A Dynamic Economy with Costly Price Adjustments

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This paper studies a general-equilibrium model of a dynamic economy with menu costs. Each firm's productivity is exposed to idiosyncratic and aggregate productivity shocks around a trend, and the money supply to monetary shocks around a trend. All consumption, pricing, and production decisions are based on optimizing behavior. There exists a staggered Markov perfect equilibrium with prices determined by a two-sided (s, S) *markup strategy. The paper analyzes the optimal markup strategy and investigates the dynamics of the price index and the aggregate output. The welfare consequences of the uncertain aggregate productivity and money supply are also examined.* (*JEL* E31, E32)

Costs of price adjustment play a prominent role in many explanations of macroeconomic fluctuations. In a static framework, George A. Akerlof and Janet L. Yellen (1985), N. Gregory Mankiw (1985), Michael Parkin (1986), and Olivier Jean Blanchard and Nobuhiro Kiyotaki (1987) demonstrate that a single unanticipated demand disturbance can have large real effects even if the cost of price adjustment is small. Laurence Ball and David Romer (1989, 1990) extend the analysis to a distribution of shocks and show that combining a small cost of price adjustment with a real rigidity can greatly increase the real effects of monetary shocks. Since the empirical study by Daniel Levy et al. (1997) indicates that costs of price adjustment are not trivial,[1] such costs together with a real rigidity are potentially a source of significant aggregate rigidities.

In spite of the popularity of the menu-cost approach to understanding aggregate rigidities, there does not yet exist a dynamic general-equilibrium model with menu costs for an uncertain environment in which all consumption, pricing, and production decisions are based on optimizing behavior. The purpose of this paper is to construct such model and to use it to study the general-equilibrium effects of both real and monetary shocks.[2]

In the model, the demand for each good is derived from utility maximization, and each firm owner chooses his price and production optimally, taking the whole future into account. The profits belong to the firm owners and affect their demand. The productivity in each firm is exposed to idiosyncratic and aggregate productivity shocks around a trend. The idiosyncratic productivity shocks affect each firm differently, while the aggregate productivity shocks affect all firms in the same way. The money supply is

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[1] The costs of price adjustment comprise 0.70 percent of revenues and 35.2 percent of net margins in five large U.S. supermarket chains.

[2] In partial equilibrium, Eytan Sheshinksi and Yoram Weiss (1977) study the optimal price strategy for a constant rate of inflation. The analysis is extended by Sheshinski and Weiss (1983) and Danziger (1984) to a stochastically increasing price level. See also Danziger (1983). In general equilibrium, the effects of a constant rate of monetary expansion on the optimal price strategy and welfare are determined by Roland Benabou (1988, 1992) and Peter A. Diamond (1993) in models with search, and by Danziger (1988) in a model with differentiated goods. Andrew S. Caplin and Daniel F. Spulber (1987) consider an uncertain, monotonically increasing money supply, but assume that firms follow price strategies that are not optimal. See also Guiseppe Bertola and Ricardo J. Caballero (1990) and Caballero and Eduardo M. R. A. Engel (1991, 1993). Caplin and John Leahy (1991) study nonmonotone monetary uncertainty with the money supply generated by a geometric Brownian motion without drift, and they too assume that firms follow price strategies that are not optimal. They extend their model in Caplin and Leahy (1997) where the firms' price strategies are optimal. However, the demand side is not modelled and there is no microfoundation for the assumed loss-of-profit function.

exposed to monetary shocks around a trend. The sign and absolute size of all the shocks are random. The analysis is facilitated by the assumption that the idiosyncratic shocks are uniformly distributed.

A main result is that there exists a staggered Markov perfect equilibrium in which prices are determined by a simple two-sided (s, S) strategy for the *markups:* A firm owner keeps his price unchanged if the markup without a price adjustment lies within the nonadjustment interval (s, S), and immediately adjusts his price if the markup without a price adjustment falls outside this interval. At a price adjustment, the new price is determined by the optimal level of the initial markup. The probability that a firm's price remains unchanged is an endogenously determined constant which is independent of when the last price adjustment occurred and of the magnitude of the shocks to the aggregate productivity and the money supply.[3] The equilibrium wage rate is proportional to both the aggregate productivity and the money supply.

It is determined how each of the parameters in the model affects the optimal markup strategy and the expected period with an unchanged price. For example, a higher cost of price adjustment and a lower trend in the money supply are accompanied by a longer expected period with an unchanged price. The markup strategy is invariant with respect to the trend in the aggregate productivity and the distribution of the aggregate productivity shocks. The reason is that both the productivity in the firm and the equilibrium wage rate are proportional to the aggregate productivity. A firm's markup for a given price is therefore unaffected by changes in the aggregate productivity. The markup strategy is also invariant with respect to the distribution of the monetary shocks. This is because the uniform distribution of the idiosyncratic productivity shocks makes the distribution of the markup with an unchanged price within the nonadjustment interval independent of the distribution of the monetary shocks.

The equilibrium distribution of the markups is stationary, with the markups equal to the optimal initial level for those firms whose prices are adjusted, and loguniformly distributed for the constant fraction of firms whose prices are unchanged. A firm's price and hence the price index is not affected by an aggregate productivity shock. The price index increases with a monetary shock, but a shock is not immediately passed fully on to the index. In the long run the price index changes proportionally to the money supply. Since the aggregate output also increases with a contemporaneous monetary shock, unanticipated movements in the price index and the aggregate output are positively correlated. This is supported by the empirical evidence in Thomas F. Cooley and Lee E. Ohanian (1991) and Julio J. Rotemberg (1996).

A firm owner's welfare is independent of the realized aggregate productivity shock and of the uncertainty of the future aggregate productivity, since the aggregate productivity does not affect a firm's markup for a given price. The welfare effect of a realized monetary shock differs between owners as it depends on the markup, and thus on the history of a firm's idiosyncratic shocks. However, on average an owner's welfare increases with a monetary shock, and an owner's welfare is unaffected by the future monetary uncertainty.

The households' welfare increases with an aggregate productivity shock since the real wage rate increases with the aggregate productivity. Their welfare also increases with a realized monetary shock, due to the initial effect on the real wage. With the chosen specification of the households' utility function, their welfare is independent of the uncertainty of the future aggregate productivity and money supply.

The remainder of the paper is organized as follows. Section I describes the model, and Section II proves the existence of a Markov perfect equilibrium. Section III establishes the comparative-static results for the optimal markup strategy and the expected period with an unchanged price. Section IV examines the dynamics of the price index and the aggregate output. Section V analyzes the welfare effects of the realized shocks to the aggregate productivity and the money supply, and of the future uncertainty of the aggregate productivity and the money supply. Section VI concludes the paper. The Appendices contain the mathematical proofs.

[3] The constant probability of an unchanged price is reminiscent of the price-setting structure in Calvo (1983). However, in the present model the probability is endogenous and the price strategy is state dependent: a firm's idiosyncratic productivity shock regulates whether, and by how much, the firm's price is adjusted.

I. The Model

A. *Firms*

Consider an economy with a unit continuum of differentiated nonstorable goods. Each good is produced by a different firm. Labor is the only input, and its marginal product at time t in a particular firm is constant and denoted by q_t. The price of a good at time t is denoted by p_t, and the wage rate by w_t.

The price-adjustment cost is independent of the size of the price adjustment, but increases with the firm's output. Specifically, the cost of price adjustment is modelled as the loss of demand by a fraction $c \in (0; 1)$ of the firm's customers.[4] Thus, if D_t denotes the total demand for a good with an unchanged price at time t, then $(1 - c)D_t$ is the total demand for a good whose price has been adjusted at time t. The firm's profit at time t is

$$\pi_t \equiv (1 - c_t)D_t\left(p_t - \frac{w_t}{q_t}\right),$$

where $c_t = 0$ if the price is unchanged and $c_t = c$ if the price is adjusted at time t.[5]

A firm owner's utility at time t is

$$\left\{\int_0^\infty [x_t(p_t)]^{1/2} f(p_t)\ dp_t\right\}^2,$$

where $x_t(p_t)$ is the owner's consumption of each of the different goods that a customer can purchase at the price p_t, and $f(p_t)$ is the endogenous density of these goods.[6] The owner spends his profit at the same t it is received. For a given profit, the owner maximizes his utility by purchasing

$$x_t(p_t) = \frac{\pi_t P_t}{p_t^2 N} \tag{1}$$

of each available good whose price is p_t, where $N \equiv \int_0^\infty f(p_t)\ dp_t$ measures the available product variety,[7] and

$$P_t \equiv N\left[\int_0^\infty \frac{1}{p_t} f(p_t)\ dp_t\right]^{-1}$$

is the price index at time t.

By substitution of his consumption from equation (1), the owner's utility at time t becomes

$$\left[\int_0^\infty \left(\frac{\pi_t P_t}{p_t^2 N}\right)^{1/2} f(p_t)\ dp_t\right]^2 = \frac{\pi_t N}{P_t}.$$

B. *Households*

There is a unit continuum of identical households. A household's utility at time t is

$$\left(2\left\{\int_0^\infty [y_t(p_t)]^{1/2} f(p_t)\ dp_t\right\}^2 - 1\right)^{1/2} \frac{1}{h_t},$$

where $y_t(p_t)$ is the household's consumption of each available good whose price is p_t, and h_t is the household's labor supply at time t. A household spends its labor income at the same t it is earned, and it chooses its consumption of the different goods and its labor supply to maximize its utility. This implies that the household purchases

[4] A possible rationale is that a price adjustment takes time, making the good unavailable for purchase in the fraction c of the time period. Assuming that a customer can visit a firm only once in each time period, that it takes the whole time period to visit all the firms, and that the order of visits is random (which is reasonable since in equilibrium the probability that a price remains unchanged is the same for all firms), a price adjustment implies that the product cannot be purchased by a fraction c of the firm's customers.

[5] It is assumed that $p_t > w_t/q_t$ so that the firm satisfies all the available demand. This is true in equilibrium.

[6] Thus, $f(p_t)$ is the density of the goods with price p_t minus c times the density of those goods whose price was adjusted to p_t at time t.

The utility function is *not* indirect as it does not assume that the consumption is optimal (but only that the owner consumes the same quantity of different goods with the same price). The prices are merely used as labels to distinguish between goods with different prices. The same holds for a household's utility function below.

[7] N is an endogenous constant in equilibrium. It equals 1 minus c times the fraction of prices that are adjusted at a given time, where the latter is a constant in equilibrium. See footnote 17.

$$y_t(p_t) = \frac{P_t^2}{p_t^2 N^2} \tag{2}$$

of each available good whose price is p_t, and supplies

$$h_t = \frac{P_t}{w_t N} \tag{3}$$

of labor.

C. *Money*

The velocity of money is unity, so equilibrium in the money market requires that the sum of all profits and wage incomes equals the money supply, which is denoted by M_t at time t. Equations (1)–(3) then imply that the total demand for a good with an unchanged price is

$$D_t = \frac{M_t P_t}{p_t^2 N}. \tag{4}$$

Substituting the total demand for a good in the firm's profit, the owner's utility at time t is

$$(1 - c_t)\left(\frac{M_t}{p_t} - \frac{w_t M_t}{q_t p_t^2}\right).$$

The utility is independent of the price index because the total demand for the firm's product is proportional to the price index and the marginal product is constant. It is also independent of N, since the inversely proportional effect of the product variety on the profit is cancelled by the proportional effect of the product variety on the utility from a given profit.

D. *Uncertainty*

The productivity in each firm changes stochastically over time, as it is exposed to both idiosyncratic and aggregate productivity shocks. The productivity in a firm at time t is $q_t = a_t b_t$, where a_t is the firm's idiosyncratic productivity factor, which is unity on average, and b_t is the aggregate productivity in the economy. A firm's idiosyncratic productivity factor evolves according to $a_t = a_{t-1} e^{\alpha_t}$, where α_t is the idiosyncratic productivity shock at time t. The idiosyncratic productivity shocks are independent across firms and over time in the same firm, and they are uniformly distributed with density $1/\phi$ on $[\underline{\alpha}; \bar{\alpha}]$, where $\underline{\alpha} \equiv \ln[\phi/(e^{\phi} - 1)]$, $\bar{\alpha} \equiv \underline{\alpha} + \phi$, and $\phi > 0$. Thus, the expected value of a_t/a_{t-1} is $(1/\phi) \int_{\underline{\alpha}}^{\bar{\alpha}} e^{\alpha_t}\, d\alpha_t = 1$: the idiosyncratic productivity shocks are not expected to change the idiosyncratic productivity factor, and a higher ϕ indicates a riskier distribution of the idiosyncratic shocks.

The aggregate productivity evolves according to $b_t = b_{t-1} e^{\beta_t + g}$, where β_t is the aggregate productivity shock at time t, and g is the trend in the aggregate productivity (and hence the trend in the productivity in each firm). The aggregate productivity shocks are independently distributed with density $\psi(\beta_t)$ on $[\underline{\beta}; \bar{\beta}]$, where $\underline{\beta} < \bar{\beta}$, and satisfy $\int_{\underline{\beta}}^{\bar{\beta}} e^{\beta_t} \psi(\beta_t)\, d\beta_t = 1$ so that the shocks are not expected to change the aggregate productivity.

The money supply also changes stochastically over time, evolving according to $M_t = M_{t-1} e^{\gamma_t + m}$, where γ_t is the monetary shock at time t, and m is the trend in the money supply. The monetary shocks are independently distributed with density $\chi(\gamma_t)$ on $[\underline{\gamma}; \bar{\gamma}]$, where $\underline{\gamma} < \bar{\gamma}$, and satisfy $\int_{\underline{\gamma}}^{\bar{\gamma}} e^{\gamma_t} \chi(\gamma_t)\, d\gamma_t = 1$ so that the shocks are not expected to change the money supply.

E. *Markov Strategies and Equilibrium*

At each t, a firm owner determines the price of his product in order to maximize his discounted expected utility

$$V_t \equiv E \sum_{\tau=t}^{\infty} e^{-\rho\tau}(1 - c_\tau)\left(\frac{M_\tau}{p_\tau} - \frac{w_t M_\tau}{q_\tau p_\tau^2}\right),$$

where $\rho > 0$ is the owner's discount rate and the expectation is taken with respect to the distribution of all future utilities. Because of the cost of price adjustment, the owner might not want to adjust his firm's price even though the previously charged price does not maximize the current profit. The price strategy maps all available information to a current price at each t. However, only Markov strategies are considered, that is, strategies for which the current price depends on only those variables that directly affect the owner's current or future utilities. At time t, these variables are q_t, w_t, M_t, and p_{t-1} (because of the price-adjustment cost incurred if the price is adjusted).

Definition: The economy is in a Markov perfect equilibrium if, at each t,

(a) Each firm owner's Markov price strategy maximizes his discounted expected utility assuming that all other firm owners follow their Markov price strategies;
(b) The aggregate demand and the aggregate supply for labor are equal.

II. The Equilibrium

An important feature of the equilibrium is that the wage rate is proportional to both the aggregate productivity and the money supply, that is, $w_t = \omega b_t M_t$, where $\omega > 0$ is a constant. In the equilibrium a firm owner therefore perceives his discounted expected utility to be

$$V_t = E \sum_{\tau=t}^{\infty} e^{-\rho\tau}(1 - c_\tau)\left(\frac{M_\tau}{p_\tau} - \frac{\omega b_\tau M_\tau^2}{q_\tau p_\tau^2}\right)$$

$$= \frac{1}{\omega} E \sum_{\tau=t}^{\infty} e^{-\rho\tau}(1 - c_\tau)a_\tau\left(\frac{1}{z_\tau} - \frac{1}{z_\tau^2}\right),$$

where $z_\tau \equiv q_\tau p_\tau / w_\tau = a_\tau p_\tau/(\omega M_\tau)$ is his markup of price over marginal cost. If price adjustments were not costly, the maximization of V_t would require that the owner always maximizes his current profit by marking up his price to the double of the marginal cost, that is, by setting $z_t = 2$ at each t. In the presence of a price-adjustment cost, always maximizing the current profit does not maximize V_t. However, a main result of this paper is that there exists a simple markup strategy that maximizes V_t for each t, even though price adjustments are costly and both the productivity and the money supply vary stochastically.

To describe the optimal strategy, let $\zeta_t \equiv q_t p_{t-1}/w_t = a_t p_{t-1}/(\omega M_t)$ denote what the markup of price over marginal cost would be if the price remains unchanged from time $t-1$ to time t. So $z_t = \zeta_t$ if the price is not adjusted at time t. Taking the markup z_t to be the owner's decision variable, the relevant state of the economy is fully characterized by a_t and ζ_t, which are the only variables that directly affect V_t. In fact, it will be shown that the optimal markup strategy does not depend on a_t, but on only ζ_t. Specifically, it is optimal for the owner to follow a two-sided (s, S) markup strategy with the initial markup I, where $s < I < S$. Since $z_t = I \Leftrightarrow p_t = \omega I M_t/a_t$, the corresponding optimal price strategy is to keep the price unchanged if $\zeta_t \in (s, S)$ and to adjust the price to $p_t = \omega I M_t/a_t$ if $\zeta_t \notin (s, S)$.

The following two assumptions are made. The first assumption assures that the idiosyncratic shocks are sufficiently dispersed that for any monetary shock there exist idiosyncratic shocks small enough that there are firms whose marginal cost $(w_t/q_t = \omega M_t/a_t)$ increases, and idiosyncratic shocks large enough that there are firms whose marginal cost decreases. To state the assumption formally, define $\sigma \equiv \min\{m + \underline{\gamma} - \underline{\alpha}; \bar{\alpha} - m - \bar{\gamma}\}$, where the first argument is the highest possible rate of change of a firm's marginal cost at the smallest possible monetary shock, and the second is minus the smallest possible rate of change of a firm's marginal cost at the highest possible monetary shock. The assumption is then as follows,

ASSUMPTION 1: $0 < \sigma$.

The second assumption captures the idea that the cost of price adjustment is sufficiently small that some of the firms experiencing an increase in their marginal cost will increase their price and some of the firms experiencing a decrease in their marginal cost will decrease their price. Together with Assumption 1, this implies that some prices are raised and other prices are lowered at each t.[8] To state the assumption precisely, let $\delta \equiv e^{m-\rho}/\phi$, $H \equiv [\frac{1}{2}(1 - \delta\sigma)(e^\sigma + 1) + \delta(e^\sigma - 1)](1 + e^{-\sigma})$, and $K \equiv 2[\frac{1}{2}(1 - \delta\sigma)(e^\sigma + 1) + \delta(e^\sigma - 1)]^2 e^{-\sigma}$. Then[9]

ASSUMPTION 2: $c < H - 1 - (1 - 2H + 2K)^{1/2}/K$.

[8] The evidence in Saul Lach and Daniel Tsiddon (1992) shows that some prices are lowered even at high rates of inflation.

[9] The right-hand side in Assumption 2 is positive (for a given δ, it vanishes in the limit as $\sigma \to 0$ and is increasing in σ), implying that there are c's satisfying Assumption 2.

To present the existence theorem, let

$$A \equiv [c(1-c)(I-1) + \tfrac{1}{4}c^2I^2]^{1/2},$$

$$B \equiv \ln\left(\frac{1-c+\frac{1}{2}cI+A}{1-c+\frac{1}{2}cI-A}\right),$$

$$Q \equiv 2 - I - \frac{\delta(BI-4A)}{(1-c)(1-\delta B)},$$

which are well defined assuming that $1 < I$ and $\delta B < 1$.

We can prove (see Appendix A),

THEOREM 1: *There exists a Markov perfect equilibrium in which each good is priced according to a two-sided* (s, S) *markup strategy with*

$$I = Q^{-1}(0),$$

$$s = \frac{I}{1-c+\frac{1}{2}cI+A},$$

$$S = \frac{I}{1-c+\frac{1}{2}cI-A}.$$

The wage rate is

$$w_t = \omega b_t M_t,$$

where $\omega = (1-c)/I^2$. *The value of the optimal markup strategy at time t is*

$$V_t = \begin{cases} a_t\left[\dfrac{1}{1-e^{-\rho}} + \left(\dfrac{S}{z_t}-1\right)\left(1-\dfrac{s}{z_t}\right)\right] & \text{if } \zeta_t \in (s, S), \\ \dfrac{a_t}{1-e^{-\rho}} & \text{if } \zeta_t \notin (s, S). \end{cases}$$

The initial markup I ensures that the discounted expected marginal utility from a change in the initial markup equals zero, and the adjustment bounds s and S are such that if the markup with an unchanged price equals one of the bounds, the discounted expected utility is the same whether the price is kept unchanged or is adjusted. Figure 1 depicts the relationship between the markup with an unchanged price and the utility at time t for the optimal two-sided markup (s, S) strategy. The lower adjustment bound s exceeds one so that all available demand is satisfied.[10] The maximum utility at time t, obtained if the price is unchanged and the markup equals two, is $\frac{1}{4}a_t/\omega$.

If the price is adjusted, the discounted expected utility is $a_t/(1-e^{-\rho})$, making the owner indifferent between receiving the stream of uncertain utilities from the optimal markup strategy, and always obtaining the certainty-equivalent utility a_t in the future. The markups that yield the utility a_t in period t, shown as s' and S' in Figure 1, are contained in the interval (s, S) of the markups where the price remains unchanged.[11] Hence, the price is unchanged not only if the markup falls in $[s', S']$, but also if it falls in (s, s') or (S', S), even though the current utility is then below the certainty-equivalent utility. This reflects the option value of keeping the price unchanged: Since price adjustments are costly, the price is kept unchanged in a wider interval than $[s', S']$ in order to reduce the likelihood that a future shock will cause the owner to regret a current price adjustment.[12]

Keeping the price unchanged when the markup falls in (s, s') makes a future price increase more likely and a future price decrease less likely, while keeping the price unchanged when the markup falls in (S', S) has the opposite effect. Since a price adjustment at time t reduces the owner's utility by $c(a_t/\omega)(1/I - 1/I^2)$, the cost of price adjustment is proportional to the idiosyncratic productivity factor a_t. A future price decrease (increase) is likely to be associated with a positive (negative) idiosyncratic productivity shock, so the owner is more willing to accept a lowering of his current utility to avoid a future price decrease than to avoid a future price increase. The optimal markup strategy therefore reduces the probability of a future price decrease relative to that of a future price increase by prescribing a lower adjustment

[10] That $s > 1$ is proved by noting that $\lim_{c\to 1} s = 1$, and Theorem 2 in the text shows that $ds/dc < 0$.

[11] The markups are found by solving $(a_t/\omega)(1/z_t - 1/z_t^2) = a_t$ and are $s' \equiv \frac{1}{2}[1-(1-4\omega)^{1/2}]/\omega$ and $S' \equiv \frac{1}{2}[1-(1-4\omega)^{1/2}]/\omega$. Since V_t increases in z_t if $z_t = \zeta_t \in (s, 2sS/(s+S))$ and decreases in z_t if $z_t = \zeta_t \in (2sS/(s+S), S)$, it follows that $s' > s$ and $S' < S$.

[12] The option effect is emphasized by Avinash Dixit (1991) and Caplin and Leahy (1997).

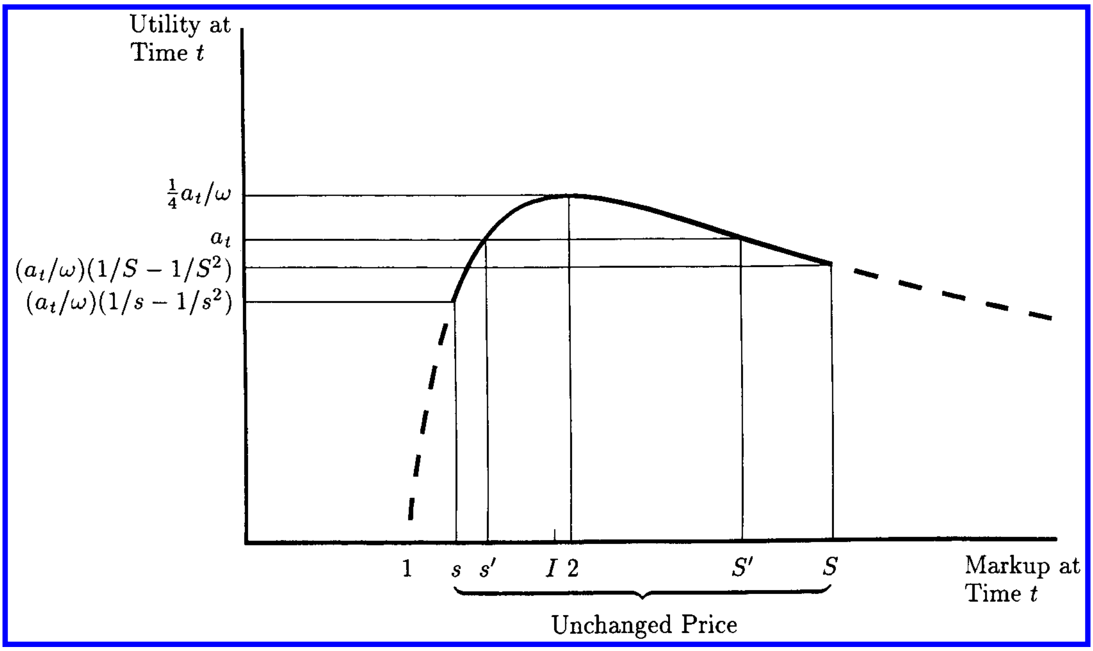

FIGURE 1. MARKUP WITH AN UNCHANGED PRICE AND THE UTILITY AT TIME t

bound s at which the current utility is less than at the upper adjustment bound S. For the same reason, the optimal markup strategy also prescribes an initial markup I which is less than two and does not maximize the current utility.[13]

The length of the nonadjustment interval for the markup is $S - s = 2IA/(1 - c)$. Since $\lim_{c\to 0}[(S - s)/c^{1/2}] = 4$, the nonadjustment interval is approximately $4c^{1/2}$ long for small c. As in the static quadratic model, a second-order small cost of price adjustment has a first-order effect on the length of the nonadjustment interval, and the derivative of the length of the nonadjustment interval with respect to the c is infinity in the limit as c approaches zero. One of the central features of the static price-adjustment model is therefore preserved by the dynamic generalization.[14]

The optimal markup strategy is invariant with respect to the trend in the aggregate productivity and with respect to the distribution of the aggregate productivity shocks. The reason is that both productivity in the firm and the equilibrium wage rate are proportional to the aggregate productivity, so that a firm's marginal cost and markup for a given price are unaffected by changes in the aggregate productivity.

The optimal markup strategy is also invariant with respect to the distribution of the monetary shocks. The uniform distribution of the idiosyncratic shocks leads to a loguniform distribution of ζ_t within the nonadjustment interval, so the distribution of the monetary shocks does not affect the distribution of ζ_t within the nonadjustment interval. It only affects the distribution of ζ_t outside the nonadjustment interval, in which case the idiosyncratic shocks cause a price adjustment for any monetary shock.

The demand function for a firm's product (4) together with the constant marginal product and the equilibrium wage rate imply that if a firm's

[13] A formal proof for $I < 2$ is that $\lim_{c\to 0} I = 2$ and Theorem 2 in the text shows that $dI/dc < 0$. The fact that $1/s - 1/s^2 - (1/S - 1/S^2) = 2A(I - 2)/I^2 < 0$ then shows that the utility is less at s than at S.

[14] The static model is obtained for $\rho \to \infty$ (or $\phi \to \infty$) and has $I = 2$, $s = 2/(1 + c^{1/2})$, and $S = 2/(1 - c^{1/2})$. See Akerlof and Yellen (1985), Mankiw (1985), and Dixit (1991). If uncertainty is modelled by a geometric Brownian motion without drift, a fourth-order small cost of price adjustment has a first-order effect on the length of the nonadjustment interval. See Dixit (1991) and Caplin and Leahy (1997).

price were always adjusted to maximize the current profit, employment in the firm would be inversely proportional to the real wage rate, independent of the aggregate productivity. Hence the aggregate demand for labor would be inversely proportional to the real wage rate. However, the price-adjustment cost makes it uneconomical to adjust the price at each t, and even when the price is adjusted, the new price is not set to maximize the firm's current profit. Nonetheless, the aggregate demand for labor is inversely proportional to the real wage rate just as if each firm's price were always adjusted to maximize the current profit.[15] Since the aggregate supply of labor is also inversely proportional to the real wage rate, the constant ω is determined so that the aggregate demand and supply for labor always coincide. As a result, the equilibrium wage rate is proportional to the aggregate productivity and to the money supply. The rate of change in the aggregate productivity and in the money supply are immediately and fully transmitted into the rate of change in the wage rate.

III. The Optimal Markup Strategy

The probability that a firm's price remains unchanged is endogenously determined. The uniform distribution of ζ_t within the nonadjustment interval implies that the probability of an unchanged price is a constant $\theta \equiv (1/\phi)\ \ln(S/s)$, independent of the time of the last price adjustment and of the current and previous shocks to the aggregate productivity and money supply. The expected period with an unchanged price increases with θ and becomes

$$\Omega \equiv \sum_{\tau=0}^{\infty} \theta^{\tau} = \frac{1}{1-\theta}.$$

Theorems 2–5 provide the comparative-static effects of all the parameters that affect the optimal markup strategy on the optimal strategy itself and on the expected period with an unchanged price (see Appendix B for proofs).

THEOREM 2: $dI/dc < 0$; $ds/dc < 0$; $dS/dc > 0$; $d\Omega/dc > 0$.

[15] See the proof of Lemma 4 in Appendix A.

The costlier it is to adjust a price, the longer an owner plans to keep his firm's price unchanged. So a higher cost of price adjustment is accompanied by a longer expected period with an unchanged price. This is obtained by decreasing the lower adjustment bound and increasing the upper adjustment bound. Since a higher cost of price adjustment makes it more important to reduce the likelihood of a future price decrease relative to that of a future price increase, it also decreases the initial markup.

THEOREM 3: $dI/d\rho > 0$; $ds/d\rho > 0$; $dS/d\rho > 0$; $d\Omega/d\rho > 0$.

The higher the discount rate, the less an owner is concerned with the future. This increases the weight of the price adjustment cost which is incurred at the beginning of a period with an unchanged price, and the expected period with an unchanged price therefore increases. Since a higher discount rate also makes it less important to reduce the likelihood of a future price decrease relative to that of a future price increase, it increases the lower and upper adjustment bounds and the initial markup (which increases the current profit).

THEOREM 4: $dI/d\phi > 0$; $ds/d\phi > 0$; $dS/d\phi > 0$; $d\Omega/d\phi < 0$.

The more uncertain the idiosyncratic shocks, the shorter is the period the price is expected to remain unchanged. Hence increased idiosyncratic riskiness reduces the influence of the future on the determination of the optimal markup strategy; the effects on the adjustment bounds and the initial markup are therefore similar to the effects of a higher discount rate. Since the optimal markup strategy is invariant with respect to the distributions of the shocks to the aggregate productivity and to the money supply, Theorem 4 underscores the necessity of differentiating between the different sources of uncertainty as they differ in their effects (or absence thereof) on the optimal markup strategy.

THEOREM 5: $dI/dm < 0$; $ds/dm < 0$; $dS/dm < 0$; $d\Omega/dm < 0$.

The higher the trend in the money supply, the larger is the idiosyncratic shock with which a future price decrease is likely to be associated, and the smaller is the idiosyncratic shock with which a future price increase is likely to be associated. Since the cost of price adjustment is proportional to the idiosyncratic productivity factor, a higher trend in the money supply makes it optimal to further reduce the likelihood of a future price decrease relative to that of a future price increase. This is obtained by reducing the adjustment bounds and the initial markup. The expected period with an unchanged price of course decreases with the trend in the money supply.[16]

IV. The Price Index and the Aggregate Output

The independence of the idiosyncratic productivity shocks implies that the price adjustments are staggered with the fraction $1 - \theta$ of the prices being adjusted at each t.[17] Since the markup is I for firms whose prices are adjusted and the markups are loguniformly distributed on (s, S) for firms whose prices are unchanged, the equilibrium distribution of markups is stationary. It is a mixed distribution where the markup I occurs with frequency $1 - \theta$ and the markups on (s, I) and (I, S) occur with loguniform density ϕ.

A firm's markup and price are independent of the aggregate productivity, since both the productivity in the firm and the equilibrium wage rate are proportional to the aggregate productivity. However, the firm's markup and price depend on the current and past money supplies, because the money supply affects the level of demand in addition to the equilibrium wage rate. As a result, the price index is independent of the aggregate productivity and can be expressed as a function of the current and past money supplies. Let $k \equiv (S - s)/(\phi I)$. It can be shown that (see Appendix C),

$$(5)\quad P_t = \frac{N}{I}\left[\frac{1}{M_t} + \frac{\theta - k + ck}{1 - c}\sum_{\tau=-\infty}^{t-1}\frac{(\theta - k)^{t-1-\tau}}{M_\tau}\right]^{-1},$$

where $\theta - k < 0 < \theta - k + ck$ and $ke^{m+\bar{\gamma}} < 1$. Since the velocity of money is unity, the aggregate output at t equals the real money balance M_t/P_t.

Figures 2A and 2B show the impulse response of a positive monetary shock at time t on the price index and the aggregate output, respectively, assuming there is no trend in the money supply and no previous or later monetary shocks. In Figure 2A the lower horizontal line indicates the price index before the shock, and the higher horizontal line the level to which the price index converges after the shock. In Figure 2B the horizontal line indicates both the aggregate output before the shock and the level to which it converges after the shock.

Generally, a positive monetary shock increases the contemporaneous and all future price indexes. A constant fraction of the prices are always adjusted, and the higher the shock, the more of the currently adjusted prices increase (and on average by more), and the less of the currently adjusted prices decrease (and on

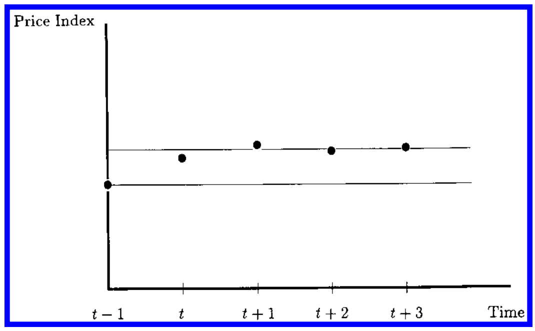

FIGURE 2A. IMPULSE RESPONSE OF A POSITIVE MONETARY SHOCK AT TIME t ON THE PRICE INDEX

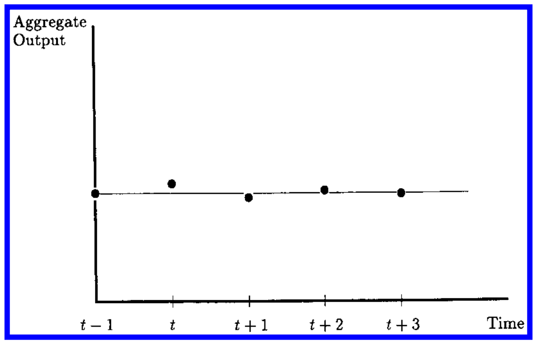

FIGURE 2B. IMPULSE RESPONSE OF A POSITIVE MONETARY SHOCK AT TIME t ON THE AGGREGATE OUTPUT

[16] Note that ρ, ϕ, and m affect the optimal markup strategy only through δ. Accordingly, $dI/d\rho = dI/d \ln \phi = -dI/dm$, and similarly for the effects on s and S. Additionally, $d\Omega/d\rho = -d\Omega/dm$.

[17] Hence, $N = 1 - c + c\theta$ is a constant.

average by less). Since the summation in equation (5) is positive, the elasticity of P_t with respect to the monetary change $e^{m+\gamma_t}$ is less than unity, which would indicate an immediate full adjustment of the price index to the monetary change. A monetary shock is therefore only partially passed on to the current price index. In the case of a positive monetary shock, the goods whose prices are adjusted have, on average, experienced smaller (i.e., more negative) idiosyncratic shocks than the goods whose prices remain unchanged, and the average relative increase in the adjusted prices exceeds the relative increase in the money supply. At time t, where the price index excludes some of the prices that are adjusted at t, the high average relative increase in the adjusted prices only partially compensates for the unchanged prices of the rest of the goods. At time $t + 1$ the index does not exclude prices because they were adjusted at time t, and P_{t+1} overcompensates for the increase in M_t. The elasticity of P_{t+1} with respect to the monetary change at t therefore exceeds unity. As a result, the price adjustments at time $t + 1$ moderate the overcompensation in P_{t+1}. At time $t + 2$, where the index does not exclude prices because they were adjusted at $t + 1$, the price adjustments that at time $t + 1$ moderated the overcompensation in P_{t+1} cause a new undercompensation in the index, but less than at time t so that the elasticity of the index is closer to unity than at time t, etc.[18] Equation (5) shows that in the long run the monetary shocks are fully passed on to the price index.

The aggregate output increases with the contemporaneous monetary shock since the price index does not fully compensate for that shock. Accordingly, a positive monetary surprise raises both the price index and the aggregate output, even if the surprise has been preceded by a long string of positive surprises. The correlation between unanticipated movements in the price index and in the aggregate output is always positive.[19]

In contrast, in Caplin and Leahy (1991, 1997) the effects of monetary changes depend on the location of the price distribution which moves within an elevator shaft. If the price distribution is at the top or bottom of the shaft, monetary changes cause a proportional change in the price index with no change in the aggregate output, while if the price distribution is in the interior of the shaft, monetary changes leave all prices unchanged while causing a proportional change in the aggregate output. In their model, positive monetary surprises increase the likelihood of future price increases for all firms in the economy, and the surprises may or may not increase the price index depending on the history of the monetary shocks.

In the present model, the idiosyncratic shocks assure that a constant fraction of the prices are always adjusted, keeping the distribution of the markups stationary. The monetary shock at time t determines in which firms prices will be adjusted at time t, and by how much. However, the size and direction of the monetary shock have no effect on the part of any future price index that stems from the firms whose prices are unchanged at time t. In the long run, the changes in the part of the price index that stems from the firms whose prices are adjusted at time t will make the whole price index change proportionally to the money supply.

V. The Welfare Effects of Shocks

A firm owner's discounted expected utility is given in Theorem 1 and is independent of the contemporaneous aggregate productivity shock

[18] The source of the fluctuations in the elasticity of the price index is that a fraction c of the adjusted prices are excluded from the current price index. The fluctuations therefore depend on c and are minor. For instance, the elasticity of P_t with respect to $e^{m+\gamma_t}$ is

$$\left[1 + \left(\frac{\theta - k + ck}{1 - c}\right) M_t \sum_{\tau=-\infty}^{t-1} \frac{(\theta - k)^{t-1-\tau}}{M_\tau}\right]^{-1} > 1 - c,$$

since $\theta - k + ck < ck$ and $M_t \sum_{\tau=-\infty}^{t-1} (\theta - k)^{t-1-\tau}/M_\tau < e^{m+\bar{\gamma}} < 1/k$.

[19] See Cooley and Ohanian (1991) and Rotemberg (1996) for empirical support on this. A monetary shock at time t causes the price index and the aggregate output to move in the opposite directions at any $t' > t$, since the sum of the elasticities of the price index and the aggregate output with respect to the monetary change is unity. The correlation between anticipated movements in the price index and in the aggregate output is therefore negative. This also finds empirical support in the above papers. However, we do not emphasize this feature of the model since the movements in the price index and the aggregate output are oscillatory and quickly damped.

Changes in the aggregate productivity lead to inversely proportional changes in the employment, so the aggregate output, like the price index, is independent of the aggregate productivity.

(and all other aspects of the aggregate productivity). The discounted expected utility is, however, dependent on the contemporaneous monetary shock through its effect on what the markup would be if the price is kept unchanged. Since ζ_t also depends on the contemporaneous idiosyncratic shock, the monetary shock affects the discounted expected utility positively for some owners, negatively for other owners, and has no effect for still other owners. On average, the discounted expected utility increases linearly with the realized monetary change.[20] Accordingly, each owner expects his welfare to increase with the monetary shock, while the future monetary uncertainty has no effect on the owner's welfare.

Turning to the households, substituting a household's consumption and labor supply in its utility function shows that the household's utility at time t is $w_t N/P_t$. Its discounted expected utility is

$$\sum_{\tau=t}^{\infty} e^{-\rho\tau} E\left(\frac{w_\tau N}{P_\tau}\right),$$

where the household's discount rate is also ρ ($\rho > g$) and the expectation is taken with respect to the distribution of all future utilities.

The households' welfare increases with the realized aggregate productivity shock (and with the trend in the aggregate productivity) since the households, as providers of labor, reap the benefit or incur the loss from the proportional change in the real wage rate that accompanies the change in the aggregate productivity. They also carry all the risks from the future fluctuations in the aggregate productivity. The households are risk averse as their utility function is strictly concave in consumption and labor. However, their (indirect) utility ($w_t N/P_t$) is proportional to the real wage rate, and hence to the aggregate productivity, as they gain from optimally adjusting their consumption and labor supply to changes in the real wage rate. Consequently, the households' welfare is not affected by the uncertainty of the future aggregate productivity.

The households' welfare also increases with the contemporaneous monetary shock. This is because the monetary shock is passed on fully to the current wage rate, but only partially to the current price index. The larger a positive monetary shock, therefore, the more the real wage rate increases, and the smaller a negative monetary shock, the more the real wage rate decreases. Since the subsequent adjustments of the price index to the shock are minor, the welfare effect of the change in the current real wage rate dominates.

The households are exposed to the risks from the future monetary uncertainty. However, since the real wage rate at any future time is linear in the monetary change caused by each of the monetary shocks that have occurred up to and including that time itself, the distribution of the future monetary shocks do not affect the households' welfare.

The independence of the households' welfare of the uncertainty of the future aggregate productivity and money supply is a consequence of the (indirect) utility being proportional to the real wage rate. An increase in the households' risk aversion, obtained by an increasing concave transformation of the (direct) utility function, would cause the uncertainty of the future aggregate productivity and money supply to have a negative impact on the households' welfare.

VI. Conclusion

This paper studies a general-equilibrium model of a dynamic economy with costly price adjustments. The productivity in each firm is exposed to idiosyncratic and aggregate productivity shocks around a trend, and the money supply is exposed to monetary shocks around a trend. All consumption, pricing, and production decisions are based on optimizing behavior. In the staggered equilibrium all prices are determined by a (s, S) markup strategy, the probability that a firm's price is unchanged is a

[20] For a given γ_{t+1}, the discounted expected value of V_{t+1} is at time t is

$$\frac{a_t}{1-e^{-\rho}} + \frac{a_t}{\phi}\int_{\gamma_{t+1}+m+\ln(s/z_t)}^{\gamma_{t+1}+m+\ln(S/z_t)} e^{\alpha_{t+1}}\left(\frac{Se^{\gamma_{t+1}+m-\alpha_{t+1}}}{z_t}-1\right)$$

$$\times\left(1-\frac{se^{\gamma_{t+1}+m-\alpha_{t+1}}}{z_t}\right)d\alpha_{t+1}$$

$$= \frac{a_t}{1-e^{-\rho}} + \frac{a_t e^{\gamma_{t+1}+m}(S+s)}{z_t\phi}\ln\left(\frac{S}{s}\right),$$

which increases linearly with $e^{\gamma_{t+1}+m}$.

constant, and the markups are loguniformly distributed for the firms whose prices are unchanged. The wage rate is proportional to the aggregate productivity and the money supply.

It would be desirable to model the money demand in more detail by allowing the agents to borrow and lend at an endogenously determined interest rate, instead of requiring them to spend all their income at the time it is earned. Aggregate productivity shocks and monetary shocks would then have additional effects on the equilibrium as the interest rate adjusts and the agents adapt to the shocks.

The general-equilibrium framework makes the model an attractive vehicle for analyzing the aggregate effects of different monetary policies. An extension that includes a less mechanic money demand would make the model even more suitable for shedding light on such important issues as whether there exists an optimal trend in the money supply, whether a transition from an inflationary to a stabilizing monetary policy will improve welfare, and how the monetary authority should respond to shocks.

The modeling of uncertainty—in particular the uniform distribution of the idiosyncratic shocks—is an attractive methodological alternative to the standard geometric Brownian motion. There is no need to keep track of complicated combinations of truncated normal distributions. The simplicity of the optimal (s, S) markup strategy is a major analytical advantage that makes it possible to determine the aggregate dynamics of the economy and to examine the welfare consequences of the realized shocks to the aggregate productivity and money supply, and of their future uncertainty. A similar formalization of uncertainty might be useful in other cases where it is important to trace the evolution of an economy with adjustment costs in which agents follow optimal adjustment strategies, e.g., in models concerned with investments or inventories.

APPENDIX A: THE EXISTENCE OF A MARKOV PERFECT EQUILIBRIUM

The first subsection establishes Lemmas 1 and 2, which are used to obtain an expression for a firm owner's discounted expected utility if $w_\tau = \omega b_\tau M_\tau$ at all τ and the price is set by a two-sided (s, S) markup strategy satisfying $\ln(S/s) \le \delta$. The second subsection determines the candidate markup strategy from the first-order conditions for an internal maximum. The third subsection proves Lemma 3, which verifies that the candidate strategy is optimal if $w_\tau = \omega b_\tau M_\tau$ at all τ. The proof consists of showing that the discounted expected utility at time t is reduced by deviating from the candidate strategy at time t and following the candidate strategy thereafter. The fourth subsection derives Lemma 4, which shows that the labor market is in equilibrium if $w_\tau = \omega b_\tau M_\tau$ at all τ and prices are set optimally. Combining Lemmas 3 and 4 proves Theorem 1.

1. *The Discounted Expected Utility from a Two-Sided (s, S) Markup Strategy*

LEMMA 1: *If $w_\tau = \omega b_\tau M_\tau$ at all τ and the price is determined by a two-sided (s, S) markup strategy with $\ln(S/s) \le \sigma$, at time t an owner's discounted expected utility until the first price adjustment after t is*

$$\frac{a_t}{\omega}\left\{\frac{1}{z_t}\left[\frac{1-\delta(1/s-1/S)}{1-\delta\ln(S/s)}\right]-\frac{1}{z_t^2}\right\} \qquad \text{if } \zeta_t \in (s, S),$$

$$\frac{a_t}{\omega}\left\{\frac{1}{I}\left[\frac{1-\delta(1/s-1/S)}{1-\delta\ln(S/s)}\right]-\frac{1}{I^2}-c\left(\frac{1}{I}-\frac{1}{I^2}\right)\right\} \qquad \text{if } \zeta_t \notin (s, S).$$

PROOF:

The Case of $\zeta_t \in (s, S)$: The Price Is Unchanged at Time t.—First, since the price is unchanged at time t, it follows that $z_t = \zeta_t$ and that the utility at t is $(a_t/\omega)(1/z_t - 1/z_t^2)$.

Next, the discounted expected value of the owner's utility at each $\tau > t$ until the next price adjustment is determined. For the price to remain unchanged through time t', it must be the case that $\zeta_\tau \in (s, S)$ for

$\tau = t + 1, t + 2, \ldots, t'$. Let $\varepsilon_\tau \equiv \gamma_\tau + m$ denote the rate of change in the money supply at time τ. Since $\zeta_\tau = z_{\tau-1}e^{\varepsilon_\tau - \alpha_\tau}$, an equivalent condition for the price to be unchanged through time t' is that $\alpha_\tau \in (\varepsilon_\tau + \ln(z_{\tau-1}/S); \varepsilon_\tau + \ln(z_{\tau-1}/s))$ for $\tau = t + 1, t + 2, \ldots, t'$. This nonadjustment interval for α_τ is contained in $(\varepsilon_\tau + \ln(s/S); \varepsilon_\tau + \ln(S/s))$, which itself is contained in $[\underline{\alpha}, \bar{\alpha}]$ on which α_τ is uniformly distributed [by the assumption that $\ln(S/s) \le \sigma$]. It follows that the discounted expected value at time $\tau - 1$ of the owner's utility at time τ before the next price adjustment is

$$\text{(A1)} \quad \frac{e^{-\rho}}{\phi}\int_{\underline{\gamma}}^{\bar{\gamma}}\left[\int_{\varepsilon_\tau+\ln(s/z_{\tau-1})}^{\varepsilon_\tau+\ln(S/z_{\tau-1})}\left(\frac{a_\tau}{\omega}\right)\left(\frac{1}{z_\tau}-\frac{1}{z_\tau^2}\right)d\alpha_\tau\right]\chi(\gamma_\tau)\,d\gamma_\tau$$

$$= \frac{a_{\tau-1}e^{-\rho}}{\omega\phi}\int_{\underline{\gamma}}^{\bar{\gamma}}\left[\int_{\varepsilon_\tau+\ln(s/z_{\tau-1})}^{\varepsilon_\tau+\ln(S/z_{\tau-1})}\left(\frac{e^{\varepsilon_\tau}}{z_{\tau-1}}-\frac{e^{2\varepsilon_\tau-\alpha_\tau}}{z_{\tau-1}^2}\right)d\alpha_\tau\right]\chi(\gamma_\tau)\,d\gamma_\tau$$

$$= \frac{a_{\tau-1}e^{m-\rho}}{z_{\tau-1}\omega\phi}\left[\ln\left(\frac{S}{s}\right)+\frac{1}{S}-\frac{1}{s}\right]\int_{\underline{\gamma}}^{\bar{\gamma}}e^{\gamma_\tau}\chi(\gamma_\tau)\,d\gamma_\tau$$

$$= \frac{a_{\tau-1}}{z_{\tau-1}\omega}\left[1-\frac{1/s-1/S}{\ln(S/s)}\right]\delta\ln\left(\frac{S}{s}\right).$$

This expression depends on $a_{\tau-1}/z_{\tau-1}$. For the realizations for which the price is unchanged at time $\tau - 1$, the discounted expected value of $a_{\tau-1}/z_{\tau-1} = (a_{\tau-2}/z_{\tau-2})e^{m+\gamma_{\tau-1}-\rho}$ at time $\tau - 2$ is $(a_{\tau-2}/z_{\tau-2})\delta \ln(S/s)$. By induction, the discounted expected value of $a_{\tau-1}/z_{\tau-1}$ from the realizations for which the price is unchanged from time t through time $\tau - 1$ is

$$\text{(A2)} \quad \frac{a_t}{z_t}\left[\delta\ln\left(\frac{S}{s}\right)\right]^{\tau-t-1}.$$

Together, (A1) and (A2) imply that the discounted expected value at time t of the owner's utility at time τ before the next price adjustment is

$$\frac{a_t}{z_t\omega}\left[1-\frac{1/s-1/S}{\ln(S/s)}\right]\left[\delta\ln\left(\frac{S}{s}\right)\right]^{\tau-t}.$$

Finally, add the utility at t and the discounted expected utility at each $\tau > t$ before the next price adjustment,[21]

$$\text{(A3)} \quad \frac{a_t}{\omega}\left(\frac{1}{z_\tau}-\frac{1}{z_\tau^2}\right)+\frac{a_t}{z_t\omega}\left[1-\frac{1/s-1/S}{\ln(S/s)}\right]\sum_{\tau=1}^{\infty}\left[\delta\ln\left(\frac{S}{s}\right)\right]^{\tau}$$

$$= \frac{a_t}{\omega}\left\{\frac{1}{z_t}-\frac{1}{z_t^2}+\frac{1}{z_t}\left[1-\frac{1/s-1/S}{\ln(S/s)}\right]\frac{\delta\ln(S/s)}{1-\delta\ln(S/s)}\right\}$$

$$= \frac{a_t}{\omega}\left\{\frac{1}{z_t}\left[\frac{1-\delta(1/s-1/S)}{1-\delta\ln(S/s)}\right]-\frac{1}{z_t^2}\right\}.$$

[21] Since the expected utility at any future τ is upward bounded by $\frac{1}{4}a_t/\omega$, it follows that $\delta \ln(S/s) < 1$.

The Case of $\zeta_t \notin (s, S)$: The Price Is Adjusted at Time t.—Since the initial markup is I, the owner's discounted expected utility until the first price adjustment after time t is obtained from (A3) by substituting $z_t = I$ and subtracting the cost of price adjustment, $(a_t/\omega)c(1/I - 1/I^2)$,

$$\frac{a_t}{\omega}\left\{\frac{1}{I}\left[\frac{1-\delta(1/s-1/S)}{1-\delta\ln(S/s)}\right]-\frac{1}{I^2}-c\left(\frac{1}{I}-\frac{1}{I^2}\right)\right\}.$$

LEMMA 2: *If $w_\tau = \omega b_\tau M_\tau$ at all τ and the price is determined by a two-sided (s, S) markup strategy with $\ln(S/s) \leq \sigma$, at time t the discounted expected value of the idiosyncratic productivity factor at the first price adjustment after t is*

$$a_t\left\{e^{-\rho}-\frac{\delta(S-s)(1-e^{-\rho})}{z_t[1-\delta\ln(S/s)]}\right\}.$$

PROOF:

If the first price adjustment after time t occurs at time T, the discounted expected value of a_T at time $T - 1$ is

$$\text{(A4)}\qquad \frac{a_{T-1}e^{-\rho}}{\phi}\int_{\underline{\gamma}}^{\bar{\gamma}}\left[\int_{\underline{\alpha}}^{\varepsilon_T+\ln(s/z_{T-1})}e^{\alpha_T}\,d\alpha_T+\int_{\varepsilon_T+\ln(S/z_{T-1})}^{\bar{\alpha}}e^{\alpha_T}\,d\alpha_T\right]\chi(\gamma_T)\,d\gamma_T$$

$$= a_{T-1}e^{-\rho}\left[1-\frac{e^m(S-s)}{z_{T-1}\phi}\int_{\underline{\gamma}}^{\bar{\gamma}}e^{\gamma_T}\chi(\gamma_T)\,d\gamma_T\right]$$

$$= a_{T-1}\left[e^{-\rho}-\frac{\delta(S-s)}{z_{T-1}}\right].$$

Thus, the discounted expected value of a_{t+1} for the realizations for which the first price adjustment after t occurs at $t + 1$ is

$$\text{(A5)}\qquad a_t\left[e^{-\rho}-\frac{\delta(S-s)}{z_t}\right].$$

If the first price adjustment after t takes place after $t + 1$, the discounted expected value of a_{T-1} at $T - 2$ is

$$\frac{a_{T-2}e^{-\rho}}{\phi}\int_{\underline{\gamma}}^{\bar{\gamma}}\left[\int_{\varepsilon_{T-1}+\ln(s/z_{T-2})}^{\varepsilon_{T-1}+\ln(S/z_{T-2})}e^{\alpha_{T-1}}\,d\alpha_{T-1}\right]\chi(\gamma_{T-1})\,d\gamma_{T-1}$$

$$= \frac{a_{T-2}e^{m-\rho}(S-s)}{z_{T-2}\phi}\int_{\underline{\gamma}}^{\bar{\gamma}}e^{\gamma_{T-1}}\chi(\gamma_{T-1})\,d\gamma_{T-1}$$

$$= \frac{a_{T-2}\delta(S-s)}{z_{T-2}}.$$

Substituting this in (A4) and using that the discounted expected value of a_{T-1}/z_{T-1} at $T - 2$ is $(a_{T-2}/z_{T-2})\delta \ln(S/s)$, the discounted expected value of a_T at $T - 2$ becomes

$$\frac{a_{T-2}\delta(S - s)}{z_{T-2}} e^{-\rho} - \frac{a_{T-2}\delta^2(S - s)}{z_{T-2}} \ln\left(\frac{S}{s}\right) = \frac{a_{T-2}\delta(S - s)}{z_{T-2}} \left[e^{-\rho} - \delta \ln\left(\frac{S}{s}\right)\right].$$

Using (A2), the discounted expected value of a_T at time t for those realizations for which the first price adjustment after t occurs after $t + 1$ becomes

$$\text{(A6)} \qquad \frac{a_t\delta(S - s)}{z_t} \left[e^{-\rho} - \delta \ln\left(\frac{S}{s}\right)\right]\left[\delta \ln\left(\frac{S}{s}\right)\right]^{\tau-t-2}.$$

Finally, add (A5) and (A6) for all $\tau > t + 1$ to obtain the discounted expected value of the productivity at the first price adjustment after time t,

$$a_t\left[e^{-\rho} - \frac{\delta(S - s)}{z_t}\right] + \frac{a_t\delta(S - s)}{z_t} \left[e^{-\rho} - \delta \ln\left(\frac{S}{s}\right)\right] \sum_{\tau=2}^{\infty} \left[\delta \ln\left(\frac{S}{s}\right)\right]^{\tau-2}$$

$$= a_t\left\{ e^{-\rho} - \frac{\delta(S - s)(1 - e^{-\rho})}{z_t[1 - \delta \ln(S/s)]}\right\}.$$

Given the assumptions of Lemmas 1 and 2, we can calculate V_t for a two-sided (s, S) markup strategy. If $\zeta_t \notin (s, S)$ and the price is adjusted at time t,

$$V_t = \frac{a_t}{\omega}\left\{\frac{1}{I}\left[\frac{1 - \delta(1/s - 1/S)}{1 - \delta \ln(S/s)}\right] - \frac{1}{I^2} - c\left(\frac{1}{I} - \frac{1}{I^2}\right)\right\} \sum_{i=0}^{\infty} \left\{ e^{-\rho} - \frac{\delta(S - s)(1 - e^{-\rho})}{I[1 - \delta \ln(S/s)]}\right\}^i$$

$$= \frac{a_t([1 - \delta(1/s - 1/S)]/\{I[1 - \delta \ln(S/s)]\} - 1/I^2 - c(1/I - 1/I^2))}{\omega(1 - e^{-\rho})(1 + \delta(S - s)/\{I[1 - \delta \ln(S/s)]\})};$$

while if $\zeta_t \in (s, S)$ and the price is unchanged at time t,

$$V_t = \frac{a_t}{\omega}\left\{\frac{1}{z_t}\left[\frac{1 - \delta(1/s - 1/S)}{1 - \delta \ln(S/s)}\right] - \frac{1}{z_t^2}\right\} + \left\{ e^{-\rho} - \frac{\delta(S - s)(1 - e^{-\rho})}{z_t[1 - \delta \ln(S/s)]}\right\} V_t^a,$$

where V_t^a denotes V_t if the price is adjusted at time t.

2. *The Candidate Markup Strategy*

The candidate two-sided (s, S) markup strategy is determined from the first-order conditions for an internal maximum for V_t. Differentiate V_t partially with respect to I, s, and S, and reduce to obtain that the candidate strategy is

$$\text{(A7)} \qquad I = Q^{-1}(0),$$

(A8) $$s = \frac{I}{1 - c + \frac{1}{2}cI + A},$$

(A9) $$S = \frac{I}{1 - c + \frac{1}{2}cI - A}.$$

Condition (A7) determines I uniquely since it implies that $\lim_{c \to 0} I = 2$ and that $dI/dc < 0$ (from Theorem 2). Conditions (A8) and (A9) then determine s and S uniquely, assuming that $I > 1$. However, since $c < 1 \Rightarrow s > 1$ [because condition (A8) shows that $\lim_{c \to 1} s = 1$, and Theorem 2 that $ds/dc < 0$] and $s > 1 \Rightarrow I > s$ [from condition (A8)], it follows that $I > 1$.[22]

The expression for V_t for a two-sided (s, S) strategy is valid only under the assumption that $\ln(S/s) \leq \sigma$. However, the candidate strategy satisfies this assumption since Theorem 2 implies that $d \ln(S/s)/dc > 0$, and the right-hand side of Assumption 2 is positive, increases in σ, and if substituted for c in conditions (A7)–(A9) yields $\ln(S/s) = \sigma$.

3. *The Dynamic Programming Problem*

LEMMA 3: *If $w_\tau = \omega b_\tau M_\tau$ at all τ, the candidate two-sided (s, S) markup strategy is optimal.*

PROOF:

Let $\tilde{V}_t$ denote a firm owner's discounted expected utility if $w_\tau = \omega b_\tau M_\tau$ at all τ and the markup strategy is optimal. Since the expected utility at a future τ is upward bounded by $\frac{1}{4} a_t/\omega$, the necessary and sufficient condition for the markup strategy to be optimal is that

$$\tilde{V}_t = \max_{z_t} \left[\frac{(1 - c_t) a_t}{\omega} \left(\frac{1}{z_t} - \frac{1}{z_t^2} \right) + e^{-\rho} E \tilde{V}_{t+1} \right],$$

where the expectation is taken with respect to the distributions of α_{t+1} and γ_{t+1}. To prove that the candidate markup strategy is optimal, it suffices to show that the value cannot be increased by deviating from the strategy at time t, and following the candidate strategy thereafter.

Substituting from conditions (A7)–(A9) and $\omega = (1 - c)/I^2$ in V_t, the discounted expected utility from the candidate strategy is

$$V_t = \begin{cases} a_t \left[\dfrac{1}{1 - e^{-\rho}} + \left(\dfrac{S}{z_t} - 1 \right) \left(1 - \dfrac{s}{z_t} \right) \right] & \text{if } \zeta_t \in (s, S), \\ \dfrac{a_t}{1 - e^{-\rho}} & \text{if } \zeta_t \notin (s, S). \end{cases}$$

The discounted expected utility at time $t + 1$ from the candidate strategy is determined by the choice of z_t and the realizations of α_{t+1} and γ_{t+1}. By a change of the time index in the expression for V_t and substituting $z_{t+1} = z_t e^{\alpha_{t+1} - \varepsilon_{t+1}}$, one obtains that V_{t+1} for the candidate strategy is

$$V_{t+1} = \begin{cases} a_t e^{\alpha_{t+1}} \left[\dfrac{1}{1 - e^{-\rho}} + \left(\dfrac{S e^{\varepsilon_{t+1} - \alpha_{t+1}}}{z_t} - 1 \right) \left(1 - \dfrac{s e^{\varepsilon_{t+1} - \alpha_{t+1}}}{z_t} \right) \right] & \text{if } \zeta_{t+1} \in (s, S), \\ \dfrac{a_t e^{\alpha_{t+1}}}{1 - e^{-\rho}} \quad \text{if } \zeta_{t+1} \notin (s, S). \end{cases}$$

[22] As pointed out in footnote 21, $\delta \ln(S/s) = \delta B < 1$.

It is simple to show that the expected value at time t of V_{t+1} from the candidate strategy does not exceed

$$a_t\left[\frac{e^{-\rho}}{1-e^{-\rho}}+\frac{I(2-I)}{z_t}\right].$$

Suppose now that $\zeta_t \in (s, S)$. If the firm owner deviates from the candidate markup strategy by adjusting the price at time t and follows the candidate markup strategy thereafter, his discounted expected utility at time t is at most

$$\text{(A10)}\qquad a_tI^2\left(\frac{1}{z_t}-\frac{1}{z_t^2}\right)+a_t\left[\frac{e^{-\rho}}{1-e^{-\rho}}+\frac{I(2-I)}{z_t}\right]$$
$$=a_t\left[\frac{1}{1-e^{-\rho}}-\left(\frac{1}{I}-\frac{1}{z_t}\right)^2\right],$$

and hence less than V_t from the candidate strategy.

Suppose instead that $\zeta_t \notin (s, S)$. The firm owner can then deviate from the candidate strategy at time t either by adjusting the price so that $z_t \neq I$ or by keeping the price unchanged. If he deviates by adjusting the price so that $z_t \neq I$ and follows the candidate strategy thereafter, then (A10) also shows that his discounted expected utility at time t is less than V_t from the candidate strategy. If he deviates by keeping the price unchanged and follows the candidate strategy thereafter, his discounted expected utility at time t is at most

$$\frac{a_t}{\omega}\left(\frac{1}{z_t}-\frac{1}{z_t^2}\right)+a_t\left[\frac{e^{-\rho}}{1-e^{-\rho}}+\frac{I(2-I)}{z_t}\right]$$
$$=a_t\left[\frac{1}{1-e^{-\rho}}+\left(\frac{S}{z_t}-1\right)\left(1-\frac{s}{z_t}\right)\right],$$

which does not exceed V_t from the candidate strategy (and is less except if $\zeta_t = s$ or $\zeta_t = S$).

Consequently, no deviation from the candidate strategy at time t can increase the owner's discounted expected utility. The two-sided (s, S) markup strategy given by conditions (A7)–(A9) is therefore optimal.

4. *The Labor-Market Equilibrium*

LEMMA 4: *If $w_\tau = \omega b_\tau M_\tau$ at each τ and each firm's price is set optimally, the aggregate demand and the aggregate supply for labor are equal.*

PROOF:

The demand for labor by a firm whose price is unchanged at time t is [see equation (4)]

$$\frac{M_tP_t}{q_tp_t^2N}=\frac{a_tP_tI^2}{(1-c)z_t^2Nw_t}=\frac{a_{t-1}P_tI^2}{(1-c)z_{t-1}^2Nw_t}\,e^{2\varepsilon_t-\alpha_t},$$

while the demand for labor by a firm whose price is adjusted at time t is

$$\frac{(1-c)M_tP_t}{q_tp_t^2N}=\frac{a_tP_t}{Nw_t}=\frac{a_{t-1}P_t}{Nw_t}\,e^{\alpha_t}.$$

At time $t - 1$, therefore, a firm's expected demand for labor at time t is

$$\frac{a_{t-1}P_t}{\phi N w_t}\int_{\underline{\alpha}}^{\bar{\alpha}} e^{\alpha_t}\,d\alpha_t + \frac{a_{t-1}P_t}{\phi N w_t}\int_{\underline{\gamma}}^{\bar{\gamma}}\left\{\int_{\varepsilon_t+\ln(s/z_{t-1})}^{\varepsilon_t+\ln(S/z_{t-1})}\left[\frac{I^2 e^{2\varepsilon_t-\alpha_t}}{(1-c)z_{t-1}^2} - e^{\alpha_t}\right]d\alpha_t\right\}\chi(\gamma_t)\,d\gamma_t$$

$$= \frac{a_{t-1}P_t}{Nw_t} + \frac{a_{t-1}P_t}{Nw_t}\int_{\underline{\gamma}}^{\bar{\gamma}}\left[\frac{I^2 e^{\varepsilon_t}}{(1-c)z_{t-1}}\left(\frac{1}{s}-\frac{1}{S}\right) - \frac{e^{\varepsilon_t}(S-s)}{z_{t-1}}\right]\chi(\gamma_t)\,d\gamma_t$$

$$= \frac{a_{t-1}P_t}{Nw_t} + \frac{a_{t-1}P_t(S-s)e^m}{z_{t-1}Nw_t}\left[\frac{I^2}{(1-c)Ss} - 1\right]$$

$$= \frac{a_{t-1}P_t}{Nw_t}.$$

Since the idiosyncratic productivity factor is unity on average, the aggregate demand for labor at time t is $P_t/(w_t N)$.

The set of households measures unity, so equation (3) also expresses the aggregate supply of labor at time t. It follows that the aggregate demand and the aggregate supply for labor are equal.

The proof of Theorem 1 is completed by combining Lemmas 3 and 4.

Appendix B: Proofs of the Comparative-Static Theorems for the Optimal Markup Strategy

THEOREM 2 [*The Cost of Price Adjustment*]: Differentiate I totally with respect to c,

$$\frac{dI}{dc} = -\frac{\partial Q/\partial c}{\partial Q/\partial I}.$$

Since

$$\frac{\partial Q}{\partial c} = -\frac{\delta A + (1-c)(1-\frac{1}{2}I)(1-\delta B)}{(1-c)^2 I(1-\delta B)} < 0,$$

$$\frac{\partial Q}{\partial I} = -\frac{1-c+c\delta B}{2(1-c)I(1-\delta B)} < 0,$$

it follows that

$$\frac{dI}{dc} = -\frac{2[\delta A + (1-c)(1-\frac{1}{2}I)(1-\delta B)]}{(1-c)[(1-c)+c\delta B]} < 0.$$

Differentiate s totally with respect to c,

$$\frac{ds}{dc} = \frac{\partial s}{\partial c} + \frac{\partial s}{\partial I}\frac{dI}{dc}.$$

Since $(1 - \frac{1}{2}I)^2 < [(\frac{1}{2} - c)(I - 1) + cI^2]^2/A^2 \Leftrightarrow 0 < (I - 1)^2$ implies that

$$\frac{\partial s}{\partial c} = \frac{s^2}{I}\left[1 - \frac{1}{2}I - \frac{(\frac{1}{2} - c)(I - 1) + cI^2}{A}\right] < 0,$$

and

$$\frac{\partial s}{\partial I} = \frac{s^2(1 - c)(A - c + \frac{1}{2}cI)}{I^2A} > \frac{s^2(1 - c)c(I - 1)}{I^2A} > 0,$$

it follows that $ds/dc < 0$.

Differentiate S totally with respect to c using that $S = 1/(\omega s)$,

$$\frac{dS}{dc} = \frac{1}{\omega s}\left(-\frac{1}{\omega}\frac{d\omega}{dc} - \frac{1}{s}\frac{ds}{dc}\right) > 0,$$

since $d\omega/dc < 0$ [otherwise it would be impossible that t is independent of c if $\zeta_t \notin (s, S)$].

Differentiate Ω totally with respect to c,

$$\frac{d\Omega}{dc} = \frac{\Omega^2}{\phi}\left(\frac{1}{S}\frac{dS}{dc} - \frac{1}{s}\frac{ds}{dc}\right) > 0$$

since $dS/dc > 0$ and $ds/dc < 0$.

THEOREM 3 [*The Discount Rate*]: Differentiate I totally with respect to ρ,

$$\frac{dI}{d\rho} = -\frac{\partial Q/\partial\rho}{\partial Q/\partial I} > 0,$$

since $\partial Q/\partial I < 0$ and

$$\frac{\partial Q}{\partial\rho} = \frac{\delta(BI - 4A)}{(1 - c)(1 - \delta B)^2} > 0.$$

Differentiate s totally with respect to ρ,

$$\frac{ds}{d\rho} = \frac{\partial s}{\partial I}\frac{dI}{d\rho} > 0,$$

since $\partial s/\partial I > 0$.

Differentiate S totally with respect to ρ,

$$\frac{dS}{d\rho} = \frac{\partial S}{\partial I}\frac{dI}{d\rho} > 0,$$

since

$$\frac{\partial S}{\partial \rho} = \frac{S^2(1-c)(A+c-\frac{1}{2}cI)}{I^2 A} > 0.$$

Differentiate Ω totally with respect to ρ,

$$\frac{d\Omega}{d\rho} = \frac{\Omega^2}{\phi}\left(\frac{1}{S}\frac{\partial S}{\partial I} - \frac{1}{s}\frac{\partial s}{\partial I}\right)\frac{dI}{d\rho} > 0,$$

since $(1/S)\partial S/\partial I - (1/s)\partial s/\partial I = c/A > 0$.

THEOREMS 4 and 5 [*The Idiosyncratic Uncertainty and the Trend in the Money Supply*]: The formulas for the optimal markup strategy and Ω reveal that ρ, ϕ, and m affect I, s, S, and Ω only through δ, except that ϕ also affects Ω separately. The proofs of Theorems 4 and 5 are therefore similar to the proof of Theorem 3, except for the effect of ϕ on Ω. To complete the proof of Theorem 4, therefore, differentiate Ω totally with respect to ϕ,

$$\frac{d\Omega}{d\phi} = \frac{\Omega^2}{\phi}\left(-\frac{B}{\phi} + \frac{1}{S}\frac{dS}{d\phi} - \frac{1}{s}\frac{ds}{d\phi}\right)$$

$$= \frac{\Omega^2}{\phi}\left(-\frac{B}{\phi} + \frac{c}{A}\frac{dI}{d\phi}\right).$$

Since

$$\frac{dI}{d\phi} = -\frac{\partial Q/\partial \phi}{\partial Q/\partial I}$$

$$= \frac{\delta(BI-4A)}{(1-\delta B)(1-c+c\delta B)\phi},$$

one obtains

$$\frac{d\Omega}{d\phi} = \frac{\Omega^2}{\phi^2}\left[-B + \frac{\delta(BI-4A)}{(1-\delta B)(1-c+c\delta B)}\right]$$

$$= \frac{\Omega^2}{\phi^2}\left[-B + \frac{2c(1-c)\delta(B-2A)}{A(1-c+c\delta B)^2}\right].$$

The term in the brackets is maximized for $\delta = (1-c)/(cB)$. Since the maximum is $-B + (B - 2A)/(BA) < 0$, it follows that $d\Omega/d\phi < 0$.

APPENDIX C: THE PRICE INDEX

The Derivation of the Price Index

The price index is the harmonic mean of the prices of the goods an individual consumes. Since a good whose price is adjusted is unavailable to the fraction c of the individuals, the price index at t is the inverse of the integral of $(1-c_t)/p_t$ of all the goods in the economy multiplied by N. To determine this integral, we consider a good with a new price at some time $\hat{t}$ and derive the expected value of $(1 - c_{\hat{t}+1})/p_{\hat{t}+1}$ for this good, then the expected value of $(1 - c_{\hat{t}+2})/p_{\hat{t}+2}$, and then, by induction, the expected value of $(1 - c_t)/p_t$. The latter is used to obtain the integral of $(1 - c_t)/p_t$ of all the goods in the economy.

Consider a good whose price is new at time $\hat{t}$. If the price is unchanged at time $\hat{t} + 1$, which happens with probability θ, then $p_{\hat{t}+1} = p_{\hat{t}}$. If the price is adjusted at time $\hat{t} + 1$, which happens with probability $1 - \theta$, then $p_{\hat{t}+1} = p_{\hat{t}} e^{\varepsilon_{\hat{t}+1} - \alpha_{\hat{t}+1}}$, in which case the expected value at $\hat{t}$ of $1/p_{\hat{t}+1}$ for a given $\varepsilon_{\hat{t}+1}$ is

$$\text{(C1)} \qquad \frac{1}{p_{\hat{t}}(1-\theta)\phi}\left[\int_{\underline{\alpha}}^{\varepsilon_{\hat{t}+1}+\ln(s/I)} e^{\alpha_{\hat{t}+1}-\varepsilon_{\hat{t}+1}}\, d\alpha_{\hat{t}+1} + \int_{\varepsilon_{\hat{t}+1}+\ln(S/I)}^{\bar{\alpha}} e^{\alpha_{\hat{t}+1}-\varepsilon_{\hat{t}+1}}\, d\alpha_{\hat{t}+1}\right]$$

$$= \frac{e^{-\varepsilon_{\hat{t}+1}} - k}{p_{\hat{t}}(1-\theta)}.$$

Accordingly, the expected value at $\hat{t}$ of $(1 - c_{\hat{t}+1})/p_{\hat{t}+1}$ for a given $\varepsilon_{\hat{t}+1}$ is

$$\text{(C2)} \qquad \frac{1}{p_{\hat{t}}}\left[\theta + (1-c)(e^{-\varepsilon_{\hat{t}+1}} - k)\right] = \frac{1}{p_{\hat{t}}}\left[(1-c)e^{-\varepsilon_{\hat{t}+1}} + \theta - k + ck\right].$$

If the price is unchanged at $\hat{t} + 2$, which again happens with probability θ, then $p_{\hat{t}+2} = p_{\hat{t}+1}$. By substituting zero for c in (C2), the expected value at $\hat{t}$ of $1/p_{\hat{t}+2}$ for a given $\varepsilon_{\hat{t}+1}$ is

$$\text{(C3)} \qquad \frac{1}{p_{\hat{t}}}\left(e^{-\varepsilon_{\hat{t}+1}} + \theta - k\right).$$

If the price is adjusted at $\hat{t} + 2$ and was also adjusted at time $\hat{t} + 1$, which happens with probability $(1 - \theta)^2$, then (C1) shows that the expected value at $\hat{t}$ of $1/p_{\hat{t}+2}$ for a given $\varepsilon_{\hat{t}+1}$ and $\varepsilon_{\hat{t}+2}$ is

$$\text{(C4)} \qquad \frac{(e^{-\varepsilon_{\hat{t}+1}} - k)(e^{-\varepsilon_{\hat{t}+2}} - k)}{p_{\hat{t}}(1-\theta)^2}.$$

If the price is adjusted at $\hat{t} + 2$ and unchanged at time $\hat{t} + 1$, which happens with probability $\theta(1 - \theta)$, then $p_{\hat{t}+2} = p_{\hat{t}} e^{\varepsilon_{\hat{t}+1} + \varepsilon_{\hat{t}+2} - \alpha_{\hat{t}+1} - \alpha_{\hat{t}+2}}$. The expected value at $\hat{t}$ of $1/p_{\hat{t}+2}$ for given $\varepsilon_{\hat{t}+1} + \varepsilon_{\hat{t}+2} - \alpha_{\hat{t}+1}$ is

$$\frac{e^{\alpha_{\hat{t}+1}-\varepsilon_{\hat{t}+1}}}{p_{\hat{t}}(1-\theta)\phi}\left[\int_{\underline{\alpha}}^{\varepsilon_{\hat{t}+2}+\ln(s/z_{\hat{t}+1})} e^{\alpha_{\hat{t}+2}-\varepsilon_{\hat{t}+2}}\, d\alpha_{\hat{t}+2} + \int_{\varepsilon_{\hat{t}+2}+\ln(S/z_{\hat{t}+1})}^{\bar{\alpha}} e^{\alpha_{\hat{t}+2}-\varepsilon_{\hat{t}+2}}\, d\alpha_{\hat{t}+2}\right] = \frac{e^{\alpha_{\hat{t}+1}-\varepsilon_{\hat{t}+1}-\varepsilon_{\hat{t}+2}} - k}{p_{\hat{t}}(1-\theta)}.$$

Since the expected value of $e^{\alpha_{\hat{t}+1} - \varepsilon_{\hat{t}+1}}$ is

$$\frac{1}{\theta\phi}\int_{\varepsilon_{\hat{t}+1}+\ln(s/I)}^{\varepsilon_{\hat{t}+1}+\ln(S/I)} e^{\alpha_{\hat{t}+1}-\varepsilon_{\hat{t}+1}}\, d\alpha_{\hat{t}+1} = \frac{S-s}{\theta\phi I} = \frac{k}{\theta},$$

the expected value at $\hat{t}$ of $1/p_{\hat{t}+2}$ for the outcomes where the price is adjusted at $\hat{t} + 2$ and unchanged at $\hat{t} + 1$ is, for a given $\varepsilon_{\hat{t}+2}$,

$$\text{(C5)} \qquad \frac{k(e^{-\varepsilon_{\hat{t}+2}} - \theta)}{p_{\hat{t}}(1-\theta)\theta}.$$

Together, (C3)–(C5) show that for given $\varepsilon_{\hat{t}+1}$ and $\varepsilon_{\hat{t}+2}$, the expected value at $\hat{t}$ of $(1 - c_{\hat{t}+2})/p_{\hat{t}+2}$ is

$$\frac{1}{p_{\hat{t}}}\{\theta(e^{-\varepsilon_{\hat{t}+1}} + \theta - k) + (1 - c)[(e^{-\varepsilon_{\hat{t}+1}} - k)(e^{-\varepsilon_{\hat{t}+2}} - k) + k(e^{-\varepsilon_{\hat{t}+2}} - \theta)]\}$$

$$= \frac{1}{p_{\hat{t}}}[(1 - c)e^{-\varepsilon_{\hat{t}+1} - \varepsilon_{\hat{t}+2}} + (\theta - k + ck)(e^{\varepsilon_{\hat{t}+1}} + \theta - k)].$$

By induction, the expected value at $\hat{t}$ of $(1 - c_t)/p_t$ becomes

$$\frac{1}{p_{\hat{t}}}\left\{(1 - c)\prod_{\tau=\hat{t}+1}^{t} e^{-\varepsilon_\tau} + (\theta - k + ck)\sum_{\tau=\hat{t}+1}^{t-1}\left[(\theta - k)^{t-1-\tau}\prod_{\tau'=\hat{t}+1}^{\tau} e^{-\varepsilon_{\tau'}}\right]\right\}.$$

Since $p_{\hat{t}} = (1 - c)M_{\hat{t}}/(a_{\hat{t}}I)$, this can be written as

$$a_{\hat{t}}I\left[\frac{1}{M_t} + \frac{\theta - k + ck}{1 - c}\sum_{\tau=\hat{t}+1}^{t-1}\frac{(\theta - k)^{t-1-\tau}}{M_\tau}\right],$$

and by letting $\hat{t} \to -\infty$, as

$$a_{-\infty}I\left[\frac{1}{M_t} + \frac{\theta - k + ck}{1 - c}\sum_{\tau=-\infty}^{t-1}\frac{(\theta - k)^{t-1-\tau}}{M_\tau}\right].$$

The average idiosyncratic productivity factor is unity, so the integral of $(1 - c_t)/p_t$ of all the goods in the economy [that is, $\int_0^\infty (1/p_t) f(p_t)\, dp_t$ in the price index], is obtained by setting $a_{-\infty} = 1$ and becomes

$$I\left[\frac{1}{M_t} + \frac{\theta - k + ck}{1 - c}\sum_{\tau=-\infty}^{t-1}\frac{(\theta - k)^{t-1-\tau}}{M_\tau}\right].$$

Taking the inverse and multiplying by N, the price index is

$$P_t \equiv \frac{N}{I}\left[\frac{1}{M_t} + \frac{\theta - k + ck}{1 - c}\sum_{\tau=-\infty}^{t-1}\frac{(\theta - k)^{t-1-\tau}}{M_\tau}\right]^{-1}.$$

Proof that $\theta - k < 0 < \theta - k + ck$.—To show that $\theta - k < 0$, let

$$F(\lambda) \equiv \ln\left(\frac{1 - c + \frac{1}{2}cI + \lambda}{1 - c + \frac{1}{2}cI - \lambda}\right) - \frac{2\lambda}{1 - c},$$

where $0 \le \lambda \le A$. Since $F(0) = 0$ and

$$\frac{dF(\lambda)}{d\lambda} = \frac{2(1-c) + cI}{(1-c+\frac{1}{2}cI)^2 - \lambda^2} - \frac{2}{1-c}$$

$$= \frac{-\frac{1}{2}c^2I^2 - (1-c)cI + 2\lambda^2}{(1-c)[(1-c+\frac{1}{2}cI)^2 - \lambda^2]}$$

$$\leq \frac{-\frac{1}{2}c^2I^2 - (1-c)cI + 2A^2}{(1-c)[(1-c+\frac{1}{2}cI)^2 - \lambda^2]} < 0,$$

it follows that $(1/\phi)F(A) = \theta - k < 0$.

To show that $0 < \theta - k + ck$, let

$$G(\lambda) \equiv \ln\left(\frac{1-c+\frac{1}{2}cI+\lambda}{1-c+\frac{1}{2}cI-\lambda}\right) - 2\lambda,$$

where $0 \leq \lambda \leq A$. Since $G(0) = 0$ and

$$\frac{dG(\lambda)}{d\lambda} = \frac{2(1-c) + cI}{(1-c+\frac{1}{2}cI)^2 - \lambda^2} - 2$$

$$= \frac{c(2 - 2c - I + 2cI - \frac{1}{2}cI^2) + 2\lambda^2}{(1-c+\frac{1}{2}cI)^2 - \lambda^2} > 0$$

(the numerator is positive because $I < 2$), it follows that $(1/\phi)G(A) = \theta - k + ck > 0$.

Proof that $ke^{m+\bar{\gamma}} < 1$.—Assumption 2 implies that $\ln(S/s) < \sigma$ and therefore that $\ln(S/I) < \sigma \Rightarrow \ln(S/I) < \bar{\alpha} - m - \bar{\gamma} \Rightarrow m + \bar{\gamma} + \ln(S/I) < \bar{\alpha}$. Consequently,

$$ke^{m+\bar{\gamma}} = \frac{1}{\phi}\int_{m+\bar{\gamma}+\ln(s/I)}^{m+\bar{\gamma}+\ln(S/I)} e^{\alpha_\tau}\, d\alpha_\tau < \frac{1}{\phi}\int_{\bar{\alpha}-\ln(S/s)}^{\bar{\alpha}} e^{\alpha_\tau}\, d\alpha_\tau < 1.$$

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