1918]	[2397, 1857, 1913]	$\frac{959}{85}$	$\frac{1857}{170}$	$\frac{131}{765}$	$\frac{17131}{1530}$
171	3	2	[1914, 1675, 1929]	[2414, 1868, 1924]	$\frac{643}{57}$	$\frac{1868}{171}$	$\frac{29}{171}$	$\frac{3829}{342}$
172	3	2	[1926, 1681, 1940]	[2431, 1879, 1935]	$\frac{485}{43}$	$\frac{1879}{172}$	$\frac{65}{387}$	$\frac{8665}{774}$
173	3	2	[1938, 1687, 1951]	[2448, 1890, 1946]	$\frac{1951}{173}$	$\frac{1890}{173}$	$\frac{259}{1557}$	$\frac{34859}{3114}$
174	3	2	[1950, 1693, 1962]	[2465, 1901, 1957]	$\frac{327}{29}$	$\frac{1901}{174}$	$\frac{43}{261}$	$\frac{5843}{522}$
175	3	2	[1962, 1699, 1973]	[2482, 1912, 1968]	$\frac{1973}{175}$	$\frac{1912}{175}$	$\frac{257}{1575}$	$\frac{35257}{3150}$
176	3	2	[1974, 1705, 1984]	[2499, 1923, 1979]	$\frac{124}{11}$	$\frac{1923}{176}$	$\frac{16}{99}$	$\frac{1108}{99}$
177	3	2	[1986, 1711, 1995]	[2516, 1934, 1990]	$\frac{665}{59}$	$\frac{1934}{177}$	$\frac{85}{531}$	$\frac{11885}{1062}$
178	3	2	[1998, 1717, 2006]	[2533, 1945, 2001]	$\frac{1003}{89}$	$\frac{1945}{178}$	$\frac{127}{801}$	$\frac{17927}{1602}$
179	3	2	[2010, 1723, 2017]	[2550, 1956, 2012]	$\frac{2017}{179}$	$\frac{1956}{179}$	$\frac{253}{1611}$	$\frac{36053}{3222}$
180	3	2	[2022, 1729, 2028]	[2567, 1967, 2023]	$\frac{169}{15}$	$\frac{1967}{180}$	$\frac{7}{45}$	$\frac{1007}{90}$
181	3	2	[2034, 1735, 2039]	[2584, 1978, 2034]	$\frac{2039}{181}$	$\frac{1978}{181}$	$\frac{251}{1629}$	$\frac{36451}{3258}$
182	3	2	[2046, 1741, 2050]	[2601, 1989, 2045]	$\frac{1025}{91}$	$\frac{153}{14}$	$\frac{125}{819}$	$\frac{18325}{1638}$
183	3	2	[2058, 1747, 2061]	[2618, 2000, 2056]	$\frac{687}{61}$	$\frac{2000}{183}$	$\frac{83}{549}$	$\frac{12283}{1098}$
184	3	2	[2070, 1753, 2072]	[2635, 2011, 2067]	$\frac{259}{23}$	$\frac{2011}{184}$	$\frac{31}{207}$	$\frac{4631}{414}$
185	3	2	[2082, 1759, 2083]	[2652, 2022, 2078]	$\frac{2083}{185}$	$\frac{2022}{185}$	$\frac{247}{1665}$	$\frac{37247}{3330}$
186	3	2	[2094, 1765, 2094]	[2669, 2033, 2089]	$\frac{349}{31}$	$\frac{2033}{186}$	$\frac{41}{279}$	$\frac{6241}{558}$
187	1	2	[2106, 1771, 2105]	[2677, 2045, 2099]	$\frac{2106}{187}$	$\frac{2045}{187}$	$\frac{41}{279}$	$\frac{6241}{558}$
188	1	2	[2118, 1777, 2116]	[2685, 2057, 2109]	$\frac{1059}{94}$	$\frac{2057}{188}$	$\frac{41}{279}$	$\frac{6241}{558}$
189	1	2	[2130, 1783, 2127]	[2693, 2069, 2119]	$\frac{710}{63}$	$\frac{2069}{189}$	$\frac{41}{279}$	$\frac{6241}{558}$
190	1	2	[2142, 1789, 2138]	[2701, 2081, 2129]	$\frac{1071}{95}$	$\frac{2081}{190}$	$\frac{41}{279}$	$\frac{6241}{558}$
191	1	2	[2154, 1795, 2149]	[2709, 2093, 2139]	$\frac{2154}{191}$	$\frac{2093}{191}$	$\frac{41}{279}$	$\frac{6241}{558}$
192	1	2	[2166, 1801, 2160]	[2717, 2105, 2149]	$\frac{361}{32}$	$\frac{2105}{192}$	$\frac{41}{279}$	$\frac{6241}{558}$
193	1	2	[2178, 1807, 2171]	[2725, 2117, 2159]	$\frac{2178}{193}$	$\frac{2117}{193}$	$\frac{41}{279}$	$\frac{6241}{558}$
194	1	2	[2190, 1813, 2182]	[2733, 2129, 2169]	$\frac{1095}{97}$	$\frac{2129}{194}$	$\frac{41}{279}$	$\frac{6241}{558}$
195	1	2	[2202, 1819, 2193]	[2741, 2141, 2179]	$\frac{734}{65}$	$\frac{2141}{195}$	$\frac{41}{279}$	$\frac{6241}{558}$
196	1	2	[2214, 1825, 2204]	[2749, 2153, 2189]	$\frac{1107}{98}$	$\frac{2153}{196}$	$\frac{41}{279}$	$\frac{6241}{558}$
197	1	2	[2226, 1831, 2215]	[2757, 2165, 2199]	$\frac{2226}{197}$	$\frac{2165}{197}$	$\frac{41}{279}$	$\frac{6241}{558}$
198	1	2	[2238, 1837, 2226]	[2765, 2177, 2209]	$\frac{373}{33}$	$\frac{2177}{198}$	$\frac{41}{279}$	$\frac{6241}{558}$

199	1	2	$[2250, 1843, 2237]$	$[2773, 2189, 2219]$	$\frac{2250}{199}$	11	$\frac{41}{279}$	$\frac{6241}{558}$
200	1	2	$[2262, 1849, 2248]$	$[2781, 2201, 2229]$	$\frac{1131}{100}$	$\frac{2201}{200}$	$\frac{41}{279}$	$\frac{6241}{558}$
201	1	2	$[2274, 1855, 2259]$	$[2789, 2213, 2239]$	$\frac{758}{67}$	$\frac{2213}{201}$	$\frac{41}{279}$	$\frac{6241}{558}$
202	1	2	$[2286, 1861, 2270]$	$[2797, 2225, 2249]$	$\frac{1143}{101}$	$\frac{2225}{202}$	$\frac{41}{279}$	$\frac{6241}{558}$
203	1	2	$[2298, 1867, 2281]$	$[2805, 2237, 2259]$	$\frac{2298}{203}$	$\frac{2237}{203}$	$\frac{41}{279}$	$\frac{6241}{558}$
204	1	2	$[2310, 1873, 2292]$	$[2813, 2249, 2269]$	$\frac{385}{34}$	$\frac{2249}{204}$	$\frac{41}{279}$	$\frac{6241}{558}$
205	1	2	$[2322, 1879, 2303]$	$[2821, 2261, 2279]$	$\frac{2322}{205}$	$\frac{2261}{205}$	$\frac{41}{279}$	$\frac{6241}{558}$
206	1	2	$[2334, 1885, 2314]$	$[2829, 2273, 2289]$	$\frac{1167}{103}$	$\frac{2273}{206}$	$\frac{41}{279}$	$\frac{6241}{558}$
207	1	2	$[2346, 1891, 2325]$	$[2837, 2285, 2299]$	$\frac{34}{3}$	$\frac{2285}{207}$	$\frac{41}{279}$	$\frac{6241}{558}$
208	1	2	$[2358, 1897, 2336]$	$[2845, 2297, 2309]$	$\frac{1179}{104}$	$\frac{2297}{208}$	$\frac{41}{279}$	$\frac{6241}{558}$
209	1	2	$[2370, 1903, 2347]$	$[2853, 2309, 2319]$	$\frac{2370}{209}$	$\frac{2309}{209}$	$\frac{41}{279}$	$\frac{6241}{558}$
210	1	2	$[2382, 1909, 2358]$	$[2861, 2321, 2329]$	$\frac{397}{35}$	$\frac{2321}{210}$	$\frac{41}{279}$	$\frac{6241}{558}$
211	1	2	$[2394, 1915, 2369]$	$[2869, 2333, 2339]$	$\frac{2394}{211}$	$\frac{2333}{211}$	$\frac{41}{279}$	$\frac{6241}{558}$
212	1	2	$[2406, 1921, 2380]$	$[2877, 2345, 2349]$	$\frac{1203}{106}$	$\frac{2345}{212}$	$\frac{41}{279}$	$\frac{6241}{558}$
213	1	2	$[2418, 1927, 2391]$	$[2885, 2357, 2359]$	$\frac{806}{71}$	$\frac{2357}{213}$	$\frac{41}{279}$	$\frac{6241}{558}$
214	1	2	$[2430, 1933, 2402]$	$[2893, 2369, 2369]$	$\frac{1215}{107}$	$\frac{2369}{214}$	$\frac{41}{279}$	$\frac{6241}{558}$
215	1	2	$[2442, 1939, 2413]$	$[2901, 2381, 2379]$	$\frac{2442}{215}$	$\frac{2379}{215}$	$\frac{41}{279}$	$\frac{6241}{558}$
216	1	3	$[2452, 1958, 2424]$	$[2909, 2393, 2389]$	$\frac{613}{54}$	$\frac{2389}{216}$	$\frac{41}{279}$	$\frac{6241}{558}$
217	1	3	$[2462, 1977, 2435]$	$[2917, 2405, 2399]$	$\frac{2462}{217}$	$\frac{2399}{217}$	$\frac{41}{279}$	$\frac{6241}{558}$
218	1	3	$[2472, 1996, 2446]$	$[2925, 2417, 2409]$	$\frac{1236}{109}$	$\frac{2409}{218}$	$\frac{41}{279}$	$\frac{6241}{558}$
219	1	3	$[2482, 2015, 2457]$	$[2933, 2429, 2419]$	$\frac{34}{3}$	$\frac{2419}{219}$	$\frac{41}{279}$	$\frac{6241}{558}$
220	1	3	$[2492, 2034, 2468]$	$[2941, 2441, 2429]$	$\frac{623}{55}$	$\frac{2429}{220}$	$\frac{41}{279}$	$\frac{6241}{558}$
221	1	3	$[2502, 2053, 2479]$	$[2949, 2453, 2439]$	$\frac{2502}{221}$	$\frac{2439}{221}$	$\frac{41}{279}$	$\frac{6241}{558}$
222	1	3	$[2512, 2072, 2490]$	$[2957, 2465, 2449]$	$\frac{1256}{111}$	$\frac{2449}{222}$	$\frac{41}{279}$	$\frac{6241}{558}$
223	1	3	$[2522, 2091, 2501]$	$[2965, 2477, 2459]$	$\frac{2522}{223}$	$\frac{2459}{223}$	$\frac{41}{279}$	$\frac{6241}{558}$
224	1	3	$[2532, 2110, 2512]$	$[2973, 2489, 2469]$	$\frac{633}{56}$	$\frac{2469}{224}$	$\frac{41}{279}$	$\frac{6241}{558}$
225	1	3	$[2542, 2129, 2523]$	$[2981, 2501, 2479]$	$\frac{2542}{225}$	$\frac{2479}{225}$	$\frac{41}{279}$	$\frac{6241}{558}$
226	1	3	$[2552, 2148, 2534]$	$[2989, 2513, 2489]$	$\frac{1276}{113}$	$\frac{2489}{226}$	$\frac{41}{279}$	$\frac{6241}{558}$
227	1	3	$[2562, 2167, 2545]$	$[2997, 2525, 2499]$	$\frac{2562}{227}$	$\frac{2499}{227}$	$\frac{41}{279}$	$\frac{6241}{558}$
228	1	3	$[2572, 2186, 2556]$	$[3005, 2537, 2509]$	$\frac{643}{57}$	$\frac{2509}{228}$	$\frac{41}{279}$	$\frac{6241}{558}$
229	1	3	$[2582, 2205, 2567]$	$[3013, 2549, 2519]$	$\frac{2582}{229}$	11	$\frac{41}{279}$	$\frac{6241}{558}$

230	1	3	[2592, 2224, 2578]	[3021, 2561, 2529]	$\frac{1296}{115}$	$\frac{2529}{230}$	$\frac{41}{279}$	$\frac{6241}{558}$
231	1	3	[2602, 2243, 2589]	[3029, 2573, 2539]	$\frac{2602}{231}$	$\frac{2539}{231}$	$\frac{41}{279}$	$\frac{6241}{558}$
232	1	3	[2612, 2262, 2600]	[3037, 2585, 2549]	$\frac{653}{58}$	$\frac{2549}{232}$	$\frac{41}{279}$	$\frac{6241}{558}$
233	1	3	[2622, 2281, 2611]	[3045, 2597, 2559]	$\frac{2622}{233}$	$\frac{2559}{233}$	$\frac{298}{2097}$	$\frac{23449}{2097}$
234	1	3	[2632, 2300, 2622]	[3053, 2609, 2569]	$\frac{1316}{117}$	$\frac{2569}{234}$	$\frac{16}{117}$	$\frac{436}{39}$
235	1	3	[2642, 2319, 2633]	[3061, 2621, 2579]	$\frac{2642}{235}$	$\frac{2579}{235}$	$\frac{278}{2115}$	$\frac{23639}{2115}$
236	1	3	[2652, 2338, 2644]	[3069, 2633, 2589]	$\frac{663}{59}$	$\frac{2589}{236}$	$\frac{67}{531}$	$\frac{11867}{1062}$
237	1	3	[2662, 2357, 2655]	[3077, 2645, 2599]	$\frac{2662}{237}$	$\frac{2599}{237}$	$\frac{86}{711}$	$\frac{7943}{711}$
238	1	3	[2672, 2376, 2666]	[3085, 2657, 2609]	$\frac{1336}{119}$	$\frac{2609}{238}$	$\frac{124}{1071}$	$\frac{11962}{1071}$
239	1	3	[2682, 2395, 2677]	[3093, 2669, 2619]	$\frac{2682}{239}$	$\frac{2619}{239}$	$\frac{238}{2151}$	$\frac{24019}{2151}$
240	1	3	[2692, 2414, 2688]	[3101, 2681, 2629]	$\frac{673}{60}$	$\frac{2629}{240}$	$\frac{19}{180}$	$\frac{4019}{360}$
241	1	3	[2702, 2433, 2699]	[3109, 2693, 2639]	$\frac{2702}{241}$	$\frac{2639}{241}$	$\frac{218}{2169}$	$\frac{24209}{2169}$
242	1	3	[2712, 2452, 2710]	[3117, 2705, 2649]	$\frac{1356}{121}$	$\frac{2649}{242}$	$\frac{104}{1089}$	$\frac{12152}{1089}$

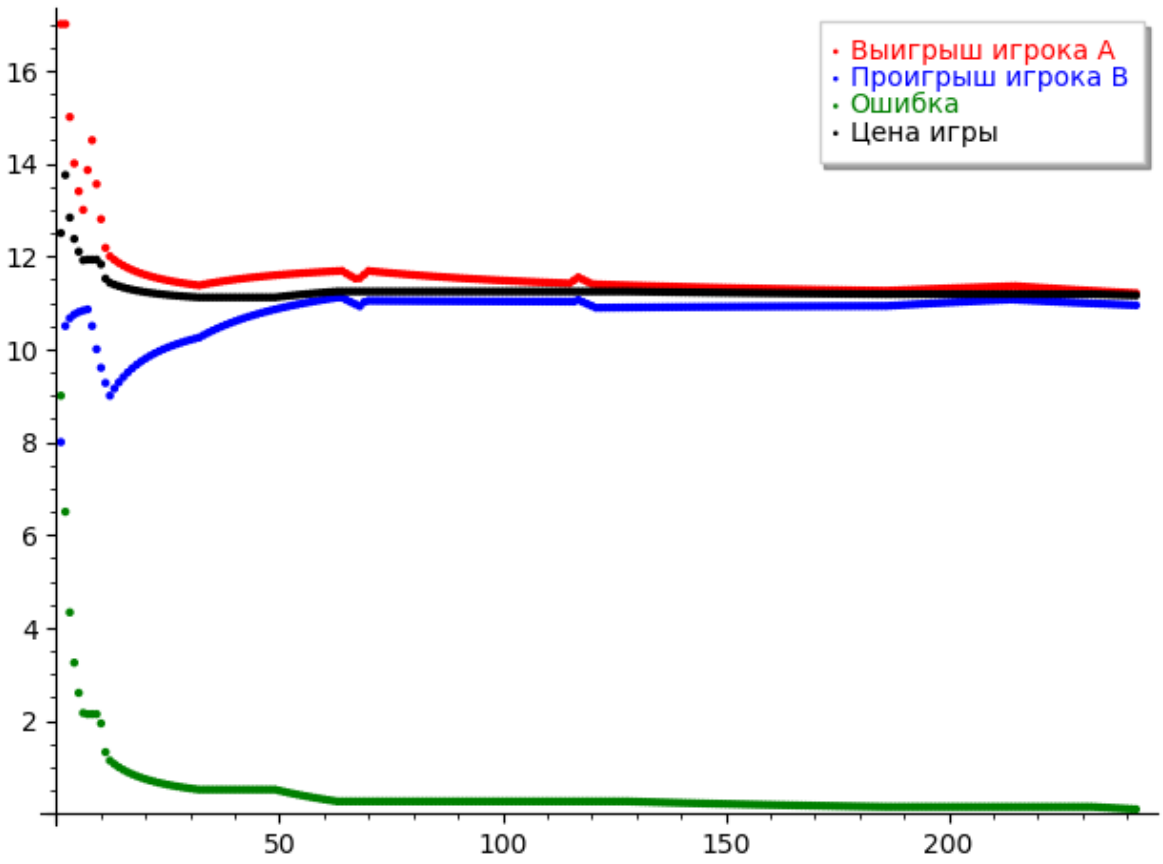
Построим графики:

```
In [9]: from sage.plot.point import Point
        a_points = point(
            [(row[0], row[5]) for row in rows[1:]],
            rgbcolor='red',
            legend_label='Выигрыш игрока A',
            legend_color='red'
        )
        b_points = point(
            [(row[0], row[6]) for row in rows[1:]],
            rgbcolor='blue',
            legend_label='Проигрыш игрока B',
            legend_color='blue'
        )
        e_points = point(
            [(row[0], row[7]) for row in rows[1:]],
            rgbcolor='green',
```

```
        legend_label='Ошибка',
        legend_color='green'
    )
    v_points = point(
        [(row[0], row[8]) for row in rows[1:]],
        rgbcolor='black',
        legend_label='Цена игры',
        legend_color='black'
    )

    a_points + b_points + e_points + v_points
```

Out[9]:



Оптимальные стратегии для игрока A:

```
In [10]: Math(latex(x_strategies))
```

```
Out[10]:
```

$$\left(\frac{93}{242} \quad \frac{5}{121} \quad \frac{139}{242} \right)$$

Оптимальные стратегии для игрока B:

```
In [11]: Math(latex(y_strategies))
```

```
Out[11]:
```

$$\left(\frac{4}{121} \quad \frac{7}{11} \quad \frac{40}{121} \right)$$

Цена игры:

```
In [12]: Math(latex(cost))
```

```
Out[12]:
```

$$\frac{12152}{1089}$$