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The Uses of Money: Money in the Theory of an Exchange Economy

By Karl Brunner and Allan H. Meltzer*

One of the oldest unresolved problems of monetary theory is to explain the use and holding of money. Resolution of the problem is central to an understanding of the difference between a monetary and a nonmonetary or barter economy, and there have been numerous attempts at resolution. The use of money or the existence of a positive demand has been made to depend on such diverse factors as the anticipation of price or interest rate changes, uncertainty, the embarrassment of default, legal restrictions, or some undefined set of "services"—such as "liquidity"—that money provides. Probably the dominant, current explanation posits the existence of a number of ostensibly separate "motives for holding money." This explanation coexists with vestiges of an earlier preferred explanation that made the use and/or holding of money depend on the "functions" performed by money, such as medium of exchange, store of value and unit of account.

Four recent developments have increased economists' interest in the problem. First, money has remained in use in a number of countries even in periods of accelerating inflation, most recently in Brazil. The continued acceptance of existing monies under conditions of ever-increasing holding cost calls into question the relevance of treating money as an asset that provides little or no return.

Second, proponents of a new international monetary unit have claimed benefits to the world economy that exceed the saving of resources arising from the substitution of paper money for commodity money. Third, recent work (see Boris Pesek and Thomas Saving, Milton Friedman and Anna Schwartz) has revived interest in the properties of a medium of exchange, in the appropriate definition of money, and in the relation between the assets that serve as medium of exchange and the assets that serve as money. Fourth, growth theorists have introduced an asset called "money" into models of economic growth. Some of their studies (Miguel Sidrauski, James Tobin (1965)) conclude that the introduction of money either reduces the community's nonmonetary real wealth and lowers the capital-labor ratio or has no effect at all on society's real wealth.[1] The economy of these growth models becomes monetized only because of a difference between private and social returns from the use or holding of money.

A large literature has developed to explain why individuals and/or societies forego consumption so as to hold an asset that adds nothing to individual wealth or utility,[2] or in some versions, has negative

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[1] Quoting Tobin, ". . . as viewed by the inhabitants of the nation individually, wealth exceeds the tangible capital stock by the size of what we might call the fiduciary issue. This is an illusion, but only one of the many fallacies of composition which are basic to any economy or any society" (1965, p. 656).

[2] This position is very old and was stated clearly by Henry Thornton, one of the best of the early monetary theorists (1965, p. 234) when discussing the holding of Bank of England notes.

"It presents to the holder no hope of future profit from the detention of it. Not only does it bear no

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marginal social product.[3] Aspects of this literature have been surveyed several times (John Gilbert, Arthur Marget, Will Mason, Don Patinkin) and summarized by Keynes (1936, ch. 15) who based three of his four motives for holding money on the principal arguments advanced by his predecessors. The main arguments, used alone or in combination, invoke 1) time, 2) uncertainty, 3) lack of synchronization of receipts and expenditures, 4) costs of transacting, and 5) the existence of non-pecuniary returns. Of these, the most common is one or another of the many versions of the synchronization argument. This argument is valid only in the simple barter economy of the textbooks.[4]

Friedman (1956) and Patinkin recognized that previous analyses yielded scarcely any information about the productivity of money. Building partly on work by Paul Samuelson (1947, pp. 118–20) they identified the productive services of money with a nonobservable, nonpecuniary return to money. But the nature of the services yielded by money and the conditions governing the marginal productivity of money have not been much explored in their work. The most extensive discussion of the coexistence of money, bonds and real capital in portfolios explains the use of money by asserting that, "Intuitively, money seems to be a more efficient carrier of nonpecuniary services . . . than bonds . . ." (Friedman (1969, p. 25)).[5]

The most casual observation suggests that a limited number of assets is used to make or receive payments in all but a few primitive societies. Very similar assets are used in very different places. The standard theory of exchange, or price, however, provides no hint as to why dominant mediums of exchange emerge.[6] Any asset or combination of assets is equally likely to be selected as a medium of exchange,

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interest, but it offers no substitute for interest; . . . the quantity held by each person is only that which the amount of payments to be effected by it renders, in his opinion, necessary."

However, in Chapter III, Thornton recognizes that resource costs are reduced when paper money replaces gold and silver and (pp. 91 ff.) he criticizes Smith, whom he credits with this argument, for failing to see that resources are also saved when bills of exchange replace notes.

[3] Recent exceptions are David Cass and Menahem Yaari and Jürg Niehans (1969) who show that social efficiency rises if the community agrees to use money. Cass and Yaari discuss the use of money as a medium of exchange (or in their terminology intermediation) and show that money is productive under the circumstances of their model. However, they restrict barter exchange to a single "double coincidence" transaction and assume that each trader knows, or can obtain at zero cost, information about the exchange ratios or prices at which all other transactors are willing to trade. Our earliest attempt to relate the productive contribution of money to the saving of costs of information was published in 1964 (pp. 258–62).

[4] It is easy to see why "lack of synchronization" does not imply that money is used and held. Consider an economy that has neither a medium of exchange nor money. If there are no costs of acquiring information, differences in the timing of receipts and payments are adjusted by issuing verbal promises in exchange for goods and, later, delivering goods. More generally, in a barter-credit economy, commitments or promises to pay "bridge the gap between receipts and payments." Gilbert discusses several versions of the "synchronization" argument and provides references to the more prominent writers and to those who stated their arguments most clearly. We have found no one who discusses barter-credit. A simple statement of the synchro-

nization argument is due to Joseph Schumpeter (p. 547) who argued that if people are paid on Saturday and stores are closed until Monday, firms hold money from Monday to Saturday and households from Saturday to Monday because "the institutional arrangement so wills it." For a comparison with current notions of the Hicksian week, see Patinkin (pp. 13–14).

[5] Friedman does not explain why money is superior to bonds as a carrier of nonpecuniary services. In his analysis, the only uncertainty is about the timing of receipts. We find nothing to prevent individuals in his model from holding interest yielding bonds and using verbal promises to pay when expenditures temporarily exceed receipts.

[6] Although specialization has long been invoked to explain exchange, economic theory explains specialization in exchange as the result of a corner solution. For much the same reason that theory offers no explanation of the concentration of the medium of exchange function in a narrow range of assets, theory provides no explanation of the specialization of traders in other inventories and types of exchange.

and the allocation of resources is not affected by the choice. In the next two sections we reconsider the services rendered by money, extend the theory of choice to include the choice of the assets individuals use as mediums of exchange and then use diagrams to illustrate the main points of our analysis.

Our argument requires the use of two postulates that are not part of conventional exchange theory. They are:

1) For each transactor in an exchange economy, the marginal cost of acquiring information, measured in units of consumption sacrificed, depends on the goods or services selected.

2) The marginal cost of acquiring information about the properties of any asset does not vary randomly within a social group and declines as the frequency with which the group uses a particular asset increases.

The two postulates are necessary and sufficient conditions for the use of a medium of exchange. They emphasize the point that will concern us throughout—that it is the uneven distribution of information, and not the existence of an undifferentiated uncertainty, that induces individuals to search for, and social groups to accept, alternatives to barter.

Social choice of an asset used as money is separate from, though not entirely independent of, individual decisions to hold money. We believe it is useful to separate the analysis of the choice of the asset used as money from the analysis of the optimal amount of money to produce and from the individual's choice of a desired money balance. Our interest here is in the individual and social choice of the assets used as money, the services money provides to individuals and societies, the relation of these services to the choice of a monetary

unit, and some implications of these decisions.

I. The Services of Money to Individual Transactors

One of the main productive activities of a household in a developed market economy is the acquisition of the goods and services consumed by the household. The provision of these goods and services requires not only the sale of income-yielding productive services but the use of resources to acquire information, arrange payments, and schedule purchases. Shopping, budgeting, and planning expenditures are productive tasks that both absorb resources and yield benefits to the skilled or knowledgeable purchasers or sellers who make advantageous exchanges. The use of a medium of exchange permits the household to economize on the amount of resources absorbed by these activities and to enjoy a larger and more diversified basket of goods and more leisure.

Potential transactors possess very incomplete information about the location and identity of other transactors, about the quality of the goods offered or demanded, or about the range of prices at which exchanges can be made. Uncertainty about quality characteristics is a main reason for the dispersion of prices of any commodity, and uneven distribution of information about the qualities of commodities increases the dispersion of prices both within the community and between bid and ask quotations.[7] Transactors can acquire information most readily and at

[7] Relatively low cost of acquiring information and a smaller dispersion of prices is one of the many meanings of the term liquidity, a property often attributed to money. Reducing the cost of acquiring information raises liquidity. We conjecture from this line of reasoning that the announcement that shares are to be moved from the over-the-counter market to an organized exchange such as the New York Stock Exchange raises the average price of the shares. The price increase is a measure of the value to the stockholders of the reduced dispersion of bid and ask prices.

lowest cost about commodities that are most widely used and best known, so the prices of these commodities have the least dispersion. When the qualities of a commodity are less certain, acquisition of information requires greater use of valuable resources, and dispersion increases. If risk aversion is prevalent, uncertainty about the properties or quality of an asset further lowers the average demand price of the asset.

The use of money reduces uncertainty and expands trade in a number of ways. One way is by providing a unit of account, or standard in which prices are expressed. If there are N commodities in a barter economy and the unit of account is randomly selected, anyone wishing to organize or participate regularly in a market must know (or be able to obtain) each of the $N(N-1)/2$ independent, exchange ratios in the barter exchange matrix. Social choice of a unit of account reduces the matrix to an Nx1 vector of exchange ratios expressed in the unit of account. The cost of acquiring, processing and storing information falls. The gain from the use of a unit of account is analogous to the gain that comes from introducing a common unit of measure such as height, weight or temperature.

The gain from using a unit of account is limited by the size of the market, but the size of the market expands after agreement on a unit of account because resources devoted to trade receive higher net returns. As information about the unit spreads and the market expands, additional private and social benefits result from the development of the market system. Additional exchange ratios are expressed in the unit; it becomes efficient to use the unit where other units were used previously.[8]

[8] An example is the spread of the decimal and dollar system or the metric system to countries where the pound or foot has long served as the standard. The

A second and considerably more important way in which the use of money reduces uncertainty and contributes to the expansion of trade and the market system is through service as the medium of exchange. The frequency with which the same unit serves both as medium of exchange and as unit of account suggests that it is efficient to perform both functions with a single unit, but the functions are distinct and require separate analysis.

To analyze the medium of exchange function, we consider a transactor who has an initial endowment of resources including his own labor time and some information about exchange ratios and qualities of commodities. He has several alternative ways of transforming his initial endowment into a preferred bundle. As in standard price theory, he can use his endowment for production, consumption, or exchange. In addition, he has two options that are neglected in traditional price theory. 1) He can use resources to increase his information about the qualities of goods and opportunities for exchange. If the transactor uses resources in this way, we say that he invests in information. 2) He can engage in indirect or roundabout methods of exchange, accepting goods with low marginal cost of acquiring information, transferring and storing, then exchanging these goods for others until he obtains an optimal bundle. We describe the resources allocated in this way as the (real) costs of transacting or exchanging. The resources used in this way are, of course, distinct from the resources exchanged.

Under conditions of uncertainty[9] about

Euro-dollar market suggests the way in which a unit of account begins to acquire the properties of a medium of exchange and other attributes of money through the efforts of private traders. Conversely, where trade between countries is small and infrequent, it is not profitable to devote resources to establishing a unit of account or medium of exchange and barter-credit predominates.

[9] Note that the uncertainty in our account differs

the quality of goods offered in exchange and about prevailing market opportunities, the costs of acquiring information and exchanging are neither zero nor identical for every good or service. Our first postulate, introduced above, recognizes that the marginal cost of the resources the transactor uses to acquire information or to carry out transactions is the amount of consumption or endowment sacrificed. This marginal cost depends on the goods or services he selects (or about which he chooses to acquire information) and is subject to substantial variance.

By choosing a sequence of transactions —a transaction chain—involving assets with low marginal cost of information, the transactor can lower the marginal cost of exchanging. He incurs transfer and carrying costs and uses existing information about the qualities of particular goods instead of investing resources to acquire information about other goods or other trading arrangements. However, transfer costs increase with the length of the transaction chain, forcing the rational transactor to compare the marginal cost of acquiring information to the marginal cost of rearranging the transaction chain and to the benefits obtained from these and alternative uses of resources.

The formal statement of our analysis involves the maximization of utility subject to a budget constraint under conditions of uncertainty about market opportunities and the qualities of goods. An individual seeks to obtain the optimum combination of goods and services by investing in information and engaging in exchange. Numerous sequences of transactions are open to him. His problem is to

find the optimal sequence of transactions and the optimal investment in information while choosing an optimal bundle of goods or consumption plan. Production of the standard type is disregarded.

Let the individual's utility function be

$$(1)\ U = U(\epsilon, v; Q)\ ; U_1 > 0; U_2 < 0; U_3 > 0$$

where ϵ is the expected bundle of goods the transactor obtains by allocating resources to the various options, v is the variability of the bundle associated with the uncertainty about market conditions, and Q is the information available to the transactor about the qualities or properties of goods. We regard Q as a partition of the state space summarizing the full range of qualities. A skilled trader or market professional has valuable information about the qualities of goods, the location of other traders and their tastes and preferences. His market opportunities differ from those of an unskilled trader or infrequent participant in the market. The more certain a transactor is about the qualities (or properties) of goods, and the more he is able to discern differences in performance or other quality characteristics, the higher his utility. The more information the transactor has about market conditions and the characteristics of goods, the lower the variability of the bundle he obtains and the higher his utility.

The budget constraint is

$$(2)\qquad R_0 = C_0 + X_0 + I_0 + S_0$$

The transactor can allocate his resources, R,[10] to reservation demand, C, to exchange,

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from the types of uncertainty emphasized in past discussions of money. The latter include uncertainty about 1) the price level, 2) the timing of receipts, and 3) interest rates (see Gilbert). For other recent attempts to relate search and costs of acquiring information to resource allocation, see Herbert Simon and George Stigler.

[10] The variables R_0, C_0, X_0, I_0, and S_0 denote diagonal nonnegative matrices. Each of the symbols is defined in the text, but some discussion of the difference between allocations to S_0 and I_0 may clarify the analysis that follows. No "generally accepted" medium of exchange is imposed on the economy discussed in the text. A transactor can forego consumption and use resources to sample the exchange ratios at which other transactors (including potential transactors) are willing to offer goods of the quality he desires for the goods he presently holds. We use the symbol I_0 to describe the resources

X, to acquire information about market opportunities, I, or to execute transactions, S. The conditions for advantageous exchange depend on the information available to the transactor. By using resources to acquire information, he lowers the amount of resources remaining available for reservation demand and trade. However, the new information reduces uncertainty about market conditions and exchange opportunities and lowers the variability of the return from trading. Optimal information is achieved when the marginal utility of resources withdrawn from consumption (reservation demand) equals the marginal utility of an improved trading position and reduced uncertainty.

Transactors' uncertainty with respect to market opportunities is conditioned by a distribution of exchange ratios governing the social exchange process.

$$(3) \qquad \pi = \pi(E \mid P)$$

The term E is a matrix of exchange ratios whose generic elements e_{ij} denote the jth unit obtained per unit of i, and P summarizes the information about market opportunities and exchange ratios available to transactors.[11] Since bid and ask prices are generally not the same in the presence of costs of acquiring and maintaining information and transferring goods, the matrix is not anti-symmetric, and $e_{ij}e_{ji} \neq 1$.

At any time the transactor knows a maximal E, denoted E^*, that describes the best market opportunities available to him if he relies on the information carried over from the past. We treat E^* as a random variable conditional on the transactors' information about market opportunities, P. There is a unique expected value of E^* corresponding to each state of knowledge about market opportunities. A transactor can improve his information about market opportunities by using a portion of his endowment to acquire additional information, i.e., by investment in information. Although a larger sample does not necessarily increase the maximal sample value, under weak constraints on π, a larger sample raises the expected value of the sample maximum and reduces the variance of the exchange ratios at which the transactor trades. We postulate that the mean and variance of the distribution, π, depend on the amount invested in information and the transaction arrangements, T and are denoted

$$\epsilon(E^* \mid I, T) \text{ and } v(E^* \mid I, T) \text{ with}$$

$$\frac{\partial \epsilon}{\partial I} > 0; \quad \frac{\partial v}{\partial I} < 0; \quad \frac{\partial^2 \epsilon}{\partial I^2} < 0; \quad \frac{\partial^2 v}{\partial I^2} > 0$$

Once a transactor allocates I_0 to improve

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allocated to sampling or search of this kind. The goods the transactor holds may include "intermediate" goods, goods acquired in previous transactions for use in later transactions. Alternatively, a transactor can seek to improve his knowledge of transaction arrangements by 1) learning about the properties of goods that other transactors desire to acquire either for consumption or as intermediate goods, or 2) by learning about the relative costs of storing or transporting goods, or 3) by learning about the costs of completing exchanges using different sets of intermediate goods. We use the symbol S_0 to summarize the resources used to acquire information about transaction arrangements and to carry out exchanges. Resources must be used for similar purposes even if a few assets are generally accepted as means of payment. "Shopping" is a common means of acquiring information about price-quality combinations. Resources must also be used to learn about the advantages of immediate payment over deferred payment, the costs and benefits of using coin or currency rather than checks in general and in specific exchanges, etc.

[11] P is a vector whose coordinates summarize market information. The coordinates depend on the individual's

and the community's information. Increased investment in information refines the available information, and reduces the generalized variance. Technological change reduces information and increases the variance of π by reducing information about the qualities of goods. The coordinates of P are the partitions, Q, of the state spaces—characterizing each individual's information. An example helps to describe Q and P. An individual's utility increases if, with unchanged market opportunities, he acquires information that permits him to develop his tastes. An ability to distinguish between Botticelli's and "paintings" increases Q but does not change other elements of P. If everyone acquires the same discriminating tastes, all Q's change and therefore P changes.

his information, only $R_0 - I_0$ of the initial endowment remains available for consumption and exchange. Exchange operations involve allocation to two distinct components, X and S, with X the bundle offered in exchange and S the resources used to produce the exchange. An exchange matrix T[12] characterizes the transformation via exchange transactions of a bundle X_0 into a new bundle, Y_1. The off-diagonal elements of the T matrix are the proportions of the ith good exchanged for the jth good. The resource cost (S) of executing a set of transactions T depends on T; $S = S(T)$. A market exchange is a transformation of the inputs X, $S(T)$ to an output $\epsilon(Y_1)$. The selection of the amounts offered in exchange, X, and the division of resources between C, X, and T depends, of course, on the information summarized by $\epsilon(E^*)$ and $v(E^*)$ and therefore on I and T.

The total volume of physical resources available to the transactor after the market transformation is expressed by the diagonal matrix $\epsilon(R_1)$[13]

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[12] T is a nonnegative hollow matrix whose element t_{ij} describes the portion of the ith good allotted in exchange for the jth good and whose diagonal elements, $t_{ii} = 0$. Also, $1 \geq t_{ij} \geq 0$ for $i \neq j$, and $\sum_j t_{ij} = 1$.

[13] The trade matrix $T'X$ can be described as a sum of matrices X^j, i.e., $T'X = \sum_j X^j$. This decomposition is uniquely determined by the form of the matrix $T'X$. The generic matrix X^j consists of zero row vectors with the exception of the jth row vector which is identical to the jth row vector of $T'X$. The index j ranges over all the rows of $T'X$. Under the circumstances specified, the constituent matrices X^j are functions of the exchange X and the transaction T, i.e., $X^j = X^j(T, X)$. Once the decomposition of $T'X$ has been defined a matrix $\epsilon(E_j^*)$ is associated with X^j. The matrix $\epsilon(E_j^*)$ consists of zero column vectors with the exception of the jth column which is identical to the jth column of the matrix $\epsilon(E^*)$. The product $X^j \cdot \epsilon(E_j^*)$ denotes a diagonal matrix with zero diagonal elements except in the jth row. This diagonal matrix describes the acquisition of the jth good by means of an exchange X and a transaction arrangement T. The sum $\sum X^j \epsilon(E_j^*)$ is a diagonal matrix describing the result of the market transformation defined by the pair (X, T) and, since E^* is a random variable, the expected value of this sum is $\sum X^j \epsilon(E_j^*)$. The result is denoted by $\epsilon(Y_1)$, i.e.,

$$\epsilon(Y_1) = \sum_j X_0^j(T_0, X_0) \cdot \epsilon[E_{j0}^*(I_0)]$$

$$(4) \qquad \epsilon(R_1) = \epsilon(Y_1) + C_0$$

and consists of the reservation demand C_0 plus the bundle $\epsilon(Y_1)$ the transactor expects to obtain via market exchange. Replacing X_0 in the description of $\epsilon(Y_1)$ (from fn. 13) we can express $\epsilon(Y_1)$ as a function

$$(5) \quad \begin{aligned} \epsilon(Y_1) &= Y_1[R_0 - C_0 - I_0 - S(T_0), I_0, T_0; \pi] \\ &= \sum_j X^j[R_0 - C_0 - I_0 - S(T_0), T_0] \\ &\quad \cdot \epsilon(E_j^*(I_0, T)) \end{aligned}$$

The function $\epsilon(Y_1)$ depends on the initial endowment R_0, the reservation demand C_0, the transaction arrangement T_0, the investment in information I_0, the distribution of exchange ratios and, therefore, on E^* via π. The mean, ϵ, and variance, v, of the transactor's utility function can now be expressed as the functions

$$(6) \quad \epsilon = \epsilon[Y_1 \mid R_0 - C_0 - I_0 - S(T_0), T_0, I_0] + C_0$$

$$(7) \quad v = v[Y_1 \mid R_0 - C_0 - I_0 - S(T_0), T_0, I_0]$$

A transactor can change his utility by three distinct types of allocation, represented in our analysis by C_0, I_0, and T_0. The transactor trades whenever the utility associated with $\epsilon = C_0 = R_0$ and $v = 0$ is less than the utility associated with some $C_0 < R_0$.

Taking derivatives of ϵ and v with respect to I_0 and solving the resulting equations shows that the optimal investment in information depends on the effects on mean, ϵ, and variance, v. Increased investment (I_0) either lowers reservation demand (C_0) or the resources devoted to exchange (S_0, X_0). With S_0 and X_0 held constant, an increase in I_0 lowers the variance $v(Y_1)$ and has two, opposing effects on the mean $\epsilon(R_1)$. Increased I_0 raises, and the associated reduction in reservation demand lowers $\epsilon(R_1)$. The investment in information raises the expected maximal ratio, expressed by $\epsilon(E^*)$. The rise in $\epsilon(E^*)$

changes the optimal amount offered in exchange, X_0, and this in turn changes C_0 and T_0 and the transactor's utility.

The transactor acquires information until the gain in utility from an increment ΔI_0 equals the loss resulting from the decrease in resources available for reservation demand and exchange imposed by the constraint

$$(8) \quad \Delta I_0 = -(\Delta C_0 + \Delta S_0 + \Delta X_0)$$

The second derivatives of ϵ and v with respect to I_0 assure that the marginal utility of investment in information decreases as investment increases. Moreover, the continued decrease of C_0 and X_0 generates an increasing loss of welfare, and the continued decrease of S_0 imposes increasingly severe constraints on the choice of transaction arrangements and (given $\partial^2 U / \partial \epsilon^2$ to form a negative definite Hessian matrix) a nondecreasing loss of utility. The allocation of resources to acquire information necessarily reaches an optimum.

A similar argument applies to the choice of transaction arrangement or reservation demand. Utility increases as more resources are allocated to market transformations, T, and the wider search for transaction arrangements, S_0. The larger S_0 and $S(T_0)$, the larger the admissible set of transaction arrangements and the greater is a transactor's opportunity to exploit differences in the marginal cost of information, associated with various goods. For example, by using indirect or roundabout

exchange arrangements that reduce uncertainty, the transactor can lower v by allocating resources to S_0 while keeping I_0 constant. The increase in S_0 (holding I_0 constant) reduces C_0 and X_0, and therefore lowers ϵ. But the allocation to S_0 permits the transactor to choose from a wider range of transaction arrangements, raising ϵ. The optimal allocation to S is reached when the gain in utility from the smaller variance and more advantageous transaction arrangements just equals the loss in utility due to a smaller C_0 and X_0.

By neglecting costs of search and exchange, the traditional analysis of barter exchange sets S and I to zero and thus omits important aspects of resource allocation and choice under uncertainty. In the usual analysis, a transactor either engages in a single, double-coincidence transaction, $C_1 = R_1$, or does not trade, $C_0 = R_0$. In our analysis, a transactor is not forced to choose between autarchy and a single, double-coincidence transaction. He can engage in a sequence of transactions, expressed by a sequence of matrices or transaction chain $[T_0, T_1, ..., T_n]$ chosen so as to exploit differences in the marginal cost of acquiring information and exchanging. The length of the transaction chain—the optimal number of exchanges—is determined jointly with the choice of commodity bundle $C_n = R_n$ that maximizes utility. The bundle, R_n, is defined recursively in equation (9); for convenience the expected value operator is omitted in

$$
\begin{aligned}
(9) \quad R_n &= Y_n + C_{n-1} \\
&= Y_n + R_{n-1} - [X_{n-1} + I_{n-1} + S_{n-1}(T_{n-1})] \\
&= Y_n + Y_{n-1} + C_{n-2} - [X_{n-1} + I_{n-1} + S_{n-1}(T_{n-1})] \\
&= Y_n + Y_{n-1} + R_{n-2} - [X_{n-1} + I_{n-1} + S_{n-1}(T_{n-1})] - [X_{n-2} + I_{n-2} + S_{n-2}(T_{n-2})]
\end{aligned}
$$

$$
R_n = \sum_{i=1}^{n} Y_i + R_0 - \sum_{i=1}^{n-1} [X_i + I_i + S_i(T_i)]
$$

the statement of the definition. Moreover, every Y_i is a function of T_{i-1}, X_{i-1} and the sequence of investments in information $(I_0. \ldots I_{i-1})$ given by

$$(10) \qquad \epsilon(Y_i) = \sum X_{i-1}^j(T_{i-1}, X_{i-1})$$
$$\cdot [E_{j,i-1}^*(I_0, I_1, \ldots, I_{i-1})]$$

as shown in footnote 13. Both ϵ_n and v_n are, therefore, functions of the transaction chain $(T_0, \ldots, T_{n-1})$, the sequence of exchanges $(X_0, \ldots, X_{n-1})$ and the sequence of investments in information $(I_0, \ldots, I_{n-1})$ according to the definition

$$(11) \qquad \epsilon_n = \epsilon(R_n); \qquad v_n = v(R_n)$$

Where information about the qualities and exchange ratios varies with the commodities, there are corresponding differences in the marginal cost of acquiring information, so transactors are not indifferent about the commodities they accept for use in subsequent exchanges. By choosing transaction arrangements T_i that induce the transactor to include in his transaction chain commodities or assets whose qualities are better known and less uncertain, the transactor can reduce the variance $v(Y_i)$ and the resources I_i that he uses to acquire information about exchange opportunities. At each step in the sequence, more resources become available for exchange and for reservation demand, so utility increases. By careful choice of the elements in the triplet C, T, I, a transactor is able to obtain the commodity bundle with mean and variance ϵ_n and v_n that maximizes utility.

Our first postulate, introduced earlier, makes the marginal cost of acquiring information depend on the good or service selected. Unless transactors ignore the information they acquire, the postulate implies that intermediate transaction matrices in the chains they select (i.e., matrices up to the last) do not exhibit a random distribution of nonzero elements. In the first matrix of the chain, clusters of nonzero elements appear most frequently in the columns identified with assets that reduce uncertainty, provide more reliable information and lower marginal information costs. In the last matrix of the chains, the nonzero elements tend to be in the rows associated with commodities that have these same properties.

The distribution of costs of acquiring and maintaining information gives the transactor an opportunity to reduce the resource cost of acquiring his preferred commodity bundle by substituting knowledge about transaction arrangements for investment in information about market conditions and the qualities of goods offered in exchange. Cost reduction occurs in two ways. First, detailed information about market conditions such as location and identification of transactors, the quality and type of commodity bundles they hold and the exchange ratios at which they trade probably decays more rapidly than knowledge about optimal transaction chains. Second as the use of an asset in exchange increases, the transactor learns more about the asset's properties. With growing use of particular transaction chains and improved knowledge of the properties of the assets exchanged, uncertainty and the variances and covariances in the general covariance matrix describing the overall density $\pi(E)$ decline.

The marginal productivity of transaction arrangements can be expressed in terms of the smaller investment in information required to keep the variance $v(R_n)$ unchanged. By choosing transaction arrangements that reduce the cost of acquiring information and exchanging, a transactor can allocate additional resources to exchange or reservation demand without raising the variance of the bundle he obtains. Indeed, if this were not so, direct exchange would predominate.

The usual discussion of a barter econ-

omy neglects the marginal productivity of transaction arrangements by limiting exchange to a single, double coincidence transaction. Once the transactor is given a choice of I and S, both the length of his optimal chain and the assets that enter depend on differences in the productivity and cost associated with alternative chains. Thus, our first postulate assures that the choice of transaction chain and of the assets used in exchange is neither random nor determined solely by the exchange— i.e., by the initial and terminal bundles.

To show that individuals' optimizing behavior leads to the social choice of a small number of medium of exchange assets, we use our second postulate. This postulate states 1) that the marginal cost of acquiring information does not vary randomly within a social group and 2) that the marginal cost declines as the frequency with which an asset is used increases. Transactors can acquire information about a particular subset of the available assets at comparatively low marginal cost once these assets are used frequently.[14]

Our second postulate implies that the transaction chains of the numerous participants in the market process exhibit some common properties. These properties can be expressed as a clustering of the non-zero elements in the columns of the first matrix and in the rows of the last matrix of the chain. The repetitive use of a relatively small number of transaction chains by the members of the social group further reduces the marginal cost to each transactor of acquiring information about the assets most frequently used. The lower costs of acquiring information and transacting induce further clustering and the

convergence of individuals' chains toward a common pattern.

There are many stages of development between double-coincidence barter and a fully monetary economy. At some stage, a few assets are used with dominant frequency in transactions. Once this stage is reached, a majority of the transaction chains consists of two matrices, the first containing a few (nonzero) columns and the second a few (nonzero) rows. Thus, money as a medium of exchange, as a transaction dominating asset, results from the opportunities offered by the distribution of incomplete information and the search by potential transactors to develop transaction chains that save resources.

The analysis also explains the emergence of specialized trading functions such as brokerage and other market arrangements. They develop from the conditions that shape the (social) convergence to a dominant medium of exchange. Where information is complete and both information and readjustment are costless, specialization of trading functions yields no economic advantages and has no utility. Where information and readjustment are not free, the situation changes. Specialized services lower the costs of acquiring information and trading by providing more complete information about the range of qualities and market conditions. With a smaller investment of resources a transactor acquires the same information, and more resources can be used for reservation demand or trade.

II. A Diagrammatic Exposition

By simplifying the argument and omitting several aspects, we can use diagrams to demonstrate some main points of our analysis. We start with a nonmonetary economy in which there are two goods. One is the initial endowment of a transactor, R_1; the other is the good, Y_2, acquired by trading in the market. We disregard the re-

[14] Differences in technological properties of commodities contribute to differences in the costs of acquiring information and transferring. Monetary literature has long recognized that portability, divisibility, durability, etc. are useful properties for assets used as mediums of exchange.
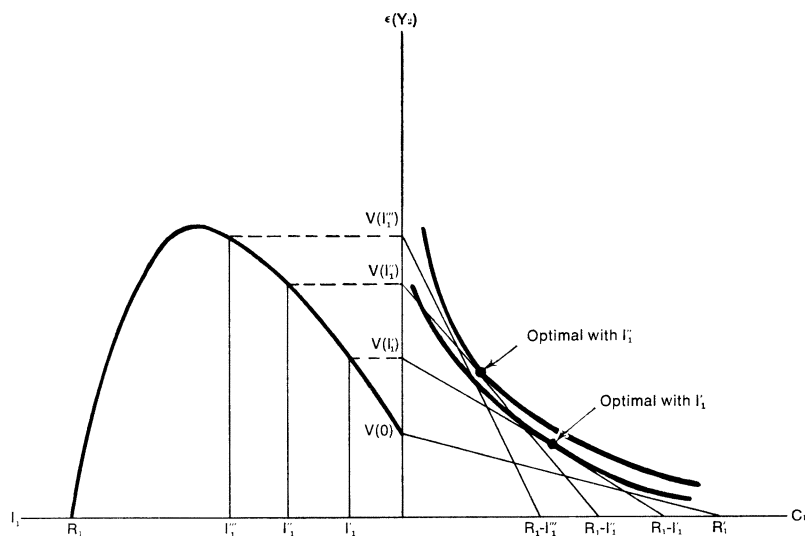
FIGURE 1

sources used in the trading process and allocate the initial endowment to reservation demand, C, to exchange in the market, X, and to the acquisition of information about market opportunities and exchange ratios, I.

$$(12) \qquad R_1 = C_1 + X_1 + I_1$$

The exchange ratio between R_1 (or C_1) and Y_2, denoted e_{12}, states the units of Y_2 obtainable per unit of R_1. Market opportunities are not known with certainty. By investing resources I_1 to acquire information, a transactor obtains a sample of potential exchange ratios. The sample has a maximal value e_{12}^*, the latter a random variable dependent on the underlying distribution. As above, we make the expectation and variance of e_{12}^* a function of investment, I.[15] Market exchange is a productive process that transforms the inputs (X_1, I_1) into an output $Y_2 = X_1 e_{12}^*(I_1)$. The value of the bundle after an exchange is $R_2 = Y_2 + C_1 e_{12}^*$.

The transactor seeks to maximize the

[15] To simplify notation we write e_{12}^* for the expectation of the maximal sample value, and where there is no change of confusion, we use Y_2 instead of $\epsilon(Y_2)$.

utility of the bundle available for consumption. The arguments of the utility function are C_1 and the expected value and variance of Y_2. The indifference curves of Figure 1 are drawn for a given value of the variance $v(Y_2)$. A change in variance moves the entire surface, or what is the same thing, requires a relabelling of the indifference curves to associate a higher level of utility with an unchanged combination of Y_2 and C_1, if the variance of Y_2 decreases and a lower level if utility of the variance increases.

Unlike the standard analysis, neither the slope nor the position of the budget line is independent of the transactor's allocation of resources. Increased investment in information raises the expectation of the maximal sample value e_{12}^* and the slope of the budget line, $-e_{12}^*$. With increased investment, the budget line becomes steeper, and the transactor obtains more Y per unit of C.

The relation between investment in information, the value of information, and the position of the budget line is developed in the left panel of Figure 1. On the horizontal axis, we measure the resources used

to acquire information on a scale running from zero at the origin to R_1, the transactor's endowment. By sacrificing a unit of real consumption and investing in information, a transactor enlarges his sample of exchange ratios and, in general, raises the value of e^*. The marginal cost of information is the consumption sacrificed. The value of the information is the gain or loss in $\epsilon(Y_2)$ obtained from the exchange of a smaller (R_1-I_1) endowment of real resources at a higher exchange ratio. The curve in this panel shows the value of information $V(I_1)$ as a function of the amount invested. The intercept of the curve on the Y_2 axis is the value of the information, in $\epsilon(Y_2)$ units, that the transactor carries over from previous trades. If he relies entirely on past experience, the best he can do is trade at the ratio $e_{12}^*(0)$; the maximum Y_2 he can obtain by exchanging all his endowment at this ratio is shown as $V(0)$. By investing in information, at a cost equal to I_1, he increases his information about market conditions. The change in the value of information is shown by the projections of the curve in the left panel on the Y_2 axis. If he invests an amount I_1', he has available for exchange only R_1-I_1', but he can exchange at the higher ratio $e_{12}^*(I_1')$ and obtain a maximum value of $Y_2 = V(I_1')$. The distance $V(I_1') - V(0)$ on the Y_2 axis is the value of the information obtained with an investment I_1'.[16]

For any investment I_1, the value of information is given by

$$(13) \qquad V(I_1) = (R_1 - I_1)e_{12}^*(I_1)$$

Inspection makes clear that if the entire endowment is used to acquire information, nothing is left for trade or consumption, so $V(R_1) = 0$. To find the value of I_1 that maximizes $V(I_1)$, we take the derivative

$$(14) \quad \frac{\partial V}{\partial I_1} = e_{12}^*(I_1)\left[\frac{R_1 - I_1}{I_1}\eta(e_{12}^*, I_1) - 1\right]$$

where $\eta(e_{12}^*, I_1)$ is the elasticity of e_{12}^* with respect to I_1. It follows from diminishing marginal productivity of investment that the increase in the expected sample value falls as investment increases, and the elasticity falls as I_1 increases. Moreover, as I_1 approaches R_1, $(R_1-I_1)/I_1$ approaches zero. The value of information V reaches a maximum when the share of the endowment invested in information is

$$(15) \qquad \frac{I_1}{R_1} = \frac{\eta}{1 + \eta} < 1$$

If the elasticity is low, investment in information raises the expected sample value and improves market opportunities very little. The maximum V then occurs at a very low value of I_1.

The optimal amount of resources invested in information is determined jointly with the optimal reservation demand and allocation to trade by maximizing utility. Differentiating

$$(16) \qquad \begin{array}{c} U = U[\epsilon(Y_2), C_1, v(Y_2)] \\ U_1, U_2 > 0; \qquad U_3 < 0 \end{array}$$

with respect to C_1 and I_1, we obtain the necessary conditions for a maximum,

$$(17) \qquad U_1 e^*(I_1) = U_2$$

$$(18) \qquad \frac{\partial V}{\partial I} = C_1 \frac{\partial e_{12}^*}{\partial I} + \frac{-U_3}{U_1}\frac{\partial v}{\partial I}$$

[16] The cost of acquiring information, I_1, is clearly visible in two places on Figure 1. One is the horizontal axis on the left panel. The other is the difference between R_1 and the intercept of any budget constraint on the C_1 axis, $R_1 - I_1$. The difference defines the maximum C_1 associated with any given I_1. As the figure is drawn, each increment to investment $(I_1'' - I_1', \ I_1' - 0)$ and each distance between intercepts of the budget line on the C_1 axis is the same size. The decreasing incremental value of information is shown by the decrease in the distance between the successive intercepts of the budget line on the Y_2 axis. The relations of I to $V(I)$ depends on the vector, P, discussed in fn. 11.

Solving these two equations for C_1 and I_1 and substituting the solutions in the budget constraint $R_1 = C_1 + I_1 + X_1$ gives the optimal allocation to reservation demand, investment in information and trade.

The first of the two necessary conditions is the familiar condition for tangency between the indifference curve and the budget line obtained in the standard analysis of consumer choice. The second equation goes beyond the standard analysis and states the condition for the optimal *position* of the budget line. The equation has two components. The first is the increment to the value of reservation demand, measured in units of Y_2, obtained from investment in information; the second is the marginal gain or loss from reducing the variance. The position of the budget line is optimal when the sum of these components equals the marginal value of information, $\partial V / \partial I_1$, measured in units of Y_2.[17]

The conditions for an optimum are shown in Figure 1. The slopes of the budget lines show the exchange ratio e_{12} for different values of I. The position of the budget lines is determined by the amount invested and the value of the information obtained. The relation between the two is given by the curve in the left panel. We disregard, for the moment, the gain or loss in utility resulting from changes in the variance. As the figure is drawn, it is suboptimal to restrict investment in information to I_1'. The transactor can reach a higher indifference curve by increasing investment to I_1''. A further increase to I_1''' lowers utility. In the absence of any effect of variance, the transacter

would choose an investment I_1'' and trade at the corresponding exchange ratio. The investment I_1'' and the associated allocations to C_1 and Y_2 represent an optimum for any transactor unconcerned by the variance, i.e., for any risk neutral transactor.

For a risk neutral transactor $U_3 \equiv 0$. For such transactors, the optimal position of the budget line is always below the projection of the maximum value of information on the $\epsilon(Y_2)$ axis. The same is true for transactors who are not risk neutral whenever $\partial v / \partial I = 0$. In these cases, the optimal value of information is always reached before the maximum value of information. This can be verified by noting that in these cases $\partial V / \partial I$, therefore $\partial e_{12}^* / \partial I$, must be positive since $\partial V / \partial I = C \cdot \partial e_{12}^* / \partial I$.

Transactors who are willing to sacrifice resources to reduce uncertainty and obtain more reliable information are important in our analysis. For them, investments in information not only shift the budget line but also change the utility surface. A reduction in variance means that there is less uncertainty about the outcome of an exchange. With uncertainty reduced, the transactor receives more utility from a smaller bundle of commodities. The effect of reducing uncertainty, can be shown in Figure 1 by relabelling each of the indifference curves so as to attach more utility to every commodity bundle or by shifting each indifference curve in Figure 1 vertically downward thereby raising the utility assigned to any Y_2, C_1 combination.

The size of the shift in the indifference curves resulting from the effect of investment in information on the variance provides another interpretation of the second component of the optimality condition. The vertical shift, measured in units of Y_2, $(-U_3 / U_1)$ $(\partial v / \partial I_1)$, is the amount of $\epsilon(Y_2)$ that provides just as much utility as the reduced variance $v(Y_2)$.

The effect of a reduction in variance on

[17] Interpretation of the second component of the second necessary condition is made clearer by noting that $-U_3 / U_1 = dY_2 / dv$ so the component can be written $[dY_2 / dv][dv / dI] = dY_2 / dI$. The left side of this last expression shows the change in Y_2 associated with an increment to investment as the product of the change in variance resulting from an increment of information weighted by the appropriate marginal rate of substitution.

utility implies that the optimal position involves more investment in information than I_1''. The bundle available for exchange and consumption declines. In terms of Figure 1, the downward shift in the indifference curves—the gain in utility from reduced variance—at first offsets the loss of utility from the decline in the bundle. Where the marginal reduction in variance $\partial v/\partial I$ and product of the marginal rate of substitution between variance and Y_2, $-U_3/U_1$, is large, the optimal value of information may not be reached until we are on the negatively sloped segment of the $V(I)$ curve. Pronounced risk aversion, a relatively large $-U_3$ raises the portion of the resources a transactor allocates to investment in information and reduction of uncertainty.

We now introduce a third good into the nonmonetary economy we have considered. This good has zero direct marginal utility for each transactor.[18] It contributes to utility indirectly, however, by improving a transactor's information and by reducing the variance of exchange ratios and thus the uncertainty about the bundles obtained in market exchange. We assume that the third good has the following properties:

$$
(19) \quad
\begin{array}{c}
\dfrac{\partial \overset{*}{e}_{13}}{\partial I} > \dfrac{\partial \overset{*}{e}_{12}}{\partial I} < \dfrac{\partial \overset{*}{e}_{32}}{\partial I} \\[2ex]
\dfrac{\partial v(\overset{*}{e}_{13}\,|\,I)}{\partial I} < \dfrac{\partial v(\overset{*}{e}_{12}\,|\,I)}{\partial I} > \dfrac{\partial v(\overset{*}{e}_{32}\,|\,I)}{\partial I}
\end{array}
$$

These conditions state that the third good reduces the transactor's cost of acquiring

[18] We do not make this assumption in the formal analysis of the previous section. The assumption is introduced here only to simplify the graphical presentation, specifically to avoid superimposing a second indifference map on the map in the figure. In the more general analysis of the previous section there are a large number of commodities and potential transaction chains. The number of chains depends on the number of commodities. Under our maximizing procedure, at least one of these chains is optimal. In the graphical analysis, V_2 represents this chain.

reliable information. With the same investment in information, he increases the expected value of the sample maxima $(\overset{*}{e}_{13}, \overset{*}{e}_{32})$ and reduces uncertainty.

The introduction of a roundabout method of exchange using a good that gives transactors increased value of information at relatively low cost offers an opportunity not previously available. Transactors are no longer restricted to choosing between allocations to reservation demand, trade, and investment in information. They may now choose transaction arrangements, i.e., they can obtain Y_2 either by exchanging part of R_1 directly at the exchange ratio $\overset{*}{e}_{12}$ or by first acquiring the new good at exchange ratio $\overset{*}{e}_{13}$ and using good 3 to obtain Y_2 at the exchange ratio $\overset{*}{e}_{32}$.

As before the initial constraint in terms of R is

$$(12) \qquad R_1 = C_1 + X_1 + I_1$$

The transactor wishes to acquire Y_2 by trading X_1 and investing I_1. The market transformation is expressed by $Y_2 = X_1 t^i(I_1)$ where

$$
(20) \quad
\begin{aligned}
t^1(I_1) &= \overset{*}{e}_{12}(I_1) \\
t^2(I_1) &= \overset{*}{e}_{13}(I_1) \cdot \overset{*}{e}_{32}(I_1)
\end{aligned}
$$

The transactor desires to maximize the utility function

$$(16) \qquad U[\epsilon(Y_2), C_1, v(Y_2)]$$

by suitable choices of C_1, I_1 and market transformation t^i, subject to the endowment constraint. The optimizing problem can be rewritten as the maximization of the function

$$U[(R_1 - C_1 - I_1)t^i, C_1, v[(R_1 - C_1 - I_1)t^i]]$$

with respect to C_1, I_1 and t^i ($i = 1, 2$). The first-order conditions for C_1 and I_1 are

$$(21) \qquad U_1 t^i = U_2$$

$$(22) \qquad \frac{\partial V(t^i)}{\partial I_1} = C_1 \frac{\partial t^i}{\partial I_1} + \frac{-U_3}{U_1} \frac{\partial v(t^i)}{\partial I_1}$$
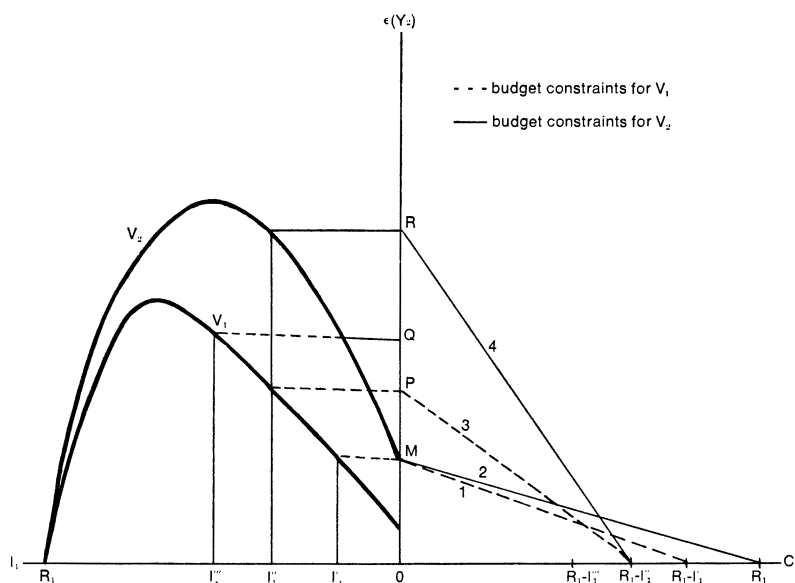
FIGURE 2

where $V(t^i) = (R_1 - I_1)t^i$, and $v(t^i) = v[X_1 \cdot t^i]$. The two equations determine C_1 and I_1 as a function of t^i. Replacement in the utility function gives

$$U[X_1(t^i)t^i, C_1(t^i), v[X_1(t^i)t^i]]$$

and the stage is set for the choice of the optimal transaction arrangement.

Figure 2 shows the choice of transaction arrangement. With this choice, the transactor determines the value of information obtained from a given investment, I_1. The curves V_1 or V_2 represent two alternative transaction arrangements. The differences in the value of the information obtained from a given investment depends on the properties of the joint conditional distribution $\pi(e_{12}^*, e_{13}^*, e_{32}^*/I_1)$ of the three exchange ratios. Since information about types of transaction chains decays slowly, the intercept of the curve V_2, describing chain t^2, is above the intercept of V_1. The diagram also shows an earlier and higher maximum for transaction chain t^2. Since V_2 lies above V_1 throughout the ascending branch of V_1, a trader gains from using

transaction arrangement t^2 throughout this range. This effect is reenforced by the shift of the indifference map resulting from the greater reduction in variance per unit of I. The selection of t^2 and the use of an asset with well-known properties permits the transactor to trade at a more favorable exchange ratio and reduces uncertainty. Thus, the choice of t^2 instead of t^1 raises the transactor's welfare.

Figure 2 shows the productivity gain resulting from the choice of transaction arrangements in two distinct ways. One measure of the gain is the smaller investment in information required to obtain information of given value. For example, the transactor can obtain information with value OM either by investing I_1' and using transaction arrangement V_1 or by making no investment in information and using transaction arrangement V_2. The productivity gain is equal to the saving of resources (I_1'). This saving permits the transactor to trade at the maximal exchange ratio represented by (the solid) budget line 2 instead of the maximal ratio

represented by (the dashed) line 1. Similarly, point OQ shows that the combination of V_1 and I_1''' produces the same value of information as the combination V_2 and I_1'. By choosing transaction arrangement V_2, the transactor reduces the cost of acquiring information of value OQ by $I_1''' - I_1'$. The saving in cost becomes available for consumption and exchange. The production of unchanged value of information with a smaller input of resources is a measure of the marginal productivity of transaction arrangement V_2.

An alternative measure of the productivity gain is the increase in the value of information obtainable with unchanged investment. The distance PR on the $\epsilon(Y_2)$ axis is the increased value of information (in units of Y_2) obtained with investment I_1'' and the choice of transaction chain V_2 instead of V_1. By using V_2 the transactor is able to trade at the ratio given by budget line 4 rather than the lower exchange ratio given by constraint 3. Thus, careful choice of transaction arrangements increases the productivity of a transactor's resources and his utility.

The results obtained in our world of three commodities apply to an n-commodity world in which there are numerous alternative transaction arrangements and many transactors. Each transaction arrangement can be represented by a curve on the left panel of Figure 2 showing the value of information obtained with a given investment of resources in information. Our first postulate implies not only that the curves differ substantially but that the optimal transaction arrangement for an individual transactor—the highest curve—involves indirect exchange, i.e., the use of an intermediate asset that reduces his cost of acquiring information about market opportunities. Our second postulate implies that what is true for the individual is true for most members of the social group. The transaction arrangements that are

most productive for the individual are also most productive for a dominant portion of the group.

The assets used to reduce the cost of acquiring information and transacting are money for the group or society. The social and private productivity of a medium of exchange results from the use of transaction chains involving assets with these properties. Where knowledge of market opportunities and the qualities of goods is neither costless to obtain nor uniformly distributed, the use of money as a medium of exchange reduces the resource cost of exchanging. In the following section, we consider the social productivity of money and some implications of the analysis for the type of assets chosen as mediums of exchange and used as money.

III. The Social Services of Money

For individuals, money is a substitute for investment in information and labor allocated to search. By using money, individuals reduce the amount of information they must acquire, process, and store, and they reduce the number of transactions in which they engage to exchange their initial endowments for optimal baskets of goods. The use of money increases the welfare of each money user by reducing uncertainty, the length of transaction chains, and the variance of price ratios and by increasing expected wealth and time available for leisure. Whatever other services create a demand for the assets that serve as mediums of exchange, their use as mediums of exchange increases demand. Individuals find it advantageous to allocate part of their wealth to money.

What is true for individuals is in this case true for society. The convergence of optimal transaction arrangements generates an aggregate demand for the assets used as mediums of exchange. The increased demand to hold inventories of these assets (money) is independent of the

previous uses of the assets and, of course, increases the (relative) prices of the assets. The average amount held in inventories depends on the prices of the assets held, the prices of alternative assets and, thus, on the relation between net marginal productivity and marginal cost.

Once inventories of money are held, payments and receipts are no longer synchronized. Lack of synchronization, however, does not explain the use of holding of money anymore than the holding of money explains the lack of synchronization. Both are a result of the superior productivity of indirect methods of exchange, the smaller resource cost of acquiring information and transacting in a monetary economy.[19]

The use of money encourages the development of the market system by lowering the costs of acquiring information and transacting. With the expansion of the market, opportunities increase for professional middlemen and specialized traders to exploit the partial and incomplete distribution of information about particular commodities. Specialized traders substitute for a wider and more general distribution of information. The use of money also affects the intertemporal allocation of resources. Deferred payments, borrowing, credit and the payments system expand

when a standardized asset with well-known properties becomes available. The reason is that transactors become more willing to enter into contracts calling for deferred payment.

The magnitude of the net social productivity of money is not constant but varies with the degree of uncertainty about market conditions, including exchange ratios and the quality of goods. Accelerated technological changes or innovations that change the qualities of goods and increase the number or types of goods raise the productivity of money. Large fluctuations in economic activity also raise costs of acquiring information and the productivity of money. Our analysis implies that the demand for mediums of exchange is higher in periods of rapid change than in periods of gradual or relatively steady change. The longer the period of steady, gradual change continues, the lower the productivity of money and the smaller the demand for assets that reduce costs of acquiring information by serving as mediums of exchange.

A stationary state or a world of steady growth are the limiting cases of economic theory. Tastes, technology, anticipations, population, and types of product are either invariant or change in a known, fully anticipated way at a steady rate. The marginal cost of acquiring information falls and in the limit approaches zero. Transaction chains no longer differ by the saving of costs of acquiring information and differ only by the costs of transfer. The main condition leading to the selection of a small group of assets as money, and therefore the main source of the distinction between money and nonmonetary assets, disappears in the stationary state or world of steady growth.[20]

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[19] In a well-developed market economy, most of the net *marginal* productivity of money probably results from the saving of costs of transacting, while the *total* productivity of money depends on the reduction in costs of acquiring information and costs of exchanging. The difference between our analysis and the usual analysis of "transaction costs" (see Baumol, Tobin (1956)) is that the *total* productivity of money in the familiar Baumol and Tobin analyses is almost the same as the *marginal* productivity in our analysis. The reason is that Baumol-Tobin transactors face fixed payment schedules, whereas our transactors optimize over all transaction arrangements. The Baumol-Tobin analysis has been used recently by Samuelson (1969), Clower and Johnson to equate the productivity of money with the "shoe leather" saved by avoiding trips to the bank or the market place. Their analyses understate the total contribution of money to wealth by neglecting the improvement in the opportunity set shown in Figure 2 and discussed in the text.

[20] In the terms of our formal analysis, the effect is conveyed by the distribution $\pi(E|P)$. The coordinates of the vector P summarizes the representative individual's information about market opportunities. The greater the frequency of change in market conditions,

Hyper-inflation and hyper-deflation provide examples of changes in the marginal cost and marginal productivity of holding and using money. In both, the increased frequency of change in market conditions increases uncertainty and the variance of exchange ratios and thus raises the marginal productivity of money. But the marginal cost of holding money increases in hyper-inflation as the rate of inflation increases. Transactors are induced to choose transaction chains that avoid the inflation tax, even if such chains use more resources for transactions.

The search for new transaction chains and the abandonment of old is commonly mistaken for a return to barter. The process does not restore either double-coincidence exchange or the random selection of mediums of exchange. Transactors concentrate their search on those transaction chains that offer at least the same expected gain in wealth as existing mediums of exchange. The assets that replace existing money may have higher marginal costs of acquiring information and transacting. If so, they must have lower marginal holding costs than the existing money.

Our analysis implies that continued and accelerated hyper-inflation eventually comes to an end. The economy gradually adopts a new money and a new set of transaction arrangements. The process of search and social convergence is not costless for the individual transactors or the society. Governments or private producers can reduce the social cost of introducing new mediums of exchange and the transition to the new transaction arrangements by introducing new assets with properties similar to the old and effectively controlling the quantity produced.

A large deflation also increases uncer-

tainty and induces transactors to reconsider transaction chains. Unlike hyper-inflation, the search for new transaction chains in a deflation does not replace existing mediums of exchange but adds new ones. The relatively high yield on money puts a premium on the search for assets that are close substitutes for the existing mediums of exchange—have similar information and transfer costs—and lower yield. New types of money evolve to supplement the existing money and increase the available stock of real balances. Adjustment to severe deflation, therefore, is not concentrated solely on the price level and rate of price change as in growth theory. New supplies of real balances are produced by the search for alternative transaction chains.

Our analysis of the use of money implies that both inflation and deflation are stable in a certain technical sense. In inflation new monies are substituted for old; in deflation supplementary monies are introduced. But the evidence, from periods of hyper-inflation and severe deflation alike, reveals the sizeable resource costs associated with the stabilizing mechanism. Hyper-inflation or deflation reach relatively high levels before transactors begin to replace existing money or add new money. The high costs that individuals are willing to pay before beginning to search for new or supplementary arrangements suggests the size of the benefits received from the use of a dominant medium of exchange.

The size of the net social productivity of money also depends on the assets selected as mediums of exchange. Once the community uses some assets as money, the private and social benefit can be increased by substituting claims against commodities for commodity money. Individuals gain from the use of substitutes for commodity money if the reduction of costs of acquiring information and transacting more than compensates for the increased

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the poorer his information. In the stationary state his information increases without offset and approaches full information. The variances and covariances of the π function decline. See fn. 11.

variability of exchange ratios. Society gains because the use of claims and fiat paper money reduces the resources used to make exchanges in three main ways. First, paper money permits society to develop a fractional reserve money system and to produce the same nominal stock of money at lower resource cost. Second, the use of claims encourages the development of privately produced money and with it the development of the payments system. The cost of acquiring information about the qualities of paper money, whether produced by government or by private producers, is lowest if the paper money starts as a claim against commodity money. When information about the paper money becomes widespread, paper money retains the property of general acceptability even if the claim against commodities is removed. Third, paper money frequently lowers the resource cost of transfer and exchange. This somewhat less than general proposition recognizes that both costs and benefits are affected and that the size of the net benefit from the use of paper money depends on the prevailing monetary arrangements.

Monetary history offers numerous examples of changes in monetary arrangements that reduced marginal costs of information or transfer for the assets used in optimal transaction chains. Coinage is one of the earliest, and bank credit cards one of the most recent steps extending the range and use of mediums of exchange by reducing these costs.[21] Suppose, however, paper money is not introduced by a central bank or government but emerges in re-sponse to the public's search for optimal transaction chains. Many different producers are induced to issue paper money as a claim against commodity money. The social benefit resulting from the use of lower cost money is partly offset by the higher cost individuals pay to acquire information. The legislation of 1844 in England and of the 1860's in the United States that reduced the number and types of notes in circulation by restricting the right to issue notes are examples of institutional changes that raised economic welfare by reducing costs of acquiring information. The requirement of par collection of checks under the Federal Reserve Act is another example.

Nothing in our analysis implies either that society converges to a single medium of exchange or that the productivity of money and the contribution of money to wealth is limited to a single monetary asset. Different types of assets—some privately, some publicly produced—appear in the transaction chains adopted by members of the group and in the transaction chains of a single transactor at different times. These differences in the choice of transaction chain reflect differences in marginal cost that depend on the type of transaction and the transactor's information. Even in highly developed economies with extensive monetary institutions, transactors can use specialized information to develop transaction arrangements that lower transfer costs by avoiding the use of money. Moreover, sectors of an economy that develop specialized information about the properties of particular assets often find it useful to develop specialized mediums of exchange.[22]

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[21] Credit cards centralize information about deposit users, reduce a seller's cost of acquiring information, encourage the separation of payments and purchases and thereby increase (relatively) the use of deposits as a medium of exchange, lowering the ratio of currency to deposits. Lowering the costs of acquiring information and transferring via deposits lowers the net marginal productivity of deposit inventories. Average inventories of deposits are reduced; velocity increases.

[22] The development of the Federal funds market is an example. Corporate mergers offer examples of the way in which the allocation of human wealth (skilled specialists) reduces cost by avoiding money as a medium of exchange. By offering the owners of the merged firm deferred equity claims of various kinds, the purchasers reduce or defer the tax liabilities of equity owners,

Our analysis suggests an extension of Gresham's law—cheap money drives out dear at fixed exchange rates—to the case of multiple mediums of exchange with variable or floating exchange rates. With floating exchange rates, stable monies drive out variable monies. Consequently, government or private producers desirous of maintaining the circulation of government and privately produced monies have found it desirable to provide arrangements for exchange of one money for the other on demand.[23] More importantly, issuers of privately produced money expand the market for their product by maintaining a fixed exchange rate. By doing so, they lower the users' cost of information and increase the demand for their product. Businessmen, bankers, and government officials have used similar reasoning to press for fixed exchange rates between national monies.

Throughout our analysis we have identified money with the medium of exchange. This usage has been criticized on two grounds. Tobin (1963) is one of the last in a long line emphasizing the existence of close substitutes for money. Tobin's criticism is part of a position discussed above, that ignores costs of acquiring information and attributes the total productivity of money to its role in synchronizing receipts and payments and reducing brokerage costs. Friedman and Schwartz (1970) also criticize the notion that money is a medium of exchange. They describe money as a "temporary abode of purchasing power" and argue that the term medium of exchange is an a priori notion devoid of empirical content. As an example of the deficiency of the medium of exchange concept, they cite the difficulty of using units of currency with large denominations or

using checkable deposits in unfamiliar environments. Their argument is insufficient for the conclusion they have drawn. The use of checks and bills of large dedenomination often involves substantial costs of acquiring information and exchanging. On our analysis, it is not surprising that neither circulates in unfamiliar environments.[24]

Defining money as a temporary abode of purchasing power does not distinguish between properties of assets or between a monetary and a barter economy in a manner independent of the medium of exchange function. As our earlier discussion shows, transactors hold intermediate goods temporarily in a barter economy—as one of the items in a transaction chain—if their best information suggests that by doing so they can make more advantageous exchanges.

The recognition of the central role of a medium of exchange does not imply that the collection of assets that serve as medium of exchange is most appropriate for explaining movements of the general price level. A definition embracing a larger collection of assets is appropriate if there are close substitutes for the medium of exchange on the supply side. In this case, slight changes in relative prices reallocate output between the medium of exchange and other assets, so the collection of assets most useful for explaining changes in the general price level differs from the assets that serve as medium of exchange. However, even if evidence suggests that a broader collection is justified empirically and the term money is used to refer to the broader collection, the significance of the medium of exchange function and its im-

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raising the owners' wealth and reducing the cost of acquisition.

[23] This is the rationale for the instant repurchase clause discussed by Pesek and Saving.

[24] Five and ten thousand dollar notes never circulated widely but were used primarily for transactions between banks where the marginal cost of acquiring information about the notes was low. With the development and extension of the Federal funds market and other lower cost means of making interbank transfers, the use of bills of large denomination declined markedly.

portance for explaining the productivity of monetary assets remains.

IV. Conclusion

The use of money remained puzzling as long as the theory of exchange was restricted to the case of perfect certainty, a world in which information about market prices and the qualities of goods and services is obtained at zero cost. Standard price theory eliminated the main reasons for the existence and use of money by confining choice to three options—production, consumption and exchange—and setting costs of acquiring information about exchange opportunities and qualities of goods to zero. With costs of executing transactions zero and information a free good, there are no costs of shopping to assure that exchanges take place at the most favorable prices and no benefits from reducing the resource cost of executing transactions and eliminating cross-hauling of commodities. Any asset is just as usable as any other for executing transactions and discharging obligations. As a result, attempts to explain the use of money generally accept some consequences of the use of money—such as lack of synchronization—as an explanation of the existence of money.

Our analysis extends the theory of exchange to include the cost of acquiring information about market arrangements, relative prices, or exchange ratios. Individuals search for those sequences of transactions, called transaction chains, that minimize the cost of acquiring information and transacting. The use of assets with peculiar technical properties and low marginal cost of acquiring information reduces these costs. Money is such an asset, and the private and social productivity of money are a direct consequence of the saving in resources that the use of money permits and of the extension of the

market system that occurs because of the reduction in the cost of making exchanges.

Money is a substitute for the specialized market skills that are part of a transactor stock of knowledge or "human wealth." Resources allocated to search and to maintaining market information can be reallocated once money is used as a medium of exchange. Trade and the market system expand, and the economy becomes increasingly monetized. More and better quality information becomes available with the expansion of the market and the opportunities for division of labor that lead to the development of professional transactors such as brokers and specialists. The use of a unit of account further reduces the cost of making exchanges.

We do not attempt to explain the holding of money except in the trivial sense that the use of an asset implies that the asset is held. Nevertheless the analysis helps to explain why money continues to be used even in periods of accelerating inflation when the cost of holding money reaches very high levels.

By analyzing the productivity of money and relating the productivity of money to the analysis of trade in an exchange economy, we clarify the meaning of the phrase "the services of money" and suggest by implication the benefits that would accrue to the world economy from the use of a medium of exchange. Since the saving in brokerage costs or "trips to the bank," that is generally presented as the total product of money, is only the marginal product of money in our analysis, the gains from using money in international as in domestic exchange are considerably larger than is generally claimed. (See Baumol, Clower, Johnson, Samuelson (1969), Tobin (1965).) Once we leave the world of certainty and costless information, both the private and social productivity of money rise; the use of money

ceases to be puzzling and becomes, instead, an implication of optimizing behavior.

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