Bài tập số 2: Chuỗi số.

Bài 1. Trong các chuỗi sau, chuỗi nào phân kỳ theo điều kiện cần của chuỗi hội tụ:

1. $\sum_{n=1}^{\infty} n$
2. $\sum_{n=1}^{\infty}\left(\frac{2n-3}{2n+1}\right)^n$
3. $\sum_{n=1}^{\infty}(-1)^n$
4. $\sum_{n=1}^{\infty}\frac{1}{2^n}$
5. $\sum_{n=1}^{\infty} a.q^n, q>0$
6. $\sum_{n=1}^{\infty}\frac{2n+3}{3n+2}$
7. $\sum_{n=1}^{\infty}\frac{n}{n+1}$
8. $\sum_{n=1}^{\infty}\left(\frac{3n+2}{3-2n}\right)^n$
9. $\sum_{n=1}^{\infty}\left(\frac{-3n}{2n+3}\right)^3$
10. $\sum_{n=1}^{\infty}\left(\frac{2n^2+3}{3n^2-1}\right)^{-2}$

Bài 2. Xét tính hội tụ và phân kỳ của các chuỗi số sau và tính tổng (nếu có):

1. $\sum_{n=1}^{\infty}\frac{1}{2^n}$
2. $\sum_{n=1}^{\infty}\frac{3}{4^n}$
3. $\sum_{n=1}^{\infty}3.2^n$
4. $\sum_{n=1}^{\infty}\frac{1}{(2n-1)(2n+1)}$
5. $\sum_{n=1}^{\infty}\ln\left(1+\frac{1}{n}\right)$.

Bài 3. Xét sự hội tụ (phân kỳ) của các chuỗi sau:

1. $\sum_{n=1}^{\infty}\frac{n}{2n+1}$
2. $\sum_{n=1}^{\infty}\frac{(-1)^n}{2n+1}$
3. $\sum_{n=1}^{\infty}\frac{\cos\frac{n\pi}{2}}{n^2+3}$
4. $\sum_{n=1}^{\infty}\frac{\sqrt{n^2+1}}{n^4+3}$
5. $\sum_{n=1}^{\infty}\frac{1}{n}$
6. $\sum_{n=1}^{\infty}\frac{1}{n^2}$
7. $\sum_{n=1}^{\infty}\frac{1}{2^n.\sqrt{n}}$
8. $\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}+2}$
9. $\sum_{n=1}^{\infty}\frac{1}{n(n+1)}$
10. $\sum_{n=1}^{\infty}\frac{n^2+1}{\sqrt{n^5+n^3+1}}$
11. $\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}$
12. $\sum_{n=2}^{\infty}\frac{1}{\sqrt{n}-\sqrt[3]{n}}$
13. $\sum_{n=1}^{\infty}\frac{1}{n(n+1)}$
14. $\sum_{n=1}^{\infty}\sin\frac{1}{n}$
15. $\sum_{n=1}^{\infty}\sin^2\frac{1}{n}$
16. $\sum_{n=2}^{\infty}\frac{1}{n^2\ln n}$
17. $\sum_{n=1}^{\infty}\frac{1}{n\ln n}$
18. $\sum_{n=1}^{\infty}\frac{1}{n2^n}$
19. $\sum_{n=1}^{\infty}\frac{n}{2^n}$
20. $\sum_{n=1}^{\infty}\left(\frac{2n^2+2n-1}{5n^2-2n+1}\right)^n$
21. $\sum_{n=1}^{\infty}\left(\frac{3n}{2n+1}\right)^n$
22. $\sum_{n=1}^{\infty}\frac{1}{(2n-1)2^{2n-1}}$
23. $\sum_{n=2}^{\infty}\frac{1}{(\ln n)^n}$
24. $\sum_{n=1}^{\infty}\left(\frac{2}{n}\right)^n$
25. $\sum_{n=1}^{\infty}\frac{n}{2^n}$
26. $\sum_{n=1}^{\infty}\frac{2^n}{n!}$
27. $\sum_{n=1}^{\infty}\frac{(n!)^2}{2^{n^2}}$
28. $\sum_{n=1}^{\infty}\frac{n^2}{n!}$
29. $\sum_{n=1}^{\infty}\frac{(n!)^2}{(2n)!}$
30. $\sum_{n=1}^{\infty}\frac{n^2}{3^n}$
31. $\sum_{n=1}^{\infty}\ln\left(1+\frac{1}{n}\right)$
32. $\sum_{n=2}^{\infty}\frac{1}{n^2-\ln n}$
33. $\sum_{n=1}^{\infty}\frac{1}{(2n+1)!}$
34. $\sum_{n=1}^{\infty}n\sin\frac{\pi}{2n}$
35. $\sum_{n=1}^{\infty}\frac{3}{2^n}$
36. $\sum_{n=1}^{\infty}\frac{(-1)^n}{n}$
37. $\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}$
38. $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n+2^n}$
39. $\sum_{n=1}^{\infty}\frac{\cos\frac{n\pi}{2}}{(n+1)(n+2)}$
40. $\sum_{n=1}^{\infty}\frac{(-1)^n\sqrt{n}}{n^2+3}$
41. $\sum_{n=2}^{\infty}\frac{(-1)^n n}{n^2-1}$
42. $\sum_{n=1}^{\infty}\frac{(-1)^n\ln n}{n}$
43. $\sum_{n=1}^{\infty}\frac{(-3)^n}{n^3}$
44. $\sum_{n=1}^{\infty}(-1)^n\sin\frac{\alpha}{n}$
45. $\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2-3}$