1

On the Rationale of Group Decision-Making

Duncan Black

Source: *Journal of Political Economy,* 56(1) (1948): 23–34.

When a decision is reached by voting or is arrived at by a group all of whose members are not in complete accord, there is no part of economic theory which applies. This paper is intended to help fill this gap; to provide a type of reasoning which will contribute to the development of the theory of trade-unions, the firm, and the cartel; and to provide the basis for a theory of the equilibrium distribution of taxation or of public expenditure. Still other uses of the theory might be not less important. For reasons of space we avoid discussion of many points that demand fuller treatment and only attempt to indicate the course of the argument.[1]

1. General Assumptions

Let us suppose that a decision is to be determined by vote of a committee. The members of the committee may meet in a single room, or they may be scattered over an area of the country as are the electors in a parliamentary constituency. Proposals are advanced, we assume, in the form of motions on a particular topic or in favor of one of a number of candidates. We do not inquire into the genesis of the motions but simply assume that given motions have been put forward. In the case of the selection of candidates, we assume that determinate candidates have offered themselves for election and that one is to be chosen by means of voting. For convenience we shall speak as if one of a number of alternative motions, and not candidates, was being selected.

To develop our theory, we must make some further assumptions. Our major assumption will be that each member of the committee ranks the motions in a definite order of preference, whatever that order may be. To take a simple illustration, if there are four motions denoted by a_1, a_2, a_3, a_4, say, before a committee, the member *A* may prefer a_2 to any of the others, may be indifferent between a_3 and a_4, and may prefer either of them to a_1.

If so, *A*'s valuation of the motions could be represented by the schedule of preferences on the left-hand side of Figure 1, in which a_2 stands highest; a_3 and a_4 next highest, each at the same level; and a_1 lowest. And similar scales could be drawn for other members of the committee with $a_1 \ldots a_4$ appearing in some definite order on each scale, though the ordering of the motions might be different on the scale of each member.

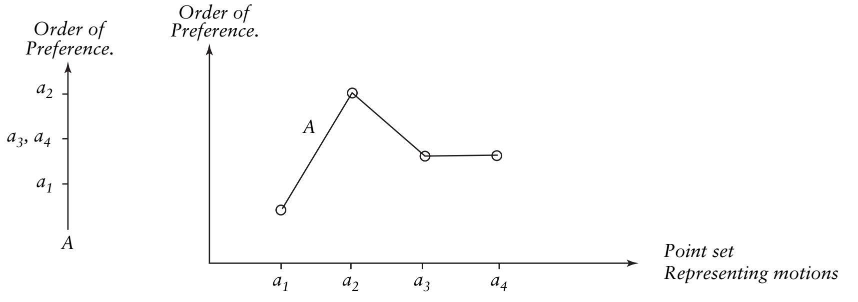


Figure 1

We are here using the theory of relative valuation of orthodox Economic Science, whether the theory of relative utility or the theory of indifference curves. The only points which have significance on the directed straight line representing a member's schedule of preferences are those at which motions are marked, and his scale really consists of a number of points placed in a certain order in relation to each other. No significance attaches to the distance between the points on the scale, and any two scales would be equivalent on which the motions occurred in the same order.

When a member values the motions before a committee in a definite order, it is reasonable to assume that, when these motions are put against each other, he votes in accordance with his valuation, i.e., in accordance with his schedule of preferences. Thus the member *A* would be assumed to vote for a_2 when it was put in a vote against a_1; or if a_3 were put against a_4 – since he is indifferent between the two and it would be irrational for him to support either against the other – he would be assumed to abstain from voting.

A member's level of preference between the different motions may also be shown by denoting the motions put forward by particular points on a horizontal axis, while we mark level of preference along the vertical axis. For instance, the same set of valuations of the individual A is shown in the right and left parts of Figure 1. The only points in the diagram having significance would be those for the values a_1, a_2, a_3, a_4, on the horizontal axis, corresponding to the motions actually put forward. We have joined these points standing at various levels of preference by straight-line segments, but this is done merely to assist the eye, since the curve would be imaginary except at the four points. In this diagram, as in the case of the preference schedule, it is only the relative heights of different points which have meaning, not their absolute heights.[2]

While a member's preference curve may be of any shape whatever, there is reason to expect that, in some important practical problems, the valuations actually carried out will tend to take the form of isolated points on single-peaked curves. This would be particularly likely to happen were the committee considering different possible sizes of a numerical quantity and choosing one size in preference to the others. It might be reaching a decision, say, with regard to the price of a product to be marketed by a firm, or the output for a future period, or the wage rate of labor, or the height of a particular tax, or the legal school-leaving age, and so on.

In such cases the committee member, in arriving at an opinion on the matter, would often try initially to judge which size is for him the optimum. Once he had arrived at his view of the optimum size, the farther any proposal departed from it on the one side or the other, the less he would favor it. The valuations carried out by the member would then take the form of points on a single-peaked or ∩-shaped curve.

In working out our theory we shall devote considerable attention to this class of curves which slope continuously upward to a peak and slope continuously downward from that peak. We shall refer to the motion corresponding to the peak of any curve – the most-preferred motion for the member concerned – as his optimum.

Another case likely to be of frequent occurrence in practice – especially, again, where the committee is selecting a particular size of a numerical quantity – is that in which the valuations carried out by a member take the form of points on a single-peaked curve with a truncated top. Such a case would arise when the individual feels uncertain as to which of two or more numerical quantities proposed represents his optimum choice. He cannot discriminate in choice between (say) two of these numerical quantities; but the farther the proposal made falls below the lower of these values, or the higher it rises above the larger of them, the less he esteems the motion concerned.

We shall work out the theory first for the case in which the members' preference curves are single-peaked, and, after that, we shall show how the answer to any problem can be obtained no matter what the shape of the members' curves may be. When any matter is being considered in a committee, only a finite number of motions will be put forward and only a finite number of valuations will be carried out by each member. If three motions or six motions were put forward, each member would be assumed to value each of them in relation to the others. When we are drawing our preference curves, however, we will draw continuous curves and – since there are an infinite number of points on any continuous curve – we imply that the person for whom the curve is drawn has carried out an evaluation of each of an infinite number of motions in regard to each of the others. This is unrealistic, it is true, but, when the theory is worked out for this case, we can easily get the answer for any case in which only a finite number of motions is put forward and valued by the members.

We assume that the committee with which we are concerned makes use of a simple majority in its voting. In practice, voting would be so conducted that, after discussion, one motion would be made and, after further discussion, another motion (an "amendment," that is) might be moved. If so, the original motion and amendment would be placed against each other in a vote. One of the two motions having been disposed of, leaving a single motion in the field, a further amendment to it might be moved; then a further vote would be taken between the survivor of the first vote and the new motion; and so on. If 2 motions were put forward, 1 vote would be taken; if 3 motions, 2 votes; and, in general, if m motions were put forward, there would be $(m - 1)$ votes.[3]

Now it will be found to simplify the development of the theory if, in the first instance, we suppose that the voting procedure is different from this. We wish to make the assumption that when m motions $a_1, a_2, \ldots a_m$ (say) have been put forward, the committee places each of these motions against every other in a vote and picks out that motion, if any, which is able to get a simple majority against *every* other motion. The motion a_1 is to be envisaged as being put against all the other motions $a_2 \ldots a_m$; a_2 will already have been put against a_1, and we assume that it will then be put against $a_3 \ldots a_m$; and so on, a_{m-1} finally being pitted against a_m. On this assumption the number of votes taken will be the number of ways of choosing 2 things out of m, i.e., $m(m - 1)/2$ votes, instead of the $(m - 1)$ votes which would be taken in practice.

This assumption enables the theory to proceed more smoothly and quickly than the assumption that only $(m - 1)$ votes are held. When we have worked out the theory on this basis, we can go on to prove that – in the class of cases in which we are mainly interested – the same answer would be given whether $m(m - 1)/2$ votes were held, as we assume, or only the $(m - 1)$ votes of reality. The assumption is a kind of theoretical scaffolding which can be discarded once it has served its turn.

These, then, are our assumptions: that in a committee m motions are put forward, that each member carries out an evaluation of each motion in regard to every other, that in the voting each motion is put against every other, and that the committee adopts as its decision ("resolution") that motion if any, which is able to get a simple majority over every other.

It can be shown that, at most, only one motion will be able to get a simple majority over every other. To prove this, let us assume that a_h is such a motion, i.e., that a_h can get a simple majority over every other. And let us assume that this is also true of some other motion, a_k. By our first assumption, however, a_h can get a simple majority over every other motion, including a_k. Therefore a_k cannot get a simple majority over a_h. Hence, at most, only one motion can get a simple majority over every other.

2. Members' Preference Curves All Single-peaked

The method of reasoning which we employ can be seen most easily from a particular example. Figure 2 shows the preference curves of the 5 members of a committee. Only part of each curve has been drawn, and the curves are supposed to extend over a common range of the horizontal axis.

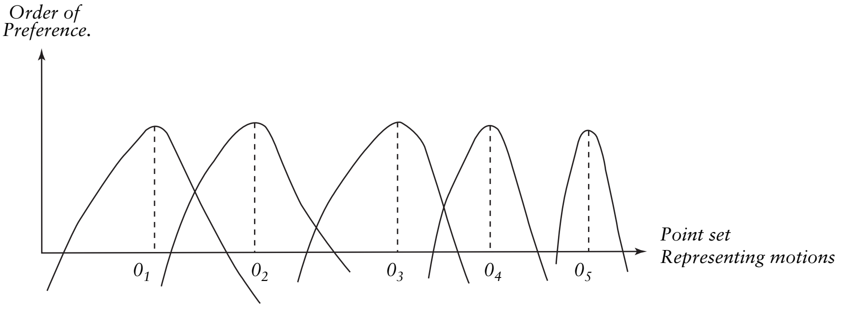


Figure 2

Then if a_h is put against a_k (where $a_h < a_k \leq O_1$), the preference curve of each member – irrespective of what its precise shape may be – is upsloping from a_h to a_k; and a_k, standing at a higher level of preference on the curve of each member, will get a 5:5 (5 out of 5) majority against a_h. If a_h is put against a_k, (where $a_h < a_k \leq O_2$), at least 4 members – viz., those with optimums at or above O_2 – will have preference curves which are upsloping from a_h to a_k; and a_k will get at least a 4:5 majority against a_h. If $a_h < a_k \leq O_3$, a_k will get at least a 3:5 majority against a_h. And similar relations hold for motions corresponding to values above O_3. If two values above O_3 are placed against each other in a vote, the nearer of the two values to O_3 will get a majority of at least 3:5 against the other.

If a value a_h (where $a_h < O_3$) is put against a value a_k (where $a_k > O_3$), before we could find which of the values would win in a vote, we would have to draw the complete preference curve for each member, find whether a_h or a_k stood higher on the preference curve of each member and count up the votes cast for a_h and a_k. But even though a value below the median optimum O_3 should defeat all values to the left of itself, and should defeat some of the values above O_3, this would be without significance. What we are looking for is that motion which can defeat every other by at least a simple majority. And we notice that the preference curves of at least 3 members are downsloping from O_3 leftward, and the preference curves of at least 3 members are downsloping from O_3 right-ward. Therefore O_3 can defeat any other value in the entire range by at least a simple majority. And, as we have already seen (end of Sec. I), this can be true of only a single value. The resolution adopted by the committee must be the motion corresponding to the value O_3.

To give the general proof, two cases must be worked out – that in which the number of members in the committee is odd and that in which it is even. We will consider each in turn.

Let there be n members in the committee, where n is odd. We suppose that an ordering of the points on the horizontal axis representing motions exists, rendering the preference curves of all members single-peaked. The points on the horizontal axis corresponding to the members' optimums are named $O_1, O_2, O_3, \ldots$, in the order of their occurrence. The middle or median optimum will be the $(n+1)/2^{th}$, and, in Figure 3, only this median optimum, the one immediately above it and the one immediately below it are shown.

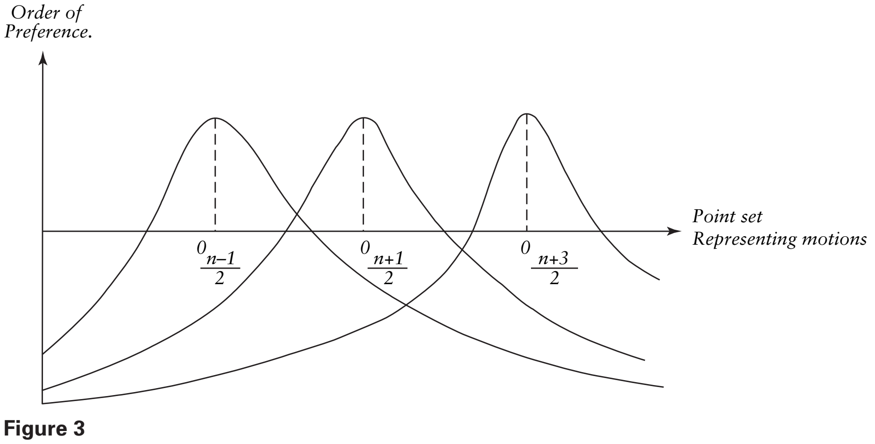


Figure 3

Then $O_{(n+1)/2}$ will be the motion adopted by the committee. Suppose $O_{(n+1)/2}$ were placed against any lower value, say, a_h. Since $(n+1)/2$ members have optimums at or above $O_{(n+1)/2}$, as we move from left to right from a_h to $O_{(n+1)/2}$, at least $(n+1)/2$ curves are up-sloping, viz., those of members with optimums at or to the right of $O_{(n+1)/2}$. At least $(n+1)/2$ members prefer $O_{(n+1)/2}$ to a_h and, in a vote against a_h, $O_{(n+1)/2}$ will get a majority of at least $(n+1)/2$: n, and this is sufficient to give it at least a simple majority. Therefore $O_{(n+1)/2}$ can get at least a simple majority against any lower value which is put against it. Similarly it can get at least a simple majority against any higher value. Thus it can get a simple majority against any other value which can be proposed. And by previous argument, it is the only value which can do so.

When the number of members, n, in the committee, is even, there may be a tie in the voting; and we will suppose that an additional person acting as chairman, in the event of a tie has the right to cast a deciding vote.

Let us suppose, first, that this member who acts as chairman has his optimum at $O_{n/2}$ or at one of the lower optimums. It can be shown that the motion corresponding to the value $O_{n/2}$ will be able to defeat any lower value (Figure 4). Let a_h be such a value, that is, $a_h < O_{n/2}$. Then $(n/2+1)$ members have optimums at or above $O_{n/2}$; and at least $(n/2+1)$ preference curves will be upsloping as we move from left to right from a_h to $O_{n/2}$. At least $(n/2+1)$ members will vote for $O_{n/2}$ against a_h, and this is sufficient to give $O_{n/2}$ a simple majority.

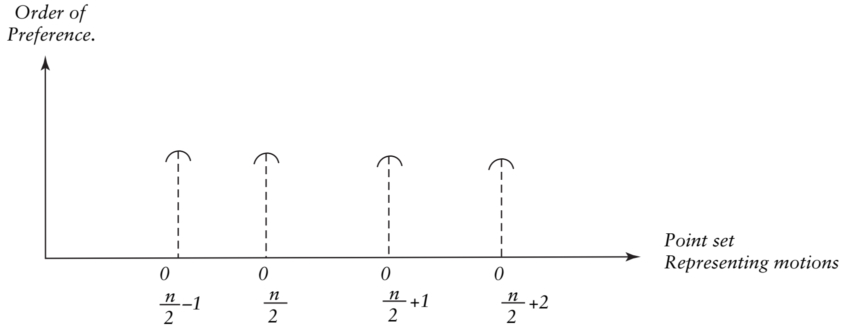


Figure 4

If $O_{n/2}$ is put against any value a_k(where $a_k > O_{n/2}$) – since there are $n/2$ optimums at or below $O_{n/2}$ – the preference curves of at least $n/2$ members will be downsloping from $O_{n/2}$ to a_k, and $O_{n/2}$ will get at least $n/2$ votes against a_k, i.e., will at least tie with a_k. In the event of a tie $O_{n/2}$ will defeat a_k with the aid of the chairman's deciding vote because, by hypothesis, his optimum is situated at or below $O_{n/2}$ and his preference curve must be downsloping from $O_{n/2}$ to a_k.

Thus, when the chairman's optimum is situated at or below $O_{n/2}$, $O_{n/2}$ will be able to get at least a simple majority against any other value which may be proposed.

Similarly when n, the number of members in the committee, is even, and the chairman's optimum is at or above $O_{(n/2)+1}$, it can be shown that $O_{(n/2)+1}$ will be able to get at least a simple majority against every other value.

One cannot leave the theorem of the preceding paragraphs without pointing out its analogy with the central principle of economics – that showing how price is fixed by demand and supply. The theorem we have proved shows that the decision adopted by the committee becomes determinate as soon as the position of one optimum – which we can refer to conveniently enough as the median optimum – is given. No matter in what manner the preference curves or optimums of the other members alter or move about, if it is given that one optimum remains the median optimum, the decision of the committee must remain fixed. The analogy with economic science is that, in the determination of price in a market, price remains unchanged so long as the point of intersection of the demand and supply curves is fixed and given, irrespective of how these curves may alter their shapes above and below that point. Or, in the version of the theory due to B öhm-Bawerk, which brings out the point very clearly, price remains unaltered so long as the "marginal pairs" of buyers and sellers and their price attitudes remain unchanged.

But the analogy exists only between the two theories; there is a marked difference in the materials to which they relate. In the case of market price, when the price of a commodity is being determined, a series of adjustments on the part of the consumers will bring into existence a state of affairs in which this commodity, and all others which they purchase, will have the same significance at the margin for each consumer. This is one of the several grand harmonies running through the material of economic life, a harmony by which no one who understands it can fail to be impressed – and by which the economists of the last generation were perhaps over-impressed. In the material of committee decisions, (or of political phenomena in general) on the other hand, no such grand harmony exists. The possibility of the persistence of disharmony and discord is as striking in the one case as is the certainty of harmony in the other.

In reaching the foregoing conclusions, we assumed that a member of the committee voted on the various motions put forward in accordance with their order on his schedule of preferences. It can be shown that, when a motion exists which would defeat every other if the members voted in this way, it is not open to any member, or any number of members acting in concert, to alter their voting so that some other motion which is more preferred by them can be adopted as the resolution of the committee.[4] It is open to them, however, to vote in such a way that no motion will be able to get a majority over all the others.

If all members voted as we have supposed, the motion adopted by the committee would be that corresponding to the median optimum, O_{med}, say. Let us suppose now that one or more members with optimums above

O_{med} – by voting otherwise than directly in accordance with their schedule of preferences – attempt to give some other value, say a_h, a majority over all the others, where $a_h > O_{med}$.

But when the members vote directly in accordance with their preference scales, those who have a_h higher on their scales than O_{med} would already be supporting a_h against O_{med} and, even so, a_h would be defeated by O_{med}. Before it could defeat O_{med}, a_h would require the support of members whose optimums lie below O_{med}. The only members who – by voting otherwise than in accordance with their scales of preferences – could make a_h the resolution of the committee, are those with optimums below O_{med}, i.e., those against whose interest it is to do so.

It would be possible, of course, for a number of members to vote so that no motion would get a majority over every other. If, for example, a sufficient number of voters with optimums above O_{med} were to vote against O_{med} when it was placed against some value which stood lower on their scales of preferences, O_{med} might be defeated. At most, therefore, a group of voters would have it in their power to prevent any resolution at all being adopted by the committee.

When the members' preference curves are single-peaked, as we suppose, it can be shown that voting between the different motions obeys the transitive property[5] and that if – of any three values a_1, a_2, $a_3 - a_1$ can defeat a_2 in a vote and a_2 can defeat a_3, then, of necessity, a_1 can defeat a_3.

This can be proved by consideration of the orderings of the points a_1, a_2, a_3, in relation to the median optimum. It can be shown that each ordering of the 4 points a_1, a_2, a_3, and the median optimum, either renders the assumption impossible that a_1 defeats a_2 and a_2 defeats a_3 or else satisfies the assumption and, at the same time, necessitates that a_1 defeats a_3.

The transitive property can easily be extended to show that, if a_1 can defeat a_2, a_2 can defeat a_3, ... and a_{l-1} can defeat a_l then a_1 can defeat a_l.

It follows from the transitive property that a_1 can defeat a_3. By hypothesis, a_3 can defeat a_4. Hence a_1 can defeat a_4. Proceeding by successive applications we can see that a_1 can defeat a_l.

In arriving at the above-mentioned results, we assumed that every motion was placed against every other and that in all $m(m-1)/2$ votes were held. We can now remove this assumption and show that the same motion will be adopted by a committee when only $(m-1)$ votes are taken as in the committee practice of real life.

For the case when n is odd, $O_{(n+1)/2}$ is one of the motions put forward and it must enter into the series of votes at some point. When it does, it will defeat the first motion which it meets. It will likewise defeat the second and every other motion which is put against it. That is, $O_{(n+1)/2}$ must enter the voting process at some stage, and, when it does, it will defeat the other motions put against it and become the decision of the committee. The conclusion we reached holds good not only for the imaginary procedure of placing every motion against every other but also for the actual committee procedure of real life. The same is true of the conclusions we reached for the case in which the number of members in the committee was even. In the committee procedure of real life $O_{n/2}$ or $O_{(n/2)+1}$ will be the motion actually adopted.

The assumption, that $m(m-1)/2$ votes were held, enabled us to give a mathematical proof which was both definite and short. But our conclusions are true independently of this assumption.

As an example of the use of this technique, we may suppose that the three directors of a monopolistic firm are fixing the price of their product for a forthcoming period. Let us further assume that neither future sales nor future costs can be calculated with certainty and that there is no possibility of a choice of price being made purely by means of cost accounting. Subjective factors enter, and varying estimates of the future position are formed by the different directors. If, on their different views of the situation, the directors' scales of preference are as shown (Figure 5), the price fixed will be that corresponding to the motion a_3.

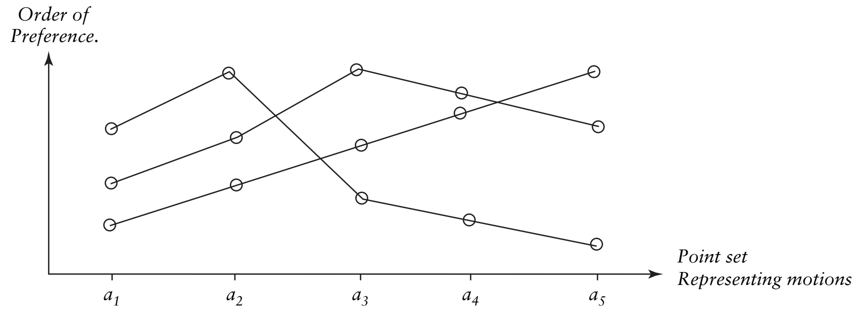


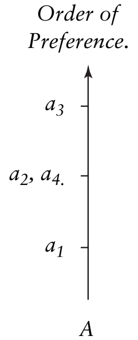
Figure 5

3. When the Members' Preference Curves Are Subject to no Restriction

When the members' preference curves are not of the single-peaked variety, a solution to any problem can always be arrived at arithmetically, provided the number of motions put forward is finite.

To begin with, we return to the assumption that every motion is placed in a vote against every other. The results of the series of votes can be shown very readily by the construction of a voting matrix.[6]

The construction of a matrix is illustrated in Figure 6, which gives the matrix corresponding to the schedule of preferences of the single member A who is voting in a committee in which the four motions $a_1 \ldots a_4$ have been put forward. Along the top row and down the left-hand column are shown the motions $a_1, \ldots a_4$. In each cell of the matrix, we record the individual's vote for one motion when it is placed against another. Looking to the topmost row of figures, when a_1 is placed against a_2, A votes for a_2, and we enter in the cell (a_1, a_2) the figures (0, 1). When a_1 is placed against a_3, he votes for a_3 and, in the cell (a_1, a_3), we enter the figures (0, 1) standing for 0 votes for and 1 against. The other cells are filled in the same way. Since A is indifferent in choice between a_2 and a_4, he will abstain from voting when a_2 is placed against a_4, and the cell (a_2, a_4) will show (0, 0). The figures in the cell (a_4, a_2) will also be (0, 0).



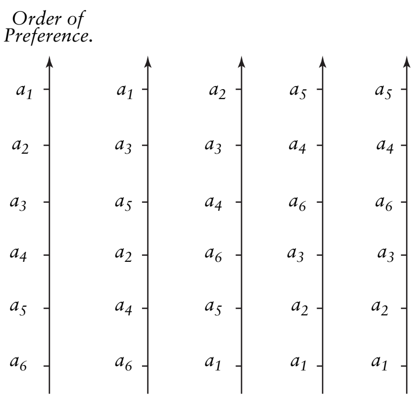
		(Against)			
		a_1	a_2	a_3	a_4
	a_1	0	(0, 1)	(0, 1)	(0, 1)
	a_2	(1, 0)	0	(0, 1)	(0, 0)
(For)	a_3	(1, 0)	(1, 0)	0	(1, 0)
	a_4	(1, 0)	(0, 0)	(0, 1)	0

Figure 6

Along the main diagonal of the matrix, instead of having cells of the usual type, we have simply placed a series of zeros and joined them by a straight line. This is to indicate that the cells $(a_1\ a_1)$, (a_2, a_2), …, which would denote that a_1 was placed against a_1, a_2 against a_2, …, have no meaning. In constructing the matrix in practice, it is usually easiest to enter these zeros along the main diagonal first and join them by a straight line.

Each row to the right of the main diagonal is a reflection in the diagonal of the column immediately beneath, with the figures in the cells reversed. Thus the cell (a_2, a_3) immediately to the right of the diagonal shows (0, 1), the reflection in the diagonal of the figure (1, 0) immediately below the diagonal. The cell (a_2, a_4), two places to the right of the diagonal, is the reflection of the cell (a_4, a_2) two places below the diagonal. The reason for this is that the figures in any cell (a_h, a_k) must be those of the cell (a_k, a_h) on the other side of the diagonal, placed in the reverse order. This feature roughly halves the work of constructing a matrix: we can fill in the figures on one side of the diagonal and then complete the matrix by reflection of these figures in the diagonal.

The construction of an individual matrix would be gratuitous labor since it merely gives, in a clumsier form, information which is shown clearly enough in the member's schedule of preference. When, however, we have a group of individuals voting on a particular topic and the preference schedule of each is known, the matrix for the group presents in very convenient form the information that we need. For instance, for the group of schedules shown in Figure 7, the accompanying matrix has been constructed precisely as described above. Along the main diagonal, as before, we enter zeros and join them by a straight line. In the cell (a_1, a_2) we enter the figure (2, 3) because on the scales of 2 members a_1 stands higher than a_2, and on the scales of the remaining 3 members a_2 stands higher. The other cells are filled in the same way and, as before, the half of the matrix on one side of the diagonal can be obtained by reversal of the frequencies in the corresponding cells on the other side.



	a_1	a_2	a_3	a_4	a_5	a_6
a_1	0	(2, 3)	(2, 3)	(2, 3)	(2, 3)	(2, 3)
a_2	(3, 2)	0	(2, 3)	(3, 2)	(2, 3)	(3, 2)
a_3	(3, 2)	(3, 2)	0	(3, 2)	(3, 2)	(3, 2)
a_4	(3, 2)	(2, 3)	(2, 3)	0	(2, 3)	(5, 0)
a_5	(3, 2)	(3, 2)	(2, 3)	(3, 2)	0	(4, 1)
a_6	(3, 2)	(2, 3)	(2, 3)	(0, 5)	(1, 4)	0

Figure 7

From the group matrix we can read off immediately that – when the motions $a_1 \ldots a_6$ are placed each against every other, as we suppose – a_3 will be able to get a simple majority over each of the other motions put forward. For this committee a_3 would be the resolution adopted.

If a motion exists which would be able to get a simple majority over all the others when the members voted directly in accordance with their schedules of preferences, it would not be open to any member or group of members – by voting in some other fashion – to bring into existence as the resolution of the committee a motion which stood higher on the scales of all of them. Proof of this proposition is almost identical with that of our earlier analysis (see above, p. 26).

If, when $m(m-1)/2$ votes are held, a motion exists which is able to get at least a simple majority over each of the other motions put forward, it can be proved, as before, that, when the members vote directly in accordance with their schedules of preferences, this would be bound to be the motion adopted even though only $(m-1)$ votes had been held.

But when the members' preference curves are not single-peaked, no motion need exist which is able to get at least a simple majority over every other. This can be seen very quickly from the accompanying group of schedules (Figure 8) in which the arrangement of the motions a_1, a_2, a_3, on the members' scales is symmetrical. When a_1 is put forward, it is defeated by a_3, which gets the votes of B and C; when a_2 is put forward, it is defeated by a_1; when a_3 is put forward, it is defeated by a_2. That is, no one of the three motions is able to get a simple majority over the other two.

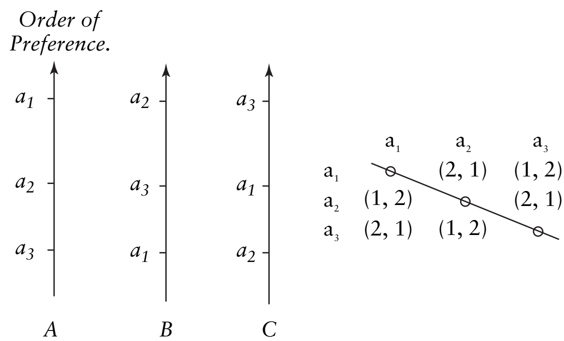


Figure 8

By writing down groups of schedules in which 6 or 7 motions are arranged in various ways and by constructing the group matrices, the reader can quickly satisfy himself that such cases – in which no motion exists which can get a simple majority over each of the others – are by no means exceptional. The greater the number of motions put forward in a committee of any given size, the greater will be the percentage of the total number of possible cases in which there exists no motion which is able to get a simple majority over each of the others.

In this state of affairs, when no one motion can obtain a simple majority over each of the others, the procedure of a committee which holds only $(m - 1)$ votes *will* arrive at the adoption of a particular motion, whereas – if the requirement were that a motion should be able to get a simple majority over every other – no motion would be adopted. The particular motion which is adopted by the committee using the procedure of practice will depend on chance – the chance of particular motions coming earlier or later into the voting process. For Figure 8, if only $(m - 1 = 2)$ votes were taken, that motion, a_1 or a_2 or a_3, would be adopted which was introduced last into the voting process. If, for example, a_1 were first put against a_2, a_2 would be eliminated; and, with the field thus cleared, a_3 would defeat a_1.

If, then, only $(m - 1)$ votes are held and if no motion exists which is able to get a simple majority over every other, we cannot read off directly from the matrix the decision adopted by the committee. But when, in addition to the matrix, we know the order in which the motions are put against one another in a vote, again we can deduce what the decision of the committee must be.

Reference to Figure 8 will show that, when the shapes of the preference curves are subject to no restriction, the transitive property does not necessarily hold good.

4. Conclusion

The technique of this paper applies irrespective of the topic to which the motions may relate. They may refer to price, quantity, or other economic phenomena; they may relate to motions put forward in regard to colonial government, to the structure of a college curriculum, and so on. The theory applies to a decision taken on any topic by means of voting – so far, of course, as the assumptions which are made correspond to reality. And it is possible to widen the assumptions, for example, to include cases of complementary valuation; to make allowance for the time element; and to cover the cases of committees making use of special majorities of any stipulated size. With these extensions in the assumptions there would be a widening of the field of phenomena to which the theory applies.

The theory, indeed, would appear to present the basis for the development of a pure science of politics. This would employ the same theory of relative valuation as economic science. It would employ a different definition of equilibrium. Equilibrium would now be defined in terms of voting, in place of the type of definition employed in economic science. We could move from the one science to the other with the alteration of a single definition. This, in the view of the writer, would be the main function of the theory. It fairly obviously, too, enables some parts of economics – those which relate to decisions taken by groups – to be carried a stage beyond their present development.

Notes

1. The theory will be set out at greater length in a forthcoming book on *The Pure Science of Politics*.
2. Cf. F. H. Knight, *Risk, Uncertainty and Profit*, pp. 68–70.
3. In addition, the motion which is selected by this process is usually put to the meeting for final acceptance or rejection. This is equivalent to putting it against the motion "that there be no change in the existing state of affairs." The step in theory to correspond to this stage in the procedure could easily be supplied.
4. If only $(m - 1)$ votes are held, this conclusion no longer holds.
5. The transitive property is defined in L. S. Stebbing, *A Modern Introduction to Logic*, pp. 112 and 168.
6. I am indebted to Dr. R. A. Newing for suggesting the use of a matrix notation.