CS 245 — Assignment #1
Spring 2006

Due Date: Tuesday, May 16 at 5pm.

Use `makeCover` to produce a cover page for your assignment and hand in your assignment in the CS 245 assignment box. Assignments are to be done individually.

1. (12 points) Translate the following sentences into propositional logic. Show the English phrase that each propositional letter represents.

   (a) Whether or not it is raining, I am going swimming.

   (b) If it rains while I am swimming, I will go hiking.

   (c) If there is a thunderstorm I'll go hiking, but I won't go swimming.

   (d) I will go hiking even though it is raining.

   (e) I will go hiking only if I do not go swimming.

   (f) I will go swimming unless there is a thunderstorm.

Solution:

I use the following prime propositions.

h – I will go hiking

r – it is raining

s – I am going swimming

t – there is a thunderstorm

(a) $(r \vee \neg r) \Rightarrow s$

(b) $(r \wedge s) \Rightarrow h$

(c) $t \Rightarrow (h \wedge \neg s)$

(d) $h \wedge r$

(e) $h \Rightarrow \neg s$

(f) $s \Leftrightarrow \neg t$

For each of the above, there are answers that are syntactically different but semantically the same. For example, (e) can be equivalently expressed as $s \Rightarrow \neg h$. As another example, (f) can be equivalently expressed as $\neg s \Leftrightarrow t$. As well, for some of the above, there are answers that differ from the above, but that can be argued to be equally valid. For example, (f) could be expressed as $\neg s \Rightarrow t$, or equivalently as $s \vee t$.

2. (6 points) For each of the following formulas, answer each of the following questions. Is the formula consistent? Is the formula a contradiction? Is the formula a tautology? Be sure to explain your answers.

   (a) $(q \vee r) \Rightarrow p$

   (b) $\neg(p \Rightarrow q) \Leftrightarrow (p \wedge \neg q)$

   (c) $(p \Rightarrow q) \Leftrightarrow \neg(\neg p \vee q)$

Solution:

Here is a truth table for the formula in (a).

p	q	r	$q \vee r$	$(q \vee r) \Rightarrow p$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	F	T
F	T	T	T	F
F	T	F	T	F
F	F	T	T	F
F	F	F	F	T

Yes, the formula is consistent since there is at least one intepretation (an assignment of true or false to the prime propositions) in which the formula is true. No, the formula is not a contradiction since it is consistent. No, the formula is not a tautology since there is at least one interpretation in which the formula is false.

Here is a truth table for the formula in (b).

p	q	$p \Rightarrow q$	$\neg(p \Rightarrow q)$	$p \wedge \neg q$	$\neg(p \Rightarrow q) \Leftrightarrow (p \wedge \neg q)$
T	T	T	F	F	T
T	F	F	T	T	T
F	T	T	F	F	T
F	F	T	F	F	T

Yes, the formula is consistent since there is at least one intepretation in which the formula is true. No, the formula is not a contradiction since it is consistent. Yes, the formula is a tautology since the formula is true in all interpretations.

Here is a truth table for the formula in (c).

p	q	$p \Rightarrow q$	$\neg p \vee q$	$\neg(\neg p \vee q)$	$(p \Rightarrow q) \Leftrightarrow \neg(\neg p \vee q)$
T	T	T	T	F	F
T	F	F	F	T	F
F	T	T	T	F	F
F	F	T	T	F	F

No, the formula is not consistent since there is no intepretation in which the formula is true. Yes, the formula is a contradiction since it is inconsistent. No, the formula is not a tautology since it is inconsistent.

3. (6 points) Do the premises logically imply the conclusion? Answer this question using a truth table and explain your answer.

$$\neg A \Rightarrow \neg C,\ \neg(A \wedge B) \ \models\ C \Rightarrow B$$

Solution:

Again, we can construct a truth table which lists every possible interpretation (assignment of true or false to each prime proposition) and check. We see below that there is one case where all of the premises are true, but the conclusion $C \Rightarrow B$ is not true. In particular, the row marked $\lhd$ is the counter example. All of the premises are true, but the conclusion is false. Therefore the premises do not logically imply $C \Rightarrow B$.

A	B	C	$\neg A \Rightarrow \neg C$	$\neg(A \wedge B)$	$(\neg A \Rightarrow \neg C) \wedge \neg(A \wedge B)$	$C \Rightarrow B$	
T	T	T	T	F	F	T	
T	T	F	T	F	F	T	
T	F	T	T	T	T	F	$\lhd$
T	F	F	T	T	T	T	
F	T	T	F	T	F	T	
F	T	F	T	T	T	T	
F	F	T	F	T	F	F	
F	F	F	T	T	T	T	

4. (6 points) Do the premises logically imply the conclusion? Answer this question using a truth table and explain your answer.

$$\neg p \Rightarrow \neg q, p \Rightarrow r \ \models \ \neg(q \wedge \neg r)$$

Solution:

The definition of logically implies states that the premises logically imply the conclusion iff every interpretation which satisfies the premises also satisfies the conclusion. We can construct a truth table which lists every possible interpretation (assignment of true or false to each prime proposition) and check. We see below that whenever all of the premises are true, the conclusion $\neg(q \wedge \neg r)$ is also true (see rows marked with $\triangleleft$). Therefore the premises logically imply $\neg(q \wedge \neg r)$.

p	q	r	$\neg p \Rightarrow \neg q$	$p \Rightarrow r$	$(\neg p \Rightarrow \neg q) \wedge (p \Rightarrow r)$	$\neg(q \wedge \neg r)$	
T	T	T	T	T	T	T	$\triangleleft$
T	T	F	T	F	F	F	
T	F	T	T	T	T	T	$\triangleleft$
T	F	F	T	F	F	T	
F	T	T	F	T	F	T	
F	T	F	F	T	F	F	
F	F	T	T	T	T	T	$\triangleleft$
F	F	F	T	T	T	T	$\triangleleft$

The premises do logically imply the conclusion.