THE EFFICIENCY WAGE HYPOTHESIS, SURPLUS LABOUR, AND THE DISTRIBUTION OF INCOME IN L.D.C.s[1]

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ONE of the 'paradoxes' of developing economies is the coexistence of unemployment with a positive (although low) wage for hired labourers. Two resolutions of the paradox are commonly offered: (*a*) There really is no unemployment. (*b*) Markets are not in equilibrium, possibly because of institutional constraints on the level of wages offered. In the rural sector, these 'institutional constraints' take the form of 'communal pressure' rather than minimum wage legislation. This paper is concerned with exploring a third alternative explanation, the efficiency wage hypothesis. The hypothesis dates back at least to the work of Leibenstein,[2] and there have been some theoretical investigations of its implications. This paper is concerned with the consequences of the efficiency wage hypothesis for the distribution of income in the rural sector and considers the effects of an increase in rural population on rural output and inequality.[3]

The economics of the rural sector—the allocation of labour, the supply of effort, the determinants of migration from the rural to the urban sector, etc.—depends critically on how the sector is organized; for instance, whether farms are individually owned or whether there are extended families, whether there is a large landless peasantry, whether individuals who migrate to the urban sector lose their rights to the land, etc.[4]

In this paper we consider several polar cases:

(*a*) The output-maximizing farm.

(*b*) The plantation farm, which maximizes rents from the land.

(*c*) The egalitarian family farm, in which income is divided equally among its members.

(*d*) The utilitarian family farm, which maximizes family social welfare.

Several striking results emerge:[1]

(i) There are important conflicts between equity and efficiency: the output in the egalitarian farm may be significantly lower than in the output-maximizing farm.

(ii) For sufficiently poor farms, complete equality may not be feasible.

(iii) Maximization of family welfare may entail some degree of inequality. We are able, however, to establish certain bounds on the degree of inequality. In particular we can think of each individual's receipts as his marginal product plus a *pro rata* share of rents. The low-wage individuals are less efficient than the high-wage individuals; they receive less than their share of the rents pro-rated on the basis of percentage of the population, but more than their share of the rents pro-rated on the basis of percentage contribution to total effective labour supply.

(iv) The social marginal product of an individual is negative in the egalitarian and utilitarian farms: as individuals migrate from the rural sector, output actually increases.

(v) In the plantation economy, working individuals have a positive marginal product, and receive a wage equal to that marginal product; but there may be considerable unemployment. This is an equilibrium; that is, the unemployed are unable to bid down the wages of the employed. The wage they receive is identical to that of those individuals who work in the output-maximizing farm.

The implication of (iv) and (v) is that the presence of a positive wage (and a corresponding positive marginal product) for working individuals in a competitive labour market cannot be taken as evidence that labour is not in surplus (as some authors seem to have done).

(vi) Raising a dollar from the rural sector, even by a lump sum tax or a land tax, reduces income in the rural sector by more than a dollar.

The paper is divided into two sections. Section 1 presents the basic model and compares the first three kinds of farms. Section 2 analyses the utilitarian farm.

1. Output-maximizing, egalitarian, and plantation farms

The efficiency wage hypothesis says that the services a labourer renders are a function of the wage he receives. One well-paid worker may do what two poorly paid workers can do. We let $\lambda(w)$ be the index of efficiency of a worker receiving a wage of w.

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[1] The exact conditions under which these results obtain are spelled out in the subsequent discussion.

We hypothesize further that λ has the shape depicted in Fig. 1. There is a region of increasing returns, where as the individual is brought above the 'starvation' level additional increments in wages result in increasing increments in efficiency, although eventually diminishing returns set in. Although many observers have claimed that the efficiency curve has the
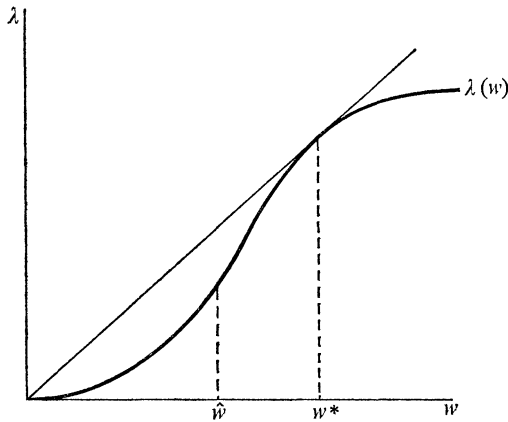
FIG. 1. The efficiency curve.

shape depicted, direct empirical evidence is hard to come by and it remains a moot question. It should be emphasized that our results do depend critically on the existence of the initial region of increasing returns.[1]

By reversing the axes of Fig. 1, we obtain in Fig. 2 an alternative interpretation of the efficiency wage curve: the wage required to obtain a given number of efficiency units from an individual. As a result we shall refer to this as the wage-requirements curve.

Certain points on the efficiency wage curve play an important role in the subsequent analysis. Let

$$\theta = \frac{w}{\lambda}$$

the wage cost per efficiency unit. Then θ is minimized when

$$\frac{1}{\lambda} - \frac{w}{\lambda^2}\lambda' = 0$$

or
$$\lambda'(w) = \frac{\lambda(w)}{w}. \tag{1.1}$$

The solution to (1.1) is often referred to as the *efficiency wage*, and will be denoted by w^*, with $\theta^* = w^*/\lambda(w^*)$.

[1] We do not wish to discuss here the direct empirical evidence in support of and against this hypothesis. We shall note, however, that the model does correctly describe many aspects of the labour market in L.D.C.s.

The value of w at the point of inflection of the efficiency wage curve we denote by $\hat{w}$.

The limiting value of θ for small levels of w, i.e. the wage per efficiency unit required to produce very small levels of efficiency, we denote by $\bar{\theta}$.

$$\bar{\theta} \equiv \lim_{w \to 0} \frac{w}{\lambda(w)}. \tag{1.2}$$

In Fig. 3, we have depicted the production function for the farm. Output in the representative farm in the rural sector is a function of the input of
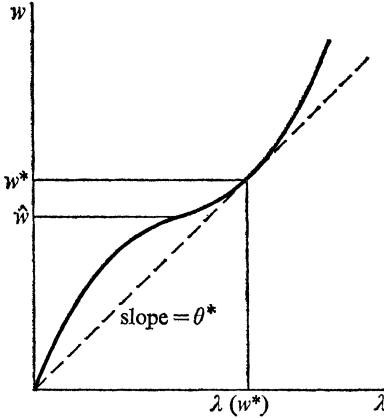

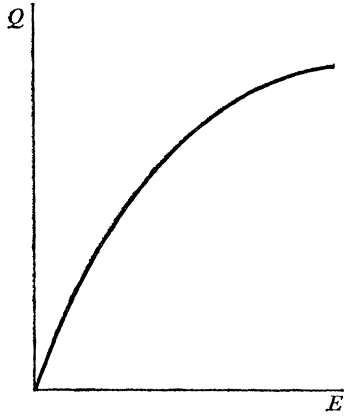FIG. 2. The wage-requirements curve.   FIG. 3. The production function.

labour services, land, capital, and other factors of production. In this paper, we focus only on labour; all other factors of production are assumed to be fixed in the short run.[1]

Let E represent the total number of efficiency units supplied on the farm, and let Q represent output. Then we represent the production function by

$$Q = G(E) \tag{1.3}$$

where $G' \geqslant 0$, $G'' \leqslant 0$; there is a positive marginal product and diminishing returns to labour.

Let $\bar{L}$ be the total number of available workers on the farm. Thus $w\bar{L}$ is the total wage requirement for obtaining $\lambda(w)\bar{L}$ efficiency units (when all individuals are paid the same wage). This is plotted in Fig. 4. The straight line OE^* represents the *minimum* food requirements for obtaining a given number of efficiency units, for $\lambda(w)\bar{L} < \lambda(w^*)\bar{L}$. This is obtained

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[1] Most of our results will still be true in the more general case where these other factors are allowed to vary, as they certainly will in the long run; our primary interest here is in the short run analysis (rather than with capital accumulation, or intersectoral capital movements) and hence the assumption of other factors being fixed may not be unreasonable. In any case, the more general analysis would obfuscate the simple points we wish to establish here.
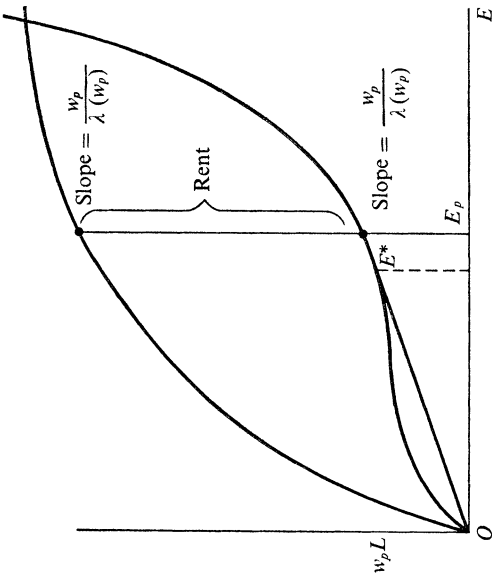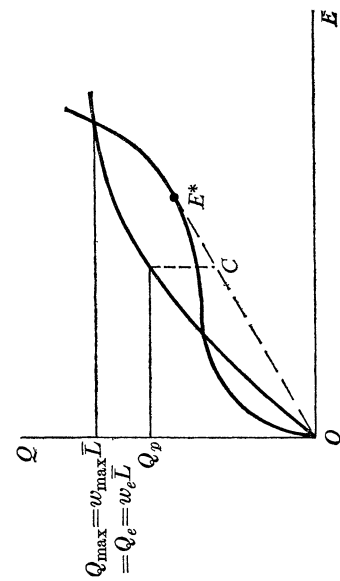
FIG. 4c. The poor economy.

FIG. 4d. The very poor economy.

FIG. 4a. The very rich economy.

FIG. 4b. The rich economy (unemployment rate in plantation economy = CE^*/OE^*).

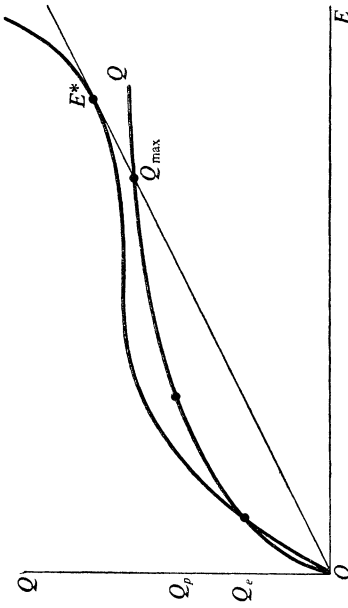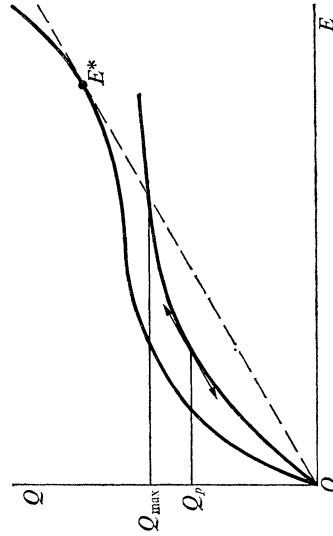by providing a fraction of the workers with a wage of w^* and the remainder with nothing. The 'convexified' wage requirements curve (the straight line OE^*) we shall refer to as the 'non-egalitarian wage requirements curve' to distinguish it from the 'egalitarian wage requirements curve' (the curve OE^*).

Considerable insight into the problem is obtained if we superimpose the wage-requirements curve (essentially Fig. 2) on to the production function Fig. 3, as is done in Fig. 4.

The output-maximizing, plantation, and egalitarian farms may now be easily described. (Each of these farms is assumed to have a given amount of land.)

(a) *Output-maximizing farm.* The output-maximizing farm finds on the production function the highest point that is feasible, i.e. that is not below the wage-requirements curve; it is, in other words, the highest intersection of the non-egalitarian wage requirements curve with the production function. We shall refer to the maximum output as $Q_{\max}$ and the corresponding wage as $w_{\max}$.

(b) *Plantation farm.* A plantation farm in a competitive economy would maximize its rents, taking the minimum wage, $\bar{w}$, at which it can obtain labour as given:

$$\max_{\{L,w\}} \left[G\big(L\lambda(w)\big) - wL \right] \qquad (1.4)$$

s.t.
$$w \geqslant \bar{w}. \qquad (1.5)$$

Let us denote the equilibrium wage for this economy by w_p (and the corresponding output by Q_p). The equilibrium involves two possibilities depending on whether at $w = w^*$, $L = \bar{L}$,

$$G'\big(\bar{L}\lambda(w^*)\big) \gtrless w^*.$$

(i) If $G'\big(\bar{L}\lambda(w^*)\big) > w^*$, at the efficiency wage there is excess demand for labour. Hence in equilibrium

$$w_p > w^*.$$

The equilibrium wage is given by

$$G'\big(\bar{L}\lambda(w_p)\big)\lambda(w_p) = w_p, \qquad (1.6)$$

i.e. the slope of the production function equals the wage cost per efficiency unit (θ). The distance between the output curve and the curve $(w/\lambda(w_p))E = \theta E$ gives the rent. Thus $E_p > E^*$ is the point where the rents are maximized, when the competitive wage at which labour can be obtained exceeds the efficiency wage w^*.

(ii) If $G'\big(\bar{L}\lambda(w^*)\big) < w^*$ at the efficiency wage, there is excess supply of labourers. Then
$$w_p = w^*$$
and
$$L < \bar{L}:$$

there is unemployment. Thus, the constraint (1.5) is not binding; even if workers were willing to work for almost nothing, their efficiency would be so low that it would not pay firms to hire them.

Equilibrium employment is given by

$$G'(L\lambda(w^*))\lambda(w^*) = w^*. \tag{1.7}$$

The slope of the production function equals the slope of the non-egalitarian wage requirements curve. Thus (w_p, L_p) maximizes rents, the distance between the output and the non-egalitarian wage requirements curves.

There is one objection which may be raised to this analysis. If the reason for the efficiency curve is, at least partially, nutritional rather than psychological, and the workers on the plantation share their income with non-working or poorer relatives, the landlord will reap, through the increased efficiency of his workers, only a part of the benefits of paying high wages. In the subsequent discussion, we shall ignore this possibility.[1]

(c) *Egalitarian farm.* The egalitarian farm divides the total output equally among its members, i.e.

$$w_e = \frac{G(\lambda(w_e)\bar{L})}{\bar{L}}. \tag{1.8}$$

The solution to (1.8) is simply given by the intersection of the wage-requirements curve and the production function, and will be denoted by (w_e, Q_e).

There are several possible configurations.

The configuration in Fig. 4a we refer to as the very rich economy, to distinguish it from the next three cases which we shall refer to as the rich, the poor, and the very poor economies. The essential feature of the very rich economy is that

$$Q_{\max} = Q_e > Q_p,$$
$$w_{\max} = w_e > w_p > w^*.$$

The output-maximizing and egalitarian farms are identical, and have a higher output and wages than the plantation farm. The wage on the plantation farm in turn is higher than the efficiency wage.

Fig. 4d represents the very poor economy. The wage-requirements curve is everywhere above the output curve; complete egalitarianism—the same wage to all workers—is not feasible. Notice, however, that the dotted line OE^* does intersect the output curve, so, with some inequality, the economy is viable. $Q_{\max}$ is now found as the intersection of dotted OE^* with the production curve, and entails a fraction of the population being unemployed, and the remaining fraction receiving a wage of w^*. The

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[1] For a more extended discussion of its implications, see J. E. Stiglitz, 'Alternative theory of wage determination and unemployment in L.D.C.'s: II. The efficiency wage hypothesis', Cowles Foundation Discussion Paper 357, Mar. 1973.

plantation farm maximizes rents at Q_p, where the slope of the production function is equal to the slope of dotted OE^*. Thus the plantation farm and the output-maximizing farm differ not in the wages they pay, but only in the number of individuals they hire.

Fig. 4b depicts the rich economy, in which in the plantation farm system there will be unemployment, even though full employment at a wage exceeding the efficiency wage is viable.
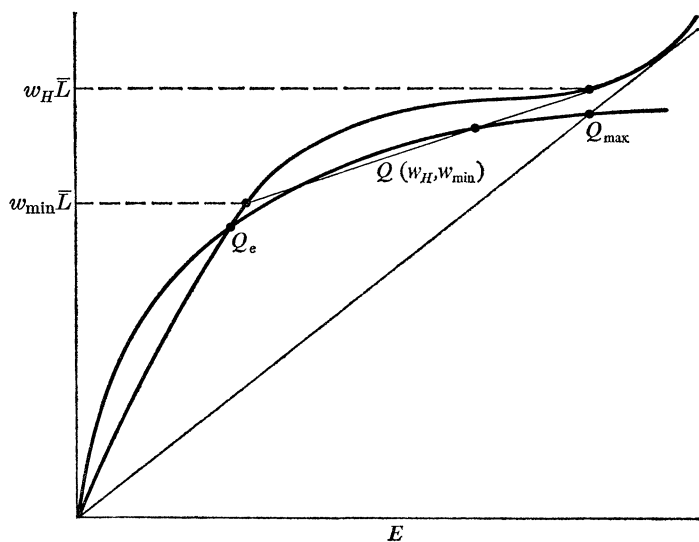
FIG. 5

Finally, Fig. 4c depicts the just poor economy in which both the completely egalitarian farm and the plantation farm have lower outputs than the output-maximizing farm; the plantation farm and the output-maximizing farm both pay the workers they hire the efficiency wage, but the output-maximizing farm hires more workers. The egalitarian farm has an output which lies between Q_{max} and Q_p.

These results suggest that although for rich economies there is no trade-off between output and equality, for the poor economies ('poor', 'very poor') there is such a trade-off.

The nature of this trade-off may be seen more clearly by considering what happens if we impose a minimum wage which all individuals must receive. In Fig. 5 we have denoted this minimum wage by w_{min}. Assume we wish to maximize the output given that all individuals receive a wage

$$w \geqslant w_{min}.$$

From Fig. 5, it is clear that for small w_{min} the solution entails a lower output than Q_{max} but a higher output than Q_e, the output on the egalitarian

farm, and that the wage received by the high-paid workers w_H is lower than the efficiency wage but greater than $\hat{w}$. As we increase w_{min}, we lower output and reduce inequality. Eventually, for high enough w_{min} (and in particular for $w_{min} > \hat{w}$), output is maximized when all individuals receive the same wage.

If we think of $w_H - w_{min}$ as a measure in inequality, then we can 'plot' output as a function of the degree of inequality, as in Fig. 6. We have also
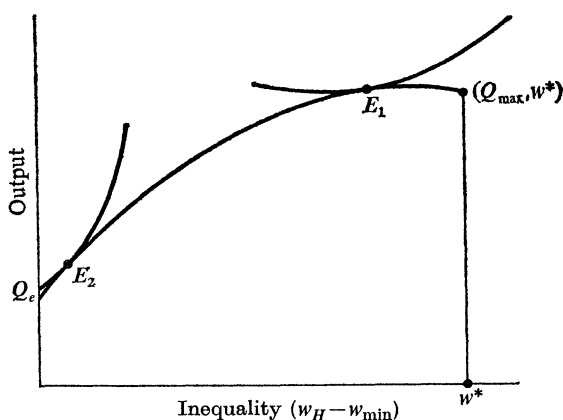

FIG. 6

plotted indifference curves between output and inequality. A family which had a low degree of inequality aversion might pick a point such as E_1 in Fig. 6, entailing an output near the maximum feasible output, while a family with a strong aversion to inequality would pick a point such as E_2 in Fig. 6, near the egalitarian farm.

It should also be clear from Fig. 5 that *in some circumstances, everyone can be made better off by the introduction of some inequality*, i.e. there may exist a feasible pair of wages (w_{min}, w_H) such that $w_H > w_{min} > w_e$.

Several other results emerge neatly from this diagrammatic analysis. First, observe that there may be multiple equilibria for the egalitarian farm; the family has a low income, so it has low productivity; and because it has low productivity, it has a low income (Fig. 7).

Secondly, consider the question of the cost of raising funds from the rural sector, say for investment in the urban sector. For simplicity, we consider only the egalitarian family farm. Assume the government imposes a lump sum tax on the farm. This is equivalent to a uniform downward shift in the production function. See Fig. 8a. Even in the rich farm, the loss in income in the rural sector exceeds the revenue raised by the government. (Similar results obtain for a proportional output tax, as Fig. 8b illustrates.)

O.E.P. 2   O

Next, consider the effect of an increase in population. This shifts the wage-requirements curve proportionately upward and to the right, as depicted in Fig. 9a. (Obviously, the efficiency wage remains unchanged.) The effect on output for an egalitarian farm can be seen clearly from the

Production
function

Wage requirements curve
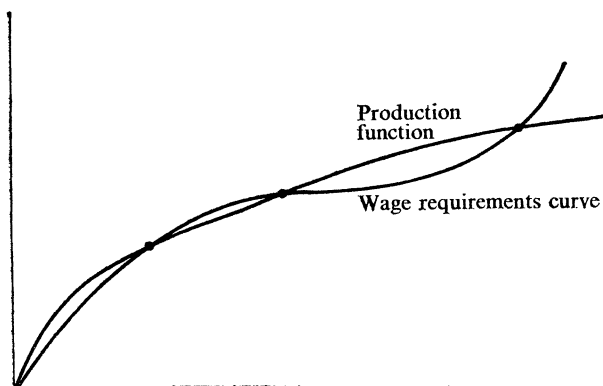
FIG. 7. Multiple equilibria.
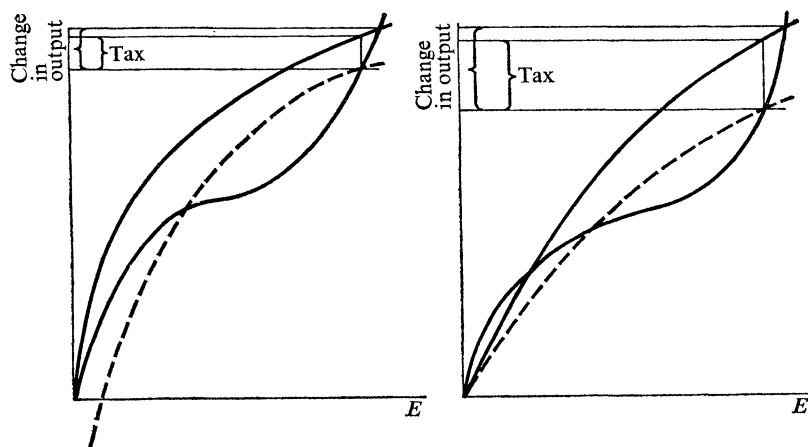
FIG. 8a.   Effects of taxes.   FIG. 8b.

diagram. If the wage in the economy had sufficiently exceeded the efficiency wage, then output increases, but if the wage had been lower than the efficiency wage, output is reduced. Thus, *for farms which are sufficiently poor that $w_e < w^*$, the social marginal productivity of a labourer is negative*.[1] For small changes in $\bar{L}$, if $w_e > w^*$, then the social marginal productivity of

[1] This argument is very different from that presented by Stiglitz, 'Rural–urban migration . . .', op. cit., and A. K. Sen, 'Peasants and dualism with or without surplus labor', *JPE* 74 (Oct. 1966), 425–50, where the social physical marginal productivity may be negative because with reduced population, workers work harder, and output actually may increase.
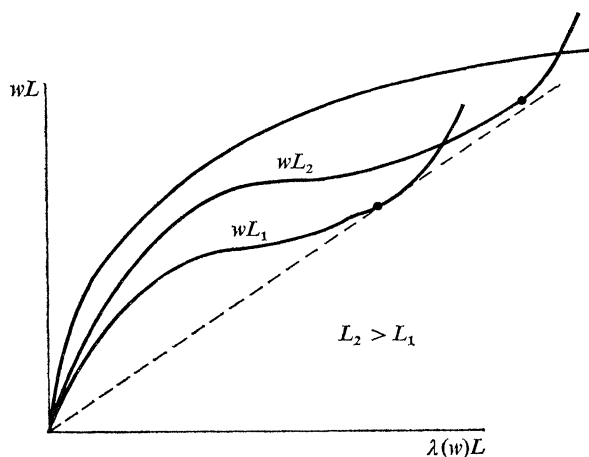
FIG. 9a. Change in labour supply.
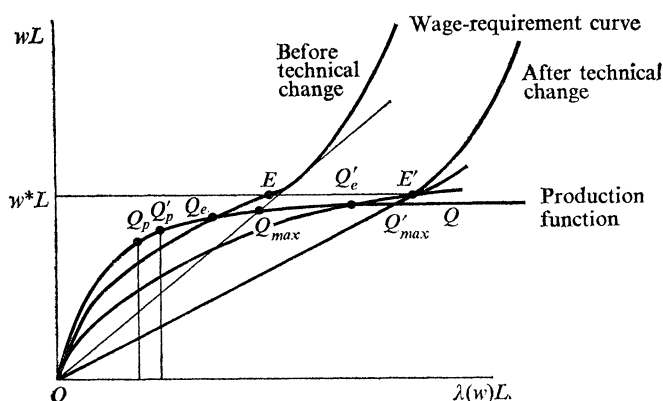
FIG. 9b. Technical change.

a labourer is positive. To see this, we take the logarithmic derivative of (1.8) to obtain

$$1+\frac{d\ln w}{d\ln L}=\frac{G'L}{G}\lambda\left(1+\frac{\lambda'w}{\lambda}\frac{d\ln w}{d\ln L}\right),$$

i.e., recalling that $E=\lambda(w)L$,

$$\frac{d\ln G}{d\ln L}=\frac{d\ln G}{d\ln E}\frac{d\ln E}{d\ln L}=\frac{d\ln G}{d\ln E}\left(\frac{d\ln\lambda}{d\ln L}+1\right)=\frac{\alpha\left(1-(\lambda'w/\lambda)\right)}{1-\alpha(\lambda'w/\lambda)}\gtreqless 0$$

as $\lambda'w/\lambda\gtreqless 1$, i.e. as $w_e\gtreqless w^*$ (using (1.1)) where $\alpha=G'E/G$, the imputed share of labour, i.e. the share of labour if marginal productivity pricing were used.[1]

It should be clear, however, that although the social marginal productivity of a labourer in the rural sector is negative, the apparent 'private'

[1] Since $d\ln w/d\ln L=-(1-\alpha)/(1-(\alpha\lambda'w/\lambda))<0$, and $\alpha<1$, it is clear that $\alpha\lambda'w/\lambda<1$.

FIG. 9c. Techniques chosen on egalitarian and plantation farm differ.

FIG. 9d. Mixtures of techniques.

marginal productivity, $G'\lambda$, is positive. Each person is contributing something on the margin to production. It is only the fact that his presence in the rural sector decreases the income *per capita*, and hence the productivity
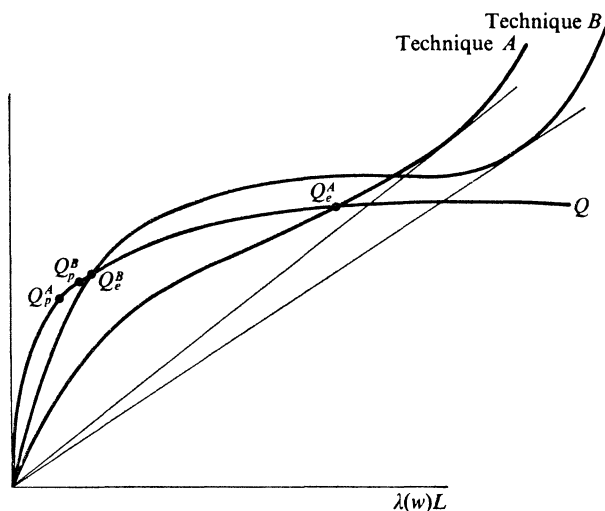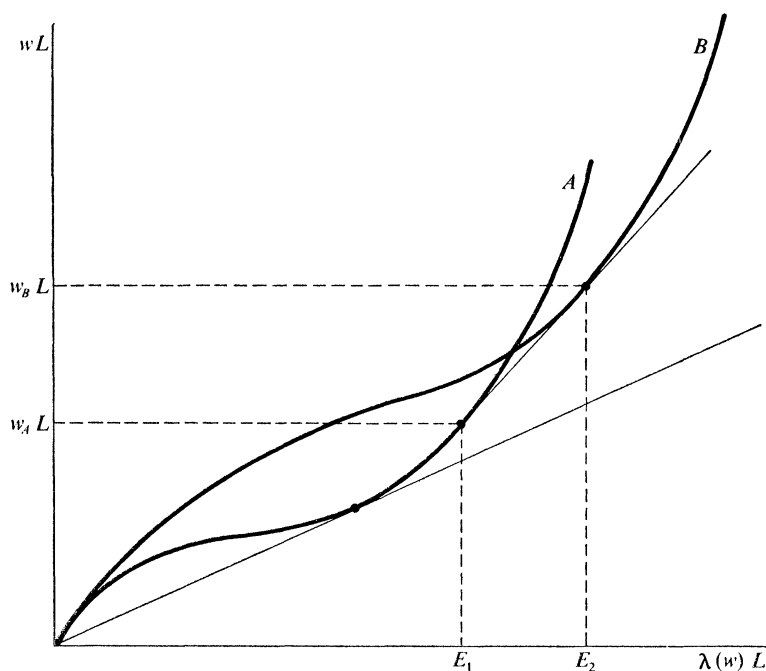
of the other workers in the rural sector, that makes his social marginal productivity negative.

For an output-maximizing or a plantation farm, in which previously there had been unemployment, there is no effect on output; the extra individual simply is added to the unemployment pool.

Next, consider the effect of a technical change which shifts the $\lambda(w)$ curve. In Fig. 9b we have depicted a case where the efficiency of a worker at each wage is increased proportionately so the efficiency wage is unchanged. This increases the equilibrium wage on the egalitarian farm, but leaves the wage on the plantation or output-maximizing farm unchanged. On the output-maximizing farm output is increased and employment is increased,[1] but on the plantation farm, output is increased, but employment may be reduced. (In the limiting case where G is not differentiable at the original situation, a small 'neutral' technical improvement will leave the effective labour unchanged, and hence will reduce employment proportionately.)

On the other hand, there can be other kinds of technical improvements which increase the efficiency wage a great deal but have a relatively small impact on the cost per unit effective labour; such a technical change will actually reduce the level of employment.

There are important differences between the farms in the choice of technique. A plantation farm (when there is unemployment) chooses whatever technique minimizes w/λ regardless of its effect on employment. A technical change which reduces $w^*/\lambda(w^*)$ will always, however, lead to an increase in output. On the other hand, the egalitarian farm will adopt an innovation if it reduces $w_e/\lambda(w_e)$. Fig. 9c illustrates a case where $w_e/\lambda(w_e)$ is increased although $w^*/\lambda(w^*)$ is reduced.

Finally, note that if two techniques can be combined, the minimum wage requirements curve appears as in Fig. 9d. For efficiency units between E_1 and E_2, a fraction of the labour force is employed on technique A, at a wage w_A greater than w_A^*, the efficiency wage of technique A, and the remaining labour force is employed using technique B, at a higher wage w_B which is greater than w_B^*.

2. Maximization of family welfare

2.1. The problem

In the previous section, we noted a trade-off between output on the family farm and 'equity'. One way of 'capturing' the family trade-off between

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[1] Since $G' > 0$, $OQ_{\max}/OE < OQ'_{\max}/OE'$. But the employment ratio in the initial situation is $OQ_{\max}/OE$ and in the final situation is $OQ'_{\max}/OE'$.

equity and efficiency is for the family to maximize a family welfare function. Let $V(w)$ be the utility associated with an income of w. We wish to find[1]

$$\max \int V(w)\, \mathrm{d}P(w) \tag{2.1}$$

where $P(w)$ is the percentage of the family workers receiving at least a wage of w,[2] subject to the constraint that

$$G\left(\int \lambda(w)\, \mathrm{d}P(w)\right) = \int w\, \mathrm{d}P(w), \tag{2.2}$$

output equals income.

2.2. The main propositions

We are able in the analysis below to provide a fairly complete characterization of the above problem.

(a) If, when income is equally distributed, labour is in surplus (i.e. its marginal product is zero), or if the economy is very rich, so that a wage in excess of the efficiency wage is feasible, or if the family is very inequality averse, then there is a single wage: the solution to the utilitarian problem is given by the egalitarian wage (e.g. 1.8).

The remainder of the analysis is concerned with characterizing those solutions which entail inequality. We let w_1 be the highest, w_2 be the wage paid to the next lower group, $w_1 > w_2 > w_3 \ldots$.

(b) The highest wage paid is less than the efficiency wage, but greater than the inflection point-wage ($\hat{w}$). This is as expected, since although an extra dollar to this group contributes more to productivity, it contributes less to 'utility' since there is diminishing marginal utility. There is a unique wage in excess of the inflection wage.

This can be seen in Fig. 10, with wages $w_1 > w_2 > \hat{w}$. Clearly by paying a wage between w_1 and w_2, output would increase and so would equality.

A similar argument cannot be made for wages less than $\hat{w}$. Indeed,

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[1] It is obvious that if V is linear, i.e. $V(w) = a + bw$, then maximization of (2.1) is equivalent to maximizing

$$\int w\, \mathrm{d}P(w)$$

which, by (2.2), is equivalent to maximizing $G(E)$, family output. If $V(w)$ is of the form $-w^{-\rho}$, then maximizing

$$-\int w^{-\rho}\, \mathrm{d}P(w)$$

is equivalent to maximizing $\left[\int w^{-\rho}\, \mathrm{d}P(w)\right]^{-1/\rho}$

and in the limit, as $\rho \to \infty$, this approaches

$$\min w.$$

For economies which are not 'too poor', we obtain the completely egalitarian solution of Section 1, where everyone receives his average product.

The integral is best interpreted as a Stieltjes integral.

[2] In most of the subsequent discussion, we shall let $P(w)$ take on any values between zero and one; obviously, if there are L individuals in the family, P can only take on values $1/L$, $2/L$, etc.; the slight loss in realism is more than compensated for by the gain in analytic tractability.

increasing inequality increases effective labour supply, so that with only a slight degree of (local) inequality aversion more than one wage less than $\hat{w}$ will be paid. For simplicity, in the subsequent discussion we will assume there is a unique wage less than $\hat{w}$.
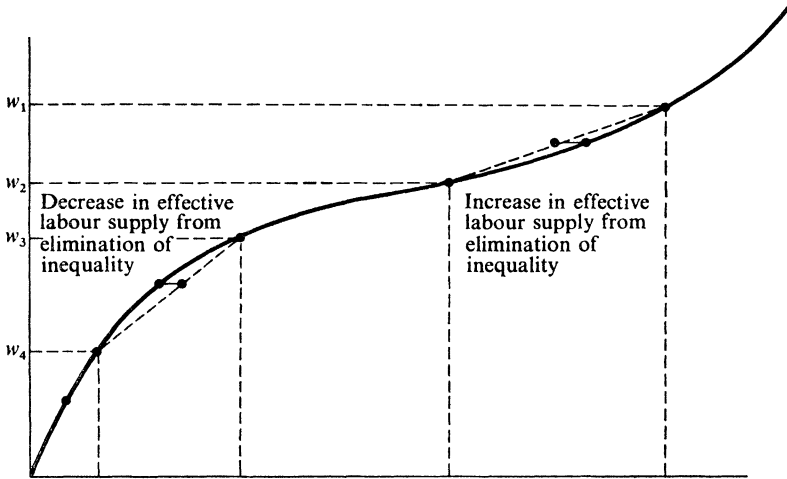
FIG. 10
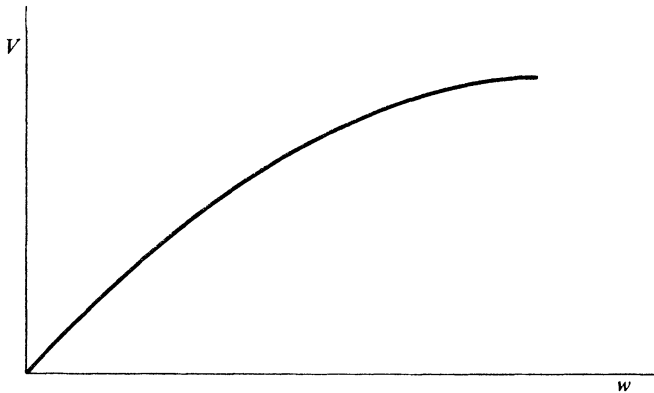
FIG. 11. Utility function with low inequality aversion for $w < \hat{w}$.

We can obtain a more precise bound on w_1: given the level of wages paid to the lower group (w_2), we showed in the previous section there was a level of wages, which we denote by w^{H*}, paid to the upper group which maximizes output. We can show that, for sufficiently large L,

$$\hat{w} < w_1 < w^{H*} < w^*. \tag{2.3}$$

As we decrease the upper wage below w^{H*}, aggregate output decreases, but inequality also decreases, and in the utilitarian calculus we are always willing to make some sacrifice in aggregate output for an increase in equality.

(Analytically this follows from the fact, established in the appendix, that in the utilitarian solution

$$\frac{1}{\lambda'(w_1)} < \frac{w_1 - w_2}{\lambda(w_1) - \lambda(w_2)}$$

while w^{H*} is that wage for which

$$\frac{1}{\lambda'(w)} = \frac{w - w_2}{\lambda(w) - \lambda(w_2)}.\Bigg)$$

FIG. 12

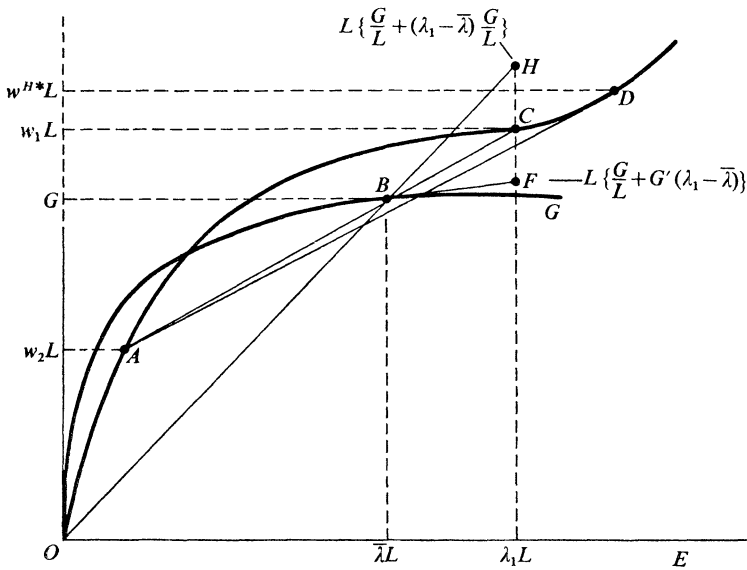(c) $$\lambda(w_1)G' + \frac{R}{L} < w_1 < \lambda(w_1)G' + \frac{\lambda(w_1)}{p\lambda(w_1) + (1-p)\lambda(w_2)}\frac{R}{L}, \qquad (2.4a)$$

$$\lambda(w_2)G' + \frac{R}{L} > w_2 > \lambda(w_2)G' + \frac{\lambda(w_2)}{p\lambda(w_1) + (1-p)\lambda(w_2)}\frac{R}{L} \qquad (2.4b)$$

where $R = G - G'E$, the implicit rent on the land, and p = proportion of population receiving a high wage. *The income received by any individual can be thought of as consisting of a wage payment, equal to his marginal productivity, $\lambda G'$, plus a share of the rent, R. Individuals in the more productive group receive more than a proportionate share of the rent.* On the other hand, they contribute more to output. *They receive less than their proportionate contribution to output.*

This result may be seen diagrammatically as follows. In Fig. 12 let A and C represent the two wages actually paid, and B the output (B lies on the production possibilities schedule). H is generated by extending OB on until $E = \lambda_1 L$; F is generated by extending the tangent to the produc-

tion function. Since $R = G - G'E$, letting $\bar{\lambda} \equiv p\lambda(w_1) + (1-p)\lambda(w_2)$, we can rewrite (2.4a) as

$$\frac{G}{L} + G'(\lambda_1 - \bar{\lambda}) < w_1 < \frac{G}{L} + (\lambda(w_1) - \bar{\lambda})\frac{G}{E}.$$

Since $w_2 > 0$, it is clear that $(w_1 - G/L)/(\lambda_1 - \bar{\lambda}) < G/E$, i.e. the slope of ABC is less than that of OBH, from which the second inequality follows. The first inequality follows from the fact that the slope of ABC is steeper than that of the production function at B (otherwise there clearly exists a feasible point which dominates B).

(d)
$$\frac{dG}{dL} < 0. \tag{2.5}$$

The social marginal productivity of a labourer in the rural sector is negative, i.e. increasing the number of workers in the rural sector reduces output.

(e)
$$\frac{dw_1}{dL} > 0, \qquad \frac{dw_2}{dL} < 0. \tag{2.6}$$

As L increases, and output falls, inequality in the rural sector increases: w_1 increases while w_2 decreases. Equality is a 'luxury' of the well-off.

2.3. On the number of wage levels paid

To see the relationship between the different wages paid to different individuals, consider two groups, with wages w_1 and w_2. Consider the experiment of giving one person at a wage of w_1 one more unit. The effective labour supply goes up by $\lambda'(w_1)$. This does not take away one full unit from the resources available to other groups, since output will go up now by $G'\lambda'(w_1)$. Hence, for product exhaustion, we need to take away *net*, say from a group with w_2 wage, an income of $1 - G'\lambda'(w_1)$. Each unit we take away *gross* leads to a reduction in effective labour supply by $\lambda'(w_2)$ and of output by $G'\lambda'(w_2)$. Hence the net revenue is $1 - G'\lambda'(w_2)$. Thus to increase the wage of a person receiving a wage of w_1 by one unit, we must reduce the wage of a person receiving a wage of w_2 by

$$\frac{1 - G'\lambda'(w_1)}{1 - G'\lambda'(w_2)}.$$

The gain in social welfare from increasing the wage of a person receiving w by a unit is $V'(w)$. Thus we require for welfare maximization

$$V'(w_1) = \frac{1 - G'\lambda'(w_1)}{1 - G'\lambda'(w_2)} V'(w_2)$$

for all w_1, w_2 *actually paid*, i.e.

$$\frac{V'(w)}{1 - G'\lambda'(w)} = \text{constant} \tag{2.7}$$

for all w actually paid.

(2.7) is plotted in Fig. 13. The logarithmic derivative of (2.7) is

$$\frac{V''}{V'}+\frac{G'\lambda''}{1-G'\lambda'}.\qquad(2.8)$$

The first term is always negative, the second is positive for $w < \hat{w}$, negative for $w > \hat{w}$.[1] Accordingly, for $w > \hat{w}$, (2.7) is declining, establishing that there can be only one wage level in excess of $\hat{w}$. As w approaches zero, (2.7) approaches a positive infinite level, provided
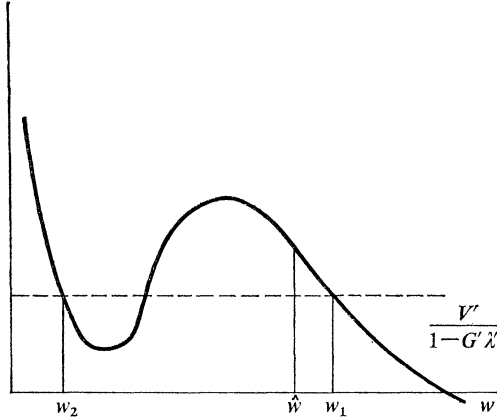
$$\lim_{w\to 0} V'(w) \to \infty$$

FIG. 13. Wage determination in family welfare maximizing farm.

(marginal utility becomes infinite as w goes to zero), since under our assumptions concerning the efficiency curve,

$$\lim_{w\to 0} \lambda'(w) \to \epsilon \geqslant 0.$$

Moreover, as $w \to \infty$, (2.7) approaches zero, if

$$\lim_{w\to\infty} V'(w) \to 0$$

and if

$$\lim_{w\to\infty} \lambda'(w) \to 0,$$

i.e. there is a maximum attainable level of efficiency.

For $0 < w < \hat{w}$, there may be any number of local minima or maxima; for instance, in Fig. 13, there are three wages at which $V'/(1-G'\lambda')$ equals the particular constant represented by the dotted line.

2.4. Reformulation of utilitarian problem

Assuming now that there are no more than two wage groups, we can reformulate our problem to read

$$\max \left(pV(w_1)+(1-p)V(w_2)\right)L\qquad(2.9)$$

[1] Assuming $G'\lambda' < 1$. Obviously, for all wages actually paid, $G'\lambda' < 1$; otherwise there is no 'revenue' cost at all in raising the wage.

subject to
$$L(pw_1+(1-p)w_2) = G((p\lambda(w_1)+(1-p)\lambda(w_2))L). \tag{2.10}$$

p is the proportion of the population in the upper group.
$$0 \leqslant p \leqslant 1.$$

The first-order conditions are simply
$$V'(w_i)+\nu(G'\lambda'-1) = 0, \quad i = 1, 2, \tag{2.11}$$
$$V(w_1)+\nu(G'\lambda(w_1)-w_1) = V(w_2)+\nu(G'\lambda(w_2)-w_2) \tag{2.12}$$

where ν is the shadow price associated with the constraint (2.10).

2.5. Proof that high wage is less than the efficiency wage

To see that $w_1 < w^*$, we multiply (2.11) by w_i and subtract the result from (2.12) to obtain
$$(V_1-V_1'w_1)-(V_2-V_2'w_2) = \nu G'[(\lambda_2-\lambda_2'w_2)-(\lambda_1-\lambda_1'w_1)] \tag{2.13}$$
where
$$V_1 \equiv V(w_1), \qquad \lambda_1 = \lambda(w_1), \ldots.$$

Since $\mathrm{d}(V-V'w)/\mathrm{d}w = -V''w > 0$, the left-hand side of (2.13) is positive, if $w_1 > w_2$. For w less than the efficiency wage w^*, $\lambda' > \lambda/w$, so $\lambda_2' > \lambda(w_2)/w_2$. Hence $\lambda_1-\lambda_1'w_1 < 0$ if the right-hand side of (2.13) is to be positive, i.e.
$$\lambda'(w_1) > \frac{\lambda(w_1)}{w_1},$$

w_1 must be less than the efficiency wage.

2.6. Conditions for egalitarian solution

This result, together with our earlier analysis, enables us to ascertain under what conditions there will be only one wage.

(a) If there is only one wage, it is clearly given by the highest value of w satisfying
$$G(\lambda(w)L) = wL.$$

If the solution is greater than w^*, the efficiency wage, clearly all individuals are better off in that solution than in the 'solution' with inequality, where $w_2 < w_1 < w^*$. In effect, when the economy is very well off, there is no trade-off between efficiency and equity, so maximization of family welfare involves complete equality.

(b) On the other hand, when the economy is very poor, i.e. if $G' = 0$, the marginal productivity of labour services is zero; then of course again there is no trade-off between equity and efficiency; as (2.8) makes clear, $V'/(1-G'\lambda')$ is a monotonically declining function of w so again there is only one wage: everyone receives his average product.

If workers get paid their average product, we have established in Section 1 that

$$\frac{\mathrm{d}E}{\mathrm{d}L} \gtrless 0 \quad \text{as} \quad L \lesseqgtr L^*$$

where

$$G(\lambda(w^*)L^*) = w^*L^*.$$

Hence $E \leqslant E^* \equiv \lambda(w^*)L^*$.

We have thus established that a necessary condition for inequality to be optimal is that

$$\bar{L} > L^*. \tag{2.14}$$

Even in these situations, if the degree of equality preference (as measured by $-V''/V'$ or $-V''w/V'$)[1] is sufficiently great, then again workers will receive their average product.[2] Our concern in this section is with the behaviour of families when the degree of equality preference is sufficiently weak that the gains in total family income induce them to give different members of the family different incomes.[3]

2.7. Further bounds on wage payments

To establish (2.4), we let S_i equal the amount by which the wage exceeds the marginal product:

$$S_i = w_i - G'\lambda_i. \tag{2.15}$$

We can rewrite (2.12) then as

$$S_1 - S_2 = \frac{V_1 - V_2}{\nu}. \tag{2.16}$$

But

$$pS_1 + (1-p)S_2 = \frac{R}{L} \tag{2.17}$$

where R/L is the 'rent' $(G - G'L)$ *per capita*. Substituting (2.17) into (2.16) we obtain

$$S_1 = (1-p)\frac{(V_1 - V_2)}{\nu} + \frac{R}{L} > \frac{R}{L},$$

$$S_2 = p\frac{(V_2 - V_1)}{\nu} + \frac{R}{L} < \frac{R}{L}.$$

On the other hand, the more productive contribute, on a *per capita* basis, more to output. If they were to receive income proportionate to their contribution to output, they would receive an amount

$$\frac{\lambda_1}{p\lambda_1 + (1-p)\lambda_2} \cdot \frac{G}{L}.$$

[1] For a discussion of the use of these as measures of degrees of equality preferences, see A. B. Atkinson, 'On the measurement of inequality', *Journal of Economic Theory*, 2 (1970), 244–63.

[2] That is, $V'/(1 - G'\lambda')$ will then be a monotonically declining function (see equation (2.8)).

[3] It is clear that what is crucial is the sign of V''/V' relative to λ'', the degree of 'increasing returns' to wages.

The difference between what the first group receives and its proportionate contribution to output is

$$w_1 - \frac{\lambda_1}{p\lambda_1+(1-p)\lambda_2}\,(pw_1+(1-p)w_2) = \frac{(1-p)w_1w_2}{p\lambda_1+(1-p)\lambda_2}\left(\frac{\lambda_2}{w_2}-\frac{\lambda_1}{w_1}\right) < 0.$$

Similarly
$$w_2 - \frac{\lambda_2}{p\lambda_1+(1-p)\lambda_1}\,(pw_1+(1-p)w_2) > 0.$$

The final two propositions come from straightforward (but tedious) differentiation of the first-order conditions for welfare maximization (see Appendix).

Concluding comments

We have analysed the implications of the efficiency wage hypothesis for the rural sector. We have shown how it can lead to a true surplus of labourers, even though all employed labourers receive a positive wage. This result has very strong implications for shadow prices of labour in the urban sector. Indeed, if this argument is correct, then the opportunity cost of hiring one labourer from the rural sector may be negative. A fuller investigation of the implications for shadow prices must, however, await another occasion.

APPENDIX

Proof that $\mathrm{d}w_1/\mathrm{d}L > 0$, $\mathrm{d}w_2/\mathrm{d}L < 0$, $\mathrm{d}Q/\mathrm{d}L < 0$.

The family's optimal decisions are described by the four equations in the four unknowns, w_1, w_2, p, and ν, and the parameter L.

$$V'(w_1)+\nu(G'\lambda'(w_1)-1) = 0 \tag{A.1}$$

$$V'(w_2)+\nu(G'\lambda'(w_2)-1) = 0 \tag{A.2}$$

$$V_1-V_2+\nu(G'(\lambda_1-\lambda_2)-(w_1-w_2)) = 0 \tag{A.3}$$

$$G((p\lambda_1+(1-p)\lambda_2)L)-(pw_1+(1-p)w_2)L = 0. \tag{A.4}$$

Totally differentiating, we obtain (after some simplification)

$$\begin{bmatrix} -(1+a_1) & -1 & -1 & -b_1 \\ -1 & -(1+a_2) & -1 & -b_2 \\ -1 & -1 & -1 & -c \\ -b_1 & -b_2 & -c & 0 \end{bmatrix} \begin{bmatrix} p\lambda'_1\,\mathrm{d}w_1 \\ (1-p)\lambda'_2\,\mathrm{d}w_2 \\ (\lambda_1-\lambda_2)\mathrm{d}p \\ \mathrm{d}\nu \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ e \end{bmatrix} [p\lambda_1+(1-p)\lambda_2]\,\mathrm{d}\ln L$$

where
$$a_i = \frac{V''_i+\nu G'\lambda''_i}{\nu\lambda'_i\,G''Lp_i} > 0, \qquad i = 1, 2$$

$$b_i = \frac{-V'_i}{\lambda'_i\nu^2G''L} > 0, \qquad i = 1, 2$$

$$c = \frac{-(V_1-V_2)}{(\lambda_1-\lambda_2)\nu^2G''L} > 0.$$

$$e = \frac{G-G'E}{[p\lambda_1+(1-p)\lambda_2]\nu G''L^2}.$$

To ascertain the signs of the various derivatives, we shall need to know the sign of $e-c$, b_1-c, and b_2-c.

$$c-e \sim \frac{V_1-V_2}{\nu(\lambda_1-\lambda_2)} - \frac{G-G'E}{(p\lambda_1+(1-p)\lambda_2)L}$$

$$= -G' + \frac{w_1-w_2}{\lambda_1-\lambda_2} - \frac{G-G'E}{E} \qquad \text{(using A.3)}$$

$$= \frac{w_1-w_2}{\lambda_1-\lambda_2} - \frac{pw_1+(1-p)w_2}{p\lambda_1+(1-p)\lambda_2} \qquad \text{(using A.4)}$$

$$\sim \frac{w_1-w_2}{w_2} - \frac{\lambda_1-\lambda_2}{\lambda_2}$$

$$\sim \frac{w_1}{\lambda_1} - \frac{w_2}{\lambda_2} < 0. \qquad \text{(since } w_2 < w_1 < w^*) \qquad \text{(A.5)}$$

$$b_1-c \sim \frac{V_1'}{\lambda_1'} - \frac{V_1-V_2}{\lambda_1-\lambda_2}$$

$$= \frac{V_1'}{\lambda_1'} - \nu\left(\frac{w_1-w_2}{\lambda_1-\lambda_2} - G'\right) \qquad \text{(using A.3)}$$

$$= \frac{V_1'}{\lambda_1'} - V_1'\left(\frac{(w_1-w_2)/(\lambda_1-\lambda_2) - G'}{1-G'\lambda_1'}\right) \qquad \text{(using A.1)}$$

$$= \frac{V_1'}{1-G'\lambda_1'}\left(\frac{1}{\lambda_1'} - \frac{w_1-w_2}{\lambda_1-\lambda_2}\right). \qquad \text{(A.6)}$$

But from the concavity of V (see Fig. 14a)

$$V_1-V_2 > V'(w_1)(w_1-w_2).$$

Hence
$$\frac{V_1'}{\lambda_1'} - \frac{V_1-V_2}{\lambda_1-\lambda_2} < V_1'\left(\frac{1}{\lambda_1'} - \frac{w_1-w_2}{\lambda_1-\lambda_2}\right). \qquad \text{(A.7)}$$

But (A.6) and (A.7) together imply

$$\frac{V_1'}{1-G'\lambda_1'}\left(\frac{1}{\lambda_1'} - \frac{w_1-w_2}{\lambda_1-\lambda_2}\right) < V_1'\left(\frac{1}{\lambda_1'} - \frac{w_1-w_2}{\lambda_1-\lambda_2}\right),$$

i.e. since, from (A.1), $G'\lambda'(w_1) < 1$,

$$\frac{1}{\lambda_1'} - \frac{w_1-w_2}{\lambda_1-\lambda_2} < 0.$$

Thus
$$b_1-c < 0.$$

Similarly,
$$b_2-c \sim \frac{V_2'}{\lambda_2'} - \frac{V_1-V_2}{\lambda_1-\lambda_2}$$

$$= \frac{V_2'}{1-G'\lambda_2'}\left(\frac{1}{\lambda_2'} - \frac{w_1-w_2}{\lambda_1-\lambda_2}\right). \qquad \text{(A.8)}$$

But from concavity of V

$$V_1-V_2 < V'(w_2)(w_1-w_2).$$

Hence
$$\frac{V_2'}{\lambda_2'} - \frac{V_1-V_2}{\lambda_1-\lambda_2} > V_2'\left(\frac{1}{\lambda_2'} - \frac{w_1-w_2}{\lambda_1-\lambda_2}\right). \qquad \text{(A.9)}$$
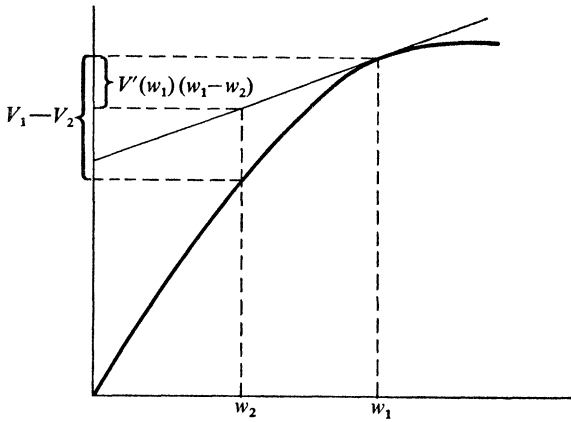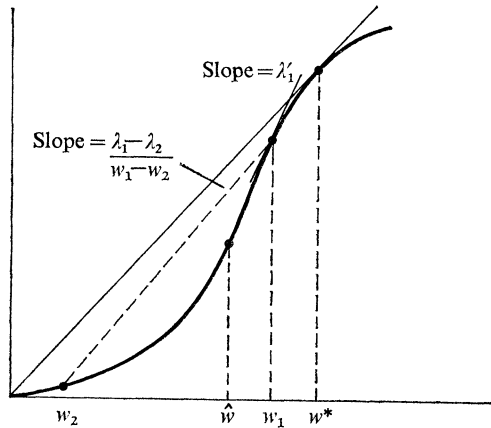
FIG. 14a

FIG. 14b

(A.8) and (A.9) imply $1/\lambda_2' - (w_1-w_2)/(\lambda_1-\lambda_2) > 0$, so

$$b_2 - c > 0.$$

Then

$$\frac{dw_1}{dL} \sim a_2(e-c)(c-b_1) > 0,$$

$$\frac{dw_2}{dL} \sim a_1(e-c)(c-b_2) < 0.$$

Finally,

$$\frac{d\ln E}{d\ln L} = 1 + \frac{1}{p\lambda_1 + (1-p)\lambda_2}\left[p\lambda_1'\frac{dw_1}{d\ln L} + (1-p)\lambda_2'\frac{dw_2}{d\ln L} + (\lambda_1-\lambda_2)\frac{dp}{d\ln L}\right]$$

$$\sim a_1 a_2 c(c-e) < 0.$$