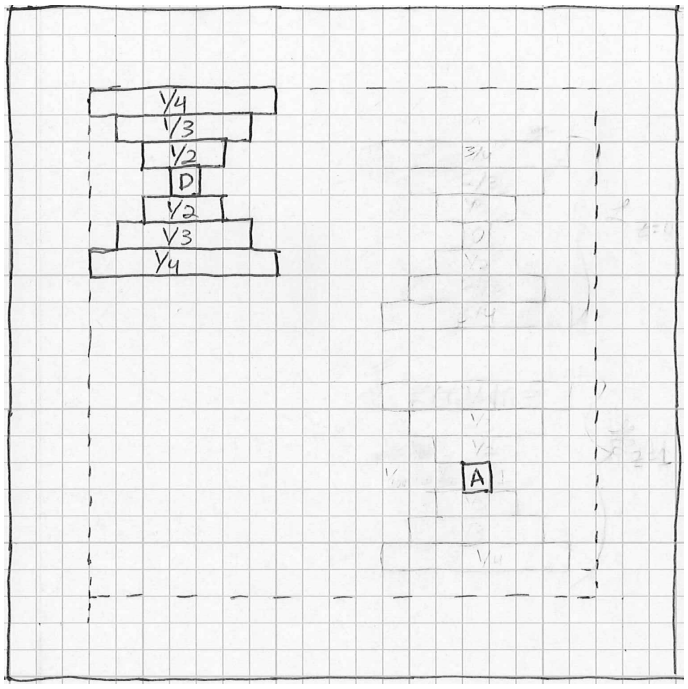
Homework 3: Information

1. (20pts) For an unfair coin with $p(H) = 0.8999$ determine the correct order of queries that should be used to determine the outcome of four coin flips. Remember that there are 16 total combinations, so there are 16 total possible states. **Turn In:** Turn in the graph that represents the order of the queries and the expected number of questions required to determine the state. (Huffman coding may be helpful here, if you wish to use it. However, to use it you must enumerate the queries you are willing to use—in this case 16 of them—prior to generating the graph. The Wikipedia article on Huffman coding is useful.

2. (20pts) Plot the Fisher information for 5 flips of a coin. **Turn In:** A plot of the Fisher information on the interval $\theta \in [0, 1]$.

3. (20pts) Compute the ergodic metric of a trajectory of $\dot{x} = \begin{bmatrix} 0 & 1 \\ -1 & -b \end{bmatrix} x$ with respect to the normally distributed distribution $\phi(x) = \det(2\pi\Sigma)^{\frac{-1}{2}} \exp\left(-\frac{1}{2}(x-\mu)^T\Sigma^{-1}(x-\mu)\right) = \mathcal{N}(x;\mu,\Sigma)$ for $\mu = 0$, $\Sigma = Diag(2,2)$, $x(0) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, a time horizon of $T = 100$ s, and $b = 0$. **Turn In:** A plot of the ergodic metric as a function of b. Is there a most ergodic choice of b? What is the most ergodic choice if you can choose both b and the time horizon T?

4. (40pts) Implement infotaxis for the localization problem of a door. (We'll say its a Tardis or whatever type of magical/science fiction door you like that doesn't have walls around it, but still does something really cool.) In the figure below, D is the location of the door and the values in the rectangles around the door denote the probability of measuring a 1 in those locations. For all other locations, the probability of measuring a 1 is 1/100.



Turn In: 4 Plots of trajectories generated using Infotaxis when you start from 4 randomly chosen locations of door and agent within the 25×25 grid.