Homework 2: Estimation

1. Implement a particle filter for the vehicle model and input from the first homework, using $dt = 0.1$ and process noise and measurement noise normally distributed with a variance of 0.05. **Turn In:** A plot of the particles every 1 second for a total of 6 seconds. Include your estimate at each time step. It should look something like the figure below:

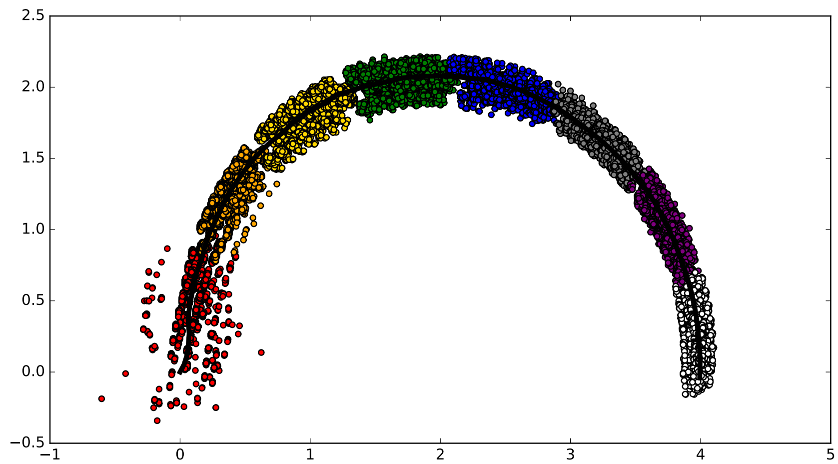

Figure 1: Each color represents a snapshot of the particles at a particular time. The black line is the state estimate of x and y every dt seconds. Notice that after 6 seconds, the particle cloud is clustered very narrowly around the estimate.

2. Implement the Kalman filter for the system

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

   with initial condition $[1, 1]$, $dt = 0.1$, measurements of x, and normally distributed process and measurement noise with variance of 0.1. **Turn In:** The evolution of the state prediction and the covariance of the state prediction for 1 second.

3. For problem 2, numerically demonstrate that the Kalman filter is indeed the optimal linear filter by simulating the the optimal filter and ten ‘nearby’ filters of your choosing for 100 sample paths and looking at the average error for each filter. **Turn In:** The plot of error versus linear filter, indicating which one is the optimal Kalman filter.