

## SOLUTION 8

Exercise Session: 27.6.2024

### Question 1

- a) Edge strength negatively correlated to edge betweenness; keeps triangles in the same community
- b) We enforced balanced cuts by minimizing normalized versions of the objective function, i.e. *RatioCut* of *Ncut*.
- c) We can calculate the first expression as

$$\mathbf{1}^T x = \sum_{i=1}^n x_i = |A| \cdot \sqrt{\frac{|\bar{A}|}{|A|}} - |\bar{A}| \cdot \sqrt{\frac{|A|}{|\bar{A}|}} = \sqrt{|\bar{A}||A|} - \sqrt{|\bar{A}||A|} = 0.$$

The second expression follows from

$$x^T x = \sum_{i=1}^n x_i^2 = |A| \cdot \frac{|\bar{A}|}{|A|} + |\bar{A}| \cdot \frac{|A|}{|\bar{A}|} = |\bar{A}| + |A| = n.$$

- d) Compute pairwise distances and input those into desired pairwise similarity measure.

### Question 2

We need to compute edge betweenness scores  $C_B(e)$  for all edges and remove the one with the highest value. The values of the edges are  $C_B(\{A, C\}) = 3$ ,  $C_B(\{B, C\}) = C_B(\{C, D\}) = 2$ ,  $C_B(\{B, D\}) = 1$ . We thus remove edge  $\{A, C\}$ , which gives us the first clustering consisting of  $V_1 = \{A\}$  and  $V_2 = \{B, C, D\}$ . [Recall that strictly following the definition of edge betweenness, we would have to consider all different pairs of nodes, so all values above would be doubled. Since we consider an undirected graph, this scaling does not matter for the decision of the Girvan-Newman algorithm.]

### Question 3

See notebook `spectral.ipynb`.