

# Computing Infrastructure

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## Part I

# Introduction

## 1 Computing Infrastructure

What is a CI? A CI is a *technological structure* that provides hardware and software for computation to other systems. This means a lot: is a joint (and heterogeneous) system that comprises both HW and SW to deliver a service. With CI is not intended the *application delivered* but the combination of elements that makes the application available and running.

So everything in between a RaspPI that does torrent seeding from a local network to the whole datacenter that runs an AWS service can be considered a computing infrastructure.

## Part II

# Hardware Infrastructures

## 2 Data Centers

### 2.1 Pros and cons

Data centers are big centralized CIs that comprise a lot of servers (to provide computations) a communication infrastructure, and a storage service.

PROS of a DC:

- Lower IT cost: renting a virtual machine is cheaper than buying / building and maintaining a whole system (for some time horizons, of course);
- High performance: virtualized resources make scaling easier, and provides optimized and finely tuned services that an in-house solution cannot provide *so* easily and fast;
- No effort software updates;
- Unlimited (!) storage capacity;
- Increased data reliability;
- Universality of accesses;
- Device indpendence.

The CONS of a DC can be found simply inverting the POV for the PROS aspects: outsourcing resources gives "someone else" the control over some crucial aspects of a CI. This is a good thing when costs (also time costs) must be reduced, but can be a bad thing when a fine grained control over a full system is needed. Also, *latency* pops in, due to the needed connection to the DC.

## 2.2 DCs and WSCs

The data center approach has been emerging in the last years. The idea behind it is that we should not "overpack" nodes of a network with all the computing capabilities required, but instead giving them a connection to such CIs. Data centers are the perfect example for this paradigm: the user interact with a client (that can be *any kind of device*) that also interact with a remote structure to provide computations. To a careful watch, SaaS and their spread are a direct consequence of this paradigm shift. Another use of DCs is using their computing capabilities not in a fragmented way to provide miriads of services, but to perform an *extremely costly computation*, like a training of a neural network.

From a DC approach we are moving to a Warehouse Scale Computers nowadays. This latter approach consists of NOT "mix up the pot" in a DC (so having a lot of heterogeneous technologies in order to achieve different tasks) but instead to "homogeneify" the cloud structure in order to do better optimization of it. Not only in the HW, but also in the SW. To centralize the DC capabilities under a single organization (as often happens) means that clients no more run their application on someone else's hardware, but instead choose from a set of precooked solutions by such vendor/organization. Is this *bad*? WSCs are just a "SaaS - oriented" Data Center Architectures, so *there's no real transition between DCs and WSCs* the only transition is in the number of *virtualized layers the client must go through to access and application*.

We can sum up the difference between DCs and WSCs in this way: where DCs are intended as a powerful collection of different servers, WSCs tries to offer a homogeneous interface that can be used also as a single server to such a collection of hardware. Still, if we consider a larger definition for DCs, WSCs are a type of DCs.

## 2.3 Architectural Overview of DCs and WSCs

A standard data center building is organized in separated modules, every one dedicated to a specific task. The four main modules types are

- Servers: computations
- Storage
- Networking: intended as the whole communication infrastructure
- Building-integrated systems:
  - Cooling system: often as powerful as the server themselves, fundamental for getting rid of excess heat
  - Power supply: integrated in the building and provided with systems to avoid power shortage
  - Failure recovery: physically realized as a building module in order to be as most fault - tolerant

Servers are organized in racks, blades or tower, and are the classical computational unit usually found in server farms. They could also not have memory attached.

Storage is the crucial long term memory for a CI. Usually built with flash SSDs and ferromagnetic mechanical disks. The memory units must provide high speed I/O capabilities, and also advanced networking power, also at software level (NAS, SAN, DAS).

The networking infrastructure is the backbone of the communication in (and from/to) a data center. Structured hierarchical approach to networking structures and organization are used to ensure security and performance.

The building itself is part of the CI.

Next sections will delve into the details of the single modules.

## 2.4 Servers

Servers are the computational compartment of a WSC. They're organized in racks (shelves) that host the computational units (pizza box computer). Servers are just computers. That's all. They got a MOBO, local primary memory, a CPU, and I/O capabilities. They can be organized as

- Towers: cheap, cooling is easy, upgrade is easy. They're also big, and they provide low density of computational power.
- Blades: also called hybrid racks, the idea is similar to the rack system, but a server is inserted *vertically* instead of *horizontally* in a dedicated place. This (together with the average smaller size of a blade server) enhances computation per volume ratio while keeping all the pros of a rack system. Blade servers have disadvantages: heat management is complex, and enclosures and envelopes are more expensive wrt racks.
- Racks: literal racks that host the pizza box shaped computational units, and accommodate all the wiring and additional connection or cooling systems. They can host also heterogeneous components, not only standard computational modules. Rack's dimension and geometry is a standard. Rack organization provides better failure handling and simplified cable management, but they're more power demanding (wrt towers) and maintenance is a mess (multiple devices must be maintained at the same time).

### 2.4.1 Hardware Accelerator

WSCs architectures and the rise of heavy computation loads (like for intense machine learning and AI training) have caused a spike in the complexity required every computation, even far beyond the Moore's law. To satisfy this need, dedicated hardware (in the form of *hardware accelerators*) are deployed in WSCs, to enhance the performance in determined fields.

**GPU** Graphical Processing Units are featured in accelerators to enhance *parallel fast computation*. Due to the architecture of them, they're also perfect to do matrix computation.

**TPU** Tensor Processing Units. These are more specialized GPUs in the machine learning direction. They're (simplifying) custom circuits to support tensor computations (matrix calculus) that are *very fast* at doing so.

**FPGA** Good ol' Field Programmable Gate Array circuits are still used to enhance performance in custom computational oriented architectures (apple afterburner...). These circuits are somewhat "fully customizable" hardware computers that if programmed in a very specific way can enhance *every kind* of possible computation.

## 2.5 Storage

Storage. Not much more. Technologies involved: HDDs and SSDs and Flash memory. Performance (and relevant) parameters:

- read / write speed and latency (or seek time)
- memory density
- security: but this is intended as an hardware security, not data security (for now)
- cost

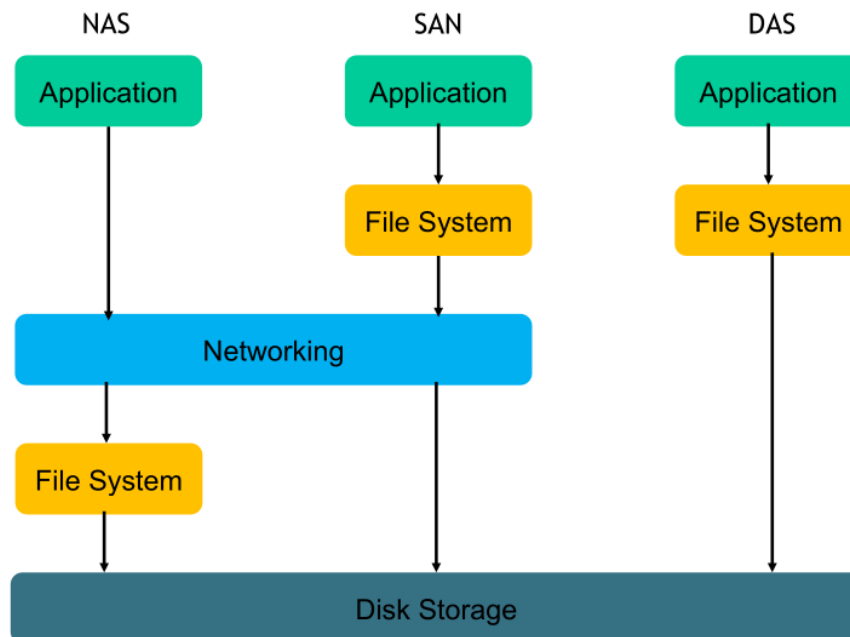
### 2.5.1 Storage Systems - DAS, NAS, SAN

- Direct Attached Storage is every device that offers storage capabilities and is directly attached to the unit processing such data in the storage. HDDs are DAS for personal computers, for example. They're directly accessible from the OS of the machine accessing it.
- Network Attached Storage is a storage system that lies *on the other end* of a network. A NAS only offers data services accessible from dedicated protocols (as FTP, SAMBA) and it's *an actual computer* that offers services.
- Storage Area Network is like NAS, but the remote storage unit are *passive* and are directly accessible by the OS (by means still of a network connection). They're *remotely accessed simple storage devices*.

**DAS** Traditional attached local storage systems. They're difficult to scale. They are difficult to manage when particular storage solutions are needed. Moreover, the OS's proprietary file sharing model / protocol / software must be used in order to share files.

**NAS** A NAS is a *full fledged computer* that's *only meant for storage services*, accessible through network protocols. It's located somewhere in the network. It's easily scalable (both augmenting the storage of a single NAS or multiplying the NAS number work).

**SAN** Disks are visible *like local storage, like DAS* to the server machina, but they're network attached. They have specialize hardware that mask the network in between, and the SAN can be accessed *directly through the filesystem*.



## 2.6 Networking

A WSC networking system can be divided in 2 separate blocks:

- an internal LAN that connects all the components (the servers) among themselves
- an interface to the internet (DMZ..?)

Obviously, the networking infrastructure in a WSC is crucial because it also connects the functional units (as the server) to the storage infrastructure and so it's *fundamental* that this connections is solid. The need of inter-server bandwidth incrases exponentially with the number of server, obviously.

A WSC internal network is usually divided in 3 levels:

1. Access: this network layer is the one that connects the servers to the network. It's embodied by the network infrastructures embedded in the server racks.
2. Aggregation: this level groups the servers together (VLAN-like) and it's usually implemented as Top Of the Rack switches, that group together all the server in a aisle (for example).
3. Core: furthest level from the single server, aggregates entities of the "aggregation" level.

## 2.7 Building and Non-IT Equipment

The way in which the hardware is arranged *inside* a DC is as crucial as the building composing the DC itself, as well as the non-IT instruments (like the



cooling system, or the energy system) that completes the DC structure.

### 2.7.1 Energy System

Powering a DC is difficult on many layers; first of all, computing infrastructures of this scale need a lot of energy, but this energy must be provided steadily and must be always available. Moreover, the power must not be subject to cuts, improvised shortages or outing. So not only the energy system of a DC must handle a lot of power, but should also handle it *with care*. Usually, DCs have a UPS that does all this work.

**The UPS** The Uninterruptible Power System is the hearth of the energy subsystem of a datacenter. It is composed of 3 main parts:

1. A socket to the external provided power. This is usually high voltage power that must be trimmed down (order of 50 kV) in order to be utilized.
2. An internal generators system that is powered up when the external power supply fails. These systems takes usually 10 to 20 seconds to power up.
3. A battery array to bridge the gap between an external power outage and the wind-up time of the backup generators.

The UPS must also "filter" the external power provided, cutting spikes and removing dangerous armonics in current. This is usually done by a AC to DC to AC converter. Then, electric power is sent to power distribution units (that resembles the one in the houses) that manages the power flow in each row or rack.

### 2.7.2 Cooling System

The cooling system removes the heat generated by the equipment. It must imply some sort of loop (thermodynamically speaking) that warms up a medium and then transfers the heat somewhere else.

There are several strategies to cool down a data center:

- Open loop: the fresh air is "sucked in" from the outside and then passively heated by the servers themselves. The hot air exhaust is let flow out from the top. In fact, this is a open-the-windows approach. Obviously this approach is very easy to set up, and requires little to no additional hardware (the additional hardware can be needed to cool down the air intake, or speed up the airflow, controlling humidity...) and can perform very well for its cost. Being so simple, it has its drawbacks: it depends from the outside temperature, the air flow is "uncontrolled"...
- Closed loop: this approach is more similar to the personal computer's one. Heat is removed from the hardware and then the medium used to remove it is chilled in another place. Why closed? Because it does not depends on external air / chiller medium to refrigerate a room. Instead, the medium is chilled in a heat exchanger (usually located in a dedicated CRAC room) and then reinsterded in the loop. Multiple loop can be exploited in order to enhance the control over the heat transfer.

- In rack cooling: manufacturers of server racks can add a device that acts as a heat exchanger (inparticular, generally it's a air to water exchanger) that sucks out the out directly from the servers.
- In row cooling: as in rack cooling, just done at row level.
- Circuit cooling: as in a personal PC, we can directly cool down circuitry (usually with liquid cooling systems) and this approach offers the most elevated performaces. However, liquid circuitry cooling system are hard to manage, other than impractical.

## Part III

# Software Infrastructure

## 3 Virtualization

Virtualization is the procedure of "making a resource available through software artifacts". This resource can range from a particular kind of processor to a whole application set or service. Virtualization is the main technology enabling cloud computing, because of the flexibility mainly, the isolation and the security offered. Server VMs are implemented without using a host OSs as the personal computer one, they rely instead on a virtual machine monitor (like HyperVisor) that manages the hardware to run multiple VMs at the same time.

### 3.1 Virtual Machines

A machine is defined as "execution environment capable of running a program". This is a very general definition, that ranges from washing machines to computing infrastructures for neural network training. A virtual machine differs from a physical machine by the way in which hardware is managed. A physical machines handles hardware by the OS, while a VM has to interface with a hardware supervisor in order to access it. "Formally", a VM is a *logical abstraction able to provide a virtualized execution environment*.

A VM must provide identical software behaviour (execution on a VM should be transparent). The VM, being composed of a mix of hardware and virtualizing/virtualized software, usually has worse performance wrt his physical counterimplementation. The VM is in charge of the translation of the virtualized software instructions into effective machine instructions.

### 3.2 Levels of Virtualization

Given the intrinsically layered structure of an operating system, it's easy to discriminate between virtualization approaches basing on the layer they address.

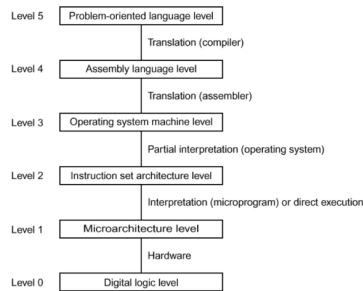


Figure 1: The layered operating system structure

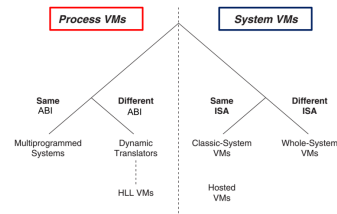


Figure 2: VMs and what they virtualize

### 3.3 Virtualization Implementation

As already said, in order to virtualize something we have to add some middleware in the software stack that carries out the virtualizing mechanism. We can put this layer *between the hardware and the OS*, or *between the operating system and the processes* as in JVM or even *between the OS and other OSs*, as in virtualbox.

As a remainder, a virtualization software should provide

- Partitioning between softwares
- Isolation, so fault tolerance and security
- Encapsulation: the state of a VM can be easily stored and moved
- HW independence. As incapsulation, but for the whole execution

### 3.4 VMM

We use three terms to define the same thing: Virtual Machine Manager, Virtual Machine Monitor and HyperVisor. They're the same thing, so *a piece of software dedicated to virtualize stuff*.

Hypervisors are the software that acts directly attached to the machine hardware and are capable of hosting an OS.

- Type 1 hypervisors provide an abstraction and a full environment to a host relying directly on the hardware.
- Type 2 hypervisors (virtualbox) provide the virtualized hardware exploiting *a host operating system*.

### 3.5 Containers

A container is a pre configured package that comprises all the software pieces needed to run a particular type of software (like the database, the support for certain languages and services). All the software comes *configured*. This approach does not fall entirely under the "virtualization" hat, because they do not depend on a OS (the container does not provide one). Instead, they rely on a container engine (as the docker engine).

## 4 Cloud Computing and Edge/Fog Computing

Virtualization paved the way to intensive cloud computing. Being able to virtualize a resource means to *have it available and accessible all the time*. It also decouples the hardware and the software.

One of the most used configuration aims to consolidate the usage of a single server, providing a dedicated environment *for each app* and they can all run in the same machine. This is a huge improvement wrt "an app for each server". On top of the hypervisor software we can also build a *distributed VM Monitor* that handles multiple VMs *across a cluster of machines*: this improves load balancing, performance and *resource usage*.

### 4.1 Cloud Computing and Services

Cloud computing is a model for enabling on demand network access to a shared pool of computing resources. The cloud approach pushed the *IaaS* paradigm to his limits: it started as "outsourcing to someone in the internet some computations" and it's "giving someone in the internet full control over our technologic stack".

#### 4.1.1 Software as a Service

#### 4.1.2 Platform as a Service

#### 4.1.3 Infrastructure as a Service

#### 4.1.4 Data as a Service

#### 4.1.5 Communication as a Service

### 4.2 Cloud Systems Taxonomy

- Public cloud are the classic cloud systems available to the users. They're large scale computing infrastructure available on a rental basis, that can be pay-as-you-go or fixed costs.
- Private cloud services are cloud systems *that are not shared with other users* so that are fully enclosed in the boundaries of an organization/company.
- Community clouds: a network of private clouds. This approach differentiates from the pure private cloud approach in the complexity of the system managed. The idea is still a single cloud platform for a set of organizations.
- Hybrid cloud: "private clouds that makes available some services".

### 4.3 Edge and Fog Computing

Edge and fog computing are two "emerging" paradigms that exploits intelligence at the edge of the network (of sensors, of data hotspots ecc) in order to provide faster communications and services.

## Part IV

# Methods

### 5 HDD and RAID Technologies

Memory and files in disks are organized in clusters to simplify the management of the data. We have:

$$\begin{cases} a = \text{actual size of a file on disk} \\ c = \text{cluster size} \\ s = \text{file size} \end{cases} \quad (1)$$

and it holds that  $a = \text{ceiling}(\frac{s}{c}) \times c$ .

#### 5.1 Data and Metadata

Data is the content stored in a mass storage device. Metadata is (as the name suggests) additional data stored in the storage device *that not contains actual content*. Metadata contains indexes, addresses, cached data that is used to manage *the actual data stored*.

#### 5.2 Magnetic Hard Drives

Magnetic disks have an internal structure and an external interface to manage files and data. The internal structure is often based in c/h/s coordinates, that are cylinder, head and sector locations. The external interface is usually composed of clusters. The traslation is carried out directly by the electronics on the disk.

##### 5.2.1 Delays

The principles on which the HDD storage is based relies on some phisical objects and their movement. The intrinsic limits of this system translates into delays in the reading of data. Four type of delays can be identified:

- Rotational: it's the time occurred to move the portion of plate under the desired head. It's related to RPM, of course
- Seek: it's the time needed to a head to move from a track to another
- Transfer: actual time to read the bytes
- Control: overhead time, related to the circuitery that manages the hardware

So, in order to calculate the actual time that passes between the issue of the read command to the presence of the desidered file on the I/O bus we have to sum all the delays.

$$T_{read} = T_{rotate} + T_{seek} + T_{transfer} + T_{overhead} \quad (2)$$

Usually, the transfer speed is usually given as transfer ratio  $\frac{MB}{sec}$ .

**Reducing Latency** Caching is the most used mechanism to make data available *faster* in order to reduce latency. Usually, at hardware level, the cache is implemented by means of a physical additional RAM memory built directly in the HDD.

**Read Cache** Often accessed data access can be reduced if there's no need to go down to the plate in order to read it all the times.

**Write Back Cache** Write buffer: writes are cached before accessing the disk, and then flushed at the end. N.B.: the "write finish" signal is issued at the end of the *caching* of the data.

**Write Through Cache** No write buffer: the "write finish" signal is issued after the write is actually written on the disk.

**Scheduling** As always, reordering operation in order to maximize efficiency is a solid approach to enhance performance, in this case reducing latency in accessing the disk. These kind of approaches (that usually prefer "near" sector) are prone to starvation, usually. Workaround to starvation problem usually are based on linearizing the traversing of the cylinders, like the elevator mechanism.

## 5.3 RAID

Redundant Array of Inexpensive Disks is a technology that exploits the parallelization of I/O bus to multiple disks in order to enhance security, write and read speed, and fault tolerance. Data are copied / distributed on multiple disks, that are presented to the OS as a single one. There are 6 (or 8) different raid configurations:

### 5.3.1 RAID 0

Data is just striped across disks. This is achieved by splitting the request to all the disks in the array. No data is replicated. RAID 0 provides high read/write speed, but worsens the reliability of the whole system: multiple disks + no backup (redundancy) options jeopardize reliability.

### 5.3.2 RAID 1

Data is mirrored. On two disks, the same copy of data is stored. This improves read speed (you can split the read of blocks) but worsens capacity and write speed. Obviously, reliability is increased. On more than 2 disks (where the mirroring is quite straightforward) raid 1 and 0 can be combined:

- RAID 0 + 1: strip then mirror. Data is striped on the first set of disks and then *mirrored striped on the second set*.
- RAID 1 + 0: mirror then strip. The data is first mirrored in two clusters and then striped in the individual clusters. The difference between 0+1 and 1+0 can be appreciated only on clusters of more than 4 disks.

Raid 0+1 and 1+0 can be imagined as a series of raid controllers.

## Part V

# Dependability

## 6 Probabilistic approach

Dependability represents the *availability performance* of a system. It encompasses

- Reliability
- Availability
- Maintainability

Dependability is approached with a statistical/probabilistic POV due to the high human component in it.

Two functions describes the system:  $F(t)$  and  $f(t)$ , respectively the cumulative function and the fault probabilistic distribution. The former represents the *unreliability* of the system analyzed. So, we can define  $R(t) = 1 - F(t)$  as the reliability function.

Probabilistics recall:

$$f(t) \rightarrow \text{probabilistic distribution} \quad (3)$$

$$F(t) = \int_0^t f(t)dt \rightarrow \text{cumulative function, unreliability} \quad (4)$$

$$R(t) = 1 - F(t) \rightarrow \text{reliability function} \quad (5)$$

To be noticed: the function  $F(t_i)$  represents the probability that component  $i$  is working at time  $t_i$  *knowing it was working at time 0*. This is to be taken in mind to make a comparison with the failure rate.

From this we can define the Mean Time To Failure for a component (that's the expected value):

$$\begin{cases} MTTF = \int_0^\infty t \cdot f(t)dt \\ MTTF = \int_0^\infty R(t)dt \end{cases} \quad (6)$$

We can also define the failure rate (mathematically, the conditioned probability):

$$\lambda(t)dt = F(t < T \leq t + dt | T > t) \quad (7)$$

So the failure rate represents the probability of failure *assuming the component was working the instant before*. This function can be seen as the number of failures in a given interval.

## 6.1 Failure Rate

Type of malfunctioning:

- Fault: physical defect or software bugs.
- Error: program incorrectness that can result from a fault.
- Failure: nonperformance of some actions that were expected. They can be result of an error.

**Properties of the failure rate** Probabilistically, failure rate is

$$\lambda(t)dt = F(t < T \leq t + dt | T > t) \quad (8)$$

So we can derive

$$\lambda(t)dt = \frac{P(t < T < t + dt \cap T > t)}{P(T > t)} \quad (9)$$

From the definition of combined probability, with P as probability. Given

$$P(t < T < t + dt) \cap P(T > t) = P(t < T < t + dt) \quad (10)$$

We obtain

$$\lambda(t)dt = \frac{f(t)dt}{R(t)} = \frac{dF(t)}{R(t)} = -\frac{dR(t)}{R(t)} \quad (11)$$

**Reliability as Weibull** Failure rate  $\lambda(t)$  function has a peculiar function shape: a *bathub* (or long U) shape. The failure rate is high in the starting period and decreases (infant mortality effect) to the constant level during the useful lifetime (constant probability of failures) and it raises again at the end (wear out period).

In fact, reliability follows the *Weibull distribution* so defined:  $y(x) = e^{-(\frac{x}{\alpha})^\beta}$ . So (due to the  $\lambda(t) = f(t)/R(t)$  relation) we obtain

$$\lambda(t) = \frac{\beta}{\alpha} \left(\frac{t}{\alpha}\right)^{\beta-1} \quad (12)$$

**Reliability as exponential** We can also model the reliability as an exponential function (so  $R(t) = e^{-\lambda t}$ ) we than obtain

$$\begin{cases} F(t) = 1 - e^{-\lambda t} \\ f(t) = \lambda e^{-\lambda t} \\ MTTF = \int_0^\infty t \cdot \lambda e^{-\lambda t} dt = \frac{1}{\lambda} \end{cases} \quad (13)$$

So we can express the reliability as function of MTTF:  $R(t) = e^{-\frac{t}{MTTF}}$ . And we all know that in certain conditions (like  $\frac{t}{MTTF} \ll 1$ ) an exponential function can be approximated to a linear function.



## 6.2 System level reliability

We can model the system reliability mainly with RBDs: Reliability Block Diagram. These simply outline the operational dependency between components. The main assumption of the RBD is that the failures are *independent*, that implies that there are no "cascading failures".

It's easy to calculate the overall reliability:

$$\begin{cases} \text{Serial components: } R_s(t) = \prod_{i=1}^n R_i(t) \\ \text{Parallel components: } R_p(t) = 1 - \prod_{i=1}^n (1 - R_i(t)) \end{cases} \quad (14)$$

The MTTF is also altered from the serialization or parallelization of components. In particular, in a serie with n identical components,  $MTTF_{serie} = \frac{MTTF}{n}$ . The general formula is the one used for parallel resistors:

$$MTTF_{serie} = \frac{1}{\sum_{i=1}^n \left( \frac{1}{MTTF_i} \right)} \quad (15)$$

For parallel system, things get complicated. The MTTF is calculated from the unreliability, integrating between 0 and  $\infty$ . The resulting formula is

$$MTTF_{parallel} = \sum_{i=1}^n (MTTF_i) - \frac{1}{\sum_{i=1}^n \left( \frac{1}{MTTF_i} \right)} \quad (16)$$

Notice that the second term of that sum is exactly  $MTTF_{serie}$  of that configuration. Again, the formula can be simplified if the components are identical:  $MTTF_{parallelidentical} = MTTF_i * \left( \sum_{i=1}^{n-1} \left( \frac{1}{n-i} \right) \right)$

**Extension of MTTF calculation to complex systems** We can apply the calculation of the *reliability* of a complex systems directly with the formulas, but the same cannot be done with MTTF. This is due to the fact that a complex system has not anymore a simple failure rate distribution, and this messes up the MTTF calculation, that's an integral on that function. So, I must pass through the reliability calculation.

## 6.3 Availability

Introducing MTTR, Mean Time To Repair. This represents the average time required to replace a failed component and "bring the system back up" after a failure. It can be the restart time (for software module) or the replace time (for an hardware component).

Another interval to be taken into consideration when talking about availability is the Mean Time Between Failure, that's just the sum of  $MTTF + MTTR$ . Availability is defined as the probability of a system to be up and running at a given instant. So  $Av = \frac{MTTF}{MTBF}$  that corresponds to

$$Availability = \frac{MTTF}{MTTF + MTTR} \quad (17)$$

The slight difference between availability and reliability takes into consideration the repairing of a system in the case it goes down. The availability function is

calculated with *the same identical formulas* used to calculate the reliability for parallel and serial components.

Talking about MTTF when taking into consideration that components can be *repaired* must be carefully handled: for example, if a parallel system has *inter-leaving* failures, it never fails entirely. A way of calculating MTTF for repairable system consists in inverting formula (9), obtaining

$$MTTF = \frac{Av * MTTR}{1 - Av} \quad (18)$$