Homework 4: Coverage

1. (50pts) Using whatever method you wish, compute a maximally ergodic trajectory for the system $\dot{x} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$ with respect to the normal distribution

$$\phi(x) = \det(2\pi\Sigma)^{\frac{-1}{2}} \exp\left(-\frac{1}{2}(x-\mu)^T\Sigma^{-1}(x-\mu)\right) = \mathcal{N}(x;\mu,\Sigma)$$

for $\mu = 0$, $\Sigma = Diag(2,2)$, $x(0) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ and time horizon of $T = 10\ s$. **Turn In:** A plot of the maximally ergodic trajectory. (This plot should definitively show coverage with respect to the distribution, but given the freedom in this problem everyone's plot should look somewhat different.)

Do ONE of the following problems.

2. (50 pts) Choice 1: Using your ergodic control capability from 1), find a scenario within the infotaxis problem from Homework 3 where ergodic control is more reliable than infotaxis. For instance, having multiple doors in the environment, or distractions that look like doors but are not doors, could work. **Turn In:** Turn in the paths for infotaxis and paths for ergodic control that illustrate the scenario you found. Also write a few sentences about how to interpret the result.

3. (50 pts) Choice 2: Using the single integrator system in the configuration $q = (x, y)$ as a starting point for control synthesis, use ergodic control to generate a Gaussian Process model of the kinematic car and then plan a parallel parking motion (like Homework 1) using the resulting learned model. That is, roughly follow these steps.

   (a) Assume that the experiments are noisy outputs (x, y) of the kinematic car model, but plan an initial exploratory movement using ergodic control based on the single integrator model. This will give you a set of data points to work with to generate a Gaussian Process model of the dynamics.

   (b) Your learned model will start out as a single integrator system and then evolve toward the kinematic car model as even more data is obtained through even more exploration. You may use any exploration method you like, but it must be in terms of (u_1, u_2)—you cannot directly sample in q without executing a trajectory.

   Turn In: Turn in the learned model and the optimized parallel parking trajectory. Also write a few sentences about how you chose your exploration method, how to interpret the quality of the learned model, and the quality of the parallel parking maneuver.

Please note that both the second and third problems are substantially under-specified, as they would be in any practical problem. Clarification of the goal will be provided in the online discussion forum, but students should be prepared to make algorithmic choices about everything from noise levels and discretization granularity, to time horizon of control.