# CS240 Assignment 3

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## Problem 1

### (a)

$c_{opt} \leq 2c_{opt} = 2\sum_{i=1}^{n} ip_i = \frac{2}{n}\sum_{i=1}^{n} i = \frac{2}{n}\frac{n(n+1)}{2} = n+1 \in \Theta(n)$

### (b)

$c_{mtf} \leq 2c_{opt} = 2\sum_{i=1}^{n} ip_i \leq 2\sum_{i=1}^{\infty} ip_i = 2\sum_{i=1}^{\infty} i\frac{1}{2^i} = 4 \in \Theta(1)$

### (c)

$c_{mtf} \leq 2c_{opt} = 2\sum_{i=1}^{n} i\frac{1}{iH_{1,n}} = \frac{2}{H_{1,n}}\sum_{i=1}^{n} 1 = \frac{2n}{H_{1,n}}$
$H_{1,n} = \sum_{j=1}^{n}\frac{1}{j} = 1 + \sum_{j=2}^{n}\frac{1}{j}$
$1 + \int_2^{n+1}\frac{dx}{x} < H_{1,n} < 1 + \int_1^n \frac{dx}{x}$
$1 + [\ln x]_2^{n+1} < H_{1,n} < 1 + [\ln x]_1^n$
$1 - \ln 2 + \ln(n+1) < H_{1,n} < 1 + \ln n$
$=> H_{1,n} \in \Theta(\ln n)$
$=> c_{mtf} \in \Theta(\frac{n}{\ln n})$

### (d)

$c_{mtf} \leq 2c_{opt} = 2\sum_{i=1}^{n} i\frac{1}{i^2 H_{2,n}} = \frac{2}{H_{2,n}}\sum_{i=1}^{n}\frac{1}{i} = \frac{2}{H_{2,n}}H_{1,n}$
$H_{2,n} = \sum_{j=1}^{n}\frac{1}{j^2} = 1 + \sum_{j=2}^{n}\frac{1}{j^2}$
$1 + \int_2^{n+1}\frac{dx}{x^2} < H_{2,n} < 1 + \int_1^n \frac{dx}{x^2}$
$1 + [\frac{-1}{x}]_2^{n+1} < H_{2,n} < 1 + [\frac{-1}{x}]_1^n$
$1 + \frac{1}{2} - \frac{1}{n+1} < H_{2,n} < 1 - \frac{1}{n} + 1$
$\frac{3}{2} - \frac{1}{n+1} < H_{2,n} < 2 - \frac{1}{n}$
$=> H_{2,n} \in \Theta(1)$
$=> c_{mtf} \in \Theta(\ln n)$

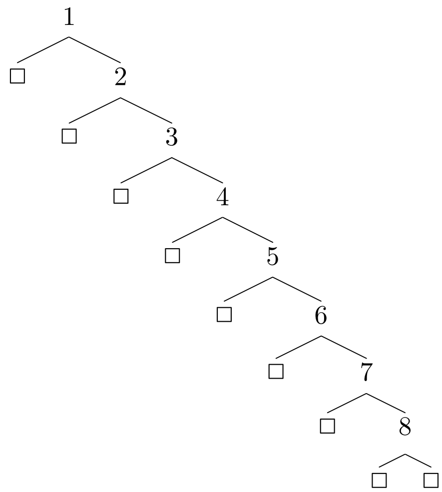
## Problem 2

### (a)

The following sequence of insertions will produce the tree in Fig 4 from an empty one:
8,7,6,5,4,3,2,1

**(b)**

1.

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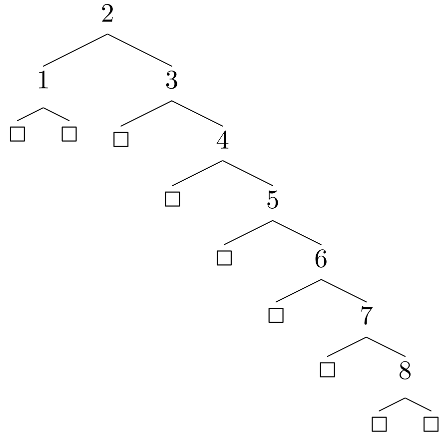


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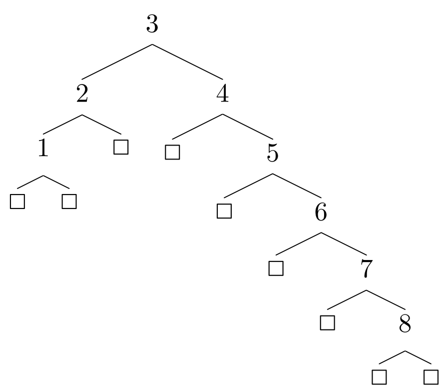

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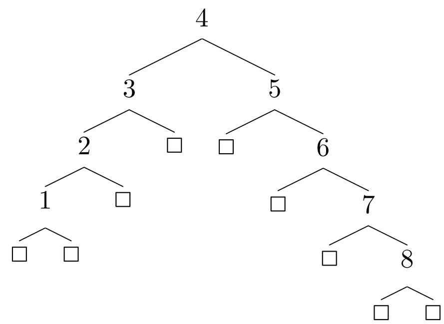


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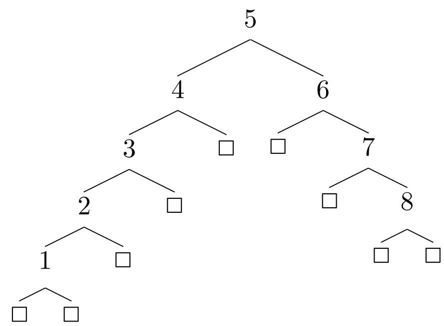


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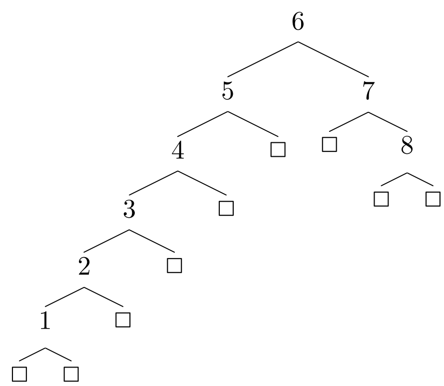


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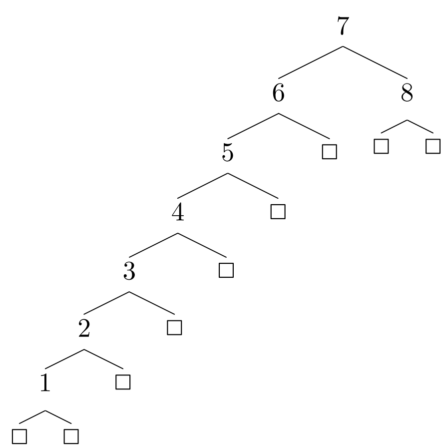

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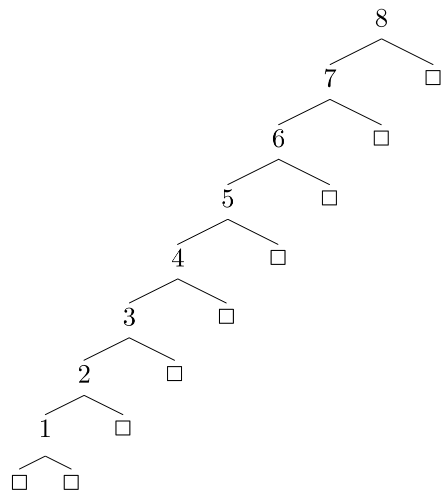

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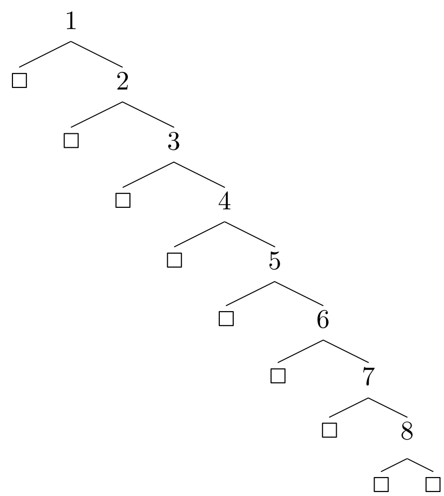

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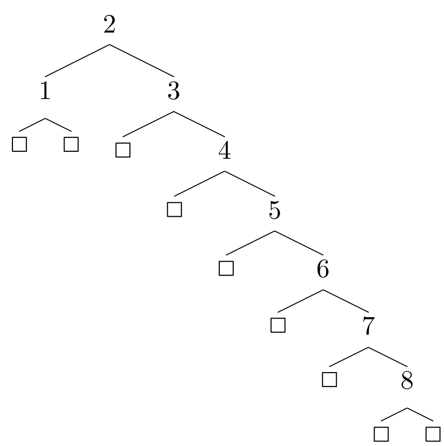

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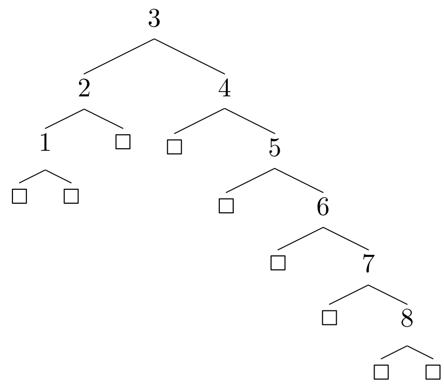


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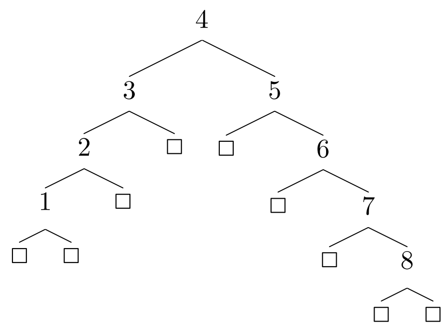


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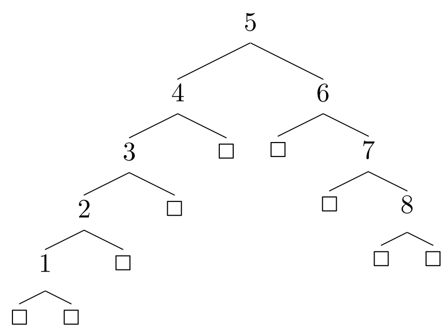
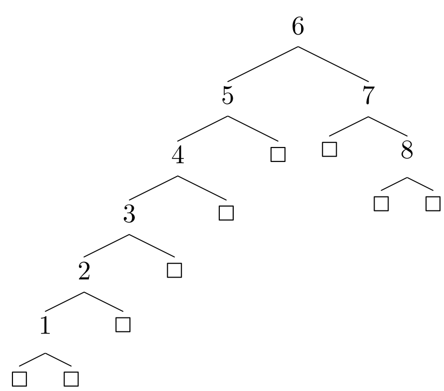



14.



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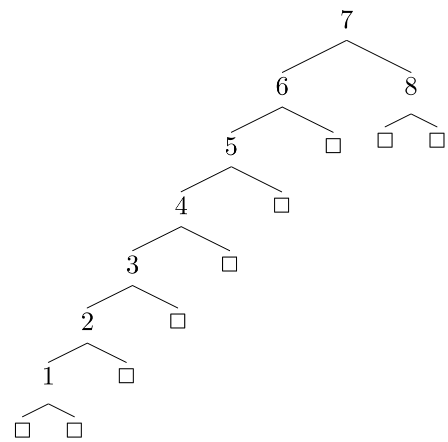
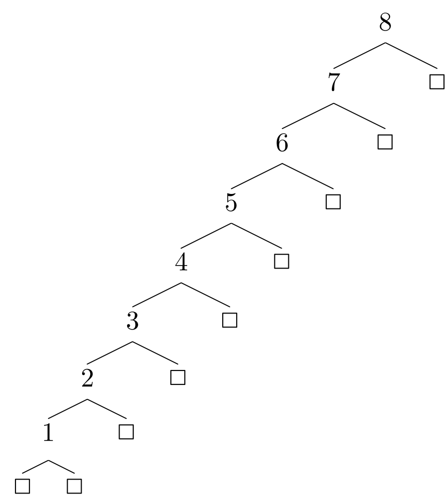
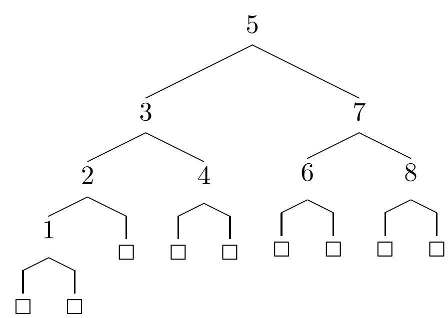



16.




## (c)




## (d)

The splay tree performs worse than the static binary search tree on the sequence of searches given. In the following sequence of 5 searches, the splay tree performs better:

1,1,1,1,1

# Problem 3

## (a)

$P(H = h) = \frac{2}{3}(\frac{1}{3})^{h-1}$
so $E(h) = \sum_{h=1}^{\infty} h\frac{2}{3}(\frac{1}{3})^{h-1} = \sum_{h=1}^{\infty} \frac{2h}{3^h} = (2)(\frac{3}{4}) = \frac{3}{2}$

## (b)

Although h decreases when t increases, the cost of the search does not go down because the path is essentially the same, except we have to go across more.

# Problem 4

The code submitted is an exact implementation of the algorithm in the slides, which works even on lists with duplicate elements (although not efficiently).