Solow's 1956 Contribution in the Context of the Harrod-Domar Model

Harald Hagemann

Starting with Adam Smith's *Wealth of Nations*, the classical economists were concerned with the long-run development of an economy. This holds in particular for Karl Marx, who in chapter 21 of volume 2 of his *Capital* elaborated a two-sector steady-state growth model, comprising a capital goods and a consumption goods sector, which functioned as a reference path that irritated friends and foes alike. Thereafter, apart from very few exceptions, growth theory stopped being a major issue for most economists until the end of World War II, when modern growth theory was born with the works of Roy Harrod (1939, 1948) and Evsey Domar (1946, 1947). Growth became almost everybody's concern, first because of fear of stagnation and soon afterward because of reflections on the remarkable growth process in the Western world since the early 1950s.

The first wave of interest in growth theory associated with the contributions of Harrod and Domar came into existence as a by-product of John Maynard Keynes's *General Theory*, as best seen in Harrod's early correspondence with Keynes and in his seminal 1939 paper, "An Essay on Dynamic Theory," published in the *Economic Journal*. Harrod as well as Domar aimed to extend Keynes's analysis into the long run by considering under what conditions a growing economy could realize full-capacity utilization and full employment. Whereas Harrod (1939, 21) regarded the

I thank Robert M. Solow, Mauro Boianovsky, and two anonymous referees for valuable suggestions.

History of Political Economy 41 (annual suppl.) DOI 10.1215/00182702-2009-017

"dynamic equilibrium" "as a necessary propaedeutic to trade-cycle study," Domar was less ambitious. The complex stability problems of a growing capitalist economy were not a central feature of his analysis. Instead he concentrated his attention on the requirements for equilibrium of supply and demand along a steady growth path. However, despite some important differences in their analyses and because of a formal identity in their results for growth equilibrium, the Harrod-Domar model of economic growth emerged as a standard textbook model.

The second wave of interest in growth theory was launched by the development of the neoclassical model by Robert Solow (1956, 1957) and Trevor Swan (1956), with such precursors as Jan Tinbergen ([1942] 1959) and James Tobin (1955).[1] In his 1942 article "Zur Theorie der langfristigen Wirtschaftsentwicklung" ("On the Theory of Trend Movements"), Tinbergen had investigated the long-run development of the economy under the influence of population growth, technical progress, and capital formation and used a linear-homogenous Cobb-Douglas function with a geometric trend factor for technological development, probably the most advanced precursor of the Solow model.

Solow's neoclassical model came into existence as a reaction to the Harrod-Domar model and some deficiencies associated with it, in particular the enormous instability problems. "An economy evolving according to Harrod-Domar rules would be expected to alternate long periods of intensifying labor shortage and long periods of unemployment" (Solow 1999, 641). The empirical observation that real economies were not as wildly unstable as the Harrod-Domar model suggested propelled Solow to develop the basic neoclassical model. He made this clear from the very beginning. "The characteristic and powerful conclusion of the Harrod-Domar line of thought is that even for the long run the economic system is at best balanced on a knife-edge of equilibrium growth" (Solow 1956, 65). Solow (1994, 45 n. 1) attributed this suspicious result to dubious assumptions, in particular the assumption of fixed-production coefficients for the economy as a whole, which implied that any deviation from growth equilibrium would cause cumulative processes to move farther away from equilibrium, and to the fact that "Harrod's exposition tended to rest on incompletely specified behavioural and expectational hypotheses." Abandoning the fixed-coefficients technology and replacing it by a technology

1. For a more detailed discussion of the contributions by Tinbergen and Tobin, see the articles by Marcel Boumans and Robert Dimand and Steven Durlauf in this volume.

that allows substitution between the two factors of production capital and labor would make the capital-labor ratio and the capital-output ratio flexible, would guarantee convergence to a balanced growth path, and would establish stability of growth equilibrium.

In the following I first present the Harrod-Domar model, which was the main initial challenge and drove Solow to develop neoclassical growth theory. Section 2 puts forward the basic neoclassical model and discusses its key characteristics. In section 3 I focus on some early reactions by Harrod and Domar as well as on the distinction between two different instability problems, namely, the divergence between the warranted and the natural rates of growth, which marked, on the one hand, the starting point for the neoclassical reaction and, on the other, the divergence between the warranted and the actual rates of growth creating a business cycle problem. The article concludes with some reflections on the problem of combining long-run and short-run issues in macroeconomics in Solow's final section ("Qualifications") in his 1956 paper and in some of his more recent comments on the cycle-trend problem.

1. The Harrod-Domar Model of Economic Growth

The approaches by Harrod and Domar both start from a fundamentally Keynesian framework and extend it to the long run, analyzing the requirements for maintaining full employment over time. According to the Harrod-Domar model, to maintain full employment, the economy must invest the amount of saving related to full-employment income every year; but that alone is not sufficient. Production capacities have to be fully utilized as well, and capital accumulation has to be synchronized with the growth of the labor force. At first, "Keynes's peculiar treatment of investment" (Domar 1957, 6) had to be overcome and the *dual character of the investment process* had to be recognized. This implies that investment not only generates income as in the Keynesian multiplier analysis but also increases the economy's productive capacity. Whereas the first effect appears on the demand side and is of a short-run nature, the latter effect appears on the supply side and is a long-run effect (over the lifetime of the capital goods). Furthermore, whereas every positive net investment I has a capacity-enhancing effect, only an increment of investment $\dot{I}$ leads to an increase in income $\dot{Y}$. Assuming that the economy initially is in a position of full-employment equilibrium, we can formulate a Domar-type model

Table 1 Domar

	Flows	Stocks
Income effect	$\dot{Y}_D = \frac{1}{s}\dot{I}$	$\dot{K} \equiv I = S = sY_D$
Supply-side capacity effect	$\dot{Y}_{cap} = \frac{1}{v_{cap}} I$	$Y_{cap} = \frac{1}{v_{cap}} K$
Equilibrium condition	$\dot{Y}_D \stackrel{!}{=} \dot{Y}_{cap}$	$Y_D \stackrel{!}{=} Y_{cap}$
Solution	$\frac{\dot{I}}{I} = \hat{I} = \frac{s}{v_{cap}} = g$	$\frac{\dot{K}}{K} = \hat{K} = \frac{s}{v_{cap}} = g$

with production coefficients either for the flow variable I or for the stock variable K and get the solution for capital accumulation, $\hat{I} = \hat{K} = s / v_{\text{cap}}$. Table 1 contains Domar's equations.

Here s denotes the constant average and marginal propensity to save, with a proportional savings function $S = sY$; v_{cap} is the capital-output ratio in growth equilibrium, that is, with full-capacity utilization and full employment. Y_{cap} is full-capacity output, Y_{D} denotes aggregate demand, and the operator $\hat{x}$ is the growth rate of a variable x.

Although Domar's solution for equilibrium growth resembles Harrod's "fundamental equation," it has to be emphasized that Domar's (1946, 140) σ, "the potential social average investment productivity," is not the reciprocal of v. Since "its magnitude depends to a very great extent on technological progress" (140), Domar's σ measures more than the increase in potential output caused by investment; it embodies the effects of the labor force and technical progress on the productive potential and thus embodies Harrod's natural rate of growth. The maintenance of full employment over time requires investment, or the capital stock, to grow at a certain constant rate. However, Domar's approach, which focuses on the consistency conditions between supply and demand in dynamic equilibrium, does not include an investment function.

Probably the greatest difference between the approaches of Harrod and Domar is the incorporation of an investment function of the accelerator type

$$I_t = v_w(D_t - Y_{t-1}), \tag{1}$$

Table 2 Harrod

	Flows	Stocks
I-S-Condition (= capital accumulation)	$I \overset{!}{=} S = sY$	$I \equiv \dot{K} = sY$
Investment function/ warranted capital stock	$\dot{K} = I_w = v_w \dot{Y}$	$K_w = v_w Y$
Solution	$\frac{\dot{Y}}{Y} = \hat{Y} = \frac{s}{v_w} = g$	$\frac{\dot{K}}{K} = \hat{K} = \frac{s}{v_w} = g$
	$\hat{I} = \hat{Y} = \hat{K} = \frac{s}{v} = g$	

where the level of investment I_t in period t equals the difference between the expected demand for overall output D_t and the actual output in the last period Y_{t-1} times the desired capital-output ratio v_w (acceleration coefficient).[2] Harrod's (1939, 14) growth model is based on the "marriage of the 'acceleration principle' and the 'multiplier' theory," where the actual output Y_t in year t will equal the investment level I_t times the multiplier $1/s$, that is,

$$Y_t = \frac{1}{s} I_t. \tag{2}$$

We can easily derive Harrod's "fundamental equation" for the equilibrium or "warranted rate of growth" in a similar way as for Domar's production coefficient version based either on flows or on stocks. Harrod's equations are in table 2.

According to Harrod (1939, 16), "The warranted rate of growth is taken to be that rate of growth which, if it occurs, will leave all parties satisfied that they have produced neither more nor less than the right amount." So far the equilibrium growth rate for investment, output (income), or the capital stock $g = s/v$ describes a capital accumulation equilibrium in which investors would be satisfied because there is no underutilization of production capacities (*full-capacity growth*). It does not necessarily

2. Harrod (1959, 452) "stressed that subject to two relatively minor reservations . . . Domar's equation is identical with mine," and identified one in the difference of Domar's σ and v_w, or what he denoted C (Harrod 1939) and C_r, respectively (Harrod 1948).

please the workers, however, because the labor market is not yet integrated. There is no reason for the warranted rate to be associated with full employment.

Growth equilibrium therefore requires that not only the full-capacity condition but also the full-employment condition be fulfilled. Assume that labor supply in an initial period $t = 0$ equals L_0 and grows with a constant rate n over time; then labor supply in period t equals

$$L_t^s = L_0 e^{nt}. \tag{3}$$

Labor demand in period t equals overall output Y_t times the labor-output coefficient u_t, that is,

$$L_t^D = u_t Y_t. \tag{4}$$

Full employment therefore requires

$$u_t Y_t = L_0 e^{nt}. \tag{5}$$

The "natural" rate of growth g_n, which marks a maximum rate of growth or ceiling for the economy in the long run, equals

$$g_n = n + m, \tag{6}$$

where m denotes the rate of labor-augmenting technical progress. Growth equilibrium, that is, the simultaneous accomplishment of full-capacity growth and full-employment growth, therefore requires the conformity of the warranted with the natural rate of growth. However, according to Harrod (1939, 30), "there is no inherent tendency for these two rates to coincide."

This creates a secular instability problem, which is thoroughly Keynesian in spirit and a kind of extension of the unemployment problem to the long run. If all four parameters s, v, n, and m are exogenously given, then an equilibrium growth path can come about only as a lucky coincidence. It is highly probable that the rate of growth s/v, warranted by saving and investment behavior, will differ from the natural rate $n + m$, which would ensure full employment. However, the two disequilibrium cases are thoroughly asymmetrical. Whereas in the case $g_n > g_w$ there is a chronic tendency toward growing unemployment, in the opposite case $g_w > g_n$ the shortage of labor that arises as soon as the full-employment barrier is encountered would have a negative impact on investment and prevent the economy from growing at the warranted rate, which probably will also affect employment negatively. In the absence of any adjustment mecha-

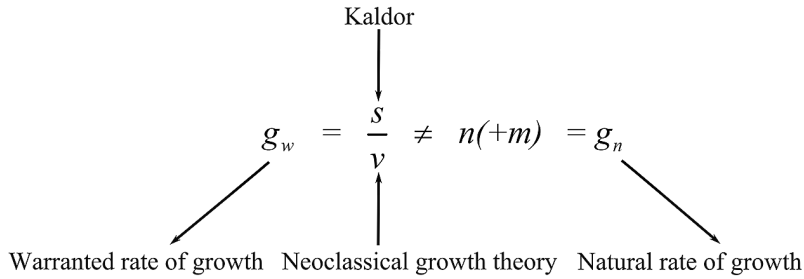

Figure 1 Potential solutions for the secular instability problem

nism to bring the warranted and the natural rates of growth closer together, economies would face most of their time either prolonged periods of rising or falling unemployment and/or periods of a decline or increase in the degree of capacity utilization. No wonder that the subsequent development of growth theory explored all four possible avenues to endogenize the parameters that allow the equalization of the warranted and the natural rates of growth. The four possibilities are indicated in figure 1. Two of these routes, which link up with the warranted rate, have become more popular. In retrospect, Solow (1994, 47 n. 2) argues that "in principle there is no reason to exclude the endogeneity of m and n. But induced changes in population growth, although an important matter in economic development, seemed not to figure essentially in the rich countries for which these models were devised. The idea of endogenous technological progress was never far below the surface." It may be recalled that migration very often depends on economic factors. Thus several Western European countries in the 1960s, after facing the full-employment ceiling, tried to overcome the shortage of labor in a period that may be interpreted as an $s/v > g_n$ constellation by hiring "guest workers" from southern Europe.[3]

As is well known, the favorite post-Keynesian solution for overcoming the secular Harrod problem was elaborated by Nicholas Kaldor (1956), who distinguishes between savings out of profits s_p and savings out of wages s_w, with $1 \geq s_p \geq s_w \geq 0$, which allows the endogenization of the overall savings propensity s. As in Keynes, the theory of income distribution has not been addressed in the basic Harrod-Domar model. In

3. According to the Federal Statistical Office, in Germany, for example, the number of guest workers increased from less than 300,000 in 1960 to nearly 2.5 million in 1973.

"Jean Baptiste" (as Samuelson teasingly called Kaldor) Kaldor's (1956, 94) second use of the multiplier principle, the level of output and employment is taken as given, and the multiplier analysis serves to develop a theory of the distribution of income between the two factors of production:

$$\frac{P}{Y} = \frac{I/Y - s_w}{s_p - s_w}, \tag{7}$$

where the share of profits in income P/Y depends on the ratio of investment to output I/Y and the two saving propensities. Despite the full-employment assumption made by Kaldor, the critical hypothesis that investment decisions are independent from saving decisions has a strong Keynesian flavor and contrasts heavily with neoclassical growth theory.

In a longer passage it becomes clear that it has been one of Kaldor's major aims in formulating a Keynesian theory of income distribution to overcome the secular Harrod problem by endogenization of the overall savings ratio, which now amounts to

$$s = \frac{I}{Y} = \left(s_p - s_w\right)\frac{P}{Y} + s_w. \tag{8}$$

"Hence the 'warranted' and the 'natural' rates of growth are not independent of one another; if profit margins are flexible, the former will adjust itself to the latter through a consequential change in P/Y" (Kaldor 1956, 97).

Whereas James Meade (1961) was open to the use of a Kaldorian saving function with different saving rates out of wages and profits and the associated adjustment mechanism within a neoclassical theory of economic growth, Solow (1994, 57) always had been skeptical and pointed out "that this way of resolving the problem did not catch on, partly for empirical reasons and partly because the mechanism seemed to require that factor prices be completely divorced from productivity considerations."

2. Solow's Neoclassical Model of Economic Growth

The very first pages of his 1956 classic elucidate how much Solow had been challenged by the secular instability problem of an incompatibility of the natural and warranted rates of growth, and the main route he took to overcome this problem, improving on the Harrod-Domar model.

> But this fundamental opposition of warranted and natural rates turns out in the end to flow from the crucial assumption that production takes place under conditions of *fixed proportions*. There is no possibility of substituting labour for capital in production. If this assumption is abandoned, the knife-edge notion of unstable balance seems to go with it. . . . The bulk of this paper is devoted to a model of long-run growth which accepts all the Harrod-Domar assumptions except that of fixed proportions. Instead I suppose that the single composite commodity is produced by labor and capital under the standard neoclassical conditions. . . . The price-wage-interest reactions play an important role in this neoclassical adjustment process, so they are analyzed too. (Solow 1956, 65–66)

Thus the neoclassical adjustment mechanism rests on *two* decisive assumptions: *substitution* between the two factors of production, capital and labor, and *flexibility* of factor prices. In contrast to Kaldor's approach, where the burden of adjustment falls on the overall propensity to save *s*, in the neoclassical model the burden of adjustment falls on the capital-output ratio *v*.

Solow had already discussed Harrod's model before. As a former research assistant of Wassily Leontief, he was well trained in input-output systems or linear models (Solow 1952). In particular, his "Note on the Price Level and Interest Rate in a Growth Model" (Solow 1953–54), where he interprets the Harrod model as a special one-commodity case of the Leontief dynamic system, shows that Solow came to Harrod via his earlier work on linear systems.[4] There he noted that neither the more complex Leontief model nor the Harrod model "has anything to say about the actual time-path that will be followed by any economic system starting from arbitrary initial conditions" (Solow 1953–54, 75). Instead, "both models define equilibrium . . . by the perpetual appropriateness of the existing capital stock to the current level of output" (75). Solow had been critical not only of the missing disequilibrium dynamics in Harrod's system but also in particular of the fact that the equilibrium conditions for output and price are separated, that is, "that Harrod's discovery of an equilibrium growth of output required no formal consideration of prices, and the development of a price-interest equilibrium" (79).

4. This is also clearly reflected in Solow's 1956 paper when he discusses the fixed-proportions example or the Harrod-Domar case as the first of three examples of production functions to solve the differential equation (13). See Solow 1956, 73–76.

In Solow's 1956 model, technological possibilities are represented by the macroeconomic production function where output Y is produced with the help of the two input factors capital K and labor L.

$$Y = F(K, L). \tag{9}$$

With constant returns to scale the quantity of output per worker $y = Y/L$ becomes a function only of capital per worker $k = K/L$. Thus we get the per capita function

$$y = f(k).^{5} \tag{10}$$

It is a one-commodity model, as are Swan's meccano sets, Meade's tons of steel (where the malleability character is more difficult to imagine than with "jelly" but comes in the backward reading of Joan Robinson's "leets"), or the Ricardian corn, which removes aggregation and capital measurement problems. The production function is "well behaved" (the later Inada conditions) and is homogeneous of the first degree, that is, it shows constant returns to scale that implies that the more general Euler theorem allows a logically consistent marginal productivity remuneration that overall income is exactly exhausted by payment to the two factors of production

$$Y = rK + wL, \tag{11}$$

with r as the rate of profit and w as the (real) wage rate. Investment is determined by the proportional saving function.

As Solow (2007, 13) has stated recently, "The Cobb-Douglas production function is a wonderful vehicle for generating instructive examples. But it has special Santa Claus properties, and we must not be misled about the generality of those examples." Among the "Santa Claus properties" are not only the evaporation of the distinction between labor-augmenting (Harrod-neutral), capital-augmenting (Solow-neutral), and

5. In Solow's original model, technical progress is widely absent. For growth equilibrium, it is necessary to assume that technical progress is of a labor-augmenting nature, that is, "neutral" in the sense of Harrod. The extension of a neutral technological change that Solow (1956, 85) discusses is a modification of (9) with increasing scale factor, that is, $Y = A(t)F(K, L)$, which is the Hicksian case of technical progress with a symmetrical productivity effect on both factors of production. However, Solow assumes a Cobb-Douglas production function, which is exactly the special case where "Hicks-neutral" technical progress (or "Solow-neutral" technical progress of the vintage capital model) coincides with the "Harrod-neutral" case. It should be pointed out, however, that, although Solow assumes a Cobb-Douglas function in his treatment of technical change, his theoretical model does only depend on a linear homogeneous function.

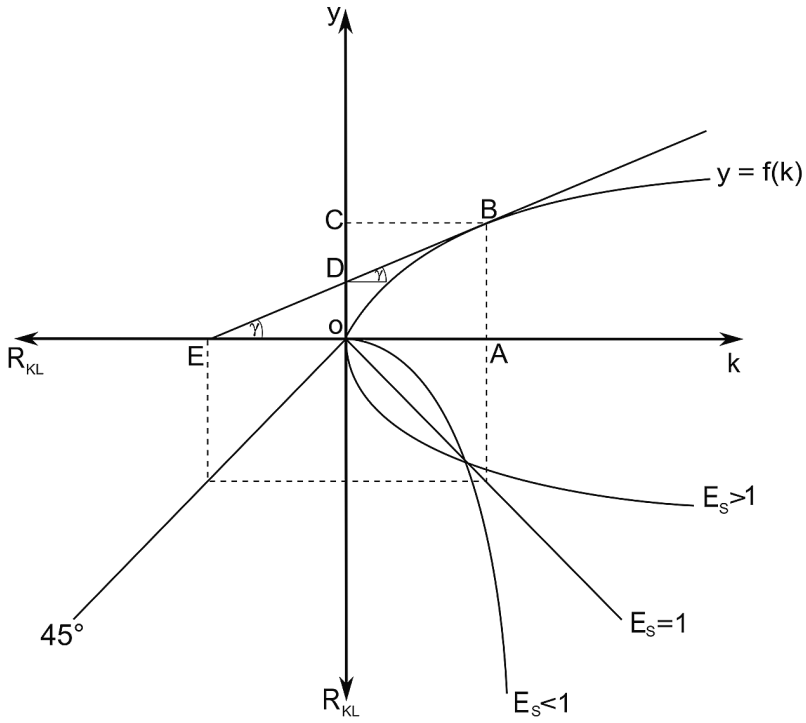

Figure 2 The central role of the elasticity of substitution

output-augmenting (Hicks-neutral) technical progress but also the constancy of the income shares of the factors of production.

The elasticity of substitution

$$E_s = \frac{dk/k}{dR_{KL}/R_{KL}} = \frac{dk/k}{d\left(\frac{W}{r}\right)/\frac{W}{r}} = \frac{d\left(\frac{0A}{0E}\right)}{\frac{0A}{0E}} \tag{12}$$

gives a measure for the easiness or difficulty of substitution between capital and labor. It addresses the central causal mechanism of neoclassical theory according to which a relative change in the (real) wage–rate of profit relation (or "factor-price" ratio) *w*/*r* leads to a relative change in the capital-labor ratio *k*. As figure 2 shows, only in the special case of a Cobb-Douglas production function with $E_s = 1$ will the shares of profit and wages in national income not be affected by substitution processes.

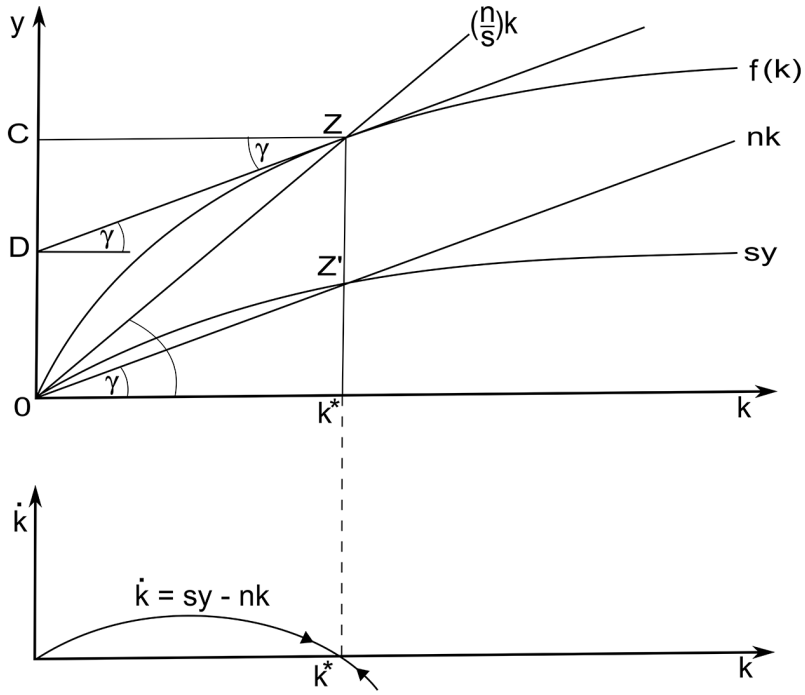

Figure 3 The fundamental equation of neoclassical economic growth

(In the figure, R_{KL} is the marginal rate of substitution between the two factors of production, tan γ indicates the rate of profit, and 0*D* represents the wage rate.) Similarly to Harrod's model of economic growth, which revolves around the warranted rate of growth derived from the marriage of the accelerator with the multiplier principle, there is also a "fundamental equation" (Solow 1956, 69) in Solow's neoclassical model of economic growth that outlines the change of the capital-labor ratio $\dot{k}$ over time. Figure 3 exhibits this "fundamental equation of neoclassical economic growth" (Jones 1975, 75–84).

Equation (10) gives output per worker *y* as a function of capital per worker or the capital-labor ratio *k*, *sy* denotes savings per capita, and *nk* denotes the investment necessary to supply the growing labor force with the same amount of capital ("capital widening"). If the initial capital stock is below the equilibrium level, that is, $sy > nk$, the investment volume increases with a declining *r*. Capital and output grow faster than the

labor force, that is, a process of "capital deepening" takes place, until the equilibrium capital-labor ratio k^* is reached. The fundamental equation of neoclassical economic growth[6] is given by

$$\dot{k} = sy - nk. \tag{13}$$

"Whatever the initial value of the capital-labor ratio, the system will develop *toward* a state of balanced growth at the natural rate" (Solow 1956, 70).

In the Solovian model, the long-run equilibrium growth rate is independent of savings and investment decisions. As Solow (1988, 308) stated in his Nobel Prize lecture, "A developing economy that succeeds in permanently increasing its saving (investment) rate will have a higher level of output than if it had not done so, and must therefore grow faster for a while. But it will not achieve a permanently higher rate of growth of output." The impact of a permanent increase of the saving rate from a lower level s_1 toward a higher level s_2 is made plain in figure 4. An increase in s leads to an upward shift of the function sy. Because of the interest rate mechanism, these higher savings are invested. During the adjustment process, capital (which now grows with the rate s_2/v) grows faster than labor (output growth is always intermediate between the growth rates of capital and labor), that is, $\hat{K} > \hat{Y} > \text{n}$, so that the capital-labor ratio is increased until a new equilibrium value is reached in k^*_2. We have a partial factor variation, where finally the diminishing returns to capital take their toll. During the adjustment process the economy experiences

- a growing capital-labor ratio k,
- a growing output per capita y,
- a growing capital-output ratio v, and
- a growing wage-profit ratio $R_{KL} = w / r$.

Although the increase in saving (investment) leads to a higher *level* of output per unit of labor input in long-run equilibrium, the steady-state *rate* of growth is independent of capital formation. This irrelevance of savings for the long-run growth rate is one of the main results of Solow's 1956 contribution. The Solovian growth model has to rely on exogenous technical progress to increase the rate of growth of output per capita in long-run equilibrium.

6. See equation (6) in Solow 1956, 69.

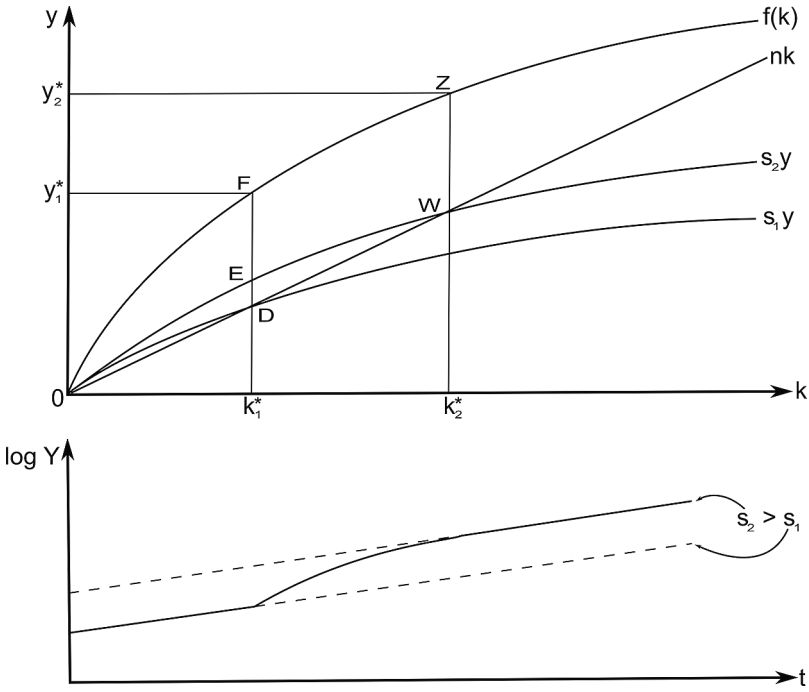

Figure 4 Effect of an increase in the saving rate

3. Early Reactions and Controversies

At the end of his presentation of the core of his neoclassical model of economic growth, Solow (1956, 73) sums up his major finding:

> The basic conclusion of this analysis is that, when production takes place under the usual neoclassical conditions of variable proportions and constant returns to scale, no simple opposition between natural and warranted rates of growth is possible. There may not be—in fact in the case of the Cobb-Douglas function there never can be—any knife-edge. The system can adjust to any given rate of growth of the labor force, and eventually approach a state of steady proportional expansion.

Solow seems to have convinced at least one of the founding fathers of the "Harrod-Domar model." Thus Domar (1957, 7–8) in the foreword to his *Essays in the Theory of Economic Growth*, published in 1957 when

he was still at Johns Hopkins before moving to MIT and becoming Solow's colleague in the subsequent year, conceded that his "model employed an inadequate production function" and that "a recent article by Robert M. Solow, which appeared in print just as I was writing these lines, has shown how a growth model can be enriched by the use of a not very complex but less rigid production function."

Domar had entered this route away from the fixed-coefficient assumption toward a greater flexibility between the factors of production already in the debate on full-capacity versus full-employment growth that took place in the same *Quarterly Journal of Economics* where Solow's famous paper was published three years later. There Domar (1953, 560) expressed some skepticism concerning two convictions in the post-Keynesian literature: that a too-high saving ratio will lead to overaccumulation of capital and prolonged unemployment, a "belief which forms the basis of Harrod's distinction between the natural and the warranted rates of growth,"[7] and for which he diagnosed a lack of empirical evidence; and that "as an analytical device, a constant input coefficient is God-sent, but it is quite a simplification and it should be used with care, particularly over longer periods of time when it is known to be subject to change" (561). Since the production universe comprises elements of flexibility, Domar (1953, 562) credited Harold Pilvin (1953) for performing "a very useful service by stating explicitly the production function implied in the existing growth models as an extreme case of the more 'normal' one—the former does not allow for substitution between factors (capital and labour), the latter does."[8]

However, Harrod, the Cambridge Keynesians, and some American Keynesians such as Robert Eisner (1958), who set out to defend "what Harrod, Domar and Hicks really said" against the neoclassical "attacks" by Tobin (1955) and Solow, remained less convinced. One target of attack was the "pre-Keynesian" character of Solow's analysis, which lacks an investment function, and consequentially has no role for motivational and behavioral patterns of entrepreneurs, including investors' expectations. Instead *ex ante* savings determine investment. On the other hand, the independence of investment decisions, sometimes guided by "animal

7. The problem of chronic depression is emphasized by Harrod (1948, vi) in his foreword to *Towards a Dynamic Economics*.

8. Leland Yeager (1954) criticized the essentially *nonmonetary* nature of the analysis of Harrod and Domar, which does not allow the drawing of far-reaching instability conclusions for real economies.

spirits" but in any case independent from savings (which have only an adaptive role to play), is the core of the Keynesian revolution. This view is also shared by the author of the first textbook of neoclassical growth theory: "Keynes's intellectual revolution was to shift economists from thinking normally in terms of a model of reality in which a dog called *savings* wagged his tail labelled *investment* to thinking in terms of a model in which a dog called *investment* wagged his tail labelled *savings*" (Meade 1975, 82). This point comes out most clearly in the critique raised by Amartya Sen (1970, 22) in the introduction to his widespread collection of selected readings in *Growth Economics*, in which he emphasizes

> The oddity of the neo-classical assumptions. . . . for in the process of adjustment a rise in the real interest rate seems to induce a higher rate of growth. One would have expected a rise in the real interest rate to cut down the rate of expansion through the investment function, but in the neo-classical model there is no investment function and investment is assumed simply to be determined by savings behaviour.

Instead of explicitly working with fixed coefficients or a Leontief technology, Harrod has never been very clear about the production-theoretic basis of his growth model. The (post-)Keynesian inflexibility of the capital-output ratio v has been substantiated less with production arguments than with the supposed inflexibility of the rate of profit (as one element of Kaldor's "stylized facts"). Thus it is equally important that the adjustment mechanism of neoclassical growth theory rests not only on substitution of the factors of production but also on the flexibility of factor prices. This decisive contrast to the post-Keynesian approach is not only pointed out by Eisner (1958) but comes out most sharply in Harrod's comment on a 1953 article by Pilvin, who made an early contribution to endogenize the capital-labor and the capital-output ratios but did not yet investigate the time path of capital intensity outside the steady state as Solow 1956 did. Here we find Harrod pointing out the centrifugal forces surrounding the warranted rate of growth, but in particular arguing in great detail against Pilvin's supposition that the rate of interest will do the appropriate job to generate an increase in the capital-labor ratio. Harrod not only refers to the problem of a liquidity trap but also states that "the postulate of a continually falling rate of interest seems rather an awkward one in a mature economy where the rate of interest is already low" (Harrod 1953, 556), and he concludes with his important doubts whether in mature economies the rate of interest has a major impact on the choice of production methods

when the producers are facing high degrees of uncertainty. These arguments elucidate that Harrod shares Keynes's fundamental skepticism concerning the required variations in the interest rate. Interestingly, Burmeister and Dobell (1970, 41), who follow Eisner (1958) in studying Harrod's position, conclude that to narrow Harrod's argument exclusively to a fixed-coefficient production function "misses the essential feature of Harrod's analysis," and on the basis of a Solovian neoclassical production function Burmeister and Dobell "have isolated the hypothesis about freely falling interest rates as the crucial consideration."

The intrinsic instability of equilibrium growth in the Harrod model has two dimensions: the divergence between the warranted and the natural rate, and the short-run instability problems that arise when the investors base their decisions on expectations of a higher or lower rate of growth than the warranted rate. The consequence is a much greater difference between the actual and the warranted rate of growth. Harrod's analysis of this instability problem leads to dramatic results because the market gives perverse signals to the investors. However, the results rely sensitively on the assumptions. This requires a thorough analysis of disequilibrium investment behavior, which is not properly elaborated by Harrod. Since Harrod originally came from trade cycle analysis, it is this cyclical stability problem that is normally more at the center of the "Harrod literature."[9]

The publication of "Second Essay in Dynamic Theory" by Harrod (1960), who sent an offprint to Solow, launched a correspondence between the two growth economists (see Young 1989, 183–85) that did not lead to greater agreement. Solow remained convinced that Harrod's "new" or "second" equation developed in that essay also presupposes a constant capital coefficient.

Solow (1988, 310) has always been well aware of the two dimensions of Harrod's instability problems, although he concedes in his Nobel Prize lecture that "I may not have been as clear then [1956] as I am now about the distinction between the two notions of instability." This becomes evident by the final section, "Qualifications," of his 1956 article, where he discusses "Keynesian" problems such as rigid wages and liquidity preference.[10] Thus he has been aware from the beginning that his neoclassical model of economic growth only "took the sting out of the first sort of

9. See, for example, Besomi 1999 and Young 1989.

10. See also Solow's reply to Eisner where Solow (1959) refers to that section and emphasizes that his main purpose had been to challenge the "tight-rope" view of economic growth.

instability," namely, the divergence between the warranted and the natural rates of growth.

I turn now to the "knife-edge" metaphor, which was used on the very first page of Solow's 1956 classic, was perpetuated in Edward Prescott's (1988, 8) remarks on Solow's contributions deserving the Nobel Prize, and is still commonly used today. What made Harrod furious, who fought a lifelong hopeless battle against "Harrod's knife-edge" (see, e.g., Harrod 1973, chap. 3), which he would have preferred to exorcize permanently from the economic literature and to replace by a corridor concept instead, was the fact that even Cambridge Keynesians such as Joan Robinson (1965)—in her "Chinese period" (Solow 2007, 4)—were using this term. In his response, Harrod (1970, 741) expressed his "hope that we shall hear no more of the 'Harrod knife-edge.'"

4. Separation or Integration of Cycle and Trend Analysis?

Things are more complicated than just separating the two stability problems into a cyclical one and a secular one. Harrod (1939, 22) himself always regarded the warranted rate of growth as intrinsically unstable, representing a "moving equilibrium," which raises the problem of interaction between the trend and the cycle to which he finally could not give a satisfying answer.

In contrast to Harrod, Solow has addressed a different conceptual problem, namely, the issue of long-run economic growth. Neoclassical growth theory of the Solow-Swan-Meade type clearly separates the cycle from the trend and focuses on the steady state. Solow (2005, 9) recently recognized this in his "Reflections on Growth Theory" in the Aghion-Durlauf *Handbook*. The point was made more explicitly than in Solow's original article in Meade's famous 1961 textbook; there, the exposition of a neoclassical growth theory was "based on the assumption of an ideally successful Keynesian policy which at every point of time manages to keep the value of investment at the desired level" (ix), which sounds a little bit strange for a neoclassical theory but serves the function of getting rid of the business cycle problem.[11]

11. In the same spirit we can read in Swan's ([1963] 1970, 205) reflections on "golden ages" that his "illustrations will be Keynesian, in the sense of the future as Keynes did, and assume either that the authorities have read the *General Theory* or that they are socialists who don't need to; in other words I assume that whatever is saved is invested."

Hahn and Matthews (1964, 805–9) may have been the first authors who, after Eisner's earlier attempt, not only clearly distinguished between two different notions of the knife-edge problem, namely, the inequality between the warranted and the natural rate of growth and the instability of the warranted rate itself, but also observed that Solow, in focusing exclusively on the first, had missed Harrod's emphasis on the second. Harrod, however, had not elaborated a convincing out-of-equilibrium dynamics in his model. As Sen's numerical example has shown, the slightest deviation from the warranted rate leads to extreme instability, which "rule[s] out the possibility of stabilizing price and interest rate movements" (Baumol [1951] 1970, 55) on the theoretical side but also is at odds with an important empirical observation: the growth process in advanced economies after the Second World War was not as wildly unstable as the Harrod-Domar model implied. As mentioned at the outset of this essay, that observation inspired Solow to develop his own neoclassical model, and one of the main reasons his model has been a success is that it projects an economy whose stability is closer to that of the real world.

Solow always has been a short-run Keynesian, as can be seen in the concluding "Qualifications" section of his 1956 contribution. However, "all the difficulties and rigidities which go into modern Keynesian income analysis have been shunted aside. It is not my contention that these problems don't exist, nor that they are of no significance in the long run" (Solow 1956, 91). However, the shunting aside opened up the opportunity for real-business-cycle theorists such as Finn Kydland and Prescott to use Solow's steady-state model, in which the economy in contrast to the Harrod model never deviates from the warranted rate, for their explanation of short-run fluctuations.

In his Nobel Prize lecture, Solow (1987, 311–12) pointed out that

> it is impossible to believe that the equilibrium growth path itself is unaffected by the short- to medium-run experience. In particular the amount and direction of capital formation is bound to be affected by the business cycle. . . . So a simultaneous analysis of trend and fluctuations really does involve an integration of long run and short run of equilibrium and disequilibrium.

Similar statements that breathe a kind of "Harrodian spirit" have been made by Solow since then, time and again. However, despite the development of such concepts as path dependency, and a great advancement in technical methods compared with the days of Harrod, this decisive

problem of combining short-, medium-, and long-run macroeconomics two decades later still has not yet been solved, and probably will not be solved in the near future.

References

Baumol, W. J. [1951] 1970. *Economic Dynamics: An Introduction*. 3rd ed. London: Macmillan.

Besomi, D. 1999. *The Making of Harrod's Dynamics*. Houndmills, U.K.: Macmillan.

Burmeister, E., and R. Dobell. 1970. *Mathematical Theories of Economic Growth*. London: Macmillan.

Domar, E. D. 1946. Capital Expansion, Rate of Growth, and Employment. *Econometrica* 14 (April): 137–47.

———. 1947. Expansion and Employment. *American Economic Review* 37 (March): 34–55.

———. 1953. Full Capacity vs. Full Employment Growth: Further Comment. *Quarterly Journal of Economics* 67.4:559–63.

———. 1957. *Essays in the Theory of Economic Growth*. New York: Oxford University Press.

Eisner, R. 1958. On Growth Models and the Neo-classical Resurgence. *Economic Journal* 68 (December): 707–21.

Hahn, F. H., and R. C. O. Matthews. 1964. The Theory of Economic Growth: A Survey. *Economic Journal* 68 (December): 779–902.

Harrod, R. F. 1939. An Essay on Dynamic Theory. *Economic Journal* 49 (March): 14–33.

———. 1948. *Towards a Dynamic Economics*. London: Macmillan.

———. 1953. Comment [on Pilvin]. *Quarterly Journal of Economics* 67.4:553–59.

———. 1959. Domar and Dynamic Economics. *Economic Journal* 69 (September): 451–64.

———. 1960. Second Essay in Dynamic Theory. *Economic Journal* 70 (June): 277–93.

———. 1970. Harrod after Twenty-One Years: A Comment. *Economic Journal* 80 (September): 737–41.

———. 1973. *Economic Dynamics*. London: Macmillan.

Jones, H. 1975. *An Introduction to Modern Theories of Economic Growth*. Sunbury-on-Thames, U.K.: Nelson.

Kaldor, N. 1956. Alternative Theories of Distribution. *Review of Economic Studies* 23.2:83–100.

Meade, J. 1961. *A Neo-Classical Theory of Economic Growth*. London: Allen and Unwin.

———. 1975. The Keynesian Revolution. In *Essays on John Maynard Keynes*, edited by M. Keynes. Cambridge: Cambridge University Press.

Pilvin, H. 1953. Full Capacity vs. Full Employment Growth. *Quarterly Journal of Economics* 67.4:545–52.

Prescott, E. C. 1988. Robert M. Solow's Neoclassical Growth Model: An Influential Contribution to Economics. *Scandinavian Journal of Economics* 90.1:7–12.

Robinson, J. 1965. Harrod's Knife-Edge. In vol. 3 of *Collected Economic Papers*, by J. Robinson. Oxford: Blackwell.

Sen, A., ed. 1970. *Growth Economics: Selected Readings.* Harmondsworth, U.K.: Penguin Books.

Solow, R. M. 1952. On the Structure of Linear Models. *Econometrica* 20 (January): 29–46.

———. 1953–54. A Note on the Price Level and Interest Rate in a Growth Model. *Review of Economic Studies* 21.1:74–79.

———. 1956. A Contribution to the Theory of Economic Growth. *Quarterly Journal of Economics* 70 (February): 65–94.

———. 1957. Technical Change and the Aggregate Production Function. *Review of Economics and Statistics* 39 (August): 312–20.

———. 1959. Is Factor Substitution a Crime, and If So, How Bad? Reply to Professor Eisner. *Economic Journal* 69 (September): 597–99.

———. 1960. Investment and Technical Progress. In *Mathematical Methods in the Social Sciences, 1959*, edited by K. Arrow, S. Karlin, and P. Suppes. Stanford, Calif.: Stanford University Press.

———. 1988. Growth Theory and After. *American Economic Review* 78.3:307–17.

———. 1994. Perspectives on Growth Theory. *Journal of Economic Perspectives* 8.1:45–54.

———. 1999. Neoclassical Growth Theory. In vol. 1A of *Handbook of Macroeconomics*, edited by J. B. Taylor and M. Woodford. Amsterdam: Elsevier.

———. 2005. Reflections on Growth Theory. In vol. 1A of *Handbook of Economic Growth*, edited by P. Aghion and S. N. Durlauf. Amsterdam: Elsevier.

———. 2007. The Last 50 Years in Growth Theory and the Next 10. *Oxford Review of Economic Policy* 23.1:3–14.

Swan, T. W. 1956. Economic Growth and Capital Accumulation. *Economic Record* 32.63:334–61.

———. [1963] 1970. Golden Ages and Production Functions. In Sen 1970.

Tinbergen, J. [1942] 1959. Zur Theorie der langfristigen Wirtschaftsentwicklung. *Weltwirtschaftliches Archiv* 55:511–49. Translated as On the Theory of Trend Movements, in *Jan Tinbergen*: *Selected Papers,* edited by L. H. Klaassen, L. M. Koyck, and H. J. Witteveen. Amsterdam: North-Holland.

Tobin, J. 1955. A Dynamic Aggregative Model. *Journal of Political Economy* 63.2:103–15.

Yeager, L. B. 1954. Some Questions about Growth Economics. *American Economic Review* 44.1:53–63.

Young, W. 1989. *Harrod and His Trade Cycle Group: The Origins and Development of the Growth Research Programme*. Houndmills, U.K.: Macmillan.