A REFORMULATION OF CERTAIN ASPECTS OF WELFARE ECONOMICS[1]

The object of the present paper is to state in a precise form the value judgments required for the derivation of the conditions of maximum economic welfare which have been advanced in the studies of the Cambridge economists,[2] Pareto and Barone, and Mr. Lerner.[3] Such a formulation, I hope, will clarify certain aspects of the contribution of these writers, and at the same time provide a basis for a more proper understanding of the principles of welfare.

I shall develop my analysis under a set of assumptions which in certain respects differ from those introduced in the welfare studies. It will be assumed throughout the discussion that the amounts of all the factors of production, other than labor, are fixed and, for convenience, non-depreciating. While a variable capital supply is included in some of the

1. I am very grateful to Mr. Paul Samuelson for suggestions on many points.

2. I use this caption to designate those economists whose names are directly attached to the Cambridge School — Marshall, Professor Pigou, Mr. Kahn — as well as others, such as Edgeworth, whose welfare analysis is in all essentials the same as that of the Cambridge group. But in the course of my discussion I shall refer mainly to the studies of the first group of economists. This will ease my task considerably, and, I believe, will involve no loss of generality.

3. The studies referred to are Marshall, Principles (all references to the Third — 1895 — Edition); Pigou, Economics of Welfare (all references to the Fourth — 1932 — Edition); Kahn, Economic Journal, March, 1935; Pareto, Cours d'Economie Politique (all references to the Lausanne — 1897 — Edition); Barone, The Ministry of Production in a Socialist State (Translated from the Italian article of the same title in Giornale degli Economisti, 1908; the translation appearing in Hayek, Collectivist Economic Planning); and Lerner, Review of Economic Studies, June and October, 1934.

welfare studies, this is not a well developed part of the analysis, and for our present purposes it will be desirable to confine to the simpler case the discussion of the evaluations required.[4] I shall assume, also, that the variables involved in the analysis — the amounts of the various commodities consumed and services performed — are infinitesimally divisible. This assumption will be interpreted more strictly than is usually done. Otherwise it is the postulate of the welfare writers, and its introduction here will involve no significant departure from their analysis. Finally, I shall assume that there are only two kinds of consumers' goods, two kinds of labor, and two factors of production other than labor in the community, and that each commodity is produced, with labor and the other factors, in a single production unit. This assumption is introduced only to simplify the notation employed. The discussion will apply, with no modification, to the many commodity, many factor, and many production unit case.[5]

I

Among the elements affecting the welfare of the community during any given period of time are the amounts of each of the factors of production, other than labor, employed in the different production units, the amounts of the various commodities consumed, the amounts of the different kinds of work done, and the production unit for which this work is performed by each individual in the community during that period of time. If we use A and B to denote the two kinds of labor; C and D to denote the two factors

4. On a simple model, similar to that of Barone, the analysis may be extended to the case of a variable capital supply.

5. The assumption that each commodity is produced in one production unit, it is true, excludes an element of "external economies" from the analysis. But in the present essay I am interested only in the maximum conditions for the community's welfare, and not in the departures from the maximum under a given institutional set-up. To the extent that, in the many production unit case, there are external economies, these will require no modification in the maximum conditions I shall present, for these conditions relate only to marginal *social* value productivities.

of production other than labor; and X and Y to denote the two consumers' goods; we may express this relationship in the form

$$(1.1)\quad W = W(x_1, y_1, a_1^x, b_1^x, a_1^y, b_1^y, \cdots, x_n, y_n, a_n^x, b_n^x, a_n^y, b_n^y, C^x, D^x, C^y, D^y, r, s, t, \cdots).$$

Here C^x and D^x are the amounts of the non-labor factors of production C and D employed in the production unit producing the consumers' good X; C^y and D^y are the amounts of these factors employed in the production unit producing the consumers' good Y; x_i and y_i are the amounts of X and Y consumed by the i^{th} individual; and a_i^x, b_i^x, a_i^y, and b_i^y are the amounts of each kind of work performed by him for each production unit during the given period of time.[6] The symbols $r, s, t, \cdots$, denote elements other than the amounts of commodities, the amounts of work of each type, and the amounts of the non-labor factors in each of the production units, affecting the welfare of the community.

Some of the elements $r, s, t, \cdots$, may affect welfare, not only directly, but indirectly through their effect on (say) the amounts of X and Y produced with any given amount of resources, e.g., the effects of a change in the weather. On the other hand, it is conceivable that variations in the amounts of commodities, the amounts of work of each type, and the amounts of non-labor factors in each of the production units also will have a direct and indirect effect on welfare; e.g., a sufficient diminution of x_i and y_i may be accompanied by an overturn of the government. But for relatively small changes in these variables, other elements in welfare, I believe, will not be significantly affected. To the extent that this is so a partial analysis is feasible.

I shall designate the function,

$$(1.2)\quad E = E(x_1, y_1, a_1^x, b_1^x, a_1^y, b_1^y, \cdots, x_n, y_n, a_n^x, b_n^x, a_n^y, b_n^y, C^x, D^x, C^y, D^y),$$

which is obtained by taking $r, s, t, \cdots$, in (1.1) as given, the Economic Welfare Function.[7]

6. I am assuming that an individual's labor time may be divided among the different types of work in any desired proportions.

7. It should be emphasized that in (1.2) other factors affecting wel-

Let us write the amounts of X and Y produced respectively by the X and Y production units as functions,

$$(1.3)\quad X=X(A^x, B^x, C^x, D^x)\,;\quad Y=Y(A^y, B^y, C^y, D^y),$$

where A^x and B^x are the amounts of the two kinds of labor and C^x and D^x are the amounts of the other two factors of production employed in the X production unit; and A^y, B^y, C^y, D^y are defined similarly for the Y production unit.

If we assume that E varies continuously with x_1, y_1, $\cdots$, we may write as a general condition for a position of maximum economic welfare that, subject to the limitations of the given technique of production and the given amounts of resources,

$$(1.4)\qquad dE=0.$$

Equation (1.4) requires that in the neighborhood of the maximum position any small adjustment will leave the welfare of the community unchanged. By use of (1.3) and (1.4) it is possible immediately to state in general terms the conditions for a maximum welfare.[8]

One group of maximum conditions relates to the consumption and supply of services by each individual in the community. They require that the marginal economic welfare of each commodity and the marginal economic diswelfare of each type of work be the same with respect to each individual in the community.[9] If we denote the marginal economic welfare of commodity X with respect to the i^{th} individual, $\frac{\partial E}{\partial x_i}$, and of Y, $\frac{\partial E}{\partial y_i}$, the first group of these con-

fare are taken as given. I do *not* assume that economic welfare is an independent element which may be added to other welfare to get total welfare.

8. The conditions I shall develop in this section are a group of necessary conditions for a maximum. They are also the conditions for any critical point, and are sufficient in number to determine the location of such a point (or points) if there is one. In section IV below I shall consider the problem of determining whether a given critical point is a maximum or not.

9. This rather awkward terminology is adopted instead of, say, the phrase marginal economic welfare *of* the i^{th} individual in order to include the possibility that an increment of X or Y given to the i^{th} individual will affect the welfare of others.

ditions requires that, for all i, and for some p, q, and ω,

$$\frac{\partial E}{\partial x_i}=\omega p \tag{1.5}$$

and

$$\frac{\partial E}{\partial y_i}=\omega q. \tag{1.6}$$

Similarly if we denote the marginal economic diswelfare of the various types of work with respect to the i^{th} individual $\frac{\partial E}{\partial a_i^x}, \frac{\partial E}{\partial b_i^x}, \frac{\partial E}{\partial a_i^y}, \frac{\partial E}{\partial b_i^y}$, the second group of these conditions requires that, for all i and for some g^x, h^x, g^y, h^y, and for the ω already chosen,

$$(1.7)\quad -\frac{\partial E}{\partial a_i^x}=\omega g^x, \qquad (1.8)\quad -\frac{\partial E}{\partial b_i^x}=\omega h^x,$$

$$(1.9)\quad -\frac{\partial E}{\partial a_i^y}=\omega g^y, \qquad (1.10)\quad -\frac{\partial E}{\partial b_i^y}=\omega h^y.$$

The minus signs and the multiplicative factor ω are inserted in these equations for convenience.

The remaining maximum conditions relate to production. They require that the economic welfare of the consumers' goods produced by a marginal increment of each type of work should equal the negative of the diswelfare of that increment of work, and that the increment of economic welfare due to the shift of a marginal unit of factors C and D from one production unit to another should equal the negative of the diswelfare caused by this adjustment. Using the notation $\frac{\partial X}{\partial A^x}$ for the marginal productivity of A^x, and a similar notation for the other marginal productivities, we may write these conditions in the form,

$$(1.11)\quad p\frac{\partial X}{\partial A^x}=g^x, \qquad (1.12)\quad p\frac{\partial X}{\partial B^x}=h^x,$$

$$(1.13)\quad q\frac{\partial Y}{\partial A^y}=g^y, \qquad (1.14)\quad q\frac{\partial Y}{\partial B^y}=h^y,$$

and,[1]

$$(1.15) \quad \omega\left(p\frac{\partial X}{\partial C^x}-q\frac{\partial Y}{\partial C^y}\right)=-\left(\frac{\partial E}{\partial C^x}-\frac{\partial E}{\partial C^y}\right),$$

$$(1.16) \quad \omega\left(p\frac{\partial X}{\partial D^x}-q\frac{\partial Y}{\partial D^y}\right)=-\left(\frac{\partial E}{\partial D^x}-\frac{\partial E}{\partial D^y}\right).$$

In equations (1.11) through (1.14), ω, which was present in all terms, has been divided out.[2]

It will be convenient to designate p the *price* of X, q the *price* of Y, and g^x, g^y, h^x, h^y, the *wage* of the types of work A^x, A^y, B^x, B^y. Equations (1.5) and (1.6) thus require that the marginal economic welfare per "dollar's worth" of each commodity, $\frac{\partial E}{\partial x_i} \cdot \frac{1}{p}$ and $\frac{\partial E}{\partial y_i} \cdot \frac{1}{q}$, be the same for each commodity and for all individuals in the community. Similarly equations (1.7) through (1.10) require that the marginal economic diswelfare per "dollar's worth" of each kind of work be the same with respect to each kind of work and each individual in the community; equations (1.11) through (1.14) require that the wages of each type of labor should equal the marginal value productivity of that type of labor;[3] and with an analogous interpretation, equations (1.15) and (1.16) require that the marginal value productivity equal the cost due to a shift in C or D from one use to another.

II

The maximum conditions presented in section I are the general conditions for a position of maximum economic

1. The derivatives on the right hand sides of (1.15) and (1.16) indicate the effect on welfare of an adjustment in C or D for which all other elements — x^i, y^i, etc.— in welfare are constant. Such an effect would arise, for example, through a positive or negative evaluation of the relative amounts and kinds of "factory smoke" emitted in the two production units for varying amounts of one or the other factors employed in each unit.

2. Strictly speaking this procedure assumes a value proposition, which we shall introduce later, to the effect that ω is unequal to zero.

3. In the present essay it will be understood that all value productivities are *social* value productivities. Compare footnote 5, p. 311, supra.

welfare for any Economic Welfare Function. The maximum conditions presented in the welfare studies relate to a particular family of welfare functions. Their derivation thus requires the introduction of restrictions on the shape of the Economic Welfare Function I have presented. Three groups of value propositions suffice for this purpose.

I shall designate the various maximum conditions derived by the names of those writers, or groups of writers, who have been especially responsible for their elucidation. For reasons which will appear I have altered somewhat the content of the conditions, and there are differences in the analyses of the various writers which must also be noted. The latter differences will be pointed out in this section and in the one following.

The Lerner Conditions

The First Group of Value Propositions: *a shift in a unit of any factor of production, other than labor, from one production unit to another would leave economic welfare unchanged, provided the amounts of all the other elements in welfare were constant.*

The First Group of Value Propositions enables us to state certain of the maximum conditions in terms of the production functions alone. From these evaluations the right hand side of (1.15) and of (1.16) must equal zero.[4] The two equations thus may be written,

$$p\frac{\partial X}{\partial C^x}=q\frac{\partial Y}{\partial C^y}, \tag{2.1}$$

$$p\frac{\partial X}{\partial D^x}=q\frac{\partial Y}{\partial D^y}, \tag{2.2}$$

and they now impose the condition that the marginal value productivity of factors other than labor be the same in every use.

Equations (2.1) and (2.2) still contain the variables p

4. The net effect on the community's welfare of the "factory smoke" arising from a shift of the non-labor factors from one use to another is zero. (Cf. footnote 1, p. 315.)

and q, which involve derivatives of the Economic Welfare Function. If we combine (2.1) and (2.2), however, we have two equations,

$$\frac{q}{p}=\frac{\partial X}{\partial C^x}\Big/\frac{\partial Y}{\partial C^y}=\frac{\partial X}{\partial D^x}\Big/\frac{\partial Y}{\partial D^y}, \tag{2.3}$$

the second of which involves only the derivatives of the production functions. It requires that in the maximum position the ratio of the marginal productivity of a factor in one use to its marginal productivity in any other use be the same for all factors of production, other than labor. The first equation of (2.3) requires that all these ratios equal the price ratio.

The significance of (2.3) for the determination of maximum welfare may be expressed in the following manner: whatever the relative evaluations of commodity X and commodity Y, that is, in Barone's terminology, whatever their ratio of equivalence, (2.3) requires that in the maximum position given that one factor C is so distributed that a small shift from one production unit to another would alter the amounts of X and Y in such a manner as to leave welfare unchanged, i.e., given that C is so distributed that $\frac{\partial X}{\partial C^x}\Big/\frac{\partial Y}{\partial C^y}$ equals the ratio of equivalence of the two commodities, then the other factors in order to be so distributed must have a ratio of marginal productivities equal to $\frac{\partial X}{\partial C^x}\Big/\frac{\partial Y}{\partial C^y}$.

The condition (2.3) can be interpreted in another manner, which however does not bring out as directly the significance of the condition for a position of maximum *welfare*. The equality of the marginal productivity ratios implies that there is no possible further adjustment for which the amount of one commodity will be increased without that of another being reduced. A shift in one factor from X to Y can at best be just compensated by a shift of another from Y to X, if (2.3) is satisfied.[5]

5. Mr. Lerner, as far as I am aware, is the only economist to present

THE PARETO-BARONE-CAMBRIDGE CONDITIONS

The Fundamental Value Propositions of Individual Preference: *if the amounts of the various commodities and types of work were constant for all individuals in the community except any i^{th} individual, and if the i^{th} individual consumed the various commodities and performed the various types of work in combinations which were indifferent to him, economic welfare would be constant.*

The First Group of Value Propositions implies that under the assumption that the amounts of the factors of production other than labor are constant, the Economic Welfare Function may be written as

$$(2.4) \quad E = E(x_1, y_1, a_1^x, b_1^x, a_1^y, b_1^y, \cdots, x_n, y_n, a_n^x, b_n^x, a_n^y, b_n^y).$$

For from these propositions a shift in C or D from one production unit to another would have no effect on welfare, if all the other elements were constant. The Fundamental Value Propositions require that E be some function of the form,

$$(2.5) \quad E = E[S^1(x_1, y_1, a_1^x, b_1^x, a_1^y, b_1^y), \cdots, S^n(x_n, y_n, a_n^x, b_n^x, a_n^y, b_n^y)],$$

where the function,

$$(2.6) \qquad S^i = S^i(x_i, y_i, a_i^x, b_i^x, a_i^y, b_i^y),$$

and interpret (2.1) and (2.2) in the form of (2.3), his interpretation being the second of the two alternatives I have noted. In the studies of Pareto, Barone, and Marshall the conditions (2.1) and (2.2) are presented with the price ratios already equated to the individual marginal rates of substitution (cf. infra). In the studies of Professor Pigou and Mr. Kahn the procedure is the same as that of Pareto, Barone, and Marshall except that these two writers include in their analysis the possibility of departures from (2.1) and (2.2) due to such effects as are discussed above in footnote 1, p. 315.

Mr. Lerner advances the conditions (2.3) for all factors of production, labor as well as non-labor (Review of Economic Studies, October, 1934, p. 57). On the face of the matter this formulation is inconsistent with Mr. Lerner's own advocacy of the supremacy of individual tastes in the sphere of consumption, and I have therefore taken the liberty to modify his conditions accordingly. The other economists also do not allow for individual preferences as between production units in their analysis.

expresses the loci of combinations of commodities consumed and work performed which are indifferent to the i^{th} individual.

The Fundamental Value Propositions enable us to state all the consumption and labor supply conditions in terms of the individual indifference functions, S^i, as ratios of (1.5), or of any other of their number. For consider the equation,

$$\frac{\partial E}{\partial x_i} \Big/ \frac{\partial E}{\partial y_i} = \frac{p}{q}, \tag{2.7}$$

obtained from (1.5) and (1.6) by division. Using the Fundamental Value Propositions,

$$\frac{\partial E}{\partial x_i} \Big/ \frac{\partial E}{\partial y_i} = \frac{\partial E}{\partial S^i}\frac{\partial S^i}{\partial x_i} \Big/ \frac{\partial E}{\partial S^i}\frac{\partial S^i}{\partial y^i} = \frac{\partial S^i}{\partial x_i} \Big/ \frac{\partial S^i}{\partial y_i}. \tag{2.8}$$

The last ratio in (2.8) is one of the slopes of the indifference locus of the i^{th} individual, or in the Hicks and Allen terminology, the marginal rate of substitution of commodity Y for commodity X.[6] Thus (2.7) requires that the marginal rate of substitution of the two commodities be the same for all individuals. By successively combining (1.5) with equations (1.7) through (1.10), the same result is obtained with respect to the other elements of welfare.

All the production conditions may now be stated in terms of the indifference functions and the production functions. For equations (1.11) through (1.14), the statement that the wage of each type of work should equal the marginal value productivity of that type of work may be interpreted to mean that the marginal product of a given type of work employed in producing a given commodity should equal the marginal rate of substitution of that commodity for that type of work. In the same manner conditions (2.2) not only require that the ratios of marginal productivities of the various factors other than labor be equal, but that these ratios should equal the marginal rate of substitution of the two commodities.

The Fundamental Value Propositions thus require that,

6. Cf. Economica, February, 1934.

whatever the ratios of equivalence between the various commodities and types of work, given that the types of work performed and commodities consumed by one individual are so fixed that for any small adjustment among them economic welfare is unchanged, i.e., given that the marginal rates of substitution and marginal productivities for this individual equal the respective ratios of equivalence, then for all other individuals to be similarly situated, their marginal rates of substitution must be the same as those of this individual. Under our implicit assumption of homogeneous factors, the respective marginal productivities of course must in any case be equal for all individuals.

Again the Fundamental Value Propositions may be interpreted also to mean that in the maximum position it is impossible to improve the situation of any one individual without rendering another worse off.[7]

The Cambridge Conditions

Let us designate

$$(2.9)\qquad m_i = px_i + qy_i - g^x a_i^x - h^x b_i^x - g^y a_i^y - h^y b_i^y,$$

7. The Pareto-Barone-Cambridge Conditions are developed by Marshall in the Principles (pp. 413–415, 526–527; Append. XIV), but the derivation of the production conditions is based upon the very simple illustrative assumption of a producer-consumer expending his capital and labor in such a manner as to maximize his utility. Under more general assumptions the conditions are developed, without the utility calculus used by Marshall, by Pareto (Cours, Vol. I, pp. 20ff., Vol. II, pp. 90ff.) and Barone (Ministry of Production), and with the utility calculus, by Professor Pigou (Economics of Welfare, particularly pp. 131–143) and Mr. Kahn (Economic Journal, March, 1935). All of these writers either develop the consumption conditions independently of their formulation of the production conditions (Marshall, Pareto) or assume the consumption conditions *ab initio* (Barone, Pigou, Kahn); and, as we shall indicate, the interpretations vary. Mr. Lerner in his study in the Review of Economic Studies, June, 1934, presents all the conditions together, and interprets them most lucidly in the second of the two senses we have pointed out.

As I have noted elsewhere (footnote 5, p. 317) none of these writers includes in his analysis individual preferences between production units. Also, Professor Pigou and Mr. Kahn include the possibility of departures from (2.3), and perhaps from (1.11), (1.12), (1.13), (1.14), for the direct effects on welfare of shifts of the factors of production from one use to another.

the Share of the i^{th} individual. In (2.9), p, q, etc. are taken proportional to the respective marginal rates of substitution. Thus m^i is defined, aside from a proportionality factor. The sum of m^i for the community as a whole is equal to the difference between the total wages and the total value of consumers' goods in the community.

The Propositions of Equal Shares: *If the Shares of any i^{th} and k^{th} individuals were equal, and if the prices and wage rates were fixed, the transfer of a small amount of the Share of i to k would leave welfare unchanged.*

The Propositions of Equal Shares enable us to state in terms of the distribution of Shares the remaining condition (1.5). According to these evaluations, if the Shares of i and k are equal, then for the price-wage situation given,

$$dE = \frac{\partial E}{\partial m_i} dm_i + \frac{\partial E}{\partial m_k} dm_k = 0, \tag{2.10}$$

for $dm_i = -dm_k$. Equation (2.10) is equivalent to the condition imposed by (1.5) that the marginal economic welfare per "dollar's worth" of X is the same for i and k.[8] Thus if the Shares of all individuals are equal, the condition (1.5) is satisfied.[9]

8. The proof is as follows:

$$\frac{\partial E}{\partial m_i} = \frac{\partial E}{\partial x_i}\frac{\partial x_i}{\partial m_i} + \frac{\partial E}{\partial y_i}\frac{\partial y_i}{\partial m_i} + \frac{\partial E}{\partial a_i^x}\frac{da_i^x}{\partial m_i} + \frac{\partial E}{\partial b_i^x}\frac{\partial b_i^x}{\partial m_i} + \frac{\partial E}{\partial a_i^y}\frac{\partial a_i^y}{\partial m_i} + \frac{\partial E}{\partial b_i^y}\frac{\partial b_i^y}{\partial m_i}$$

By (2.9)

$$1 = p\frac{\partial x_i}{\partial m_i} + q\frac{\partial y_i}{\partial m_i} - g^x\frac{\partial a_i^x}{\partial m_i} - h^x\frac{\partial b_i^x}{\partial m_i} - g^y\frac{\partial a_i^y}{\partial m_i} - h^y\frac{\partial b_i^y}{\partial m_i}.$$

Using this equation (2.7), and similar equations for the commodities and services,

$$\frac{\partial E}{\partial m_i} = \frac{\partial E}{\partial x_i} \cdot \frac{1}{P}.$$

9. Among the welfare studies the Cambridge Conditions are the distinctive characteristic of the writings of the members of the Cambridge School. They are advanced in the works of all the Cambridge economists, and in none of the other welfare studies we have considered. But certain qualifications must be noted.

The Cambridge economists require an equal distribution of incomes, $(px_i + qy_i)$, rather than of Shares as the condition for equality of the marginal economic welfare per "dollar" for all individuals (with quali-

The three groups of value propositions are not only sufficient for the derivation of the maximum conditions presented in the welfare studies. They are necessary for this procedure. For it is possible, and I shall leave the development of the argument to the reader, to deduce from the maximum conditions presented the restriction imposed upon the Economic Welfare Function by the value judgments introduced.

But it should be noted that the particular value judgments I have stated are not necessary to the welfare analysis. They are essential only for the establishment of a particular group of maximum conditions. If the production functions and individual indifference functions are known, they provide sufficient information concerning the Economic Welfare Function for the determination of the maximum position,

fications which we shall note directly, cf. Kahn, Economic Journal, March, 1935, pp. 1, 2; Pigou, Economics of Welfare, pp. 82ff.; Marshall, Principles, p. 795). If it is assumed that the amounts of the various types of labor performed by each individual in the community are given, this condition is of course the same as ours. But otherwise for a requirement of equal incomes there is unlikely to be any position which satisfied all the conditions for a maximum. For it would be necessary that in the neighborhood of the maximum position the marginal productivity and marginal diswelfare of each type of work be zero.

[The condition of equal incomes is not necessarily inconsistent with the other postulates. There might be some indifference functions and production functions such that all the maximum conditions are satisfied. But it may be noted here, in general, as a minimum requirement that the various conditions must be consistent with each other. Compare Lange, Review of Economic Studies, October, 1936, pp. 64, 65, and Lerner, ibid., p. 73.]

For convenience I have presented the Cambridge Conditions in a rather simple form. In a more elaborate exposition of the conditions advanced by the Cambridge economists I should have to introduce — and on *a priori* grounds I believe it desirable to introduce — modifications in the distribution of Shares for changes in the price-wage situation which might affect different individuals differently — some moving to a more preferable position, and others to a less preferable one — and for other special differences between individuals. (Cf. Marshall's reference to the distribution of *wealth*, op. cit., pp. 527, 595, and Pigou's reference to the distribution of the *Dividend*, op. cit., p. 89; but cf. also Kahn's reference to the distribution of *money incomes*, p. cit., pp. 1, 2.)

if it exists.[1] In general, any set of value propositions which is sufficient for the evaluation of all alternatives may be introduced, and for each of these sets of propositions there corresponds a maximum position. The number of sets is infinite, and in any particular case the selection of one of them must be determined by its compatibility with the values prevailing in the community the welfare of which is being studied. For only if the welfare principles are based upon prevailing values, can they be relevant to the activity of the community in question. But the determination of prevailing values for a given community, while I regard it as both a proper and necessary task for the economist, and of the same general character as the investigation of the indifference functions for individuals, is a project which I shall not undertake here. For the present I do not attempt more than the presentation of the values current in economic literature in a form for which empirical investigation is feasible.[2]

III

The formulation I have used to derive the maximum conditions of economic welfare differs in several respects from that of the welfare studies. It will be desirable to review briefly the relevant points of the various expositions, and the departures of the present essay from them. I shall continue to use the set of assumptions stated on page 310.

1. Cf. footnote 8, p. 313

2. This conception of the basis for the welfare principles should meet Professor Robbins' requirement that the economist take the values of the community as data. But in so far as I urge that the economist also *study* these data it represents perhaps a more positive attitude than might be inferred as desirable from his essays. (The Nature and Significance of Economics, London, 1932, particularly chapter VI.) Whether the approach will prove a fruitful one remains to be seen.

It may be noted that tho Professor Robbins is averse to the study of indifference curves (pp. 96ff.) his own analysis requires an assumption that a movement of labor from one use to another is indifferent to the laborer and that a shift of other factors of production is indifferent to the community. Without these assumptions, for which I can see no *a priori* justification, his whole discussion of alternative *indifferent* uses, and his references to the most adequate satisfaction of demand from a given amount of means are without basis.

In the Cambridge analysis[3] the welfare of the community, stated symbolically,[4] is an aggregate of the form,[5]

$$\overline{E} = \Sigma U^i(x_i, y_i, a_i^x, b_i^x, a_i^y, b_i^y). \tag{3.1}$$

In this expression U^i is some function of the indifference function, S^i, and measures the satisfactions derived by the i^{th} individual from x_i, y_i, a_i^x, b_i^x, a_i^y, b_i^y. If individual temperaments are about the same, that is, if individuals are capable of equal satisfactions, the marginal utilities or derivatives of the utility functions of different individuals, it is assumed, will be equal for an equal distribution of Shares.[6]

It is possible to derive all the maximum conditions, in specific terms, from the equation

$$\Sigma dU^i = 0. \tag{3.2}$$

The technique used by the Cambridge economists is less direct and varies in certain respects. For our present purposes these procedural differences are of little special interest, but it will facilitate our discussion of the analysis of Pareto and Barone if we append the following notes.

Marshall develops the Pareto-Barone-Cambridge consumption and labor supply conditions separately from the rest of his analysis.[7] These conditions are that for some price-wage situation p, q, g^x, h^x, g^y, h^y, and for all i,

$$w^i = \frac{U_1^i}{p} = \frac{U_2^i}{q} = \frac{-U_3^i}{g^x} = \frac{-U_4^i}{h^x} = \frac{-U_5^i}{g^y} = \frac{-U_6^i}{h^y} \tag{3.3}$$

3. The passages in the Cambridge studies which are particularly informative as to the Cambridge concept of welfare are Marshall, op. cit., pp. 80ff., 200ff., 527, 804; Pigou, op. cit., pp. 10–11, 87, 97; Kahn, op. cit., pp. 1, 2, 19; and also Edgeworth, Papers Relating to Political Economy, Vol. II, p. 102 (from the Economic Journal, 1897).

4. Aside from Marshall's appendices, the exposition of Marshall, Professor Pigou, and Mr. Kahn is non-mathematical, but the few relationships we discuss here may be presented most conveniently in a mathematical form. This will also facilitate comparison with the studies of Pareto and Barone.

5. In the analyses of Professor Pigou and Mr. Kahn some modification of (3.1) would be introduced to take care of the direct effects on aggregate welfare of shifts of factors of production from one use to another (cf. footnote 1, p. 315).

6. With the qualifications of footnote 9, p. 321.

7. Cf. the references in footnote 7, p. 320.

In (3.3), w^i is the marginal utility of money to the i^{th} individual and U_1^i, U_2^i, U_3^i, etc., are the marginal utilities of the various commodities and disutilities of the various types of work. In Marshall's exposition it is shown that, for any given amounts of X, Y, A^x, B^x, A^y, B^y, if the conditions (3.3) are not satisfied some U^i can be increased without any other being decreased. Thus for (3.2) to hold, (3.3) must be satisfied. Professor Pigou and Mr. Kahn do not develop the conditions (3.3), but assume them *ab initio* in their analysis.

If the conditions (3.3) are satisfied, (3.2) may be written in the form

$$(3.4) \qquad \Sigma w^i \Delta_i = 0,$$

where

$$(3.5) \qquad \Delta_i = p dx_i + q dy_i - g^x da_i^x - h^x db_i^x - g^y da_i^y - h^y db_i^y.$$

The remaining conditions again may be derived from (3.4). However, in Mr. Kahn's reformulation of Professor Pigou's analysis,[8] it is assumed also that the Shares are distributed equally, and the remaining conditions are developed from the requirement that

$$(3.6) \qquad \Sigma \Delta_i = 0.$$

The summation in (3.6), with certain qualifications, is Professor Pigou's index of the National Dividend.[9] The procedures of Professor Pigou and Marshall differ from this, but the variances need not be elaborated here.[1]

Pareto and Barone also assume initially that conditions (3.3) are satisfied, but Pareto like Marshall shows in an early section of his work that, otherwise, it is possible to increase the *ophélimité* of some individuals without that of any others being decreased.[2] To develop the remaining con-

8. Economic Journal, March, 1935.

9. Professor Pigou's index does not include cost elements; it relates to large adjustments — whence the problem of backward and forward comparisons; and it is expressed as a percentage of the total value product at the initial position. Cf. Economics of Welfare, Chap. VI.

1. But cf. section IV, infra.

2. Cours, Vol. I, pp. 20ff.

ditions, aside from the Cambridge Conditions, Pareto expressedly avoids the use of (3.2) on the ground that

> nous ne pouvons ni comparer ni sommer celles-ci [dU^1, dU^2, etc.], car nous ignorons le rapport des unités en lesquelles elles sont exprimées.[3]

Instead Pareto proceeds directly to (3.6) and deduces the maximum conditions for production from it. In this, evidently for the same reason, Barone follows.[4] Neither Pareto nor Barone introduces the Cambridge Conditions into his analysis. Pareto merely assumes that the shares are distributed "suivant la régle qu'il plaira d'adopter," or in a "manière convenable,"[5] and Barone that they are distributed according to some "ethical criteria."[6]

The basis for developing production conditions directly from (3.6), for Pareto, is that this equation will assure that if the quantities of products

> étaient convenablement distribuées, il en resulterait un maximum d'ophélimité pour chaque individu dont se compose la société.[7]

Barone adopts the requirement that the sum be zero because this

> means that every other series of equivalents different from that which accords with this definition would make that sum negative. That is to say, either it causes a decline in the welfare of all, or if some decline while others are raised, the gain of the latter is less than the loss of the former (so that even taking all their gain from those who gained in the change, reducing them to their former position, to give it completely to those who lost, the latter would always remain in a worse position than their preceding one without the situation of others being improved).[8]

Mr. Lerner, in the first of his two studies on welfare, advances as a criterion for a maximum position the condition that it should be impossible in this position to increase the welfare of one individual without decreasing that of another. From this criterion he develops graphically the

3. Ibid., Vol. II, p. 93.
4. Cf. Ministry of Production, p. 246.
5. Cours, Vol. II, pp. 91, 93, 94.
6. Op. cit., p. 265.
7. Op. cit., pp. 93, 94.
8. Op. cit., p. 271.

first two groups of maximum conditions. Like Pareto and Barone he does not introduce the Cambridge Conditions into his analysis but, as he indicates, ignores the problem of distribution.[9] In his later paper Mr. Lerner presents the first group of maximum conditions alone, on the basis of the criterion for a maximum that it should be impossible to increase the production of one commodity without decreasing that of another.[1]

In my opinion the utility calculus introduced by the Cambridge economists is not a useful tool for welfare economics. The approach does not provide an alternative to the introduction of value judgments. First of all, the comparison of the utilities of different individuals must involve an evaluation of the relative economic positions of these individuals. No extension of the methods of measuring utilities will dispense with the necessity for the introduction of value propositions to give these utilities a common dimension. Secondly, the evaluation of the different commodities cannot be avoided, even tho this evaluation may consist only in a decision to accept the evaluations of the individual members of the community. And finally, whether the direct effects on aggregate utility of a shift of factors of production from one use to another are given a zero value, as in Marshall's analysis, or a significant one, as in the analyses of Professor Pigou and Mr. Kahn,[2] alternatives are involved, and accordingly value judgments must be introduced.

While the utility calculus does not dispense with value judgments, the manner in which these value judgments are introduced is a misleading one. Statements as to the aggregative character of total welfare, or as to the equality of marginal utilities when there is an equal distribution of Shares, provided temperaments are about the same, do have the ring of *factual* propositions, and are likely to obscure the

9. Review of Economic Studies, June, 1934.
1. Ibid., October, 1934.
2. Cf. footnote 1, p. 315 and footnote 3, p. 324.

evaluations implied. The note by Mr. Kahn, in reference to his own formulation of the maximum conditions for economic welfare, that

> many will share Mr. Dobbs' suspicion "that to strive after such a maximum is very much like looking in a dark room for a black hat which may be entirely subjective after all." [3]

is not one to reassure the reader as to the nature of the welfare principles derived in this manner. To the extent that the utility calculus does conceal the rôle of value judgments in the derivation of welfare principles, the criticism directed against the Cambridge procedure by Professor Robbins and other students of economics[4] is not without justification.

The approach, it must also be noted, requires a group of value propositions additional to those I have presented. Insofar as the Cambridge economists require that the economic welfare of the community be an *aggregate* of individual welfares, value judgments must be introduced to the effect that each individual contributes independently to the total welfare. These value propositions, which imply the complete measurability of the economic welfare function aside from an arbitrary origin and a scalar constant, are not necessary for the derivation of the maximum conditions, and accordingly are not essential to the analysis.[5]

The derivation of conditions of maximum economic welfare without the summation of individual utilities, by Pareto, Barone, and Mr. Lerner, is a stride forward from the Cambridge formulation. Pareto's exposition of the basis for the procedure is somewhat ambiguous. Properly stated, the argument for developing production conditions directly from (3.6) is the same as that used in developing consumption conditions. The increment Δ_i in (3.5) indicates the prefer-

3. Economic Journal, March, 1935, footnote, p. 2.

4. Cf. Robbins, The Nature and Significance of Economic Science (London, 1932); Sutton, C., Economic Journal, March, 1937.

5. Lange's discussion of utility determinateness (Review of Economic Studies, June, 1934.) errs insofar as it implies that welfare economics requires the summation of the independently measurable utilities of individuals, i.e., his second utility postulate.

ence direction of the i^{th} individual.[6] If Δ_i is positive, the i^{th} individual moves to a preferable position. The condition that $\Sigma\Delta_i$ be equal to zero does not assure that the *ophélimité* of each individual be a maximum, but that it be impossible to improve the position of one individual without making that of another worse. This, disregarding the misleading comparison of losses and gains, is the interpretation of Barone, and it is also the condition for a maximum used by Mr. Lerner.

But in avoiding the addition of utilities, Pareto, Barone, and Mr. Lerner also exclude the Cambridge Conditions from their analysis. None of the writers indicates his reasons for the exclusion, and I believe it has not proved an advantageous one. The first two groups of value propositions are introduced in the studies of Pareto and Barone by the use of, and the argument as to the use of, (3.6) as a basis for deriving maximum conditions, and in the analysis of Mr. Lerner by the criteria adopted for a maximum. In this respect the formulations differ little from that of the Cambridge economists. With the accompanying statements by Pareto and Barone that *the distribution of Shares* is decided on the basis of some "ethical criteria" or "rule," or with the complete exclusion of the problem by Mr. Lerner, this approach is not more conducive to an apprehension of the value content of the first two groups of maximum conditions. In the case of Mr. Lerner's study a misinterpretation does in fact appear. For in his analysis the first group of maximum conditions are advanced as objective in a sense which clearly implies that they require no value judgments for their derivation.[7]

Further, it must be emphasized, tho the point is surely an obvious one, that unless the Cambridge Conditions, or a modified form of these conditions, is introduced there is no reason in general why it is more preferable to have the other two groups of conditions satisfied than otherwise. Placing $\Sigma\Delta_i$ equal to zero does not assure that there are

6. Cf. Allen, Economica, May, 1932.
7. Review of Economic Studies, October, 1934, p. 57.

no other positions for which welfare is greater, but only that there are no other positions for which the welfare of one individual is greater without that of another being less. In general if the third group of maximum conditions is not satisfied, it is just as likely as not that any position for which $\Sigma\Delta_i$ does not equal zero will be *more* desirable than any position for which it does equal zero.

In the Pareto-Barone analysis, tho not in that of Mr. Lerner, there is reason to believe that, in a general form, the third group of maximum conditions is assumed to be satisfied. While the distribution of Shares is not specified, it is consistent with some "ethical criteria," or "rule." Whatever the rule is, it should follow that in the maximum position the marginal economic welfare "per dollar" with respect to all individuals is the same. Otherwise, in the light of that rule, some other distribution would be preferable. If this interpretation is correct, the special exposition used by Pareto and Barone to support their derivation of maximum conditions is inappropriate. In (3.6) it is true that each dollar does not express the same amount of utility in the Cambridge sense, since the value propositions of independence are not introduced. But each dollar does express the same amount of welfare. The argument used to place (3.6) equal to zero is thus not the Pareto-Barone one, but that if it were unequal to zero, a further adjustment increasing the summation would be possible, and this would directly increase welfare, *regardless* of whether the position of some individuals were improved and that of others worsened by the change.[8]

IV

I have noted elsewhere that the conditions for a maximum welfare which are presented in sections I and II are the conditions for any critical point. They are sufficient to inform us whether or not we are at the top or bottom of a hill, or at the top with respect to one variable, and the bottom with respect to another. The requirement for a

8. This argument is more fully developed in section IV, infra.

maximum position is that it be possible to reach the position from any neighboring point by a series of positive adjustments. For the determination of such a position, it is necessary to know the sign (+, −, 0) of any increment of welfare.

In the welfare studies the sign of dE is specified only for limited groups of adjustments. It will be of interest to note these conditions, and the value judgments required, tho I shall not review again the formulations of the various writers.

(1) If we assume that all the conditions for a critical point are satisfied, except those relating to the distribution of the factors of production between different uses, one additional group of value judgments gives us sufficient information concerning the shape of the Economic Welfare Function to determine the sign of an increment of welfare. These value propositions are: *if all individuals except any i^{th} individual remain in positions which are indifferent to them, and if the i^{th} individual moves to a position which is preferable to him, economic welfare increases.* If we denote a more preferable position by a positive movement of S^i, these value propositions require that

$$\frac{\partial E}{\partial S^i} > 0, \tag{4.1}$$

for any i. Let us write from (2.5),

$$\begin{aligned} dE = \sum \frac{\partial E}{\partial x_i} dx_i + \frac{\partial E}{\partial y_i} dy_i + \frac{\partial E}{\partial a_i^x} da_i^x + \frac{\partial E}{\partial b_i^x} db_i^x \\ + \frac{\partial E}{\partial a_i^y} da_i^y + \frac{\partial E}{\partial b_i^y} db_i^y. \end{aligned} \tag{4.2}$$

Using the equations (1.5) through (1.10), and the notation of (3.5),

$$dE = \omega \Sigma \Delta_i. \tag{4.3}$$

By (4.1) and the equations (1.5) through (1.10), ω must have the same sign as the price-wage rates in Δ_i. We shall take this sign as positive. Thus if the Shares are distributed equally, and if the prices and wage rates are proportionate to the marginal rates of substitution of the different kinds of commodities and types of work, economic welfare has the sign of Professor Pigou's index of the National Dividend. It will be increased by any adjustment which has as a result the movement of factors of production to a position of higher marginal value productivity.

(2) If the assumption that the Cambridge Conditions are satisfied is relaxed, (4.3) may be written in the form

$$dE = \Sigma \omega^i \Delta_i \tag{4.4}$$

where ω^i is the marginal economic welfare per dollar with respect to the i^{th} individual. Using the evaluation in (4.1) it follows that, for any adjustment for which no Δ_i decreases and some Δ_i increases, economic welfare will increase.

(3) Continuing to use the assumptions of (2), let us write

$$\lambda_{ik} = \frac{\omega^i}{\omega^k}, \tag{4.5}$$

and

$$dE = \omega^k \Sigma \lambda_{ik} \Delta_i. \tag{4.6}$$

Let us introduce the value propositions: *for a given price-wage situation, and any i and k, if the Share of i is greater than that of k, a decrease in the Share of k would have to be accompanied by a larger increase in the Share of i, for economic welfare to remain unchanged.* Since it can be shown that if the Share of the i^{th} individual increases by dm_i a concomitant decrease, $-\lambda_{ik} dm_i$, in the share of the k^{th} will leave economic welfare unchanged,[9] these value propositions require that λ_{ik} be less than unity. It follows that, for any given adjustment, if $\Sigma\Delta_i$ is positive, and if Δ_i does not vary with m_i, or if it decreases with m_i, economic welfare will increase. In other words, if the change in the National Dividend is not counteracted by a change in its distribution, the welfare of the community will be increased, even if some Δ_i increase and others decrease.

The adjustments in (1) are those considered by Mr. Kahn;

9. This relationship follows immediately from the equations:

$$dE = \frac{\partial E}{\partial m_i} dm_i + \frac{\partial E}{\partial m_k} dm_k = \omega^i dm_i + \omega^k dm_k.$$

in (2) by Pareto, Barone, and Mr. Lerner; and in (3) by Marshall and Professor Pigou. As Professor Pigou has pointed out,[1] the sign of an increment of welfare for some adjustments is left undetermined in his analysis. To determine the sign of dE for all adjustments, all the λ's would have to be evaluated, and a similar group of value judgments for the case where prices and wages are not proportional to the marginal rates of substitution would have to be introduced. On *a priori* grounds there is no reason why more information should not be obtained, since the comparison involved in evaluating the λ's is the same as that required for the Value Propositions of Equal Shares. For some additional and fairly rough evaluations, the range of adjustments included can be extended considerably, tho an element of uncertainty is involved. Two such approximations, perhaps, are of sufficient interest to note, tho they are not introduced in the welfare studies.

(4) The assumptions of (2) are retained. Let us suppose that with respect to some individual, say the k^{th},

$$\Sigma\lambda_{ik}=N \tag{4.7}$$

the sum being taken for all i. Thus ω^k is the average ω If we write

$$\alpha_i=\lambda_{ik}-1;\ \beta_i=\Delta_i-\frac{\Sigma\Delta_i}{N}; \tag{4.8}$$

then

$$dE=\omega^k(\Sigma\alpha_i\beta_i+\Sigma\Delta_i). \tag{4.9}$$

The first term in the brackets may be regarded as an index of the distribution of the National Dividend. It follows immediately from (4.9) that (*a*) if Δ_i is positively correlated with λ_{ik}, dE will increase with an increase in the Dividend and conversely; (*b*) if the coefficient of variation of the ω's is less than one hundred per cent, that is, if the standard deviation of λ_{ik} is less than unity, and if the coefficient of variation of Δ_i is also less than one hundred per cent, dE

1. Economics of Welfare, p. 645.

will have the sign of the index of the Dividend *regardless* of changes in its distribution.[2]

To determine precisely whether the conditions enumerated are satisfied, of course, would require a complete evaluation of the λ's. But the following rough evaluations would be sufficient to assure the likelihood of the results. For (*a*), it must be possible to say that "on the average" the change in distribution does not affect the "poor" more than the "rich" or vice versa. For (*b*) it is necessary to conceive of an individual or group of individuals who are, on the whole, in an average position from the point of view of welfare, and to determine whether, for a given position, ω^i "on the average" is likely to be somewhat less than twice the marginal economic welfare per "dollar" for the average individuals, that is, less than twice ω^k. (This should be stated in terms of the average shift in Shares for which welfare remains unchanged.) If it is determined that such a position is occupied, it would be likely that if tastes did not vary greatly — that is, if the relative variation of Δ_i were not very large — dE would increase for an increase in the Dividend. Since, however, the relative variation of Δ_i would ordinarily become excessively large as $\Sigma\Delta_i$ approached zero, it would be highly uncertain, for adjustments close to the maximum, whether or not an unfavorable change in distribution would obliterate the change in the Dividend.

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2. From (4.9),

$$dE = \omega^k(Nr_{\lambda\Delta}\sigma_\lambda\sigma_\Delta + \Sigma\Delta)$$
$$= \omega^k(Nr_{\lambda\Delta}\sigma_\lambda\sigma_\Delta/\Sigma\Delta + 1)\Sigma\Delta.$$

The proposition (*a*) follows immediately, and (*b*) is based on the fact that $r_{\lambda\Delta}$ must be less than unity.