

STRĀTUMN

CRYPTO  
NIGHT

CP-ABE

Ciphertext-Policy Attribute-Based Encryption

MOURAD BEJI

—  
07.11.2018

# Intro



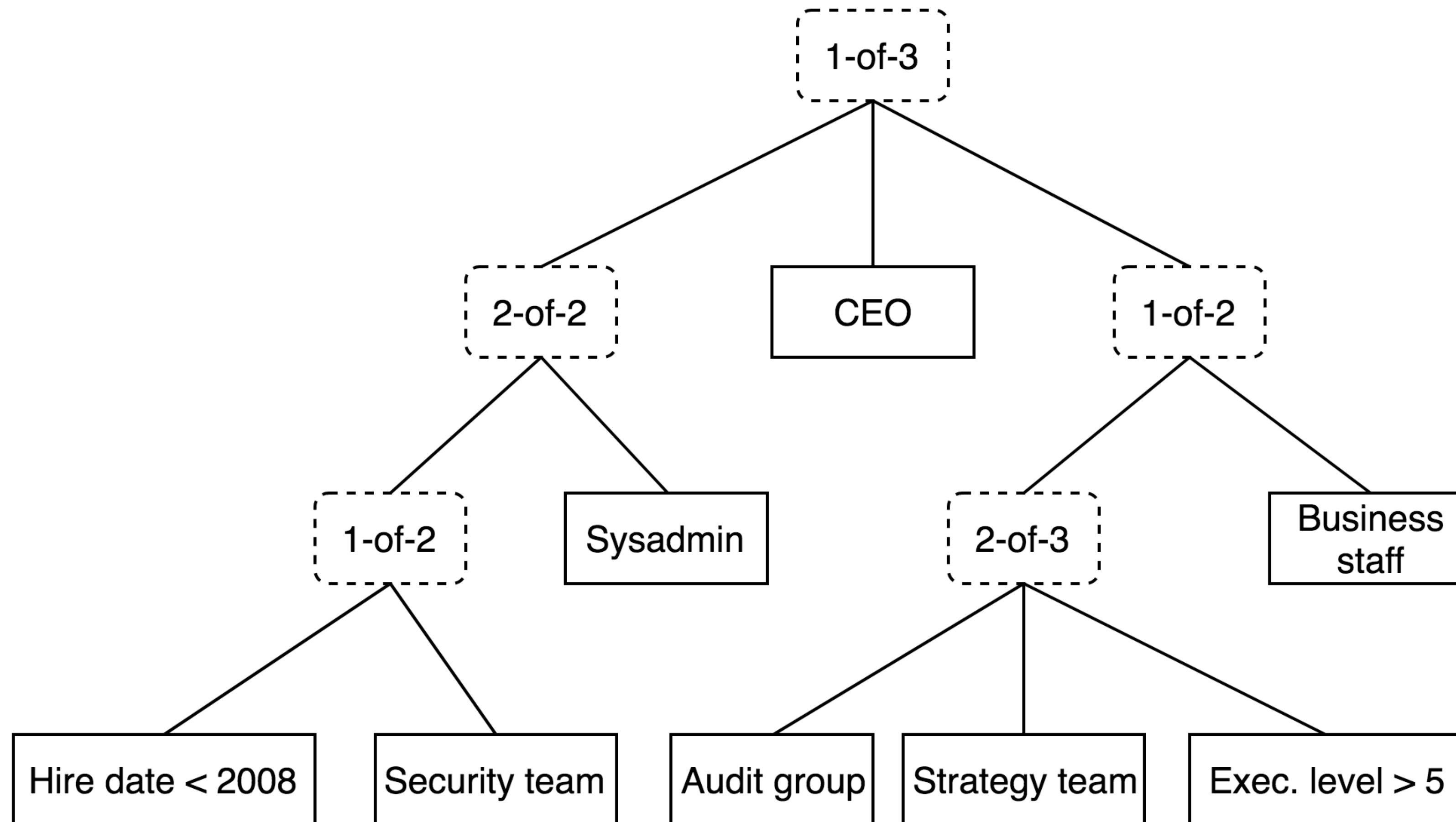
- define fine-grain access policies for data encryption and recovery
- usually the server that has the data controls the access to it
  - But more and more data is distributed (cloud) => untrusted/semi-trusted servers, higher chances of compromising data
- p2p networks: no centralized server after system setup

We don't want to encrypt the file with an exhaustive list of all possible recipients

Solution: access control should be inherent to the cryptographic scheme

# Access structure

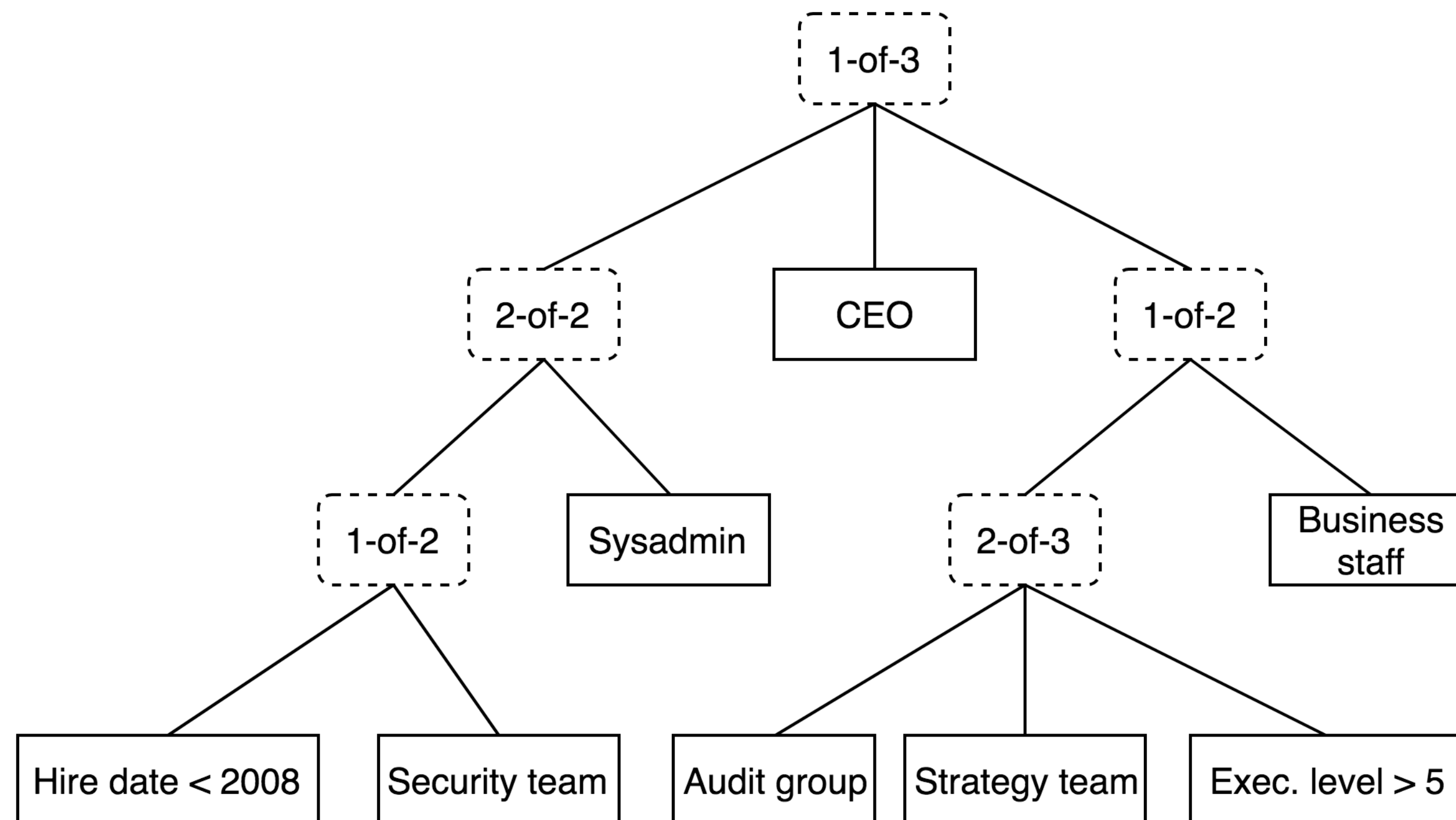
# Access structure



# Access structure




((Hire date < 2008 **OR** security team) **AND** Sysadmin) **OR**  
CEO **OR**  
(**2 OF** (Executive level  $\geq$  5, Audit group, Strategy team) **AND** Business staff)



# CP-ABE

A. Sahai, B. Waters

# CP-ABE



- Each user has a secret key =  $f(\text{attributes})$
- Data is encrypted using an policy = access structure
- Decryption is possible if attributes verify the policy tree

# CP-ABE Algorithms



- Setup:  $1^\lambda \rightarrow (PK, MSK)$
- KeyGen:  $(MSK, \mathcal{S}) \rightarrow SK$
- Encrypt:  $(PK, plaintext, \mathbb{A}) \rightarrow ciphertext$
- Decrypt:  $(PK, ciphertext, SK) \rightarrow \begin{cases} plaintext & \text{if } \mathcal{S} \text{ verifies } \mathbb{A} \\ \perp & \text{otherwise} \end{cases}$

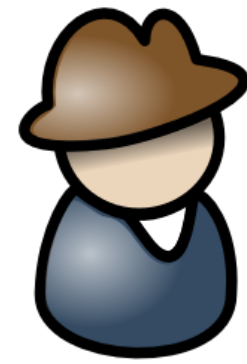


# CP-ABE Algorithms

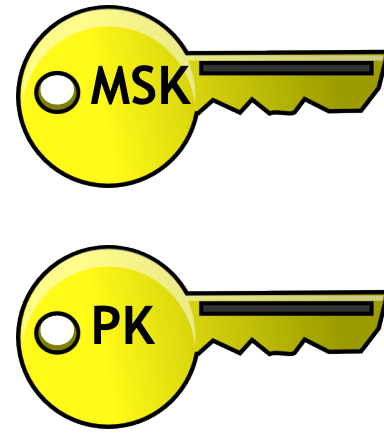


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- Delegate  $(SK, \tilde{\mathcal{S}}) \rightarrow \tilde{SK}$  if  $\tilde{\mathcal{S}} \subset \mathcal{S}$

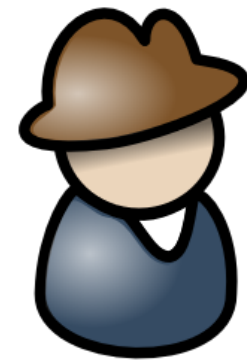
# CP-ABE Protocol - Setup



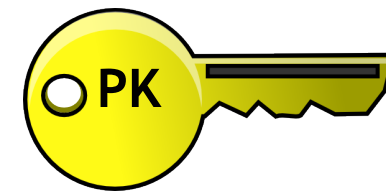
Key issuer



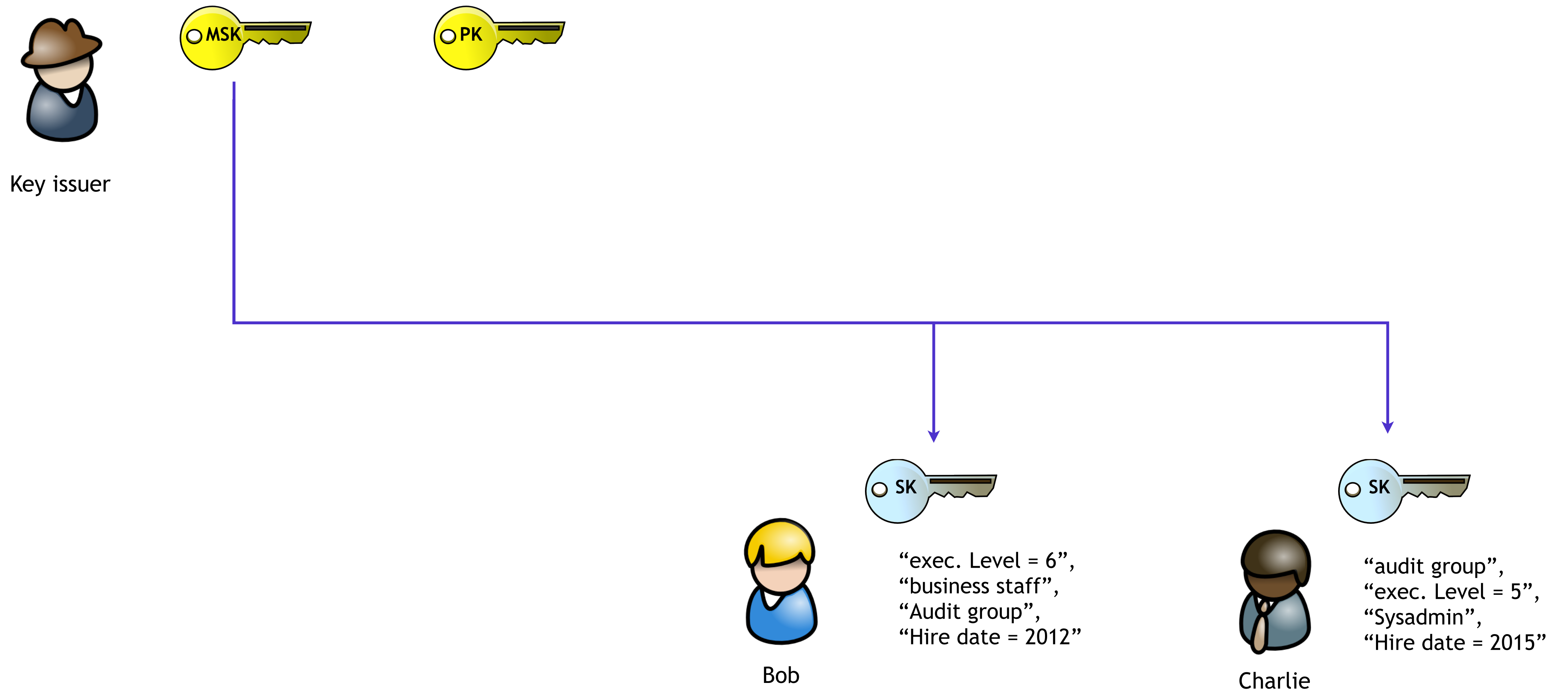
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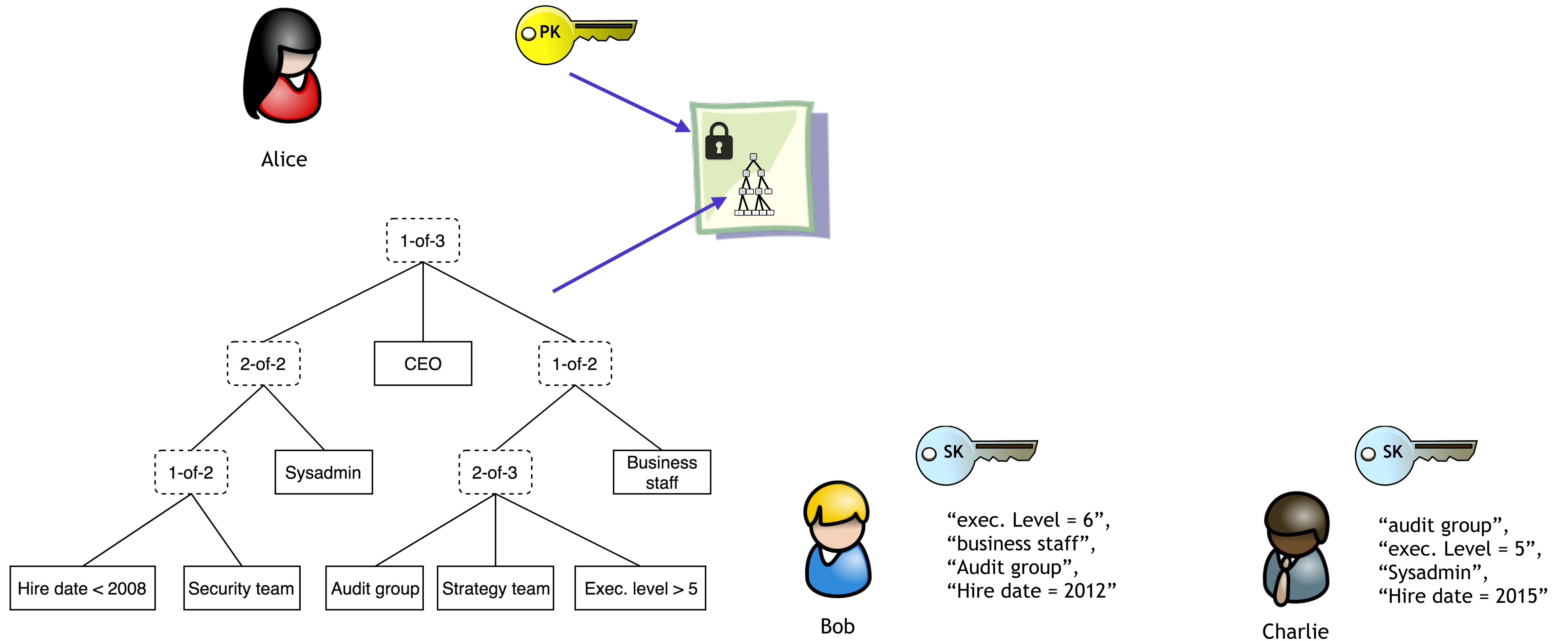
Key issuer



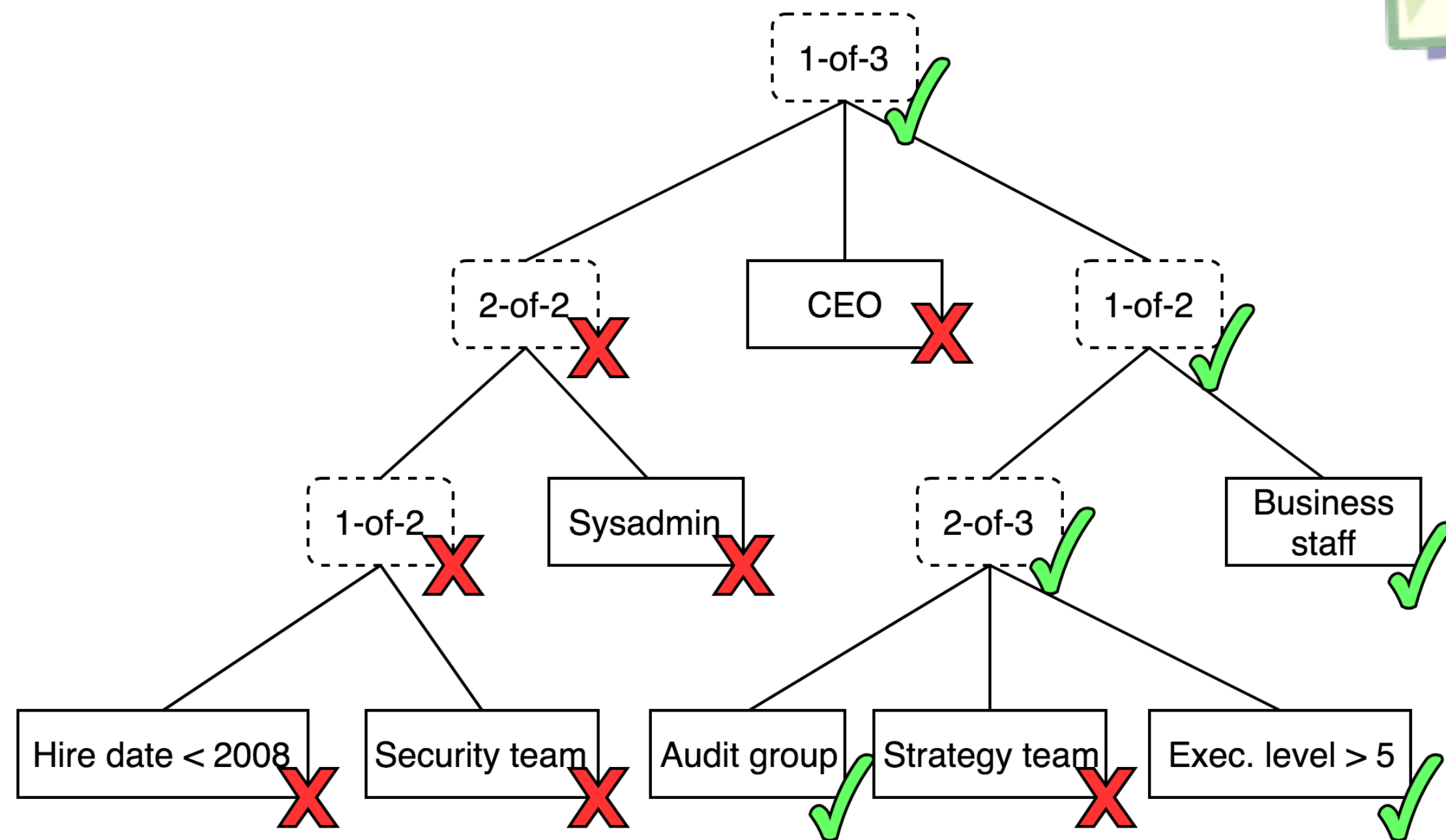
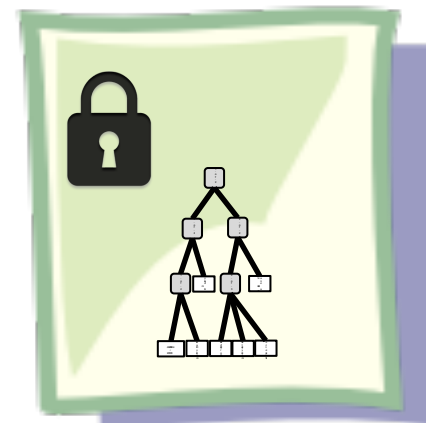
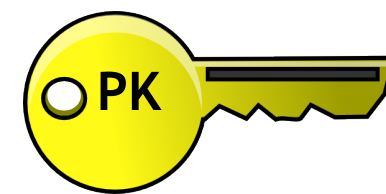
# CP-ABE Protocol - Key Generation



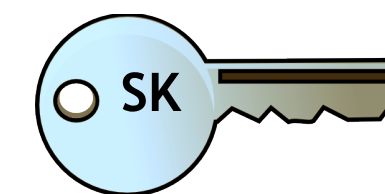
# CP-ABE Protocol - Encryption



# CP-ABE Protocol - Decryption



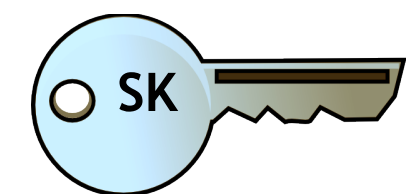
Bob



“exec. Level = 6”,  
“business staff”,  
“Audit group”,  
“Hire date = 2012”

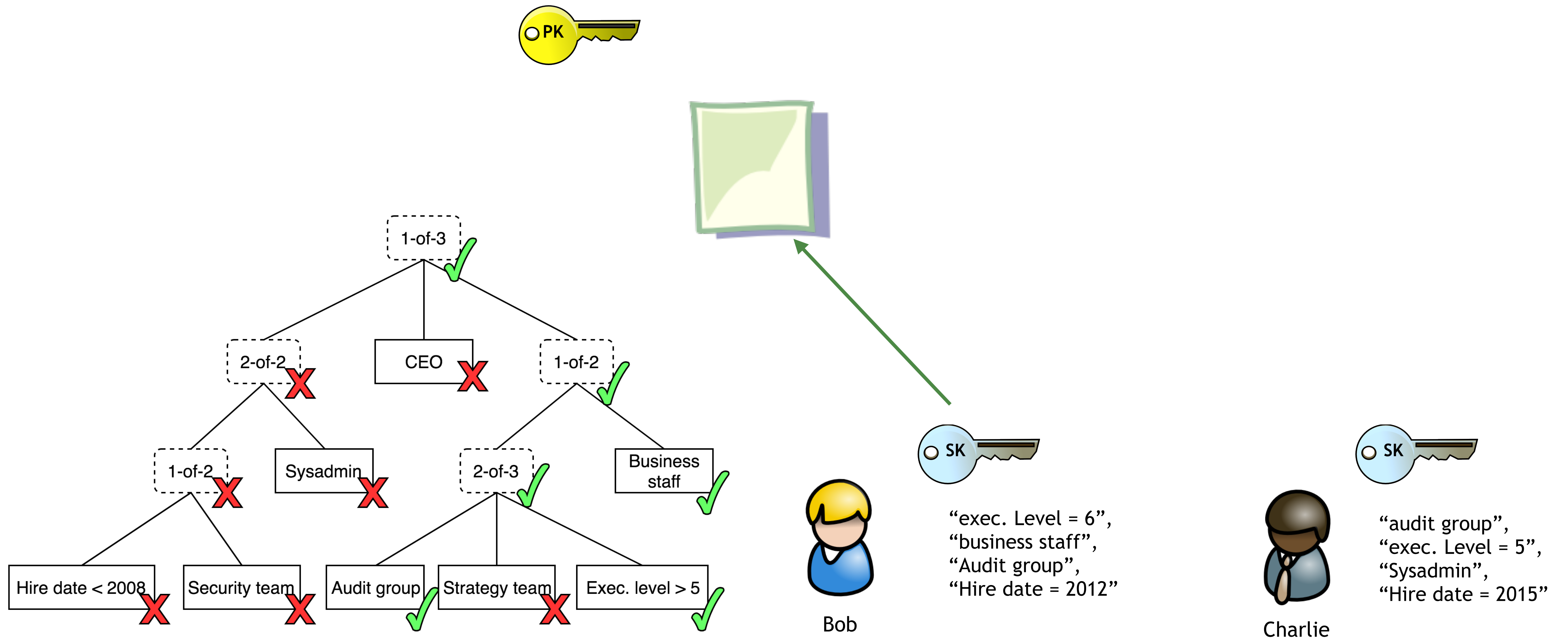


Charlie



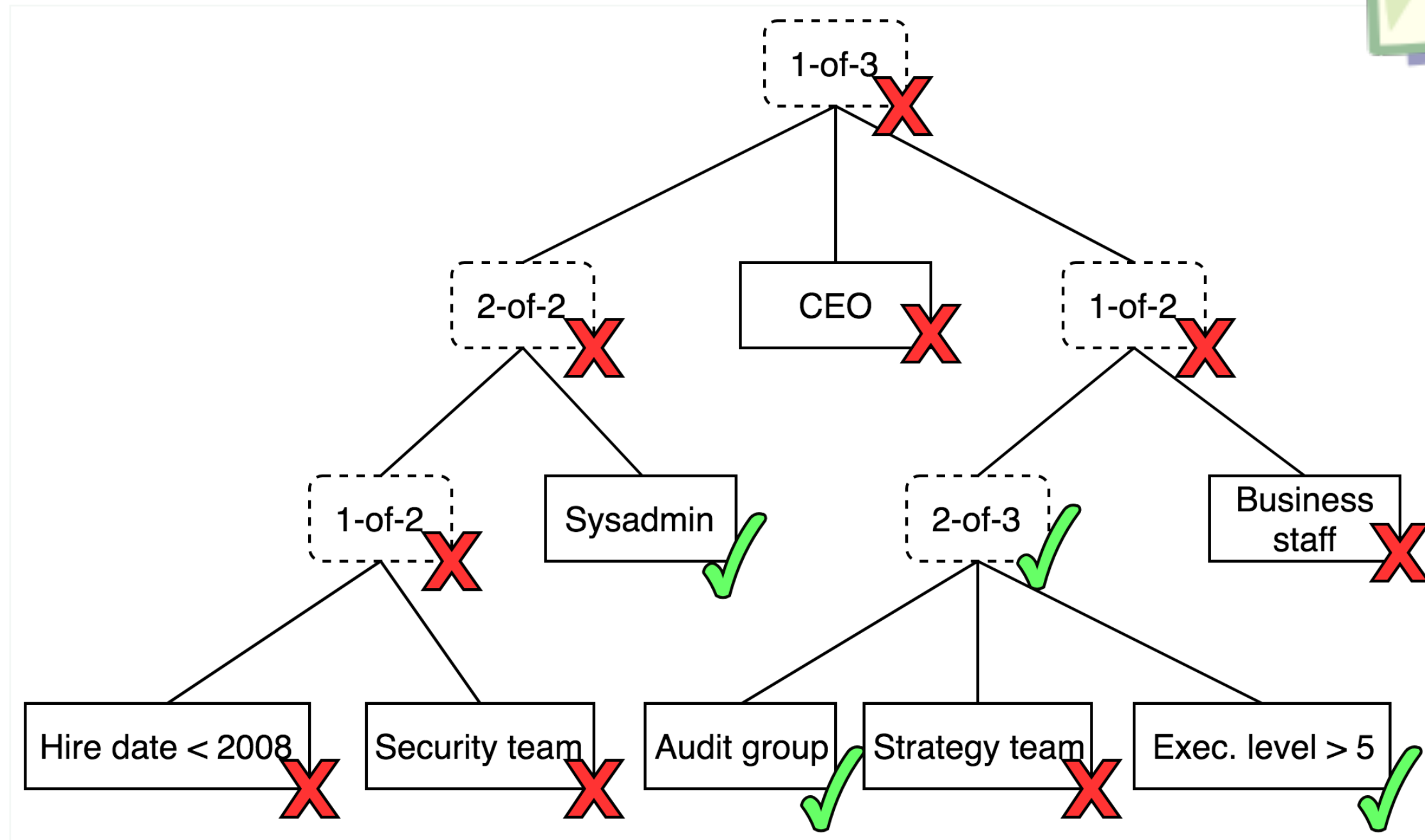
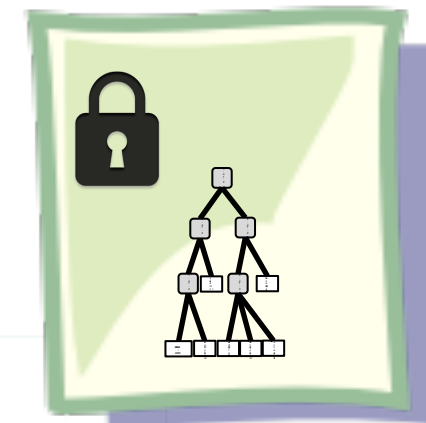
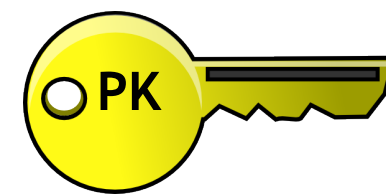
“audit group”,  
“exec. Level = 5”,  
“Sysadmin”,  
“Hire date = 2015”

# CP-ABE Protocol - Decryption





# CP-ABE Protocol - Decryption



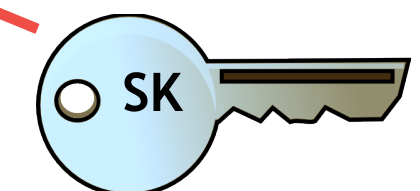
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“exec. Level = 6”,  
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Charlie



“audit group”,  
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# Cryptography



- CP-ABE is based on Pairing Based Cryptography on elliptic curves
- Chosen Ciphertext Attack secure
- Collusion resistance:  
If multiple users collude, they should be able to decrypt a ciphertext if and only if at least one of the users could decrypt it on his own.  
**solution:** randomizing users secret keys so that they cannot be combined.

# Implementation

OpenABE project by Zeutro, written in C++

<https://www.zeutro.com>

<https://github.com/zeutro/openabe>

```
1. mourad@Fortress: /tmp/test (zsh)
→ test echo The plaintext data > file.txt
→ test oabe_setup -s CP
writing 497 bytes to mpk.cpabe
writing 149 bytes to msk.cpabe
→ test oabe_keygen -s CP -i "business_team | hire_date = 2012 | exec_level = 42" -o bob
functional key input: business_team | hire_date = 2012 | exec_level = 42
→ test oabe_keygen -s CP -i "exec_level = 50 | hire_date = 2007 | security_team" -o charlie
functional key input: exec_level = 50 | hire_date = 2007 | security_team
→ test oabe_enc -s CP -i file.txt -o ciphertext -e "((hire_date < 2008 OR business_team) AND exec_level > 9000) OR security_team"
input file: file.txt
encryption functional input: ((hire_date < 2008 OR business_team) AND exec_level > 9000) OR security_team
→ test oabe_dec -s CP -k bob.key -i ciphertext.cpabe -o bob_dec.txt
ciphertext: ciphertext.cpabe
user's SK file: bob.key
abe/zcontextcca.cpp:decrypt:613: 'Error occurred during decryption'
caught exception: Error occurred during decryption
→ test oabe_dec -s CP -k charlie.key -i ciphertext.cpabe -o charlie_dec.txt
ciphertext: ciphertext.cpabe
user's SK file: charlie.key
→ test cat charlie_dec.txt
The plaintext data
```

# Performance



- KeyGen is linear in number of attributes associated with the key it is issuing
- Encryption is linear in number of leaf nodes in the policy tree
- Decryption perf depends on the particular policy tree shape, but roughly linear in the number of leaf nodes of the policy

# Drawbacks



Revocation is complex:

- Many users have the same attributes, how do we exclude one of them ?
- Use expiration dates in the keys => continuous update of the SKs, not so fine-grained revocation and non retroactive revocation.

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Centralized key issuer(s):

- Need to be trusted with full power
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Based on discrete log => not quantum secure

# Variants

# Different types of ABE



- Ciphertext-Policy ABE: keys are bound to a set of attributes and the file is encrypted following an access policy  
Role based access control
- Key-Policy ABE: file has a set of attributes attached to it, keys are generated using an access policy  
Content based access control

KP vs CP: the intelligence (the one who actually defined the policy) is assumed to be with the key issuer vs encryptor



# Different types of ABE



- Multi-Authority ABE: Different authorities, each one is responsible for a subset of all attributes
- IR-CP-ABE: Identity revokable CP-ABE (W. Wang et al. - 2017)
  - KeyGen:  $(MSK, \mathcal{S}, ID) \rightarrow SK$
  - Encrypt:  $(PK, plaintext, \mathbb{A}, \{ID_j\}) \rightarrow ciphertext$   
where  $\{ID_j\}$  is the set of revoked IDs

# Functional Encryption



generalization of public-key encryption in which possessing a secret key allows one to learn a function of what the ciphertext is encrypting.

- Setup:  $1^\lambda \rightarrow (MK, PK)$
- KeyGen:  $(MK, k) \rightarrow SK$ , where  $k \in K$  the key space
- Enc:  $(PK, m) \rightarrow c$
- Dec:  $(SK, c) \rightarrow F(k, m)$  where  $F$  defined over  $(K, M)$

# Functional Encryption



Dec:  $(SK, c) \rightarrow F(k, m)$  where  $F$  defined over  $(K, M)$

$K = \{0,1\}^n$   $n$  bit strings representing  $n$  boolean variables  $\vec{z} = (z_1, \dots, z_n)$

$M = (I, X)$  where  $I =$  set of all poly-sized bool formulas over  $n$  variables  
*i.e.*  $m = (\phi, msg)$

$$F(k, (\phi, msg)) = \begin{cases} m & \text{if } \phi(k) = 1 \\ \perp & \text{otherwise} \end{cases}$$

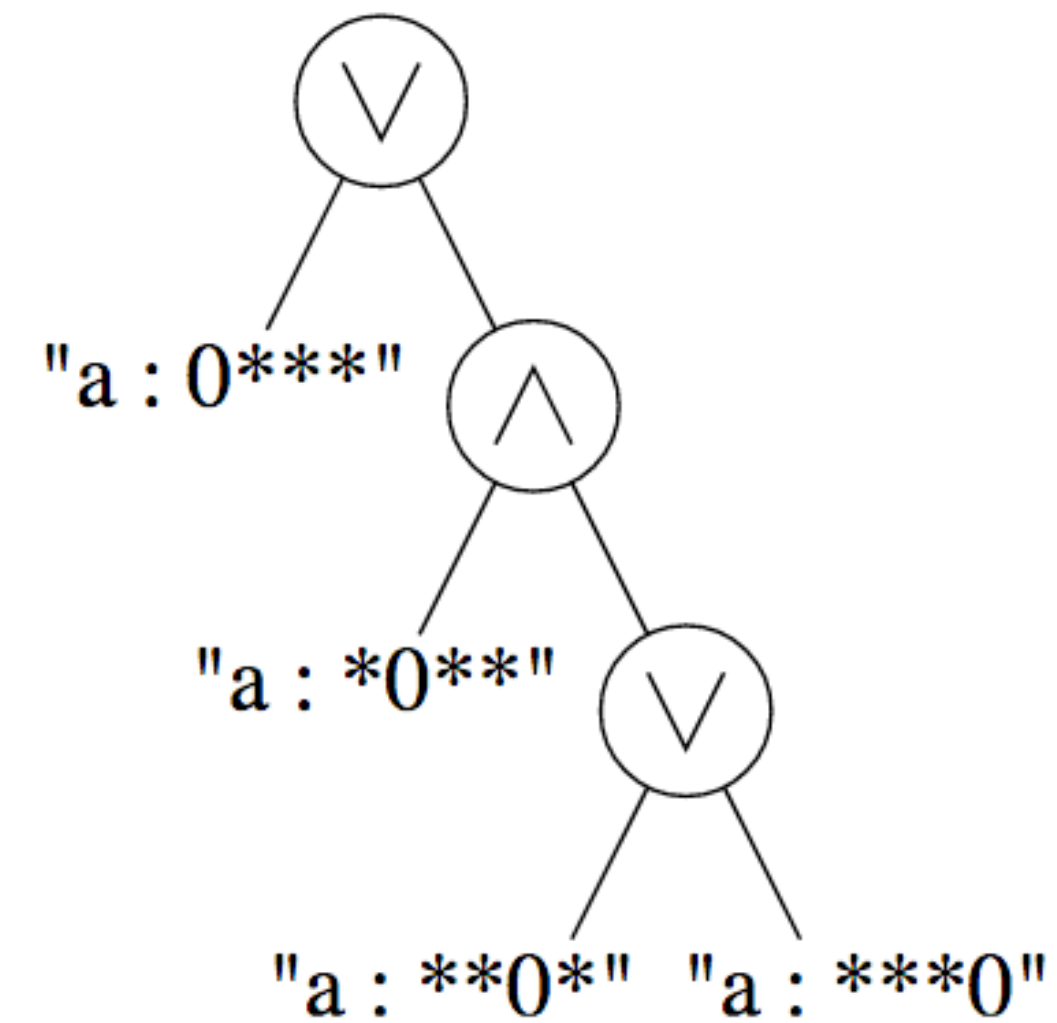
# Questions ?

# Appendices

# Appendix I: Access structure



Policy tree implementing integer comparison " $a < 11$ "



# Appendix II: Cryptography



## Setup

For a bilinear group  $\mathbb{G}$  of prime order  $p$ , generator  $g$  and pairing  $e$  and a randomly chosen  $\alpha, \beta \in \mathbb{Z}_p$

$$\begin{cases} MPK = \mathbb{G}, g, h = g^\beta, f = g^{1/\beta}, e(g, g)^\alpha \\ MSK = \beta, g^\alpha \end{cases}$$

# Appendix II: Cryptography



## Key Generation

Pick random,  $r \in \mathbb{Z}_p$ ,  $r_j \in \mathbb{Z}_p$ ,  $\forall j \in \mathcal{S}$

$$SK = g^{(\alpha+r)/\beta}, \forall j \in \mathcal{S} : g^r \cdot H(j)^{r_j}, g^{r_j}$$



# Appendix II: Cryptography

## Encryption

To encrypt a message  $M$  following an access tree  $\mathcal{T}$ ,  
for each node  $x \in \mathcal{T}$ , we create  $q_x \in \mathbb{Z}_p[X]$  recursively s.t.

- $d_x = \deg(q_x) = k_x - 1$  where  $k_x$  is the threshold value of the gate at node  $x$
- $q_{root}(0) = s \in \mathbb{Z}_p$  chosen randomly, and set  $d_{root}$  other points randomly
- for  $x \neq root$ ,  $q_x(0) = q_{parent(x)}(index(x))$  and set  $d_x$  other points randomly

$$ciphertext = (\mathcal{T}, M \cdot e(g, g)^{\alpha s}, h^s, \forall y \in Y : g^{q_y(0)}, H(att(y))^{q_y(0)})$$

where  $Y$  is the set of leafs of the tree (the actual attributes)

# Appendix II: Cryptography

## Decryption

To decrypt a ciphertext  $CT = (\mathcal{T}, \tilde{C}, C, \forall y \in Y : C_y, C'_y)$   
with a secret key  $SK = (d, \forall j \in \mathcal{S} : D_j, D'_j)$   
we define recursively  $DecryptNode(CT, SK, x)$

- if  $x$  leaf,  $DecryptNode(CT, SK, x) = \begin{cases} \frac{e(D_{att(x)}, C_x)}{e(D'_{att(x)}, C'_x)} = e(g, g)^{rq_x(0)} & \text{if } att(x) \in \mathcal{S} \\ \perp & \text{if } att(x) \notin \mathcal{S} \end{cases}$

- Then for  $x \in \mathcal{T}$ , if we have a  $k_x$ -sized subset  $\mathcal{S}_x$  of  $x$ 's children that verify  $F_z = DecryptNode(CT, SK, z) \neq \perp$  for  $z \in \mathcal{S}_x$

$$DecryptNode(CT, SK, x) = \prod_{z \in \mathcal{S}_x} F_z^{\Delta_{i, \mathcal{S}_x}(0)} = e(g, g)^{rq_x(0)}$$

- if we go back to the root, we find  $F_{root} = e(g, g)^{rs}$

$$M = \tilde{C} / (e(C, D) / F_{root}) = \tilde{C} / \left( e \left( h^s, g^{(\alpha+r)/\beta} \right) / e(g, g)^{rs} \right)$$