Name: Your name here
Due: 2024/10/14

Homework 5

Be sure to submit **both** the .pdf and .qmd file to Canvas by Monday, October 14th at 11:59 pm.

0. [1 pt] With whom did you work on this assignment?

1. [12 pt] The purpose of this question is to demonstrate that there exist time series structures that Holt-Winters cannot estimate very well. To demonstrate this idea, we are going to simulate data from an AR(2) process with $\alpha_1 = 1.1$ and $\alpha_2 = -.3$.

   a) [1 pt] Write the hypothetical AR(2) model in proper notation.

   b) [1 pt] Express the model from part a in terms of the backshift operator.

   c) [1 pt] Determine whether the model is stationary.

   d) [3 pt] Simulate 12 years of monthly data from an AR(2) process with $\alpha_1 = .8$ and $\alpha_2 = -.1$. Convert the synthetic data to a `ts` object and plot the resulting series.

   e) [1 pt] Create an ACF plot of the synthetic series and comment what the plot suggests about the autocorrelation.

   f) [1 pt] Create an PACF plot of the synthetic series and comment on what the plot suggests about the autocorrelation.

   g) [2 pt] Fit a Holt-Winters model to the synthetic series and plot the results.

   h) [2 pt] Create an ACF plot of the residuals from the Holt-Winters model and comment on whether the model has accounted for the residual autocorrelation.

2. [2 pt] Using the characteristic equation, show that a random walk is not stationary.

3. [3 pt] Determine for what values of α an AR(1) process is stationary.

4. [6 pt] The purpose of this question is to prove one of the second order properties of an AR(1) process.

   a) [1 pt] Express an AR(1) process in terms of the backshift operator.

   b) [3 pt] Using the expression from part a, show that $x_t = \sum_{i=0}^{\infty} \alpha^i w_{t-i}$. It is likely helpful to know that $(1-B)^{-1} = 1 + B + B^2 + ...$ by Binomial expansion.

   c) [2 pt] Using the expression from part b, show that the $E(x_t) = 0$. Recall that $E\left(\sum_{i=1}^{n} X_i\right) = \sum_{i=1}^{n} E(X_i)$ and that $E(cX) = cE(X)$ if c is a constant.