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# Nobel Laureate
# The Many Other Allais Paradoxes

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*"Maurice Allais was awarded the Nobel Prize!"*
*"Maurice Allais was awarded . . . just for the Allais paradox?! Did he do anything else?"*

When the decision of the Nobel Committee was made public, in the fall of 1988, I imagine that this conversational exchange (or variants of it) took place quite a few times. Indeed, the work of Maurice Allais is often reduced to the 1952 "paradox," even within the profession of economics. I would like to show here that such an opinion is incorrect and unfair to Allais; in fact, very important parts of his work remain virtually unknown to most economists. As Paul Samuelson (1983) wrote, "Had Allais' earliest writings been in English, a whole generation of economic theory would have taken a different course." Perhaps the Nobel award will encourage economists to examine his work more closely.

Let me first give a quick portrait of Maurice Allais, and then list some of his major contributions to the theory of markets, to the theory of capital, to decision making under risk and to monetary dynamics. To be sure, Allais's work is considerably broader than these categories. In 1959 he received two major awards, one in France and one in the United States, for his contributions to physics,[1] and he has also done research in political science and a few other

[1]In particular, the awards were the 1959 Galabert Prize of the French Astronautical Society, and the 1959 Award of the Gravity Research Foundation in the United States.

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areas. But we shall leave those topics aside, although they testify to the vast intellectual culture of Maurice Allais. Someone once said to me, "Allais is the epitome of the 19th century scientist. He reads everything and works, alone, in several disciplines..."

But where did this "19th century scientist" come from?

## A Quick Portrait: Chance and Destiny

Maurice Allais had to struggle through his secondary education. His parents ran a very modest milkshop in Paris. After the death of his father in World War I, his mother kept a baby linen shop in the southern part of the French capital. However, Allais managed to rank first in almost every field at school—the sort of thing which doesn't necessarily make everyone friendly to you!—proving thus to be gifted and hard-working.

Usually a young man is happy to receive his *baccalauréat* at the end of his secondary education. Allais wasn't. So he took *two baccalauréats*: one with a mathematics major and one with a philosophy major. In fact, he was fascinated by history and considered applying to the Ecole des Chartes, the top school for paleographers and historians in France. By mere chance, a discussion with his professor of mathematics convinced him to enter a program in science and then to apply to the Ecole Polytechnique, the most competitive engineering school in France, originally founded by Napoleon for multi-purpose (hence the name) military engineers. Maurice Allais prepared for the entrance examination under the most unfavorable conditions: his desk was under the shop's roof, his bedroom was, at night, the shop itself! Nevertheless, he entered the school (1931), graduated as number one in his class in 1933 and received, in the same year, two prizes from the French Academy of Sciences.[2] Subsequently, he graduated from the Ecole Nationale Supérieure des Mines de Paris, the very competitive finishing school for mining engineers, with a strong tradition in economics and a special curriculum for graduates of the Ecole Polytechnique.

Encouraged by the two prizes of the French Academy of Sciences, Maurice Allais wished then to become a researcher in physics. However by stroke of fate, no such positions were available in 1936. The young man had thus to take a job as an engineer in the then newly nationalized coal-mining sector in France: within five *départements* (administrative districts) he had to supervise the management of the mining plants and the regulation of the transportation system, particularly the railway system.

During the following period, he observed three important things: First, the regulations he was supposed to enforce on his job were usually ineffective and most of the time he was not able to follow closely enough each case to which they applied. Second, relying on information given in specialized newspapers to

[2] They were the 1933 Laplace Prize and the 1933 Rivot Prize of the French Academy of Sciences.

**Maurice Allais**

bet on horse races during his leisure time could lead either to high gains, in monetary as well as in psychological terms, but with a high probability of ruin; or to expected gains which looked unattractive if the probability of ruin was to be kept low, so that some tradeoff had to be determined for a player to be rational. Third, and most importantly, the effects of the Great Depression—which were felt in France later than in the United States or in England—were socially and intellectually distressing. These three observations induced him to explore further the economic science of the time.

However, World War II broke out and this last project had to be delayed: Maurice Allais, as a lieutenant in the French army, was in charge of an artillery battery in the Alps. It was not before the 1940 armistice was signed that he returned to civilian life, under the German occupation of France, and resumed his investigation of economics. In this field, he considered himself as an amateur, who needed a firm theoretical background on which to act, in the immediate as well as in the future periods (he was very much concerned with what would happen in the after-war period). He was impressed by Irving Fisher and even more so by Vilfredo Pareto, whom he considers to be the greatest of all the authors whose works he would become familiar with. But he decided that economics, as it then stood, needed to be strengthened. He thus became a Professor at the Ecole Nationale Superieure des Mines de Paris (1944) and a Researcher at CNRS (1946).

This biographical sketch helps us to imagine how Allais's courageous personality, rationalist philosophy and inclination to pay tribute to physics and history emerged and inspired his interests and methodology in economic research. He later made clear to his students the few principles on which his economic methodology rests: 1) make reference to original thinkers only; 2) never accept any theory if it has not been successfully checked on empirical data; 3) look for invariants in space and time as much as possible; 4) make use of mathematics only as a way of expressing a theory rigorously (and particularly of explicitly stating the hypotheses on which it rests), but never admit that the mathematical content of a paper is a significant index of quality; and 5) aim at developing synthetic views.

## Theory of Markets and Efficiency with Applications

In this domain, two series of contributions have been made by Allais: his 1943 book, *In Search of a Science of Economics* (*A la Recherche d'une Discipline économique*) is an amazingly rich and comprehensive contribution to equilibrium and economic efficiency theory.[3] The second series of contributions is contained in several articles (from 1967 onwards) and in another major book, *General Theory of Surpluses* (*La Théorie générale des Surplus*), which was published in 1981. It constitutes a revision of Maurice Allais's view of basic economic activity. The first of these books was explicitly mentioned in the Nobel award decision, the second was commented upon during the Prize presentation ceremony (Stahl, 1988). Instead of trying to summarize them, let me pick and choose some of the major innovations Maurice Allais brought to the theory of markets and economic efficiency in these works.

Allais (1943–1952) contains an amazing number of original contributions, some of them incompletely explored even today. At the time, they represented decisive improvements. For example, the two equivalence theorems between market equilibrium (under pure competition and perfect market conditions) and Pareto optimality were already known in a static, timeless, environment (although not widely known), but had not been proved in a rigorous way, even in these static models. Allais recognized this fact and offered two alternative rigorous proofs for these theorems (pp. 617–627, no. 265–266; pp. 628–635, no. 267–268) taking into account second order conditions within a static as well as within an intertemporal model. In fact, he combined both propositions into a single one: competitive prices and an adequately chosen distribution of income

[3]The book consisted of 852 oversized pages. Due to the circumstances and the potential costs, the work was not printed, but only typewritten and duplicated, and published through individual subscriptions: In 1943, the C.N.R.S. (The French counterpart to the American NSF) had declined to grant a subsidy for publication. In 1952, the C.N.R.S. granted a subsidy, but the costs were so high that the book was simply photocopied, with the addition of a 63-page printed preface. This second version is known under the name of *Traité d'Economie Pure*, Paris, Imprimerie Nationale 1952. We thus refer to this work as Allais (1943–1952). To give precise references, we indicate page(s) and paragraph(s), so that the reader can retrieve the corresponding text easily in another edition. Indeed, the first printed edition of the book will be published in 1991 in Paris by the new publishing company *Les Editions Clement Juglar*.

*Table 1*
**Works by Maurice Allais**

*A la Recherche d'une Discipline Economique, Premieře partie: L'Economie pure.* Paris: Ateliers Industria, 1943. [*Traité d'Economie Pure.* Paris: Imprimerie Nationale, 1952. (63 printed pages of introduction + preceding book). First entirely printed edition-Paris Editions Clément Juglar, forthcoming.]

*Economie Pure et Rendement Social.* Paris: Sirey, 1945.

"Le problème de la Coordination des Transports et la Théorie économique," *Revue d'Economie Politique*, 1947a, *57*, 212–271.

*Economie et Intérêt.* Paris, Imprimerie Nationale, 1947b. [New Edition: Paris: Les Editions Clément Juglar, forthcoming.]

"L'Extension des Théories de l'Equilibre économique général et du Rendement social au cas du Risque," *Econometrica*, October 1953a, *21*, 269–290.

*La Gestion des Houllères nationalisées et la Théorie economique.* Paris: Imprimerie Nationale, 1953b.

"La Psychologie de l'Homme rationnel devant le Risque, la Théorie et l'Experience," *Journal de la Societé de Statistique de Paris*, Janvier-Mars 1953c, 47–73.

"Le Comportement de l'Homme rationnel devant le Risque, Critique des Postulats et Axiomes de l'Ecole Américaine," *Econometrica*, 1953d, *21*, 503–546.

"Fondements d'une Théorie positive des Choix comportant un Risque et Critique des Postulats et Axiomes de l'Ecole Américaine," *Actes du Colloque International sur le Risque*, Volume 40 of the "Colloques Internationaux" series. Paris: Editions du CNRS, 1953e, pp. 257–332.

*Illustration de la Théorie monétaire des Cycles économiques par des Modèles non linéaires*, mimeo, Centre d'Analyse Economique Ecole Nationale Supérieure des Mines de Paris, 1953f.

"Illustration de la Théorie monétaire des Cycles économiques par des Modèles non linéaires" (shortened version), *Econometrica*, 1954, *22*, 116–120.

"Explication des Cycles économiques par un Modèle non linéaire à Régulation retardée," *Metroeconomica*, 1956a, *8*, 4–83.

"Explication des Cycles économiques par un Modèle non linéaire à Régulation retardée, Memoire complémentaire," *Les Modèles Dynamiques en Econométrie*, Volume 62 of the "Colloques Internationaux" series. Paris: Editions du CNRS, 1956b, pp. 259–308.

"Method of Appraising Economic Prospects of Mining Exploration over Large Territories, Algerian Sahara Case Study," *Management Science*, 1957, *3*, 285–347.

"Influence du Coefficient capitalistique sur le Revenu réel par Tête," *Bulletin de l'Institut International de Statistique*, 1960, *38*, 1–70.

"The Influence of the Capital-Output Ratio in Real National Income," *Econometrica*, October 1962, *30*, 700–728. (Paper given as the Walras-Bowley Lecture at the 1961 meeting of the Econometric Society.) Reproduced in *Readings in Welfare Economics*, American Economic Association, 1969, *XII*, pp. 682–714.

"Options de la Politique tarifaire dans les Transports," (with del Viscovo, Duquesne de la Vinelle, Oort, and Seidenfus). *Travaux du Comité d'Experts pour l'Etude des Options de la Politique tarifaire dans les Transports*, Brussels, EEC Studies, Transportation Series, 1965a.

"The Role of Capital in Economic Development," *The Econometric Approach to Development Planning.* Amsterdam: North-Holland, and Chicago: Rand MacNally, 1965b, pp. 697–1002.

"Reformulation de la Théorie quantitative de la Monnaie," *Bulletin SEDEIS*, 1965c, *supplement to 928.*

"A Restatement of the Quantity Theory of Money," *American Economic Review*, 1966, *56*, 1123–1156.

"Some Analytical and Practical Aspects of the Theory of Capital." In Malinvaud, E., and M. Bacharach, eds., *Activity Analysis in the Theory of Growth and Planning.* London: Macmillan, and New York: St. Martin's Press, 1967.

*Monnaie et Développement*, 7 volumes, mimeo, Ecole Nationale Supérieure des Mines de Paris, 1968.

"Le Condizioni dell' Efficienza nell' Economia." In Angeli, Franco, ed., *Programmazione e Progresso Economico.* Milan: Central Studi e Ricerche su Problemi Economico Sociali, 1969a, pp. 13–153, 169–181, 207–208 and 221–242.

*Table 1*
**Continued**

"Growth and Inflation," *Journal of Money, Credit and Banking*, 1969b, 355–462.
"A Reply to Michael Darby's Comments on Allais's Restatement of the Quantity Theory of Money," *American Economic Review*, 1970, *60*, 447–456.
"Les Théories de l'Equilibre économique général et de l'Efficacité maximale, Impasses récentes et nouvelles Perspectives," *Revue d'Economie Politique*, 1971, *81*, 331–409. English translation in: Schwodiauer, G., ed., *Equilibrium and Disequilibrium in Economic Theory*. Dordrecht: Reidel, 1977, pp. 129-201.
"Forgetfulness and Interest," *Journal of Money, Credit and Banking*, 1972, 40–71.
"The Psychological Rate of Interest," *Journal of Money, Credit and Banking*, 1974, 285–331.
"The Hereditary and Relativistic Formulation of the Demand for Money: Circular Reasoning or a Real Structural Relation, A Reply to Scadding's Note," *American Economic Review*, 1975, *65*, 454-464.
*L'Impôt sur le Capital et la Réforme monetaire*. Paris: Hermann, 1977.
"The So-Called Allais Paradox and Rational Decisions Under Uncertainty." In Allais, M., and O. Hagen, eds., *Expected Utility Hypotheses and the Allais Paradox*. Dordrecht: Reidel, 1979, pp. 437–681.
"La théorie générale des Surplus," *Economies et Sociétés*, 1981, *15*, no. 1-2-3 (Band I) and No. 4-5 (Band II). 2nd edition, Grenoble: Presses Universitaires de Grenoble, 1989.
"Fréquence, Probabilité et Hasard," *Journal de la Société de Statistique de Paris*, 1983, *124*, no. 2, pp. 70–102, no. 3., pp. 144–221.
"The Concepts of Surplus and Loss and the Reformulation of the Theories of Stable Economic Equilibrium and Maximum Efficiency." In Baranzini, M., and R. Scazzieri, eds., *Foundations of Economics, Structures of Inquiry and Economic Theory*. London: Basil Blackwell, 1984a, pp. 135–174.
"The Foundations of the Theory of Utility and Risk." In Hagen, O., and F. Wenstøp, eds., *Progress in Decision Theory*. Reidel: Dordrecht, 1984b, pp. 3–131.
"Determination of Cardinal Utility According to an Intrinsic Invariant Model." In Daboni, L., A. Montesano, and M. Lines, eds., *Recent Developments in the Foundations of Utility and Risk Theory*. Reidel: Dordrecht, 1986a, pp. 83–120.
"The Empirical Approaches of the Hereditary and Relativistic Theory of the Demand for Money: Results, Interpretation, Criticisms and Rejoinders," *Economia della Scelte Publiche*, Fondazione Luigi Einaudi, 1986b, *1*, pp. 3–83.
"Economic Surplus and the Equimarginal Principle," *The New Palgrave, A Dictionary of Economics*, Volume 2. London: Macmillan, 1987a, pp. 62–69.
"Allais's Paradox," *The New Palgrave, A Dictionary of Economics*, Volume I. London: Macmillan, 1987b, pp. 78–80.
"The Credit Mechanism and its Implications." In Feiwel, G., ed., *Essays in Honor of Kenneth Arrow*. London: Macmillan, 1987c, pp. 491–561.
"The General Theory of Random Choices in Relation to the Invariant Cardinal Utility Function and the Specific Probability Function: The $(U, \phi)$ Model, a General Overview." In Munier, B., ed., *Risk, Decision and Rationality*. Reidel: Dordrecht, 1988, pp. 231–289.
"L'Economie des Infrastructures de Transport et les Fondements du Calcul Economique," *Revue d'Economie Politique* (Special Issue, edited by Bertrand R. Munier and G. Terny, *Vingt ans de Calcul Economique Public: Bilan et Perspectives*), 1989a, pp. 159–203.
*Autoportraits, une vie, une oeuvre*. Paris: Montchrestien, 1989b.
"My Life Philosophy," *The American Economist*, 1989c, *33*, 3–17.
"An Outline of my Main Contributions to Economic Science," Nobel Lecture, *Theory and Decision*, 1990a, *30*.
*Pour la Réforme de la Fiscalité*. Paris: Clément Juglar, 1990b.
*Pour l'Indexation*. Paris: Clément Juglar, 1990c.
"Cardinal Utility: History, Empirical Findings and Applications, An Overview," *Theory and Decision*, 1991.

constitute a necessary and sufficient condition for the economy to be at one particular point of the Pareto efficient frontier. Allais made clear the correspondence between income distribution and any given point of the utility frontier.

*In Search of a Science of Economics* also squarely faces the difficulties encountered in real economies. Thus, Allais recognizes the existence of nonconvex technologies, and gives necessary and sufficient conditions to manage plants in sectors like electricity supply, coal production and railways transportation in a way that will produce the competitive prices required in his main proposition for welfare efficiency. Not without a glimpse of regret for physics, Allais called this a "theory of maximum social efficiency" ("théorie du rendement social maximum"). Later on, to avoid any misunderstanding, he used the term of "maximum economic efficiency."

May one say, consequently, that Allais is yet another economist who asserts that free markets solve every (or almost every) economic problem? Not at all. His personal experience in the Depression and his forceful respect of facts led him to introduce many reservations.

First, Allais admits (and advocated in 1943) that natural monopolies should be governmental property. In any case, he would at least demand strict regulations regarding such firms (pp. 656–657, no. 273).

Secondly, Allais makes clear in all his writings, then and since, that one has to fight sources of unearned incomes (*revenus non gagnés*). These include all possible sorts of pure rents, including seniorage on money issuing. Allais advocated several policy measures to fight such unearned incomes.

For pure rent, Allais first took Walras's position: nationalize all land (pp. 383–389, no. 163) and have the government rent it to farmers and other users. Consistently but more generally, he refused to co-sign the declaration of the Mount Pelerin Society in 1947 because F. von Hayek would not accept any compromise on private property and excluded every possibility of even limited collective property. After 1948, however, Allais admitted that the nationalization of land could impair political as well as economic freedom and changed his opinion. Thus, Allais (1977) argued that a tax should be levied on all privately owned physical durable goods (as explained below) to reconcile private property of land with the need to avoid rents being privately appropriated. In predominantly socialist postwar France, Allais was considered a staunch *libéral* economist, bluntly advocating free markets, but even so, no conservative journal would agree at that time to publish his proposal for a general tax on physical assets to replace every income tax. The paper was finally published by the socialist party's newspaper *Le Populaire*, thus showing Allais's substantial political independence!

On the other hand, Allais never changed opinion on seniorage: the issuing of money should be exclusively the central bank's responsibility and all the seniorage should thus go to the government. On this issue, Allais is completely opposed to von Hayek and in possibly substantial disagreement with Milton Friedman, for the consequence of Allais's view (1987c) is that no private bank could create money.

Allais's third major reservation about market efficiency is that, even if real capitalist economies were freed of institutional imperfections (among which the above-mentioned are quite important) and managed according to Allais's prescriptions, they would not attain Pareto optimality. The reasons behind this opinion seem to be, beyond transaction costs and inadequate income distribution, that Allais doubts the capacity of economic agents to perceive unbiased intertemporal rates of substitution (pp. 661–662, no. 277). In this respect, Allais seems closer to Herbert Simon than to any of the new classical economists, and certainly does not subscribe to anything like a rational expectations hypothesis.

But these reservations about markets, which belong intimately to Allais's philosophical view, his *weltanschauung* of the economic world, do not contradict the fact that Allais fiercely opposed collectivism, as well as all types of government interventions which could imperil decentralized market mechanisms. One has to make clear that in France, until at least the early 1960s, such an attitude led to being labeled as the bad pupil in class. Being little gifted for retreat, or even for compromise, Allais expressed his views even more bluntly and squarely and in some cases, this made communication difficult.

From the beginning of his first book, Allais has focused on intertemporal problems, so the question of how the economy moves towards a Pareto optimum (or close to it) has been a central problem in his research. It is little known that the first theorems on global stability in a *tâtonnement* process were proved by Allais in 1943 in the quite general framework of his model. Negishi (1962) mentioned it, however, in his survey of the stability of economic equilibrium.

But Allais (pp. 564–565, no. 228) was not satisfied with a *tâtonnement* process. In such a process, economic agents first simulate (without making any move) the relative supplies and demands situations of all other actors as functions of the price system, and move only after they have found a feasible solution to the problem. Such a *tâtonnement* view may be realistic in some auction markets, where someone actually plays the role of the Walrasian auctioneer (the "*commissaire priseur*") and announces increasingly higher prices until demand equals supply and only then calls for fulfillment of the transaction. But in practice, very few markets function in such an institutional setting. To be rigorous, then, the *tâtonnement* process requires some hypothesis—in today's terms we might say a common knowledge hypothesis—which can substitute for the absence of a Walrasian auctioneer. But Allais considers such hypotheses as artificial and thus of limited interest (p. 640, no. 270, footnotes 3 and 9).

To tackle this question, Allais designed an out-of-equilibrium process for an economy to evolve towards some Pareto optimum. He introduced, as a completely new idea in 1943, the concept of "maximum possibilities in the space of utilities," which is now familiar to many students in Economics 1 courses under the name of utility possibilities frontier. Allais imagined that exchanges take place effectively between two or more agents, at a price which is mutually agreeable for them, and in such a way that some utility increase or "surplus" accrues to each party in the exchange. Under such circumstances, no market price need exist during all periods where these out-of-equilibrium exchanges take place. Allais (p. 640, no. 270) undertakes to show that when every possibility of such an exchange between agents has been exhausted, an equilibrium (which is also a Pareto optimum) has been reached and that then, but only then, mutually agreeable prices for any set of agents in the economy do coincide.[4] To Allais in 1943, this was only a model of a dynamic trajectory of the economy towards an equilibrium. However, his dissatisfaction with Walrasian economics became such that he changed his view, as will be described in a moment.

It is difficult for us today to realize how revolutionary and paradoxical many of Allais's practical recommendations appeared in postwar Europe, and particularly in France. For example, in *The Problem of Transportation Coordination and Economic Theory* (*Le Problème de la Coordination des Transports et la Théorie Economique*), published in 1947(a), Allais explains that pricing transportation in terms of the value of the goods to be carried is inefficient in terms of social welfare. Such pricing habits, called ad valorem, were then commonplace. When he argued that pricing rules of railways should take into account peak–load periods as opposed to ordinary periods, his proposal triggered acrimonious comments of the type: "It is intellectually elegant and perhaps even correct, but it is certainly impractical." Yet today, such peak–load pricing is current practice in many countries of the world, including France.

Allais created a similar stir in 1949, when he wrote a report (published as Allais, 1953b) on the pricing policy of coal and the management practices in that nationalized French industry. After having made some (today standard) marginalist recommendations, he went on to argue that inefficient mining plants should be closed and, if necessary, some coal imported from the United States. Imagine suggesting that prices of certain types of coal had to be increased to foster profits while inflation was already in double digits! Imagine arguing that employment has to be reduced in coal-mining to maximize welfare, and that coal should be imported from the United States, while Communist trade unions were dominating labor! This certainly has little to do with deriving nice equations for the intellectual satisfaction of colleagues and students!

In the 1960s, Allais headed a committee of the EEC on transportation infrastructure and again argued that optimal pricing of such facilities had usually little to do with their cost but mainly to do with the rate of saturation of their capacity (the classical welfare problem of the optimal toll on a bridge, originally solved in 1844 by another engineer-economist named Dupuit, who

[4]This is not what is called a non-*tâtonnement* process towards equilibrium, for when exchanges in out-of-equilibrium conditions do effectively take place in those models, there is always a market price for each good, even before equilibrium has been reached.

also influenced Allais strongly). There, it is by no means clear that such recommendations are being followed nowadays. Indeed, a current practice allows tolls to be levied on turnpike traffic to reimburse funds borrowed to build the facilities, and to be then removed (examples are only too easy to find in France). According to such policies, tolls are levied when there is (most of the time) overcapacity and removed when there is complete saturation!

While doing this type of work, Allais was uneasy about total and average cost computations, about whether every input and output had a unique market price, and about the convex technology hypotheses. In the fall of 1967, he expressed his views on the shortcomings of standard Walrasian economics in a paper published in Italian (Allais, 1969a). In 1971, at the conference of French-speaking economists held in Lausanne in Walras's honor, he expressed his rejection of the Walrasian model even more forcefully (Allais, 1971). He argued that the assumptions of convex technologies and of a single price for every good on its market were both unrealistic. In addition, he argued that precise computations of marginal costs are very difficult to perform, according to his experience in the sectors of energy and transportation.

Allais then recalled his 1943 model of agents searching for a maximum surplus, outside of equilibrium, by exchanging commodities at commonly agreed-upon prices. This model became more than a dynamic procedure to attain an equilibrium state. Rather, it became a basis for a new paradigm of economic activity, which Allais calls markets (with a "s") economics (*l'Economie de marchés*).

In analyzing a markets economy, one does not depend on continuity, derivability and convexity hypotheses. Furthermore, the concept of price becomes secondary, since the central motivation for economic activity is the search for surpluses. Once the possibility of surplus has been exhausted for all sets of agents in the economy, an equilibrium will have been reached. This markets equilibrium will lie on the utility frontier and be a Pareto optimum unless there is some external constraint (like a social taboo or some misplaced regulation) which would turn our problem into a second-best situation. (These external constraints are, of course, different than the basic constraint that no more resources can be taken out of the market than have been brought to it.) At any markets equilibrium, there will be a unique price for each commodity—rates of exchange have to be equal, if possibilities of mutually surplus-forming exchanges are exhausted—but this markets equilibrium need not coincide with the Walrasian market equilibrium.[5]

To be sure, free competition will still be a necessary condition for general efficiency, but it will have an impact on businesses through the general need to offer any product at a lower price (thus more easily agreed-upon with con-

[5]If we represent the functioning of a markets economy within an Edgeworth box, for example, budget constraints are not defined once and for all like in a Walrasian economy. Thus, income and welfare distribution between consumers will therefore differ in general from what they can be under such a Walrasian market equilibrium.

sumers) than competitors in each transaction, rather than through mechanical marginal cost pricing and efficient allocation of inputs. Indeed, input prices themselves are (consistently) not uniquely defined, except in an equilibrium state.

Clearly, the markets economy allows many useful syntheses. As one final example, Allais (1981–89, p. 421) shows that the losses in general welfare when moving slightly away from the utility frontier because of taxes or tariffs are only "of the second order and in any case relatively small when compared to the gains resulting from the stronger struggling for efficiency which results in turn from liberalization of exchanges and thus from competitive pressure." However, Allais rejects any rigid doctrinal position here and admits some moderate and temporary tariffs, to achieve bearable conditions of shifting labor from one activity into another, for example. As the search for synthesis has always been so important to Allais, it is little wonder that he extended his investigation of general intertemporal equilibrium to the specifics of growth theory.

## Theory of Capital

In the 1930s, capital theory consisted mainly of models that, although interesting, were either intuitive and incompletely specified or oversimplified, and thus unsatisfactory. In particular, the models of that time either dealt with interdependencies between agents or with interdependencies between periods. Allais undertook to study interdependencies between both agents and periods, in a more general setting.

Already in *In the Search for an Economic Science*, Allais (1943–52) made clear that the determination of intertemporal equilibria and of the interest rate heavily depended on the completeness or incompleteness of markets, even under the assumption of perfect expectations. But his general views on interest, intertemporal allocation of resources and growth were expressed in another major book, also explicitly mentioned in the Nobel Foundation award, *Economie et Intérêt* (Allais, 1947b, in particular ch. 7, 8, 9, 10 and annex II) and in four main articles elaborating on this fundamental contribution which appeared during the 1960s (Allais, 1960, 1962, 1965b, 1967).

On the consumption side, the most important contribution of Allais (1947b, ch. 10 and annex II) went unnoticed until E. Malinvaud (1986) drew attention to it. It refers to the essential point that all consumers, at a given date, are not all at the same stage in their life cycle: young people make choices which will have consequences for their future life, while old people benefit from their former decisions. But economists have to take into account the fact that young consumers of today will eventually become old ones, interacting then with new young consumers in an economic environment which will depend upon today's investments. This is nothing else than what economists have come to call the "overlapping generations model," usually attributed to Samuelson (1958), but

Allais had explicitly studied it eleven years earlier. One might add that Allais considered more general cases than Samuelson did later, although Allais did not focus on the special case treated by Samuelson in his "demographic" interpretation of the interest rate (Malinvaud, 1986, 1987).

Indeed, the fundamental point Allais made in 1947 is that, contrary to the conclusion drawn from models relying on a single "representative" consumer, the intertemporal preferences of consumers and the intertemporal production functions of firms are not sufficient to determine the interest rate. Why is this? Because, says Allais, a single consumer can always decide for himself how much (in terms of discounted income) he wants to spend in his youth and how much in his old age. But young people cannot decide how much of the global income should be allocated to themselves, as opposed to their elders, during their youth, and how much during the later part of their life. Nor can old people decide on the distribution of the global income which should prevail between themselves and the younger generation. The indeterminacy can only be waived if there is some collective rule of fair division between generations.

Allais (1947b, p. 592) ventures to suggest that "there are cases where, from a common interest point of view, the spontaneous equilibrium to which free competition leads should be discarded, and this is simply because competition cannot take into account individual preferences but for a given generation at a given point of time, not for the other generations, indeed not even for the considered generation at a later point of time." This calls for government intervention, which may in this case help to retrieve the situation "which would have been obtained spontaneously... if individuals would have had a clear and objective knowledge of their future preferences as well as of the advantages brought by the maximization of social productivity" (p. 593).

The latter question had led Allais to another major contribution. In chapter 7 of *Economie et Intérêt*, Allais undertakes to show that, for a stationary state (*régime permanent*), the optimal value of the interest rate should be zero. This condition of maximization of real national income—unheard of at the time—is nothing but the "golden rule" established independently by E. S. Phelps (1961) 14 years later, in the special case of a stationary economy.

Intuitively, Allais's golden rule can be visualized with the following example. Assume we grow potatoes by using plows which we also produce. To maximize the quantity harvested, we have to find some optimal allocation of disposable labor between building plows (for next year's usage) and using them this year to grow the potatoes. Obviously, extreme allocations are less favorable. The question is: how can the labor market spontaneously produce the optimal allocation of labor between the two tasks? To insure equilibrium, we have to hold salaries of the two types of workers proportional to their marginal physical productivities at any point in time. On the other hand, to insure maximization of output, these marginal productivities have to be equal. But remember there is a one-year lag between the respective outputs of the two types of labor, the respective salaries being thus in a $(1 + i)$ to 1 ratio. If quantities and prices stay constant over time (stationary economy hypothesis), it can be seen that these conditions can hold only if the interest rate is zero. If not, the market would shift more and more labor towards using the plows, to the expense of the labor producing them. Indeed, producers might lose some output value by doing so, but would save large interest costs. Consequently, global output would not be maximized anymore.[6]

The rule was later extended to the general case, where the economy is not necessarily in a stationary state, but may be growing (in the 1961 Walras-Bowley lecture at the Econometric Society meeting, published as Allais, 1962). The optimal interest rate has then to be equal to the rate of growth. Several other analytical and practical implications of the model were derived by Allais (1965b, 1967).[7]

As the great synthesis on interest rate theory developed in Chapter 10 of *Economie et Intérêt* points out, an interest rate plays four different roles: 1) it influences the value of accumulated capital at a given time; 2) it influences the savings-consumption ratio; 3) it influences the capitalistic intensiveness of production processes; and 4) it represents the price of holding liquid assets.

Allais's contribution to the first three aspects have already been mentioned. In the last one, studied in Chapter 8 of this inexhaustible book, Allais (1947b) managed to develop the essentials of Baumol's celebrated model of cash transaction demand in two footnotes written for completeness (footnotes 11, pp. 238–239, and 12, pp. 240–241). This contribution also went unnoticed until the present author mentioned it (Munier, 1986, p. 33); it was later reported by W. Baumol and J. Tobin (1989).

## Decision Theory Under Risk

When Allais heard of Von Neumann and Morgenstern's 1944 book, *Theory of Games and Economic Behavior*, he recalled his pre-war observations on horse-race betting and had an intuition that an important aspect of the psychology of

[6]Let $Y$ the quantity of potatoes harvested, $X$ the quantity of labor used to harvest, $Q$ the quantity of labor used to produce plows (for next year's use), $P$ the number of plows in use. Let $y$, $x$, $q$ and $p$ be the corresponding prices. No time index is needed, because of the stationary economy hypothesis. The production function for potatoes is $Y = f(X, P)$, the one for plows is $P = g(Q)$. Total labor constraint is $X + Q =$ Constant. Market equilibrium implies $f'(X)/x = f'(Q)/q$. Maximizing $Y = f(X, P)$ under $P = g(Q)$ and $X + Q = Ct$ implies $f'(X) = f'(Q)$. The latter equality together with the market equilibrium conditions implies clearly $x = q$. But $x = q(1 + i)$, where $i$ is the interest rate. This is possible only if $i = 0$.

[7]To establish these results, Allais made use of the "characteristic function" relating the average time elapsing in a given economy between the use of inputs and the availability of outputs in the capitalistic process. This kind of analytical tool had been used by S. Jevons (1871), and similar ideas developed in some Austrian writings, notably in F. von Hayek (1931). But this instrument hasn't been used since Allais's contributions. The "characteristic function" should be rehabilitated and taught again in economics courses, for it has a considerable importance in economic dynamics, as can be seen from Allais (1965b) in particular.

risk was missing in the new theory. More precisely, Allais had the impression that the independence axiom was far too strong to give an accurate account of what individual attitude towards risk was like.

To test his intuition, he designed a questionnaire, part of which gave rise to what was to become famous under the somewhat misleading name of the "Allais Paradox." Consider two pairs of prospects defined as follows:

$a_1$: certainty of receiving 100 millions Francs
$a_2$: 10 percent chance of receiving 500 millions Francs
89 percent chance of receiving 100 millions Francs
1 percent chance of receiving 0

and:

$a_3$: 11 percent chance of receiving 100 millions Francs
89 percent chance of receiving 0
$a_4$: 10 percent chance of receiving 500 millions Francs
90 percent chance of receiving 0

The reader of this paper should stop at this point. Take a moment for introspection and ask what your own choice would be between $a_1$ and $a_2$, on one hand, and between $a_3$ and $a_4$ on the other hand.

Allais and Darmois organized a conference in Paris in 1952 on mathematical economics and risk. The above questions were put to several participants by Allais, and in particular to authors of the Neumann-Savage expected utility theory—which Allais calls the "neo-Bernoullian" theory—to researchers supporting the theory, like B. de Finetti, as well as to other persons. Most of these individuals, including Savage himself, preferred $a_1$ to $a_2$ in the first pair and $a_4$ to $a_3$ in the second one. Later on, Allais submitted the questionnaire containing, among others, the above questions to a number of colleagues and students. About 65 percent of them made similar choices.

Where is the "paradox"? Strictly speaking, nowhere. But these results conflict with the generally accepted view of the expected utility hypothesis, where a decision-maker should weight the utility received in each possible event by the probability of that event. Indeed, to be consistent, any expected utility maximizer who prefers $a_1$ to $a_2$ should also prefer $a_3$ to $a_4$, while any expected utility maximizer who prefers $a_2$ to $a_1$ should prefer $a_4$ to $a_3$. This can be straightforwardly shown by the simple calculation of expected utility for $a_1$, $a_2$, $a_3$ and $a_4$, where the probabilities of the outcomes are decomposed into combinations of 0.01, 0.10 and 0.89 probabilities.[8]

Why, then, does such a large majority fall into the paradox? Because, says Allais, decision makers change their attitude towards risk in the vicinity of

[8] Under expected utility, $a_1$ should be valued at $0.01U(100) + 0.89U(100) + 0.10U(100)$, while $a_2$ should be valued at $0.01U(0) + 0.89U(100) + 0.10U(500)$. Thus, preferring $a_1$ to $a_2$ entails, by substraction: $0.01U(100) + 0.10U(100) > 0.01U(0) + 0.10U(500)$. Similarly, $a_3$ should be valued at $0.01U(100) + 0.89U(0)$, and $a_4$ at $0.01U(0) + 0.10U(500) + 0.89U(0)$. Thus, preferring $a_4$ to $a_3$ entails: $0.01U(0) + 0.10U(500) > 0.01U(100) + 0.10U(100)$, an obvious contradiction with preferring $a_1$ to $a_2$.

certainty when the amounts at stake are important. A very strong risk aversion is associated with the choice of $a_1$ (which is a *certain* option) over $a_2$. Such a strong feeling doesn't come into play when choosing between $a_3$ and $a_4$.

Allais had put the finger on what is now termed the "certainty effect" in experimental decision science. But he suspected, without being able to prove it immediately, that the lesson was more general: attitudes towards risk change not only from an individual to another, but also from a given individual between different patterns of risk.[9] This poses a challenge to von Neumann-Morgenstern utility theory, for attitude towards risk is exhibited in that framework only by the curvature of the utility function. In that theory, attitudes towards risk can change from an individual to another, since each individual has his or her own utility function, but *not between different patterns of risk for the same individual.*

This proposition has been since then corroborated by many experimental findings and empirical observations, obtained in different countries by different researchers of different backgrounds and cultures. Yet, as Machina (1983) has shown, most of these experiments reveal that attitude towards risk of a given individual changes *in a systematic way*, not just through random errors or misperceptions, with the risky patterns the decision maker is confronted with.

In this case, as is well known, Allais's contribution did not go unnoticed. Alas, a terrible ambiguity prevailed in the 1950s and the 1960s, which turned many scientific discussions on this point into a dialogue of the deaf.[10] In 1979, however, Allais and Hagen published a book which contained a much more extensive account of Allais's experimental findings and views than his 1953(d) article and which triggered a revival of interest in the debate. The latter has

[9] It is particularly interesting to remark that, when Allais tried to construct the von Neumann-Morgenstern utility index of the subject having answered his questionnaire, he found different functions according to the question considered for *all subjects*. This suggests that this very concept of utility wants to accomplish too many things simultaneously to be stable (see the next footnote for further discussion), whereas stability should be the first property required in a choice-generating function.

[10] For example, let us recall the Allais Paradox problem: $a_1$, $a_2$, $a_3$ and $a_4$ are well-defined lotteries. Von Neumann-Morgenstern utility ranks these lotteries *directly* through some function $W$ assigning to each lottery a real number (a so-called "cardinal" Neumannian utility value), so that we can compare these real numbers in lieu of comparing the lotteries. What Allais (following Daniel Bernoulli) envisions as "cardinal" utility requires a much simpler operation: it merely consists of saying that each individual assigns to a given sum of money a "psychological value" which is his own, depending upon personal wealth and other characteristics. Clearly, a sum of money is not a lottery: one cannot claim that in evaluating it, I am showing my attitude towards risk! When we say in everyday economics that the expected utility of a lottery is the probability-weighted sum of the utilities of the possible gains, we are being rigorous within a Bernoulli-Allais framework, but not if we are within a von Neumann-Morgenstern framework. In the latter case, what we ought to say is that the utility of the lottery is equal (due to the set of axioms von Neumann and Morgenstern use) to the probability-weighted sum of the utilities of the lotteries respectively giving with probability one each of the possible outcomes. The lack of rigor in everyday language makes it easy to confuse the concepts, but they are indeed quite different. In the Bernoulli-Allais case, the "curvature" of the utility curve represents only the diminishing marginal valuation of income. But in the von Neumann-Morgenstern case, it claims to represent *at the same time* the attitude towards risk (Pratt, 1964). The expression "cardinal utility" needs thus to be qualified every time it is used, which wasn't realized in the 1950s and in the 1960s. Hence the unproductive discussion at the time.

consequently taken a more precise direction: now that the Neumann and Allais constructs have been carefully distinguished, the question is which is more valid empirically and epistemologically. This in turn leads to new problems (Machina, 1987), new models (Munier, 1989) and new debates (Machina, 1989).

Meanwhile, Allais (1984b, 1986a, 1988, 1991) made his views more explicit and testable in several important contributions. He insisted that one has to clearly distinguish between marginal valuation of income (or of wealth), on one hand, and attitude towards risk on the other hand, and to assign each concept a separate analytical tool. Thus, in his 1986 $(U, \phi)$ model (published in 1988), he suggests that, in the same way as the (Bernoulli-Allais) utility function modifies the scale of the monetary outcomes to reflect marginal valuation of income, a "specific" probability function $\phi(\cdot)$ distorts the scale of the cumulative probability distribution (to reflect attitude towards risk in the specific risk-situation faced by the individual). A lottery is then valued as some expression of the "distorted" probabilities, linearly weighted by the corresponding utilities.[11] When the specific probability function $\phi(\cdot)$ reduces to the identity function—that is, in the special case in which there is no probability distortion —this is traditional expected utility.

Similar expressions, using a different utility function in each case, have been independently found by Quiggin (1982) while working explicitly on a suggestion from Allais, and by Yaari (1987). This has become known as the "anticipated utility" hypothesis (Munier, forthcoming), and it could be the ultimate result of Allais's contribution to decision under risk. It can be regarded as one particular and elegant analytical expression of the original idea of Allais —recall the prewar experience on horse race betting—to balance expected layoff in psychological value and the probability of ruin.

A direct expression of the same idea was put in practice by Allais in a study intended to rationalize mining exploration in the Saharan desert. The study he conducted (Allais, 1957) won him the Lanchester Prize in Operations Research, awarded by The Johns Hopkins University. This approach to risk psychology was used also, under a slightly more restrictive specification, to extend the general equilibrium and maximum efficiency results to a risky environment. Indeed, at the same 1952 conference already mentioned, K. J. Arrow and Allais presented alternative models on this very question. In Arrow's model, money has a specific usefulness only in the (admittedly likely) case where equities are less numerous than the states of the world. But what do Allais's main contributions to monetary theory look like?

[11]The preference function $A$ for a lottery $P$ can then be expressed as:

$$A(P) = U_1 + \phi(1 - P_1)(U_2 - U_1) + \phi(1 - P_1 - P_2)(U_3 - U_2) + \ldots + \phi(P_n)(U_n - U_{n-1})$$

which is the same as expected utility as soon as $\phi(P) = P$. Yaari's preference function is the special case where $U(x) = x$.

## Money and Monetary Dynamics

In monetary economics, Allais definitely belongs to the quantity theory tradition, but with two immediate provisos. First, Allais develops the demand side of the theory extensively, in such a way that it becomes much less mechanical than had been suggested in the past. One could venture to say that Allais finds traditional money demand theory too close to classical mechanics and, although he is fond of physics, wants all the same to look within that discipline for a more sophisticated inspiration to his theory: he finds it in relativity theory. Hence the concepts of "psychological" time and "psychological interest rate," to be explained in a moment. Second, Allais thinks that the importance of lags in the realm of money management has been overlooked. Unlike physicists, economists have largely ignored reaction periods.

Let us take a quick look at these two aspects of his monetary contributions. The Allais theory of demand for money has been given the name of HRL theory by its author. *H* stands for "hereditary," *R* for "relativistic," and *L* for "logistic." The reader will admit that this calls for some explanation. We can disregard the *L*, which has to do with the (logistic) form chosen by Allais on the basis of his axioms for some function in the model, and which is of secondary importance here. But what about the *H* and the *R*?

As to the *H*, Allais believes that the behavior of individuals is literally conditioned by their memories of the past, in contrast with most authors who relate it to some form of expectation of the future. A theory of collective memory was thus needed in economics. Allais suggested that we forget the past at a decreasing rate. Our "rate of oblivion" (or of "forgetfulness," as some translators of "*taux d'oubli*" have put it) works indeed in a symmetrical way to a "pure" discount rate; that is, we forget more and more about past figures the more distant the latter are in the past, exactly as we discount future incomes more heavily the more distant they are in the future. But this is not the whole story.

Indeed, the *R* comes in at this point. The time we usually count in economics is physical time. However, Allais considers two different types of time: the standard physical one and the "psychological" one, within which we can unclose the real "constants" of economic activity. Within that reference frame of the psychological time, the rate of oblivion and the "pure" discount rate (which Allais calls "psychological rate of interest") will be constant and equal. In the same way, within the frame of reference of "psychological time," the ratio of money demand to global expenditure is constant: thus, the Pigovian tradition of relative desired money balances finds, with Allais's HRL theory, a new and sexy youth. On another hand, the rate of oblivion in the standard physical time is variable: the more rapidly economic magnitudes evolve within that frame of reference, the more rapidly people forget (the higher their rate of oblivion). Thus, in a hyperinflation we forget very quickly what happened last month or even yesterday, whereas in a slow-growing economy with stable prices, our monetary behavior is conditioned to a greater extent by what happened some months ago.

In these different psychological processes (as well as in the physical ones), the economic magnitude Allais focuses on is the idea of "global expenditure"; that is, the sum of all funds spent by individuals within a unit of time, like a year. In each particular circumstance, Allais will find a "proxy" for global expenditure when testing the model.

Now, the problem is how to relate the psychological rate of interest and the standard rate of interest. The answer is that in a stationary economy both are equal, and thus also equal to the rate of oblivion. I would describe this as a situation when our memory-diagram is "flat," for all magnitudes, and in particular global expenditure. The higher the memorized rates of growth of past global expenditure, the higher the interest rate.[12]

This admittedly subtle model of money demand allows a prediction of both the interest rate and the stock of money in a given economy. The predictive power of the model appears quite strong in comparison with existing models (for numerous tests by Allais, see Allais, 1965c, 1966, 1972, 1974; for a test on U.S. money stock data, see Bethenod, 1979; for a test on U.S. interest rate data, see Durand, 1977). As these tests have been performed, at least prior to the 1970s, with only one or (more generally) two (depending on the variables selected for the tests) degrees of freedom, such results have stirred some suspicion. Some observers have mentioned potential spuriousness in the correlations; others, potential circular reasoning (Darby, 1970; J. L. Scadding, 1972). Allais (1970, 1975, 1986b) has convincingly countered these arguments.

This theory of the demand for money amounts, as the reader may now grasp, to a subtle restatement of the quantity of money, in which the velocity of money circulation, far from being a mere constant, is a stable function which incorporates memorized past events in the way we have seen.

The second aspect of Allais's monetary contributions is a dynamic theory of business cycles based on the gap between the money stock and money demand, through what Allais (1968, vol. 1, pp. 75–82; vol. 2, pp. 132–134) has called his

[12]If $D$ is set for total expenditure and $M_D$ is the desired nominal balances, real relative balances are $MD/D$ which is assumed constant within the reference frame of psychological time. If $\phi_0$ is some parameter to be estimated, $Z$ the "psychological rate of expansion" (that is, what our memory has withheld from past rates of growth of the economy) and $\psi(\cdot)$ a logistic function, if $\chi_0$ is the rate of oblivion $\chi(\cdot)$ for $Z = 0$ and $i(t)$ the interest rate at (standard) time $t$, Allais's basic equations can be written as:

(A) heredity $$M_d = \phi_0 \cdot D \cdot \psi(Z), \text{ with } \psi(Z) = \frac{2}{1 + e^2}.$$

(B) relativity $$i(t) = \frac{\chi_0}{\psi(Z_t)} = \chi(t).$$

Allais shows that the following differential equation can then be used to test the theory:

(C) $$\frac{dZ}{dt} + \frac{\chi_0}{2}(1 + e^{Z_t}) = \frac{I}{D_t} \cdot \frac{dD_t}{dt}.$$

"Fundamental Equation of Monetary Dynamics."[13] In this equation, fluctuations of total expenditure are directly related to the difference between money stock and money demand, and to the money supply in a broad sense (including the indebtedness of the economic agents outside the banking sector), the relation depending itself on a reaction lag, valued as the average period between expenditure decisions of economic agents and on the velocity of circulation.

Here is Allais's monetarism: money supply management is at the root of the regulation of economic activity. It is all the more difficult to perform because it is a lagged regulation process. If private banks do make profits in issuing money, part of which is of the seniorage type, there is every reason to fear that they will ignore the duty of regulation. Allais (1977, 1987c) consequently explains the monetary conditions for an efficiently working markets economy. First, banks should be differentiated into lending banks (financial intermediaries for saving term deposits) and deposit banks (cash intermediaries), the creation of money staying within the hands of the sole central bank with seniorage accruing to the government, as well as the regulation of economic activity through the money supply. Second, income tax could be done away with, and replaced by a general tax on all durable physical assets (Allais, 1990b). Finally, all future commitments should be indexed in real value, which amounts to using a stable unit of account (Allais, 1990c).

One can see here the influence Irving Fisher had on Allais's views, as Allais himself has always claimed. But one can also see the originality and the synthetic vision which characterize Allais's various contributions to economics.[14]

## Concluding Remarks

The Nobel award explicitly mentioned the outstanding contributions made in two of Allais's books (1943–52 and 1947b). But it is no easy task to separate these achievements from the other challenging views—the other "paradoxes" —which Allais expressed in several areas of economic, history, sociology and the philosophy of probability and determinism. All these hold tightly together. Reading Allais is like wandering through an intellectual garden picking surprising fruits from different trees. I hope to have made clear that, even if a reader gets temporarily lost in this extensive garden, he will never be bored.

[13]Let $V$ be the velocity of circulation, $E$ the indebtedness of the economic agents outside the banking sector, $T$ the defined reaction lag and $M$ the money supply. Using the previous notations, the "fundamental equation of monetary dynamics" is written as:

$$\frac{1}{D}\cdot\frac{dD}{dt}=\frac{1}{VT^2}\left(\frac{M-M_D}{M}\right)+\frac{1}{V\cdot T}\left(\frac{1}{M}\cdot\frac{dM}{dt}\right)+\frac{1}{T}\frac{E}{D}\left(\frac{1}{E}\cdot\frac{dE}{dt}\right).$$

[14]The interested reader, who would like to see a rigorous and more complete account of Allais's works and thought, should see Munier (1991).

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