Day 10 - Autoregressive models

Introduction

We continue our discussion of basic stochastic models, introducing the generating autoregressive model of order p.

```
# packages
library(tidyverse)
library(lubridate)
```

Review

Read in the Bozeman air quality data set, create a time series object, and filter the time series to only observations between September 15^{th} and September 20^{th}. Plot the resulting time series.

```
# load air quality data
bz_air <- readRDS("mt_pm25_sept2020.rds")
```

Autoregressive models

Note

A time series $\{x_t\}$ is an ______________________________ or AR(p) if

$$x_t = \alpha_1 x_{t-1} + \alpha_2 x_{t-2} + \cdots + \alpha_p x_{t-p} + w_t$$

where $\{w_t\}$ is white noise and the α_i are the model parameters. It can be shown that the AR(p) model can be expressed as a polynomial of order p in the backshift operator:

$$\theta_p(B)x_t = (1 - \alpha_1 B - \alpha_2 B^2 - \cdots - \alpha_p B^p)x_t = w_t$$

A few notes on AR(p) models:

- The ______________________________ is a special case of AR(1) with $\alpha_1 = 1$.

- The ______________________________ model is a special case of an AR process with $\alpha_i = \alpha(1-\alpha)^i$ as p approaches infinity.

- A prediction at time t is given by

$$\hat{x}_t = \hat{\alpha}_1 x_{t-1} + \hat{\alpha}_2 x_{t-2} + \cdots + \hat{\alpha}_p x_{t-p}$$

- Model parameters are estimated by minimizing the ______________________________.

Is an AR(p) process stationary?

Note

The equation $\theta_p(B) = 0$ is called the ____. The roots of the ____ may be used to determine whether an AR(p) process is stationary.

If all roots of the ____ exceed 1 in magnitude, the model is stationary. You may use the `polyroot` function in R to find the roots of polynomials.

Example: Determine whether the AR(1) model $x_t = \frac{1}{2}x_{t-1} + w_t$ is stationary.

Example: Determine whether the AR(2) model $x_t = x_{t-1} - \frac{1}{4}x_{t-2} + w_t$ is stationary.

Example: Determine whether the AR(2) model $x_t = \frac{1}{2}x_{t-1} - \frac{1}{2}x_{t-2} + w_t$ is stationary.

AR(1) processes

A time series x_t is an AR(1) process if

The second-order properties of an AR(1) process are:

$$\mu(t) = 0$$
$$\gamma_k = \frac{\alpha^k \sigma^2}{(1-\alpha^2)}$$
$$\rho_k = \alpha^k$$

How can we simulate an AR(1) process?

Note

The ____ at lag k is the correlation that results after removing the effect of any correlation due to terms at shorter lags.

Fitting an AR(p) process

To fit an AR(p) process, we use the `ar` function in R. To select the order of the AR process, R minimizes the AIC.

$$AIC = 2 \cdot (-\text{log-likelihood} + \text{number of parameters})$$

Closing remarks

- The stochastic models discussed this week (white noise, random walks, and AR(p) processes) are not very useful for forecasting on their own, and are unlikely to compete with procedures like Holt-Winters.
- In practice, these stochastic models are combined with other techniques (regression, moving average, and integrated moving averages) to construct powerful forcasting techniques.
- The AR(p) process is one component of the ARIMA model, which we will discuss towards the end of the semester.