Lecture Introduction to Network Science
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Solution 6

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Question 1

a) Topology changes frequently and massively; changes are an integral part of the system; nodes are fixed, only edges change over time

b) Subpaths of optimal temporal paths can be suboptimal (see examples on Slide 20).

c) The starting time of a fastest path needs to be one of the time points contained in S. By computing earliest-arrival paths P_t for each $t \in S$ we minimize for the end time of temporal paths. Recall that fastest paths minimize for duration, which is defined as the difference of end and start time of a temporal path. We have thus reduced our set of possible fastest paths to the set of all P_t's, because this set considers all possible starting times of fastest paths and already optimizes for `end`, which appears in our objective function $\texttt{dur}(P) = \texttt{end}(P) - \texttt{start}(P)$. Therefore, we can simply pick the one path P^* from all P_t's that minimizes duration.

Question 2

We list the possible paths with duration and distance:

1. $(A, B, 1, 2), (B, D, 3, 3) \quad \text{dur} = 5, \text{dist} = 5$
2. $(A, B, 1, 2), (B, C, 4, 1), (C, D, 8, 1) \quad \text{dur} = 8, \text{dist} = 4$
3. $(A, B, 2, 2), (B, C, 4, 1), (C, D, 8, 1) \quad \text{dur} = 7, \text{dist} = 4$
4. $(A, C, 3, 1), (C, D, 8, 1) \quad \text{dur} = 6, \text{dist} = 2$
5. $(A, C, 4, 1), (C, D, 8, 1) \quad \text{dur} = 5, \text{dist} = 2$

Based on the above calculations, we conclude that Path 1 is an earliest-arrival path, Path 5) is a latest-departure path, Paths 1 and 5 are fastest paths, and Paths 4 and 5 are shortest paths.

Question 3

See jupyter notebook `fastest_paths.ipynb`.