CS 245 — Assignment #2
Spring 2006

Due Date: Tuesday, May 23 at 5pm.

Use `makeCover` to produce a cover page for your assignment and hand in your assignment in the CS 245 assignment box. Assignments are to be done individually.

1. (3 points) Give the formula $\neg(p \wedge (\neg(\neg r \wedge (s \vee p)) \Rightarrow \neg(\neg p \Rightarrow q)))$ in conjunctive normal form (CNF) and prove the equivalence of the two formulas by giving a transformational proof. Be sure to simplify, by using the appropriate logical laws, so that in each clause there are no duplicate literals or contradicting literals.

2. (10 points) Consider the fragments of code given on the left and the right below, where P1, P2, P3, P4, and P5 are blocks of code.

Fragment #1

```
if( NOT a OR b ) {
   if( NOT a AND NOT b ) {
      P1
   }
   else
   if( b ) {
      P2
   }
   else {
      P3
   }
}
else {
   P4
}

if( NOT a OR b OR (a AND NOT b) ) {
   P5
}
```

Fragment #2

```
if( a AND NOT b ) {
   P4
}
else
if( NOT a AND NOT b ) {
   P1
}
else {
   P2
}

P5
```

(a) For Fragment #1, express in propositional logic the conditions under which each of the blocks of code P1, P2, P3, P4, and P5 will be executed. Do not simplify.

(b) For Fragment #2, express in propositional logic the conditions under which each of the blocks of code P1, P2, P4, and P5 will be executed. Do not simplify.

(c) Give transformational proofs to show that Fragment #1 and Fragment #2 have the same behavior. For any unreachable (dead) code, give a transformational proof that the condition under which the code would be executed are a contradiction (equivalent to false). For any reachable code, give a transformational proof that the conditions under which the code would be executed are equivalent in both fragments.

3. (12 points) For each of the following arguments, determine whether the argument is valid or invalid. If the argument is valid, prove it using Natural Deduction. If the argument is invalid, provide a counter example and demonstrate that the argument is invalid.

(a) $p \vee \neg q,\ \neg r \Rightarrow \neg\neg q,\ r \Rightarrow \neg s,\ \neg\neg s \quad \vdash \quad p$

(b) $(p \vee q) \Rightarrow r \quad \vdash \quad q \Rightarrow r$

(c) $p \Rightarrow r,\ q \Rightarrow s \quad \vdash \quad (p \vee q) \Rightarrow (r \vee s)$

(d) $\neg p \Rightarrow q,\ \neg r \Rightarrow s,\ \neg q \vee s \quad \vdash \quad p \vee r$

(e) $\neg p \Rightarrow q,\ \neg r \Rightarrow s,\ \neg q \vee \neg s \quad \vdash \quad p \vee r$