CS 245 — Assignment #8

Spring 2006

Due Date: Tuesday, July 18 at 5pm.

Use `makeCover` to produce a cover page for your assignment and hand in your assignment in the CS 245 assignment box. Assignments are to be done individually.

For each of the following triples (pre-condition, program, post-condition), prove that the triple is satisfied under partial correctness. Use natural deduction or transformational proof techniques to prove any implied conditions.

1. (8 points)

$(\!|x = x_0 \quad \wedge \quad y = y_0|\!)$
```
x = x + y;
y = x - y;
x = x - y;
```
$(\!|x = y_0 \quad \wedge \quad y = x_0|\!)$

$(\!|x = x_0 \quad \wedge \quad y = y_0|\!)$

$(\!|((x + y) - ((x + y) - y)) = y_0 \quad \wedge \quad ((x + y) - y) = x_0|\!)$ `implied (algebra)`

`x = x + y;`

$(\!|(x - (x - y)) = y_0 \quad \wedge \quad (x - y) = x_0|\!)$ `assignment`

`y = x - y;`

$(\!|(x - y) = y_0 \quad \wedge \quad y = x_0|\!)$ `assignment`

`x = x - y;`

$(\!|x = y_0 \quad \wedge \quad y = x_0|\!)$ `assignment`

2. (17 points)

$(\!|\mathbf{true}|\!)$

```
if (x > y)      m = x + 2 * y;
else if (x < y) m = x + 3 * y;
else            m = x + 4 * y;
```

$(\!|(x > y \;\wedge\; m = x + 2y) \;\vee\; (x < y \;\wedge\; m = x + 3y) \;\vee\; (x = y \;\wedge\; m = x + 4y)|\!)$

Let $s(m, x, y)$ denote $(x > y \wedge m = x + 2y) \vee (x < y \wedge m = x + 3y) \vee (x = y \wedge m = x + 4y)$

$(\!|\mathbf{true}|\!)$	
`if (x > y)`	
$\quad(\!|x > y|\!)$	`if-then-else`
$\quad(\!|s(x + 2y,\ x,\ y)|\!)$	`implied (1)`
$\quad$`m = x + 2 * y;`	
$\quad(\!|s(m,\ x,\ y)|\!)$	`assignment`
`else if (x < y)`	
$\quad(\!|\neg(x > y) \;\wedge\; x < y|\!)$	`if-then-else`
$\quad(\!|s(x + 3y,\ x,\ y)|\!)$	`implied (2)`
$\quad$`m = x + 3 * y;`	
$\quad(\!|s(m,\ x,\ y)|\!)$	`assignment`
`else`	
$\quad(\!|\neg(x > y) \;\wedge\; \neg(x < y)|\!)$	`if-then-else`
$\quad(\!|s(x + 4y,\ x,\ y)|\!)$	`implied (3)`
$\quad$`m = x + 4 * y;`	
$\quad(\!|s(m,\ x,\ y)|\!)$	`assignment`
$(\!|s(m,\ x,\ y)|\!)$	`if-then-else`

Proof of 1: $x > y \Rightarrow s(x + 2y, x, y)$
I.e., $x > y$

$$\Rightarrow$$

$$(x > y \wedge x + 2y = x + 2y) \vee (x < y \wedge x + 2y = x + 3y) \vee (x = y \wedge x + 2y = x + 4y)$$

1.	$x > y$	assumption
2.	$x + 2y = x + 2y$	$=$_I
3.	$x > y \wedge x + 2y = x + 2y$	$1, 2, \wedge$_I
4.	$(x > y \wedge x + 2y = x + 2y) \vee$	
	$(x < y \wedge x + 2y = x + 3y) \vee$	
	$(x = y \wedge x + 2y = x + 4y)$	$3, \vee$_I
5.	(line 1) $\Rightarrow$ (line 4)	$1 - 4, \Rightarrow$_I

Proof of 2: $\neg(x > y) \wedge x < y \Rightarrow s(x + 3y, x, y)$
I.e., $\neg(x > y) \wedge x < y$

$$\Rightarrow$$

$$(x > y \wedge x + 3y = x + 2y) \vee (x < y \wedge x + 3y = x + 3y) \vee (x = y \wedge x + 3y = x + 4y)$$

1.	$\neg(x > y) \wedge x < y$	assumption
2.	$x < y$	$1, \wedge$_E
3.	$x + 3y = x + 3y$	$=$_I
4.	$x < y \wedge x + 3y = x + 3y$	$2, 3, \wedge$_I
5.	$(x > y \wedge x + 3y = x + 2y) \vee$	
	$(x < y \wedge x + 3y = x + 3y) \vee$	
	$(x = y \wedge x + 3y = x + 4y)$	$4, \vee$_I
6.	(line 1) $\Rightarrow$ (line 5)	$1 - 5, \Rightarrow$_I

Proof of 3: $\neg(x > y) \wedge \neg(x < y) \Rightarrow s(x + 4y, x, y)$
I.e., $\neg(x > y) \wedge \neg(x < y)$

$$\Rightarrow$$

$$(x > y \wedge x + 4y = x + 2y) \vee (x < y \wedge x + 4y = x + 3y) \vee (x = y \wedge x + 4y = x + 4y)$$

1.	$\neg(x > y) \wedge \neg(x < y)$	assumption
2.	$x = y$	$1,$ algebra
3.	$x + 4y = x + 4y$	$=$_I
4.	$x = y \wedge x + 4y = x + 4y$	$2, 3, \wedge$_I
5.	$(x > y \wedge x + 4y = x + 2y) \vee$	
	$(x < y \wedge x + 4y = x + 3y) \vee$	
	$(x = y \wedge x + 4y = x + 4y)$	$3, \vee$_I
6.	(line 1) $\Rightarrow$ (line 5)	$1 - 5, \Rightarrow$_I