Short-Run Equilibrium Dynamics of Unemployment, Vacancies, and Real Wages

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In this paper, I study the dynamics of adjustment in a labor market, following an exogenous shock to the real value of output. Some of the stylized facts of business cycles, with which the predictions of the model are consistent, include first, real wages do not fully reflect fluctuations in the real value of labor's marginal product, so real profits fluctuate more than real wages. Second, unemployment responds to output shocks, but its response is slow. Finally, in countries where there are good data on vacancies, like Britain, we observe that vacancies respond more quickly to shocks and with greater amplitude than unemployment.

Several authors have constructed models to explain why output shocks are absorbed partly by real wages and partly by unemployment (the empirical regularity in the United States is discussed by Robert Hall, 1980). Implicit contract models (Costas Azariadis, 1979; Oliver Hart, 1983) have successfully explained why real wages may not reflect output shocks, and the models with asymmetric information and severance pay have also had some success in explaining fluctuations in unemployment. Bargaining models (Ian McDonald and Robert Solow, 1981) and efficiency wage models (Janet Yellen, 1984) appear to be more successful in explaining fluctuations in unemployment, but formalizations are still in their infancy. The models have not yet been subjected to the same scrutiny as implicit contract and earlier models.

One feature shared by all these models is that they are static. They explain how real wages and employment respond to shocks in a comparative-static framework but say nothing about the adjustment path from one equilibrium to the next. Also, the models say nothing about job vacancies, either in equilibrium or during the adjustment process. By contrast, this paper takes the view that by modeling job vacancies explicitly, one can learn more about the behavior of unemployment and real wages, both in equilibrium and during the adjustment to equilibrium. Thus, the model developed below is explicitly dynamic, and in it job vacancies play a critical role in the transmission of output shocks to real wages and unemployment.

A job vacancy indicates a willingness by a firm to hire a worker.[1] It is equivalent to unemployment of capital, so just as workers move between the states of employment and unemployment, jobs move between the states of occupancy and vacancy. I model the interaction of vacancies and unemployment by using ideas from equilibrium search theory, where there is continuous wage recontracting and perfect anticipation of the adjustment paths of all endogenous variables. Job vacancies enter the model via their influence on job contacts, which depend on the number of firms looking for workers. Some firms may not wish to hire and so they may not be actively engaged in the search process. Only firms with job vacancies are actively engaged in search, so the number of job contacts and

*London School of Economics, Houghton Street, London WC2A 2AE. Work on this paper was started at NBER's Summer Institute in July 1983 and later completed at the Industrial Relations Section of Princeton University; I thank both institutions for their hospitality and financial support. I also thank the referee and seminar participants at the universities of Boston, Chicago, Iowa, Wisconsin, and Yale for helpful comments.

[1]For more discussion of the concept of vacancies, with empirical evidence for the United States and Britain, see, respectively, Katharine Abraham (1983) and my paper with Richard Jackman and Richard Layard (1983). Earlier contributions incorporating vacancy-unemployment interactions (but without explicit micro models and with a peripheral role for wages) include Charles Holt and Martin David (1966) and Bent Hansen (1970).

the flow of workers out of unemployment depend on the number of job vacancies.

The static properties of the model closely resemble those of other models cited: real wages do not fully reflect output shocks and unemployment absorbs some of the shocks. In order to derive an adjustment path from this setup, I make use of the observation that vacancies respond more quickly to shocks than unemployment. When desired employment rises, firms open up new vacancies to indicate their willingness to hire more labor, and unemployment falls when labor takes up these vacancies. Similarly when desired employment falls, firms withdraw their vacancies, making it more difficult for unemployed workers to find jobs. Hence, changes in vacancies lead changes in unemployment. The critical assumption that I make in this paper is that vacancies are a nonpredetermined, fully flexible variable, whereas unemployment is predetermined, except for additions to it that result from upward movements in reservation wages. Wages are also nonpredetermined with continuous recontracting.

The assumption concerning vacancies is obviously extreme, since opening up new vacancies often requires the acquisition of new capital, which may not be readily available. However, if the existence of a job vacancy is a prerequisite to a hiring, vacancies must respond at least as quickly to shocks as unemployment, whatever the length of time required to open up a vacancy. The assumption that I make here enables the development of a simple model whose dynamic behavior is governed by this differential speed of response, and not by any time lags in the acquisition of capital.

I consider the effects of a multiplicative shock to the value of output and to capital costs, under the assumption of perfect foresight. Changes in unemployment result from successful job contacts and from job separations, making unemployment a relatively sluggish predetermined variable. Vacancies and wages adjust continuously so as to ensure that during adjustment the economy stays on a unique perfect-foresight adjustment path.

The assumptions of the model imply that the unique adjustment path is characterized by a constant vacancy-unemployment ratio and constant real wages. Hence these variables change only in response to news about shocks and not during the adjustment following the news. Moreover, real wages do not change by as much as the value of output and capital costs, inducing firms to change the number of job vacancies. Following this initial response to the shock, the number of job contacts changes, leading to simultaneous changes in vacancies and unemployment until the economy reaches its new equilibrium.[2] The model predicts that vacancies overshoot their long-run equilibrium value when expansionary or contractionary shocks are first observed. Following the initial impact of the shock, they change in the same direction as unemployment, tracing anticlockwise loops in vacancy-unemployment space.

The model also predicts that the time-series of unemployment, vacancies, and real wages will be characterized by asymmetries. The response of unemployment to a negative shock is faster than the response to a positive shock. The reason for this is that, when the shock is negative, reservation wages and profits do not fall by as much as actual wages and profits, leading to a number of immediate job separations. There are no corresponding immediate job matchings when the shock is positive, because the process that brings together firms and workers is time consuming. For the same reason, vacancies overshoot by less when the shock is negative, and mean observed wages respond less at the onset of a negative shock than at the onset of a positive one. The job separations when the shock is negative re-

[2] Thus, in time-series data, the model predicts that changes in productivity-corrected real wages should be small and uncorrelated, whereas unemployment should follow an autoregressive process. These predictions are consistent with the U.S. and U.K. data of Joseph Altonji and Orley Ashenfelter (1980), and with the U.S. real-wage data of Ashenfelter and David Card (1982).

move from the market low-wage jobs, raising the mean wage.[3]

Of crucial importance in the derivation of these results is the fact that labor has some alternative use to production, whose real return is not sensitive to expansionary or contractionary output shocks. This alternative use determines partly the workers' threat point in the wage bargain, introducing an element of inflexibility in wages.[4] Firms' threat points are determined partly by the cost of idle capital, which I assume varies in proportion to the value of output. This assumption, however, is not important for the qualitative impact of output shocks.

Sections I, II, and III develop the equations describing the behavior of unemployment, vacancies, and wages. Section IV derives the reservation prices and discusses job rejection. Section V brings the elements together and derives the unique perfect foresight path for this economy. Finally, Section VI describes the economy's response to a multiplicative output shock.

I. Unemployment

Consider an economy consisting of a fixed labor force (which is used as the normalizing constant) and of a variable number of jobs. At any point in time, a fraction $1-u$ of the labor force is employed and the remaining fraction u is unemployed and looking for a job. The variable u, as well as the other endogenous variables, are functions of time, but the time notation will be suppressed for convenience. Steady-state equilibrium values will be distinguished by a bar over the relevant variable. The number of jobs as a fraction of the labor force is $1-u+v$, where v denotes the job vacancies which are waiting for workers to arrive. It is assumed that no firm without a vacancy can take on a worker; that is, having a job vacancy is a prerequisite for participating in the search process that brings together jobs and workers.

There are frictions in the labor market, which make it impossible for all the unemployed to find jobs instantaneously. I shall not model explicitly the source of these frictions.[5] Instead, it will be assumed that the implication of these frictions is that the typical unemployed worker is faced with a probability p of making a contact with a firm with a job vacancy, the typical vacancy is contacted by a worker with probability q and both p and q lie strictly between 0 and 1. It is plausible to assume that these contact probabilities depend on the relative number of firms and workers engaged in job search. Writing θ for the ratio of vacancies to unemployment (v/u ratio) we assume that both p and q are continuous differentiable functions of θ, with $\partial p/\partial\theta \geq 0$ and $\partial q/\partial\theta \leq 0$.[6]

There is a simple relationship between p and q, because, by definition, the number of unemployed workers who make contacts with jobs must be equal to the number of vacancies that are contacted by workers. Thus, with a typical contact probability of p and with u job searchers, the number of workers who make contacts is up; similarly, the number of job vacancies that are contacted by workers is vq. Hence, $up = vq$, or, using the θ notation,

$$p(\theta) = \theta q(\theta).$$

The assumption that $\partial p/\partial\theta \geq 0$ and $\partial q/\partial\theta \leq 0$ implies that the elasticities of p and q

[3] Recently, Salih Neftci (1984) provided evidence for the asymmetric behavior of unemployment which is consistent with the predictions derived here.

[4] McDonald and Solow make a similar argument for wage inflexibility in their union bargaining model.

[5] The literature on imperfect information and job search discusses extensively the sources of these frictions. See, for example, Edmund Phelps (1972).

[6] Assuming that p and q depend on the ratio θ and not on the absolute levels of u and v imposes some homogeneity restrictions on the process of search which, in some models, have strong implications for the efficiency of equilibrium outcomes. See, for example, my 1984 paper, where I used a steady-state version of this model to study the efficiency of job rejection in the presence of search externalities. In this model, homogeneity simplifies the exposition without materially affecting the analysis. There are also some reasons to believe that homogeneity is a plausible restriction a priori; see, for example, Stephen Nickell (1979), where the probability p is successfully estimated as a function of the v/u ratio, and Hall (1979) where a homogeneous search process is explicitly derived.

with respect to θ must be less than 1 in absolute value.

Not all job-worker pairs are equally productive. I assume that when a worker and a vacancy come together they immediately establish their net output from a potential match. Let this be y units and suppose that y is a random drawing from a cumulative density function $F(y)$. The density function is identical for all jobs and workers, so the reason for different productivities is the difference in the efficiency of the job match, not in general skills or technologies. I assume that y is a constant flow per unit time and that it is parametric.

When a firm and a worker meet, they agree whether to form a job match, or whether to reject it and try again. All potential job matches with nonnegative surplus are made. The surplus is divided according to a wage function whose derivation I shall describe in Section III. For the moment, let x denote the productivity that yields zero surplus, so all jobs matches with productivity $y \geq x$ are made, whereas job matches with productivity $y < x$ are rejected. The reservation productivity x is a nonpredetermined choice variable.

The job matches that are made at each point in time equal the fraction of contacts that yield productivity at least as high as x. Let

$$a = \int_x^\infty dF(y).$$

Then, in a large market the number of job matches made is simply *aup*; the number of contacts made is *up*, and a is the fraction of acceptable contacts.

Additions to unemployment take place exogenously, at the flow rate $s(1-u)$. The jobs broken up are selected randomly, so each job-worker pair is faced with an exogenous separation probability s, regardless of the productivity of the match or the time it was formed. These assumptions, although strong, are needed to make the analysis tractable. Out of the steady state, separations may also take place endogenously as a result of changes in the reservation productivity x. The exogenous separation process assumed may be justified by appeal to firm-specific shocks that cancel out in aggregate, such as structural shifts in final demand or random obsolescence (and breakdown) of machines.

Unemployment changes in response to the flows in and out of jobs. The rate at which it changes is given by

$$\dot{u} = (1-u)s - a(x)p(\theta)u, \tag{1}$$

where a dot denotes a time derivative. The variables u, θ, and x are functions of time, but s is assumed to be constant. Equation (1) makes unemployment a sluggish variable, and it is one of the fundamental dynamic equations of the model.[7]

II. Vacancies

The number of job vacancies is determined by firms, in response to the expected profit from a new vacancy. For simplicity, I shall use the terms firm and job interchangeably; that is, it will be assumed that there are constant returns to scale without substitution possibilities between labor and capital after the installation of capital, so each firm may be modeled as having only one job. Intuitively, a job may be thought of as a machine that could be operated by one worker. Firms could acquire machines for a fixed rental, and in order to engage in the search process that leads to job matches, they must have an idle machine. The number of machines in existence depends on the expected profit from an extra machine, given the expected duration of a job vacancy and the profit from production. Machines can be brought into use, rented and scrapped instantaneously, so the number of job vacancies is a perfectly flexible nonpredetermined variable.

Let us assume that apart from the foregone profit that a firm suffers when it has a vacancy, it has to bear also a cost k per unit time. The cost k is assumed to be a flow in order to simplify the exposition; it represents

[7]Some exogenous changes may induce once-and-for-all step changes in unemployment, and at these points equation (1) is not defined. See Section V below.

the fixed cost of machines that has to be borne regardless of whether jobs are filled or not, and any other labor-recruitment costs that the firm may have. Since net output y is parametric, it may be thought, in general, as being net of the part of k that has to be borne when the machine is occupied. Other costs netted out of y (but not out of k) include the cost of raw materials.

The flow of profit from a job with productivity y is given by $y - w(y)$, where $w(y)$ is the wage rate. Wages are chosen by firms and workers after they meet according to a Nash-bargaining rule, in a way described in the next section. Let V be the asset value of a vacancy (i.e., an idle machine) and J^e the expected asset value of a filled job, and suppose there is a perfect capital market with fixed interest rate r. Then, the asset value of a vacancy must yield a net return that is equal to the yield from the vacancy, plus the expected capital gain from finding a worker:

$$rV - \dot{V} = -k + aq(J^e - V). \tag{2}$$

In capital market equilibrium, the free-market yield rV must be equal to what the firm expects to get from an idle machine: an appreciation $\dot{V}$, a net cost k, and a probability aq of filling the job, and so of making a capital gain of $J^e - V$. The expected capital value of a filled job is the conditional expectation

$$J^e = E(J(y)|y \geq x),$$

where $J(y)$ is the expected asset value of a job with productivity y. In general, the asset value of a job will depend on the productivity of the job match, and since only job matches at least as productive as x are accepted, the best a firm can do when calculating the expected profit from a job match is to take the conditional expectation of $J(y)$.

Firms will acquire machines for as long as $V > 0$, and will scrap them when $V < 0$. Hence, if machines can be bought and sold without lags, equilibrium implies $V = 0$. This yields an equilibrium restriction on J^e:

$$J^e = k/a(x)q(\theta). \tag{3}$$

Equation (3) holds at all times, both in and out of steady-state equilibrium.

The asset value of an occupied machine that produces output y, $J(y)$, satisfies a condition similar to (2). Recalling that the profit from the occupied machine is $y - w$ and that the probability of losing a worker is s, we obtain

$$rJ(y) - \dot{J}(y) = y - w(y) + s(V - J(y)). \tag{4}$$

Since $V = 0$, this simplifies to

$$\dot{J}(y) = -(y - w(y)) + (r + s)J(y). \tag{5}$$

Conditions (3) and (5) may be solved for θ in terms of a and $w(y)$, which are also endogenous. With knowledge of θ, we can obtain u and v from (1) and the definition of $\theta = v/u$. However, it is more convenient to treat θ as the unknown rather than v, and I shall be doing this in the development of the model. The next task is to specify the wage function $w(y)$ and the reservation productivity x that determines a. As I will show, they may be expressed as functions of θ and the exogenous variables, so their determining conditions are, like (3) and (5), independent of the levels of u and v.

III. Wages

Wages are assumed to be determined by Nash bargains between the meeting firm and worker and to be perfectly flexible; that is, there can be continuous renegotiation and recontracting.[8] The (generalized) Nash rule says that the surplus from a job match is divided between the firm and the worker according to a fixed parameter β, $0 \leq \beta \leq 1$. The surplus enjoyed by firms when the productivity of the job is y is $J(y) - V$, which in equilibrium is simply equal to $J(y)$. In order to calculate the surplus enjoyed by workers, we need to derive expressions for the worker's

[8]Similar rules for wages, but in a steady-state equilibrium only and without variations in productivities, were also discussed by Peter Diamond (1982).

net worth (asset value) when occupied and when idle.

Let U be the worker's asset value when he is unemployed and $W(y)$ be his asset value when he is employed in a job producing output y. Then, if b denotes the worker's return when he is not producing (including any unemployment benefits and net of out-of-pocket search costs), U satisfies

$$(6) \qquad rU - \dot{U} = b + ap(W^e - U).$$

where b is assumed to be parametric,[9] and ap is the transition probability for workers, as already defined. The conditional expectation $W^e = E(W(y)|y \geq x)$, and it gives the worker's expected net worth from a job, given that all jobs with productivity below x are rejected.

If a worker is in a job with productivity y, his net worth $W(y)$ satisfies

$$(7) \quad rW(y) - \dot{W}(y) = w(y) + s(U - W(y))$$

This equation has the same interpretation as (6): the worker receives $w(y)$ from the job (which will in general depend on y) and faces a probability s of returning to unemployment with reward U.

The worker's net surplus from a job match with productivity y is $W(y) - U$. I assume that the worker gets a fraction β of the total surplus $W(y) + J(y) - U - V$. Hence, wages are fixed so as to satisfy the condition

$$(8) \quad W(y) - U = (\beta/(1-\beta))(J(y) - V).$$

In equation (8), U acts as the worker's threat point in the wage bargain, and V acts as the firm's threat point. The best that each side can do if they fail to agree on a sharing rule is to search optimally for another match. Then, the parameter β may be interpreted as a coefficient measuring bargaining strength independently of the relative position of the two sides' threat points. In the symmetric Nash case, examined by Peter Diamond, $\beta = 1/2$.

To derive an equation for wages subtract (6) from (7), and rearrange to obtain

$$(9) \quad (r+s)(W(y) - U) - (\dot{W}(y) - \dot{U}) = w(y) - b - ap(W^e - U).$$

Substitute now (8) into (9), noting that (8) implies

$$\dot{W}(y) - \dot{U} = (\beta/(1-\beta))(\dot{J}(y) - \dot{V}).$$

The result is

$$(10) \quad (\beta/(1-\beta))[(r+s)(J(y) - V) - (\dot{J}(y) - \dot{V})] = w(y) - b - ap(W^e - U).$$

But (2) and (4) imply

$$(r+s)(J(y) - V) - (\dot{J}(y) - \dot{V}) = y + k - w(y) - aq(J^e - V).$$

Hence (10) becomes, after rearranging,

$$(11) \quad w = (1-\beta)b + \beta(y+k) + (1-\beta)ap(W^e - U) - \beta aq(J^e - V).$$

Thus, workers receive a payment b, plus a fraction β of the net surplus from the job $y + k - b$, plus an amount depending on perceptions of the gains from employment elsewhere, minus an amount depending on perceptions of the firm's gains from recruiting another worker to the job. If perceptions are correct then (8) implies $W^e - U = (\beta/(1-\beta))(J^e - V)$, and in equilibrium $V = 0$, J^e is given by (3), and by definition $q = p/\theta$. Hence the wage equation (11) becomes

$$(12) \quad w(y) = (1-\beta)b + \beta(y + \theta k).$$

Equation (12) is the generalized Nash wage equation, holding out of steady-state equilibrium, but only when perceptions of wages

[9] That is, we ignore variations in the intensity of search, whereby a worker may increase his contact probability p by spending more time searching (thus lowering b).

and profits elsewhere are correct and when the expected profits from a vacancy are zero. In the steady state, this equation becomes

$$\overline{w}(y)=(1-\beta)b+\beta(y+\bar{\theta}k).$$

Interestingly, $w(y)$ does not depend on the rate of growth of u or of any other variable. This property simplifies the short-run dynamic analysis of the model and has some strong implications for its behavior. Also, with correct perceptions and the zero profit condition $V=0$, what goes on in the rest of the market (for example, the distribution of productivities or the rate of structural change) influences $w(y)$ only indirectly, through the vacancy-unemployment ratio θ. If $V\neq 0$, the mean productivity y^e and other parameters also influence $w(y)$ directly through a third term, which vanishes only if $u=v$ (see my earlier paper).

With $V=0$, any exogenous change that raises the vacancy-unemployment ratio raises wages because it improves the worker's threat point in the wage bargain, relative to the firm's. For similar reasons, an increase in the worker's return from nonmarket activities b, or an increase in the firm's vacancy costs k, also increase wages for given θ. The former improves the worker's threat point, whereas the latter deteriorates the firm's threat point. But when account is taken of the effect of b and k on θ (see equation (20) below), then the effect of k on wages is reversed: an increase in the cost of idle capital reduces wages, so as to compensate firms for the higher nonlabor costs. Finally, a higher match-specific productivity y, given the distribution of productivities $F(y)$, implies higher wages and profits, with wages rising at a rate β and profits at a rate $1-\beta$. If the higher productivity is not specific to the match but general, in equilibrium workers receive more than a fraction β of the increase, because a general increase in productivities increases the vacancy-unemployment ratio (see Section VI).

IV. Job Rejection

Now, I am in a position to describe the choice of reservation productivity by firms and workers, and so close the model.

Suppose a firm and a worker meet and discover that the productivity of their match is equal to y. If the firm accepts to form the match, it will enjoy an expected return $J(y)$, whereas if it rejects it, it will search again and so enjoy the expected returns from a vacancy, V. Hence firms will be willing to accept all matches which satisfy $J(y)\geq V$, or, since in equilibrium $V=0$, $J(y)\geq 0$. It follows that the reservation productivity x satisfies

$$J(x)=0. \tag{13}$$

Similarly, if a worker accepts a job match with productivity y, he will enjoy returns $W(y)$, whereas if he rejects it he will go back to net worth U. Hence workers accept all matches that satisfy $W(y)\geq U$, giving a reservation x satisfying $W(x)=U$. By the equilibrium wage condition (8), the x that solves (13) also satisfies the worker's condition: hence firms and workers agree about the reservation job, and to obtain its properties we need only consider (13).

The equilibrium x is immediately obtained from (5) by substituting in it $y=x$ and making use of (13). Then $w(x)=x$, and using the wage equation (12), we get

$$x=b+(\beta/(1-\beta))\theta k. \tag{14}$$

Equation (14) closes the system. It is in unfamiliar form, compared with reservation-wage formulas derived elsewhere, because it already incorporates the equilibrium condition $V=0$. Firms and workers, of course, ignore the effect that their actions have on equilibrium, so it is not true to say that when they choose their reservation productivity they simply look at b, β, θ, and k, and calculate it according to (14). But after they make their choice of reservation productivity, the creation and closure of jobs in the market as a whole will ensure that their chosen x behaves according to (14). The form of (14) is the most convenient one for our purposes because, like the wage rate in (12), x is expressed as a linear function of θ, and of no other endogenous variable.

Equation (14) suggests that any exogenous change that increases the vacancy-unemployment ratio θ increases the reservation productivity. Thus, despite the fact that both workers and firms search and they both agree on which job matches to reject, higher job availability is associated with more job rejection. The asymmetry that gives rise to this result is the assumption that the size of the labor force is fixed, whereas the number of jobs is variable. This puts firms at a disadvantage in the job-matching process, so in equilibrium the marginal job with productivity x pays the entire product as wages. In partial equilibrium models of search where only workers reject jobs, higher job availability induces workers to select higher reservation wages, so the same relation holds between job availability and job rejection.[10]

V. Equilibrium and Short-Run Dynamics

Equations (3), (5), (12), and (14) contain four unknowns: $J(y)$, θ, $w(y)$, and x. We may eliminate the wage equation (12) by substituting into (5), to obtain

$$\dot{J}(y) = -(1-\beta)(y-b)+\beta\theta k + (r+s)J(y). \tag{15}$$

Hence, equations (3), (14), and (15) may now be solved for the three unknowns $J(y)$, θ, and x.

I first express (15) as a linearized differential equation in θ, by making use of (3) and (14). Using bars to denote steady-state values, I write (3) as

$$J^e = \frac{k}{a(\bar{x})q(\bar{\theta})} - \frac{kq'(\bar{\theta})}{a(\bar{x})q(\bar{\theta})^2}(\theta-\bar{\theta}) - \frac{ka'(\bar{x})}{a(\bar{x})^2 q(\bar{\theta})}(x-\bar{x}). \tag{16}$$

But from (14), $x-\bar{x} = (\beta/(1-\beta))k(\theta-\bar{\theta})$, so substituting into (16), we get

$$J^e = \frac{k}{a(\bar{x})q(\bar{\theta})} - \frac{k}{a(\bar{x})q(\bar{\theta})} \times \left[\frac{q'(\bar{\theta})}{q(\bar{\theta})} + \frac{a'(\bar{x})}{a(\bar{x})}\frac{\beta}{1-\beta}k\right](\theta-\bar{\theta}). \tag{17}$$

Taking now conditional expectations of (15), we obtain

$$\dot{J}^e = -(1-\beta)(y^e-b) + \beta\theta k + (r+s)J^e. \tag{18}$$

Substution of J^e and $\dot{J}^e$ from (17) into (18) yields the equation in θ:

$$\dot{\theta} = \left[(1-\beta)(y^e-b)-(r+s)\frac{k}{aq}\right] \times \left[\frac{q'}{q}+\frac{a'}{a}\frac{\beta}{1-\beta}k\right]^{-1}\frac{aq}{k} - (r+s)\bar{\theta} + \left[r+s-\beta aq\left(\frac{q'}{q}+\frac{a'}{a}\frac{\beta}{1-\beta}k\right)^{-1}\right]\theta. \tag{19}$$

The arguments of a and q have been omitted for notational convenience, it being understood that all coefficients are evaluated at $\bar{\theta}$.

Equation (19) is a fixed-coefficients differential equation in θ. Since $q'(\theta)<0$, $a'(x)<0$, the coefficient of θ in (19) is positive, making it an unstable equation. Hence, the only perfect foresight solution for θ is its steady-state value $\bar{\theta}$. If $\theta \neq \bar{\theta}$, the system will diverge on an explosive path. Thus, if any parameter of the system changes, θ will adjust immediately to its new steady-state value. This steady-state value is given by (19) when $\dot{\theta}=0$ and $\theta=\bar{\theta}$:

$$\frac{r+s}{a(\bar{x})q(\bar{\theta})} + \beta\bar{\theta} = (1-\beta)\frac{y^e-b}{k}. \tag{20}$$

Equation (20) gives the value of θ both in and out of steady-state equilibrium. By the wage equation (12) and the reservation productivity equation (14), both wages and the reservation productivity are also always at

[10] If $V=0$ is not built into the derivation of the reservation productivity level, the formula giving x is very similar to the standard reservation-wage formula derived in partial models, except that it depends on both the worker's and the firm's costs and returns, and not only on the worker's. See my earlier paper. Then, higher θ values are not necessarily associated with more job rejection, because with a fixed number of jobs ($V \neq 0$) firms and workers are treated with full symmetry.

their steady-state values, $\overline{w}(y)$ and $\bar{x}$. But unemployment is a predetermined variable and adjusts according to (1):

$$(21) \qquad \dot{u}=s-[s+a(\bar{x})p(\bar{\theta})]u.$$

During the economy's adjustment, both x and θ are at their steady-state values so, since θ is defined as the v/u ratio, vacancies change along with unemployment, in such a way as to ensure that θ is constant. This simply requires that vacancies change in the same proportion as unemployment; that is, $\dot{v}/v=\dot{u}/u$, or, using the θ notation, $\dot{v}=\bar{\theta}\dot{u}$.

Now, it was argued in Section I that additions to unemployment may take place also endogenously, following a rise in the reservation productivity x for given distribution $F(y)$. This possibility introduces a further element into the unemployment dynamics of equation (21), which, because of the behavior of x, is easy to deal with. Thus, if following a parametric change, some jobs are no longer viable, the firm and worker are involved separately, creating an immediate inflow into unemployment. Following this impact change, unemployment changes smoothly according to (21).

In contrast to increases in unemployment, there are no impact changes when the reservation productivity falls. It is not possible for unmatched firms and workers to come together and form jobs which they had rejected earlier, other than through the search process. Thus, unemployment behaves asymmetrically on impact: following a rise in x relative to the distribution of y, there is an immediate rise in unemployment followed by smooth adjustment; following a fall in x relative to $F(y)$, unemployment changes smoothly from the start.

I shall illustrate the properties of this model by considering its response to a shock that may reasonably be argued to correspond to what we typically observe over a business cycle.

VI. Response to Output Shocks

Consider a multiplicative real shock to the output from each job, y, and to the cost of a vacancy k. Thus, output and capital costs rise in equal proportion, but the worker's alternative return b does not rise. This causes a differential change in the net returns that workers and firms get from employment, producing some real responses of the endogenous variables.

It could be argued that this change in relative costs and returns is the main channel through which the business cycle affects the labor market: there is a general rise in output prices and costs, unaccompanied (in the short run at least) by changes in the value of the workers' alternative return (compare McDonald and Solow, p. 896). As I show, the response of real wages, number of jobs and unemployment to this kind of shock simulates the observed stylized changes of these variables over a typical cycle. In particular, real wages fluctuate by less than output and other costs, and, as a consequence, the v/u ratio rises in the peak and falls in the trough, and unemployment falls in the peak and rises in the trough.[11]

Suppose then y and k depend on a parameter h, and write

$$y(h)=(1+h)y; \quad k(h)=(1+h)k.$$

I evaluate the effect of (unanticipated permanent) changes in h at the point $h=0$.[12] It is

[11] I show below that the system is neutral to a multiplicative shift in y, k, and b, so the response to a multiplicative shift in y and k is the same (but of opposite sign) as the response to a multiplicative shift in b. This seems to suggest that the effects of changes in unemployment benefits, which can be represented by changes in b, are similar to the effects of the cycle. Two caveats should be noted here. First, cyclical shocks are short-lived, whereas the effects of changes in benefits are thought to be permanent. This does not make a qualitative difference to modeling the two kinds of shocks, but it does make a difference when it comes to an interpretation of the data. Second, if a multiplicative shock to y and k persists (like, for example, technological changes do) it is unlikely that b will remain unaffected. The value of workers' time in nonmarket activities now is not what it was in the last century, surely because of large technological improvements in production and rises in the standard of living.

[12] The modeling of cyclical shocks as a series of unanticipated permanent shocks in a perfect foresight model, such as the one of this paper, may be criticized on the grounds that agents will eventually realize the regularity of the shocks and incorporate them into their decisions. The motivation for the modeling of shocks in

convenient to derive first the effect of changes in h on y^e and a. To do this, we need to write expressions for $a(h)$ and $y^e(h)$ for a new distribution function $g(y)$, which lies to the right of the old distribution $f(y)$ and it is also more spread out. Consider first $a(h)$.[13] By definition,

$$a(h)=\int_{x(h)} g(y)\,dy,$$

whereas at $h=0$, $a=\int_x f(y)\,dy.$

To compare the two expressions, let us express $g(y)$ in terms of $f(y)$, by noting that the change from $f(y)$ to $g(y)$ is achieved by multiplying y by $(1+h)$. The fact that densities must integrate to 1 implies

$$1=\int g(y)\,dy=c\int f\left(\frac{y}{1+h}\right)dy,$$

where c is a normalizing constant. Let $z=y/(1+h)$, hence

$$1=c\int f(z)(1+h)\,dz=c(1+h),$$

and so $c=1/(1+h)$.

Now, returning to the expression for $a(h)$, we have

$$a(h)=\frac{1}{1+h}\int_{x(h)} f\left(\frac{y}{1+h}\right)dy,$$

or, changing the variable of integration to z,

$$a(h)=\int_{x(h)/(1+h)} f(z)\,dz.$$

By a similar argument, we can derive

$$y^e(h)=a(h)^{-1}\int_{x(h)/(1+h)} z(1+h)f(z)\,dz.$$

Hence, differentiating $a(h)$ and $y^e(h)$ with respect to h, and evaluating at $h=0$, we obtain

$$\frac{\partial a}{\partial h}=-\left(\frac{\partial x}{\partial h}-x\right)f(x); \tag{22}$$

$$\frac{\partial y^e}{\partial h}=y^e-\frac{1}{a}(y^e-x)\frac{\partial a}{\partial h}. \tag{23}$$

The acceptance probability changes only if the reservation productivity does not change in the same proportion as the exogenous productivities. Similarly, the mean conditional productivity changes by the same proportion as each individual productivity only if the acceptance probability is unaffected by the change.

To see now the crucial role played by wages in inducing real responses to changes in h, consider the unemployment equation (21). Unemployment will respond to changes in h if $a(x)$ or $p(\theta)$ respond to them. But in the derivation of x in (14) we saw that in equilibrium $x=w(x)$. Therefore, if wages change in the same proportion as productivities, x will also change in the same proportion, and from (22) we get that $a(x)$ will be unaffected by the change.

Similarly, we derived θ from (3) and (5), by noting that along the perfect foresight path $\dot{\theta}=0$. Hence $\dot{J}=0$, and (3) and (5) give

$$q(\theta)=\frac{(r+s)k}{a(x)(y^e-w^e)}.$$

If wages change in the same proportion as y and k, $a(x)$ is unaffected, so from (23) and an equivalent expression for w^e, we obtain that y^e and w^e also change in the same proportion. Hence, with $a(x)$ constant, and y^e, w^e, and k changing by the same proportion, $q(\theta)$ and θ must be unaffected by the change in h.

It follows then that neither unemployment nor vacancies will change if wages change in the same proportion as output and costs. But

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this paper is, first, that forecasts of the timing and intensity of shocks are not good, even though agents may know that some shocks will occur, so shocks contain most of their news value when they occur. Second, the paths of unemployment, vacancies, and real wages between turning points are dominated by endogenous adjustments following substantial shocks that disturb the equilibrium, rather than by responses to frequent serially correlated shocks.

[13] I am indebted to a referee for suggesting the method of derivation of $a(h)$ that follows.

the wage equation (12) implies that wages will change in the same proportion as y and k only if b changes in the same proportion too, for then θ is constant. Hence, an equiproportional change in b, y, and k is absorbed entirely by wages, and has no employment effects. But if the worker's alternative return b is not as sensitive to the shocks as productivity and capital costs, wages will not respond fully to the shocks and there will be real effects.

To derive the full effects of the shock when b is fixed, I return to the condition determining θ. This is given by equation (20), rewritten as (noting that $q = p/\theta$)

$$(24)\qquad \frac{(r+s)\theta}{a(h)p(\theta)} + \beta\theta = (1-\beta)\frac{y^e(h)-b}{k(1+h)}.$$

Differentiation with respect to h yields

$$(25)\qquad \left[\frac{r+s}{ap}(1-\eta)+\beta\right]\frac{\partial\theta}{\partial h} = \frac{(1-\beta)b}{k},$$

where η is the elasticity of $p(\theta)$ and it is less than 1. Hence $\partial\theta/\partial h > 0$ unambiguously.

The reservation productivity rises too, since, from (14),

$$(26)\qquad \frac{\partial x}{\partial h} = \frac{\beta}{1-\beta}\left(k\frac{\partial\theta}{\partial h} + \theta k\right) > 0.$$

But the increase in x is not as big as the increase in y and so the acceptance probability rises. Substitution of (26) into (22) yields

$$\frac{\partial a}{\partial h} = \left(-\frac{\beta}{1-\beta}k\frac{\partial\theta}{\partial h} + b\right)f(x),$$

and by making use of (25), we obtain

$$(27)\qquad [(r+s)(1-\eta)/(ap)+\beta]\,\partial a/\partial h = (r+s)(1-\eta)bf(x)/(ap) > 0.$$

The results that I have derived so far take place instantaneously, as soon as the change in h is realized. The equations that I have used hold both in and out of steady-state equilibrium, so the instantaneously observed changes in θ, x, and a are also the final changes. By contrast, unemployment does not change on impact if a increases: it rises if a decreases, as the now unacceptable jobs break up. But following these impact changes, unemployment changes in response to the changes in a and $p(\theta)$ according to (21). If $h > 0$, unemployment starts falling towards a new steady-state equilibrium, since both contact probabilities $p(\theta)$ and acceptance probabilities $a(x)$ rise. If $h < 0$ unemployment rises.

Returning now to the wage equation (12), we find that the response of wages to the shift parameter h is

$$(28)\qquad \frac{\partial w(y)}{\partial h} = \beta(y+\theta k) + \beta k\frac{\partial\theta}{\partial h}.$$

The change in wages is proportionally less than the change in y or k, despite the term $\beta k\,\partial\theta/\partial h$. By making use of (25), it can be shown that

$$\frac{\partial w(y)}{\partial h}\frac{1}{w(y)} < \frac{\partial y}{\partial h}\frac{1}{y} = 1.$$

Profits, as a result, rise by more than in proportion, so firms increase the number of job offers. Also, because the rewards available to firms and workers who reject jobs are relatively less than before, they are more willing to accept jobs than before. Thus, in terms of wages and output, what we observe in the market is an increase in the value of output accompanied by a partial response of real wages, leading to more job vacancies and more job acceptances.

Following these changes unemployment starts falling, as both job contacts and job acceptances increase. The lower unemployment leads to a decrease in the number of jobs taken up (since the number of searchers falls with every fall in unemployment) and an increase in the number of job separations (because the employment rate increases). Eventually a new equilibrium is reached with lower unemployment, higher wages and profits, and higher v/u ratio.

Vacancies increase at first on impact so as to drive θ up to its new equilibrium level. Then, as unemployment starts to fall vacancies fall too, since, with a constant θ, $\dot{v} = \bar{\theta}\dot{u}$.

Thus, in the first phase of expansion, vacancies overshoot their new equilibrium value. It is not possible to say whether the final equilibrium value of vacancies is above or below the old equilibrium value. At the point $\dot{u}=0$, we have

$$\partial\bar{v}/\partial h=\bar{u}\partial\theta/\partial h+\theta\partial\bar{u}/\partial h, \tag{29}$$

where, from (21),

$$\frac{\partial\bar{u}}{\partial h}=-\frac{p\bar{u}}{s+ap}\frac{\partial a}{\partial h}-\eta\frac{pa\bar{u}}{\theta(s+ap)}\frac{\partial\theta}{\partial h}. \tag{30}$$

Hence, substituting into (29), we obtain

$$\frac{\partial\bar{v}}{\partial h}=-\frac{p\bar{u}\theta}{s+ap}\frac{\partial a}{\partial h}+\frac{s\bar{u}+(1-\eta)pa\bar{u}}{s+ap}\frac{\partial\theta}{\partial h}. \tag{31}$$

This cannot in general be signed, if only because $\partial a/\partial h$ depends on $f(x)$, which makes a comparison between the two terms in (31) inconclusive. Equation (31) points to the fact that there are two effects on equilibrium vacancies. A positive effect from the increase in θ, and a negative one from the increase in the acceptable job matches. In general we cannot say which effect dominates, but if $f(x)$ is small, the positive effect through θ will dominate.[14]

The behavior of unemployment and vacancies in response to a positive and negative shock h is shown diagrammatically in Figure 1. The $\dot{u}=0$ locus slopes downwards under the assumption

$$-ap'(\theta)+p(\beta/(1-\beta))kf(x)<0.$$

That is, when the effect of vacancies on unemployment through job availability dominates the effect through the increase in

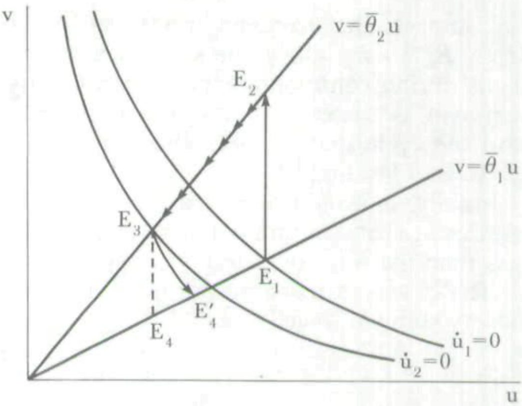

FIGURE 1. THE EFFECT OF MULTIPLICATIVE OUTPUT SHOCKS ON VACANCIES AND UNEMPLOYMENT

reservation productivities (holding output and all other variables constant; see the discussion in the preceding paragraph and in fn. 14). Under the reasonable restriction $p''(\theta)\le 0$, the locus is also convex to the origin if the job availability effect dominates, but this is not important for our purposes. The stable trajectory (saddlepath) is the $\theta=\bar{\theta}$ locus, shown by the straight line through the origin.

Suppose the economy is initially at E_1, and a positive multiplicative shock takes place. Then, on impact, θ rises (say from θ_1 to θ_2) pivoting the stable trajectory to the left. The economy jumps on impact from E_1 to E_2, on the new trajectory. The $\dot{u}=0$ locus also shifts inwards, because of the increase in θ and $a(x)$, say from $\dot{u}_1=0$ to $\dot{u}_2=0$. At point E_2 both vacancies and unemployment start falling towards the new long-run equilibrium E_3, where unemployment is lower, but vacancies may be lower or higher than at E_1.

Now if, starting at E_3, there is a negative shock of a similar magnitude the curves shift back, so the new equilibrium is at E_1. But now the point reached on impact is not E_4, but some point to the right of it, like E_4'. The reason for this is that at the time of the negative shock, the number of acceptable job matches falls. So there are some jobs which are no longer acceptable, and these break up immediately, leading to an immediate increase in unemployment. Following this ini-

[14]This kind of ambiguity frequently arises in models of search, except that it is discussed only in relation to unemployment. Here it does not arise on the unemployment side because the cause of a rise in θ is a positive output shock, which increases the acceptance rate despite the increase in θ. The common conjecture is that the effect through job availability normally dominates.

tial impact, the economy travels smoothly from E_4' to E_1, along the stable trajectory. Thus during contractions the economy's adjustment is faster than during expansions, and the cycle that it traces about the $\dot{u}=0$ curve is of less amplitude.[15]

Finally, it should be noted that observed mean wages and output in the market change less than the wage or output of a typical job, y. The observed mean output is y^e and the observed mean wage is

$$w^e = (1-\beta)b + \beta(y^e + \theta k). \tag{32}$$

It follows immediately from (23) that the proportional change in y^e is less than 1: the number of acceptable jobs increases (if $h > 0$) adding some low productivity jobs to those already in the market. As a result, there is another dampening effect on mean wages, besides $(1-\beta)b$. Like the effect of a on unemployment, this dampening effect operates with a lag in an expansion, leading to a mild overshooting of mean productivity and wages in the first stages of expansion. But in a contraction it operates without lag, so the response of mean wages and productivity on impact is less in contraction than in expansion.

VII. Conclusions

Early search theory was effectively criticized for its exclusive reliance on supply-of-labor responses and for not providing an adequate theory of wages (see, for example, James Tobin, 1972). The model presented in this paper meets both of these criticisms by treating supply and demand symmetrically, and by relying on vacancies as a link between output shocks and unemployment. Thus, wages are endogenously determined by bargaining at the individual level and they are fully flexible, so firms and workers always agree about which jobs to accept and which to reject. But the supply of vacancies is determined exclusively by profit-maximizing firms, so workers may find themselves unemployed for lengthy periods of time because of limited vacancy availability.

The comparative static predictions of the model are similar to those of contract theories, and other theories that developed partly in response to the criticisms of early search theory. A shock to labor's marginal and average product is absorbed partly by real wages and partly by employment. The role of some alternative return to labor that is insulated from the shock is crucial in this prediction. But unlike other recent theory, the model can shed light on the adjustment paths of the endogenous variables. Search considerations suggest that unemployment is a sluggish (predetermined) variable and if we add to this the assumption of nonpredetermined vacancies (and also of nonpredetermined real wages and reservation prices) we can derive unique perfect-foresight adjustment paths for all the endogenous variables. The role of vacancies in pushing the economy to its unique perfect-foresight path is crucial: at the onset of expansions and recessions, vacancies have to overshoot their equilibrium value and return subsequently to equilibrium, tied to unemployment. By contrast, real wages and reservation prices change only in response to news, and even then they do not fully reflect the output shocks. Thus, in contrast to unemployment which is highly serially correlated, changes in (productivity-corrected) real wages should be small and uncorrelated, unless the shocks are serially correlated. Moreover, the model predicts that the response of unemployment to a negative shock should be faster than its response to a positive shock. Both of these predictions are consistent with the time series data on real wages and unemployment (Altonji and Ashenfelter; Neftci).

A possible objection to the model is that although its predictions conform to the commonly observed anti-clockwise loops in vacancy-unemployment space in Britain, it predicts too much response to news. Thus, empirically vacancies do not seem to overshoot their equilibrium value to the extent

[15] The $\dot{u}=0$ curve is often referred to as the Beveridge curve. The argument of this paper suggests that the Beveridge curve is not stable over the cycle, as is normally assumed. The evidence concerning the British curve in the 1960's and 1970's is discussed in my paper with Jackman and Layard. Hansen also derived the anti-clockwise loops in $u-v$ space, but he relied on *ad hoc* adjustment equations with a speculative component in vacancy decisions, giving him the lead of vacancies over unemployment.

that the model predicts, and some may argue that real wages sometimes behave like a predetermined variable. However, it would need a much more careful study of the data than now available to establish the validity of these propositions; for example, there is no analysis (to my knowledge) of the effects of news in labor markets, of the kind that one finds in recent research on asset markets. But in addition, more sluggishness may be incorporated into the model by realizing that there may be predetermined elements in some aspects of vacancy decisions. For example, the extent to which firms can open up new vacancies in response to news may be limited by their ability to acquire new capital. Then, in expansions the initial jump in vacancies may be checked, and changes in the vacancy-unemployment ratio may continue after the revelation of news. Real wages would then change sluggishly during adjustment, despite continuous recontracting, because of their dependence on the vacancy-unemployment ratio.

A formal model incorporating these elements would have to deal with the asymmetries that are likely to arise in booms and recessions. The availability of capital will not introduce sluggishness into vacancy decisions in a recession, though it might do in a boom. This paper has made a start at the formal analysis of short-run dynamics by making the simple assumption of predetermined unemployment vs. nonpredetermined vacancies. Generalizing this assumption may shed more light on the extent of fluctuations in response to news in labor markets, but it is not likely to alter the general patterns of adjustment derived.[16]

[16] If output shocks are to some extent anticipated, as they might if they are associated with regular cycles, the response of vacancies and real wages to them will also be less pronounced and more spread out. Less than perfect anticipations have the effect of spreading out the revelation of news, with predictable results.

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