CS245 Assignment 9

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Question 1

Loop Invariant:
$\{z = i^2\}$

$\{n \geq 1\}$
$\{1 = 1^2\}$ Implied (obvious)
$i = 1$;
$\{1 = i^2\}$ Assignment
$z = 1$;
$\{z = i^2\}$ Assignment
while (i != n) {
    $\{z = i^2 \wedge i \neq n\}$ While
    $\{z + (2(i+1) - 1) = (i+1)^2\}$ Implied (a)
    i = i + 1;
    $\{z + (2i - 1) = i^2\}$ Assignment
    z = z + (2*i - 1);
    $\{z = i^2\}$ Assignment
}
$\{(z = i^2) \wedge (i = n)\}$ While
$\{z = n^2\}$ Implied (obvious)

Implied (a)

1	$(z = i^2) \wedge (i \neq n)$	Assumption
2	$z = i^2$	1,$\wedge$_E
3	$z + (2(i+1) - 1) = z + (2(i+1) - 1)$	=_I
4	$z + (2(i+1) - 1) = z + 2i + 1$	3,Algebra
5	$z + (2(i+1) - 1) = i^2 + 2i + 1$	2,4,=_E
6	$z + (2(i+1) - 1) = (i+1)^2$	5,Algebra
7	$Line1 \Rightarrow Line6$	1-6,$\Rightarrow$_I

Question 2

Loop Invariant:
$\{(\forall k.\ 1 \leq k \leq i-1 \Rightarrow max \geq A[k])$

Case 1

Case: $e_2 \geq e_1$
$\{n \geq 2\}$
$\{(\forall k.\ 1 \leq k \leq 1 \Rightarrow A[1] \geq A[k]) \wedge (2 \leq n)\}$ Implied (a)
max = A[1];
$\{(\forall k.\ 1 \leq k \leq 1 \Rightarrow max \geq A[k]) \wedge (2 \leq n)\}$ Assignment
for i = 2 to n {
  $\{(\forall k.\ 1 \leq k \leq i-1 \Rightarrow max \geq A[k]) \wedge (2 \leq i) \wedge (i \leq n)\}$ For-Loop
  $\{(\forall k.\ 1 \leq k \leq i-1 \Rightarrow max \geq A[k]) \wedge (2 \leq i) \wedge (i \leq n)\}$ Implied (obvious)
  if (max < A[i]) {
    $\{(\forall k.\ 1 \leq k \leq i-1 \Rightarrow max \geq A[k]) \wedge (2 \leq i) \wedge (i \leq n) \wedge (max < A[i])\}$ If-Then
    $\{(\forall k.\ 1 \leq k \leq i \Rightarrow A[i] \geq A[k]\}$ Implied (b)
    max = A[i];
    $\{(\forall k.\ 1 \leq k \leq i \Rightarrow max \geq A[k]\}$ Assignment
  }
  $\{\forall k.\ 1 \leq k \leq i \Rightarrow max \geq A[k]\}$ If-Then
  $\{\forall k.\ 1 \leq k \leq i \Rightarrow max \geq A[k]\}$ Implied (obvious)
}
$\{(\forall k.\ 1 \leq k \leq i-1 \Rightarrow max \geq A[k]) \wedge (i = n+1)\}$ For-Loop
$\{(\forall k.\ 1 \leq k \leq n \Rightarrow max \geq A[k])\}$ Implied (obvious)

Implied (a)

We will first show that:
$(\forall k.\ 1 \leq k \leq 1 \Rightarrow A[1] \geq A[k])$ is a tautology.

1	$\forall k.\ 1 \leq k \leq 1$	Assumption
2	$k = 1$	1, Algebra
3	$A[1] = A[1]$	$=$_I
4	$A[1] \geq A[1]$	3, Algebra
5	$A[1] \geq A[k]$	2,4, $=$_E
6	$\forall k.\ 1 \leq k \leq 1 \Rightarrow A[1] \geq A[k]$	1-5 $\Rightarrow$_I

$(n \geq 2) \Rightarrow ((\forall k.\ 1 \leq k \leq 1 \Rightarrow A[1] \geq A[k]) \wedge (2 \leq n))$

1	$\forall k.\ 1 \leq k \leq 1 \Rightarrow A[1] \geq A[k]$	Premise (proven above)
2	$n \geq 2$	Assumption
3	$(\forall k.\ 1 \leq k \leq 1 \Rightarrow A[1] \geq A[k]) \wedge (n \geq 2)$	1,2,$\wedge$_I
4	$(n \geq 2) \Rightarrow ((\forall k.\ 1 \leq k \leq 1 \Rightarrow A[1] \geq A[k]) \wedge (n \geq 2))$	2-3,$\Rightarrow$_I

Implied (b)

$((\forall k.\ \ 1 \leq k \leq i-1 \Rightarrow max \geq A[k]) \wedge (2 \leq i) \wedge (i \leq n) \wedge (max < A[i])) \Rightarrow (\forall k.\ \ 1 \leq k \leq i \Rightarrow A[i] \geq A[k])$

1	$(\forall k.\ \ 1 \leq k \leq i-1 \Rightarrow max \geq A[k]) \wedge (2 \leq i) \wedge (i \leq n) \wedge (max < A[i])$	Assumption
2	$\forall k.\ \ 1 \leq k \leq i-1 \Rightarrow max \geq A[k]$	1, $\wedge$_E
3	$max < A[i]$	1,$\wedge$_E
4	$A[i] \geq max$	3,Algebra
5	$A[i] \geq A[i]$	Algebra
6	$\forall k.\ \ 1 \leq k \leq i-1 \Rightarrow A[i] \geq A[k]$	2,4,=_E
7	$\forall k.\ \ 1 \leq k \leq i \Rightarrow A[i] \geq A[k]$	5,6,Algebra
8	$Line1 \Rightarrow Line7$	1-7,$\Rightarrow$_I

Case 2

Case: $e_1 > e_2$
$\{n = 1\}$
$\{(\forall k.\ \ 1 \leq k \leq 1 \Rightarrow A[1] \geq A[k]) \wedge (n = 1)\}$ Implied (a)
max = A[1];
$\{(\forall k.\ \ 1 \leq k \leq 1 \Rightarrow max \geq A[k]) \wedge (n = 1)\}$ Assignment
for i = 2 to n {
    if (max < A[i]) {
        max = A[i];
    }
}
$\{(\forall k.\ \ 1 \leq k \leq i-1 \Rightarrow max \geq A[k]) \wedge (i = n+1)\}$ For-Loop
$\{(\forall k.\ \ 1 \leq k \leq n \Rightarrow max \geq A[k])\}$ Implied (obvious)

Implied (a)

We will first show that:
$(\forall k.\ \ 1 \leq k \leq 1 \Rightarrow A[1] \geq A[k])$ is a tautology.

1	$\forall k.\ \ 1 \leq k \leq 1$	Assumption
2	$k = 1$	1, Algebra
3	$A[1] = A[1]$	=_I
4	$A[1] \geq A[1]$	3, Algebra
5	$A[1] \geq A[k]$	2,4, =_E
6	$\forall k.\ \ 1 \leq k \leq 1 \Rightarrow A[1] \geq A[k]$	1-5 $\Rightarrow$_I

$\{(n = 1) \Rightarrow ((\forall k.\ \ 1 \leq k \leq 1 \Rightarrow A[1] \geq A[k]) \wedge (n = 1))$

1	$(n = 1)$	Assumption
2	$\forall k.\ \ 1 \leq k \leq 1 \Rightarrow A[1] \geq A[k]$	Premise (proven above)
3	$(\forall k.\ \ 1 \leq k \leq 1 \Rightarrow A[1] \geq A[k]) \wedge (n = 1)$	1,2,$\wedge$_I
4	$Line1 \Rightarrow Line3$	1-3,$\Rightarrow$_I