CS 245 — Assignment #6
Spring 2006

Due Date: Tuesday, July 4 at 5pm.

Use `makeCover` to produce a cover page for your assignment and hand in your assignment in the CS 245 assignment box. Assignments are to be done individually.

1. (10 points) For each of the following $p(n)$, prove $\forall n : \mathbb{N} \bullet p(n)$ using mathematical induction.

   (a) Let $p(n)$ be $\exists c \bullet T(n) \leq c(n^2 + n)$, for $n \geq 1$, where

   $$\begin{aligned} \mathrm{T}(n) &= 1 && \text{if } n = 1 \\ \mathrm{T}(n) &= \mathrm{T}(n-1) + n && \text{if } n > 1 \end{aligned}$$

   (b) Let $p(n)$ be $\exists b \bullet \exists c \bullet T(2^n) = b2^n - c$, for $n \geq 0$, where

   $$\begin{aligned} \mathrm{T}(2^n) &= 1 && \text{if } n = 0 \\ \mathrm{T}(2^n) &= \mathrm{T}(2^{n-1}) + \mathrm{T}(2^{n-1}) + 1 && \text{if } n > 0 \end{aligned}$$

2. (10 points) Prove the following set equalities and subset relations.

   (a) $A \cup (A' \cap B) = A \cup B$

   (b) $A \cap (A \cup B) \subseteq A$

3. (10 points) Using appropriate sets, formalize the following sentences. Your answers must not contain logical quantifiers.

   (a) Computer Science students are not History students.

   (b) No student is a Psychology student and not an Arts student.

   (c) Any student who is not a Computer Science student is a Psychology student.

   (d) History students are Arts students.

   Prove that (d) logically follows from (a)-(c) using natural deduction. In your proof you may use *any* of the inference rules from propositional and predicate logic and you may use any of the definitions given on the summary sheet for set theory.