Applications: Pattern Matching algorithms

CS240: Data Structures and Data Management
Slide Set 16

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Outline

Pattern Matching

- Search for a string in a large body of text
- T – The text being searched within
- P – The pattern being searched for
- Applications:
  - Information Retrieval
  - Compression
  - Data-Mining, classification

Definitions

- Σ – The alphabet
- Often T is written as $T[0..n-1]$
- Often P is written as $P[0..m-1]$
- Return first i such that

$$P[j] = T[i+j] \qquad \text{for } 0 \leq j \leq m-1$$

- Return -1 if no such i exists
  - Define T_i as $T[i..(i+m-1)]$
  - Trying to find a $T_i = P$
- Example:
  - $P =$ `Waldo`
  - $T =$ `Where's Waldo in the Land of Giants?`

Naive Algorithm

- Brute-Force

$Naive(P[0..m-1], T[0..n-1])$

for $i \leftarrow 0$ *to* $n - m$ **do**
  for $j \leftarrow 0$ *to* $m - 1$ **do**
    if $T[i + j] = P[j]$ **then**
      if $j = m - 1$ **then**
        return i
      end if
    else
      break *out of inner (j) loop*
    end if
  end for
end for
return -1

Example

- Example: $P =$ `abba`
- We will only ever show the explicit character comparisons

a	b	b	b	a	b	a	b	b	a	b
a	b	b	a							
	a									
		a								
			a							
				a	b	b				
					a					
						a	b	b	a	b

Match at position $i = 6$.

- What is the worst possible input?
  $P = a^{m-1}b$, $T = a^n$
- Worst case performance
  $(n - m + 1)m \in O(nm)$,
  so $O(n^2)$ if $m = \frac{n}{2}$.

Outline

Rabin-Karp

Less iterations of inner loop in the naive algorithm, through quick hashing test to eliminate some candidates.

- Compute pattern's fingerprint $h(P)$
- For each i compute $h(T[i, \ldots, i-1+m])$:
  1. if $h(T[i, \ldots, i-1+m]) = h(P)$ then check each position
  2. $h(T[i, \ldots, i-1+m]) \neq h(P)$ then move to next position
- $h(T[i, \ldots, i-1+m])$ need to be computed quickly.

Example

$\Sigma = \{0, \ldots, 9\}$ and $h(S) =$ sum of the digits in S.

$$\begin{aligned} h(T[i, \ldots, i-1+m]) &= T[i] + T[i+1] + \ldots + T[i+m-1] \\ &= h(T[i-1, \ldots, i-2+m]) \\ &\quad - T[i-1] + T[i-1+m] \end{aligned}$$

Example

$P = 1991$
$h(P) = 20$

	3	8	5	6	8	1	1	9	9	2	1	9	9	1
h	22	27	**20**	16	19	**20**	21	21	21	21	**20**			
			1											
						1	9							
											1	9	9	1

- Time spent computing hash values: $O(m + n)$
- Time spent comparing characters, assuming k collisions:
- Worst case performance: $O(n + km)$
- Typical results: $O(n)$ with $O(\frac{n}{m})$ hits

Better Signatures

- Use polynomial hashing function (slight variation)
- c_i is the numeric value of the i'th character

$$h(P) = \sum_{i=0}^{m-1} c_i \cdot r^{m-1-i}$$

- Note that r should be greater than the maximum c_i
- Computing $h(T_i)$ given $h(T_{i-1})$:

$$h(T_i) = \left(h(T_{i-1}) \pmod{r^{m-1}}\right) \cdot r + T[i+m-1]$$

- Generally work modulo a large prime

Improvements

- Do not just work entirely with character arrays
- Preprocess either P or T, and build a new data structure for P or T and then search
- Example: Multiple searches over a fixed body of text: Preprocess T once, and use for all future searches

Outline

Knuth-Morris-Pratt

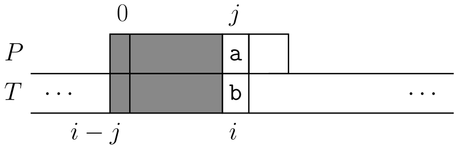
- When naive mismatches, we advance the string by one: can we skip ahead more than one character?
- Suppose $P =$ `abcdcaba` and mismatch on the last character of P:

$T =$	a	b	c	d	c	a	b	c	?	?	?
	a	b	c	d	c	a	b	a			
						a	b	c	d	c	a

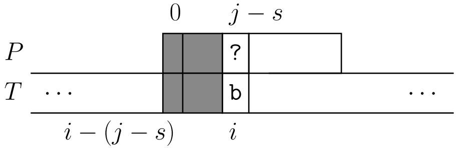
- How far can we safely move ahead, reusing knowledge from previous matches.

Matching Algorithm

- Keep an index into each string
  - i is an index in T while j is an index in P
  - The first character of T's substring is always at $i - j$
- If we mismatch:



- Shift P ahead s places, and retest at the same spot



- **Note**: decreasing j has the effect of shifting P

KMP Failure Function

- What is the correct shift value?
- For $0 \leq x \leq m-1$ we define failure function as:

$$f(x) \begin{cases} 0 & \text{if } x = 0 \\ \text{length of longest prefix of } P & \\ \text{that is a suffix of } P[1..x] & \text{if } x > 0 \end{cases}$$

- Consider $P =$ `abacaba`

x	$P[1..x]$	P	$f(x)$
0	—	abacaba	0
1	b	abacaba	0
2	ba	abacaba	1
3	bac	abacaba	0
4	baca	abacaba	1
5	bacab	abacaba	2
6	bacaba	abacaba	3

Main Algorithm

$KMP(P[0..m-1], T[0..n-1])$

$f \leftarrow$ KMPFailure(P)

$i \leftarrow 0$

$j \leftarrow 0$

while ($i < n$) **do**

  if $T[i] = P[j]$ **then**

    if $(j = m-1)$ **then**

      return $i - j$

    end if

    $i \leftarrow i + 1$

    $j \leftarrow j + 1$

  else if ($j > 0$) **then**

    $j \leftarrow f[j-1]$

  else

    $i \leftarrow i + 1$

  end if

end while

return -1

Example

$P = \texttt{abacaba}$

$T = \underline{\texttt{abaxyabacabb}}\texttt{aababacaba}$

0	1	2	3	4	5	6	7	8	9	10	11
a	b	a	x	y	a	b	a	c	a	b	b
a	b	a	c								
		(a)	b								
			a								
				a							
					a	b	a	c	a	b	a
									(a)	(b)	a

Exercise: continue with $T = \texttt{abaxyabacabb}\underline{\texttt{aababacaba}}$

Computing Failure Function

KMPFailure$(P[0..m-1])$

$f[0] \leftarrow 0$
$i \leftarrow 1$
$j \leftarrow 0$
while $(\ i < m\ )$ **do**
  if $(\ P[i] = P[j]\ )$ **then**
    $f[i] \leftarrow j + 1$
    $i \leftarrow i + 1$
    $j \leftarrow j + 1$
  else if $(\ j > 0\ )$ **then**
    $j \leftarrow f[j-1]$
  else
    $f[i] \leftarrow 0$
    $i \leftarrow i + 1$
  end if
end while
return f

Analysis

- What is the running time to compute the failing function?
  $O(m)$
- What is the running time of KMP?
  Each iteration we either:
  1. increase i and j by one
  2. decrease j by at least one
  3. increase only i by one

  (1) and (3) execute in time at most n,
  and (2) executes no more often than (1),
  hence a total of $2n$ comparison

Hence a total running time of $O(m + n)$.

Example

$P =$ `ababbababa`

0	1	2	3	4	5	6	7	8	9
a	b	a	b	b	a	b	a	b	a
		a	b	a					
				a					
					a	b	a	b	b
									a

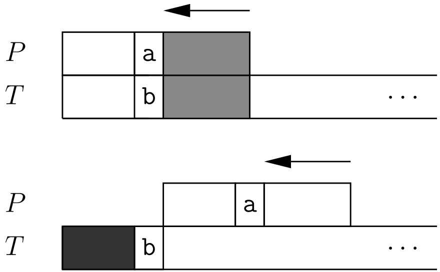
$f(x)$:

0	1	2	3	4	5	6	7	8	9
0	0	1	2	0	1	2	3	4	3

Outline

Boyer-Moore

- Try matching P backwards!
- Still shift to the right
- Can skip large parts of T entirely
- Example:



- Use two heuristics to decide how far to shift.
- Pick whichever gives the furthest shift.

Character Jump Heuristic

- Look at character c we mismatched with in T
- If c is not in P shift all the way past
- Otherwise shift P to line up the **last** occurrence of c in P with the one in T
- **Note**: We should always shift at least one
- Use an array which for each character of the alphabet indicates the position where to jump.

Examples

P	=	a	l	d	o								
T	=	w	h	e	r	e	i	s	w	a	l	d	o
					o								
									o				
										a	l	d	o

6 comparisons

P	=	m	o	o	r	e					
T	=	b	o	y	e	r	m	o	o	r	e
						e					
						(r)	e				
							(m)	o	o	r	e

6 comparisons

Partial Match Heuristic

- Similarly to KMP shift function, line up characters already matched in P with an occurrence further left.
- Use an array which for each position of the pattern indicates the position where to jump.

Examples:

P	=	b	a	n	a	n	a		
T	=	b	o	b	a	n	a	n	a
				n	a	n	a		
				b	a	n	a	n	a
P	=	b	o	b	o				
T	=	r	o	b	o	t	i	c	s
		b	o	b	o				
							o		

Results

- Worst-case running time $\in O(n + m + |\Sigma|)$
- Works very well when m is large
- Alphabet should not be too small
- On typical English text B-M probes approximately 25% of the characters in T

Outline

Summary

- Naive: $O(nm)$
- Rabin-Karp: Hashing, still $O(nm)$
- Knuth-Morris-Pratt Large/Intelligent Skip-Ahead, $2n + O(m)$
- Boyer-Moore Backward Matching + Intelligent Skip-Ahead, $O(n + m + |\Sigma|)$ in worst case, sublinear in practice
- If more time, preprocess Pattern to produce automata $n + O(m|\Sigma|)$.

Reading Materials

	Topic	GT	CLRS
	Rabin-Karp	418–421	906–922
	KMP+BM	422-428	923–930

- GT = Algorithm Design, by Goodrich & Tamassia
- CLRS = Introduction to Algorithms, by Cormen, Leisersen, Rivest & Stein