CS 245 Midterm Exam

Spring 2005, Instructor: P. van Beek
Thursday, June 9, 4:30pm-6:30pm

INSTRUCTIONS

- No books, no notes, no computers, no calculators.
- Time limit: 2 hours
- Write the exam that has your name on it.
- Explain your answers.
- This exam has 14 pages (including this cover page). Check that your exam is complete.

Question	Mark	Max	Marker
1		10	
2		10	
3		10	
4		10	
5		10	
6		10	
7		10	
8		10	
9		10	
10		10	
Total		100	

1 (10 Marks) Short Answer

Explain your answers.

(a) What does $p_1, p_2, \ldots, p_n \vdash q$ mean?

(b) Can we prove that a propositional logic formula p is a tautology using a transformational proof? If not, why not? If yes, what would we prove about formula p?

(c) If a formula p is consistent, is it possible for $\neg p$ to be a contradiction?

2 (10 Marks) Propositional Logic: Formalization

Formalize the following sentences in propositional logic. Show the English phrase that each prime proposition represents.

(a) If the waves are high, I will go to the beach.

(b) While the waves are high, I will go surfing.

(c) I will go swimming even though the waves are high.

(d) I will make sand castles only if I don't go swimming or diving.

(e) I won't go swimming unless I remember to bring my towel and bathing suit.

3 (10 Marks) Propositional Logic: Logical Implication

Does the premise logically imply the conclusion? Explain your answer.

$$\neg(\neg c \Rightarrow b) \lor \neg a \quad \models \quad \neg a \lor b$$

4 (10 Marks) Propositional Logic: Transformational Proof

Prove the following using transformational proof:

$$\neg((a \Rightarrow \neg(c \Rightarrow d)) \land (a \Rightarrow (d \lor \neg c))) \quad \Longleftrightarrow \quad a$$

$\Longleftrightarrow$

$\Longleftrightarrow$

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5 (10 Marks) Propositional Logic: Natural deduction

Answer this question by giving a natural deduction proof. Do not use any logical laws from transformational proof in your natural deduction proof.

Given $p \Rightarrow (q \lor r),\ q \Rightarrow s,\ \neg(r \lor s)$, show $\neg p$ using natural deduction.

6 (10 Marks) Propositional Logic: Natural Deduction

Answer this question by giving a natural deduction proof. Do not use any logical laws from transformational proof in your natural deduction proof.

Given $(p \Rightarrow q) \vee (\neg q \Rightarrow r)$ show $p \Rightarrow (q \vee r)$ using natural deduction.

7 (10 Marks) Propositional Logic: Semantic Tableaux

Using a semantic tableaux, show whether the following set of formulas is consistent. Explain. Do not use any logical laws from transformational proof in your semantic tableaux.

$$p \Rightarrow q,\ p \Rightarrow r,\ \neg((\neg q \vee \neg r) \Rightarrow r)$$

8 (10 Marks) Propositional Logic: Semantic Tableaux

Answer this question by giving a semantic tableaux proof. Do not use any logical laws from transformational proof in your semantic tableaux proof.

Given $p \lor q,\ r \lor s$, show $(\neg p \Rightarrow q) \land (\neg r \Rightarrow s)$ using semantic tableaux.

9 (10 Marks) Predicate Logic: Formalization

Formalize the following sentences in predicate logic. Be sure to give the intended meaning of each of the constants and predicates that you use. Do not use types in your answer.

(a) All criminals are disliked by everyone.

(b) Only criminals are disliked by everyone.

(c) Criminals are disliked by everyone except their mothers.

10 (10 Marks) Predicate Logic: Interpretations

Consider the following sentences in predicate logic.

(i) $\exists x \bullet \exists y \bullet p(x) \land p(y) \land (x \neq y)$

(ii) $\forall x \bullet \forall y \bullet (p(x) \land p(y) \land (x \neq y)) \Rightarrow r(x, y)$

(iii) $\exists x \bullet r(x, x)$

Show that (iii) does not logically follow from (i) and (ii) by providing a counter-example. Carefully explain why your counter-example really is a counter-example. (Note: your answer must include an *interpretation* in order to receive any marks.)

Propositional Logic
Summary of Logical Laws for Transformational Proofs

Commutativity

$$p \wedge q \iff q \wedge p$$
$$p \vee q \iff q \vee p$$
$$p \Leftrightarrow q \iff q \Leftrightarrow p$$

Associativity

$$p \wedge (q \wedge r) \iff (p \wedge q) \wedge r$$
$$p \vee (q \vee r) \iff (p \vee q) \vee r$$

Distributivity

$$p \vee (q \wedge r) \iff (p \vee q) \wedge (p \vee r)$$
$$p \wedge (q \vee r) \iff (p \wedge q) \vee (p \wedge r)$$

De Morgan

$$\neg(p \wedge q) \iff \neg p \vee \neg q$$
$$\neg(p \vee q) \iff \neg p \wedge \neg q$$

Negation

$$\neg(\neg p) \iff p$$

Excluded Middle

$$p \vee \neg p \iff \mathbf{true}$$

Contradiction

$$p \wedge \neg p \iff \mathbf{false}$$

Implication

$$p \Rightarrow q \iff \neg p \vee q$$

Contrapositive

$$p \Rightarrow q \iff \neg q \Rightarrow \neg p$$

Equivalence

$$p \Leftrightarrow q \iff (p \Rightarrow q) \wedge (q \Rightarrow p)$$

Idempotence

$$p \vee p \iff p$$
$$p \wedge p \iff p$$

Simplification I

$$p \wedge \mathbf{true} \iff p$$
$$p \vee \mathbf{true} \iff \mathbf{true}$$
$$p \wedge \mathbf{false} \iff \mathbf{false}$$
$$p \vee \mathbf{false} \iff p$$

Simplification II

$$p \vee (p \wedge q) \iff p$$
$$p \wedge (p \vee q) \iff p$$

Propositional Logic
Summary of Inference Rules for Natural Deduction

Conjunction: ∧_I

$$\frac{\begin{array}{l} p \\ q \end{array}}{p \wedge q}$$

Simplification: ∧_E

$$\frac{p \wedge q}{p} \qquad \frac{p \wedge q}{q}$$

Addition: ∨_I

$$\frac{p}{p \vee q} \qquad \frac{q}{p \vee q}$$

Disjunctive syllogism: ∨_E

$$\frac{\begin{array}{l} p \vee q \\ \neg p \end{array}}{q} \qquad \frac{\begin{array}{l} p \vee q \\ \neg q \end{array}}{p}$$

Modus ponens: ⇒ _E

$$\frac{\begin{array}{l} p \Rightarrow q \\ p \end{array}}{q}$$

Modus tollens: ⇒ _E

$$\frac{\begin{array}{l} p \Rightarrow q \\ \neg q \end{array}}{\neg p}$$

Contradiction: ¬_E

$$\frac{\begin{array}{l} p \\ \neg p \end{array}}{q}$$

Double negation: ¬_E

$$\frac{\neg\neg p}{p}$$

Transitivity of equivalence: ⇔ _I

$$\frac{\begin{array}{l} p \Leftrightarrow q \\ q \Leftrightarrow r \end{array}}{p \Leftrightarrow r}$$

Laws of equivalence: ⇔ _E

$$\frac{p \Leftrightarrow q}{p \Rightarrow q}$$

$$\frac{p \Leftrightarrow q}{q \Rightarrow p}$$

Deduction theorem: ⇒ _I

$$\frac{\left[\begin{array}{l} r \\ \vdots \\ q \end{array}\right.}{r \Rightarrow q}$$

Reductio ad absurdum: ¬_I

$$\frac{\left[\begin{array}{c} r \\ \vdots \\ \textbf{false} \end{array}\right.}{\neg r}$$

Propositional Logic
Summary of Rules for Semantic Tableaux

AND

$$p \wedge q$$
$$p$$
$$q$$

NOT-AND

$$\neg(p \wedge q)$$
$$\neg p \qquad\qquad \neg q$$

OR

$$p \vee q$$
$$p \qquad\qquad q$$

NOT-OR

$$\neg(p \vee q)$$
$$\neg p$$
$$\neg q$$

IMPLIES

$$p \Rightarrow q$$
$$\neg p \qquad\qquad q$$

NOT-IMPLIES

$$\neg(p \Rightarrow q)$$
$$p$$
$$\neg q$$

NOT-NOT

$$\neg\neg p$$
$$p$$

IFF

$$p \Leftrightarrow q$$
$$p \wedge q \qquad\qquad \neg p \wedge \neg q$$

NOT-IFF

$$\neg(p \Leftrightarrow q)$$
$$p \wedge \neg q \qquad\qquad \neg p \wedge q$$