CS 245 — Assignment #4
Spring 2006

Due Date: Tuesday, June 6 at 5pm.

Use `makeCover` to produce a cover page for your assignment and hand in your assignment in the CS 245 assignment box. Assignments are to be done individually.

1. (15 points) Consider the following murder mystery.

   (a) Someone who lives at Wisteria Lodge murdered Aunt Agatha.
   (b) Aunt Agatha, Beatrice, and Charles live at Wisteria Lodge and nobody else lives there.
   (c) Beatrice is the only person Aunt Agatha doesn't hate.
   (d) Aunt Agatha hates no-one that Charles hates.
   (e) Beatrice hates everyone unless that person is richer than Aunt Agatha.
   (f) Beatrice hates everyone Aunt Agatha hates.
   (g) Aunt Agatha and Beatrice are not the same person.
   (h) Everyone has someone that they don't hate.
   (i) A murder victim is always hated by their murderer.
   (j) A murderer is never richer than his victim.

   Express each of the clues above in predicate logic. Do not use types in your formalization. Be sure to give the intended meaning of each of the constants and predicates that you use (see the next question for an example of what to specify). In the next assignment, we will prove who murdered Aunt Agatha.

   Intended meaning of constants and predicates.

   Constants:

   Agatha, Beatrice, Charles – intended meaning is clear

   Predicates:

murdered(x, y)	is true if and only if x murdered y
hates(x, y)	is true if and only if x hates y
lodge(x)	is true if and only if x lives at Wisteria Lodge
richer(x, y)	is true if and only if x is richer than y

Formalization of the clues in predicate logic.

(a) $\exists x \bullet$ lodge$(x) \land$ murdered$(x$, Agatha$)$

(b) lodge(Agatha) $\land$ lodge(Beatrice) $\land$ lodge(Charles) $\land$
$\forall x \bullet$ lodge$(x) \Rightarrow (x =$ Agatha $\lor\ x =$ Beatrice $\lor\ x =$ Charles$)$

(c) $\forall x \bullet \neg$hates(Agatha, $x) \Rightarrow (x =$ Beatrice$)$

(d) $\forall x \bullet$ hates(Charles, $x) \Rightarrow \neg$hates(Agatha, x)

(e) $\forall x \bullet$ hates(Beatrice, $x) \lor$ richer$(x$, Agatha$)$

(f) $\forall x \bullet$ hates(Agatha, $x) \Rightarrow$ hates(Beatrice, x)

(g) $\neg$(Agatha $=$ Beatrice)

(h) $\forall x \bullet \exists y \bullet \neg$hates$(x, y)$

(i) $\forall x \bullet \forall y \bullet$ murdered$(x, y) \Rightarrow$ hates(x, y)

(j) $\forall x \bullet \forall y \bullet$ murdered$(x, y) \Rightarrow \neg$richer(x, y)

2. (10 points) Consider a world with five toy wooden blocks. Suppose that we use the constants A, B, C, D, E to denote the five blocks and the predicates clear, green, on, and onTable with the following intended meanings:

clear(x)	is true iff x is clear (has no block on it)
green(x)	is true iff x is the color green
on(x, y)	is true iff x is resting on y
onTable(x)	is true iff x is resting on the table

Suppose there is a blocks world scene of which we do not know the entire configuration but only a part of it, and we describe it as,

1. on(A, B)
2. on(B, C)
3. on(D, E)
4. onTable(C)
5. clear(D)
6. green(A)
7. $\neg$green(C)
8. $\forall x \bullet \neg$on(x, x)
9. $\forall x \bullet \forall y \bullet$ on$(x, y) \Rightarrow \neg$clear(y)
10. $\forall x \bullet \forall y \bullet$ on$(x, y) \Rightarrow \neg$onTable(x)

Let (1)–(10) above be the set of premises. Consider the four possible conclusions (a)–(d) below. For one of the four possible conclusions, you may simply state that the formula logically follows from the premises. For the remaining three possible conclusions, show that the argument is invalid by providing a counter example (i.e., show that the conclusion does not logically follow from the premises by giving an interpretation in which the premises are satisfied but the conclusion is not satisfied).

(a) Is block A not clear?
I.e., $\neg$clear(A)

(b) Is block A clear?
I.e., clear(A)

(c) Is it the case that all of the blocks on the table are not green?
I.e., $\forall x \bullet \text{onTable}(x) \Rightarrow \neg\text{green}(x)$

(d) Is there a block that is not green that has a block on it that is green?
I.e., $\exists x \bullet \exists y \bullet \neg\text{green}(x) \wedge \text{on}(y, x) \wedge \text{green}(y)$

You may specify an interpretation by listing only the ground predicates that are true in your interpretation, and then stating that everything else is false. In addition to specifying an interpretation by specifying what is true and what is false, include a drawing or picture of the blocks world scene that illustrates your interpretation.

I will begin by giving two interpretations that satisfy the premises.

Interpretation 1. Formally, the interpretation is given by,

> The following are true:
> on(A, B), on(B, C), on(D, E),
> clear(A), clear(D),
> onTable(C), onTable(E),
> green(A), green(E).
> Everything else is false.

We can verify that each of the premises is satisfied. A picture of the interpretation is given on the last page.

Interpretation 2. Formally, the interpretation is given by,

> The following are true:
> on(D, E), on(E, A), on(A, B), on(B, C),
> clear(D),
> onTable(C),
> green(A), green(E).
> Everything else is false.

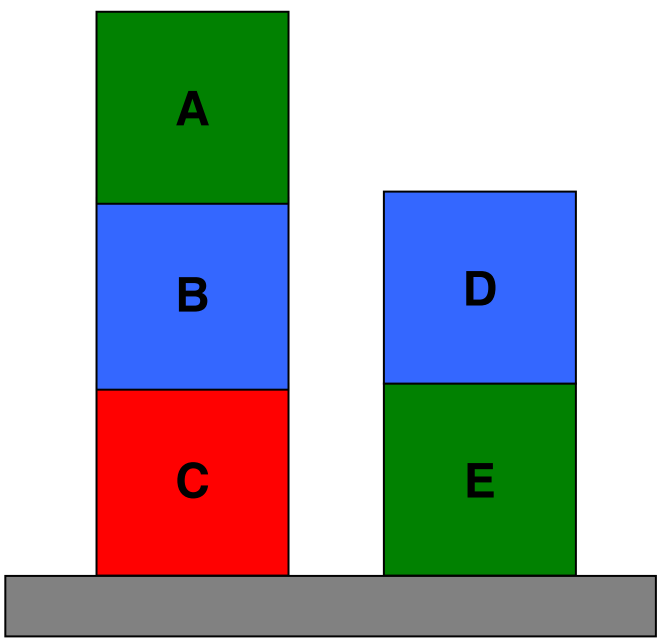
We can verify that each of the premises is satisfied. A picture of the interpretation is given on the last page.

(a) $\neg$clear(A) is not a logical consequence of the premises as Interpretation 1 (above) is a counter example. All of the premises are true, but the conclusion $\neg$clear(A) is false.

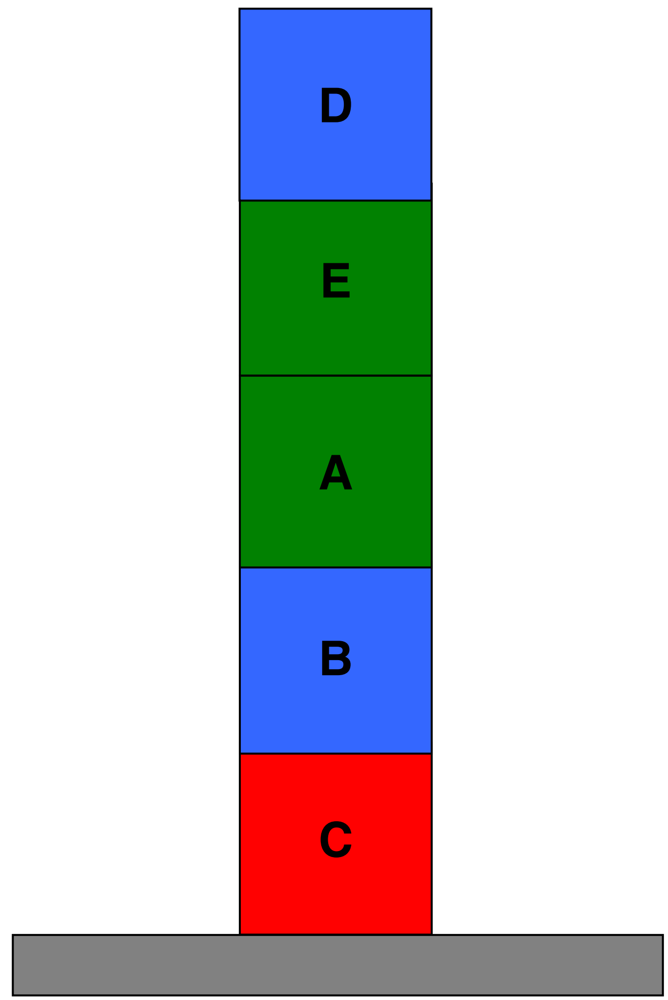
(b) clear(A) is not a logical consequence of the premises as Interpretation 2 (above) is a counter example. All of the premises are true, but the conclusion clear(A) is false.

(c) $\forall x \bullet \text{onTable}(x) \Rightarrow \neg\text{green}(x)$ is not a logical consequence of the premises as Interpretation 1 (above) is a counter example. All of the premises are true, but the conclusion is false as not all of the blocks that are on the table are not green. In particular, E is a block that is on the table and is green.

(d) $\exists x \bullet \exists y \bullet \neg\text{green}(x) \wedge \text{on}(y, x) \wedge \text{green}(y)$ is a logical consequence.



(Interpretation 1)



(Interpretation 2)