RICHARD T. ELY LECTURE

The Design of Mechanisms for Resource Allocation

By LEONID HURWICZ*

Traditionally, economic analysis treats the economic system as one of the givens. The term "design" in the title is meant to stress that the structure of the economic system is to be regarded as an unknown. An unknown in what problem? Typically, that of finding a system that would be, in a sense to be specified, superior to the existing one. The idea of searching for a better system is at least as ancient as Plato's *Republic*, but it is only recently that tools have become available for a systematic, analytical approach to such search procedures. This new approach refuses to accept the institutional status quo of a particular time and place as the only legitimate object of interest and yet recognizes constraints that disqualify naive utopias.

A wealth of ideas, originating in disciplines as diverse as computer theory, public administration, games, and control sciences, has, in my view, opened up an exciting new frontier of economic analysis. It is the purpose of this paper to survey some of the accomplishments and to consider outstanding unsolved problems and desirable directions for future efforts.

It is not by accident that the terms "analytical" and "institutional" were only a few words apart in the preceding statement of scientific goals of our inquiry. In the past, especially in the nineteenth century, cleavage developed between analysts who tended to focus on the competitive and monopolistic market models and institutionalists who, either as historians or as reformers, felt the need for a broader framework, but found the existing analytical tools inadequate for their purposes. It is perhaps symbolic that a lecture named after "the father of institutional economics in the United States" (see H. C. Taylor) should provide a forum for a step toward synthesis of the two approaches.

I should make clear that I do not regard Richard T. Ely as a hundred percent kindred spirit. One reason for this may be seen from the following quotation of his views (1884) on mathematical economics:

> No mention has been made of the younger 'mathematical school' of political economists, of whom the chief representatives are Stanley Jevons . . . and Léon Walras . . . , because it is difficult to see in their mathematico-economical works anything more than a not very successful attempt to develop further the older abstract political economy. Any advance of the science due to the mathematical character of their method has certainly not yet become widely known, and the writer is much inclined to believe that the works which have advocated the application of mathematics to economics form no essential part of the development of economic literature. Certain unreal conceptions and a few definitions are used as bases for mathematical deductions. [p. 60]

Yet I find much to agree with Ely in his broader scientific objectives. However, I shall not go so far as to propose reinstatement into the bylaws of the American

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Economic Association the platform provisions which he proposed (1887) when the Association was being formed:

> 1. We regard the state as an educational and ethical agency whose positive aid is an indispensable condition of human progress. While we recognize the necessity of individual initiative in industrial life, we hold that the doctrine of *laissez-faire* is unsafe in politics and unsound in morals; and that it suggests an inadequate explanation of the relations between the state and the citizens. [pp. 6–7]

What I do sympathize with in Ely's attitude is the desire to view the economic system as a variable and to go beyond analytical frameworks that were unable to cope with this problem. A sharp statement illustrative of the "activist" point of view of that era (quoted by Ely who, however, characterizes it as too narrow) is the following definition, due to the Belgian Emile de Laveleye, dating from 1882: "Political economy may . . . be defined as the science which determines what laws men ought to adopt in order that they may, with the least possible exertion, procure the greatest abundance of things useful for the satisfaction of their wants, may distribute them justly and consume them rationally." I do feel that Ely underestimated the potential of development of theory (and of mathematical theory in particular) to help in this endeavor; but given the lag of a better part of a century in this development, perhaps he can be forgiven.

In what follows I want to focus on developments that are relatively recent, primarily those of the last two decades, and characterized by at least an attempt at rigorous mathematical formulation. First, however, I want to acknowledge the value of work that preceded the recent period.

In spirit, I regard the Utopians, and Utopian socialists in particular, as the initiators of what one might call an "activist" (as well as critical) attitude toward the social system in general, and the economic system in particular. They were, in a sense, the first systems designers in the social sphere. Marx, Engels, and their followers broke with the Utopian socialists. An unfortunate byproduct was the neglect of problems of resource allocation in the ("historically inevitable") socialist economy of the future, with Kautsky something of an exception. In the late nineteenth century there were, however, nonsocialist (and even antisocialist) economists who tackled the problem in a remarkably objective spirit, among them Pareto, Boehm-Bawerk, and von Wieser.[1] Barone's now famous 1908 paper was at least partly stimulated by Pareto's earlier analysis.

A "second round" of discussion was largely provoked by von Mises' skepticism as to even a theoretical feasibility of rational allocation under socialism. Oskar Lange's contribution to the debate in the 1930's is well known, but there was a remarkable earlier reply by Jacob Marschak in 1924.

While Lange's line was to be that socialism is as capable of playing the perfectly competitive game as is capitalism, Marschak took the opposite view: capitalism is a world not of perfect competition but of monopolies and cartels, which (in a Schumpeterian spirit) has its good points,

[1] It is striking that Pareto (1896–97, p. 58), in addition to pointing out the theoretical feasibility of rational allocation in what he called a collectivist régime (it would have the same coefficients of production as free competition), also dealt with costs of operating the system: "A second approximation will take account of the expense of putting the mechanism of free competition in full play, and will compare this expense with that necessary for establishing some other new mechanism which society may wish to test." Similarly, in the *Manuel* (p. 364), Pareto points out the need to compare the expense on the entrepreneurs and proprietors under the system of private property with that of state employees under collectivism. But, he says, "pure economics does not give us a truly decisive basis for choosing between organization based on private property and a socialist organization."

especially in the realm of dynamics. Marschak expected similar phenomena under the brand of democratic "socialism" he had in mind and was not depressed by the prospect. He felt that the advantages of imperfect competition would carry over into collectivism. The real issues would be not rational economic calculation, but motivation, stimulation of initiative, and intensity of effort—under an egalitarian system where managers would be democratically elected. (Still, he regarded these problems as less severe than those of "centralistic" socialism.)

In the 1930's, two major lines of development are relevant. One line was the work of Lange, Lerner, and others on resource allocation in a socialist economy, the slightly earlier (1929) paper by F. M. Taylor on trial and error methods, Hotelling's contribution on marginal cost pricing and consumer-producer surplus, and the "new welfare economics" of Hicks, Kaldor, Scitovsky, and others. The other line of development, started in the 1940's, was the mathematization of "classical welfare economics" by Lange (1942), Allais, Debreu, and Koopmans (1957), with the Arrow–Hahn book a recent entry in this series.

I have so far been stressing the ideas oriented toward redesigning the economy of a nation or similar collectivity. But with the enormous growth of private enterprises and governmental bodies, similar issues arise in determining the relationships between the headquarters of a firm and its divisions, or a ministry (department) and its components. Most of the proposed mechanisms are highly relevant in such circumstances, but the team theory model may be particularly appropriate.

There is also a close relationship with information theory and with problems of administrative organization. For linking up resource allocation with information processing and organization, major credit must go to Marschak's development of economic theory of information and to Herbert Simon for his work on organization and economic behavior.

Also one should not forget a pioneering effort toward an abstract formalization undertaken by J. B. Kruskal, Jr., and Allen Newell in "A Model for Organization Theory," circulated at Rand Corporation in 1950 but, I believe, never published.

It has been said, only half in jest, that the theory of organization is a field rich in definitions, but short on results in the form of theorems. This is no longer true. There are two categories of results. On the one hand, quite a few specific allocation mechanisms have been invented and their properties, such as feasibility, optimality, and convergence, rigorously established. On the other hand, there are also some more general results, dealing with the possibility or impossibility of various types of decentralized mechanisms, depending on the environments with which they must cope. We shall mention a few of these results, following the sampling of the specific mechanisms and the discussion of a framework required for formulating the more general questions.

I. Specific Mechanisms Whose Properties Have Been Investigated

As promised, we shall now sample, although very incompletely, some of the wealth of specific mechanisms that have been formulated in a rigorous way, mostly during the last two decades—but without forgetting the crucial influences of their less formal predecessors.

We shall confine ourselves to procedures that have been formulated with sufficient precision to avoid ambiguity as to which economic agent says what to whom and when. This makes it possible to determine the informational requirements, as well as convergence and optimality properties.

A major impetus was given to the design

of such mechanisms by these developments of the 1940's:

1) activity analysis and linear programming (including the simplex method)—Dantzig, Kantorovitch, Koopmans;

2) game theory, including the iterative solution procedures—von Neumann and Morgenstern, George Brown, Julia Robinson;

3) discoveries concerning the relationships connecting programming (linear or nonlinear), two-person zero sum games, and the long known Lagrange multipliers—Gale, Kuhn, Tucker.

While in economics one deals with goal conflicts due to multiplicity of consumers, linear and nonlinear programming models usually presuppose a single well-defined objective function to be, say, maximized, i.e., a situation corresponding to an economy with a single consumer. So it is not surprising that the mechanisms designed under the influence of programming theory dealt to a large extent with one-objective-function problems and thus failed to face the crucial issue of goal conflict. Nevertheless, one should not underrate their usefulness as a necessary step on the road to the harder multi-objective problem, since the difficulties of the simpler situations do not disappear when goal conflicts are introduced.

We can distinguish two strands here: one, a rather close relative of the programming approach; the other, "team theory," (J. Marschak, Radner, Groves) more closely related to the theory of statistical inference and decision making. We shall concentrate on the former.

We thus have a situation where there is only one consumer (individual, firm, or even nation) and, hence, only a single utility function to be maximized, but a multiplicity of producers and resource holders. The technological relations (production functions) and limits of resource availability constitute the constraints subject to which maximization must be carried out.

Two difficulties make the problem nontrivial: calculation and information transfer. First, consider the calculation of the maximizing values for the variables of the problem. Assuming even that all the relevant information concerning the parameters of the problem is in the hands of a computing agency, this agency needs a well-defined computational procedure (algorithm) to find solutions.

For linear economies, the simplex method is such an algorithm. For "smooth" (nonlinear) unconstrained maximization problems resembling the task of groping one's way in the dark to the top of a hill, the obvious idea of moving uphill is embodied in the notion of gradient (or "steepest ascent") processes. Equally evident is the fact that a valley between the spot one is at and the peak of the hill would cause trouble; hence, the success of gradient procedures depends on the curvature characteristics of the terrain, a natural requirement being that the hill be dome-like (technically, a strictly concave function).

A natural extension of the gradient process idea was suggested by the famous Kuhn-Tucker theorem associating with a constrained maximum a saddle point, i.e., a maximum-minimum point of the so-called Lagrangean expression; thus the search for a maximum with respect to certain decision variables was converted into a mixture of maximizing and minimizing tasks to each of which one could apply the gradient idea (groping upward in terms of the decision variables and downward with respect to certain auxiliary variables—Lagrange multipliers to the mathematician, shadow prices to the economist). *Statically*, the Kuhn-Tucker result required little more than the concavity (not necessarily strict) of the relevant functions and, hence, was applicable to linear problems as well as the typical smoothly curved pictures of classical economics (Pareto, Hicks). But we shall see that dynamics was more troublesome.

Even when there is an algorithm suitable for calculations by an agency to whom all the data are available, it may be that the information processing capacity of any one such agency is inadequate because of the size of the problem (the number and complexity of constraints, objectives, and variables). If there are several potential information-processing agencies (and here every human brain qualifies to some extent), we may be saved by devising computing procedures which parcel out the work among them; these are called decomposition algorithms. (In recent years, related problems have been studied in connection with the design of electronic computer utilization under the label of multi-processing.)

It should be recalled that one of Hayek's (1935, p. 212) chief points in summing up the state of the debate concerning the feasibility of a centralized socialist solution was that the number of variables and equations would be "at least in the hundreds of thousands" and the required equation solving "a task which, with any of the means known at present, could not be carried out in a lifetime. And yet these decisions would . . . have to be made continuously . . . ." The market-simulation procedure developed by Lange and Lerner may be viewed as an early example of a decomposition algorithm.

From the point of view of the economics of information processing it is clear that a parceling out of the task may be advantageous even if single agency capacity constraints have not been reached; this may well lower the resource cost and cut down the time required for the completion of the computing process.

But another informational consideration, stressed by Hayek (1935, 1945) has gained special prominence: the difficulty of placing all the relevant information in the hands of a single agency because information is dispersed throughout the economy. A natural assumption is that, initially, each economic unit has information about itself only: consumers about their respective preferences, producers about their technologies, and resource holders about the resources. An attempt to transfer all this information to a single agency before it starts its calculations is regarded as either impossible (in the sense that much information would be lost) or too costly in relation to the existing accuracy requirements. (One reason for this difficulty is that even the individual units have the required information only in potential form, except for situations corresponding to their past experience: e.g., firms know only certain parts of their production functions. It is easier to use "localized" procedures which require an exploration only of the relevant parts of the individual units' maps; but such localization is impossible if whole maps are to be conveyed to the single computing agency at the beginning of the computing process.)

If the economic units which initially are the only ones with information about themselves are also capable of carrying out calculations, it is natural to seek computing procedures that would both minimize the need for information transfers and also parcel out the tasks of calculation. This is what informationally decentralized procedures are meant to accomplish.

In the 1930's it would have been most natural to start with "smooth" strictly concave economies (diminishing marginal utilities and returns), without kinks or corner solutions. But around 1950, linear models were in fashion. Furthermore, the simplex method was available and proved to be convergent. Since the simplex method, applied to the economy as a whole, lacked informational decentralization, a search for an alternative was bound to occur. For the economist, an obvious candidate was a simulated (perfectly competitive) market process à la Lange, Lerner, and (specifically in the context of a linear economy) the Koopmans model with a helmsman representing the consumer, production managers maximizing profits, and resource custodians adjusting prices according to excess demand.

From a static point of view, the equilibrium of such a process would be optimal. But if the initially proposed process and quantities were "wrong," would there be convergence to equilibrium? To make this question meaningful, one must specify the dynamics of the adjustment process, e.g., by how much prices are to be raised per unit of time given the magnitude of excess demand, etc. A pioneering model of this type is due to Samuelson (1949) who postulated a system of differential equations in which prices vary proportionately to excess demand, and resource use rises when low resource prices yield positive profits. He immediately noted that this dynamic system would behave like a frictionless pendulum, i.e., would not converge to an equilibrium position. Whether we are thinking of computations or of designing an economy, we must look further.[2]

Samuelson's discovery posed a challenge: can an informationally decentralized convergent allocation process be designed for linear economies? (Of course, there is also the problem of designing such processes for economies with increasing returns. But here the difficulties are bound to be serious, since the competitive mechanism lacks even the usual static properties.)

One line of attack involves the replacement of the fixed (that is, parametric) price idea by that of a price *schedule* and is applicable not only in the linear (constant returns) case, but also under increasing returns. (See the modified Lagrangean Arrow-Hurwicz approach below.) Another approach, to be discussed first, retains the parametric prices; it grew out of linear programming techniques, with the Dantzig-Wolfe decomposition method its earliest example.

The Dantzig-Wolfe economy has special features which provide scope for the decomposition approach. These are the usual features of a resource allocation model without technological externalities, in which the objective function is a sum of the contributions of the individual units and in which certain resources must be utilized by all units. (Both the objective function and the overall constraints are "additively separable.") The mechanism may be viewed as a dialogue between the producing units (who know their technologies and contributions to the objective function) and a "center" which knows the total resources available. One aspect of the dialogue is that the center proposes tentative resource prices and the producing units develop corresponding profit-maximizing production programs (with prices treated parametrically). In the light of these programs, the center revises the proposed prices. Because of the linear character of the economy, both the center

[2] Samuelson himself suggested "a little intelligent speculation or foresight." (Interestingly enough, a 1972 contribution by Groves to dynamic team theory exploits a similar idea to increase the average performance of a "truncated" Lange-Lerner process.) In addition to this dynamic defect of the competitive mechanism in a linear economy, several recent papers have also studied what is perceived to be a static defect: even when the prices are correct (i.e., at their competitive equilibrium values), the producer will typically be indifferent between "socially correct" and "incorrect" actions because both types of actions may be on the same iso-profit line. Bessière and Sautter speak of absence of "separability" and Jennergren makes a similar finding. However, with "incorrect" individual actions there would not be equilibrium; this would become evident to the price setters, because excess demand would be different from zero. Since excess demand must be checked to know whether the price is "correct," the requirement of "separability" in the above sense seems too strong. Nevertheless, it is of some interest to note that where individual profit optima are unique (e.g., with strictly concave production functions), "separability" is present.

and the producing units can use linear-programming (primal and dual) techniques, and an equilibrium is reached in a finite number of steps. So far, we may regard the algorithm as a variety of the market (parametric price) process. But there is a difference. The final allocation will not necessarily correspond to the final production programs of the producing units. Rather, the center will "order" each producing unit to undertake a program which "mixes" (averages) the final proposal with several previous ones.[3]

As pointed out by Baumol and Fabian, the procedure can be extended to situations where constraints pertaining to single producing units are nonlinear, while the overall constraints pertaining to resources needed by all units remain linear. In this case, however, the units must have a computational algorithm for their nonlinear problem since the simplex method can no longer be used. For other nonlinear economies, and especially those with increasing returns, different processes had to be sought.

Before we look at those, however, let us examine a mechanism designed specifically to guide a *linear* economy but in a manner that partly reverses the roles played by the "center" and the "periphery" (the producing units), the process due to Kornai and Lipták. The assumption concerning the economy, as in the Dantzig-Wolfe model, is that of "block angularity," i.e., there are subsets of constraints each pertaining to a given sector and also resource constraints affecting the whole economy. In the dialogue, the center proposes allotments of scarce resources to the various sectors; then each sector responds with shadow prices (marginal rates of substitution) minimizing the value of the allotment subject to sectoral dual constraints (nonprofit condition for every sectoral activity). The center's aim, on the other hand, is to maximize the contributions of the sectors to the objective function, i.e., to maximize the value of the allocated resources at the shadow prices received from the sectors, subject to the limitation of available resource totals.

Taking advantage of the equivalence of linear-programming programs and games, Kornai and Lipták, by structuring the dialogue as a fictitious game, are able to establish convergence to an equilibrium with any desired degree of accuracy, though (unlike in the Dantzig-Wolfe procedure) without reaching the equilibrium in a finite number of steps. In addition to the latter disadvantage, it has also been pointed out (by Jennergren) that the Kornai-Lipták procedure is not completely informationally decentralized, since each sector's resource sectoral allotments must be large enough ("evaluable") to assure the existence of a feasible solution for that sector.

One advantage claimed for the Kornai-Lipták procedure is that it may be computationally manageable where alternative decomposition algorithms are not. I regret that I have not had an opportunity to look into this question. But another feature of this process is of great interest to the economist. This is the fact that the center, instead of simulating the market as does the Taylor-Lange-Lerner mechanism, specifies quantitative input and output targets or restrictions, while the sectors supply the center with productivity information in the form of shadow prices. This appears more in line with many observed planning practices and thus may provide a useful descriptive model.

[3] The need for such averaging is due to the fact that a firm's final program in the Dantzig-Wolfe procedure can be a profit-maximizing "corner," while "social optimality" may require the utilization of a "non-corner" profit-maximizing production program. In effect, by considering the various averages of the producing unit's programs, the center determines the whole set of profit-maximizing programs and, from among those, picks out the socially optimal one. (See Baumol and Fabian.)

There are several other mechanisms, in general designed for nonlinear economies, which are also of the "quantity-guided" type (as distinct from the "price-guided" type), that is, where the center sends out messages concerning quantities (e.g., targets) and the periphery (the producing units, sectors) responds with marginal entities or shadow prices. Informationally, since the center is sending different quantity messages to different sectors, its total signals are of higher dimensionality than in price-guided systems where the same message (price vector) goes out to all sectors. (We must bear in mind that each quantity vector has the same dimension as the price vector.) Whether this difference is significant is somewhat controversial; the negative has been strongly expressed by Marglin who has constructed quantity-guided (called by him "command") counterparts of certain price-guided processes. One of Marglin's mechanisms requires the center to allocate the scarce inputs on the basis of information obtained from the producing units concerning their marginal productivities and their excess demands. Adjustment ceases when aggregate excess demand is zero and the marginal productivities of producers are equalized. (A similar process was proposed by Heal in 1969.)

A process, characterized by a mixture of price- and quantity-guided elements and also due to Heal (1971), is particularly interesting because (with some qualifications) it converges to optima even for nonconvex economies, in particular for increasing returns. An essential informational feature is that certain functions of each producing unit's marginal productivities (roughly, its shadow prices for particular resources) must be conveyed to the center. The center can then calculate improved resource allotments, or else it may calculate and send to the units a resource price (the same for all units) and so enable them to determine their respective resource requirements. The latter option is, of course, informationally more decentralized: it requires fewer message transfers from the center to the periphery, and fewer computations are carried out at the center. (Marglin had a similar process for a more restricted class of economies. It is not clear whether his process could be adapted to corner maxima in nonconvex cases.)

Heal's process has the further merit that if the initial allocation is feasible, so are all the later ones, thus satisfying a Malinvaud postulate. (The same seems true of Marglin's process, although, unlike Heal, he does not assume the initial position to be feasible.) The maintenance of feasibility is simple in models without intermediate goods because the procedure always allocates all available resources and producers are required to stay on their efficient frontiers. The matter gets more complicated when intermediate goods are introduced and only special cases appear to have been dealt with so far by this approach.

Processes in which the center specifies quantities and the peripheral units convey their individual marginal rates have also been used in models where public goods are among those to be allocated. I shall mention three treatments of this case, due to Drèze and de la Vallée Poussin, Malinvaud (1970), and Aoki. In the versions known to me each is somewhat specialized: Malinvaud and Drèze-Poussin have only one producer but many consumers, Aoki only one consumer; also, Drèze-Poussin deal primarily with the case of only one private good, although they indicate how the results may be generalized to more. Aoki's economy is closest to those we have been considering so far because it has only one consumer (the center), hence, no income distribution problems; it also has many producers. His mechanism uses price-guidance for private goods and quantity-guidance for public goods. A producer develops production plans that maximize net revenue given the central "guidelines" (prices for the private goods, quantities for the public goods), and conveys to the center his demands for private goods and marginal evaluations, including marginal cost, for public goods. The center, in turn, adjusts the price of each private good according to the difference between its marginal utility and price (as in the Arrow-Hurwicz gradient process); the targets for public goods are increased in proportion to the net aggregate of marginal valuations (users' minus producers'); thus the center combines the functions of the helmsman and resource custodian of the Koopmans model for private goods with target setting for public goods. The other two processes have the same adjustment rule for public goods targets, but differ in other respects. In particular, they specify the rules of income distribution.

All three processes converge (at least locally) under suitable convexity assumptions concerning the environment. Somewhat paradoxically, Malinvaud's process does not seem to satisfy his desiderata of feasibility maintenance and monotonicity, while the other two do.

I shall now go back to the price-guided processes, but more briefly because they came earlier and are better known. Here again, I shall focus on the one consumer case. For a linear economy, Koopmans (1951) described the functioning of such a mechanism in the spirit of the Taylor-Lange-Lerner rules by setting up an "allocation game" to be played, in an informationally decentralized manner, by a helmsman (setting the prices of final goods and thus representing consumer preferences), commodity custodians (adjusting the prices of resources according to excess demand), and activity managers who determine the production programs. Koopmans' adjustment rule (similar to Samuelson's) is that managers expand profitable activities and curtail those bringing losses; in a constant returns economy this is equivalent to profit maximization. Koopmans stressed that "the dynamic aspects of these rules have on purpose been left vague." We know that Samuelson's experiment in this direction yielded nonconvergent oscillations.

On the other hand, in an economy where all functions (including the utility indicator) are strictly concave (i.e., we have diminishing returns) similar rules produce a process with the desired stability properties. Utilizing the notion of gradient approach to the saddle point of the Lagrangean expression, Arrow and Hurwicz (1960) used the following rules: the helmsman, taking the prices of desired commodities as given, changes each final demand at a rate equal to the difference between its marginal utility and price; each manager, again taking prices as given, changes the scale of his process in proportion to its marginal profitability; each commodity custodian varies the price of his commodity in proportion to excess demand. (I am omitting modifications pertaining to corners and zero prices.) A limiting form of such a process is the price-adjustment method in which prices are varied as before, but both the helmsman and each manager reach (as against merely moving toward) the values of their decision variables which maximize their respective objective functions: for the helmsman, the difference between utility and price; for the manager, the level of profit. The familiar Walrasian competitive process is a variant of such price-adjustment in which the demand for final goods is determined by utility maximization subject to a budget constraint with specified income or wealth.

Although the gradient process is informationally decentralized and converges to an optimum under strict concavity assumptions, it has certain disadvantages. To begin with, it is formulated in continuous time, while realistic mechanisms operate more naturally by iterations, i.e., with a discrete time parameter.

But it is possible to construct a discrete time parameter counterpart of the gradient process. In fact, this was done by Uzawa and further elaborated by Malinvaud (1967) who, however, pointed out another undesirable feature of the gradient process: although it converges to a feasible solution, its "interim" proposals are, in general, not feasible. In other words, while the process pushes the participants toward compatibility in their claims on resources, they may be demanding either more or less than the total available while the process is going on. Thus if the gradient process were to be interrupted at a finite time, there might be a problem of reconciling incompatible claims.

Malinvaud then formulated a desideratum, viz., that (discrete time parameter) adjustment processes yield feasible solutions after a finite number of iterations. He then proceeded to construct two processes (for different environments) which satisfied this desideratum. In both cases he assumed that the initial proposal, serving as the point of departure for the iterations, was feasible. In effect, he was paying a "price" for the feasibility of all his interim proposals—namely, he assumed that the center had an additional piece of information: a feasible point of departure; the Arrow-Hurwicz gradient process, on the other hand, was designed for situations where this information was not available to the center.

It is worthwhile to become acquainted with Malinvaud's second procedure. The center proposes prices to the producing units which, in turn, determine production plans maximizing the value of the firm's output in terms of those prices. The center then builds up its picture of each unit's production set (see Fig. 1) by taking all convex mixtures of its previous proposed input-output vectors, together with the initial feasible vector, assumed known to the center. (Since the production sets are assumed convex, this yields an increasing subset of the unit's true production set. But, again, there is an informational price to be paid: the center must accumulate, on a disaggregated basis, all past proposals from the units.) Treating its pictures of the production sets as if they were the actual sets, the center then maximizes its utility function subject to the resource availability constraint and proposes a new set of prices corresponding to the relevant marginal rates of substitution.

It could perhaps happen that, even after several iterations, the only production program compatible with resource constraints is the one originally assumed known to the center; or that, even if new feasible programs are generated, their utility is no higher than that of the original known feasible allocation. But it was shown by Malinvaud that, as the number of iterations goes to infinity, the utility associated with the corresponding plans tends to the upper bound of its feasible values.

Roughly speaking, the center constructs plans which would be nearly optimal for the economy if its images of the individual production sets were sufficiently close to correct. The informational price paid is the need for building up these images; in effect, the information concerning the production functions is being transferred to the center, although on an installment plan. This differs from the usual informationally decentralized procedures where the center does not accumulate such information and never knows more than the structure of the production set in the neighborhood of the current proposal.

The process also illustrates another desideratum formulated by Malinvaud: that the utility of successive proposals should not decrease as iterations progress ("monotonicity"). The preceding process obviously satisfies this requirement because earlier proposals are always among the available alternatives during the utility maximization phase of the center's calculations. (Heal's process also has the monotonicity property when the initial proposals are feasible.)

A procedure, which is a sort of dual to that of Malinvaud, has been proposed by Weitzman. While Malinvaud's center is rather timid and only considers plans known to be feasible for the units, Weitzman's central planning agency constructs imaginary production sets it knows to be too ambitious, formulates targets that are, in general, infeasible and then lets the units scale down the proposals to feasible levels. (See Fig. 2.) Also, the units provide the center with respective marginal rates (shadow prices) as a basis for subsequent central targets. Here, again, the center must accumulate all previous information concerning the structure of the individual production sets. Convergence is assured (even in a finite number of steps

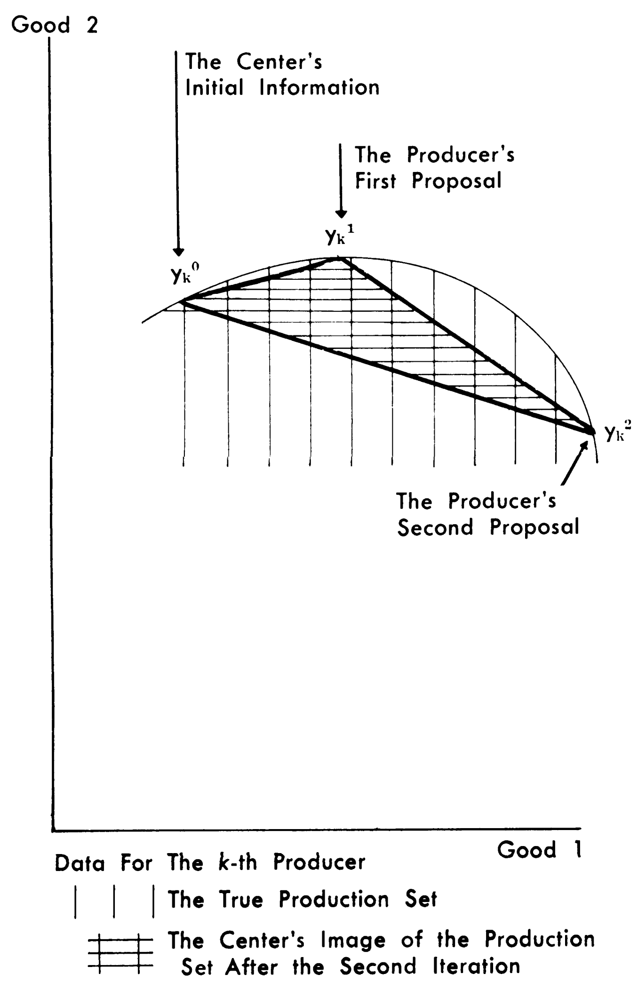

FIGURE 1

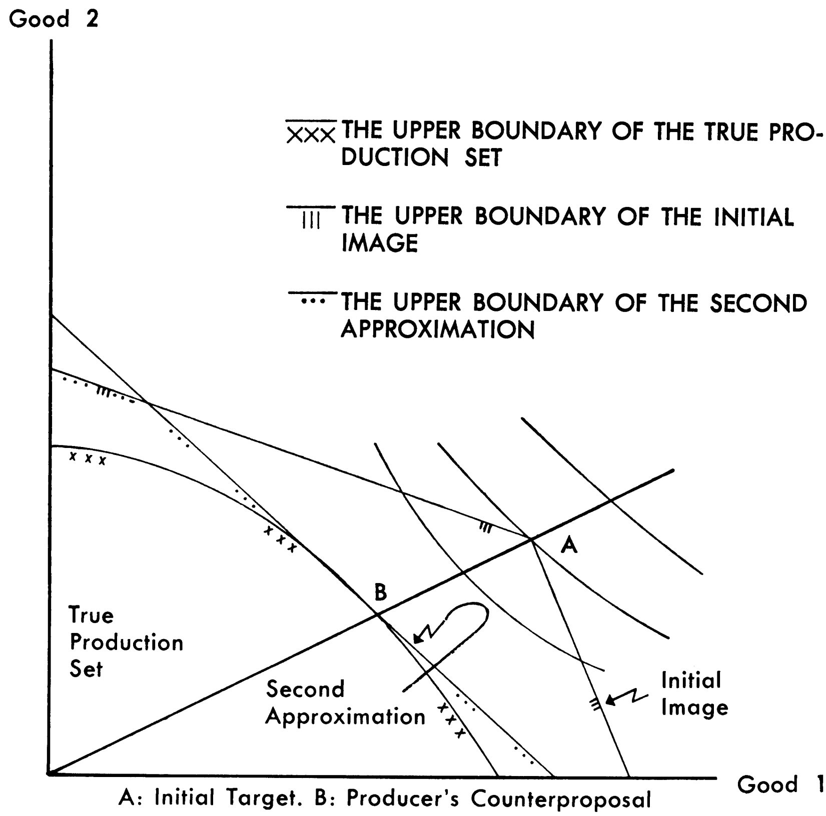

FIGURE 2

when the production sets are polyhedral). But the center's proposals will, in general, be infeasible for the units, and the units' counterproposals may not be compatible with resource availabilities. Hence, termination after a finite number of iterations may give rise to the same problems of feasibility brought out by Malinvaud with reference to gradient processes.

Perhaps a less ambitious form of the Malinvaud feasibility postulate would be acceptable: that the process should not depart from feasibility once it has encountered a feasible point in the process of iterations and that, in any case, it should converge to optimality, hence, to a feasible point. I have not explored Malinvaud's processes in the light of this modified postulate. It seems, however, that the following procedure would satisfy the modified postulate: use the gradient process when starting from an infeasible set of proposals, but switch to a Malinvaud type mechanism as soon as a feasible point is reached.

We may note that the feasibility problem arises for any Walrasian tâtonnement process which is terminated at a finite time. Typically, supply will not equal demand for all goods and, if actual allocation is to be made at the termination time, one must have a way of resolving conflicts. When demand exceeds supply, one can think of prorating the available goods or using the first come first served principle. However, this will not, in general, yield a feasible solution. Even for a pure exchange economy some individuals might be getting less than subsistence requirements; with production involving intermediate goods the situation would be all the more difficult.

Informational considerations aside, both cumulative procedures (Malinvaud's and Weitzman's) rely on the convexity of production sets. Malinvaud's other price-guided process (related to Taylor's suggestions) assumes a Leontief-Samuelson constant returns economy. What about decentralized resource allocation in *nonconvex* environments?

We have already seen one procedure designed to cope with such situations, that of Heal (1971). This process is shown to converge to a "critical point" of the optimization problem, i.e., a point where first order conditions for maximization are satisfied. Under conditions of convexity, such points will, of course, be (at least local) maxima. But without convexity, they no longer need be even local maxima. Now in Heal's process, if one starts from a feasible point which is not a critical point, the subsequent points will also be feasible and have a rising utility; hence, the point to which the process converges cannot be a local minimum, but it need not be a maximum. (See Figures 3 and 4.) However, if the starting point happened to be a local minimum, it seems that the rules of the process would generate an equilibrium there. It may be that this difficulty could

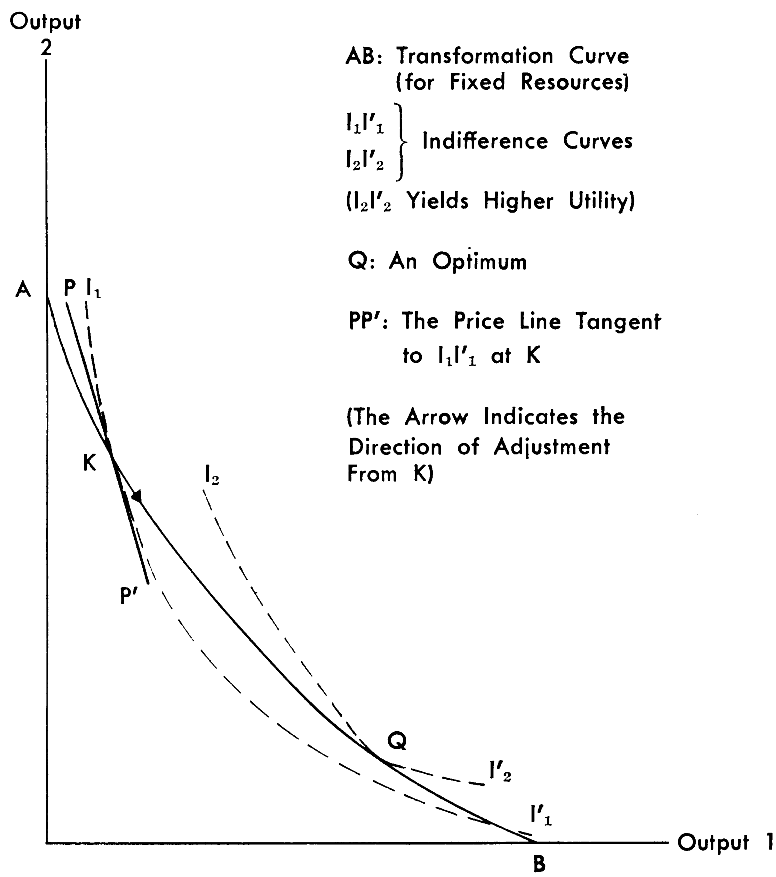

FIGURE 3

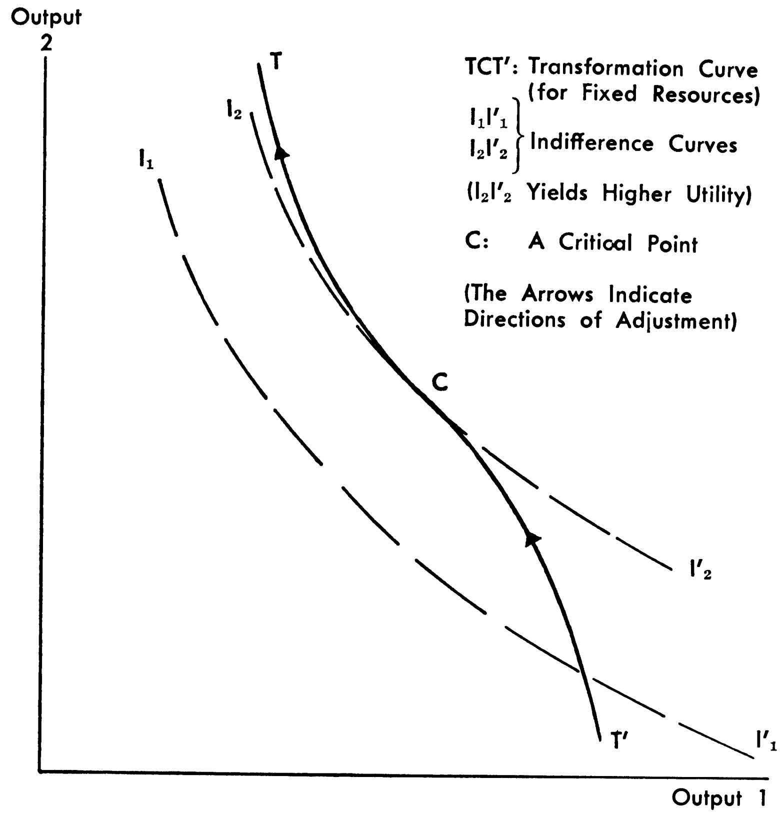

FIGURE 4

be avoided by some modification of Heal's rules (e.g., by tâtonnement in the neighborhood of any critical point).

An alternative was proposed by Arrow and Hurwicz (1956, 1958, 1960). Mathematically, it amounts to rewriting the nonconcave constraints in such a manner that the *modified Lagrangean* expression becomes locally strictly concave in the activity variables so that a local saddle point is created and the gradient method can be used, at least locally. (There are many ways of bringing about this modification.) Hence, we then have available to us a gradient process which is guaranteed to converge to a local optimum, not merely to a local critical point. Admittedly, convergence to a global optimum would be preferable. However, in the absence of concavity properties, one cannot hope to guarantee convergence to a global optimum on the basis of local first order properties only! The modification just described can also be applied to produce *strict* concavity in linear situations. We thus have a solution of the dilemma of nonconvergent oscillations encountered by Samuelson in the linear case.

It should be admitted that computational experiments along these lines (T. Marschak, 1958) did not turn out encouraging. Nevertheless, the economic interpretation of the modified Lagrangean (Arrow-Hurwicz) process seems of interest. Here the Lagrange multipliers are no longer interpreted as prices. In fact, the custodian announces a *price schedule* (of which the Lagrange multiplier is a parameter) with the price depending on the quantity purchased. The helmsman and the managers must perform their marginal calculations on the basis of such schedules. The custodians adjust the schedules by varying the Lagrangean parameters according to supply and demand conditions. Thus optimal allocation is achieved through monopolistic practices and imperfect competition. (Jennergren recently explored the use of price schedules in allocative processes for concave, including linear, environments. Heal made a related suggestion.)

It is not surprising that monopolistic elements should appear in a process that simulates market phenomena, since we know that perfectly competitive equilibrium need not exist in nonconvex situations and even where it exists in linear cases it has certain undesirable indeterminacies. It should be noted that the modified approach just described is informationally less decentralized than was the case in a world of concave functions without the monopolistic modification. It is a major problem of the general theory to what extent such informational losses are unavoidable.

A very different price-guided process was suggested by Radner and developed by Groves (1969, 1972). Their framework is that of team theory, with a simple common goal (total output maximization) and randomly fluctuating productivity parameters. The production functions are classical (differentiable and concave, or even quadratic). One of the adjustment processes considered is in the spirit of Lange-Lerner procedures. Others have information structures ranging from complete communication to absence of communication.

The distinguishing feature of the model is that allocative decisions are made without waiting for equilibrium to be reached, hence usually on the basis of only partial information. Also, there is a departure from the Lange-Lerner rules in that the production manager's decisions do not maximize profits, although profit figures are used for informational purposes.

Among the remarkable findings is the fact that the asymptotic value of communication per enterprise (as the number of enterprises increases) is as good under Radner's "One-Stage-Lange-Lerner" process as it would have been under complete communication.

Although the results so far obtained are valid only for rather specialized models, they show how to go beyond traditional equilibrium analysis, while exploiting ideas developed in equilibrium oriented models.

II. Basic Concepts

We may think of economists as taking part in a contest to design a "superior" mechanism, with some submitting entries before it has been clarified what would qualify as a mechanism and which mechanisms would be considered "superior," or even feasible.

It is fortunate that our predecessors did not wait for such definitions. In the process of submitting their entries, they have provided examples of mechanisms, the foremost among them competitive and marginal cost pricing, and, by now, many others which can be used as guides in constructing a definition. Similarly, theorems concerning the efficiency and optimality properties of these mechanisms suggest possible classification criteria.

One could, of course, get by without formalizing what one means by a resource allocation mechanism. But it is then impossible to determine to what extent the various desiderata implicit in the past debates are compatible with one another, or what the "tradeoffs" are among them. Also, in searching for alternatives to known mechanisms for nonclassical situations (indivisibilities, increasing returns, externalities, public goods), it helps to have a rigorous formulation of what a resource allocation mechanism is and which of its features are desirable. Notions of a mechanism different from that given below may well have greater merit. Our picture of a mechanism will in fact sacrifice possible greater generality to gain definiteness and simplicity. Nevertheless, it will be general enough to embrace (as it stands or with minor modifications) most of the economic systems we wish to study.

It is the function of a resource allocation mechanism to guide the economic agents (producers, consumers, bankers, and others) in decisions that determine the flow of resources. Simplifying to the utmost, we may imagine each agent having in front of him a console with one or more dials to set; the selection of dial settings by all agents determines uniquely the flow of goods and services (trade vector) between every pair of agents and also each agent's production (input-output vector), his "trade with nature." Not all dial settings are possible and some are possible only in conjunction with other dial settings. Thus the feasibility of a complex of actions (a specified combination of dial settings for all agents) can be split into individual feasibility and compatibility. (In our standard models, any point on a given agent's production function is individually feasible for this agent, but two individually feasible input-output vectors of two firms may be incompatible if one calls for an input which the other does not propose to supply.)

It is natural to demand that the mechanism should guide the agents toward actions which are at least feasible, and even that can be difficult. Yet in classical welfare economics we require more than feasibility, viz., such attributes as efficiency or optimality. After decades of meanderings, we are fairly clear on our options—from efficiency in production (as defined by Koopmans), through optimality (introduced by Pareto under the label of maximum ophelimity), to the maximization of a social welfare function (as defined by Bergson, Samuelson, or Arrow). From our point of view, these different attributes have an important feature in common: they are defined independently of the mechanism. An optimality criterion which presupposes a particular mechanism cannot serve as a legitimate criterion for comparison with other mechanisms.

Specifically, whether an allocation is or is not optimal depends on its feasibility and on the individual preferences, with feasibility determined by the individual endowments and the technology. The individual endowments, the technology, and preferences, taken together, are referred to as the *environment*. More generally, the environment is defined as the set of circumstances that cannot be changed either by the designer of the mechanism or by the agents (participants).[4]

To simplify matters, we shall confine ourselves to mechanisms analogous to the tâtonnement process where a period of dialogue without action is followed by decisions as to resource flow (production and exchange). Ultimately, we shall need a more general theory in which dialogue, decisions, and actions overlap in time, as in the nontâtonnement processes.

The participants in the dialogue consti-

[4] For instance, in a multiperiod model, where inventive activity is a controllable factor of production, the environment is given not by the existing technology, but rather by the relationship between inventive activity and the resulting production function in later periods. This interpretation fits the more general definition of environment if the latter relationship cannot be altered by the decision makers. Similarly, the existing preferences do not constitute the environment in its generalized sense when tastes are malleable, e.g., on grounds adduced by Galbraith. But if the responsiveness of tastes to influences such as advertising cannot be influenced by decision makers, it should be regarded as part of the environment. (To make meaningful welfare judgments possible, some underlying values or "true preferences" would have to be postulated.)

tute a broader group than the "doers" (consumers, producers, etc.), since they may include governments, planning agencies, central banks, or unions. If we excluded such participants, we should obviously be unable to develop a theory general enough to encompass centrally planned or even mixed economies. The dialogue is an exchange of messages between the participants. The nature and contents of the messages vary from mechanism to mechanism. They may be proposals of actions, bids, offers, plans of resource flow for the whole economy, or they may contain information about the environment (preferences, technology, resource endowments). The totality of messages permissible under a given mechanism constitute its *language*.

The mechanism specifies rules according to which, given the information available to him at a given time, a participant sends messages to others. The information consists of messages previously received, as well as some (direct) knowledge of environment, and are called *response rules* because they govern the message response to messages previously received. (By "direct" is meant knowledge of the environment derived from sources other than messages previously received.) To provide for a transition from dialogue to decisions and actions, the mechanism must also have an *outcome rule* which specifies what actions are to be taken given the course of the dialogue. The rules may be deterministic or probabilistic; mathematically they are expressed as functions.

Both market phenomena and command systems can be fitted into this schema. Thus in the Walrasian tâtonnement process the language consists of prices and quantities demanded or supplied by the various agents. If the model contains an "auctioneer," his response function calls for price changes proportional to aggregate excess demand, while the response functions of others require them to convey their excess demands given the prices called out by the auctioneer. In an extreme version of a "pure command" system, the dialogue starts with the peripheral agents sending to the center messages describing their respective components of the environment (e.g., their resource holdings and production functions), whereupon the center, after suitable data processing and calculations, sends to the peripheral agents the orders for action. In this command system the outcome rule is clear: to carry out the orders received. In the Walrasian tâtonnement process, the matter is a bit more complicated. One must wait until equilibrium is somehow established—i.e., everyone is repeating his previous message. Then the outcome rule is to carry out exchanges according to the equilibrium bids made.

The languages of the two mechanisms are also different. In the Walrasian process, messages are the proposed prices and commodity bundles. Namely, a message can be regarded as an ordered sequence of numbers, as many as there are goods in the system, i.e., as a vector whose dimension is that of the commodity space. In the "pure command" system, a message may contain the description of a production function or of a preference map. Since it may take arbitrarily many parameters to specify a production function, no á priori upper bound can be imposed here on how many numbers might have to be transmitted in such a process. The language of the command process is much larger than that of the Walrasian process. We must remember, however, that the pure command process is finished after only two exchanges of information while the tâtonnement may go on a long time.

The purpose of the two examples has been not to compare their merits, but to illustrate the meaning of the terms used in describing the resource allocation mechanisms we shall be dealing with. We shall call such mechanisms *adjustment processes*.

Thus, an adjustment process is specified by its language, response rules, and the outcome rule. (As we shall see later, "distributional parameter settings" also enter the picture.)

With this formulation, given a class of environments which the designer must cover, there is a well-defined family of adjustment processes that can at all be constructed with any specified language. This is so because, mathematically speaking, the class of environments and the language delimit the domains and ranges of the response and outcome functions (rules), and the class of functions with any given domain and range is well-defined. This opens the door to impossibility or possibility theorems concerning the family of adjustment processes that use specified languages and cover a given class of environments.

(The resemblance to Arrow's social welfare function problem is purely formal: in both cases one is investigating the family of functional relations with specified domains and ranges. However, the functions—rules—defining an adjustment process bear no direct relationship to the social welfare functions.)

We have already mentioned one natural desideratum for adjustment processes—that they yield feasible outcomes. More ambitiously, one may ask that the outcomes be optimal. A process whose outcomes are optimal for a given class of environments will be called *nonwasteful over that class*. (For instance, a competitive process is nonwasteful, in a Pareto optimality sense, over environments that are free of externalities and of locally saturated preferences.) But it is crucial to realize that nonwastefulness, although appealing, is not enough. After all, a process lacking any positions of equilibrium would have to be classified as (vacuously) nonwasteful. So, we would wish to require that the process possess *some* equilibrium position for every environment of the class it is designed to cover.

Yet even this is insufficient. What if the equilibria of the process always tend to favor one group of participants at the expense of another? The victims would hardly be comforted by the fact that the outcomes are always Pareto optimal.

Here again the experience of classical welfare economics points the way. Whether we study Arrow, Debreu, or Koopmans, we find not one but two basic theorems. One establishes nonwastefulness. The other asserts that any Pareto optimum can be attained as a competitive equilibrium. A precise statement of the latter property turns out to be complicated, however. For we know that, given the option—always available to resource owners—of not trading and not producing, ordinary competitive equilibrium cannot yield levels of satisfaction below the original ones. Hence, inferior optima could never be reached and the second theorem might seem false. But the theorem refers not merely to a market process. When interpreted within the framework of a private ownership system, it envisages a market process preceded by transfers of resource holdings and (in a production economy) of claims to profits (Allais, Debreu, Arrow, Arrow and Hahn). It is these transfers that can lead to optima that are inferior for some of the participants.

Perhaps the most important point here is that the mechanism can no longer be viewed as accepting as given the distributional aspects of the system. Rather, it consists of two parts: a setting of what we may call the *distributional parameters* (resource holdings, profit shares), followed by a tâtonnement procedure. The property of the competitive mechanism expressed by the second classical welfare economics theorem, which we shall call *unbiasedness*, amounts to this: given any conceivable Pareto optimal position of the economy, there exists a setting of the distributional parameters which would make that position an equilibrium of the process. (Again this property holds *over some* specified *class of environments*.) Thus if there is an optimal allocation that cannot be made into equilibrium by any setting of the distributional parameters, the process fails the test of unbiasedness.

In a pure exchange economy there is no difficulty in interpreting the meaning of setting distributional parameters: it is merely a matter of reshuffling the initial endowment. But in the presence of production, the matter is more complicated. In a private ownership economy, profit shares can be shifted. But in searching for alternative systems of different institutional nature, we do not want to commit ourselves to considering profits or incomes as well-defined concepts, since both involve prices, and prices may not be defined in certain institutional structures. (This, incidentally, shows some of the difficulties of even formulating the problem without implicitly presupposing marketlike conditions.) Yet it is possible to provide a definition free of institutional limitations by interpreting the distributional parameters as guaranteeing to each participant a minimum level of satisfaction corresponding to a particular resource allocation. The manner in which this guarantee is implemented does involve institutional arrangements.

One more requirement. Ideally, we might demand that the rules of the process send the economy to a uniquely determined allocation. But this turns out to be difficult to accomplish and even the competitive mechanism may yield multiple equilibrium allocations compatible with a *particular* equilibrium price. Since, however, all these allocations happen to have the same utility for all participants, the indeterminacy is acceptable. We shall call a process *essentially single-valued* if any equilibrium indeterminacies are of this trivial nature. (Note that this does not require that allocations corresponding to *different* equilibrium prices, when there are multiple equilibria, yield the same satisfactions to all participants. Ruled out is only a situation in which, corresponding to a particular equilibrium price, there would be alternative allocations yielding different levels of satisfaction.)

We are now ready to formulate the basic set of performance requirements for processes to be considered: that they be nonwasteful, unbiased, and essentially single-valued. This trio of requirements will be, for short, referred to as *(Pareto-) satisfactoriness*. Whether the requirements are or are not met by a given process depends on the environments in which it is asked to operate. The environments which have the properties stated in the classical welfare economics theorems are, naturally, called classical. Thus *classical environments* are free from externalities or indivisibilities, their sets are convex, etc. In particular, increasing returns are not classical!

With this long introduction, the two great welfare economics propositions may briefly be stated thus: the competitive process is Pareto-satisfactory over classical environments.

This statement immediately provokes the question: what about the nonclassical environments? Can we find Pareto-satisfactory processes for them too? For all environments, or just some?

These questions would be meaningless if some restrictions were not imposed on the information (or incentive) structure of the processes to be considered. The explicit recognition of the role played by the information structure is one of the major accomplishments of Jacob Marschak and Roy Radner in their work on the theory of teams. An analysis of informational issues, with emphasis on feasibility, timing, and costs, with particular reference to the issue of decentralization, is due to Thomas Marschak. Oniki has pioneered in comparisons of numbers of messages that flow in different processes.

The formulation I have adopted here is conceptually much less statisfactory. I consider certain specific restrictions on the informational aspects of the process and then, as a first step, I ask whether for a given class of environments (wider than classical) one can design Pareto-satisfactory processes. (These restrictions are satisfied by the competitive process, so there is no problem with regard to classical environments. But it is proper to ask whether there could be processes that are informationally "even better" than the competitive process. In terms of certain criteria of dimensionality of messages, the answer obtained independently by Hurwicz (1972b), by Mount and Reiter and—from a somewhat different point of view—by Sonnenschein is in the negative: in a specified sense, one cannot do "better.")

As a second step, adjustment processes can be compared with respect to the required "size" of the language (e.g., the cardinality or dimensionality of the message space), or according to the fineness of the perceptions they call for (informational efficiency), and other aspects of information processing effort or expense. These concepts and their relationships have been studied by Kanemitsu (1966) and by Reiter (1972*a*, 1972*b*).

But back to the "first step," the formulation of requirements. To express them, it is necessary to distinguish between information obtained directly (say, by one's own observation) and through messages received. My first postulate is that participants have direct information only about themselves, not about others. I have never found a good term for this property, but currently I refer to it as *privacy* (of the adjustment process).

It is clear that the restriction on direct information would be virtually meaningless if arbitrary messages were permitted, for then "in one move" any information not available directly could be obtained through a message. Thus the *language* (the set of permissible messages) must somehow be restricted. It turns out from mathematical considerations (something I had not been fully aware of in my original work) that restrictions on the language may also turn out to be ineffective unless certain conditions (akin to but stronger than continuity) are imposed on the response and outcome functions.

The totality of these and certain other conditions (including some pertaining to the language) I have labeled *informational decentralization*. There is, of course, some arbitrariness in such a definition, but it enables us to formulate a simple "yes or no" question: how broad is the class of environments for which Pareto-satisfactory, informationally-decentralized processes can be designed? In particular, can informationally-decentralized processes be designed for the class of all environments free of externalities (*decomposable environments*)?

III. Some Results

With all this machinery, we have finally managed to ask a question. Are there answers? Indeed there are, some positive, some negative.

To begin with, it is possible to construct processes that are informationally decentralized for all decomposable environments. As for nondecomposable environments, it seems that one would have to say "no," but this has not as yet been rigorously established. It can be shown that informational requirements increase when nondecomposability (externalities) enters. While this is hardly surprising, the fact that informational decentralization *is* possible for the class of all decomposable environments, was, to me at least, a surprise, particularly because of the well-known difficulties that arise in the presence of indivisibilities. Indivisibilities do make trouble, but not to the point of making informational decentralization impossible.

To show this, it is sufficient to exhibit an informationally-decentralized process which is Pareto-satisfactory for all decomposable environments. In fact, there exists at least one, although with certain limitations and defects. The process works roughly as follows. A message is a listing, (or description) of trades (with others or with nature) that a participant is prepared to engage in. The response is a counterproposal listing all those trades that are better, or at least as good, from the respondent's point of view. (You can see why this has been called the *greed process*.) The process does qualify as informationally decentralized because the privacy requirement is satisfied, since the participant needs to know only his own characteristics; and the language, that of trade sets, also fits the definition. (There is no denying, of course, that the messages used are very "heavy" and complex.)

Furthermore, it is simple to show that this process (which is only defined for decomposable environments) yields optimal equilibria and any optimum can be reached by it. It is Pareto-satisfactory. But this is only a static property. Dynamically, the greed process is terrible. Unless we start it from an optimal position, it oscillates indefinitely with constant amplitude. So we get more ambitious and look for a *convergent* Pareto-satisfactory process.

It is possible to "fix up" the greed process by building into it a certain amount of "inertia," i.e., dependence on earlier values. Kanemitsu (1970) showed that such a greed-inertia combination does result in convergence in *continuous* decomposable environments. This is quite a broad class (including increasing returns and other nonconvexities), but it rules out indivisibilities. Whether one can generalize this to discrete (indivisible) economies is, I believe, an open question.

So, we have not yet exhibited a *convergent* informationally-decentralized process that is Pareto-satisfactory for *all* decomposable environments!

At this point, the idea of randomness, not a stranger to search procedures, comes to the rescue. By introducing an element of randomness, we can obtain convergence (although, I must admit, in a probabilistic sense only) and also simplify the structure of messages. This process, developed by Hurwicz, Radner, and Reiter, is particularly simple when all goods are *indivisible*. Let me describe it for the two-trader Edgeworth Box case.

At each time point, each participant picks at random (but according to a fixed probability distribution) a trade that will leave him at least as well off as he is at present. If the two trade proposals happen to be compatible, the bargain is sealed, and the traders start all over again from the newly reached point. It is clear that the process has the privacy property: one needs to know only one's preferences and subsistence requirements; in fact, the process, even in the more general situations (with any number of traders and production) is informationally decentralized. Unbiasedness is also intuitively clear: one can pick any optimal allocation as a point of departure and the rules make it impossible to get away; thus there is a (probabilistic) equilibrium. It is not obvious that the traders won't "get stuck" at some nonoptimal position, in which case the process would be wasteful. But a certain amount of mathematics leads to the conclusion that we are protected from such a fate with probability one.

And note that in the indivisible goods case the messages here are not sets but single trade proposals. Definitely very superior to the greed process.

So, the indivisible case turns out to be relatively simple, although one cannot make any practical claims until the speed of convergence has been investigated. But the usually well-behaved, continuous, *divisible* environment makes trouble here, since under the just-stated rules the probability of encounter would be zero: the proverbial needle in the proverbial haystack. So there has to be a modification: when commodities are perfectly divisible, a proposal will consist of not only the point picked at random but also a certain neighborhood of it (say, a square or a cube), with the further unattractive proviso that any part of the neighborhood whose utility is below that of the original endowment is cut off by the participant before the proposal is sent out. Now if the two (neighborhood) proposals fail to meet, one must try again; if they do meet, a point is chosen at random from the intersection area and the game goes on. Under rather mild assumptions of topological nature, probabilistic convergence can again be proved and the process is Pareto-satisfactory. It also qualifies as informationally decentralized, but it is clear that the informational burden is much greater than in the indivisible case, since the cut off neighborhoods might be very irregular in shape.

Facing such complexities of the process, it is natural to ask whether these are merely due to lack of cleverness on our part. Can we hope to design a process with all the nice properties of this one but with a smaller informational burden? A. P. Lerner raised this point in his *Economics of Control* after he proposed dealing with indivisibilities by a consumer-producer surplus type of criterion. His answer was that "the necessity of making unreliable estimates is in the nature of the problem and not in the method of solving it. . . . The same estimates and guesses must be made in *any* economy where knowledge is imperfect and where large decisions have to be made" (p. 198).

I believe that Lerner was right, but even now, almost thirty years later, we do not have a rigorous proof of this contention. The main difficulty is conceptual. We must define precisely what would be considered informationally less burdensome. Namely, we need a concept of informational efficiency, somewhat similar to production efficiency—a partial ordering criterion. Once the informational efficiency ordering is available, we can interpret Lerner's question (although he stated it as an assertion) as follows: is a given proposed process (e.g., that embodying his criterion) informationally *maximally* efficient among, say, informationally-decentralized processes that are Pareto-satisfactory for all decomposable environments? Although some informational-efficiency criteria have been suggested, we do not as yet have a definitive answer to Lerner's question.

But there are partial answers based on the criterion of message space size which is related to informational efficiency (see Reiter 1972b). In particular, it has been shown that, in the absence of convexity or monotonicity restrictions on the environment, there is no upper bound to the number of auxiliary parameters that must be used in a privacy-preserving Pareto-satisfactory process (Hurwicz, 1972b). This comes close to confirming Lerner's view. Note the contrast with classical environments where the number of auxiliary parameters need only be as high as the number of commodities—as when prices are such parameters. (As already mentioned, fewer auxiliary parameters would not work; the competitive mechanism is "dimensionally efficient.")

When the assumption of decomposability of the environment is abandoned (i.e., externalities are permitted), the preceding processes are not even defined. But when they are redefined so as to cover nondecomposabilities (Ledyard, 1968), they may lose their informationally-decentralized character.

For a particular class of nondecomposable situations, however, an interesting possibility emerges. It is well known that it may be possible formally to eliminate externalities by enlarging the commodity space (although this is based on implicit observability assumptions). Now Starrett has pointed out that such enlargement may introduce nonconvexities in a previously convex world. Namely, we have either fewer goods and convexity but with externalities, or more goods and no externalities, but no convexity. Thus the competitive mechanism fails either way, because of externalities or because of nonconvexities. Still, not all is lost. If continuity is not impaired, a probabilistic process such as that outlined above might provide a decentralized Pareto-satisfactory and convergent solution.

IV. Incentive Compatibility

So far we have been asking whether it is possible to design process rules which, if followed, would have certain desirable consequences. Where the answer is in the negative, we at least know our limitations. But if a process with the desired properties is found, the question arises whether one could expect the participants to follow the rules, since there is a possibility of collusions or of individual departures from the prescribed norms. We shall consider only the latter.

Whether a certain configuration of behavior patterns is compatible with the participants' "natural inclinations" is a problem in the theory of games, in this case noncooperative games without side payments. There are differences of opinion as to what constitutes a reasonable concept of solution for such games. For the sake of definiteness we shall adopt that of a Nash equilibrium which seems to be a good formalization of many intuitive ideas prevailing among economists. A configuration of behavior patterns constituting a Nash equilibrium will be referred to as (individual-) *incentive-compatible*. Such a configuration is present if no participant finds it advantageous to depart from his behavior pattern so long as the others do not. For instance, in a bilateral monopoly situation competitive behavior is not (usually) incentive-compatible because when one participant is a price-taker it would be to the other's advantage to be a price-setter (monopolist) rather than a price-taker.

One might, however, assume that there exists an enforcement system (carrots or sticks) which makes it unattractive for the participants openly to defy the prescribed rules. But suppose that the enforcing agency has no knowledge of the characteristics of the individual participants (their preferences, technologies, or endowments). It is then conceivable that the participants would "cheat" without openly violating the rules. A participant could try to "cheat" by doing what the rules would have required him to do had his characteristics been different from what they are—i.e., he could "pretend" to be poorer than he is, or less efficient, or less eager for certain goods. (It is important to understand that he would not be doing this directly by uttering false statements, but indirectly by behaving inappropriately according to the rules for his true characteristics.)

Economists have long been alerted to this issue by Samuelson (1954) in the context of the allocation problem for public goods. But, in fact, a similar problem arises in a "nonatomistic" world of pure exchange of exclusively private goods. Consider, for instance, an economy consisting of two traders, as conventionally represented by an Edgeworth Box, with classical strictly convex indifference curves and a positive nonoptimal initial endowment of both goods for both traders. We already noted that if they were both told to behave as price-takers it would pay one of them to violate this rule if he could get away with it. Now we assume that he cannot violate the rule openly, but he can "pretend" to have preferences different from his true ones. The question is whether he could think up for himself a false (but convex and monotone) preference map which would be more advantageous for him than his true one, assuming that he will follow the rules of price-taking according to the false map while the other trader plays the game honestly. It is easily shown that the answer is in the affirmative. Thus, in such a situation, the rules of perfect competition are not incentive-compatible.

It follows that the enforcing authority could not hope to maintain competitive (price-taking) behavior without directly checking up on the participants' characteristics, thus transgressing the requirements of informational decentralization. Perfect competition is not incentive-compatible.

But might there not exist some other set of rules to generate a process avoiding this difficulty? We have so far seen only that the conventional parametric price mechanism will not do the job, and there are many alternative mechanisms one could experiment with. But the same trouble will be present in *any* process yielding Pareto-optimal outcomes and giving the participants the option of remaining at their initial endowments if they so desire (the "no-trade option"). For it turns out that, given any such process, the non-law-abiding trader can profit by behaving in a way that successfully misrepresents his true characteristics (Hurwicz, 1972a). Thus, in a nonatomistic exchange economy, there can be no incentive-compatible mechanism with optimal equilibria and the no-trade option—even though there are no public goods! (See also Ledyard 1972.)

These results show that the difficulty is due not to our lack of inventiveness, but to a fundamental conflict among such mechanism attributes as the optimality of equilibria, incentive-compatibility of the rules, and the requirements of informational decentralization. Concessions must be made in at least one of these directions.

The possibility of successful false revelation of individual characteristics is particularly important in an economy with production. The issue has come up in many contexts. It has been raised as an objection to the Lange-Lerner mechanism. It has also been recognized in connection with situations where targets and norms are set, whether for enterprise managers in a Soviet type economy or for workers on a piece rate under capitalism. In all of these cases there is a "superior" and a "subordinate," and the latter has an incentive to depress the norms when the penalty for failure to reach a target is severe.

In analyzing the situation, one should distinguish cases in which the subordinate's reward depends only on activities observed by the superior from those where, as in Leibenstein's X-efficiency model, the subordinate's satisfaction is affected by factors under his control which cannot be observed by the superior. More realistically, one may assume that the superior can observe more or less, but only at a cost. Furthermore, if we think of the superior as a central authority and the subordinate as a manager of a plant, it is reasonable to suppose that the subordinate possesses technological knowledge while the superior knows the (social) objective function and, perhaps, resource availabilities.

Under such circumstances the superior

faces a dilemma. If he gives his subordinate a great deal of autonomy and does not make an effort at observation, the subordinate will maximize his own rather than the superior's objective function. If the superior makes the costly effort at observation and deprives the subordinate of autonomy, he is likely to give wrong orders because of his technological ignorance. A possible solution is a reward structure which brings the objectives of the two parties closer together (e.g., output sharing); this may give the subordinate a motivation consonant with the superior's and would make him use his technological knowledge to maximize the superior's objective function if he were given sufficient freedom to select the correct actions. The loss to the superior from sharing may in some cases exceed the gains in efficiency. But we can see that it is meaningful to seek an optimum combination of observation effort, delegation of authority (grant of autonomy), and reward structure.

The example is instructive because it shows the difference in the conceptual structure needed for the investigation of the interplay of authority, incentive, information, and performance issues. Where only performance and information were being investigated, the object of study was the mechanism or adjustment process. But from the viewpoint of incentive and authority structure, it is helpful to note (as does Camacho) that the actual responses of the participants may be regarded as the resultant of two factors: the official rules ("the régime") and the behavioral characteristics of the participants. In general, the rules limit the responses of the participants, but do not prescribe them uniquely. Given the rules, together with the punishment and reward structure for transgressions, the actual responses are determined by the participants' behavioral characteristics. *Laissez-faire* may be defined as a régime granting the participants maximal leeway; at the other extreme (and equally unrealistic) would be an economy in which the rules make all participants into preprogrammed automata.

Aside from value judgments and preferences for freedom of decision making, one can regard the degree of restrictiveness of the régime, together with the reward structure and informational activities (insofar as these can be varied by the designer) as unknowns of the problem. The objective is to attain optimal performance. The behavioral characteristics of the participants and the class of environments for which the system is being designed are the givens. (Examples of this type, but with information structure regarded as given, were worked out by Camacho.)

Let us go back to consider the incentive-compatibility issue in a production economy, assuming complete observability of all activities (but not necessarily of all characteristics!). A very simple example involves two persons: a farmer producing wheat and a laborer. The farmer wants to maximize the amount of wheat after paying off the laborer (in wheat); the laborer's net satisfaction is measured by the difference between the amount of wheat received and an index of disutility of labor. If the production function and the disutility curve have their classical shapes, there will be a unique Pareto-optimum at a point where the marginal disutility of labor equals its marginal productivity. Perfect competition would produce this resource allocation. But it will be found that the farmer can get more wheat by misrepresenting his production function, namely, by underestimating the marginal productivity at a point below the optimum output level. (It is assumed that the laborer remains a wagetaker.)

Thus again perfect competition is not incentive-compatible. However, unlike under pure exchange, it *is* possible here to devise a reward scheme that would eliminate the incentive toward misrepresentation on the part of one participant. For in this simple farmer-laborer example it so happens that Pareto optimality is equivalent to the maximization of the difference between the total output and the total disutility of labor. (This disutility, under the assumptions made, can be considered cardinally measurable). Thus if the two participants, the farmer and the laborer, are promised fixed shares of this difference (say, one-third for the farmer, two-thirds for the laborer), with the laborer in addition getting an amount of wheat just sufficient to compensate him for the disutility of labor, the farmer will have an interest in reaching the Pareto-optimum. Thus one could ask them to play the market game, but only for the purpose of determining the proper labor input level, following which a noncompetitive "surplus-sharing" rule would be implemented.

Here the manager (farmer) would have an incentive to be truthful. Thus we have a situation where perfect competition is not incentive-compatible for either party but for the manager another process, departing from perfect competition in its distributive aspects, is. However, I rather doubt that this is typical. In more elaborate situations, with the managers' utilities depending on the product mix of the economy, informationally-decentralized mechanisms that are incentive-compatible on management side may well fail to exist. But in "team" type situations (with scalar additive outputs) sharing formulas will be safe against misrepresenttation, while competitive price mechanisms in general will not. (Related questions of incentive-compatibility were examined by Groves, 1969.)

The incentives for truthful revelation of incentives when public goods are being allocated were examined by Drèze and de la Vallée Poussin. Unfortunately, their conclusions are not directly comparable with those we have reached for the world of private goods. They do find that, in terms of their criterion, at an equilibrium of the process, all consumers have an incentive to reveal their preferences correctly. However, their criterion is an *instantaneous* change in utility, while the criterion used here (and, it seems, that implicit in Samuelson's argument) refers to the utility of the *final* outcome.[5]

Heal points out that an incentive-compatible reward structure (on the output side) can be obtained for his (1971) process in the case of one producer when the center's utility function is positively homogeneous. The producer would not pay for the centrally allocated resources, but would get paid for the outputs, with prices equal to the marginal utilities of the outputs. In this case the utility is a strictly increasing function of the value of output; hence, if the producer were rewarded, say, by a fixed share of the value of output, he would find it to his advantage to maximize the center's utility. Unfortunately, where there are several firms, the one-to-one relationship is between utility and the value of outputs *aggregated* over all producing units; this does not imply *individual* incentive-compatibility.

Jennergren shows the incentive for misrepresentation in the Dantzig-Wolfe decomposition procedure, specifically an incentive to "hide" a part of the (feasible) production set. Both Marglin and Jennergren stress the incentive toward "cheating" under the (parametric) price mechanism in the case of small numbers of producing units. For the "atomistic case," Marglin suggests a combination of a "command" (quantity-guided) system in the search for an optimum with a profit-

[5] For the case of one private good, Drèze-Poussin find correct revelation the only good strategy in the minimax (as distinct from Nash) sense.

maximization motive to provide suitable incentives. This is, in a sense, the reverse of the system considered above for the farmer-laborer example.

From the preceding discussion it is evident that the incentive structure is largely determined by what the participants can achieve for themselves by their free actions; this in turn depends on such institutional phenomena as private property, rules for the distribution of profits, or the freedom not to trade. A tool appropriate for the analysis of such phenomena is the characteristic function of a game defined by von Neumann and Morgenstern. Shapley and Shubik carried out a study of different institutional property arrangements, including feudalism, sharecropping, and the village commune, by constructing the corresponding characteristic functions and exploring the different versions of game solutions (von Neumann-Morgenstern solutions, the core, the Shapley value). Thus a significant step is taken toward a formalization of the distributional aspects of the economic system.[6] (The distributional parameters introduced above, as well as the no-trade option, may be regarded as special cases of a similar approach.)

In the context of distributional issues, the conflict between informational decentralization and incentival considerations is illustrated by a model due to Pazner and Schmeidler. Their objective is to show that preassigned income distribution can be attained without sacrificing either Pareto optimality or informational decentralization. To accomplish this, they postulate a central agency distributing money (purchasing power) to individuals in accordance with the desired income distribution. Also, there are central agencies in charge of allocation of manpower. These custodians of labor operate as if they owned it. The workers can buy their leisure back from the manpower agency, but they have no freedom of choice where to work. The total supply of labor is assumed fixed and independent of the rewards. In the absence of individual checking procedures and given ordinary human motivations, such an assumption may be difficult to maintain.

[6] See also Shubik (1962) and Shapley (1972).

V. Conclusion

The proper integration of the information and incentive aspects of resource allocation models is perhaps the major unsolved problem in the theory of mechanism design. Many other questions also remain unanswered. Nevertheless, I think this survey shows that economic analysis has broken out of its traditional limits in at least two important ways: (1) devising specific new mechanisms; and (2) exploring the constraints and tradeoffs to which the design of mechanisms is subject.

The new mechanisms are somewhat like synthetic chemicals: even if not usable for practical purposes, they can be studied in a pure form and so contribute to our understanding of the difficulties and potentialities of design. The design point of view enlarges our field of vision and helps economics avoid a narrow focus on the status quo, whether East or West.

We have made significant progress in understanding the problems of designing resource allocation mechanisms. But the field is still in its infancy because these are hard problems.

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