Humidity And Dew Point Discussion

David Snyder, StratOhio

Abstract

I follow Daniels and Alberty [1966] derivation of the Clausius-Clapeyron Equation to determine water vapor saturation pressures, i.e. 100% humidity levels, as a function of temperature. I use this to calculate the Dew Point from temperature and relative humidity, values easily obtained from solid state sensors.   Then I look at the Magnus formula, with coefficients suggested by Alduchov and Eskridge [1995] and calculate Dew Points from them.   For my initial purposes either approach is good enough, but the Alduchov and Eskridge [1995] approach has a wider accurate temperature range and is intended for high altitude measurements.

Introduction

I would like take <u>atmospheric relative humidity measurements</u>, and convert them to an absolute humidity. Absolute Humidity can be described by the density of water, either as a number density or mass density, as a partial pressure of water vapor, or as a Dew Point, T_d, i.e. the temperature at which the water content is saturated.

The relative humidity, $H_R(T)$, is described as the ratio of the water content of the atmosphere, E, to the amount the amount that it could hold, the amount when saturated, $E_S(T)$.

$$H_R(T) = E/E_S(T) \qquad (1)$$

from the definition of Dew Point,
$$E = E_S(T_d) = H_R(T) * E_S(T) \qquad (2)$$

There are three approaches, mechanisms to understanding the saturated water content. 1) Air saturated over a pool of water, 2) Air saturated over a sheet of ice, and 3) Water condensing out of a cooled atmosphere. One hopes they all yield the same result, but can conceive of differences between ice, and a super cooled pool of water due to the higher heat of vaporization, or differences between atmospheric water condensing into snow vs rain.

Clapeyron Equation

Following Daniels and Alberty (1966), D&A, chapter 4.

The condition for saturation, i.e. equilibrium between liquid and vapor, is that the Gibb's Free Energy, G, for both phases is the same. $G_l = G_v$. For an incremental change in temperature and pressure while maintaining saturation,

$$(\delta G_l/\delta P)_T \, dP + (\delta G_l/\delta T)_p \, dT = (\delta G_v/\delta P)_T \, dP + (\delta G_v/\delta T)_p \, dT \qquad (3)$$

and

$$V_l \, dP - S_l \, dT = V_v \, dP - S_v \, dT \qquad (4)$$

$$dP/dT = S_v - S_l \ / \ V_v - V_l \qquad (5)$$

but $S_v - S_l = \Delta H_{vap} / T$ where H_{vap} is the enthapy of vaporization,

Which gives the Claperon Equation,

$$dP/dT = \Delta H_{vap} / T (V_v - V_l) \qquad (6)$$

Enthalpy and Volumes can be replaced by the specific enthalpy and specific volumes.

Clausius - Clapeyron Equation

For an ideal gas, and where the density, mol/volume, of the liquid is much greater than the density of vapor, this can be rewritten,

$$dP/dT = P \Delta H_{vap} / R T^2 \qquad (7)$$

Where T is relative to absolute zero, Kelvin. Replacing dP/P with d ln P.

$$d \ln P = (\Delta H_{vap} / R T^2) dT \qquad (8)$$

for ΔH_{vap} independent of temperature (or weakly dependent, and small temperature ranges) this can be integrated to yield the Clausius-Clapeyron equation.

$$\ln P/P_0 = (\Delta H_{vap} / R)(1/T_0 - 1/T) \qquad (9)$$

and,

$$P(T)/P_0 = \exp(\Delta H_{vap} / R T_0)\exp(-\Delta H_{vap} / R T) \qquad (10)$$

For my purposes this gives the saturation vapor pressure as a function of temperature. It has a form of $P(T) = P(T_0) A(T_0) \exp(-\beta/T)$. Pressure units depend only on the units of the reference pressure.

A review of the assumptions involved are that this is for water vapor over water. That is, it is for water vapor evaporating from a pool of water under near vacuum conditions. Additional gas, atmospheric pressure, may interfere with the enthalpy of vaporization, but I assume the P(T) is close enough to the saturation partial pressure of water. I assume water vapor behaves like an ideal gas. And that the enthalpy of vaporization is independent of temperature, or at least the has small changes over the temperature range involved.

From D&A problem 4.24, for H_2O at 0C (273.15K), $\Delta H_{vap} = 595$ cal g^{-1}, P = 4.58 mm. R= 1.987 cal deg^{-1} mol^{-1}. At 18 g/mol, $\Delta H_{vap} / R = 5390.0$ deg. P(t) = 4.58 * 3.714E5 * exp(-5390/(t+273.15)), where t = temperature in Celsius.

Dew Point

The relative humidity is ratio of the water content to the saturated water content, usually expressed in percent.

$$U_{rh} = P(T_d)/ P_s(T) \qquad (11)$$

the dew point is the temperature, T_d, at which that water content is the saturation content.

$$P(T_d) = U_{rh} * P_s(T)$$

from (10) above,

$$P(T_d) = P_0\, A(T_0)\, \exp(-\beta/T_d) \qquad (12)$$

or

$$\exp(-\beta/T_d) = (U_{rh} * P_s(T)) / (P_0\, A(T_0))$$

and

$$\exp(-\beta/T_d) = U_{rh}\, \exp(-\beta/T) \qquad (13)$$

$$-\beta/T_d = \ln(U_{rh}) - \beta/T$$

Finally

$$T_d = \beta\, T / (\beta - T \ln(U_{rh})) \qquad (14)$$

where $\beta = \Delta H_{vap} / R$. Since $U_{rh} < 1$, T_d will be $< T$. Note that in this case T is in °K. Note that the original reference pressure and temperature cancel out of the evaluation.

To test this I took values of water saturation pressure from a table in the range of 30C to 0C. By taking 30C as a reference, the relative humidity can be calculated for the respective dew points. This gives relative humidities from 100% to 14%. Plugging these relative humidities into (14), with $\lambda = 5390$ deg^{-1}, I constructed the table below. For this limited test the reverse calculated dew points agree with the dew points within ¼ degree. It looks like the formula is doing something right.

Temp(C)	P_{sat}(mb)	U_{rh}(30C)	T_d calc
30	42.5	1.00	30.00
25	31.7	0.75	25.08
20	23.4	0.55	20.16
15	17.1	0.40	15.23
10	12.3	0.29	10.24
0	6.1	0.14	0.16

Table 1: Comparison of Dew points to Eqn 14.

Alduchov and Eskridge

Alduchov and Eskridge [1995], A&E, fit contants p_0, β, and λ, to the Magnus [1844] formula,

$$e_w(t) = p_0\, \exp((\beta*t)/(\lambda+t)) \qquad (18)$$

where t is temperature in Celsius and $e_w(t)$ is the saturation water vapor pressure. In their equation 21, they recommend $p_0 = 6.1094$ mbarr, $\beta = 17.625$, and $\lambda = 243.04°$, for standard surface and upper air applications in the temperature range -40° – 50° C with a relative error of <.4% compared to other models. The accuracy degrades below -40° C.

An enhancement factor can be used to account for pressure effects, i.e. the partial pressure of water vapor. But this factor appears to be negligible for atmospheric pressures.

I have seen it suggested that the Magnus formula is a solution to the Clausius-Clapeyron equation, but I don't see it. That does not invalidate its use as an approximation that gets around some of the assumptions of (10).

Magnus Dew Point

Lawrence (2005) gives a solution for the dew point from the Magnus Formula.

$$p(t_d) = p0 \ U_{rh}(t) \ exp((\beta * t)/(\lambda + t)) \qquad (19)$$

or

$$exp((\beta * t_d)/(\lambda + t_d)) = U_{rh}(t) \ exp((\beta * t)/(\lambda + t))$$

$$(\beta * t_d)/(\lambda + t_d) = ln(U_{rh}(t)) + ((\beta * t)/(\lambda + t))$$

$$\beta * t_d = (\lambda + t_d) (\ ln(U_{rh}(t) + ((\beta * t)/(\lambda + t)) \ )$$

$$t_d (\ \beta - (\ ln(U_{rh}(t) + ((\beta * t)/(\lambda - t)))) = \ \lambda (\ ln(U_{rh}(t) + ((\beta * t)/(\lambda - t)) \ )$$

$$t_d \ = \ \lambda (\ ln(U_{rh}(t) + ((\beta * t)/(\lambda - t)) \ ) / (\ \beta - \ ln(U_{rh}(t) - ((\beta * t)/(\lambda - t)) \ ) \qquad (19)$$

The reference pressure again cancels out, and I expect some of the other joint factors are reduced.

Sensiron

Sensiron, a manufacturer of solid state sensors, has a white paper, *Introduction to Humidity*.
It recommends, like A&E above, using the Magnus formula.

$$e_w(t) = p_0 \ exp(\ (\beta * t)/(\lambda - t))$$

Where t is ºC, p_0 = 6.112 hPa, β = 17.62, λ = 243.12 for water, close to the A&E constants.

The dew point, t_d, is given by,

$$t_d(t, U_{rh}) = \ \lambda * (\ ln(U_{rh}/100\%) \ + (\ (\beta*t)/(\lambda+t) \ ) \ ) / (\ \beta - \ ln(U_{rh}/100\%) \ - (\ (\beta*t)/(\lambda+t) \ ) \ )) \qquad (20)$$

where U_{rh} is the relative humidity in percent.

Conclusions

The table below is a comparison between water vapor pressure from tables, constant ΔH equation (10) and (18). It shows the A&E parameters do indeed work better than the simple formulation.

For room temperature work either approach seems good enough, but equation (18) is accurate overt a wider temperature range. My guess is that it compensates somewhat for the Enthalpy of vaporization temperature dependence.

Temp (C)	Pvap (mb)	ΔH_{vap}	A&E(1995)
-40		0.21	0.19
-20		1.28	1.26
0	6.1	6.11	6.11
10	12.3	12.26	12.26
20	23.4	23.47	23.33
30	42.5	43.04	42.37
50	123.4	129.35	123.61
70	311.8	341.94	314.51
100	1013.2	1,208.88	1,040.77

Table 2: Comparison of tabulated water vapor pressures to model values.

Following, is a comparison of Dew Point calculations to tabulated water vapor pressures.

Temp(C)	P_{sat}(mb)	U_{rh}(30C)	T_d ΔH_{vap}(0)	A&E (1995)
30	42.5	1.00	30.00	30.00
25	31.7	0.75	25.08	24.99
20	23.4	0.55	20.16	19.99
15	17.1	0.40	15.23	15.02
10	12.3	0.29	10.24	10.00
0	6.1	0.14	0.16	-0.06

Table 3: Dew point calculations at 30C from the models.

Again, for room temperature either approach seem good enough. For hand calculations, I would use equation 14.   The reference pressure and temperature mostly cancels out of the formulations making them more forgiving over a wider range. For hand calculations, I would use equation 14. However, especially for high altitude calculations the Magnus- Alduchov formulation is recommended. For automated data processing the calculation penalty is minimal.

References

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