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This paper argues that the "scale effects" prediction of many recent R & D–based models of growth is inconsistent with the time-series evidence from industrialized economies. A modified version of the Romer model that is consistent with this evidence is proposed, but the extended model alters a key implication usually found in endogenous growth theory. Although growth in the extended model is generated endogenously through R & D, the long-run growth rate depends only on parameters that are usually taken to be exogenous, including the rate of population growth.

I. Introduction

Recently the endogenous growth literature has turned to a class of models in which growth is driven by technological change that results from the research and development efforts of profit-maximizing agents, with the implication that subsidies to R & D, and perhaps other government policies, may influence the long-run rate of economic growth. Important contributions to this literature include Romer (1990), Grossman and Helpman (1991a, 1991b, 1991c), and Aghion and Howitt (1992).

This paper makes several points, the most fundamental of which are straightforward and easy to illustrate. In brief, virtually all the R & D–based models in the literature (and certainly those referenced

759

above) share a prediction of "scale effects": if the level of resources devoted to R & D—measured, say, by the number of scientists engaged in R & D—is doubled, then the per capita growth rate of output should also double, at least in the steady state. Empirically, of course, such a prediction receives little support. The number of scientists engaged in R & D in advanced countries has grown dramatically over the last 40 years (because of population growth and an increase in the intensity of R & D), and growth rates either have exhibited a constant mean or have even declined on average. For example, according to the National Science Foundation (1989), the number of scientists and engineers engaged in R & D in the United States has grown from under 200,000 in 1950 to nearly 1 million by 1987; per capita growth rates in the United States exhibit nothing remotely similar to this fivefold increase. The prediction of scale effects is clearly at odds with empirical evidence. Therefore, virtually all the R & D–based growth models in the literature are inconsistent with this simple observation.

Presumably, however, this inconsistency is not detrimental to the spirit of the R & D–based growth literature. The prediction of scale effects has not been widely emphasized (although see the notable exceptions of Grossman and Helpman [1991a] and Rivera-Batiz and Romer [1991]), and perhaps a plausible model could be constructed that eliminates this prediction while maintaining the other features of the R & D–based models. Section III of the paper attempts to construct such a model. The key result from this section, however, is that eliminating the prediction of scale effects typically alters other implications of the R & D–based growth models; in particular, eliminating the scale effects induces a return to Solow-like implications for long-run growth. In the extended model, long-run per capita growth depends only on parameters that are usually taken to be exogenous and is therefore independent of policy changes such as subsidies to R & D or subsidies to capital accumulation. Specifically, the steady-state growth rate depends on the growth rate of inventions, which in turn depends on the (exogenous) rate of population growth, reflecting an intuitive link between innovations and scientists: inventions require inventors. However, as in the Romer/Grossman-Helpman/Aghion-Howitt models and in contrast to the Solow (1956) model, growth in the extended model arises endogenously through R & D.

These results suggest a refinement of the term "endogenous growth." Growth in the model is endogenous in the sense that technological progress, which generates long-run growth, results from R & D undertaken by profit-maximizing agents. However, long-run growth is not endogenous, as it was in the AK models and in the

Romer/Grossman-Helpman/Aghion-Howitt models, in the sense that traditional policy changes have long-run growth effects. A term such as "semi-endogenous" growth might usefully be applied to the model developed here.

The paper is organized as follows. Section II summarizes a basic objection to the prediction of "scale effects" associated with the Romer/Grossman-Helpman/Aghion-Howitt models. Section III proposes an extension of these models that eliminates scale effects but also implies that the long-run growth rate is a function of parameters usually taken to be invariant to government policy. Section IV characterizes the welfare properties of the decentralized model and notes that they are driven primarily by the monopolistic competition associated with innovation rather than by the externalities associated with R & D, suggesting that this aspect of the model should be examined more carefully in future research. Section V analyzes the transition dynamics of the model, and Section VI discusses the overall plausibility of the model as a description of economic growth. Section VII concludes the paper.

II. The R & D Equation and the Problem of Scale Effects

The R & D–based models in the endogenous growth literature by Romer (1990), Grossman and Helpman (1991a, 1991b, 1991c), Aghion and Howitt (1992), and others share the counterfactual prediction of "scale effects": an increase in the *level* of resources devoted to R & D should increase the *growth rate* of the economy. Jones (1995) discusses in detail the evidence against these intertemporal scale effects. Because this observation represents the point of departure in this paper, it is worth reviewing briefly.

The essence of the "scale effects" prediction in the Romer/Grossman-Helpman/Aghion-Howitt models is summarized in the following two equations:

$$Y = K^{1-\alpha}(AL_y)^\alpha \tag{1}$$

and

$$\frac{\dot{A}}{A} = \delta L_A, \tag{2}$$

where Y is output, A is productivity or knowledge, and K is capital. Labor is used either to produce output (L_Y) or to search for new knowledge (L_A). Equation (1) is a standard production function, and equation (2) typifies the R & D equation in the R & D–based endogenous growth models.

The source of scale effects is the R & D equation in (2). This equation implies that total factor productivity (TFP) growth will be proportional to the number of units of labor devoted to R & D. With a constant share of labor devoted to R & D, this rate will be proportional to the size of the labor force, a result found in the Romer/ Grossman-Helpman/Aghion-Howitt models and many other R & D–based models.[1] With a bit more structure to the model (i.e., so that the capital/output ratio is constant in the steady state), this growth rate ties down the growth rate of output per worker.

The prediction that the growth rate of the economy is proportional to the size of its labor force is easily falsified. The most obvious evidence against it is to be found in the historical experience of the advanced economies: the size of the labor force has grown dramatically over the last 25 (or 50 or 100) years, but average growth rates have been relatively constant or have even declined.

The evidence against the R & D equation, the source of the scale effects, is equally compelling. Figure 1 plots the number of scientists and engineers engaged in R & D for the United States together with annual TFP growth rates for the private business sector. The amount of labor engaged in R & D grows by more than a factor of five, from about 160,000 in 1950 to nearly 1 million by 1988. A similar pattern can be seen in France, West Germany, and Japan. In contrast, the pattern of average TFP growth rates is well known: TFP growth for the postwar period is relatively constant or even declining. One might worry about the relevant unit of observation (the world vs. a single country) or the lags associated with R & D, but it should be clear from the figure that these concerns cannot overturn the rejection of scale effects.[2] The assumption embedded in the R & D equation that the growth rate of the economy is proportional to the level of resources devoted to R & D is obviously false.

An important alternative specification of the R & D equation that, at least on the surface, maintains the key results of the Romer/Grossman-Helpman/Aghion-Howitt models without imposing scale effects assumes that TFP growth depends on the share of labor devoted to R & D rather than on the quantity:

$$\frac{\dot{A}}{A} = \delta \frac{L_A}{L} = \delta s. \tag{3}$$

With a specification such as (3), it is easy to see that R & D drives TFP growth and that subsidies to R & D that increase s will raise the

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[1] Strictly speaking, the input to R & D in the literature is not always labor. For example, in Romer (1990) it is human capital. However, with merely a change of terminology, the same criticism applies.

[2] Jones (1995) provides a more detailed discussion of these and other concerns.
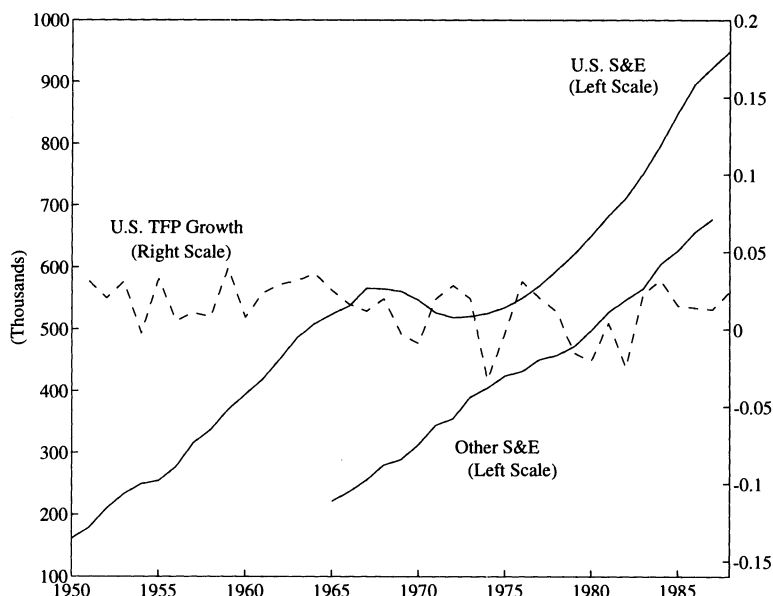
FIG. 1.—Scientists and engineers engaged in R & D and U.S. TFP growth. Source: The number of scientists and engineers engaged in R & D is taken from National Science Foundation (1989) and various issues of the *Statistical Abstract of the U.S. Economy*. TFP growth rates are calculated using the private business sector data in Bureau of Labor Statistics (1991). "Other S&E" is the sum of scientists and engineers engaged in R & D for France, West Germany, and Japan.

steady-state growth rate. However, this specification is unsatisfactory for a number of reasons. First, equation (3) is inconsistent with the microfoundations of the R & D models developed by Romer/Grossman-Helpman/Aghion-Howitt. These microfoundations imply that new ideas are discovered by individuals so that the number of innovations is inherently tied to the number of persons engaged in R & D. A specification devoid of scale such as (3) has the counterfactual implication that an economy with only one unit of labor can produce as many innovations (or at least can generate equivalent TFP growth) as an economy with 1 million units of labor.

The empirical evidence against equation (3) is also compelling. Figure 2 graphs the share of labor devoted to R & D for the United States, revealing that equation (3) is also inconsistent with the lack of increase in TFP growth rates: the share of labor devoted to R & D shows a strong positive trend in the postwar period. For example, the share grows from about 0.25 percent in the United States in 1950 to nearly 0.80 percent by 1988, an increase of over threefold. (The evidence from France, Germany, and Japan is similar.)
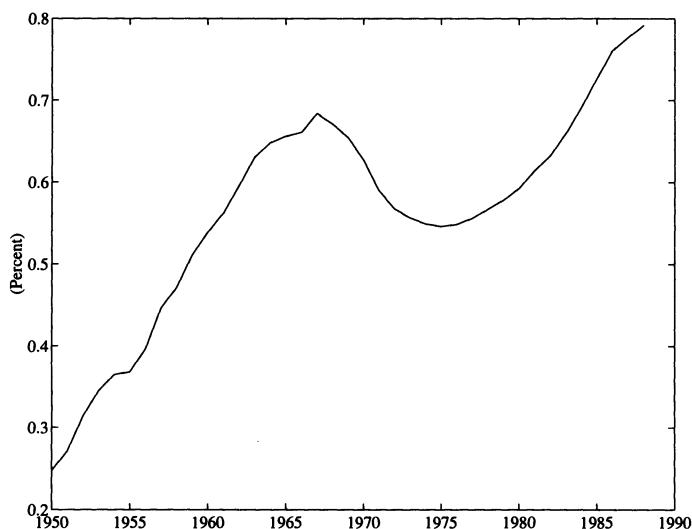
FIG. 2.—U.S. scientists and engineers engaged in R & D as a share of total labor force. Source: The number of scientists and engineers engaged in R & D is taken from National Science Foundation (1989) and various issues of the *Statistical Abstract of the U.S. Economy*. Labor force data are taken from Summers and Heston (1991).

The prediction of scale effects associated with the R & D equation in (2) is detrimental to the R & D–based models in the endogenous growth literature. The models of Romer/Grossman-Helpman/ Aghion-Howitt and others are all easily rejected because of this prediction. However, apart from this problem, the R & D–based models are intuitively very appealing. Growth arises as a result of intentional innovation by rational, profit-maximizing agents, and the models have strong microfoundations. Because of this appeal, it is desirable to find a way to maintain the basic structure of these models while eliminating the prediction of scale effects. This is the goal of the remainder of this paper.

III. An R & D–Based Model of Semi-endogenous Growth

Consider once again the specification of the R & D equation. How does R & D affect productivity growth? Following Romer/Grossman-Helpman/Aghion-Howitt, define A to be the stock of knowledge or technology in an economy. Knowledge is simply the accumulation of ideas, and ideas are developed by people. In the simplest model, then, the change in knowledge $\dot{A}$ will be equal to the number of

people attempting to discover new ideas multiplied by the rate at which R & D generates new ideas:

$$\dot{A} = \delta L_A. \tag{4}$$

Such a specification could be given microfoundations by appealing to a Poisson process governing the arrival rate δ.

One might expect the rate at which scientists discover new ideas to be a function of the amount of knowledge in the economy. For instance, if there are positive spillovers in the production of knowledge, δ would be increasing in the level of A. The discovery of calculus, the invention of the transistor, and the creation of semiconductors are all examples of major innovations that most likely raised the productivity of the scientists who followed. Alternatively, perhaps the most obvious ideas are discovered first so that the probability that a person engaged in R & D discovers a new idea is decreasing in the level of knowledge.[3] Parameterizing the arrival rate δ, we get

$$\delta = \delta A^{\phi}. \tag{5}$$

In this equation, $\phi < 0$ corresponds to the case referred to in the productivity literature as "fishing out," in which the rate of innovation decreases with the level of knowledge; $\phi > 0$ corresponds to the positive external returns case. A value of $\phi = 0$ represents the useful benchmark of constant returns to scale (zero external returns) in which the arrival rate of new ideas is independent of the stock of knowledge. Notice that these effects will be external to the individual scientist so that ϕ measures the degree of externalities across time in the R & D process.

Finally, consider the possibility that at a point in time the duplication and overlap of research reduce the total number of innovations produced by L_A units of labor. That is, suppose that it is not L_A but rather L_A^{λ}, $0 < \lambda \le 1$, that belongs in the R & D equation.[4] Incorporating this change into (4) and (5) yields the R & D equation:

$$\dot{A} = \delta L_A A^{\phi} l_A^{\lambda-1}, \tag{6}$$

where $l_A = L_A$ in equilibrium, but l_A captures the externalities occurring because of duplication in the R & D process.[5] When $\phi = 1$

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[3] A different interpretation of this setup is that the value of new ideas, rather than the number, depends on the stock of knowledge.

[4] This specification of diminishing returns at a point in time was suggested in Romer (1990) and has been considered by Kortum (1992) and Stokey (1992).

[5] The incorporation of $\lambda < 1$ need not reflect externalities. For example, perhaps the addition of labor into the R & D process at a point in time requires the use of less skilled scientists. However, for the modeling purposes below, λ will be assumed to measure duplication externalities.

and $\lambda = 1$, this equation reduces to the R & D equation assumed in the Romer/Grossman-Helpman/Aghion-Howitt models.[6]

One might add other features to the basic specification of the R & D equation in (6). For instance, perhaps computers and other forms of capital play a complementary role in the discovery of knowledge, in which case capital belongs in the R & D equation. Because the main results are robust to such changes, I shall omit capital from the R & D equation.

In part, this discussion is meant to demonstrate that the assumption of $\phi = 1$ in the Romer/Grossman-Helpman/Aghion-Howitt models is arbitrary.[7] An assumption of $\phi = 0$ might actually seem most natural since, as Romer (1990) argues, whether there are increasing or diminishing returns to R & D is in part a philosophical question. One might possibly make a plausible case for increasing returns to R & D so that $\phi > 0$. However, $\phi = 1$ represents a completely arbitrary degree of increasing returns and, as was argued in the previous section, is inconsistent with a broad range of time-series data on R & D and TFP growth. In what follows, I shall impose the restriction $\phi < 1$ and show that this justifiable assumption leads to a model in which a balanced growth path is consistent with an increasing number of persons devoted to R & D.

The Decentralized Model

Apart from the specification of the R & D equation, the growth model considered here is very similar to the one in Romer (1990).[8] The economy consists of three sectors. First, a final-goods sector produces

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[6] Similar specifications have been considered elsewhere but have not been emphasized as plausible models of long-run growth. Grossman and Helpman (1991a) consider a specification analogous to (6) in which the size of the population is held fixed and note that per capita growth dies out asymptotically for the case considered below. Judd (1985) employs a specification with $\phi = 0$ to discuss the effects of patents on innovative activity.

[7] Of course, it is an important assumption from the standpoint of these models in that it generates "endogenous" growth in the traditional sense.

[8] The specific method used here to relax the assumption that innovations increase productivity proportionally will not generate a closed-form solution in the models of Grossman and Helpman (1991b) and Aghion and Howitt (1992) because of the functional form assumptions in those models. However, I conjecture that a closely related method will work. Instead of reducing the effect of an innovation on productivity, one can let the amount of labor required to discover a new innovation grow with the level of productivity. To see the merit of this approach, consider a simple analogy. If the discovery of knowledge is thought of as climbing a ladder in which each rung is a new innovation, then the first method (the one used in this paper) corresponds to reducing a climber's vertical speed (measured in feet, not rungs, per unit of time). The second method corresponds to widening the space between each rung by ever-increasing amounts while keeping vertical speed constant. Both methods have the effect of making the ladder more difficult to climb. I thank Robert Barro for this insight.

the consumption/capital good using labor and a collection of pro-
ducer durables as inputs. Second, a collection of monopoly firms in
the intermediate-goods sector transforms capital into producer dura-
bles using designs discovered by the third sector, the R & D sector.
In the R & D sector, individuals take advantage of the existing stock
of knowledge, A, to invent new designs for producer durables and
sell these designs to the intermediate sector. In this framework, the
stock of knowledge corresponds to the subset of the real line denoting
the producer durables for which designs have been invented.

Because the decentralized setup closely follows Romer (1990), the
main exposition of the decentralized model is presented in the Ap-
pendix. Here, I shall focus on two aspects of the model that are of
particular interest, the derivation of the steady-state growth rate and
the share of labor invested in R & D by the decentralized economy.

The steady-state growth rate is derived easily from the R & D equa-
tion, and relaxing the assumption of $\phi = 1$ generates a balanced
growth path in the presence of an increasing labor force. To see this,
rewrite equation (6) in terms of the growth of the stock of knowledge
to get

$$\frac{\dot{A}}{A} = \delta \frac{L_A^{\lambda}}{A^{1-\phi}}. \tag{7}$$

Along the balanced growth path, the growth rate of knowledge is
constant by definition. This will be consistent with an increasing
amount of labor engaged in R & D provided that L_A^{λ} and $A^{1-\phi}$ grow
at the same rate, a restriction that will naturally tie down the growth
rate of knowledge. In each of the Romer/Grossman-Helpman/
Aghion-Howitt models, this or a similar strategy is sufficient to elimi-
nate the scale effects.

Differentiating both sides of equation (7) allows us to solve explic-
itly for the balanced path growth rate of knowledge. Moreover, as
discussed in the Appendix, this growth rate pins down all the other
interesting growth rates in the model:

$$g_A = g_y = g_c = g_k = g \equiv \frac{\lambda n}{1 - \phi}, \tag{8}$$

where n is the growth rate of the labor force, g_x represents the growth
rate of the placeholder x, y is output per worker, c is per capita
consumption, and k is the capital/labor ratio. Equation (8) states that
the growth rate of the economy in the steady state depends only on
the growth rate of the labor force and the parameters ϕ and λ, which
determine the external returns (as well as the returns to scale) in the
R & D sector. Notice that if we follow Romer/Grossman-Helpman/
Aghion-Howitt and assume $\phi = 1$, no balanced growth path exists

in this economy because L is growing.[9] Assuming $\phi < 1$ eliminates the devastating scale effects of the Romer/Grossman-Helpman/Aghion-Howitt models, and these scale effects are replaced by an intuitive dependence on the *growth rate* of the labor force rather than on its *level*.

To see this intuition, consider the case in which $\phi = 0$ and $\lambda = 1$ so there are no externalities to R & D. In this case, the rate of innovation is independent of the stock of knowledge so that

$$\dot{A} = \delta L_A. \tag{9}$$

If the labor force engaged in R & D were constant, the constant number of new innovations in each period would constitute a decreasing percentage of productivity over time. When there is very little knowledge in the economy, a new idea has a dramatic effect in percentage terms on the total amount of knowledge. However, once the economy has accumulated a large stock of knowledge, each new idea has only a small impact in percentage terms. If the number of new ideas is constant over time, then eventually the percentage increment in knowledge due to new ideas will go to zero. Growth will cease asymptotically.

Now suppose that instead of being constant, the labor force grows at some exogenous rate n. In the case of $\phi = 0$, the number of new ideas is also growing at rate n. For this to generate a balanced growth path, though, the number of new innovations must always represent a constant fraction of the stock of knowledge. But this is just another way of saying that the number of new innovations and the stock of knowledge must grow at the same rate. Since the number of new innovations is proportional to the labor engaged in R & D, the growth rate of productivity is then inextricably tied to the growth rate of the labor force. This is the intuition behind equation (8).[10]

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[9] More generally, if capital is included as an input to the R & D equation, the condition for a balanced growth path to exist in the presence of population growth is that the returns to scale for accumulable inputs in the R & D equation be less than unity. The parallel to the original AK-style models is obvious.

[10] The prediction that per capita growth depends on the growth rate of population can be found in Arrow (1962), but for a different reason. In that model, externalities to capital accumulation lead to increasing returns to scale in production, as in $Y = K^{\alpha+\beta}L^{1-\alpha}$, $0 < \alpha < 1$, $0 < \beta < 1$. Romer (1987) showed that with constant population and sufficiently strong externalities so that $\alpha + \beta = 1$, growth would not cease asymptotically. As in the R & D models, however, population growth in this simple AK model will cause growth rates to increase without bound. The Arrow model has $\alpha + \beta < 1$ so that the steady-state rate of per capita output growth is proportional to population growth. In this sense, the present paper relates to the Romer/Grossman-Helpman/Aghion-Howitt papers in the same way that Arrow (1962) relates to Romer (1987).

This result is found in a similar setting in Nordhaus (1969). I thank Sam Kortum for this reference.

It is also important to note what does *not* determine steady-state growth in this model. Steady-state growth is invariant to government tax policy, including investment tax credits and R & D subsidies. This result is immediately obvious from the derivation of equation (8). That equation hinges on taking logs and differentiating both sides of equation (7), which will necessarily be independent of a constant subsidy or tax. Therefore, such taxes and subsidies will never have growth effects in this model.

These results contrast sharply with the results of the Romer/Grossman-Helpman/Aghion-Howitt models in which the steady-state growth rate depends endogenously on policy variables such as subsidies to R & D. This endogeneity hinges critically on the assumption that $\phi = 1$, which is strongly rejected by the time-series evidence on TFP growth and R & D. Once this assumption is relaxed to generate results consistent with the time-series evidence, the implications of the Romer/Grossman-Helpman/Aghion-Howitt models change substantially. Long-run growth in the extended model depends only on population growth and the degree of external returns to R & D, parameters that are typically assumed to be exogenous.[11]

The Appendix also derives the share of labor employed by the R & D sector in the decentralized economy:

$$s^{DC} \equiv \frac{L_A}{L} = \frac{1}{1 + \psi^{DC}},$$

$$\psi^{DC} \equiv \frac{1}{1 - \alpha} \left[\frac{\rho(1 - \phi)}{\lambda n} + \frac{1}{\sigma} \right]. \tag{10}$$

According to this equation, the steady-state share of labor devoted to R & D depends on several parameters within the model. A higher steady-state growth rate $\lambda n / (1 - \phi)$ is associated with a larger share of labor in R & D. Notice, however, that the causality runs completely from growth to R & D and not vice versa. A lower rate of time preference or a higher intertemporal elasticity of substitution also leads to an increase in the share of labor devoted to R & D along the balanced growth path. An increase in the R & D productivity parameter δ will have no effect on the steady-state labor share of the R & D sector, but it is easy to show that a wage subsidy to labor engaged in this activity will increase the share of labor devoted to R & D.[12]

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[11] Kremer (1993) extends the results in this paper by allowing the level of population to be determined endogenously in a Malthusian way by the level of output.

[12] If ξ is the rate of the proportional wage subsidy to R & D, then the steady-state share of labor in R & D will be given by $\tilde{s}^{DC} = 1/(1 + \tilde{\psi}^{DC})$, where $\tilde{\psi}^{DC} = \psi^{DC}/(1 + \xi)$.

IV. Welfare and the Social Planner Problem

To gauge the welfare properties of the decentralized solution, consider the social planner formulation of this growth model. A representative consumer solves

$$\max_{c_t, L_{At}} \int_0^\infty e^{-\rho t} u(c_t) dt, \quad c \equiv \frac{C}{L} \tag{11}$$

subject to the following constraints:

$$Y = (A L_Y)^\alpha K^{1-\alpha}, \tag{12}$$

$$\dot{K} = Y - C, \tag{13}$$

$$\dot{A} = \delta L_A^\lambda A^\phi, \tag{14}$$

and

$$L_A + L_y = L, \quad \frac{\dot{L}}{L} = n. \tag{15}$$

Setting up the usual Hamiltonian and solving this program reveal that growth in the steady state is given once again by

$$g_y = g_c = g_k = g_A = g \equiv \frac{\lambda n}{1 - \phi}. \tag{16}$$

Thus the steady-state growth rate in the decentralized model is the same as that in the social optimum, despite the presence of externalities in the R & D sector and monopoly behavior in the intermediate-goods sector.

In the social planner formulation, the share of labor devoted to R & D along the balanced growth path is affected by the externalities and the imperfect competition. The socially optimal share of labor is given by

$$s^{SP} \equiv \frac{L_A}{L} = \frac{1}{1 + \psi^{SP}},$$

$$\psi^{SP} \equiv \frac{1}{\lambda} \left[\frac{\rho(1 - \phi)}{\lambda n} + \frac{1}{\sigma} - \phi \right]. \tag{17}$$

When this solution is compared to the decentralized solution in equation (10), the steady-state share of labor devoted to R & D in the social optimum differs for three reasons. First, the presence of an additional " $-\phi$ " term in the social optimum reflects the incorporation of the externalities to R & D. When these externalities are positive, too little R & D is undertaken in the decentralized equilibrium because agents do not take into account the increase in the value of

future R & D that their discoveries impart. On the other hand, if there are diminishing returns to R & D so that φ is negative, there may actually be too much R & D in the decentralized equilibrium.

Second, the parameter λ enters the socially optimal share of labor in R & D reflecting the externalities at a point in time due to the duplication of research. Other things equal, the presence of λ < 1 will cause the decentralized economy to overinvest in R & D because of the negative externality.

Finally, the decentralized share of labor in R & D differs from the social optimum because of the monopoly markup over marginal cost in the sale of producer durables to the final sector, reflected by the presence of $1/(1 - \alpha)$ in equation (10). This effect causes too little labor to be devoted to R & D along the balanced growth path in the decentralized model. Note that if there are no external returns in the R & D equation so that φ is zero and λ is one, this is the only effect present and R & D is too low in equilibrium. Since taxes and subsidies to R & D can affect this share, there is room in this model for tax policy to be welfare improving. Whether or not the appropriate policy is a tax or a subsidy, though, generally depends on the sign of φ and the magnitude of λ.

Figure 3 plots the difference between the socially optimal share of labor in R & D and the share devoted to R & D by the decentralized economy in the steady state. Parameter values for this calibration exercise are $\alpha = 2/3$, $\rho = .03$, $n = .02$, and $\sigma = 1$. The parameter φ is allowed to take on values in the range $[-1, 1)$, and λ takes values in the range $(0, 1]$. The most surprising result of this exercise is that for nearly all these parameter values the decentralized economy underinvests in R & D. For example, even when $\phi = -1$ and $\lambda = .5$ so that there are large negative externalities to the R & D process (both at a point in time and across time), the decentralized economy underinvests in R & D. When the decentralized economy does over-invest in R & D, the amount of overinvestment is small. The reason is that for plausible parameter values, the monopoly effect in this model is large and dominates the negative externalities. For example, if the labor share is 2/3, the monopoly markup for intermediate goods $1/(1 - \alpha)$ is 300 percent.

This effect is documented more carefully in figure 4, which plots the gap between s^{SP} and s^{DC} as a function of φ and α for λ in {1.00, .75, .50, .25}. Even if $\lambda = .5$, relatively low labor shares (below $\alpha = .6$) are required to generate overinvestment in R & D. Alternatively, values of λ of .25 or below can generate overinvestment in R & D, as shown in figure 4d.

Together, figures 3 and 4 suggest that the decentralized economy underinvests in R & D relative to the social optimum for plausible
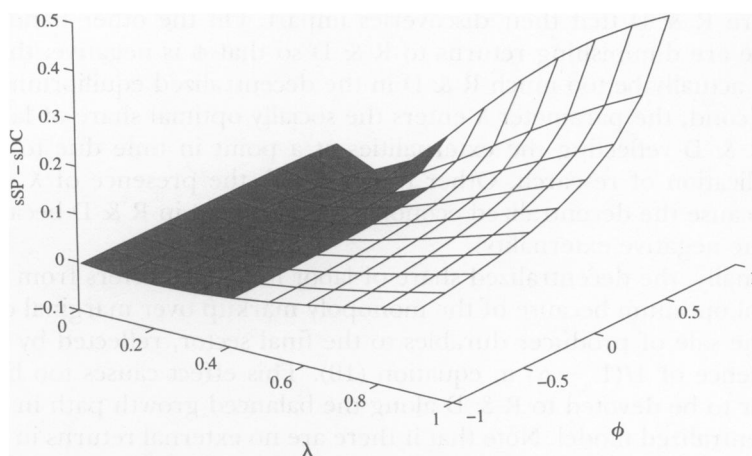
FIG. 3.—Difference in R & D shares: social optimum vs. the decentralized economy. Source: Author's calculations. Negative values are shaded.
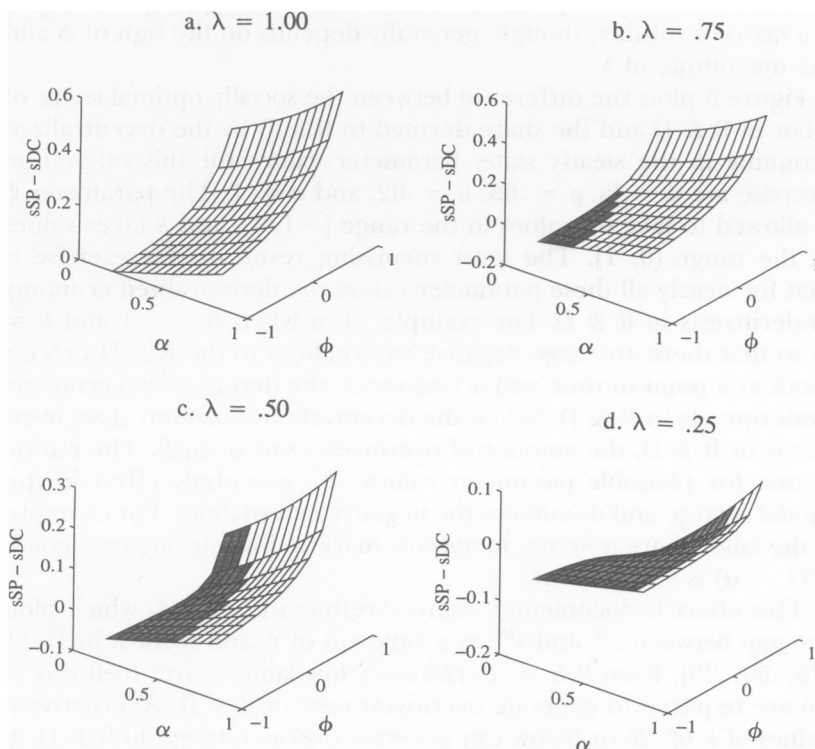

FIG. 4.—Difference in R & D shares: social optimum vs. the decentralized economy. Source: Author's calculations. The labor share (α) ranges from .2 to .85 in the figure. Negative values are shaded.

values of α and that this feature of the model is relatively insensitive to the parameters ϕ and λ. This perhaps surprising result suggests that the interaction between market structure and patents is possibly more important than the degree of externalities in the R & D process itself. This aspect of the model has not been explored carefully in the endogenous growth literature, but the evidence here highlights it as important for future research.

V. The Transitory Effect of an Increase in R & D Investment

One of the main results in the model above is that a permanent increase in the R & D share does not have a permanent effect on the growth rate. Nevertheless, such a change clearly has a level effect and raises the growth rate along a transition path to the new steady state. Intuitively, the length of the transition path is likely to depend on the parameter ϕ, and the closer ϕ is to one, the longer the transition path. This intuition is made more precise below.

We now depart from the rest of the paper and assume that both the R & D share of labor and the physical investment rate are constant and are given exogenously. This reduces the dimensionality of the problem and simplifies the analysis.[13] Williams (1994) provides numerical results that support the results derived below when both the physical investment rate and the R & D share are determined endogenously.

To analyze the transition dynamics of the model, define $x \equiv \dot{A}/A$ and $z \equiv Y/K$. These variables will be constant in the steady state. We shall consider experiments in which one-time shocks raise either the R & D share or the physical investment rate, perturbing x and z away from their steady-state values. The behavior of the growth rate of per capita output g_y can be written as a function of these variables as in the following three equations:

$$
\begin{aligned}
g_y &= \alpha x + (1 - \alpha)iz - (1 - \alpha)(n + d) \\
&= \alpha(x - x^*) + (1 - \alpha)i(z - z^*) + g^*,
\end{aligned}
\tag{18}
$$

$$
\frac{\dot{x}}{x} = -(1 - \phi)(x - x^*),
\tag{19}
$$

and

$$
\frac{\dot{z}}{z} = \alpha(x - x^*) - \alpha i(z - z^*),
\tag{20}
$$

[13] In the context of capital accumulation, Sato (1966) and Mankiw, Romer, and Weil (1992) examine transition dynamics for the case of a constant saving rate. Barro and Sala-i-Martin (1992) endogenize the saving rate and find similar results.

where i is the physical investment share of output, d is a constant exponential rate of depreciation for physical capital,[14] and an asterisk denotes steady-state values: $g^* = \lambda n/(1 - \phi)$, $x^* = g^*$, and $z^* = (x^* + n + d)/i$. Equation (18) is derived by log-differentiating the production function and substituting in from the dynamic budget constraint for capital accumulation. Equation (19) is obtained by differentiating the R & D equation. Equation (20) is obtained by combining equation (18) with the dynamic budget constraint for capital accumulation.

The dynamic system defined by (19) and (20) is nonlinear, but it is obviously globally stable for $\phi < 1$, the relevant case. The nonlinear system can be linearized around its steady state to yield the local transition dynamics for the growth rate of output per capita:

$$
\begin{aligned}
g_y = g^* &+ \alpha(x_0 - x^*)e^{-bt} \\
&+ (1 - \alpha)i(z_0 - z^*)e^{-at} \qquad\qquad (21) \\
&+ (1 - \alpha)\frac{a}{a - b}(x_0 - x^*)(e^{-bt} - e^{-at}),
\end{aligned}
$$

where $b \equiv (1 - \phi)x^*$ and $a \equiv \alpha(x^* + n + d)$, and x_0 and z_0 are starting values for x and z. The transition dynamics for output growth involve three components. The first term reflects the direct transition dynamics that occur when the growth rate of knowledge (or TFP) deviates from its steady-state value. The speed of convergence for TFP growth is given by the parameter b, which is equal to the steady-state growth rate itself in the benchmark case of $\phi = 0$ and is even smaller when ϕ is positive. If the steady-state growth rate of the economy is 2 percent, then the half-life for the transition dynamics of TFP growth is 35 years for $\phi = 0$.

The second term of equation (21) captures the transition dynamics associated with capital accumulation. The speed of convergence for this term is the parameter a, which is familiar from the work by Mankiw et al. (1992) and others. If labor's share of final output is $2/3$ and $x^* + n + d$ is something like 9 percent, then this term is approximately 6 percent, resulting in the rapid convergence commonly associated with the neoclassical growth model.

The third term of equation (21) incorporates the interaction between technological change and the marginal product of capital. Technological change raises the marginal product of capital, offsetting the usual diminishing returns as the economy grows.

There are two experiments to consider in the context of the transi-

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[14] I introduce nonzero depreciation of capital here because it is important for examining the speed of convergence.

tion dynamics here. Both will be presumed to occur starting from the steady state. The first experiment is a permanent increase in the investment rate. Because of the strong assumption that capital does not enter the production function for knowledge, a change in the investment rate affects the dynamics of growth only via the second term. The rapid convergence back to the steady state found in the neoclassical model is preserved in this special case.

The second experiment to consider is a permanent increase in R & D. This change will affect both of our state variables x and z and is therefore more complicated. For this experiment, it is helpful to have parameter values in mind. In particular, assume that the labor share of final output is $2/3$ and that the economy begins with 1 percent of its labor in the R & D sector. Also, assume $n = .02$, $g^* = .02$, and $d = .05$. The experiment will be a permanent increase in the R & D share from 1 percent to 2 percent.[15]

The results from this experiment are presented in table 1 and figure 5 for various combinations of λ and ϕ consistent with a steady-state growth rate of 2 percent. Several results are worth noting. First, though the half-lives of TFP growth are already long, the half-lives of output per worker are even longer. This is almost entirely due to the interaction term (the third term) in equation (21); in this experiment the change in the R & D share is so small that the effect on the capital/output ratio is negligible. For $\phi = 0$, the half-life for TFP growth is 35 years and the half-life for labor productivity growth is 62 years. Second, as ϕ rises, the half-life rises considerably, consistent with the observation that as ϕ goes to one the transition path becomes infinitely long. When $\phi = .5$, the two half-lives rise to 69 and 120 years, respectively. Third, though the transition paths are very long, the magnitude of the effect of raising the R & D share on growth depends crucially on λ. For $\lambda = 1$, the effect of the doubling of the R & D share is to raise the initial growth rate by a factor of $4/3$. However, for small values of λ, the initial impact on growth itself becomes small. Finally, figure 5 indicates why the half-lives for labor productivity growth are longer. The interaction term causes the labor productivity growth rate to rise for a number of years as the marginal product of capital increases before it begins falling inevitably to the original steady-state value.[16]

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[15] The basic results are similar when other plausible parameter values are chosen.

[16] One potential concern with these results is that they may overstate the half-lives, either because they are calculated when the economy is very close to its steady state or because of the linearization of nonlinear dynamics. On the first point, note that while the half-life calculation for labor productivity growth depends on the starting position, this is not true of the half-life calculation for TFP growth, where the dynamics follow a simple exponential path. On the second point, the differential equation in

TABLE 1

RESPONSE TO A PERMANENT INCREASE IN THE R & D SHARE FROM
1 PERCENT TO 2 PERCENT

(ϕ, λ)	Initial Growth Rate of A	Half-Life of A Growth	Initial Growth Rate of Y/L	Half-Life of Y/L Growth
(.00, 1.00)	.0333	35	.0331	62
(.50, .50)	.0255	69	.0253	120
(.75, .25)	.0225	139	.0223	242
(.90, .10)	.0210	347	.0208	674

NOTE.—Half-lives are reported in years and are based on a linearized approximation. The steady-state growth rate of both A and Y/L is 2 percent.
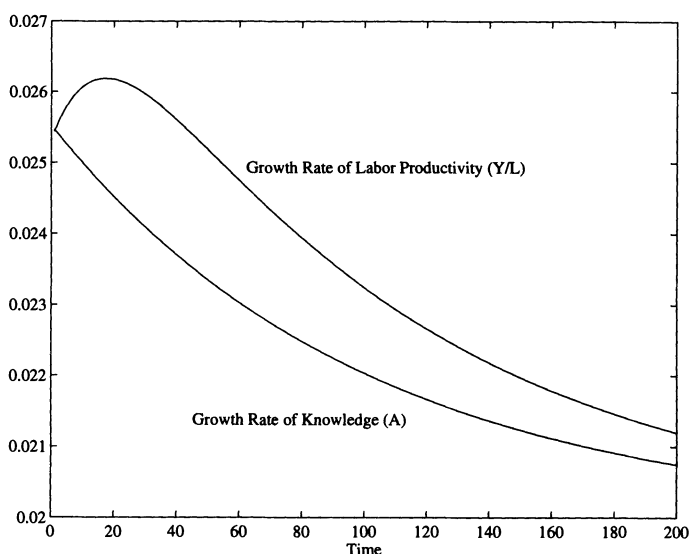

FIG. 5.—Growth rates along the transition path. The growth rates represent the response to a permanent increase in the R & D share from 1 percent to 2 percent for the case of ϕ = .5 and λ = .5.

There is a trade-off in the model between the length of the transition dynamics (governed by ϕ) and the magnitude of the effects (governed by λ). Matching a steady-state growth rate for a given rate of population growth pins down a linear relationship between λ and ϕ.

(19) governing the growth rate of knowledge x can actually be solved analytically with no approximation. In this case, the half-life calculations produce numbers slightly smaller than those in the linear approximation, but the spirit of the results is unchanged. For example, instead of 35 or 69 years in the first two cases, the nonlinearized calculation produces half-lives of 25 and 51 years.

Large values of ϕ (i.e., close to one) require small values of λ, and vice versa. As shown in table 1, when a large value of ϕ generates a long transition path, the initial impact of an increase in R & D on growth is very small because of the small value of λ. It is interesting to note that the impact of R & D on growth is most pronounced for values of ϕ that are small, despite the shorter length of the transition path. Overall, this brief analysis of transition dynamics suggests that the effects of R & D, although they do not last forever, may still be very important.

Both the analysis of transition dynamics and the question of over- versus underinvestment in R & D indicate that obtaining empirical estimates of the λ and ϕ parameters is an important next step. A substantial amount of research in the micro productivity literature has focused on estimating knowledge spillovers and knowledge pro- duction functions (see Griliches [1992] and Nadiri [1993] for recent reviews of this literature). In particular, work by Adams (1990), Kor- tum (1992), and Caballero and Jaffe (1993) using industry-level data is potentially very useful. Jones (1994) provides estimates of λ and ϕ from aggregate data and compares these estimates to results in the productivity literature. Overall, the conclusion is that it is difficult to separately identify the effects of λ and ϕ. Detailed work at the indus- try level combining the approach of Caballero and Jaffe with a speci- fication of the R & D equation that avoids scale effects appears to be necessary to pin down both parameters simultaneously.

VI. Is This a Plausible Model of Economic Growth?

The implication of this model that population growth is a fundamen- tal driving force behind per capita output growth is superficially easy to reject (though perhaps not as easy to reject as the implication that per capita growth is driven by the level of population or human capital). For example, a number of cross-country studies have shown that population growth and per capita output growth are either un- correlated or even negatively correlated (see, e.g., De Long and Sum- mers 1991; Mankiw et al. 1992). However, this evidence does not necessarily contradict the model for at least three reasons. First, the model describes the economic growth of an entity that depends on the creation of new ideas for growth. In this sense it is much more reasonable to apply this model to advanced economies than to econo- mies that can grow by transferring existing ideas across countries. Second, even among advanced economies, issues of technology trans- fer vastly complicate cross-sectional inference. Comparing the United States and its population growth rate to Japan and its population

growth rate would neglect the potentially large flow of ideas that occurs between these countries. The world (or at least the part of the world that is growing by pushing out the technological frontier) is potentially a more appropriate unit of observation. Finally, the model actually relates the growth in the effective number of researchers, rather than in population, to economic growth. As shown in figure 2, these numbers have not been equivalent for the United States over the postwar era: R & D intensity appears to have increased. Jones (1994) explores this difference in more detail.[17]

Then what is a test of the model? Despite the difficulties associated with relating population growth to economic growth in large cross sections of countries, the essence of the model still contains very strong and testable restrictions. I have argued that the implication of intertemporal scale effects associated with the Romer/Grossman-Helpman/Aghion-Howitt models is markedly inconsistent with time-series evidence. However, appropriately interpreted, the model still contains a very strong prediction of scale effects. Consider two economies that are both at approximately the same level of development (i.e., that have the same level of knowledge A). Furthermore, suppose that these economies are technologically distinct in the sense that they do not exchange ideas either with each other or with other economies. The R & D equation in (7) predicts that the economy with more researchers should grow faster. The growth differences may not last forever, but as we saw in the previous section, the transition dynamics can be very long. That is, cross-sectional scale effects over relatively long transition periods are preserved within the model. Having more people to draw on to create ideas is a significant advantage that should be detectable under the right circumstances.

Kremer (1993) provides evidence of this effect by examining a natural experiment associated with the melting of the polar ice caps around 10,000 B.C. The ensuing destruction of land bridges eliminated nearly all contact between the Old World, the Americas, Australia, Tasmania, and Flinders Island. Kremer argues that the subsequent economic development of these regions until they were technologically reintegrated around the time of Christopher Columbus's voyage is broadly consistent with the kind of cross-sectional scale effects predicted here. Examination of evidence such as this may help in determining the plausibility of R & D–based growth models.

[17] The rise in R & D intensity is itself somewhat of a puzzle in the context of the model discussed here. Along a balanced growth path, nothing in the model delivers a rising R & D intensity. One possibility is that these increases occur for reasons outside the model, e.g., as a reflection of changing tax policies or a strengthening of property rights. Another possibility is that much of the rise merely reflects the relabeling of individuals who previously were not officially counted as "scientists and engineers engaged in R & D."

VII. Conclusion

This paper begins with the simple observation that we do not see the intertemporal scale effects that are strongly predicted by the R & D–based models of growth of Romer (1990), Grossman and Helpman (1991a, 1991b, 1991c), Aghion and Howitt (1992), and others. This literature implies that growth rates should be monotonically increasing in the level of resources devoted to R & D. But despite enormous increases in the amount of R & D over the last 40 years, we see nothing like the large increases in growth rates that should occur.

The "semi-endogenous" R & D–based growth model proposed here is consistent with these empirical observations and produces a number of interesting results. First, unlike the AK-style models and the R & D models of Romer/Grossman-Helpman/Aghion-Howitt, this model predicts that the growth rate is determined by parameters that are typically viewed as invariant to policy manipulation. Growth in the economy is tied directly to growth in productivity, which in turn depends on the discovery of new designs through R & D. Individuals are the critical input into the discovery of new designs, and the growth rate of the economy depends crucially on the growth rate of the labor force, an exogenous variable.

As in the Solow model, subsidies to R & D and to capital accumulation have no long-run growth effects in this model, but rather affect growth only along the transition path to the new steady state. This result runs counter to much of the existing literature but, as argued in this paper and in Jones (1995), is perfectly consistent with the time-series evidence for advanced OECD economies. Eliminating the dependence of the long-run growth rate on policy appears to be necessary to reconcile the R & D–based models of endogenous growth with the joint time-series behavior of R & D and TFP growth.

On the other hand, the model differs from the Solow model in a crucial respect. Although the growth rate of the economy turns out to be a function of parameters that are typically thought of as exogenous, growth in this model is endogenous in the sense that it derives from the pursuit of new technologies by rational, profit-maximizing agents. The model provides a well-defined answer to the question, Why do economies exhibit sustained per capita income growth?

Appendix

The Decentralized Model

The final-goods sector produces the consumption good Y using labor L_Y and a collection of intermediate inputs x, taking the available variety of intermedi-

ate inputs A as given:

$$Y = L_Y^{\alpha} \int_0^A x_i^{1-\alpha} \, di. \tag{A1}$$

This production function characterizes technological change as increasing variety, in the tradition of Dixit and Stiglitz (1977) and Ethier (1982). Invention corresponds to the discovery of a new variety of producer durables that provides an alternative way of producing the final consumer good. In this way, the fatal onset of diminishing returns is continually postponed by the creation of new inputs.

Since final-goods production is constant returns to scale, without loss we can consider a single price-taking firm when solving for the competitive outcome. When the price of Y is normalized to unity in every period, profit maximization yields the following conditional demand functions:

$$w = \alpha \frac{Y}{L_Y} \tag{A2}$$

and

$$p_i = (1 - \alpha)L_Y^{\alpha}x_i^{-\alpha} \quad \forall \, i, \tag{A3}$$

where w represents the wage paid to labor in the final-goods sector, and p_i is the rental price of producer durable i.

The intermediate sector is composed of an infinite number of firms on the interval $[0, A]$ that have purchased a design from the R & D sector and now act as monopolists in the production of their particular variety. Capital is rented at rate r for the period, and a firm that has purchased a design can then effortlessly transform each unit of capital into a single unit of the intermediate input. We assume for simplicity that producer durables can be transformed costlessly back into capital at the end of the period and that no depreciation takes place. Each of the intermediate firms, then, solves the following problem every period:

$$\max_x p(x)x - rx. \tag{A4}$$

Because these firms are monopolists, they see the downward-sloping demand curve for their producer durables generated in the final-goods sector. The result is a standard monopoly problem with constant marginal cost and constant elasticity of demand, and it is readily solved to yield the following equations for price, quantity, and profit π:

$$\bar{p}_i = \bar{p} \equiv \frac{r}{1 - \alpha} \quad \forall \, i, \tag{A5}$$

$$\bar{x}_i = \bar{x} \equiv \left[\frac{(1 - \alpha)L_Y^{\alpha}}{\bar{p}} \right]^{1/\alpha} \quad \forall \, i, \tag{A6}$$

and

$$\bar{\pi}_i = \bar{\pi} \equiv \alpha \bar{p} \bar{x} = \alpha(1 - \alpha)\frac{Y}{A} \quad \forall\, i. \tag{A7}$$

These equations demonstrate that each intermediate firm sets the same price and sells the same quantity of its producer durable. This observation and the fact that the total stock of producer durables is related to the capital stock by

$$K = \int_0^A \bar{x}\, di = A\bar{x} \tag{A8}$$

yield the production function for the final-goods sector given previously in equation (1). Combining this result with equations (A3) and (A7) shows that capital is underpaid relative to the competitive case in order to compensate the R & D labor:

$$r = (1 - \alpha)^2 \frac{Y}{K}. \tag{A9}$$

Finally, consider the production of new designs in the R & D sector. We assume that labor engages in R & D to search for new designs and succeeds according to the specification in equation (4), where $\bar{\delta} = \delta l_A^{\lambda-1} A^\phi$ is taken as given by an individual researcher. Any individual is allowed to enter the R & D sector and prospect for new designs, so that labor must receive the same compensation in its two uses:

$$w = P_A \delta A^\phi L_A^{\lambda-1}. \tag{A10}$$

The upstream decision by intermediate firms to produce a producer durable hinges on the difference between the cost of purchasing the patent from the R & D sector, P_A, and the monopoly rents that can be obtained in exchange. With this knowledge, the monopolistically competitive R & D sector sets the price P_A to extract the present discounted value of the intermediate sector's monopoly profit. Since all durables yield the same profit in every period, all designs, regardless of age, trade for the same price P_A at a point in time. Then the following simple arbitrage equation must hold:

$$r = \frac{\bar{\pi}}{P_A} + \frac{\dot{P}_A}{P_A}. \tag{A11}$$

This equation says that the R & D sector charges a price for its designs that is just sufficient to make the intermediate monopolists indifferent between purchasing the design to produce the durable good and not undertaking any production at all. The dividend rate π/P_A and the capital gain exactly meet the required rate of return on investment r.

To close the model, we examine the consumption decision. Following the usual convention, we shall assume that this decision can be characterized by a representative consumer maximizing an additively separable utility function

subject to the dynamic budget constraint:

$$\max_{c_t} \int_0^\infty e^{-\rho t} u(c_t) dt \qquad (A12)$$

subject to

$$\dot{K} = rK + wL_y + wL_A - P_A\dot{A} + A\overline{\pi} - C, \qquad (A13)$$

where C is consumption and c is C/L. Assuming that the utility function $u(\cdot)$ exhibits constant relative risk aversion equal to $1/\sigma$, we can write the first-order condition for the consumer's problem as

$$\frac{\dot{c}}{c} = \sigma(r - \rho - n). \qquad (A14)$$

Equilibrium and Balanced Growth in the Decentralized Model

In the perfect-foresight equilibrium of the decentralized model, all agents take as given the time paths of variables that they do not control: consumers take the time paths of wages and interest rates as given; labor working in the R & D sector takes the stock of knowledge A and the resale price of a new design as given; intermediate-goods producers take the price of designs and the demand for producer durables as given; and so forth. Equilibrium is then characterized by the condition that supply equals demand for all relevant quantities.

At this point we restrict our attention to the perfect-foresight equilibrium balanced growth path, also referred to as the steady state. The balanced growth path equilibrium will be defined as the perfect-foresight equilibrium in which the growth rates of all variables in the model are constant. The R & D equation pins down the growth rate of A, as discussed in the text. Then the usual arguments lead to the fact that the growth rates of c, y, k, and A are all given by the same rate g: the constancy of y/k and the dynamic budget constraint mean that c and y grow at the same rate; the fact that technological change is labor augmenting means that all these growth rates are equal to the growth rate of A, as given in equation (8) above.

The decentralized share of labor going to R & D is obtained as follows. Labor will enter the R & D sector until the marginal cost, the wage, is equal to the (private) marginal benefit, $P_A\delta$. With a slight rewriting,

$$w = P_A \frac{\dot{A}}{L_A}. \qquad (A15)$$

If we multiply both sides by L_A, this is just the zero-profit condition. Substituting from the arbitrage equation for P_A then gives

$$wL_A = \frac{g}{r - (\dot{P}_A/P_A)} A\overline{\pi}. \qquad (A16)$$

Total profits $A\overline{\pi}$ are proportional to output, which is in turn proportional to wL_y. This leads to the ratio $L_A/L_y = s/(1 - s)$ entering the equation, which

can then be solved to yield the equation for s^{DC} in the text, after we note that both r and the growth rate of design prices are constant.

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