NBER WORKING PAPER SERIES

SELF-FULFILLING EXPECTATIONS AND FLUCTUATIONS IN AGGREGATE DEMAND

Michael Woodford

Working Paper No. 3361

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
May 1990

This paper is part of NBER's research program in Economic Fluctuations. Any opinions expressed are those of the author and not those of the National Bureau of Economic Research.

NBER Working Paper #3361
May 1990

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ABSTRACT

The paper presents an intertemporal general equilibrium model with rationing in the product market, in which stationary sunspot equilibria are shown to exist, indicating the possibility of fluctuations in economic activity simply due to self-fulfilling variations in economic agents' expectations. Specifically, revised expectations about future aggregate demand change current investment demand, which (amplified by a "multiplier" process) then affects current aggregate demand. Parameter values required for endogenous fluctuations are discussed, as well as quantitative properties of the fluctuations predicted. Countercyclical stabilization policies are shown to rule out such equilibria.

Michael Woodford
Department of Economics
University of Chicago
1126 E. 59th Street
Chicago, IL 60637

A long tradition of attempts to explain recurrent cyclical fluctuations in business activity assigned a central role to shifts in the "degree of optimism" or "confidence" on the part of economic actors -- changes in outlook that were not required by any objective change in economic circumstances. Writers who assigned an important role to expectations as an autonomous causal factor include John Stuart Mill, John Mills, Alfred and Mary Marshall, Frederick Lavington, and Arthur Pigou. The culmination of this tradition was, of course, the *General Theory* of John Maynard Keynes, in which aggregate fluctuations are argued to be driven mainly by fluctuations in investment spending, which in turn is volatile largely because of its sensitivity to volatile expectations, the "animal spirits" of entrepreneurs.

Changes in expectations have not, however, been assigned so large a role in the explanation of business fluctuations in the period since Keynes wrote. There are probably several reasons. For one, in the postwar heyday of positivist social science, there was an evident reluctance to appeal to "subjective" factors. Thus Alvin Hansen (1964, p. 288) writes approvingly of Tugan-Baranovsky's rejection of an important role for expectational factors: "In the end it is cold objective facts that control, not simply psychological moods of optimism and pessimism. The hour of reckoning comes sooner or later."

Apart from this, there is doubtless a fear that free use of the hypothesis of expectational instability makes things too easy -- any event, it might be argued, can be "explained" after the fact by positing an arbitrary shift in expectations, but nothing can be predicted in advance. This is an understandable concern, but is unfair to the earlier literature on expectational instability. Changes in expectations were not invoked simply as a *deus ex machina*, allowing the analyst to evade any responsibility for narrative continuity in his account of economic events. On the contrary, there is frequently an insistence upon the extent to which a change in expectations, once begun, produces effects that confirm and strengthen that very belief. Lavington, for example, writes that an initial increase in the confidence of some producers, "whether or no this confidence is justified," leads to actions that are themselves "a real cause of increased confidence on the part of many other producers," so that a boom results (1921, pp. 171-172).

In other words, changes in beliefs become important in generating fluctuations in circumstances in which they tend to be *self-fulfilling*. Accordingly, this literature emphasizes the role of particular economic structures in creating the conditions under which revisions of expectations can become self-fulfilling, and gives detailed discussions of the particular sequence of events consequent upon such a trajectory once the initial perturbation from the economy's stationary state occurs. While little attempt is made to predict why the initial triggering event occurs or even what sort of event it must be, there are many predictions about the typical course of a "business cycle" and about the kind of economies that should be subject to instability of that sort.

The view that expectations ought not be assigned any independent explanatory role is sometimes thought to follow from the rational expectations hypothesis; for it is assumed that if expectations change in the absence of any change in economic "fundamentals", they must be biased. But in fact, it is possible even in a rational expectations equilibrium for expectations to change in response to a random event that does not affect fundamentals; the very fact that people change their expectations and hence their actions in response to the event can make it rational to change one's forecast when it occurs. David Cass and Karl Shell (1983) call such rational expectations equilibria "sunspot equilibria". In grounding the possibility of a causal role for revisions of expectations upon an underlying indeterminacy of rational expectations, the modern literature again formalizes a theme of the older writers, as represented in particular by Keynes' comparison of the stock market to a "beauty contest" (1936, pp. 154-158).

Some have argued that economic models with multiple equilibria should be judged unsatisfactory for that very reason, and so should be either assumed not to describe or the world, or should be supplemented by a selection criterion -- such as Bennett McCallum's (1983) "minimum state variable solution"-- which picks out a unique equilibrium, that presumably will not involve any fluctuations in response to "sunspot" variables. Behind such a proposal is the idea that ruling out such equilibria *a priori* imposes more discipline upon the process of economic explanation, by establishing a tighter link between assumptions about underlying structure and the predictions generated about aggregate fluctuations. But such reasoning is hardly persuasive. First, it is undesirable to rely upon an arbitrary selection principle that has no interpretation in terms of a causal mechanism the realism of which could be independently verified. But even more crucially, it is unfair to characterize reliance upon a multiplicity of rational expectations equilibria as making the predictions of economic theory too little specific, if the alternative strategy for explaining aggregate fluctuations is to assume continual random shifts in preferences and/or technology as needed in order to account for the fluctuations. Indeed, the latter approach allows the theorist many more degrees of freedom in explaining observed patterns of variation than does the one I argue for here. For only under certain conditions will sunspot equilibria be possible, while one can always obtain fluctuating equilibria if one posits fluctuating fundamentals; and insofar as one is free to assume any statistical properties one likes for the fluctuating fundamentals, the range of possible types of equilibrium fluctuations that are consistent with a given basic model becomes very large.

Another general ground for reluctance to consider models with multiple equilibria is the feeling that in such models determinate predictions about the consequences of policy interventions are not possible, so that economic theory ceases to provide any guidance for policy. It is true that the same sort of economic structures that allow for sunspot equilibria will also render indeterminate the response to many kinds of policy changes, and this even if one neglects to consider the possibility of sunspot equilibria, as Roger Farmer and I (1984) have shown previously. But this does not mean that such a model makes no useful predictions about policy choices. For it remains possible to distinguish between policy regimes or institutional arrangements that allow for sunspot equilibria and those that do not, and the choice of policies or institutions of the latter sort, in order to rule out one possible source of aggregate instability, may iself be an appropriate object of public policy.

In the present paper, I illustrate these points by presenting a simple model in which equilibrium fluctuations are possible in response to self-fulfilling revisions of expectations, in the absence of any stochastic variation in either government policies or preferences and technology. The model is an example of an equilibrium business cycle theory of the kind called for by Robert Lucas (1980), in that the objectives and constraints of all economic decision makers are made explicit, as well as the way in which a given agent's constraints are the result of the decisions made by others. This means that the model's empirical relevance can be cross-checked against microeconomic studies of household, firm and industry behavior, and not simply in terms of its ability to successfully account for the observed character of fluctuations in aggregate variables. Furthermore, the basic structure of the model assumed here is dictated by a desire to account for certain basic features of aggregate fluctuations, quite apart from any presumption that they represent expectational phenomena rather than a response to changing fundamentals. I show that stationary sunspot equilibria are possible in such an economy under certain conditions, and show that the empirical plausibility of those conditions can be evaluated entirely with reference to features of the economy unrelated to the existence of aggegate fluctuations, such as quantitative properties of the economy's long run growth trend. I then show that despite the existence, under those conditions, of a large multiplicity of rational expectations equilibria, the model makes a large number of precise quantitative predictions about the character of equilibrium fluctuations if they occur, and the parameters determining these predictions can all be identified empirically by reference to either microeconomic studies or to the long run growth path, in much the same way as Finn Kydland and Edward Prescott (1982) are able to "calibrate" their business cycle model. Even in the very simple version of the model presented here, in which many extreme specifications are adopted in order to simply the exposition, the predictions regarding the co-movements of aggegate variables in fluctuating equilibria capture many important features of observed business cycles. Finally, I use this example to illustrate a number of general points about the kind of economic structures in which self-fulfilling revisions of expectations are possible, and about the consequences of such an explanation of aggregate fluctuations for stabilization policy.

An Imperfectly Competitive Model of Aggregate Fluctuations

The model of aggregate fluctuations that I sketch here derives from an endogenous cycle model developed by Duncan Foley (1987). In this model, a random event that leads people to expect high aggregate demand in the future increases current investment demand; the increased investment demand then makes current aggregate demand higher. Furthermore, if the "sunspot" variable that triggers optimistic expectations is positively serially correlated, then it is possible for the change in expectations to be correct, insofar as the realization that triggers optimism also means a higher probability that such a realization will occur in the future, resulting in high aggregate demand then. Thus people's adherence to a particular theory about the significance of that realization leads them to act in a way that *makes that theory true*, although other theories -- e.g., the belief that the sunspot variable has no significance at all for future aggregate demand -- would equally well prove self-fulfilling if adhered to. In this way fluctuations in economic activity can result from changes in expectations cued by arbitrary random events, and yet be consistent with rational expectations equilibrium.

Elements of a Model

Robert Hall (1986, 1987) has argued that imperfect competition in firms' product markets is necessary to explain several features of observed aggregate fluctuations. In particular, he argues that the existence of prices above marginal cost can account for the procyclical movement of the "Solow residual" -- a measure of productivity growth based upon the growth accounting of Robert Solow (1957) -- without having to assume true productivity shocks that happen to be strongly correlated across industries. Stated alternatively, pricing above marginal cost can explain why the elasticity of output with respect to variation in labor inputs is so much larger than labor's share of value added. Hall computes estimates of the mark-up of prices above marginal cost for some fifty two-digit industries, and finds estimated mark-up ratios of over 1.5 for more than half of the industries considered.

The existence of such large mark-ups raises the question of why one does not observe higher rates of profitability for U.S. corporations. Hall argues that firms with market power are typically operating on a decreasing portion of their average cost curves, due to Chamberlinian monopolistic competition between firms with fixed costs or a minimum practical scale of operation. As evidence for the existence of such fixed costs Hall shows that for most industries in his sample, the return to capital would be negative if output were priced at marginal cost. Hall (1989) also shows that his "cost based productivity residual" -- a measure related to the Solow residual, but with the property that even with pricing above marginal cost, it should be uncorrelated with sources of output fluctuations other than true productivity growth, if firms choose capacity optimally and the technology exhibits constant returns to scale -- moves in the same direction as output in the case of oil price shocks or military spending shocks. This too is evidence in favor of excess capacity due to Chamberlinian competition.

The model presented here adopts both of these specifications: pricing above marginal cost, and free entry combined with fixed costs of production. However, it goes beyond Hall's suggestions in assuming that the repesentative firm faces not just a downward sloping demand curve, but a kinked demand curve. Such a demand curve can result in the case of sequential search by buyers, as Joseph Stiglitz (1987) has shown, if there are increasing costs of additional search and a sufficiently large number of competitors. The kink is at the price charged by all other firms. Demand is relatively elastic above this price because buyers -- who know the distribution of prices charged by all firms but not the prices of individual firms -- are in this case certain of finding a lower price if they search again. But demand is relatively inelastic below the market price, for few buyers from other firms would be induced to continue searching, as they would expect to have to search many times in order to find the one low-price firm.

The appeal of such a specification is as follows. A crucial fact that any business cycle theory must explain is why output is not predetermined by the existing level of productive capacity, i.e., how cyclical variations in capacity utilization are possible. Since the output of almost all industries is procyclical (see, e.g., Table 3 in Hall (1986)), a model in which productive factors are bid away from countercyclical sectors to procyclical sectors in booms is inadequate to explain this fact. Accordingly, I restrict my attention to a one-sector economy, and for the sake of simplicity I suppose that labor is the only factor that is variable in the short run. Then in a competitive economy, equilibrium requires production by the representative firm at a level Q statisfying

$$w(L(Q))\ L'(Q) = 1 \tag{1}$$

where $w(L)$ is the real wage required to induce labor supply L, and $L(Q)$ is the labor required to produce Q given existing capacity. The solution to (1) will be at least locally unique for generic specifications of these functions, and if w', L', $L'' > 0$, as would usually be assumed, the solution will be unique. Thus supply factors determine the equilibrium level of output quite independently of any demand considerations.

But over the business cycle we observe variations in the level of production that cannot be accounted for solely in terms of increases in productive capacity as a result of investment or decreases due to depreciation. One possible explanation is shifts in the labor supply relation $w(L)$, whether due to money illusion, non-indexed wage contracts, or the intertemporal substitution in labor supply emphasized by many equilibrium business cycle theories. All of these suggestions founder upon the observation that real wages do not move countercyclically -- as was pointed out soon after Keynes' suggestion that a rigid money wage could explain supply variations -- though this would be required by such a model if $L'' > 0$. An alternative explanation would be that the $L'(Q)$ relation shifts due to productivity shocks, as in the model of Kydland and Prescott. But, as Hall (1986) notes, it is hard to believe that such shocks should be significantly correlated across sectors in general. And even granting variation in the overall pace of technological innovation, such innovation would seem most often to be "embodied" in new capital goods, rather than to shift the production function for existing capital goods. Thus technical change should shift investment demand, but not the $L'(Q)$ function given existing productive capacity, and hence not equilibrium supply. Furthermore, as Julio Rotemberg and I (1989) have shown, the increase in employment and output that follows an innovation in real military purchases (a reasonable proxy for an exogenous change in aggregate demand) is not associated with a decline in real wages but, if anything, an increase. This cannot plausibly be explained by postulating either that military purchases shift the aggregate production function or that innovations in military purchases result from shocks to technology.

Nor does the mere allowance for market power change the argument much. In the case of a smooth, downward sloping demand curve, (1) becomes

$$w(L(Q))\,L'(Q) \;=\; 1 - e^{-1} \tag{2}$$

where e is the elasticity of demand. Shifts in the demand curve (specifically, in the quantity demanded when each firm prices at the price charged by all other firms) still cannot bring about any change in equilibrium supply, except insofar as they happen to coincide with changes in the elasticity of demand at that price. While there are various reasons why cyclical variation in demand elasticity might occur (Stiglitz (1984) and Rotemberg and Woodford (1989) discuss a number of possibilities), the assumption of a kinked demand curve is by far the simplest solution, and will allow the complete equilibrium dynamics to be analyzed in an especially simple way. In this case (2) becomes

$$1 - \underline{e}^{-1} \;\leq\; w(L(Q))\,L'(Q) \;\leq\; 1 - \bar{e}^{-1} \tag{3}$$

where $\underline{e}$ is the elasticity of demand for prices just below the kink, and $\bar{e}$ is the elasticity just above the kink. Hence it is possible for output to vary over a certain interval, in response to shifts in demand, without any change having occurred in either labor supply behavior or the production technology.

One immediate consequence of considering equilibria of this kind is that the equilibrium price of the price-setting game among firms is indeterminate. That is, given any particular demand conditions, there will exist an entire interval of prices that result in demand Q satisfying (3). It will be true of each such price that it will be rational for each firm to charge that price if it expects all other firms to; hence there is a continuum of Nash equilibria. This indeterminacy provides one way in which "sunspot" events could affect economic activity -- they might change firms' expectations about which price other firms will charge, a change in price that will actually come about if all firms expect it.

This is not, however, the kind of self-fulfilling expectations with which I am primarily concerned here. For one thing, my results here will be more interesting if one supposes that sunspot equilibria of a similar general character would also occur in the case of price rigidity of other kinds, rather than being strongly dependent upon a kinked demand curve. Furthermore, the type of sunspot equilibrium just described may not be robust under small changes in assumptions about agents' information. Suppose that there is a large number of firms, each of which observes the realization of the "sunspot" variable with a small amount of idiosyncratic error. It is not clear that there can exist an equilibrium in which each firm faces a kinked demand curve, if firms change their prices in response to their observation of the sunspot variable; for then each firm would expect its competitors to be charging a distribution of different prices. But Stiglitz-type equilibria would still exist in which the price charged is independent of the sunspot realization. The sunspot equilibria to be considered below, in which a sunspot event can create expectations of higher future aggregate demand that result in higher investment demand currently, *are* robust in this sense. No firm, in order to invest more in response to its observation of the signal, need believe that other firms observe exactly the same signal; it is sufficient that each firm's signal be sufficiently strongly correlated with the average signal that firms will receive in the future.

I will assume, then, that all firms charge a price p^*, that, because of the existence of a stable pricing convention, is the price each expects all others to charge. I will assume not only that this price does not change in response to arbitrary events observed by the sellers, but that it does not change in response to variations in aggregate demand. That is, firms increase the quantity they produce in response to an observed change in demand, rather than changing the price they charge. (I will, of course, have to assume that demand fluctuations are small enough so that this is consistent with (3) always being satisfied.) This will, again, be only one of many possible Nash equilibria -- varying degrees of price rigidity or flexibility could equally be self-fulfilling -- but it has the advantage, among the possible equilibria, of being especially robust under modifications of the information structure. A firm observing only the change in its own sales may not be able to infer the change in aggregate demand, if there are relative demand shocks as well; but under the rigid-price convention, it need only observe its own sales in order to know both what price to charge and how much to produce. Furthermore, the rigid-price equilibrium captures (while exaggerating) an important feature of observed aggregate fluctuations, which is that many prices remain fixed in money terms for quite long periods despite large changes in the quantities produced and sold.[1] It would also provide an explanation of how monetary policy can affect real activity, although that application will not be taken up here.[2]

Equilibrium Conditions

I assume an economy made up of a large number of identical representative households, each of which seeks to maximize the expected value of

$$\sum_{t=0}^{\infty} \beta^t U(c_t - v(L_t),\, m_t) \tag{4}$$

where c_t is consumption in period t, L_t is labor supplied, and m_t is real money balances held at the end of the period. Here β is a discount factor between zero and one, v is an increasing convex function, and U is a concave function, increasing in both arguments, such that $U_{cm} > 0$. The assumption made as to the way in which the disutility of labor enters implies that labor supply will depend solely upon the current real wage, simplifying the intertemporal linkages in the model. The labor supply curve can then be written

$$w_t = v'(L_t) \tag{5}$$

As is discussed further below, the introduction of liquidity services from money balances is simply a way of generating a demand for the asset in terms of which the pricing convention is defined; the asset need not be fiat money, although this seems the most realistic of the possible interpretations of the model. The assumptions regarding U_{cm} then reflect the idea that the need for real balances is positively related to the level of expenditures undertaken by an individual.[3]

I also assume a constant money supply M per capita. It then follows from the existence of a pricing convention of the sort described above that in equilibrium each household must choose to hold real balances in the same quantity $m^* = M/p^*$ each period. Given this, and the constant price level, one can show that in any equilibrium households choose to vary their consumption demand with their labor supply so that

$$c_t = e^* + v(L_t) \tag{6}$$

at all times, where e^* is a constant. (See the Appendix for details of this derivation.) This means that both arguments of U are constants in equilibrium, and hence that the marginal utility of consumption is a constant. Households will therefore be risk-neutral in their portfolio choices, and in equilibrium the expected real return on all assets not yielding liquidity services will have to be β^{-1}.

I assume that each firm, in order to produce, requires $\bar{K}$ units of capital goods, purchased the previous period; the firm then has a labor requirement $L(Q)$ if Q is the output to be produced. I assume that a fraction $\delta \leq 1$ of the capital depreciates after one period's use, while the remainder is indistinguishable from new capital goods. I ignore here any irreversibility of investment decisions -- that is, I assume that the $(1-\delta)\bar{K}$ units of capital left over after one period's production can be withdrawn and transferred to another firm if necessary. This somewhat artificial assumption can be dispensed with in a more complicated model in which a firm's labor requirement is a decreasing function of its capital stock, rather than each firm's capital having to be exactly $\bar{K}$. Similar results are obtained in that case. I also assume that the capital requirement $\bar{K}$ for a firm may be divided among several households who own shares in it, so that each household's capital accumulation decision involves a continuous choice variable. Finally, I assume that the number of firms in the economy is so large that the indivisibility of investment can be ignored for the aggregate economy as well.

The gross real return on capital in period t is then

$$r_t = \bar{K}^{-1}(Q_t - w_t L(Q_t)) + (1-\delta)$$

where Q_t is the demand per firm in period t, and w_t is the real wage. It is this quantity that must have an expected value of β^{-1}, conditional on information available in period t-1, when the investment decision is made.

I assume that total demand (aggregating the consumption and investment demands of all households) is allocated proportionally among all existing firms. That is, $Q_t = Y_t \bar{K}/K_t$, where Y_t is aggregate demand in period t, and K_t is the aggregate capital stock. The labor supply relation (5) then allows us to determine the equilibrium real wage w_t as a function of K_t and Y_t, which in turn allows us to express the gross returns to capital r_t as $r(K_t, Y_t)$. The level of capital stock K_{t+1} must be chosen in period t to satisfy

(7) $$E_t\, r(K_{t+1}, Y_{t+1}) = \beta^{-1}$$

Condition (7) is an "accelerator" relationship determining period t investment on the basis of expectations regarding period t+1 aggregate demand.

Finally, given aggregate labor demand (and hence equilibrium labor supply) in period t as a function of of K_t and Y_t, condition (5) gives consumption demand $c(K_t, Y_t)$ as well. The national income accounting identity is then

$$Y_t = c(K_t, Y_t) + K_{t+1} - (1-\delta)K_t$$

Solving this for Y_t yields a function of the form

(8) $$Y_t = f(K_t, K_{t+1})$$

that is increasing in K_{t+1}. Condition (8) is a "multiplier" relationship determining period t aggregate demand as a function of investment demand. Substituting (8) into (7) yields an equilibrium condition of the form

(9) $$E_t\, F(K_{t+1}, K_{t+2}) = 0$$

relating the choice of K_{t+1} to the probability distribution for K_{t+2} (the investment decision in period t+1) conditional on period t information. As is shown below, for empirically plausible parameter values, the function F is such that an expectation of a higher desired capital stock K_{t+2} in the following period leads to choice of a higher capital stock K_{t+1} in period t.

In any rational expectations equilibrium, the process by which the capital stock evolves must satisfy (9). And conversely, any process for the evolution of the capital stock satisfying (9), such that the implied level of demand per firm remains always within the bounds (3), and such that the capital stock remains forever bounded (so as to avoid any possible violation of the transversality condition for infinite horizon optimization), represents a rational expectations equilibrium. For any specification of a process for the capital stock satisfying (9), one can derive the evolution of aggregate demand using (8), and from this the evolution of labor demand, consumption demand, and so on. Hence we can restrict our attention to the question of what processes for the evolution of the capital stock satisfy (9).

Indeterminacy and Sunspot Equilibria

It is useful to begin by considering the set of *perfect foresight* equilibria, i.e., the set of equilibria in which all variables evolve deterministically, consistent with a given initial capital stock K_0. We can graph the deterministic relationship that must exist between K_{t+1} and K_{t+2} (for periods t = 0, 1, ...) as in Figure 1. A perfect foresight equilibrium is a sequence of capital stocks $\{K_1, K_2, ...\}$ such that $F(K_1, K_2) = F(K_2, K_3) = \ldots = 0$, or in other words that form a sequence of the kind depicted in the Figure.

Now when the F = 0 curve cuts the 45° line from above, as shown, there exists an entire interval of possible values for K_1, each of which begins a sequence that converges asymptotically to the steady state capital stock K^*. Let us suppose furthermore that the steady state involves a level of demand that satisfies both parts of (3) with strict inequality. Then for any choice of K_1 close enough to K^*, the associated sequence of capital stocks remains forever close enough to K^* for the associated levels of demand to always satisfy (3). Hence there must exist a continuum of equally acceptable perfect foresight equilibria. In all of these the

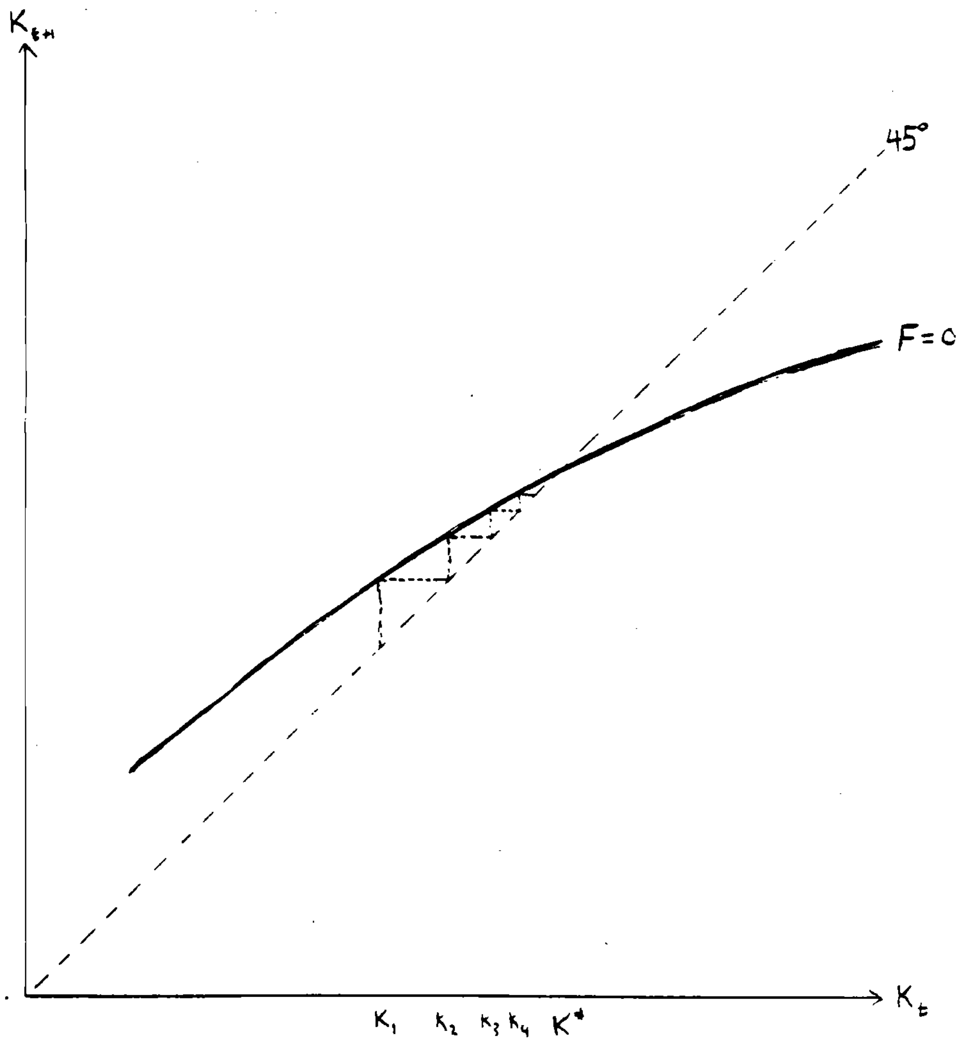

Figure 1.

equilibrium capital stock asymptotically approaches K^*, but it may start out either above that level or below it.

It is important to observe that all of these equilibria are equally consistent with the *same* initial capital stock K_0, as long as K_0 is close enough to K^* for (3) to be satisfied in period zero as well. The initial capital stock that happens to be in existence when the economy begins does *not* uniquely determine the subsequent perfect foresight equilibrium path. That is because there is no requirement that $F(K_0, K_1) = 0$ among the equilibrium conditions. It is not necessary for the existence of a perfect foresight equilibrium that the level of aggregate demand in period zero should render optimal the quantity of capital that happens to exist in period zero, since that quantity is simply an exogenous datum.

This indicates how it is possible for investment booms or slumps to come about simply due to a change in expectations. The choice of a high level of capital stock K_1 is possible in equilibrium, if people have come to expect a high desired capital stock K_2 in the following period, which in turn can be sustained by high expectations regarding K_3, and so on; but equally well one could have a low choice of K_1, due to low expectations regarding K_2, and so on. The infinite regress of expectations creates an indeterminacy of the same sort asserted by Keynes in his discussion of the long term rate of interest (1936, pp. 202-204).

In order for the above construction to work, of course, it is necessary that the graph of $F = 0$ cut the 45° line from above, as shown. This occurs if and only if

$$|F_2(K^*,K^*)| > |F_1(K^*,K^*)| \tag{10}$$

This condition is discussed further below and is shown not to require especially extreme parameter values.

Now let us consider the possibility of "sunspot" equilibria, i.e., rational expectations equilibria in which economic activity fluctuates in response to arbitrary random events that do not affect the equilibrium conditions directly. It is obvious that such equilibria are possible when (10) holds, for a random event could choose the level of K_1, with the evolution of the capital stock from then on being deterministic as above. That is, a "sunspot" realization could determine which of the many possible perfect foresight equilibria people expect to occur (and hence bring about).

But the effect of sunspot events need not be limited to such an initial selection. (Since we haven't observed a point in time at which our economy began, we wouldn't expect in that case to ever see a sunspot event mattering.) For just as the capital stock K_0 cannot force a particular equilibrium level of aggregate demand in period zero, so in later periods, once the capital stock K_t is in place there is nothing to prevent a sunspot event that changes people's expectations from causing a level of aggregate demand in period t that makes investors unhappy *ex post* about the amount of capital invested in in period t-1. In a rational expectations equilibrium the surprises cannot be systematic -- the *ex post* gross rate of return must be sometimes less than β^{-1}, sometimes greater -- but they may continually occur, and need not be due to any change in fundamentals.

Because we are interested in explaining repetitive "business cycles", it is of particular interest to consider the possibility of random fluctuations in economic activity that are stationary (in the sense that unconditional moments for all state variables are time invariant). This amounts to asking what stationary stochastic processes for $\{K_t\}$ will satisfy (9) at all times. In the case of such a stationary solution, in which the capital stock (and other variables) are not simply constant, one speaks of a *stationary sunspot equilibrium*.

One can demonstrate the possibility of such equilibria as follows.[4] Suppose that the *ex post* gross rate of return on capital in period t exceeds β^{-1} by an amount $F(K_t, K_{t+1}) = \varepsilon_t$, where ε_t is an independently and identically distributed random variable, with mean zero, a positive variance, and a bounded support $[\underline{\varepsilon}, \bar{\varepsilon}]$, the value of which is not realized until period t. One can solve for the level of investment required to bring about this rate of return, and write the result

$$K_{t+1} = G(K_t, \varepsilon_t) \tag{11}$$

(A unique solution exists, at least for K near K* and ε near zero; by choosing the bounds $[\underline{\varepsilon}, \bar{\varepsilon}]$ small enough, we can be sure that G is defined for all K_t in some interval I containing K*.)

Equation (11) then describes a Markov process for the evolution of the capital stock, assuming that the value of K_{t+1} obtained from (11) remains always within the interval I on which G is defined, and the Markov process is such that (9) is always satisfied. For any value of ε near zero, the function $G(\ .\ ,\varepsilon)$ will represent only a small perturbation of the curve shown in Figure 1. If condition (10) holds at the deterministic steady state, the perturbed curves, like the original curve, will cut the 45° line from above, as shown in Figure 2. It then follows, as shown in the Figure, that there will be an interval $[\underline{K}, \overline{K}]$, containing a neighborhood of K^*, and such that if K_t is in the interval, K_{t+1} is as well. This insures that (11) describes a well-defined Markov process. It also suffices to guarantee the existence of an invariant distribution over the interval, preserved by the Markovian dynamics, so that the stochastic process followed by the capital stock is indeed stationary. The stationary character of the implied fluctuations is perhaps clearest if one substitutes recursively into equation (11) to obtain an expression for the desired capital stock as a function solely of the history of sunspot realizations:

$$\begin{aligned} K_{t+1} &= G(K_t, \varepsilon_t) = G(G(K_{t-1}, \varepsilon_{t-1}), \varepsilon_t) \\ &= G(G(G(K_{t-2}, \varepsilon_{t-2}), \varepsilon_{t-1}), \varepsilon_t) = \ldots \\ &= g(\varepsilon_t, \varepsilon_{t-1}, \varepsilon_{t-2}, \ldots) \end{aligned}$$

When the curves look the way they are shown in Figures 1 and 2, g will be an increasing function of each of each of its arguments, with progressively less dependence upon realizations farther in the past.

The logic of the effect of the sunspot realizations upon the economy is as follows. Suppose that everyone expects aggregate investment to depend upon the history of sunspot realizations in the way represented by the function g. And suppose that in period t a positive value for ε_t is observed. Then, insofar as people expect the aggregate choice of K_{t+2} to be made in accordance with the above formula, the high realization for ε_t shifts the distribution of possible values of K_{t+2} toward higher values. This leads to a shift of the distribution of possible values for future aggregate demand, Y_{t+1}, toward higher values, through the multiplier relationship (8), and this leads to an increased desired capital stock K_{t+1}, through the

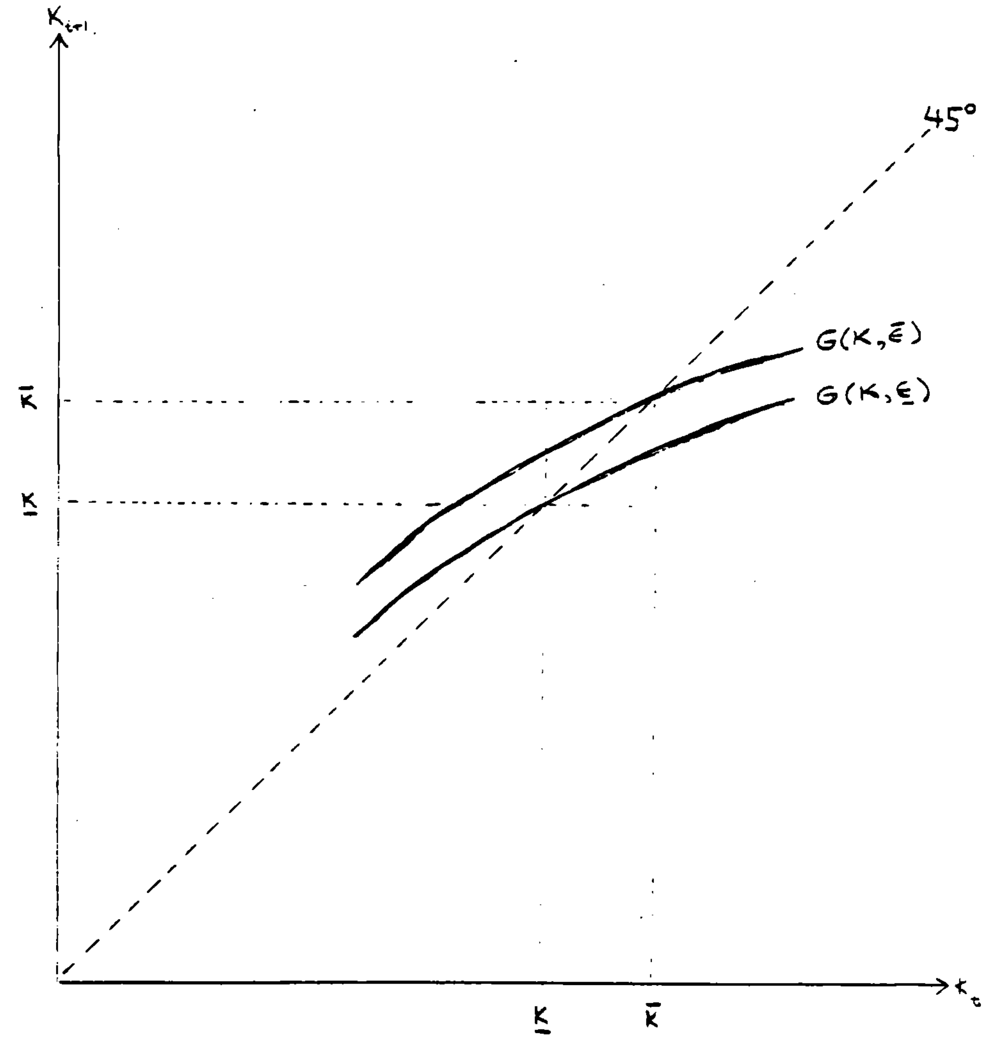
K_{t+1}
45°
$G(K,\bar{\epsilon})$
$G(K,\underline{\epsilon})$
$\bar{K}$
$\underline{K}$
K_t
$\underline{K}$
$\bar{K}$

Figure 2.

accelerator relationship (7). Thus K_{t+1} is chosen higher in response to the observation of a positive ε_t, in the way indicated by the function g. Thus belief that investment will respond in the future to the sunspot history in the way indicated by this function makes that belief correct in the present.

Parameter Values Required for Sunspot Equilibria

Thus far I have spoken simply of theoretical possibilities; it is useful to consider under what circumstances such phenomena could actually occur and whether the parameter values required are within the range of empirical plausibility. In particular, it is appropriate to ask whether condition (10) is at all plausible.

Differentiation of the equilibrium conditions shows that

$$F_1 = [e_s(\mu-1)]^{-1}[1-\delta+y-s_w y\mu] - \beta^{-1}$$

$$F_2 = 1 - [e_s(\mu-1)]^{-1}$$

where μ is the mark-up ratio, e_s is the elasticity of labor supply, y is the ratio of output per period to the capital stock, and s_w is the share of wages in national income, all evaluated at the deterministic steady state. If we let the "periods" represent quarters, then reasonable parameter values, based on typical values for aggregate U.S. data, would be about $y = .1$, $s_w = .7$, $\delta = .015$ (to obtain a 15% share of investment in total GNP, in the steady state), and $\beta^{-1} = 1.015$ (since corporate profits plus net interest make up about 15% of the gross domestic product of nonfinancial corporations). The figure for β^{-1} (representing a 6% annual rate of time preference) is also required by the others if (7) is to hold in the steady state.

Following Hall (1986, 1987), one might estimate μ by the requirement that μs_w equal the observed elasticity of output with respect to changes in hours of labor employed, which is to say, a quantity slightly above one. This would suggest $\mu = 1.5$ as a reasonable value for the aggregate economy. With these values, (10) is satisfied as long as $e_s < 1.96$. All studies of individual labor supply estimate supply elasticities much less than this. If one supposes that Hall's method overestimates the mark-up of prices over marginal costs (e.g., because of other contributions to marginal cost in addition to labor costs), the range of labor supply elasticities consistent with (10) can be even larger; for instance, if one assumes $\mu = 1.25$, (10) is satisfied as long as $e_s < 3.96$. Hence the model of stationary sunspot equilibria sketched here does not require particularly implausible parameter values, unlike some of the earliest theoretical examples described in the literature.

Conditions Necessary for Endogenous Fluctuations

Before considering further the realism of the model just presented as an account of actual business cycles, it is useful to reflect further on exactly which features of the model play a crucial role in generating endogenous fluctuations. Such an inquiry can help us to have a better sense of the extent to which the sort of mechanism at work in this example is likely to be at all robust under alternative institutional specifications. In addition, the present example can be used to illustrate some general points about the conditions under which sunspot equilibria are possible that may be of use in sharpening our theoretical understanding even if the example is ultimately judged not to be empirically realistic itself.

The Role of Fiat Money

The first fully specified economic model in which sunspot equilibria were shown to be possible, and still the best known example, is the overlapping generations model of fiat money.[5] A number of other equilibrium models of fiat money are also known to allow the existence of sunspot equilibria.[6] This has led to perhaps too great an identification of the possibility of sunspot equilibria with the problem of the indeterminacy of the equilibrium value of an intrinsically valueless asset, or with the paradoxical features of a good whose utility depends upon its price. Such a conclusion might lead to worry that the possibility of sunspot equilibria is a property only of certain relatively unsatisfactory models of money and might not characterize more realistic models of the role of money in the economy.

In fact, while the model described above involves valued fiat money, the sunspot fluctuations do not involve changes in the value of money. And the character of the equilibria would be unchanged if we assumed instead that the pricing convention fixes the price of the produced good in terms of some non-produced durable good that yields a real service flow. One would then interpret m_t to refer to the quantity of this durable good held by the representative agent at the end of period t, m^* to the fixed quantity of the good in existence, and the argument m_t in the utility function to the real flow of services received from the good, which need not be related to any role as a means of payment. This reinterpretation of the above model clarifies the fact that the mechanism responsible for the possibility of self-fulfilling expectations has nothing to do with the way in which money is assumed to provide liquidity services.

Absence of Contingent Claims Markets

Early examples in the literature have also fostered a widespread impression that sunspot equilibria can exist only if there do not exist markets for securities contingent upon sunspot events, that would allow people to insure themselves against the sunspot fluctuations. This view, in turn, has led some to suggest that the existence of sunspot fluctuations of any economic significance would provide an incentive for such markets to be created, suppressing the sunspot equilibria. Hence it is argued that theoretical examples of sunspot equilibria are of no practical significance.

It is true that in the overlapping generations example treated by Costas Azariadis (1981), introduction of contingent claims markets would prevent the existence of any sunspot equilibria.[7] And in overlapping generations models generally, it is true that if there exists an asset ("land") -- even in an extremely small quantity -- that pays off a stream of returns that is greater or equal to some positive fraction of the entire infinite horizon endowment of the economy, then the introduction of complete contingent claims markets will prevent the existence of sunspot equilibria.[8]

Nonetheless, in examples like the one presented here, the absence of securities contingent upon sunspot realizations has *nothing* to do with the existence of sunspot equilibria. We could introduce competitive trading in a complete set of such securities, and the set of possible equilibrium allocations of resources would remain *exactly the same*. For the markets for these securities would have to clear at prices such that the representative household holds a zero net position in each of them. This fact is, in my view, an important reason to believe that the sort of mechanism that generates sunspot equilibria in the model presented here is more likely to be of practical significance than is that represented by the overlapping generations examples.

The Role of Market Imperfections

The assumption that the product markets fail to be perfectly competitive, due to the costs of acquiring information about prices, *is* crucial for the existence of sunspot equilibria in the above example. To see this, consider instead an economy with the same kind of households and firms, with the same preferences and technology as above, but assume that the price of the produced good is set so as to always clear a competitive spot market. Let us assume also the "durable good" interpretation, rather than the "fiat money" interpretation of the non-produced good, since in this case households have standard preferences defined over quantities of goods independently of their prices. In that case, sunspot equilibria would plainly be impossible.

Consider first the case of complete markets, including markets for claims contingent upon sunspot histories. Then it is possible to prove a First Welfare Theorem, so that the allocation of resources associated with an equilibrium would have to maximize the expected utility of the representative household, and so equilibrium must be unique. But by the argument given above, the same set of equilibria exists even if there are no markets for contingent securities, so equilibrium is unique in that case as well. Furthermore, the unique equilibrium cannot be a sunspot equilibrium, since, assuming a convex aggregate production technology[9] and risk-averse households, it cannot be optimal to randomize the allocation of resources. Hence one conclusion from the example presented above is that failure of the price mechanism to clear goods markets can have the effect, not only of allowing an inefficiently low level of utilization of productive capacity, but of allowing the existence of endogenous fluctuations due to self-fulfilling expectations, when this would otherwise not occur.

By a similar argument, it can be shown that *some* sort of market imperfection is necessary in order for sunspot equilibria to be possible, in any representative agent model (under standard assumptions of concavity). For, by an argument first made by Cass and Shell (1983), under these assumptions a sunspot equilibrium is necessarily not Pareto optimal, and hence can exist only under conditions that prevent one from proving a First Welfare Theorem. But the range of kinds of market imperfections that can make sunspot equilibria possible is very wide, and need not have anything to do with price rigidity.[10]

The Predicted Character of Aggregate Fluctuations

I have indicated in the introduction that sunspot theories in fact make detailed quantitative predictions about the character of aggregate fluctuations that should be observed, insofar as the theory is correct and the fluctuations observed are not substantially due to exogenous shocks to the economy. I now wish to illustrate what I mean, using as an example the simple multiplier-accelerator theory sketched above.

When condition (10) holds, there exists a very large set of stochastic processes for the aggregate capital stock each of which satisfies (9). (Among others, there is a large set of solutions of the form (11), with different distributions for the random shock ε.) But this does not mean that the theory places few restrictions upon observed aggregate data. First of all, there is a system of stochastic difference equations -- including equations (6) through (9) above -- that must be satisfied by *any* equilibrium, and the restrictions implied by these equations can be tested econometrically.

But this might not seem an entirely satisfactory resolution of the question of the theory's empirical content; tests of individual equation specifications of this sort might be thought to reveal a certain amount about the realism of the preference specification (4), the realism of the model of competition among firms assumed here, and so on, without revealing much about whether or not self-fulfilling expectations alone can explain the sort of aggregate fluctuations that occur. From this point of view it is useful to consider the predictions of the model as a whole about summary statistics of the kind considered by Kydland and Prescott -- which variables should move together and what their relative variances should be, what patterns of serial correlation should be observed -- under the assumption that there are no other shocks to any part of the model.

The success or failure of predictions of this kind is, of course, difficult to interpret. Because the predictions depend upon so many different aspects of the complete model specification, on the one hand, and because none of the individual predictions are uniquely consequences of the sort of specifications assumed, on the other, it is difficult to say to what extent a particular mixture of sucess and failure at mimicking observed statistics either confirms or casts doubt upon either the overall modelling strategy or the particular specifications used. One can only hope to have some sense of this after the corresponding predictions of many alternative models have been calculated for comparison. Yet the body of work now existing within the "real business cycle theory" program gives one a certain baseline for comparison.

Furthermore, as Kydland and Prescott have emphasized, considerable discipline is provided in one's search for successful specifications if, in accordance with the arguments of Lucas (1980), one adopts an "equilibrium" modelling strategy. This means allowing for as few as possible "free parameters" that can be fit only to data on aggregate fluctuations, as opposed to parameters with a microeconomic interpretation that can in principle be estimated from other data sets. The model proposed here is of this sort, and below I emphasize the degree to which precise quantitative predictions about fluctuations are obtained once the model's parameters have been "calibrated" on the basis of other kinds of facts about the U.S. economy. It is worth pointing out in this connection that acceptance of Lucas' general methodological point does not require one to assume *a priori* the existence of perfectly competitive markets, insofar as parameters such as our mark-up factor μ are also parameters that can be estimated from microeconomic data sets rather than "free parameters" in Lucas' sense.

The model presented here differs from that considered by Kydland and Prescott in that it possesses a large multiplicity of equilibria while theirs has a unique equilibrium. But this model nonetheless allows detailed predictions about the co-movements of various variables in equilibrium fluctuations, because all stationary sunspot equilibria in which the fluctuations around the deterministic steady state are *small enough* for a linear approximation to the equilibrium conditions to be valid will *look alike* in this respect -- only the *amplitude* of the fluctuations differs across equilibria, to a linear approximation. (For all stochastic solutions to the *linearized* equilibrium conditions are scaled up or scaled down versions of a single solution.[11]) And the lack of a theoretical prediction about what the amplitude of fluctuations must be puts sunspot theories in no worse a position than any theory that posits exogenous shocks to tastes or technology whose magnitude cannot be determined other than from the amplitude of aggregate fluctuations.

Persistence

A central fact about observed fluctuations in aggregate activity is their persistence -- i.e., positive serial correlation. This is also a property of equilibrium fluctuations in the above model, and the model makes a precise prediction about the degree of serial correlation, i.e., about the time scale over which the fluctuations decay. Linearizing (9) around the deterministic steady state yields

$$E_t[\lambda K_{t+1} - K_{t+2}] = 0$$

where $\lambda = -F_1/F_2$, and so in all stationary sunspot equilibria involving small enough fluctuations around the steady state, the capital stock follows a first-order autoregressive process

$$(12) \qquad K_{t+2} = \lambda K_{t+1} + \varepsilon_{t+1}$$

where $\{\varepsilon_t\}$ is an identically and independently distributed mean zero sunspot process. (From here on, I write K_t for the *deviation* of the aggregate capital stock from its steady state value, and likewise for other variables.) Under (10), $|\lambda| < 1$, and λ represents the degree of serial correlation of the stationary process (12).

Using the above formulas for the derivatives of F, we can obtain a prediction regarding the value of λ. For the parameter values assumed above, and assuming that $0 < e_s < 1$, λ is predicted to be between .94 and .98. Thus the model predicts substantial persistence in fluctuations of the capital stock about its steady state value. This is not surprising, since it is the expectation of a high desired capital stock in the future that sustains a high capital stock in the present.

We can similarly obtain quantitative predictions regarding the serial correlation properties of output fluctuations. Linearizing (8) and substituting into it (12) yields an equation of the form

$$(13) \qquad Y_{t+1} = (\mu-1)^{-1}[\rho K_{t+1} + \varepsilon_{t+1}]$$

This shows that the same sort of sunspot event in period t+1 that increases investment demand increases output, and the multiplier is $(\mu-1)^{-1}$, as noted earlier.[12] We can use (13) along with (12) to trace out a complete impulse response function for output in response to a sunspot event.

This prediction of the model is not appealing. Under the assumptions about parameter values made above, ρ must be very small and may even be negative. Hence the theory predicts a return of GNP to nearly its steady state level in the quarter following an innovation in the output process, or even a reversal in sign, due to the immediate damping effect on subsequent investment demand of the large capital stock accumulated during the first quarter of a boom. But of course in actual U.S. data one observes a "hump shaped" impulse response function in which output is even farther from its trend level in the second quarter than in the first, and returns to the trend level only slowly. This undesirable prediction may be due to our oversimplified assumptions here about technology; with delivery lags in the investment technology, the effects on output of an investment boom might not be canceled so quickly.[13]

Hours, Real Wages, and Productivity

The same sort of revision of expectations that increases current output increases current labor demand, and, as noted above, the percentage increase in hours will equal the precentage increase in output divided by μs_w. Hence the model predicts procyclical hours, and, if the parameters are "calibrated" as above, it predicts the correct amount of cyclical variation in hours. (This is because, following Hall, we have estimated the average level of marginal cost on the basis of a comparison of the variability of hours to that of output.) Because of (5), this requires a procyclical real wage as well, another desirable feature of the model.

The question remains, of course, whether the model predicts the observed (quite small) degree of covariation of real wages with output. If we choose e_s large enough, we can get this prediction right, but the value of e_s required may be larger than is consistent with (10), and, in any event, the value required will be much larger than is consistent with panel studies of labor supply or with the observed long run trends in hours and real wages. Doubtless a serious consideration of this issue would, as with other equilibrium business cycle models, lead us to consider a more complex preference specification than (4), so that short- and long-run labor supply elasticities could be different.

Another possible solution is to argue that the observed cyclical variation in wages understates the cyclical variation in the shadow value of hours, because both prices and quantities are determined by long-term contracts between workers and firms rather than in a spot market. We can recast our model in these terms -- assuming that the contract between workers and firms specifies an efficient allocation of resources -- by simply assuming that each representative agent operates firms at which he employs himself, rather than buying or selling labor time in a spot market. The equilibrium conditions are in this case exactly the same as above, except that (5) does not occur, as there is no determination of a market wage.

Hence sunspot fluctuations continue to be possible in equilibrium, under the conditions discussed above, and have the properties here described. If one supposes that the average level of wages specified in the long-term contracts is equal to the average shadow value of hours, it continues to be correct to "calibrate" the parameter above called s_w on the basis of the observed average share of wages.

Average labor productivity is predicted to be slightly procyclical if μs_w is slightly greater than one, as assumed above. The fact that it need not be strongly procyclical is an advantage of this type of model over the technology-shock model of Kydland and Prescott (1982), which, as McCallum (1989) has noted, predicts more covariance of productivity with output variations than is observed.

Another type of productivity measure, the "Solow residual", is predicted to be procyclical as well, and to a much greater degree. A favorable revision of expectations will result in an apparent productivity increase in the amount of

$$\Delta Y_{t+1} - s_w \Delta L_{t+1} = (\mu Y^*)^{-1} \varepsilon_{t+1}$$

where Δ refers to the change in the logarithm of a variable. If $\mu = 1.5$, the apparent productivity increase will account for one-third of the percentage increase in output. The model proposed here -- which is that of Hall (1986) -- successfully accounts for the fact that the Solow residual has a greater covariance with output movements than does average labor productivity, an observation that is hard to explain if, following Kydland and Prescott, one interprets the Solow residual as a production function shift.

Hall's (1989) "cost-based productivity residual" -- in which labor's share in total income is replaced by labor's share in total costs -- is also predicted to be procyclical. In fact, neglecting terms of second order in the size of the revision of expectations ε_{t+1}, it will have the same value as the Solow residual, for in our model entry occurs until capital costs have exhausted all *ex ante* pure profits, so that the share of wages in total costs equals their share in total income. Hence the model proposed here would explain Hall's results.[14]

Matthew Shapiro's (1987) "dual" productivity residual

$$s_w \Delta w_{t+1} + (1-s_w)\Delta Y_{t+1}$$

should also increase when expectations are revised favorably, since both output and real wages increase. This measure is accordingly predicted to be strongly positively correlated with the Solow residual, as Shapiro finds. Shapiro finds, in fact, a coefficient near one when he regresses the Solow residual on the dual residual, and finds the difference between the two residuals not to be significantly correlated with GNP growth. This he interprets as evidence in favor of an interpretation of the Solow residual as mainly a production function shift, since in this case the two residuals should be equal. But the model presented here also predicts that the dual residual should be nearly equal to the Solow residual -- despite an assumption of no shifts in the production function at all -- if we choose e_s large enough, or assume long-term contracts in which real wages vary sufficiently little in response to short-term changes in the shadow value of hours, to account for the low cyclical variability of the real wage. Even assuming a correctly specified cost function, Shapiro's residual identifies true technology shifts only insofar as output price moves proportionally with changes in marginal cost of production; the countercyclical price-to-marginal cost margin associated with our model of rationing in the output market (or, similarly, a model such as Rotemberg and Woodford (1989)) results in procyclical movements of the Shapiro residual even if technology (and hence the cost function) does not shift.

Consumption and Investment Spending

Because of (6), the revision of expectations that leads to an increase in hours of labor employed must increase consumption as well. Hence both consumption spending and investment spending are predicted to be procyclical in this model. Their predicted relative variability also agrees reasonably well with observation. It follows from (6) that percentage fluctuations in consumption will be $s_w y/(y-\delta)$ times as large as percentage fluctuations in hours, or, given the parameter values used above, about .8 times as large; this is only slightly larger than the ratio observed for detrended U.S. aggregate data. However, the figure referred to (which is

McCallum's) refers to total consumption, including consumer durables purchases; non-durables purchases are considerably less cyclical, and our model has not modeled any durable aspect of consumption purchases. As a result, the variability of consumption in the calibrated model is really too high to correspond with reality. This doubtless results from a preference specification (4) in which variation in hours worked has a big effect on the marginal utility of consumption (leisure and consumption are quite substitutable). Smaller fluctuations in consumption could easily be arranged by choosing a different utility function. What is less clear is the extent to which preferences that make consumption less variable would still be consistent with existence of stationary sunspot equilibria, insofar as a large consumption response to an increase in investment demand is necessary in order to have a strong "multiplier" effect in (8), which is in turn crucial for creating a high degree of sensitivity of economic activity to expectations. It is possible that in a more complex (and more realistic) model, spending on durables would have to be explicitly modeled and would play a critical role in creating the strong "multiplier" effect needed to allow for self-fulfilling expectations.

It similarly follows from (12) and (13) that percentage fluctuations in investment should be 5 to 7 times as large as percentage fluctuations in output, given the parameter values used above (depending upon one's choice for e_5); the lower figure is about right for U.S. data.

Interest Rates and Asset Prices

As noted above, the *ex ante* real rate of interest is constant in the fluctuating equilibria of this model. The exact constancy of the interest rate is, of course, an artifact of the special kind of preferences assumed in (4); it is easy to vary this assumption and obtain some cyclical variation. Since most studies indicate little cyclical variation in *ex ante* real rates, while the Kydland-Prescott model predicts sizeable procyclical movements in the real rate (McCallum (1989)), the prediction of little variation is an advantage of the present model.

The model also predicts procyclical movements of stock prices, if these are taken to represent the value of existing capital goods, since a revision of expectations that increases aggregate demand Y_t also increases the *ex post* returns to capital $r(K_t, Y_t)$. The variation in the value of capital goods relative to current output, of course, can only be unexpected -- and hence cannot persist for more than a "period" -- because of the assumption of a one-sector technology with no adjustment costs for changes in the capital stock. This is another respect in which the predictions of the model could probably be made more realistic by introducing delivery lags for new capital goods.

Implications for Stabilization Policy

If we were to conclude that some important part of actual aggregate fluctuations is due to self-fulfilling revisions of expectations -- rather than representing a response to exogenous shocks to economic fundamentals -- this would necessarily have important consequences for the way we conceive the aims of stabilization policy. First, it might well create an increased presumption that the degree of fluctuations observed is undesirable, and so that a policy that succeeded in preventing such fluctuations, at least insofar as it does not require interventions so extreme as to create large costs of their own, could improve welfare. If there are stochastic shocks to preferences or technology, the optimal response of the economy will surely involve some sort of business cycle, and in the case of the best-known example of a business cycle theory of this kind, that of Kydland and Prescott, the aggregate fluctuations that occur under *laissez-faire* maximize the welfare of the representative agent. In a "sunspot" model, by contrast, the fluctuations surely are not efficient, as noted above.

But it is important not to assume from such a consideration alone that a stabilization scheme necessarily improves welfare. Even a scheme that succeeds in eliminating all sunspot equilibria while not interfering with the economy's deterministic steady state, like that discussed below, does not necessarily increase welfare. For the argument of Cass and Shell demonstrates only that sunspot equilibria are necessarily not Pareto optimal. In a model with market imperfections other than the mere absence of markets for securities contingent upon sunspot realizations, like the model presented here, there will in general be *no* Pareto optimal equilibria, and there is no reason to assume that the deterministic steady state is better than any of the sunspot equilibria. The model presented here illustrates this point. In the fluctuating equilibria, as in the steady state, $c_t - v(L_t) = e^*$ and $m_t = m^*$ at all times, so that the utility obtained by the representative agent is the same in all equilibria.

An important role for self-fulfilling expectations in generating aggregate instability would also create an increased presumption in favor of the possibility of reducing the amplitude of aggregate fluctuations through a properly designed stabilization policy, should that be a goal. It would also suggest the possibility of doing so with relatively little continuing intervention in markets, assuming that the government is able to credibly commit itself to intervene should instability develop. If business cycles are due to exogenous shocks to fundamentals, and equilibrium is unique, then alteration of the cyclical pattern that occurs in equilibrium will require significant alteration of the incentives faced by private parties and hence significant government intervention in the marketplace at all times. But if fluctuations are due to self-fulfilling expectations, then a simple commitment to a policy of intervening in order to stabilize *if it ever were necessary* could, by assuring people that fluctuations will not occur, prevent the occasion for intervention from ever arising.

A simple example of a stabilization policy that would suffice to rule out sunspot equilibria in the type of economy discussed above is as follows. Let us suppose that the government purchases a quantity of goods G_t in period t, financed through a lump sum tax on households in that same period, and that government demand is distributed equally across firms, just as with private demand. Then aggregate demand will be given by

$$Y_t = c(K_t, Y_t) + K_{t+1} - (1-\delta)K_t + G_t$$

which can be solved to yield

$$Y_t = f(K_t, K_{t+1} + G_t)$$

where the function f is as in (8). Equilibrium condition (9) accordingly becomes

$$E_t F(K_{t+1}, K_{t+2} + G_{t+1}) = 0$$

Now if the government commits itself to a policy of choosing G_{t+1} to offset fluctuations in private investment spending one-for-one, so that $K_{t+2} + G_{t+1} = \hat{A}$ regardless of the sunspot history, then if people understand this the capital stock K_{t+1} will be chosen at a level independent of the sunspot history as well. But then, in the only rational expectations equilibrium, the capital stock $\hat{K}$ will always be chosen, where $F(\hat{K}, \hat{A}) = 0$, and the level of government spending each period will be $\hat{G} = \hat{A} - \hat{K}$. From this will follow constant values for consumption, labor supply, the real wage, and so on. The policy is completely stabilizing, and without the level of government spending having to vary in equilibrium. If one chooses $\hat{A} = K^*$, then $\hat{K} = K^*$ as well, and the unique rational expectations equilibrium is the steady state that would exist in the absence of government spending.

It may be objected that the success of such a policy depends critically both upon rational expectations and, perhaps more doubtfully, upon the credibility of a government commitment that is never acted upon. This is surely true, and in reality one might suppose that from time to time the private sector would fail to be convinced that everyone is convinced that ... ad infinitum, and so choose a level of investment greater or lower than that associated with the steady state. This is plausible, and so the government probably would have to demonstrate its commitment to stabilize aggregate demand from time to time; but the amount of ongoing intervention that would be required is surely much less in a case like this than in one where active government intervention is required even in rational expectations equilibrium.

A more serious objection is that, as noted above, the mere suppression of the sunspot equilibria does not, in the present case, achieve a higher level of welfare for the representative agent. It is necessary to supplement such a policy with a policy to alter incentives so as to achieve a more efficient steady state allocation. We cannot address this issue here because the model presented above really admits of a continuum of deterministic steady states -- one for each possible value of the conventional price level p^* that results in a level of steady state demand satisfying (3) -- and so we must consider whether a policy intended to alter the steady state could not also result in a shift in p*. This issue cannot be resolved without a deeper theory of how firms should come to coordinate their price expectations.

APPENDIX

The representative household seeks to maximize the expected value of (4) subject to a sequence of budget constraints of the form

(A.1) $$m_t = m_{t-1} + w_t L_t + r_t K_t - c_t - K_{t+1}$$

together with the constraints $c_t, m_t, K_{t+1}, L_t \geq 0$. Stochastic processes for $\{c_t, m_t, K_{t+1}, L_t\}$ maximize expected utility if they satisfy the first-order conditions

(A.2) $$U_c(e_t, m_t) - U_m(e_t, m_t) = \beta\, E_t[U_c(e_{t+1}, m_{t+1})]$$

(A.3) $$U_c(e_t, m_t) = \beta\, E_t[r_{t+1}\, U_c(e_{t+1}, m_{t+1})]$$

(A.4) $$w_t = v'(L_t)$$

at all times, along with (A.1) and the transversality condition

(A.5) $$\lim_{t\to\infty} \beta^t\, E_0[U_c(e_t, m_t)\,(m_t + K_{t+1})] = 0$$

Here we write e_t for the utility argument $c_t - v(L_t)$, and U_c, U_m for the derivatives of U with respect to its first and second arguments respectively. Note that (A.4) is just the labor supply relation (5) given in the text.

Equation (A.2) can be thought of as a money demand equation for period t,

$$f(e_t, m_t) = i_t$$

where $f(e, m) = U_m(e, m)/[U_c(e, m) - U_m(e, m)]$ and i_t is the equilibrium nominal rate of interest on a one-period riskless bond issued in period t, i.e.,

(A.6) $$i_t = \frac{U_c(e_t, m_t)}{\beta E_t[U_c(e_{t+1}, m_{t+1})]} - 1$$

If $U_{cm} > 0$ as assumed in the text, the demand for real balances implied by (A.2) is a decreasing function of the nominal interest rate, as in conventional specifications. The demand for real balances will also be an increasing function of e_t, a conventional notion insofar as it implies that more consumption purchases result in greater demand for real balances. This specification is less conventional in its implication that money demand is also increased when leisure is greater.

In an equilibrium, the demand for real balances must always equal m^*, the constant level of real balances determined by the constant money supply M and the conventional price level p^*. Then (A.2) determines e_t as a function of expectations at time t regarding the distribution of values that may be taken by e_{t+1}. One solution is $e_t = e^*$ forever, where e^* satisfies

$$(\beta^{-1}-1)\, U_c(e^*, m^*) = U_m(e^*, m^*)$$

A solution $e^* > 0$ plainly exists, under standard boundary assumptions on U_c and U_m, if $U_{cm} > 0$, as assumed in the text. This solution then implies equation (6) in the text.

It can also be shown that $e_t = e^*$ is the only solution to (A.2) that can characterize a rational expectations equilibrium in which all state variables remain forever close to their values in the deterministic steady state of the economy (which assumption in turn is crucial to the use in the text of a linearization of the equilibrium conditions in deriving quantitiative predictions about the sunspot fluctuations). For if e_t were ever to exceed e^*, even by an infinitesimal amount, i_t would have to exceed its steady state value $(\beta^{-1}-1)$, in order for money demand not to exceed m^*. But this interest rate in excess of $\beta^{-1}-1$ would be possible, given (A.6), only insofar as $U_c(e_{t+1}, m^*)$ is expected at time t to be, on average, even smaller than $U_c(e_t, m^*)$. This is possible only if e_{t+1} can be even larger than e_t, which would require i_{t+1} to be even larger than i_t, and so on. Expectations about the more distant future must diverge progressively farther from the steady state values. Hence the unique solution remaining forever near the steady state is $e_t = e^*$ forever, which implies equation (6) in the text.

Since $(e_t, m_t) = (e^*, m^*)$ is constant forever, U_c is constant, and (A.3) reduces to

$$E_t[r_{t+1}] = \beta^{-1}$$

which then implies the "accelerator" relationship (7) in the text.

Note that the assumption made about U_{cm} does not make it more likely that sunspot equilibria will occur. To the contrary, if one supposed that U_{cm} was sufficiently negative, it would be possible to have stationary sunspot equilibria even without any of the market imperfections discussed below; there would be sunspot solutions to equation (A.2), corresponding to self-fulfilling fluctuations in the money demand curve, that would produce interest rate fluctuations that would in turn affect the allocation of resources, albeit not through the kind of mechanism stressed here. Indeed, sunspot equilibria could be generated through this channel even with perfectly competitive output markets and flexible prices. I wish, however, to deflect attention from this possibility, because it depends upon a money demand specification that seems unrealistic, i.e., one in which higher interest rates increase, rather than decrease money demand. For a sunspot event that results in a temporary reduction of e_t below e* must result in an expectation that e_t will rise, and so by (A.6) in a higher than average level of interest rates. But the increase in interest rates can cause a temporary reduction in e_t only if it *increases* the demand for real balances, so that a decline in e_t is required to clear the money market.

FOOTNOTES

* I would like to thank Duncan Foley, Paul Romer, and Andrei Shleifer for many helpful discussions of this material, and the National Science Foundation for research support.

1. For recent surveys of the evidence, see Carlton (1986), Blanchard (1987), and Rotemberg (1987).

2. For a discussion of monetary non-neutrality in a similar model see Woglom (1982).

3. It should also be noted that the role of this assumption is not to make it easier for self-fulfilling expectations to be possible; see further discussion in the Appendix.

4. The method used here is developed in Farmer and Woodford (1984), which paper provides additional details of the construction. Other methods that could also be used to demonstrate the existence of stationary sunspot equilibria in this model are illustrated in Woodford (1986a, 1986b).

5. Early analyses of sunspot equilibria in this model include Shell (1977), Azariadis (1981), and Azariadis and Guesnerie (1986).

6. See Woodford (1988).

7. Cass and Shell (1989) exhibit a non-monetary overlapping generations model in which sunspot equilibria can occur even with complete contingent claims markets. But this example is not robust to the introduction of a small amount of an asset of the kind discussed in the text.

8. In such a case the complete endowment of the economy must have a finite value in any equilibrium, in order to allow the asset to have a finite value. This allows one to prove a First Welfare Theorem in the standard manner. But since sunspot equilibria cannot be Pareto optimal, as noted below, there cannot be any sunspot equilibria in such a case.

9. Although each firm's production set is non-convex due to an indivisibility (it must purchase $\overline{\mathbf{K}}$ capital goods or produce nothing), the aggregate production possibility set, i.e., the set of 3-tuples (K,L,Y) such that $L \geq (K/\overline{\mathbf{K}})\, L(\overline{\mathbf{K}}Y/K)$, is convex if $L(Q)$ is a convex function.

10. For some other examples involving institutional assumptions of possible interest to macroeconomic theory, compare the models of Diamond and Fudenberg (1987), Smith (1988a, 1988b), Shleifer (1986), and Woodford (1986a, 1988). Diamond and Fudenberg do not demonstrate the existence of sunspot equilibria in their model, but instead, like Foley (1987), show the possibility of perfect foresight equilibrium cycles. However, the evident indeterminacy of perfect foresight equilibrium in their model indicates the possibility of stationary sunspot equilibria as well. For further discussion, see Woodford (1984).

11. For demonstration of the validity of the method of linearization used here, see Woodford (1986b).

12. Note that it is possible to have an extremely large investment multiplier, even in a model where consumption decisions solve an infinite horizon optimization problem, conumers are never liquidity constrained, and the real rate of interest is low and constant -- assumptions

generally thought to lead to "permanent income hypothesis" type consumption behavior and hence a small multiplier. The reason is because an increase in current aggregate demand can increase current consumption spending via another channel than the effect of higher current income on the lifetime budget constraint; higher demand for the produced good leads to higher labor demand, and a household that supplies more labor has a higher marginal utility for current consumption, under the assumption made in (4) that leisure and consumption are substitutes.

13. A "hump shaped" impulse response function of the kind actually observed would be consistent with the model presented here if we were free to assign ρ a large positive value in order to fit the observed pattern of output fluctuations. But "calibration" of the model's parameters on the basis of data for long run trend values prevents us from doing this -- an example of how much sharper the predictions of the model become using this procedure.

14. Strictly speaking, Hall's results are for fluctuations that are correlated with certain exogenous shocks to the economy such as changes in military spending, and hence that presumably do not represent sunspot phenomena. However, the model presented here would also allow demand shocks of this kind to affect output as well, and the predicted covariance of the Hall residual with output is the same in the case of demand fluctuations due to shocks to fundamentals as in the case of demand fluctuations due to self-fulfilling revisions of expectations.

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