Lecture Introduction to Network Science
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BIONETS

Solution 4

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Question 1

Let $G = (V, E)$ be a graph with $n = |V|$ nodes:

a) Degree centrality: $\min = 0,\ \max = n - 1$ (assuming there are no self-loops).

b) Eigenvector centrality: $\min = 0,\ \max = 1$.

c) PageRank centrality: $\min = 0,\ \max = 1$.

d) Closeness centrality: $\min = 0,\ \max = \frac{1}{n-1}$.

Question 2

a) The transition matrix is given by

$$A = \begin{pmatrix} 0 & 0 & 0 & 0 & 0.5 \\ 0.5 & 0 & 0 & 0 & 0 \\ 0.5 & 0.5 & 0 & 0 & 0.5 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0.5 & 0 & 1 & 0 \end{pmatrix}.$$

b) The PageRank transition matrix for $d = 0.5$ is given by

$$B = \frac{1}{2}A + \frac{1}{2}\begin{pmatrix} \frac{1}{5} & \cdots & \frac{1}{5} \\ \vdots & & \vdots \\ \frac{1}{5} & \cdots & \frac{1}{5} \end{pmatrix}.$$

c) Yes, because the transition matrix is built from a strongly connected graph.

d) Yes, due to inserted “teleportation edges” the PageRank transition matrix always “relates” to a strongly connected graph.

Question 3

See the jupyter notebook `betweenness.ipynb`.