Markowitz's "Portfolio Selection": A Fifty-Year Retrospective

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Editor's Note: The Editor wishes to thank Mark Rubinstein for agreeing to prepare this retrospective, and for bringing to the task his unique erudition and perspective.

THIS YEAR MARKS the fiftieth anniversary of the publication of Harry Markowitz's landmark paper, "Portfolio Selection," which appeared in the March 1952 issue of the *Journal of Finance*. With the hindsight of many years, we can see that this was the moment of the birth of modern financial economics. Although the baby had a healthy delivery, it had to grow into its teenage years before a hint of its full promise became apparent.

What has always impressed me most about Markowitz's 1952 paper is that it seemed to come out of nowhere. Compared to the work of his 1990 co-Nobel Prize winners (Sharpe primarily for his paper on the capital asset pricing model and Miller for his paper on capital structure), Markowitz's paper seems to have more of this flavor. In 1676, Sir Isaac Newton wrote his friend Robert Hooke, "If I have seen further it is by standing on the shoulders of giants" (Newton (1959)) and that is true of Markowitz as well, but, like Newton, he certainly saw a long distance given the height of those shoulders.

Markowitz was hardly the first to consider the desirability of diversification. Daniel Bernoulli in his famous 1738 article about the St. Petersburg Paradox argues by example that risk-averse investors will want to diversify: " . . . it is advisable to divide goods which are exposed to some small danger into several portions rather than to risk them all together" (Bernoulli 1954). As Markowitz (1999) himself points out in his historical review of portfolio theory, Bernoulli is also not the first to appreciate the benefits of diversification. For example, in *The Merchant of Venice*, Act I, Scene I, William Shakespeare has Antonio say:

> " . . . I thank my fortune for it,
> My ventures are not in one bottom trusted,
> Nor to one place; nor is my whole estate
> Upon the fortune of this present year . . . "

Although this turns out to be a mistaken security, Antonio rests easy at the beginning of the play because he is diversified across ships, places, and time.

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Variance may have first been suggested as a measure of economic risk by Irving Fisher in *The Nature of Capital and Income* (1906). Jacob Marschak (1938) suggested using the means and the covariance matrix of consumption of commodities as a first order approximation in measuring utility. Even though Marschak actually supervised Markowitz's dissertation, he never mentioned this earlier work to Markowitz, presumably because he felt it not sufficiently related.

In his Nobel Prize autobiography, Markowitz (1991) writes "The basic concepts of portfolio theory came to me one afternoon in the library while reading John Burr Williams' *The Theory of Investment Value*." Williams was remarkably prescient. He provided the first derivation of the "Gordon growth formula," the Modigliani-Miller capital structure irrelevancy theorem, and strongly advocated the dividend discount model. But Williams had very little to say about the effects of risk on valuation (pp. 67–70), because he believed that all risk could be diversified away:

> The customary way to find the value of a risky security has been to add a "premium for risk" to the pure rate of interest, and then use the sum as the interest rate for discounting future receipts. . . . Strictly speaking, however, there is no risk in buying the bond in question if its price is right. Given adequate diversification, gains on such purchases will offset loses, and a return at the pure interest rate will be obtained. Thus the *net risk* turns out to be nil. (pp. 67–69)

Other authors, seduced by Jacob Bernoulli's (1713) law of large numbers, were led to a similar conclusion.

Markowitz had the brilliant insight that, while diversification would reduce risk, it would not generally eliminate it. Markowitz's paper is the first mathematical formalization of the idea of diversification of investments: the financial version of "the whole is greater than the sum of its parts." Through diversification, risk can be reduced (but not generally eliminated) without changing expected portfolio return. Markowitz postulates that an investor should maximize expected portfolio return (μ_P) while minimizing portfolio variance of return (σ_P^2).

Probably the most important aspect of Markowitz's work was to show that it is not a security's own risk that is important to an investor, but rather the contribution the security makes to the variance of his entire *portfolio*—and that this was primarily a question of its covariance with all the other securities in his portfolio. This follows from the relation between the variance of the return of a portfolio (σ_P^2) and the variance of return of its constituent securities (σ_j^2 for $j = 1,2,\ldots, m$):

$$\sigma_P^2 = \Sigma_j x_j^2 \sigma_j^2 + \Sigma_j \Sigma_{k \neq j} x_j x_k \rho_{jk} \sigma_j \sigma_k$$

where the x_j are the portfolio proportions (that is, the fraction of the total value of the portfolio held in security j so that $\Sigma_j x_j = 1$) and ρ_{jk} is the

correlation of the returns of securities j and k. Therefore, $\rho_{jk}\sigma_j\sigma_k$ is the covariance of their returns. Markowitz's 1952 paper seems to contain the first occurrence of this equation in a published paper on financial economics.

So the decision to hold a security should not be made simply by comparing its expected return and variance to others, but rather the decision to hold any security would depend on what other securities the investor wants to hold. Securities could not be properly evaluated in isolation, but only as a group. This perspective was clearly missing from Williams (1938) and from Graham and Dodd (1934). Indeed, even as late as in the revised 1962 version of the latter, it received scant comment.

Roy (1952) independently sets down the same equation relating portfolio variance of return to the variances of return of the constituent securities. He develops a similar mean-variance efficient set. Whereas Markowitz left it up to the investor to choose where along the efficient set he would invest, Roy advised choosing the single portfolio in the mean-variance efficient set that maximizes $(\mu_P - d)/\sigma_P^2$ where d is a "disaster level" return the investor places a high priority on not falling below. Many years later, comparing Roy's paper to his own, Markowitz (1999) charitably writes "On the basis of Markowitz (1952), I am often called the father of modern portfolio theory (MPT), but Roy can claim an equal share of this honor."

Along with Tobin (1958), the best work on portfolio theory in the 1950s after the publication of Markowitz's paper was by Markowitz himself in his 1959 book on portfolio selection. Here he provides an extended and detailed development of Markowitz's (1952) mean-variance model of portfolio choice, purposely designed for access by readers with a modest quantitative background. In view of the then recently completed work of von Neumann and Morgenstern (1947) and Savage (1954), Markowitz also strove to find a way to reconcile his mean-variance criterion with the maximization of the expected utility of wealth after many reinvestment periods.

The book also foreshadows several avenues of future research. (1) Markowitz advises using the strategy of maximizing the expected logarithmic utility of return each period for investors with a long-term horizon, and he develops a useful quadratic approximation to this strategy that allows the investor to choose portfolios based on mean and variance. (2) Markowitz actually recommends semi-variance as a replacement for variance as a measure of risk on the grounds that it is realistically superior and investigates its properties and optimal portfolio computing procedures. (3) He outlines the diagonal or market model in an extended footnote that later, at Markowitz's suggestion, Sharpe (1963) would develop more fully. (4) Insisting that the investor choose his portfolio to maximize his expected utility according to the Savage (1954) axioms, he compares several alternative measures of risk: standard deviation, semi-variance, expected value of loss, expected absolute deviation, probability of loss and maximum loss. (5) Markowitz lays out how to solve the multi-period expected utility of consumption problem by using the backwards recursive technique of dynamic programming, used subsequently by Phelps (1962) and then by many others to solve the multi-period problem.

For much of the 1950s and into the 1960s, while Markowitz was off developing the use of sparse matrices (a term he coined) and SIMSCRIPT (a computer language designed to implement simulations), academics in finance slowly began to take Markowitz seriously. By 1970, Markowitz assessed the major subsequent discoveries that his 1959 book had not encompassed:

> As compared to later analyses, the chapter 13 consumption-investment game was in discrete time rather than in continuous time (as in Merton 1969), did not reflect the discovery of myopic utility functions (as did Mossin 1968 and Samuelson 1969), and did not consider the behavior of a market populated by consumer/investors playing this game. (as in Sharpe (1964))

Markowitz was interested in decision rules that he could recommend to rational investors, that is normative modeling. So he has spent a great deal of time working out numerical algorithms for implementing calculation of mean-variance efficient sets. Strangely, he has not seemed to take much interest in the prescriptive extension of his work taken by Sharpe (1964) and others who asked what would happen if everyone in the economy actually followed Markowitz's advice.

Markowitz's approach is now commonplace among institutional portfolio managers who use it both to structure their portfolios and measure their performance. It has been generalized and refined in innumerable ways, and is even being used to manage the portfolios of ordinary investors. Its prescriptive extension has led to increasingly refined theories of the effects of risk on valuation. Indeed, the ideas in his 1952 paper have become so interwoven into financial economics that they can no longer be disentangled.

Near the end of his reign in 14 AD, the Roman emperor Augustus could boast that he had found Rome a city of brick and left it a city of marble. Markowitz can boast that he found the field of finance awash in the imprecision of English and left it with the scientific precision and insight made possible only by mathematics.

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