CS 245 — Assignment #3
Spring 2005

Due Date: Tuesday, May 30 at 5pm.

Use `makeCover` to produce a cover page for your assignment and hand in your assignment in the CS 245 assignment box. Assignments are to be done individually.

1. (10 points) Consider the following,

   - $\neg r \Rightarrow \neg p,\ \neg(q \vee r),\ s \Rightarrow (p \vee q) \quad \models \quad \neg(\neg q \wedge \neg r)$
   - $\neg r \Rightarrow \neg p,\ \neg(q \vee r),\ s \Rightarrow (p \vee q) \quad \models \quad \neg s$

   (a) Using a semantic tableaux, show whether the premises are consistent. Explain.

   (b) For each of the above, determine whether the conclusion logically follows from the premises. If the conclusion does logically follow from the premises, prove it using Natural Deduction. If the conclusion does not logically follow from the premises, provide a counter example that demonstrates it does not.

1. a)

1. $\neg r \Rightarrow \neg p$ premise
2. $\neg(q \vee r)$ premise
3. $s \Rightarrow (p \vee q)$ premise

NOT-OR, 2

4. $\neg q$
5. $\neg r$

IMPLIES, 1

6. $\neg\neg r$ — CLOSED, 5, 6

7. $\neg p$

IMPLIES, 3

8. $\neg s$

9. $p \vee q$

OR, 9

10. p — CLOSED, 7, 10

11. q — CLOSED, 4, 11

The premises are consistent since it is not possible to close all the branches. The premises are consistent when $\neg p$, $\neg q$, $\neg r$, and $\neg s$ are all true; i.e., when

p is false r is false
q is false s is false

- $\neg r \Rightarrow \neg p,\ \neg(q \vee r),\ s \Rightarrow (p \vee q) \quad \models \quad \neg(\neg q \wedge \neg r)$

  The conclusion does not logically follow from the premises. A counter-example is: p is false, q is false, r is false, and s is false. The premises are all true in this interpretation, but the conclusion is false. Hence, the conclusion does not logically follow from the premises.

- $\neg r \Rightarrow \neg p,\ \neg(q \vee r),\ s \Rightarrow (p \vee q) \quad \models \quad \neg s$

  The conclusion logically follows from the premises. Here is a natural deduction proof.

1.		$\neg r \Rightarrow \neg p$	premise
2.		$\neg(q \vee r)$	premise
3.		$s \Rightarrow (p \vee q)$	premise
4.		s	assumption
5.		$p \vee q$	$3, 4, \Rightarrow$_E
6.		q	assumption
7.		$q \vee r$	$6, \vee$_I
8.		**false**	$2, 7, \neg$_E
9.		$\neg q$	$6 - 8, \neg$_I
10.		p	$5, 9, \vee$_E
11.		$\neg p$	assumption
12.		**false**	$10, 11, \neg$_E
13.		$\neg\neg p$	$11 - 12, \neg$_I
14.		$\neg\neg r$	$1, 13, \Rightarrow$_E
15.		r	$14, \neg$_E
16.		$q \vee r$	$15, \vee$_I
17.		**false**	$2, 16, \neg$_E
18.		$\neg s$	$4 - 17, \neg$_I

2. (15 points) For each of the following, determine whether the conclusion logically follows from the premises. If the conclusion does logically follow from the premises, prove it using Semantic Tableaux. If the conclusion does not logically follow from the premises, provide a counter example that demonstrates it does not.

   (a) $p \Rightarrow \neg r,\ \neg\neg r,\ s \vee \neg t,\ \neg t \Rightarrow p \quad \models \quad s$

   (b) $\neg p \Rightarrow r,\ \neg s \Rightarrow t,\ \neg t \vee \neg r \quad \models \quad s \vee p$

   (c) $\neg p \Rightarrow \neg(s \vee t) \quad \models \quad s \Rightarrow p$

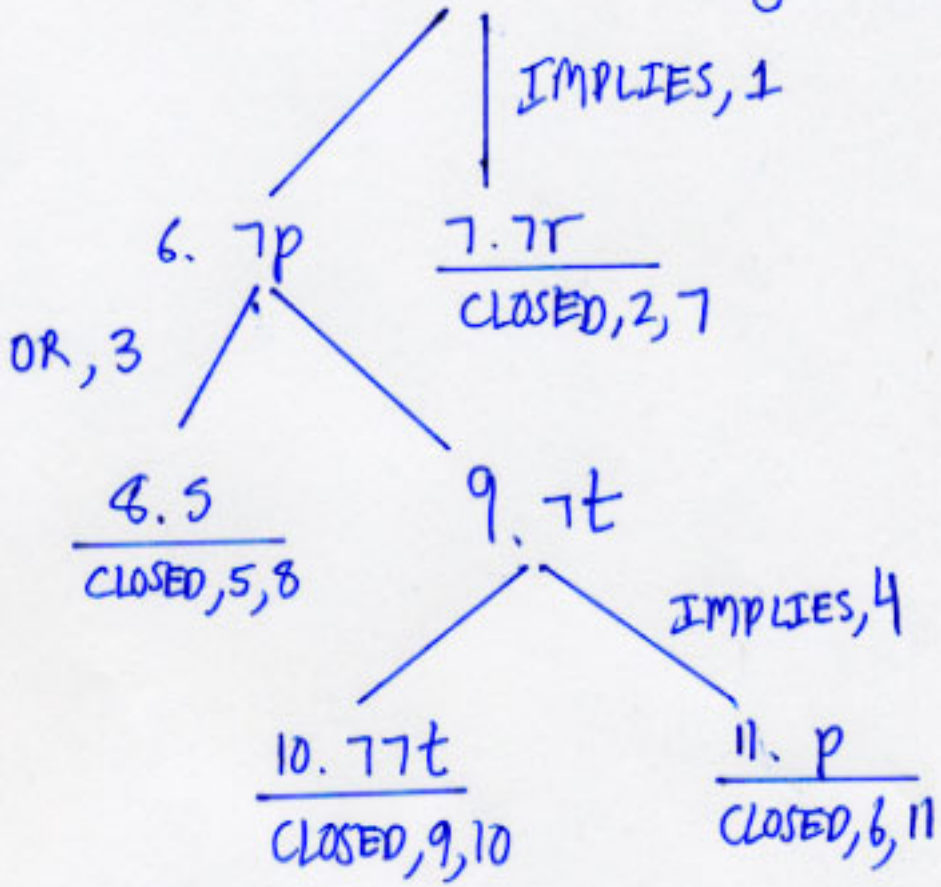
   (d) $\neg p \Leftrightarrow q,\ \neg r \wedge s,\ (t \vee s) \Rightarrow (\neg p \Rightarrow r) \quad \models \quad \neg q$

   (e) $s \Rightarrow p,\ t \Rightarrow r \quad \models \quad \neg(p \vee r) \Rightarrow \neg(s \vee t)$

Each of the above arguments is valid. For each of them, I give a semantic tableaux proof (see following pages). In each semantic tableaux, because all of the branches are closed, the premises together with the negated conclusion are inconsistent, and we can conclude the original (un-negated) conclusion.

2a)

1. $p \Rightarrow \neg r$ premise
2. $\neg\neg r$ premise
3. $s \vee \neg t$ premise
4. $\neg t \Rightarrow p$ premise
5. $\neg s$ negated conclusion



2b)

1. $\neg p \Rightarrow r$ premise
2. $\neg s \Rightarrow t$ premise
3. $\neg t \vee \neg r$ premise
4. $\neg(s \vee p)$ negated conclusion

NOT-OR, 4

5. $\neg s$
6. $\neg p$

IMPLIES, 1

7. $\neg\neg p$ — CLOSED, 6, 7

8. r

IMPLIES, 2

9. $\neg\neg s$ — CLOSED, 5, 9

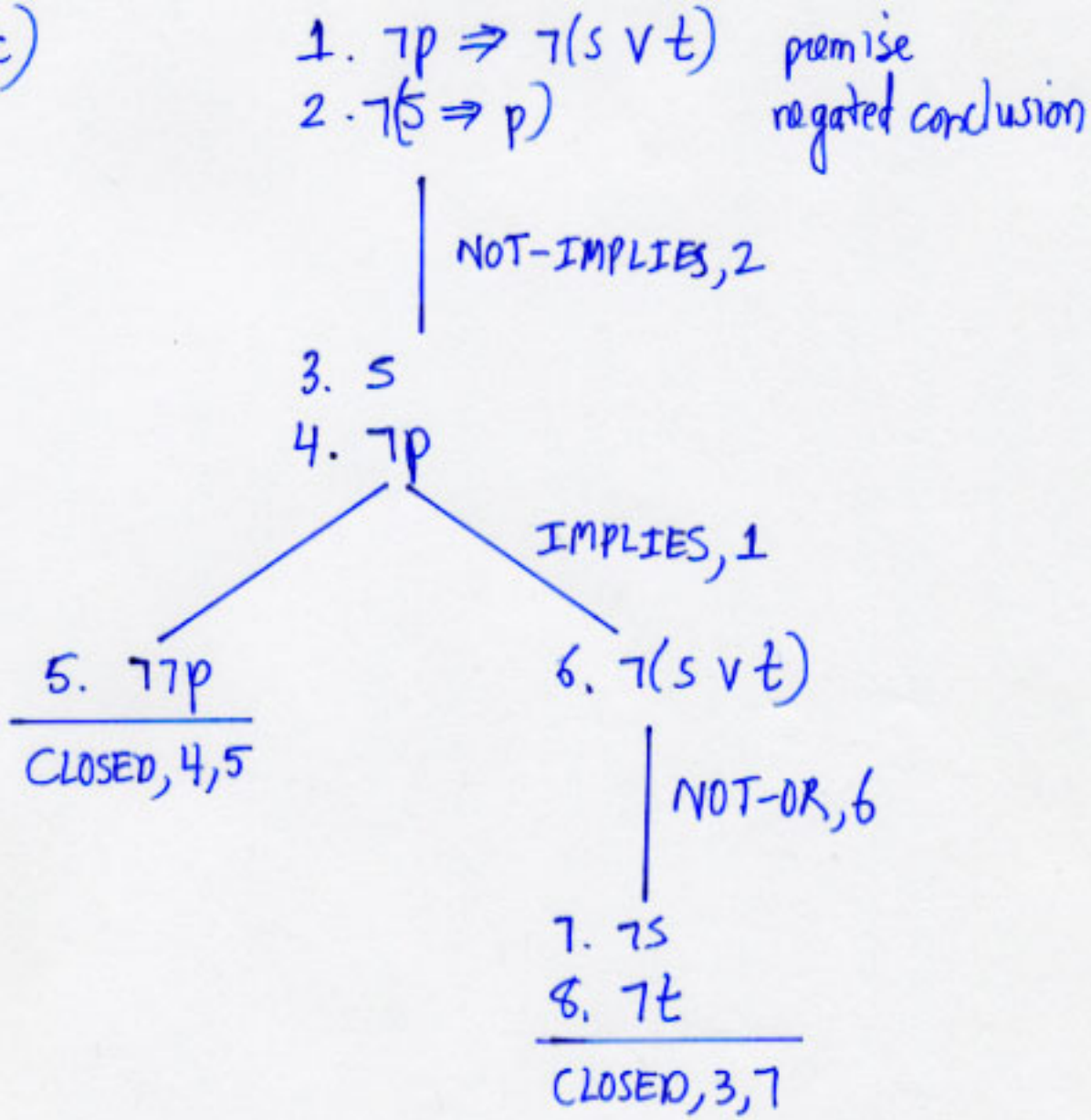
10. t

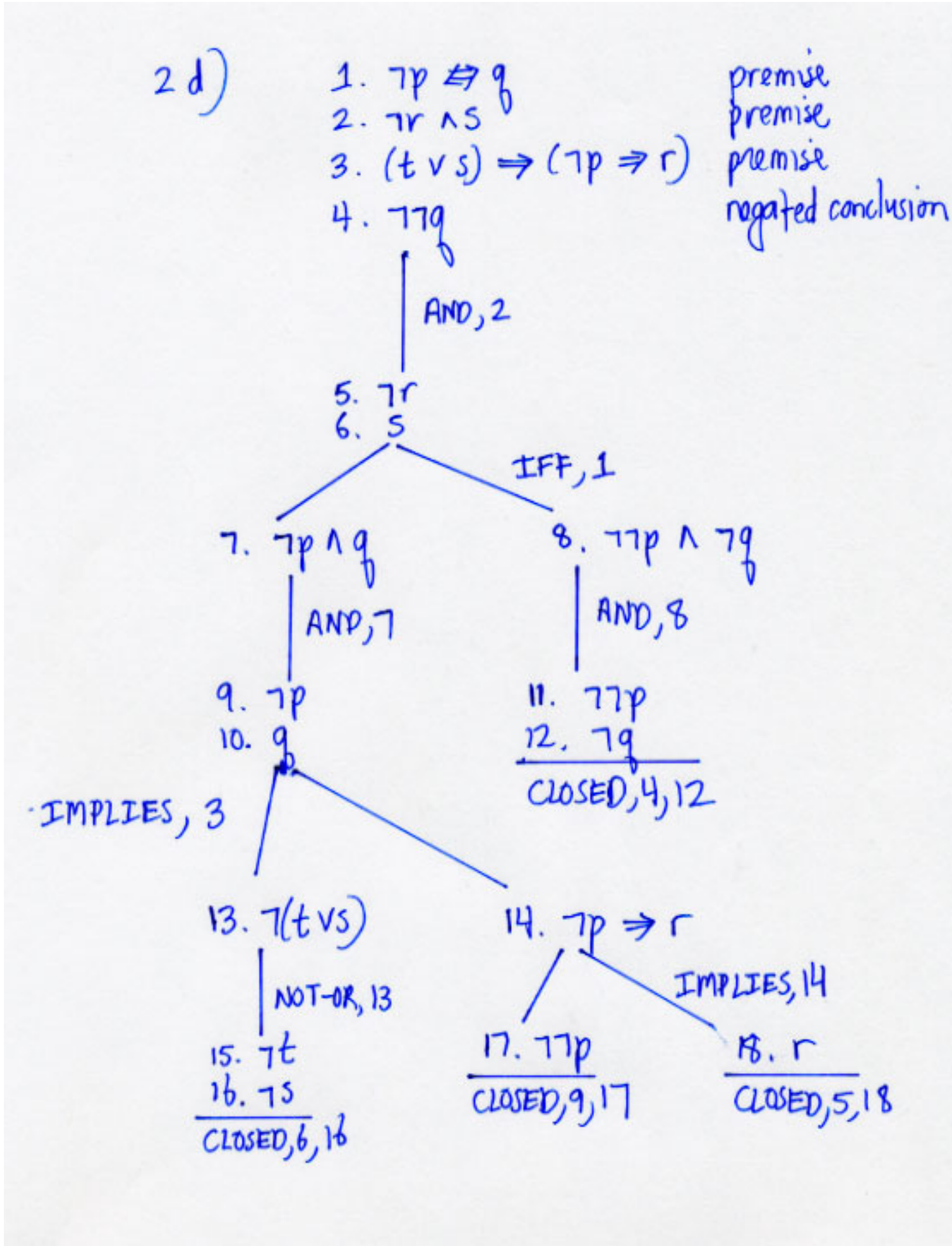
OR, 3

11. $\neg t$ — CLOSED, 10, 11

12. $\neg r$ — CLOSED, 8, 12

2c)


1. ¬p ⇒ ¬(s v t)
premise
2. ¬(s ⇒ p)
negated conclusion
NOT-IMPLIES, 2
3. s
4. ¬p
IMPLIES, 1
5. ¬¬p
CLOSED, 4, 5
6. ¬(s v t)
NOT-OR, 6
7. ¬s
8. ¬t
CLOSED, 3, 7


2 d)
1. ¬p ⇔ q
2. ¬r ∧ s
3. (t ∨ s) ⇒ (¬p ⇒ r)
4. ¬¬q
premise
premise
premise
negated conclusion
AND, 2
5. ¬r
6. s
IFF, 1
7. ¬p ∧ q
8. ¬¬p ∧ ¬q
AND, 7
AND, 8
9. ¬p
10. q
11. ¬¬p
12. ¬q
CLOSED, 4, 12
IMPLIES, 3
13. ¬(t ∨ s)
14. ¬p ⇒ r
NOT-OR, 13
IMPLIES, 14
15. ¬t
16. ¬s
17. ¬¬p
18. r
CLOSED, 6, 16
CLOSED, 9, 17
CLOSED, 5, 18

2e)

1. $s \Rightarrow p$ — premise
2. $t \Rightarrow r$ — premise
3. $\neg(\neg(p \vee r) \Rightarrow \neg(s \vee t))$ — neg. concl.

| NOT-IMPLIES, 3

4. $\neg(p \vee r)$
5. $\neg\neg(s \vee t)$

| NOT-OR, 4

6. $\neg p$
7. $\neg r$

IMPLIES, 1.

8. $\neg s$

| NOT-NOT, 5

10. $s \vee t$

OR, 10

11. s — CLOSED, 8, 11

12. t

IMPLIES, 2

13. $\neg t$ — CLOSED, 12, 13

14. r — CLOSED, 7, 14

9. p — CLOSED, 6, 9