Machine Learning for econometrics

Event studies: Causal methods for panel data

Matthieu Doutreligne March, 11th, 2025

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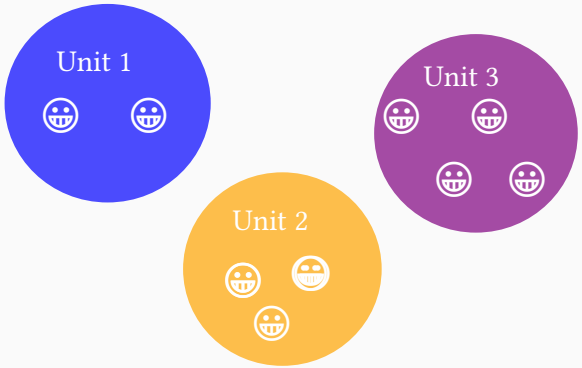
Motivation

Setup - Estimation of the effect of a treatment when data is

- Aggregated: country-level data such as employment rate, GDP...



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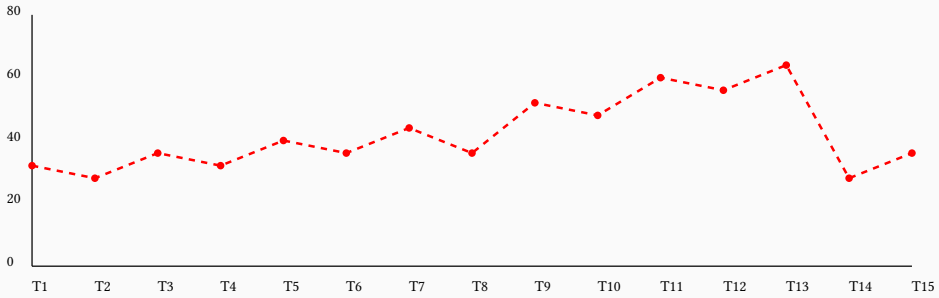

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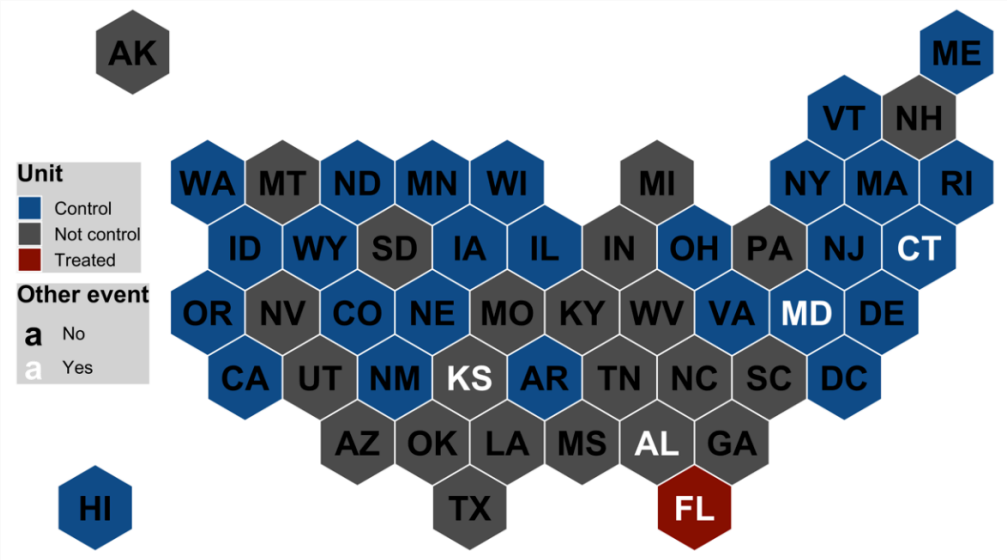

Figure from (Degli Esposti et al., 2020)

Setup - Estimation of the effect of a treatment when data is

- Aggregated: country-level data such as employment rate, GDP…
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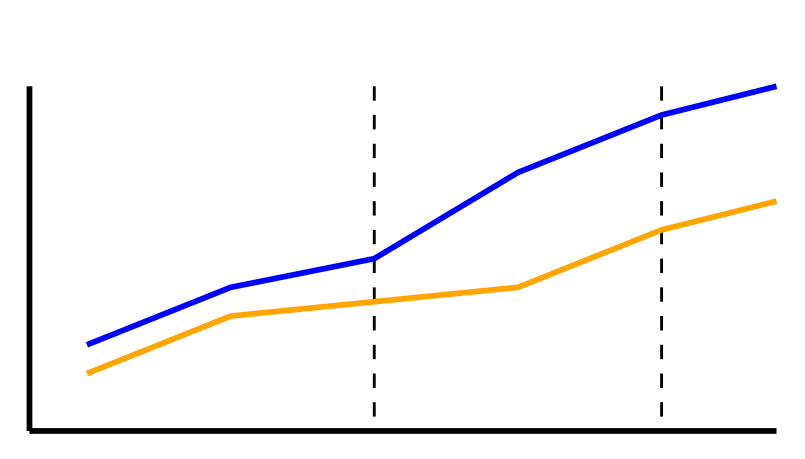
- Staggered adoption of the treatment: units adopt the policy/treatment at different times…



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This setup is known as:

Panel data, event studies, longitudinal data, time-series data.

Examples of event studies

Archetypal questions

- Did the new marketing campaign had an effect on the sales of a product?
- Did the new tax policy had an effect on the consumption of a specific product?
- Did the guidelines on the prescription of a specific drug had an effect on the practices?

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Modern examples

- What is the effect of the extension of Medicaid on mortality? (Miller et al., 2019)
- What is the effect of Europe's protected area policies (*Natura 2000*) on vegetation cover and on economic activity? (Grupp et al., 2023)
- Which policies achieved major carbon emission reductions? (Stechemesser et al., 2024)

Setup: event studies are quasi-experiment

Quasi-experiment

A situation where the treatment is not randomly assigned by the researcher but by nature or society.
It should introduce *some* randomness in the treatment assignment: enforcing treatment exogeneity, i.e. ignorability (i.e. unconfoundedness).

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Other quasi-experiment designs

- Instrumental variables: a variable that is correlated with the treatment but not with the outcome.
- Regression discontinuity design: the treatment is assigned based on a threshold of a continuous variable.

Reminder on difference-in-differences

History

- First documented example (though not formalized): John Snow showing how cholera spread through the water in London (Snow, 1855)[1]
- Modern usage introduced formally by (Ashenfelter, 1978), applied to labor economics

[1] Good description: https://mixtape.scunning.com/09-difference_in_differences#john-snows-cholera-hypothesis

History

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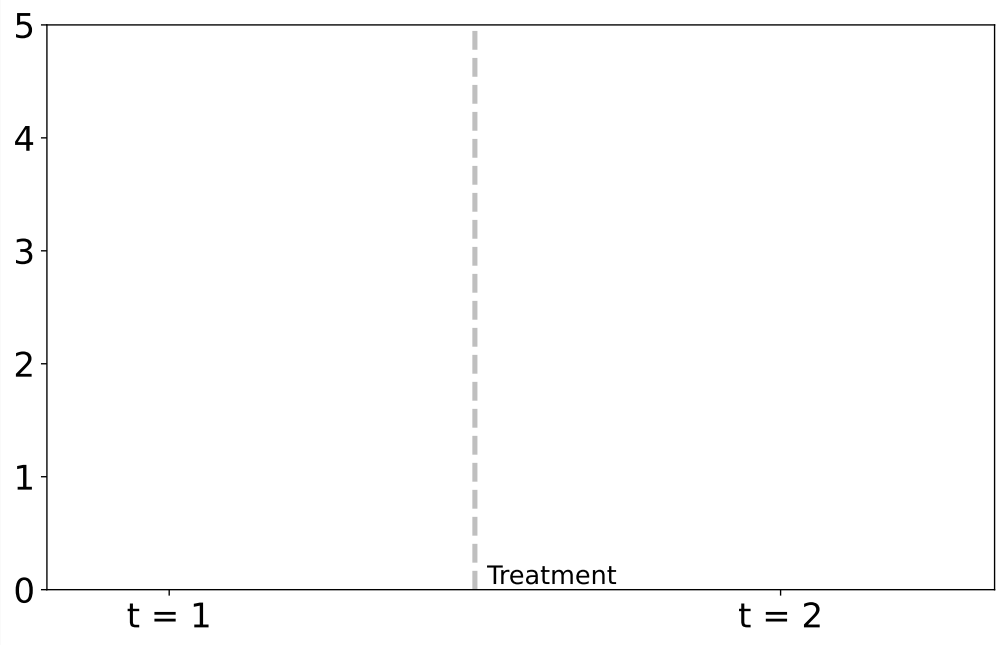
Idea

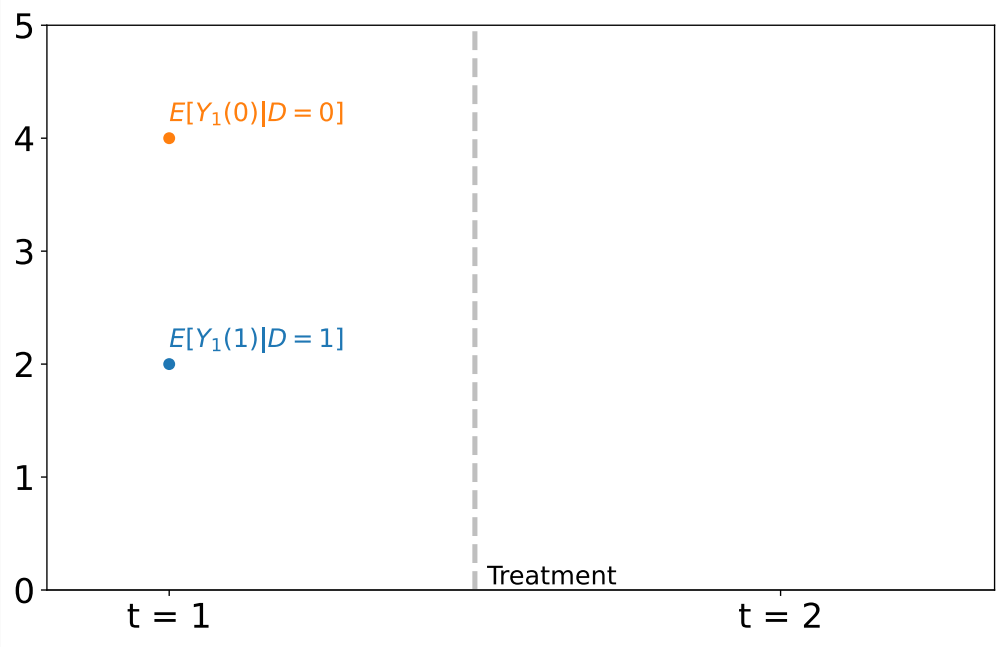
- Contrast the temporal effect of the treated unit with the control unit temporal effect.
- The difference between the two differences is the treatment effect.

[1] Good description: https://mixtape.scunning.com/09-difference_in_differences#john-snows-cholera-hypothesis

Two period of times: t=1, t=2



Potential outcomes: $Y_t(d)$ where $d = \{0, 1\}$ is the treatment at period 2



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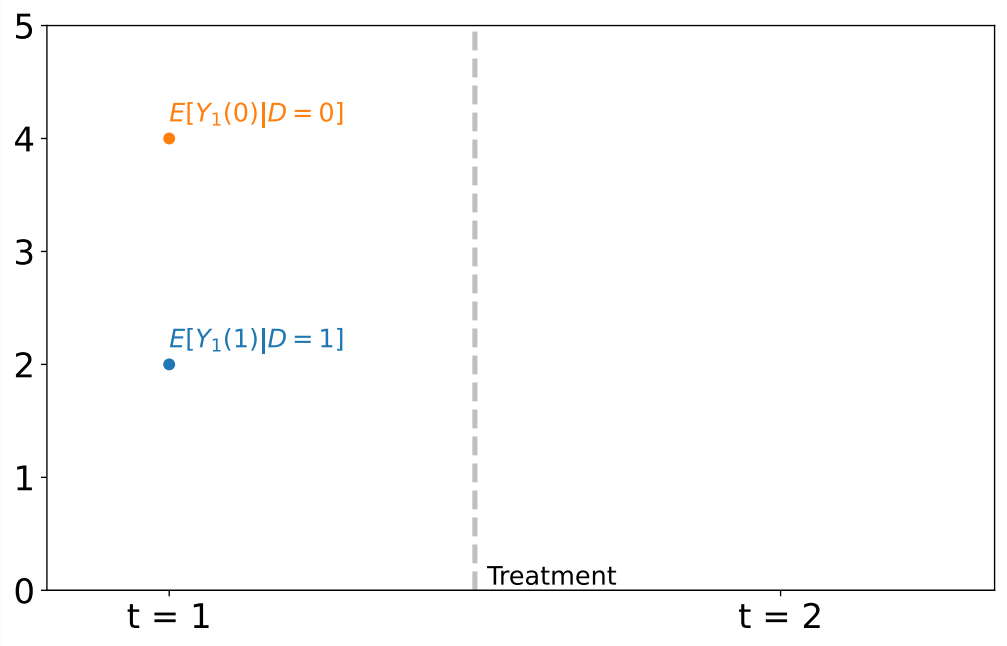
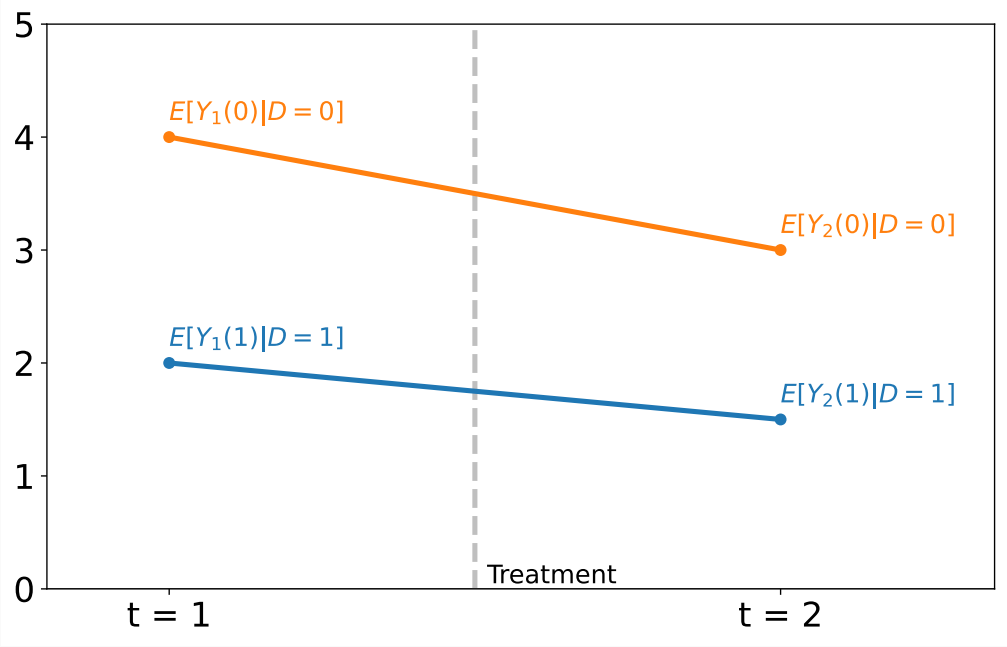
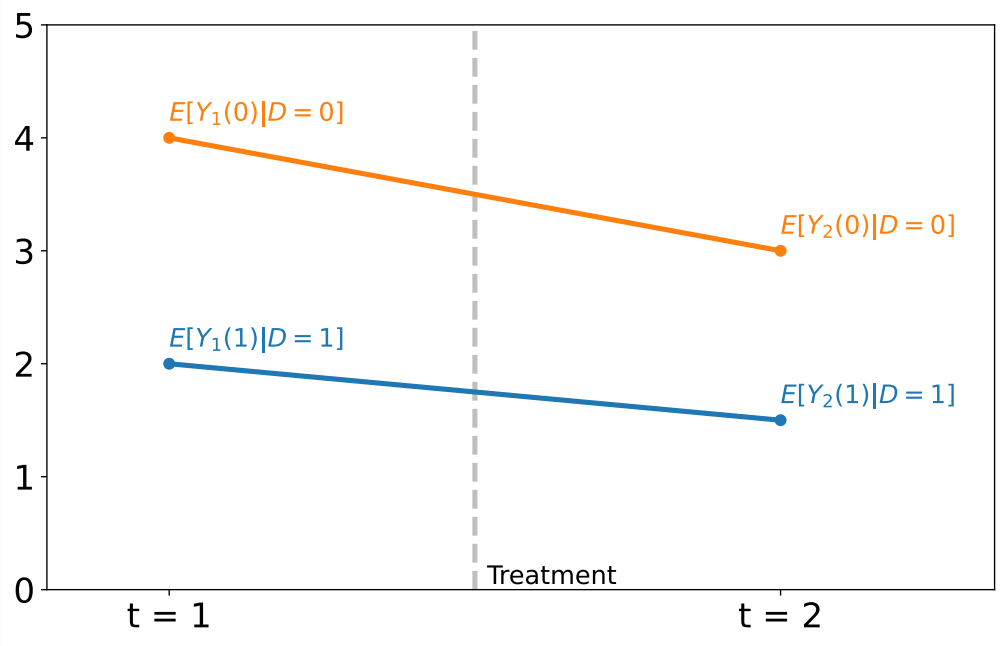


⚠️ $\mathbb{E}[Y_t(0)] = \underbrace{\mathbb{E}[Y_t(0) \mid D = 0]}_{\text{observed}}\mathbb{P}(D = 0) + \underbrace{[Y_t(0) \mid D = 1]}_{\text{counterfactual}}\mathbb{P}(D = 1)$

Plotting all observed data



Our target is the average treatment effect on the treated (ATT)



$$\tau_{\text{ATT}} = \mathbb{E}[Y_2(1)|\ D=1] - \mathbb{E}[Y_2(0)|\ D=1]$$

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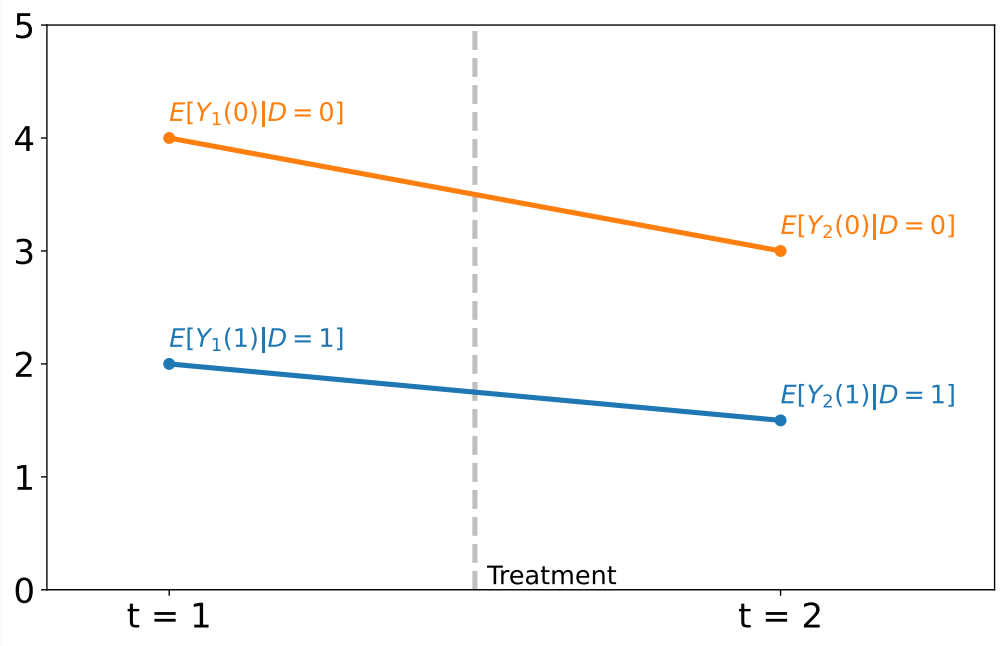
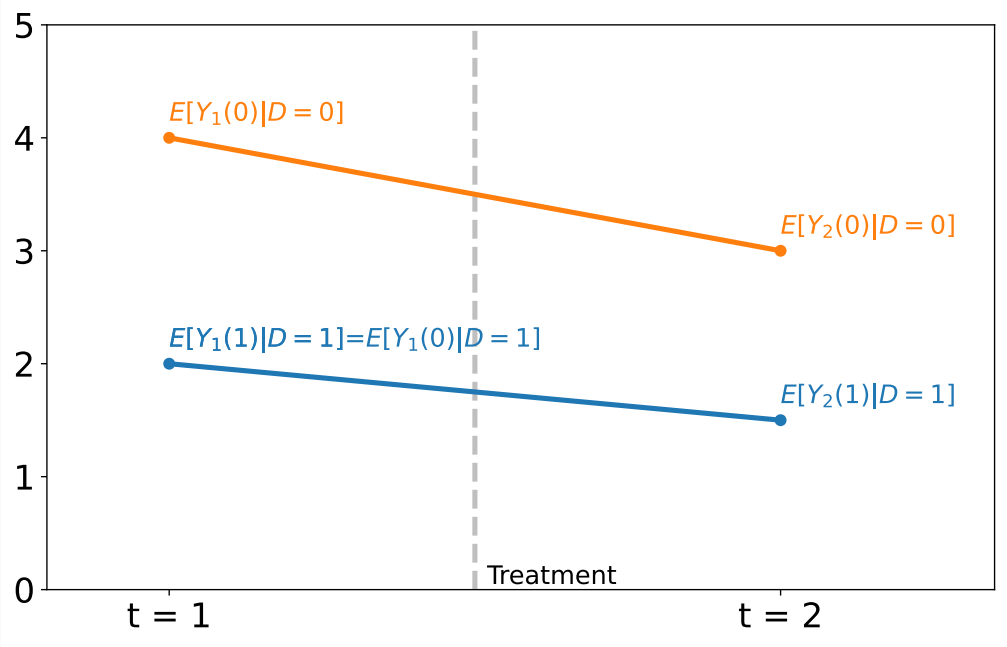


$$\tau_{\text{ATT}} = \underbrace{[Y_2(1)|\ D=1]}_{\text{treated outcome for t=2}} - \underbrace{\mathbb{E}[Y_2(0)|\ D=1]}_{\text{unobserved counterfactual}}$$

First assumption, no anticipation of the treatment

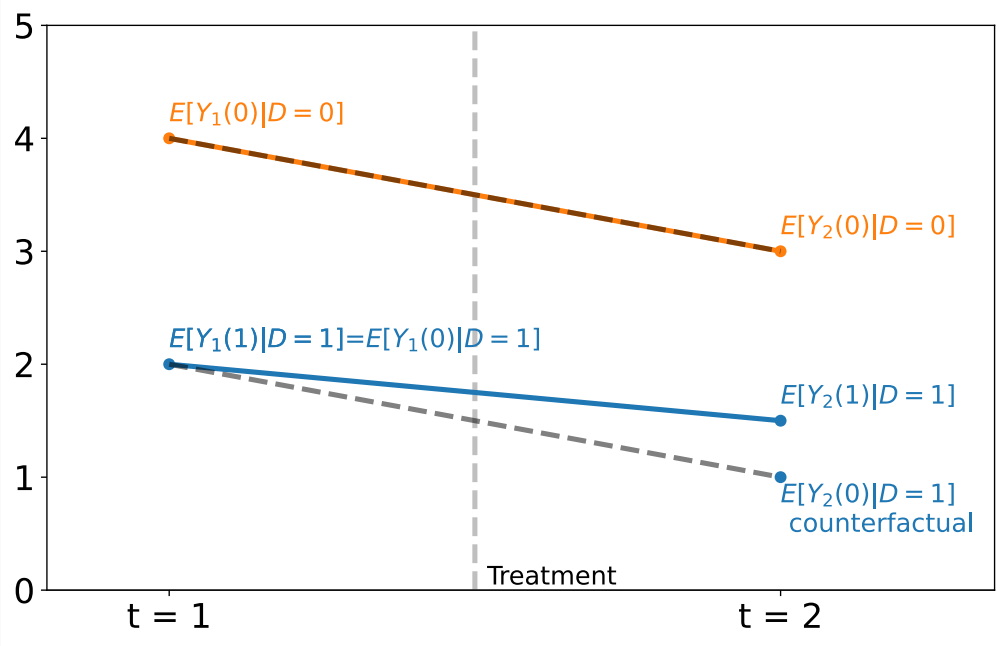
$$\mathbb{E}[Y_1(1)|D=1] = \mathbb{E}[Y_1(0)|D=1]$$



ie. No effect of the treatment before implementation.

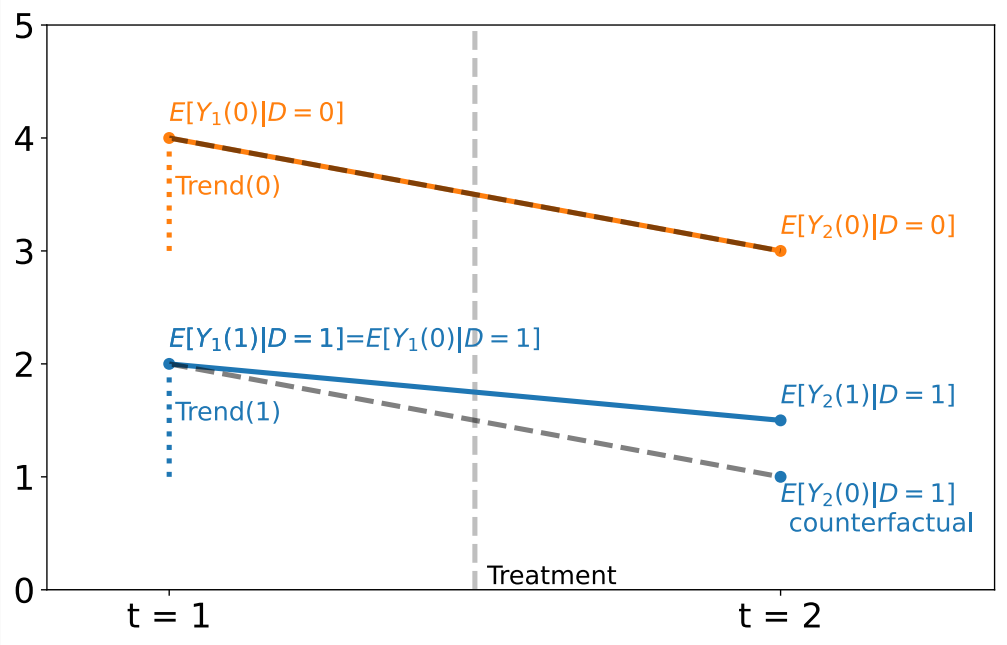
Second assumption, parallel trends

$$\mathbb{E}[Y_2(0) - Y_1(0) \mid D = 1] = \mathbb{E}[Y_2(0) - Y_1(0) \mid D = 0]$$



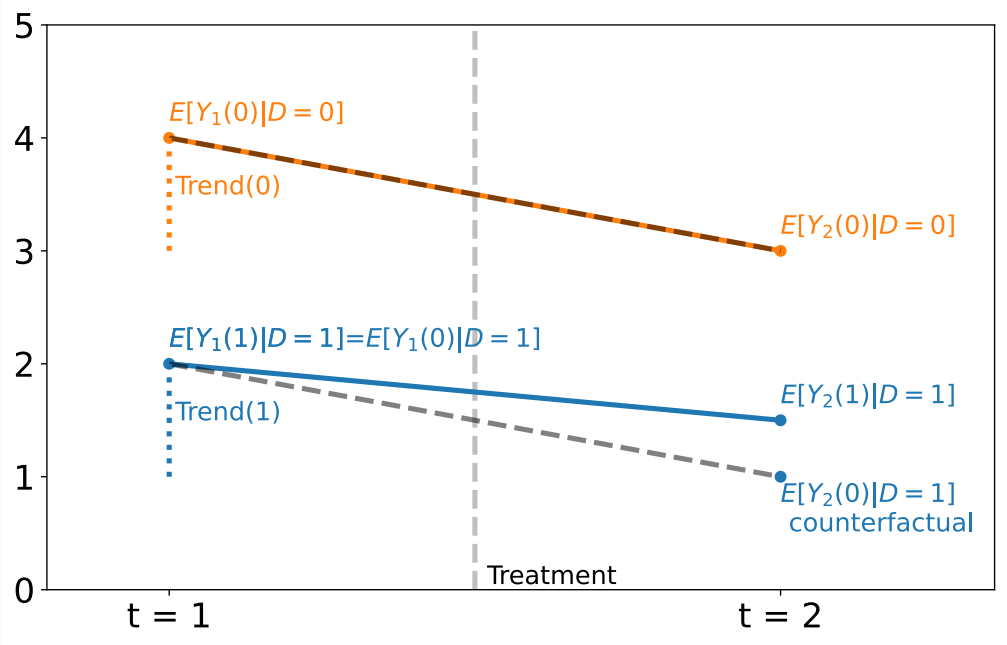
Difference-in-differences

$$\underbrace{[Y_2(0) - Y_1(0) \mid D = 1]}_{\text{Trend}(1)} = \underbrace{\mathbb{E}[Y_2(0) - Y_1(0) \mid D = 0]}_{\text{Trend}(0)}$$



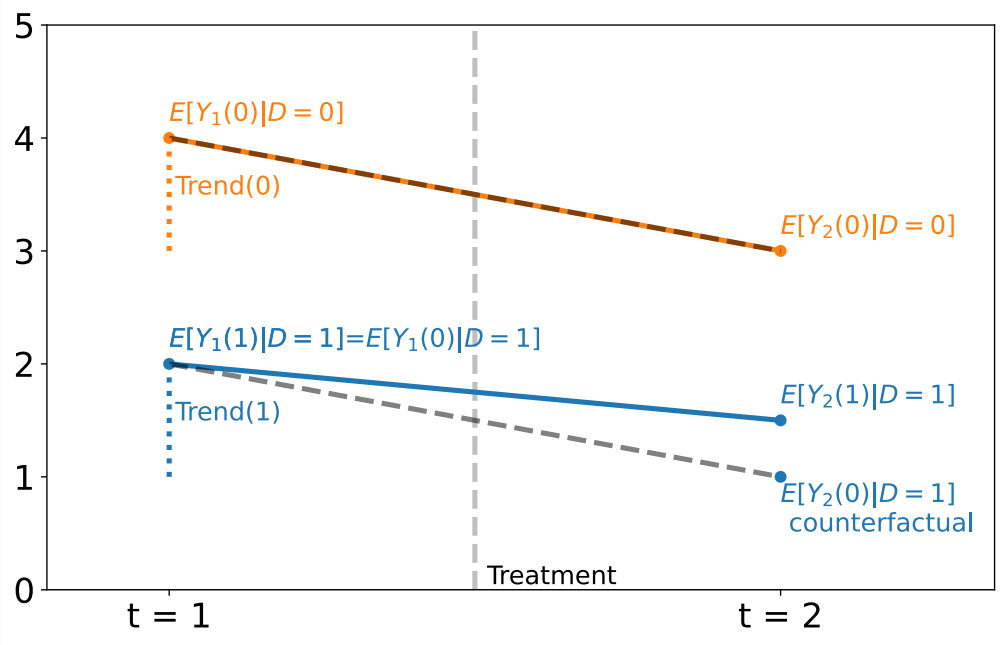
Isolating target unobserved counterfactual

$$\mathbb{E}[Y_2(0) \mid D=1] = \underbrace{\mathbb{E}[Y_1(0) \mid D=1]}_{\text{unobserved counterfactual}} + \underbrace{\mathbb{E}[Y_2(0) - Y_1(0) \mid D=0]}_{\text{Trend}(0)}$$



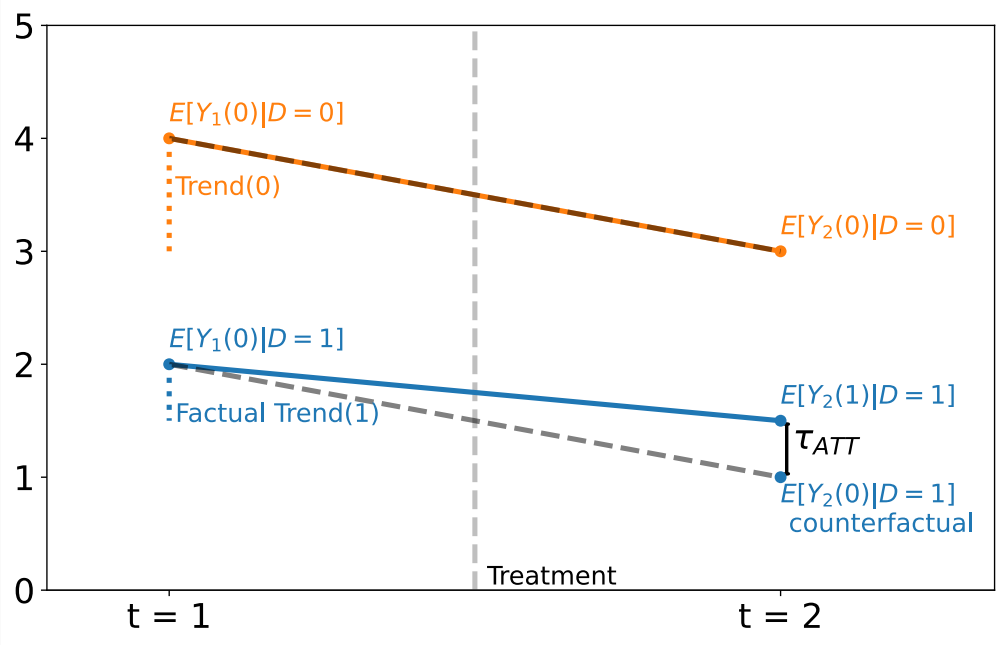
Second assumption, parallel trends

$$\mathbb{E}[Y_2(0) \mid D=1] = \underbrace{\mathbb{E}[Y_1(0) \mid D=1]}_{\mathbb{E}[Y_1(1)|D=1] \text{ (no anticipation)}} + \mathbb{E}[Y_2(0) - Y_1(0) \mid D=0]$$



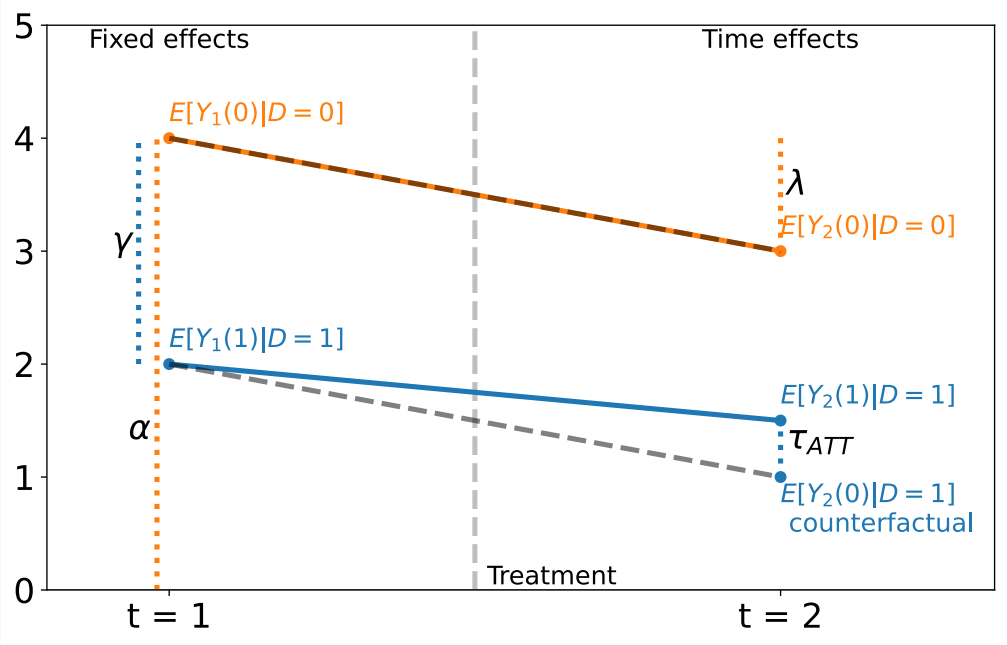
Identification of ATT

$$\tau_{\text{ATT}} = \mathbb{E}[Y_2(1)|\ D=1] - \mathbb{E}[Y_2(0)|\ D=1]$$
$$= \underbrace{\mathbb{E}[Y_2(1)|\ D=1] - \mathbb{E}[Y_1(1)|D=1]}_{\text{Factual Trend}(1)} - \underbrace{\mathbb{E}[Y_2(0)|D=0] - \mathbb{E}[Y_1(0)|D=0]}_{\text{Trend}(0)}$$



Estimation: link with two way fixed effect (TWFE)

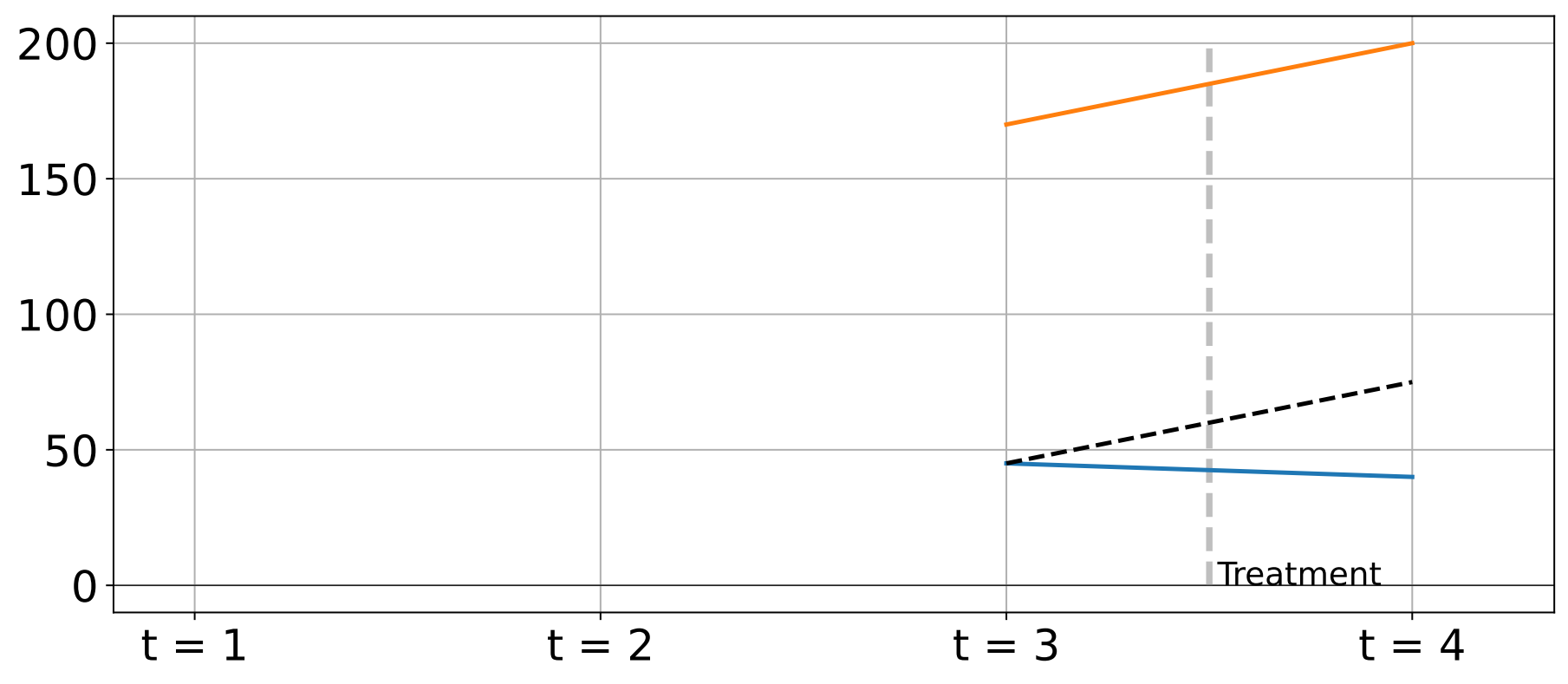
$$Y = \alpha + \gamma D + \lambda \mathbb{1}(t = 2) + \tau_{\text{ATT}} D \mathbb{1}(t = 2)$$



⚠️ Mechanic link: works only under parallel trends and no anticipation assumptions.

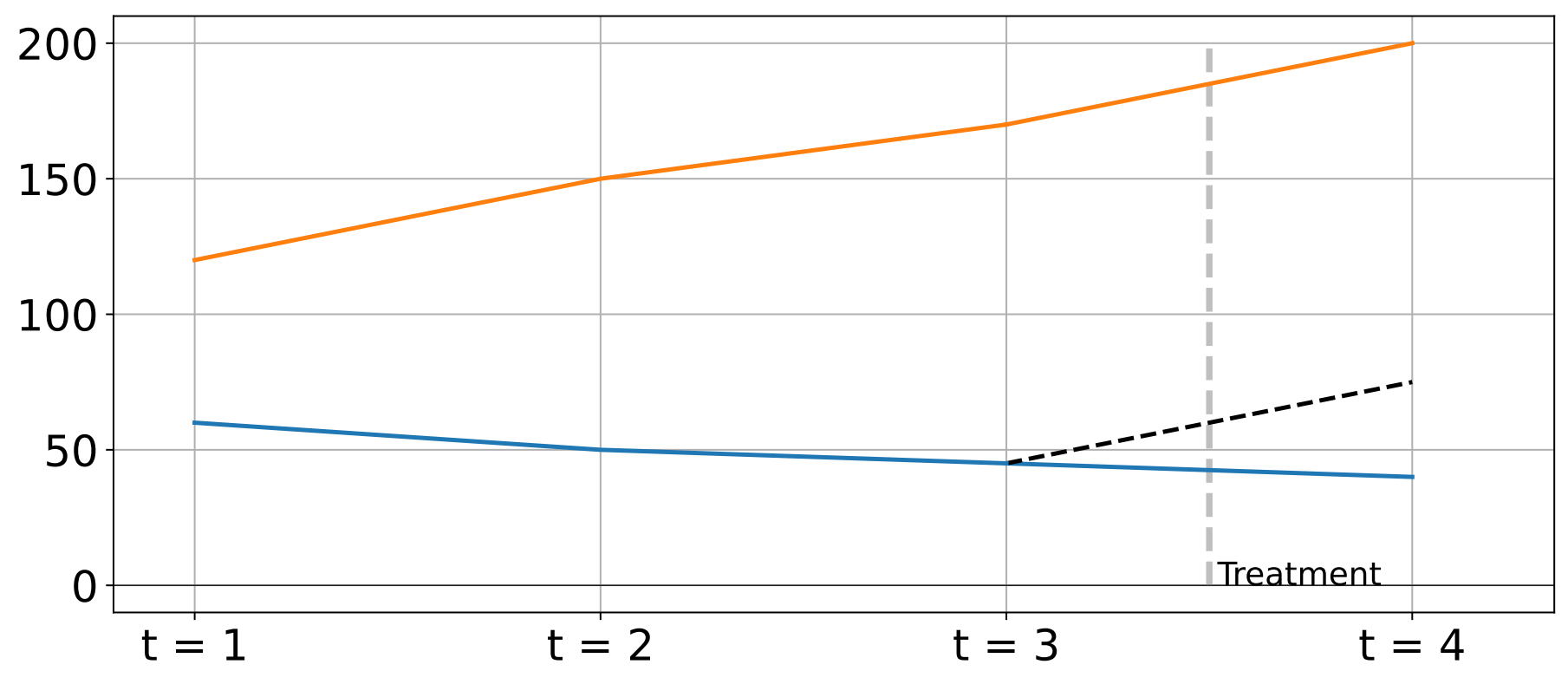
Failure of the parallel trend assumption

Seems like the treatment decreases the outcome!



Failure of the parallel trend assumption

Oups...



DID estimator for more than two time units

Target estimand: sample average treatment effect on the treated (SATT)

$$\tau_{\text{SATT}} = \frac{1}{|\{i:D_i=1\}|}\sum_{i:D_i=1}\frac{1}{T-H}\sum_{t=H+1}^{T} Y_{it}(1) - Y_{it}(0)$$

DID estimator

$$\widehat{\tau_{\text{DID}}} = \frac{1}{|\{i:D_i=1\}|}\sum_{i:D_i=1}\left[\frac{1}{T-H}\sum_{t=H+1}^{T} Y_{it} - \frac{1}{H}\sum_{t=1}^{H} Y_{it}\right] - \frac{1}{|\{i:D_i=0\}|}\sum_{i:D_i=0}\left[\frac{1}{T-H}\sum_{t=H+1}^{T} Y_{it} - \frac{1}{H}\sum_{t=1}^{H} Y_{it}\right]$$

Assumption

No anticipation of the treatment: $Y_{it}(0) = Y_{it}(1) \forall t = 1, ..., H.$

Parallel trend: $\mathbb{E}[Y_{it}(0, \infty) - Y_{i1}(0, \infty)] = \beta_t, t = 2, ..., T.$

See (Wager, 2024) for a clear proof of consistancy.

DID: Take-away

Pros

- Extremely common in economics and quite simple to implement.
- Can be extended to (Wager, 2024)
  - more than two time periods: exact same formulation
  - staggered adoption of the treatment: a bit more complex

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- Does not account for heterogeneity of treatment effect over time (De Chaisemartin & d'Haultfoeuille, 2020).

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Can we force robust to the parallel trend assumption?

Synthetic controls

Synthetic Control Methods (SCM)

Introduced by (Abadie & Gardeazabal, 2003) and (Abadie et al., 2010).

Quick introduction in (Bonander et al., 2021), technical overiew in (Abadie, 2021),

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Idea

Find a weighted average of controls that predicts well the treated unit outcome before treatment.

Example

What is the effect of tobacco tax on cigarettes sales? (Abadie et al., 2010)

Context

1988: 25-cent tax/pack of cigarettes + ban of on cigarette vending machines in public areas accessible by juveniles + ban on single cigarettes sales.

Synthetic control example: California's Proposition 99 (Abadie et al., 2010)

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- Outcome, $Y_{j,t}$: cigarette sales per capita in state j

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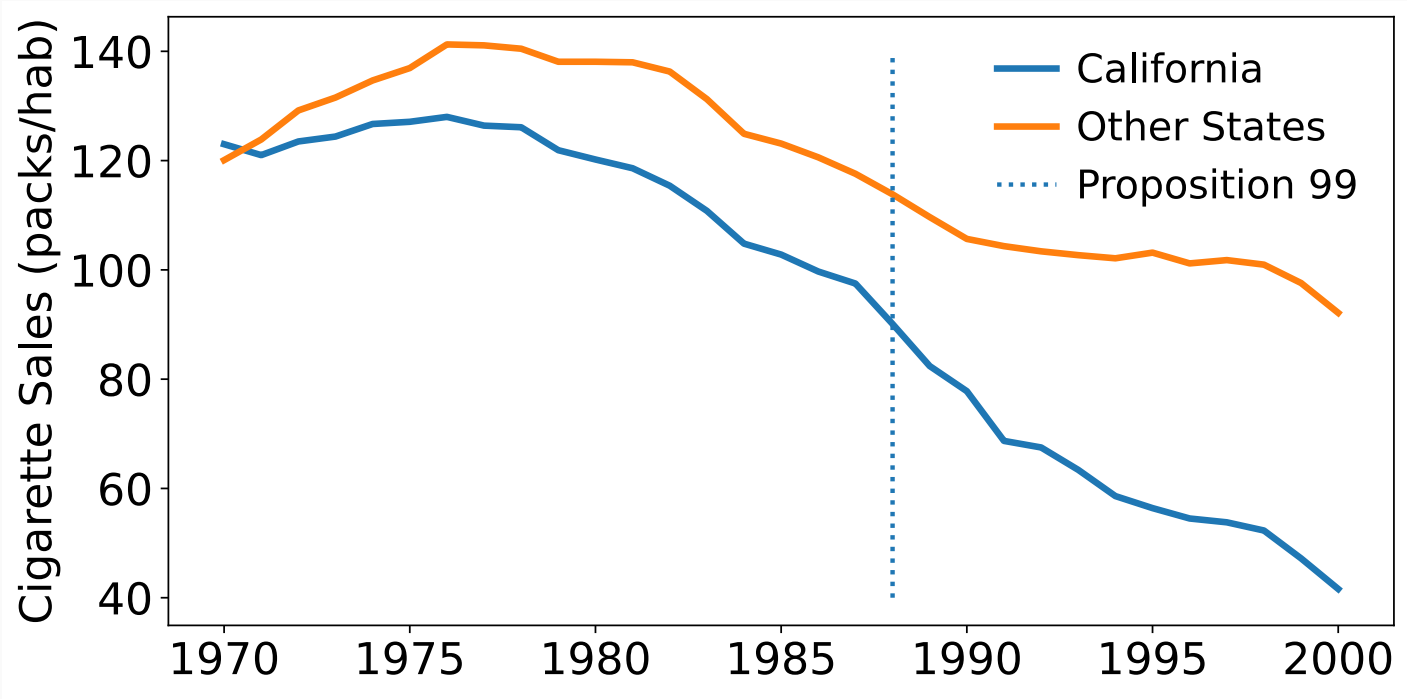
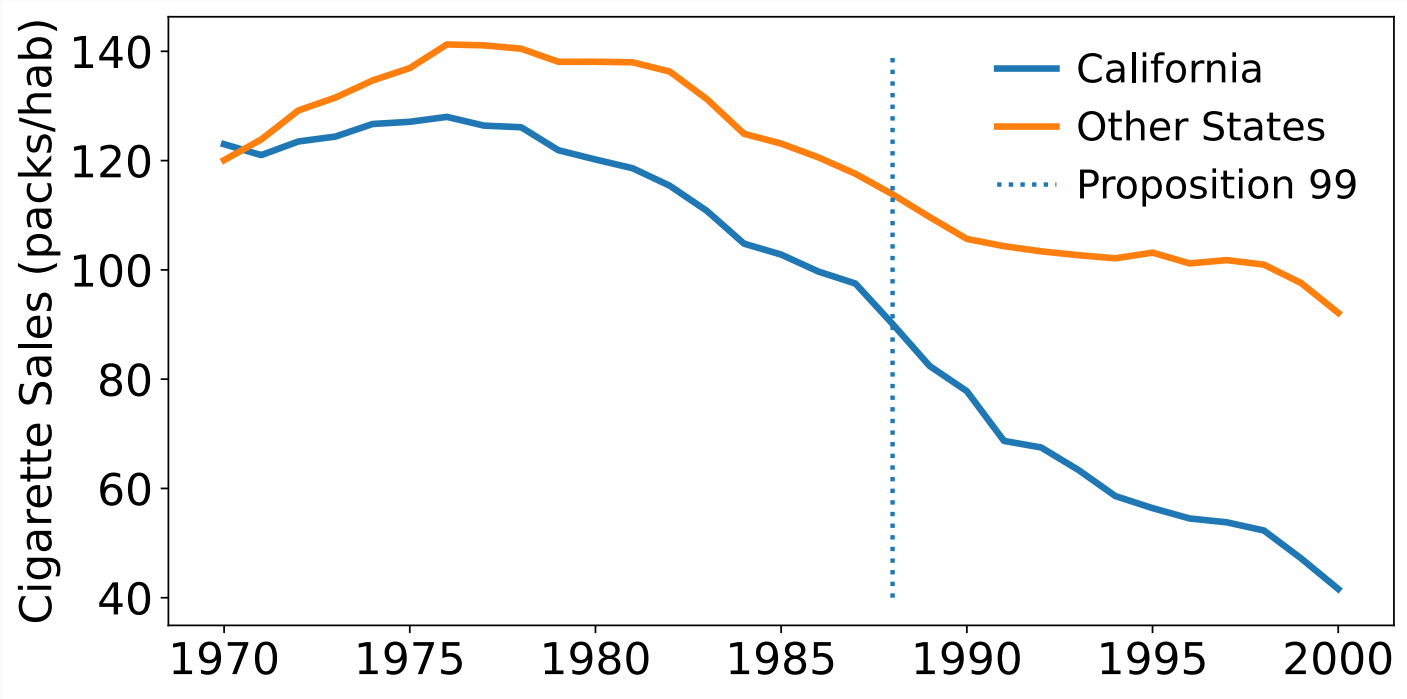
- Time period: $t \in \{1, ..T\} = \{1970, ..2000\}$ and treatment time $T_0 = 1988$
- Covariates $X_{j,t}$: cigarette price, previous cigarette sales.

Synthetic control example: plot the data



Synthetic control example: plot the data



😲 Decrease in cigarette sales in California.

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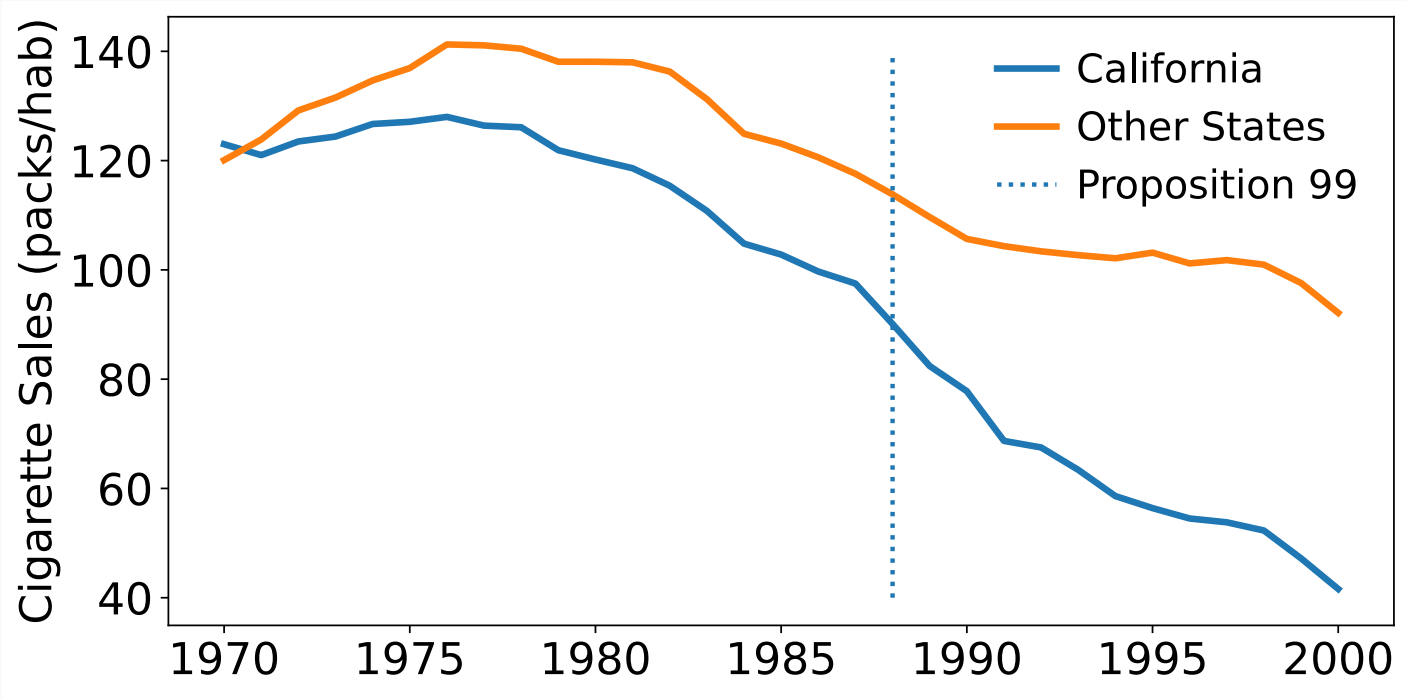

😲 Decrease in cigarette sales in California.

🤔 Decrease began before the treatment and occured also for other states.

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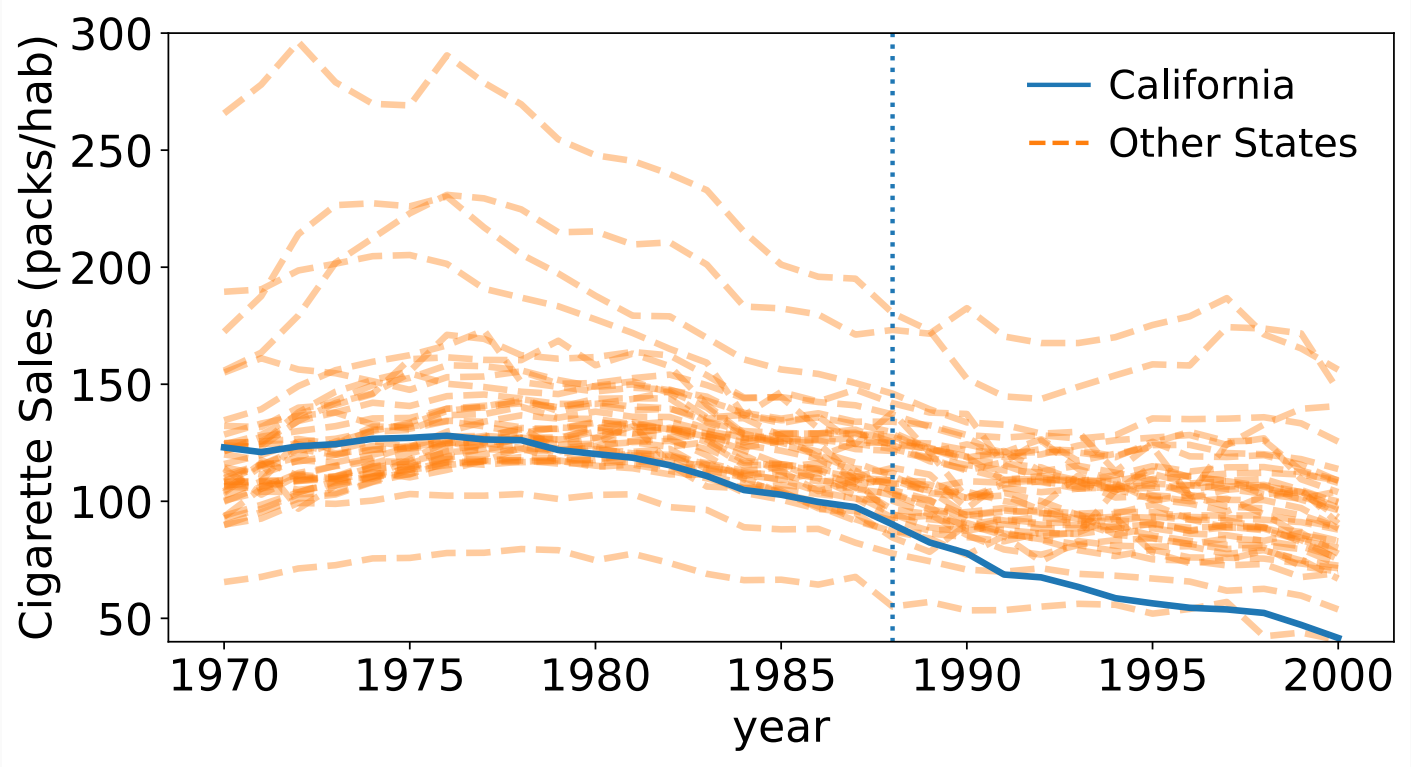

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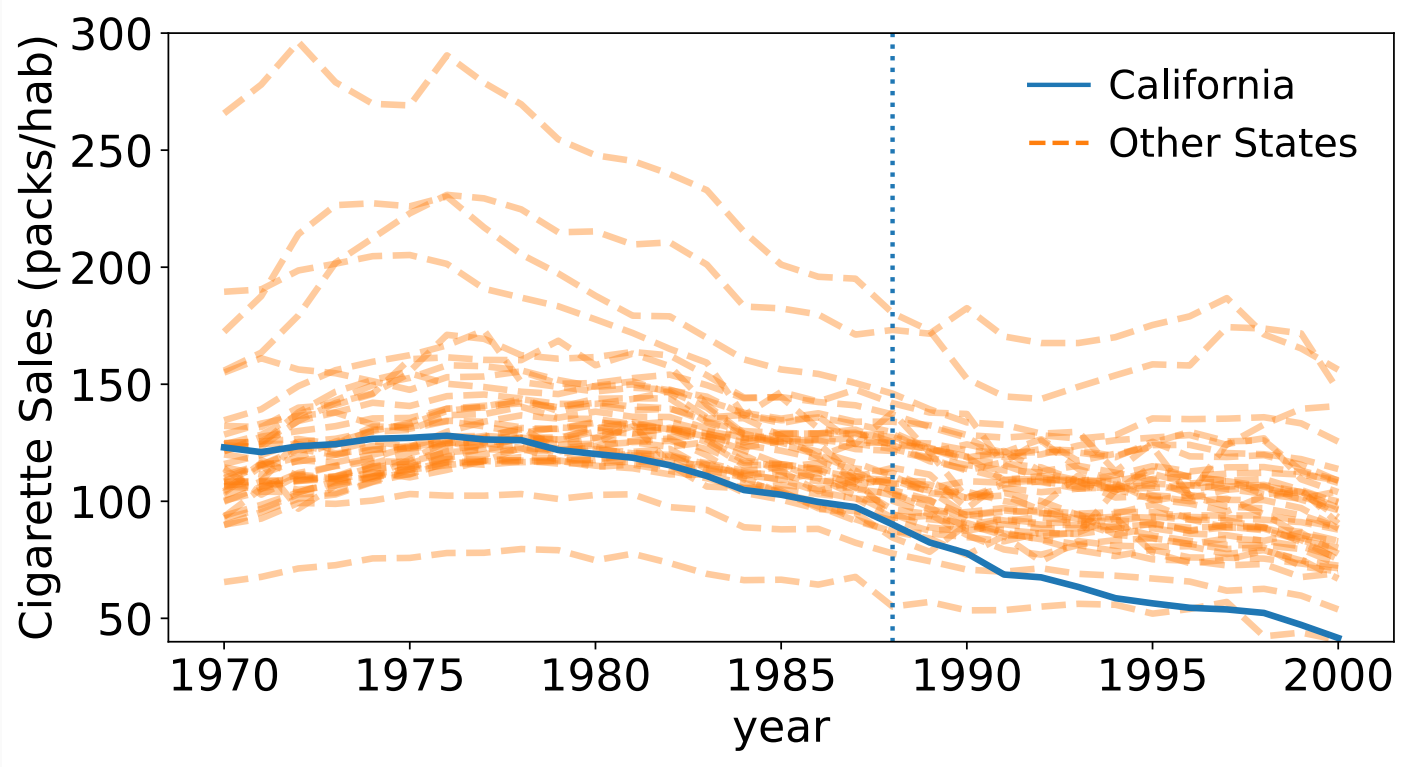
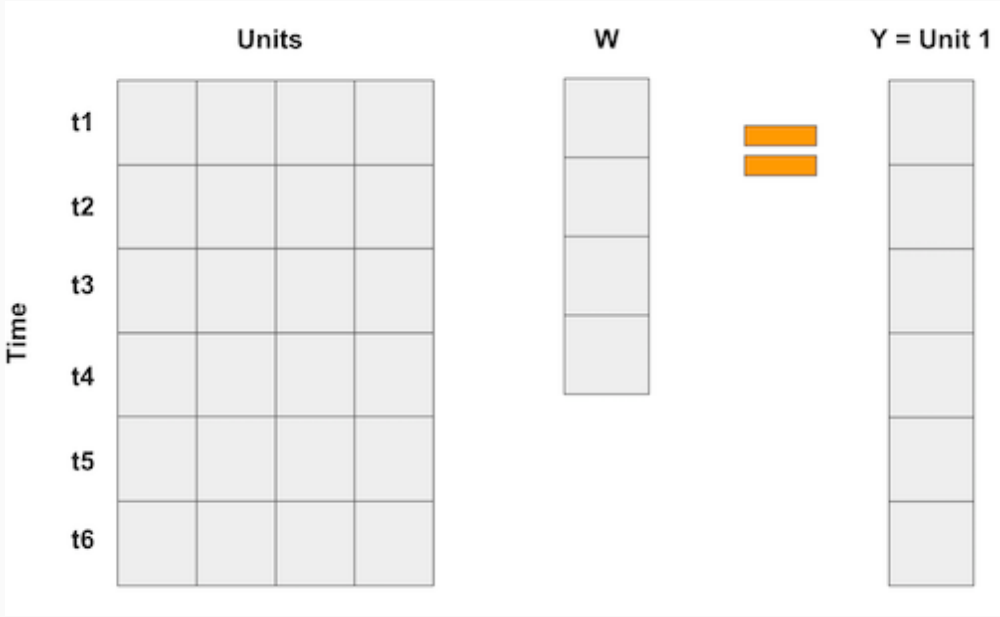


💡 Force parallel trends with a weighted average of other states that predicts well the pre-treatment trend of California (before $T_0 = 1988$).

Synthetic control as weighted average of control outcomes

Build a predictor for $Y_{1,t}$ (California):

$$\hat{Y}_{1,t} = \sum_{j=2}^{n_0+1} \hat{w}_j Y_{j,t}$$



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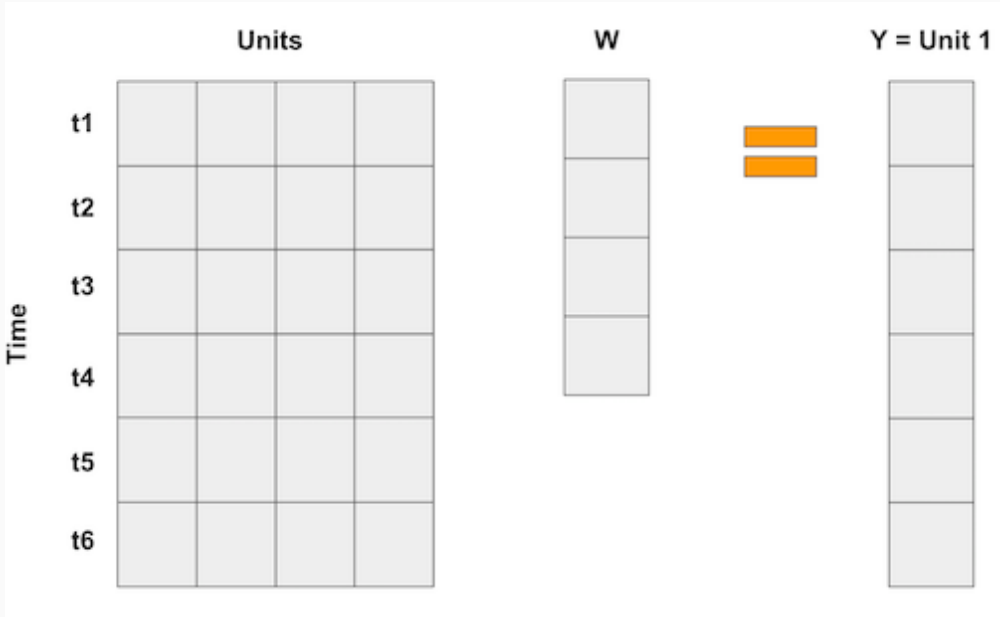
🤔 How to choose these weights?



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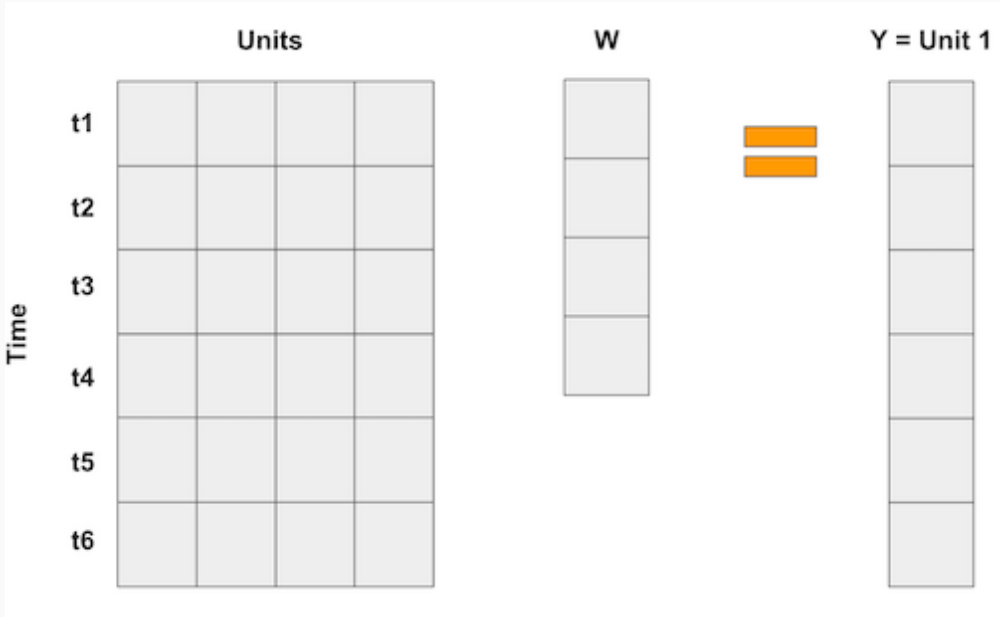
💡 Minimize some distance between the treated and the controls.



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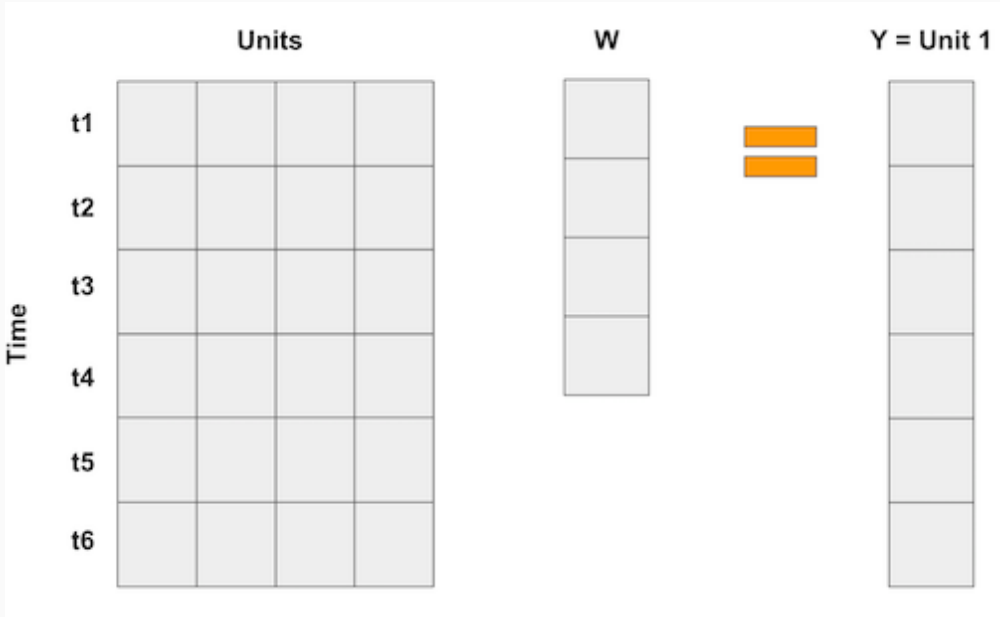
🤓 This is called a balancing estimator: kind of Inverse Probability Weighting.

Cf. (Wager, 2024, chapter 7) for details on balancing estimators.



Synthetic controls: minimization problem

Characteristics

Pre-treatment characteristics concatenate pre-treatment outcomes and other pre-treatment predictors Z_1 eg. cigarette prices:

$$X_{\text{treat}} = X_1 = \begin{pmatrix} Y_{1,1} \\ Y_{1,2} \\ \dots \\ Y_{1,T_0} \\ Z_1 \end{pmatrix} \in \mathbb{R}^{p \times 1} \qquad X_{\text{control}} = \left(X_2, .., X_{n_0+1}\right) \in \mathbb{R}^{p \times n_0}$$

Minimization problem

Synthetic controls: minimization problem

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$$\text{where } \|X\|_V = \sqrt{X^T V X} \quad \text{with} \quad V \in \text{diag}(\mathbb{R}^p)$$

V gives feature importance (prior knowledge).

Synthetic controls: minimization problem

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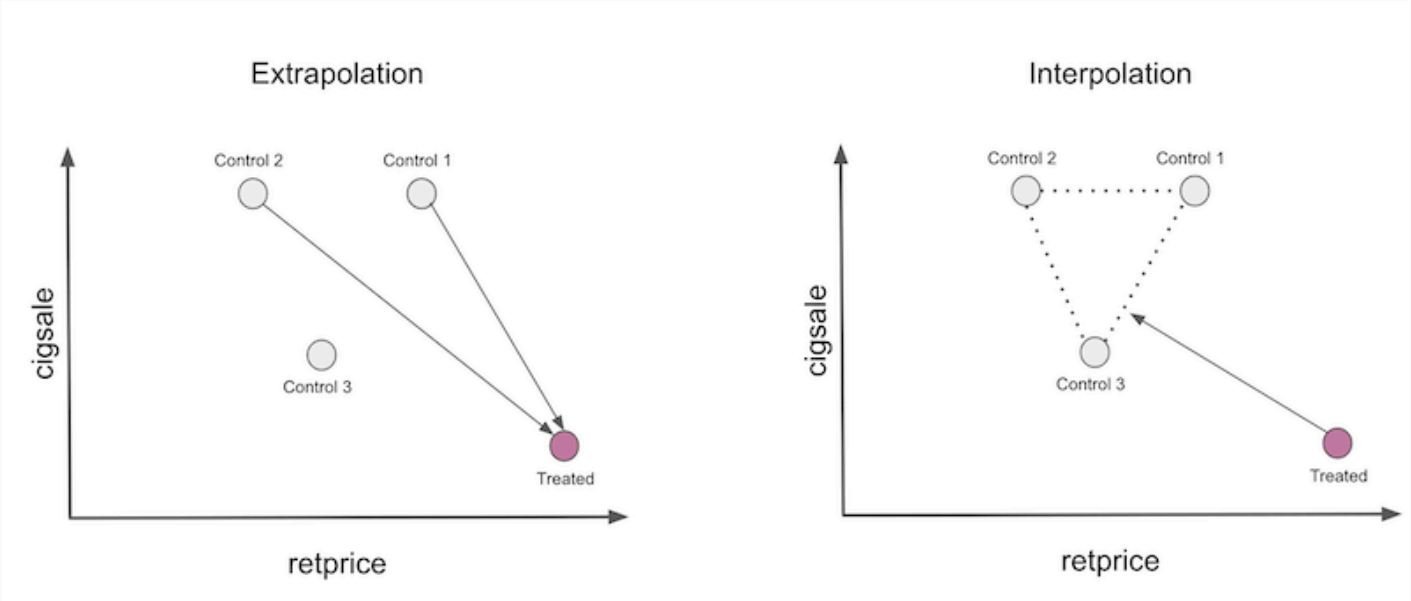
Minimization problem with constraints

$$w^* = \text{argmin}_w \ \|X_{\text{treat}} - X_{\text{control}} w\|_V^2$$
$$s.t.\ w_j \geq 0,$$
$$\sum_{j=2}^{n_0+1} w_j = 1$$

Synthetic controls: Why choose positive weights summing to one?

This is called interpolation (vs extrapolation)



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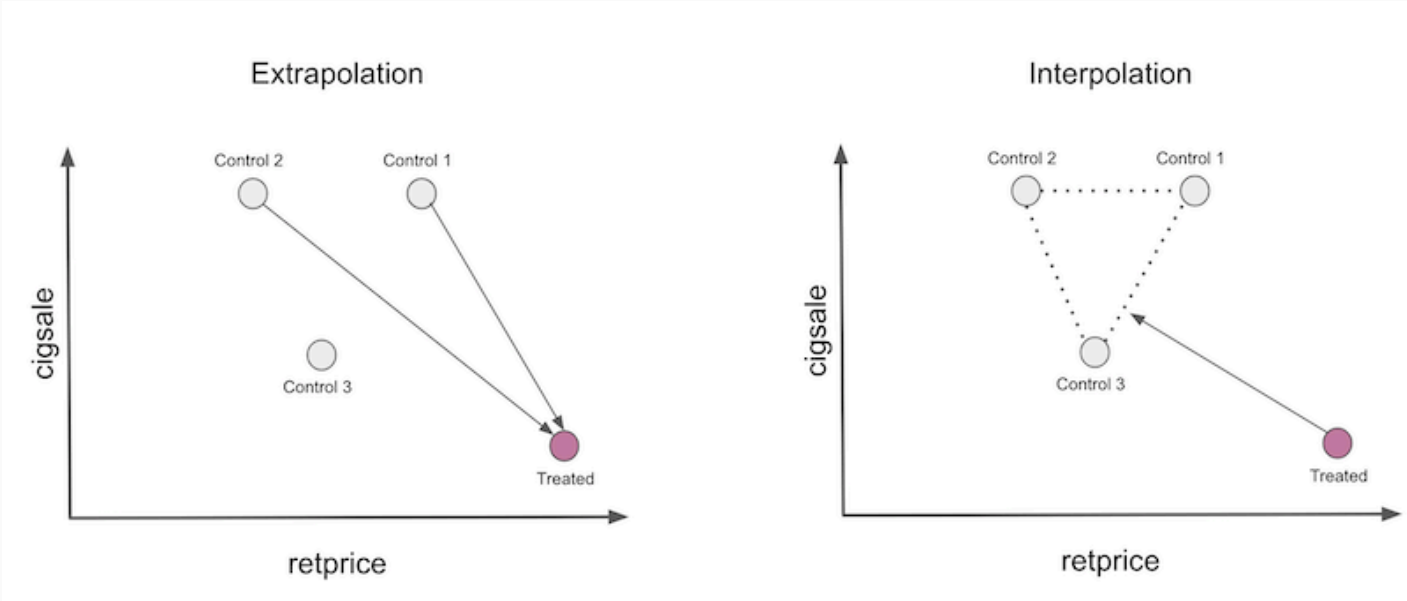

Interpolation enforces regularization, thus limits overfitting

Same kind of regularization than L1 norm in Lasso: avoid extreme coefficients.

Synthetic controls: Extrapolation failure with unconstrained weights

$p = 2 \times T_0$ covariates:

$$X_j = \begin{pmatrix} Y_{j,1} \\ \dots \\ Y_{j,T_0} \\ Z_{j,1} \\ \dots \\ Z_{j,T_0} \end{pmatrix}^T \in R^{2T_0}$$

Y cigarette sales, Z cigarette prices.

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Model: $\underbrace{X_{\text{treat}}}_{p \times 1} \sim \underbrace{X_{\text{control}}}_{p \times n_0} \underbrace{w}_{n_0}$

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Y cigarette sales, Z cigarette prices.

-> Estimation with OLS

Prediction: $\hat{Y}_{\text{synth}} = (Y_{t,j})_{\substack{t=1..T \\ j=2..n_0+1}} \hat{w}$

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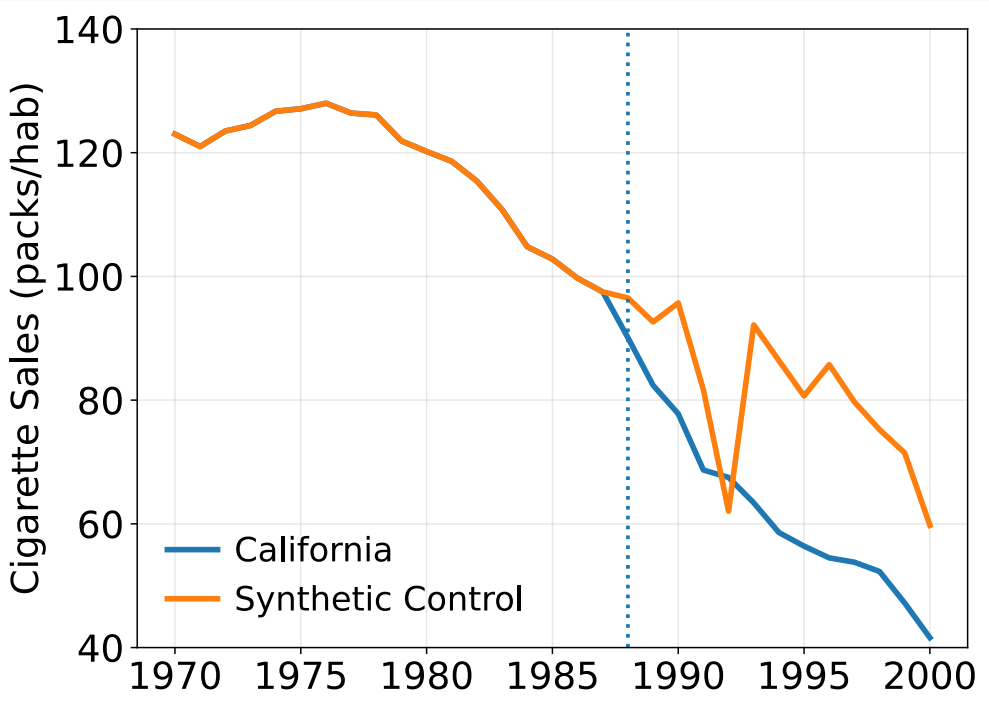
😭 **Overfitting**



Synthetic controls: How to choose the predictor weights V?

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2. Rescale by the variance of the predictors:
   $V = \text{diag}\left(\text{var}\left(Y_{j,1}\right)^{-1}, .., \text{var}\left(Y_{j,T_0}\right)^{-1}, \text{var}\left(Z_{j,1}\right)^{-1}, .., \text{var}\left(Z_{j,T_0}\right)^{-1}\right)$.

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3. Minimize pre-treatment mean squared prediction error (MSPE) of the treated unit:

$$\text{MSPE}(V) = \sum_{t=1}^{T_0} \left[Y_{1,t} - \sum_{j=2}^{n_0+1} w_j^*(V) Y_{j,t}\right]^2 = \| \left(Y_{1,t}\right)_{t=1..T_0} - \left(Y_{j,t}\right)_{\substack{j=2..n_0+1 \\ t=1..T_0}}^{T} w^*(V) \|_2^2$$

This solution is solved by running two optimization problems successively:

- Inner optimization solving $w^*(V) = \text{argmin}_w \ \|X_{\text{treat}} - X_{\text{control}} w\|_V^2$
- Outer optimization solving $V^* = \text{argmin}_V \ \text{MSPE}(V)$

Synthetic controls: estimation without the outer optimization problem

Same covariates:
$X_j = \left(Y_{j,1}, ..., Y_{j,T_0}, Z_{j,1}, ..., Z_{j,T_0}\right)$

Y cigarette sales, Z cigarette prices.

SCM minization with
$V = I_p$, ie. $\|X\|_V = \|X\|_2$.

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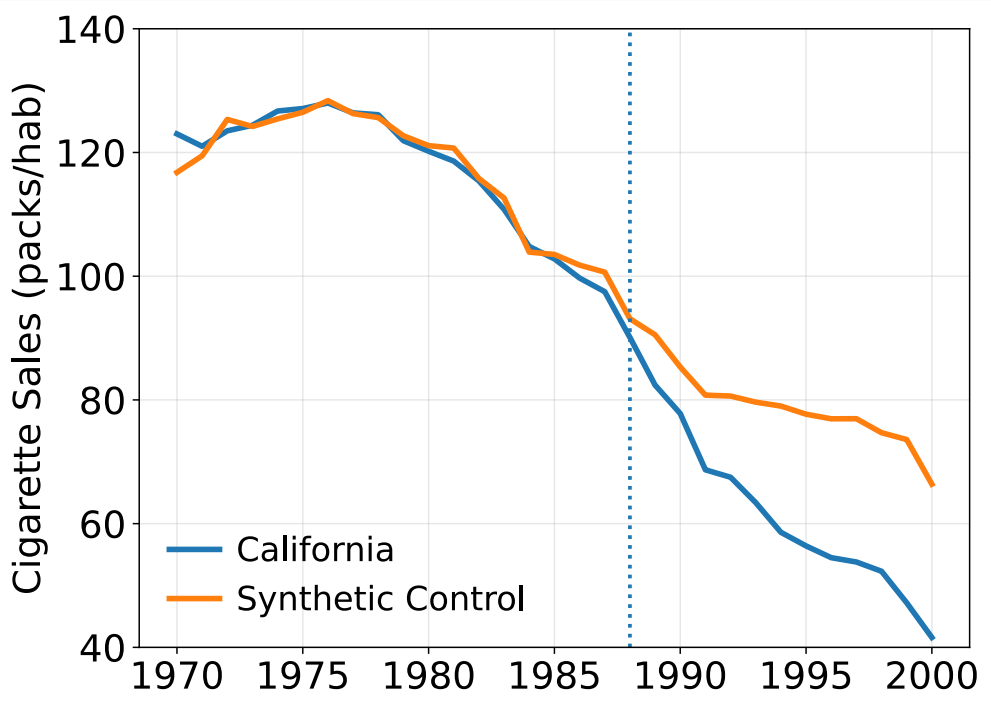
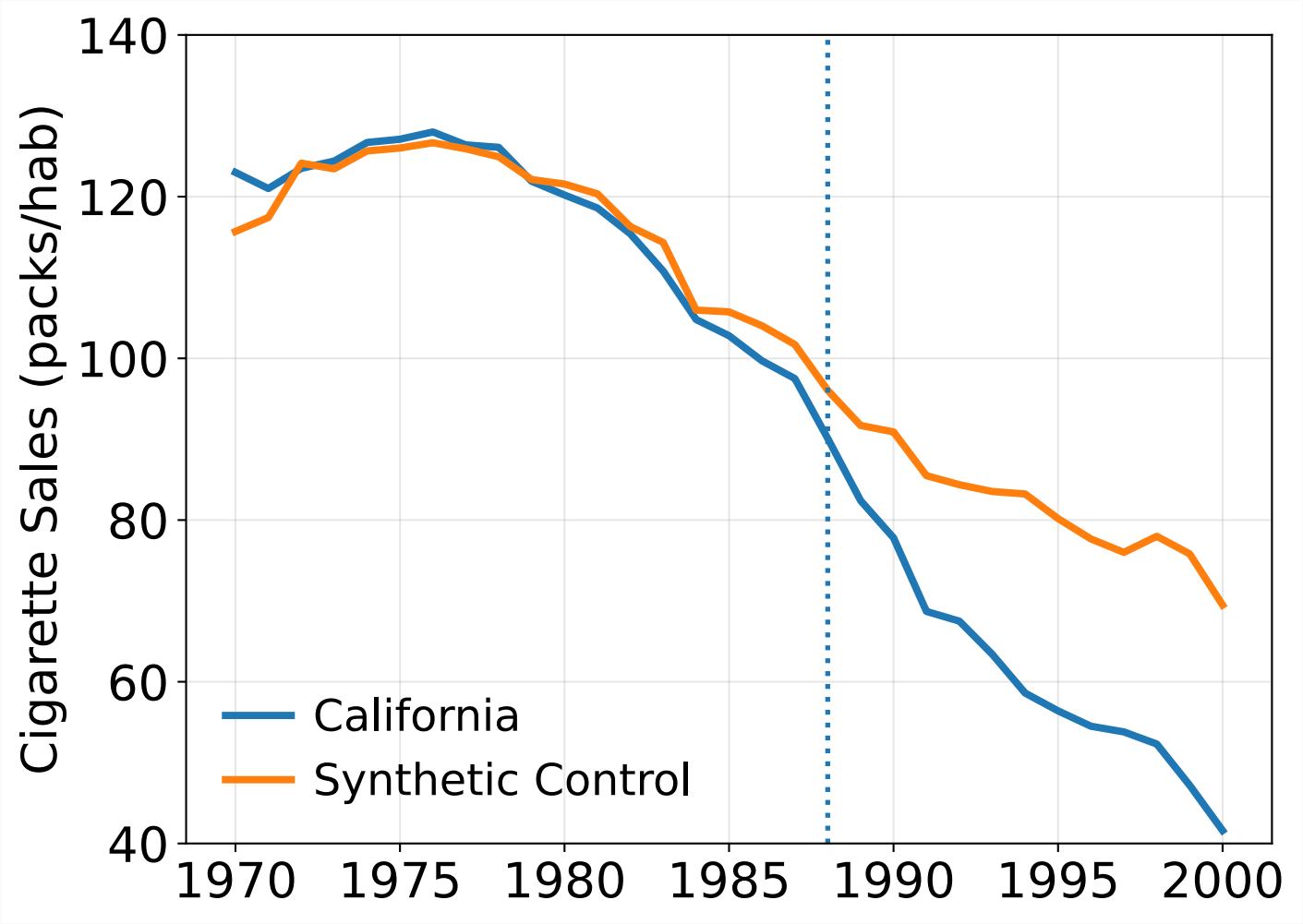


Synthetic controls: estimation with the outer optimization problem



Synthetic controls: inference

Variability does not come from the variability of the outcomes (often aggregated)

Usually, aggregates are not very noisy (once deseasonalized)...

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... but from the variability of the chosen control units

Treatment assignment introduces more noise than outcome variability.

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... but from the variability of the chosen control units

Treatment assignment introduces more noise than outcome variability.

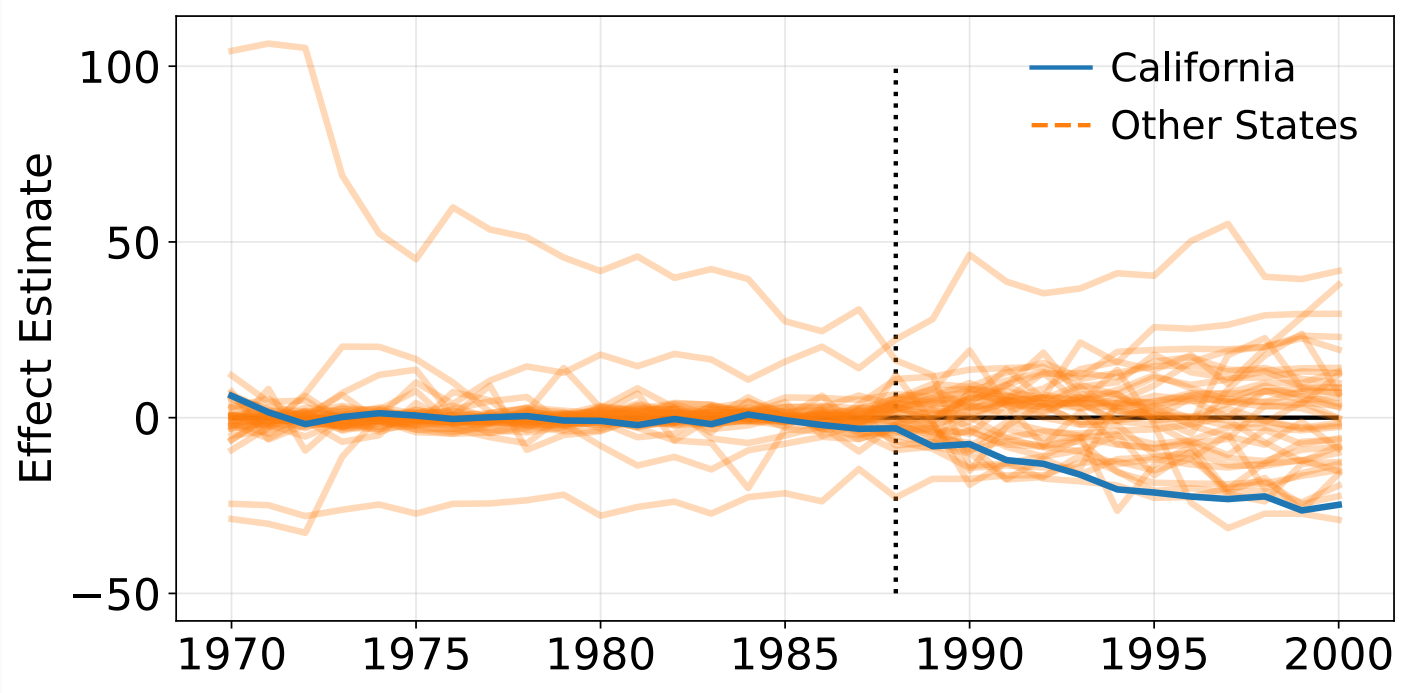
(Abadie et al., 2010) introduced the placebo test to assess the variability of the synthetic control.

There is also a modern approach on inference for SCM based on Conformal prediction (Chernozhukov et al., 2021) (cf. supplementary material slides for intuition).

Idea of placebo tests, also called Fisher's Exact tests

- Permute the treated and control exhaustively.
- For each unit, we pretend it is the treated while the others are the control: we call it a placebo
- Compute the synthetic control for each placebo: it should be close to zero.

Example of placebo test: For all 38 control states



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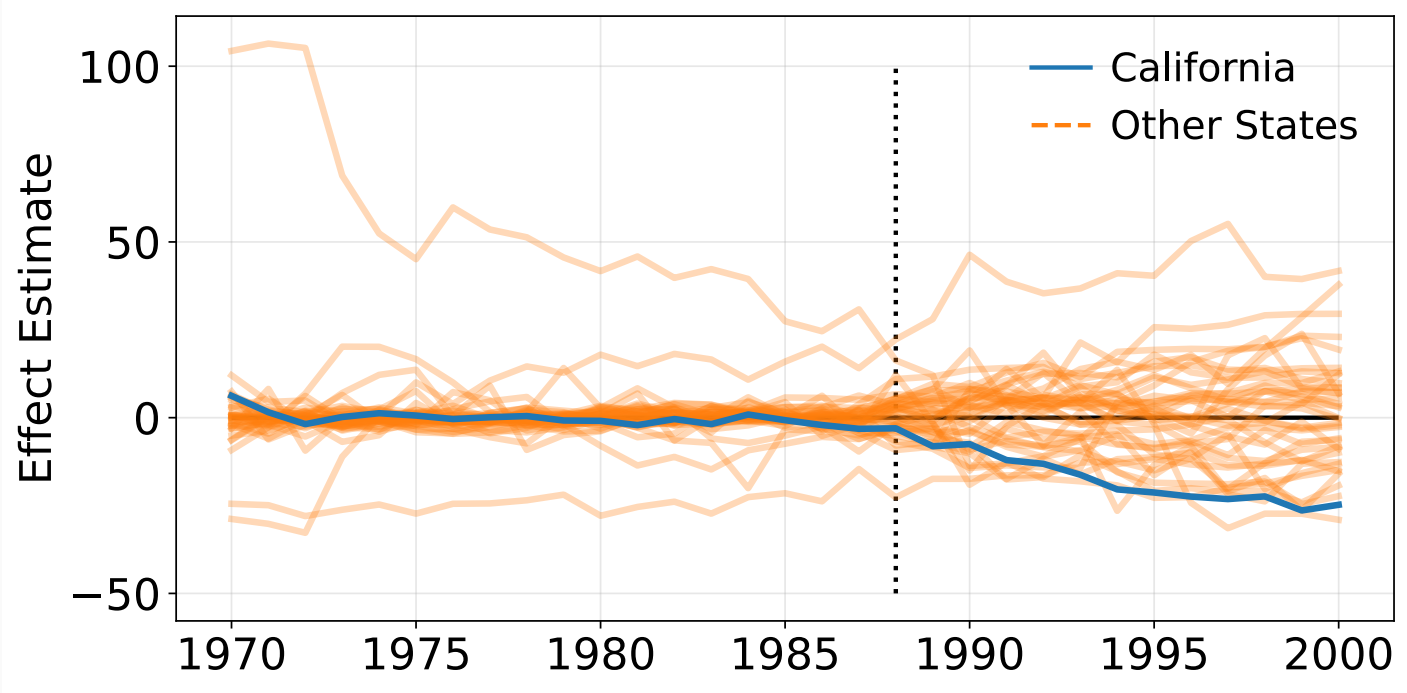
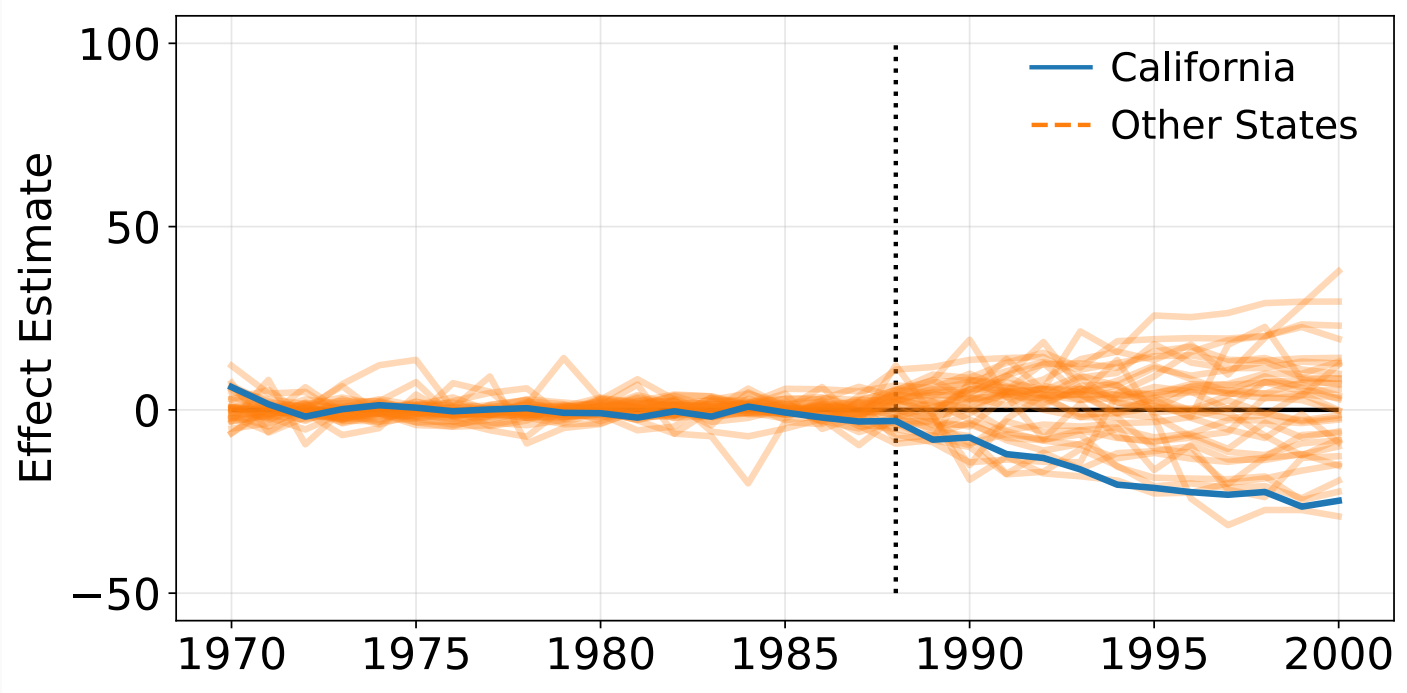


- More variance after the treatment for California than before.
- Some states have pre-treatment trends which are hard to predict : Expected because interpolation not possible for them.

Example of placebo test: Focus on 34 controls with good pre-treatment fit



Removing states above the 90 percentiles of the distribution of the pre-treatment fit.

Example of placebo tests: distribution

California absolute cumulative effect

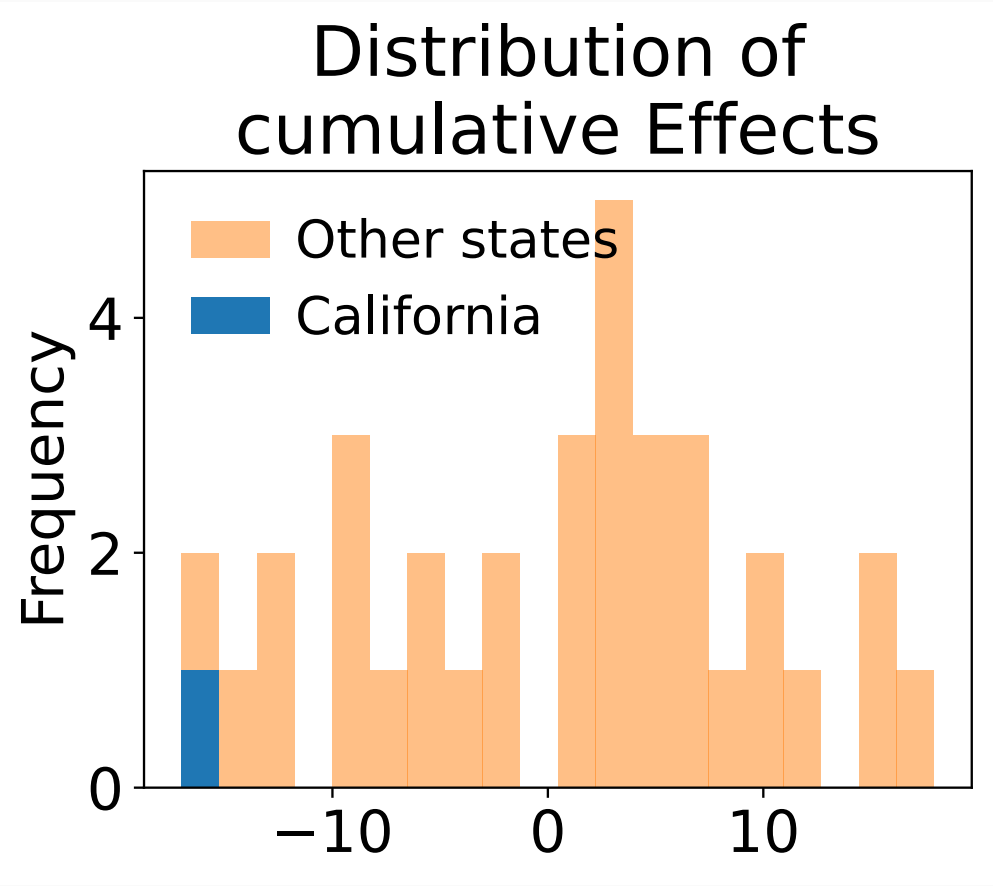
$$\hat{\tau}_{\text{scm, california}} = -17.00$$

Get a p-value

In how many repetitions is California's effect above the placebo effect ?

$$p = 1 - \frac{1}{n_0} \sum_{j=2}^{n_0} \mathbb{1}\left(|\hat{\tau}_{\text{scm, california}}| > |\hat{\tau}_{\text{scm},j}| \right)$$

$$= 0.029$$

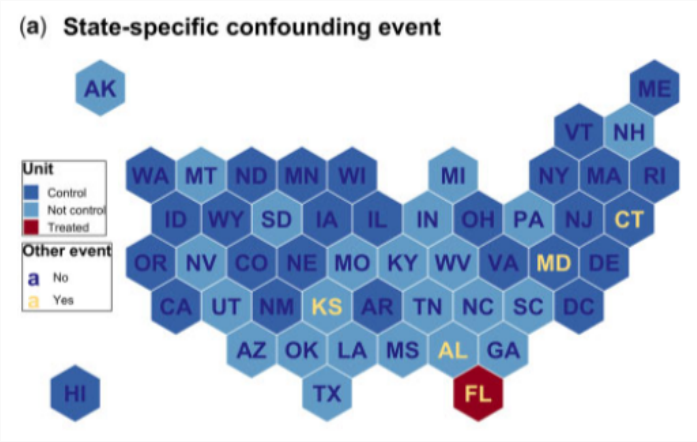


Failure of synthetic controls: confounding events

Confounding event : affecting the outcome for the treated unit and only part of the controls.

(Degli Esposti et al., 2020) setup:

- Population: US states
- Intervention: Stand Your Ground law in Florida (october 2005)
- Comparator: Other states without SYG laws
- Outcome: homicide rate



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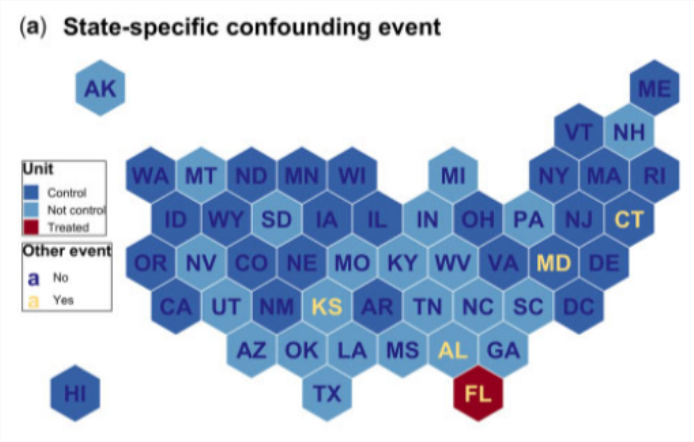
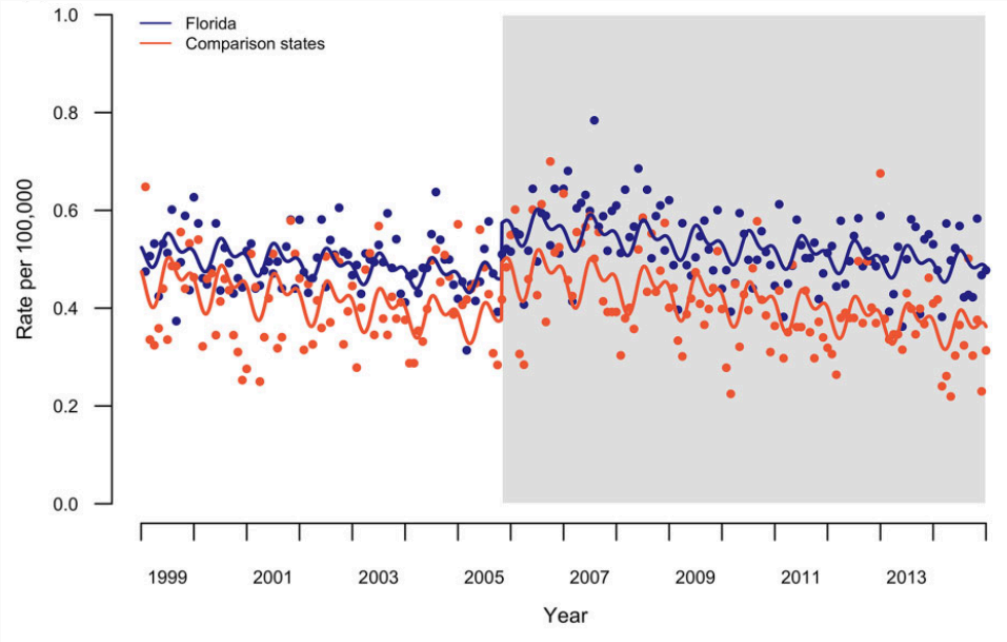


Failure when confounding event has an impact on outcomes for only some control states and the treated (KS, MD, AL, CT, FL): $\mathbb{1}[t > T_0]$ so no treatment effect.

Synthetic controls failure: appropriate controls

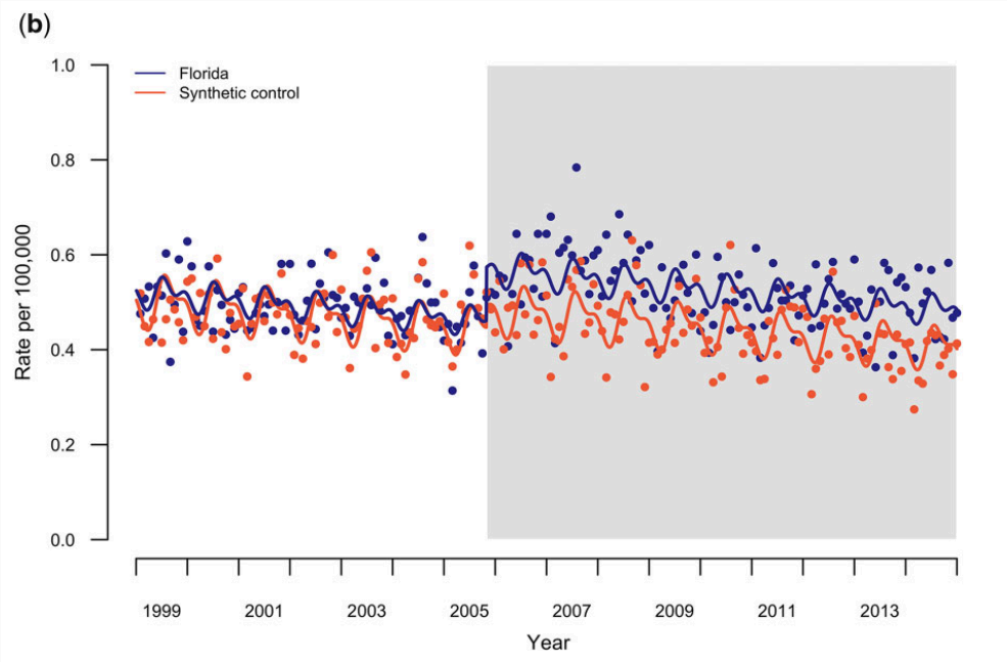
Focus only on states affected by the confounding events



Comparison states: KS, MD, AL, CT -> also affected by the counfounding event.

🤩 We would conclude to no effect of the treatment.

Synthetic controls failure: data-driven controls with all comparison states



SCM matches pre-treatment trends, fails to detect the confounding event.

🤯 We would falsely conclude to a positive treatment effect.

Synthetic controls: Take-away

Pros

- More convincing for parallel trends assumption.
- Handle multiple time periods.
- Data driven.
- Gives confidence intervals thanks to placebo test.

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See (Arkhangelsky et al., 2021) for discussions.

Interrupted time-series: methods without a control group

Interrupted Time Series: intuition

Setup

- One treated unit, no control unit.
- Multiple time periods.
- Sometimes, predictors are availables: there are called exogeneous covariates.

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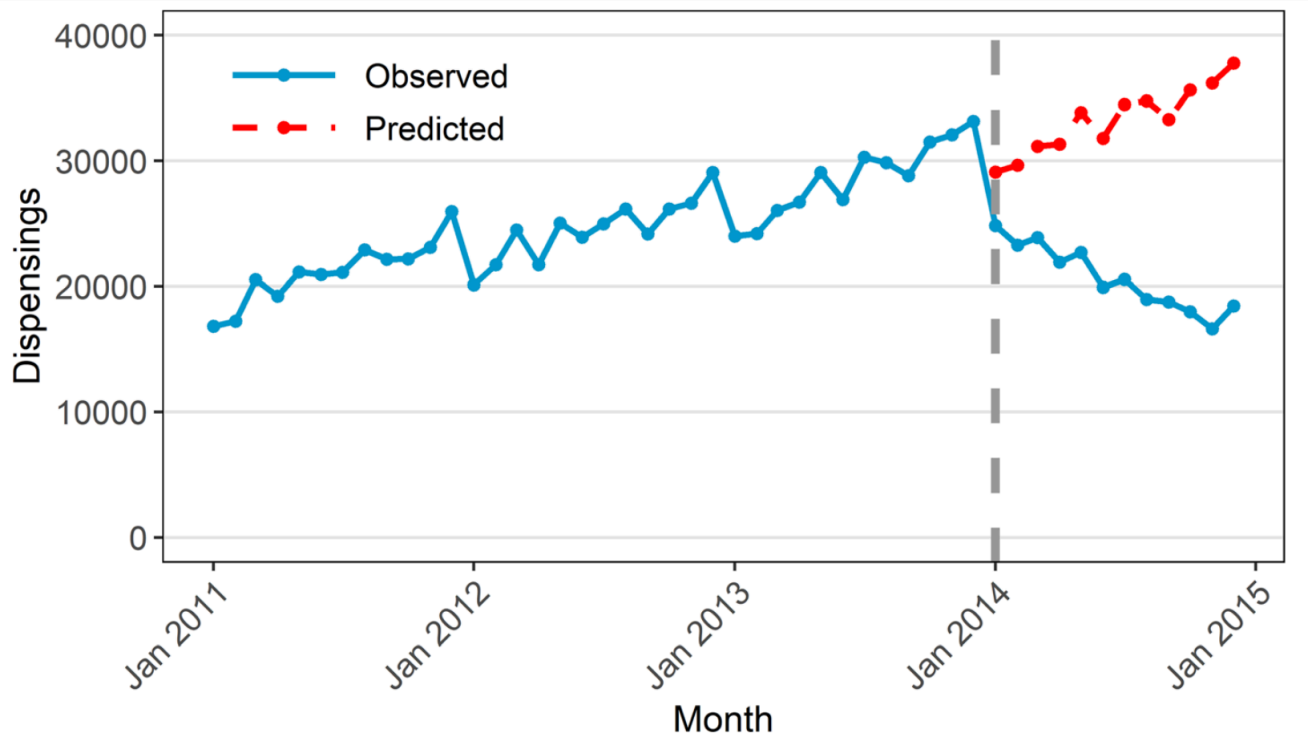
Intuition

- Model the pre-treatment trend: $Y_{t(1)}$ for $t < T_0$
- Predict post-treatment trend as the control: $\hat{Y}_t(0)$ for $t > T_0$
- Obtain treatment effect by taking the difference between observed and predicted post-treatment observations: $Y_t(1) - \hat{Y}_t(0)$

Interrupted Time Series: illustration from (Schaffer et al., 2021)



Y_t: Dispensations of quetiapine, an anti-psychotic medicine.

Treatment: Restriction of the conditions under which quetiapine could be subsidised.

Tools

- ARIMA models: AutoRegressive Integrated Moving Average

Motivation of ARIMA

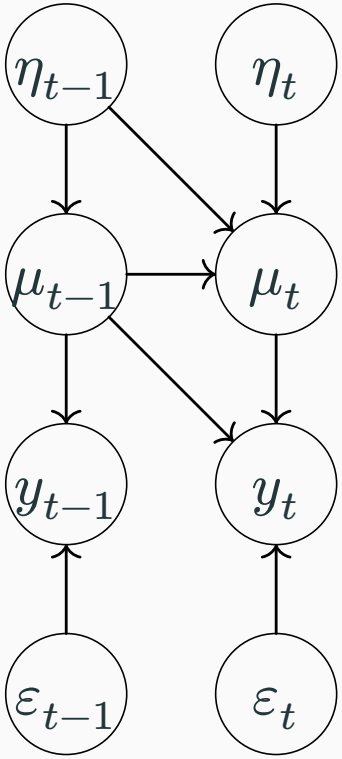
- Structure of autodependance between observation (auto-regression, moving average),
- Linear trends,
- Seasonality.

Good reference

Forecasting (fpp3): Principles and Practice, chapter 8

What is a (linear) state space model?

- Two (sometimes multi-dimensional) components: the state μ_t and the observation y_t.



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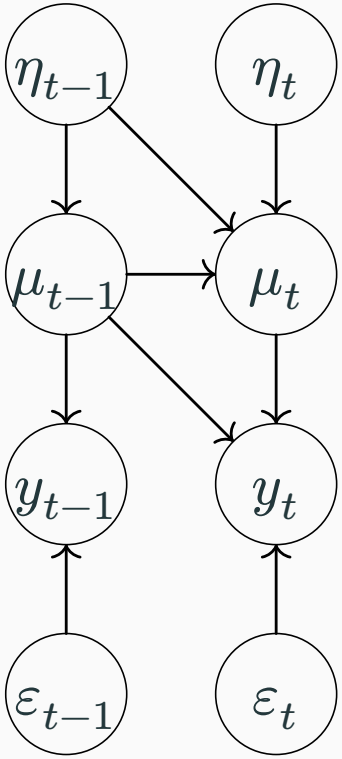
- State, ie. latent (unobserved) variable:

$$\mu_t = \overbrace{T_t}^{\text{Transition matrix}} \mu_{t-1} + \overbrace{R_t}^{\text{Transition matrix}} \underbrace{\eta_t}_{\text{gaussian white noise}}$$



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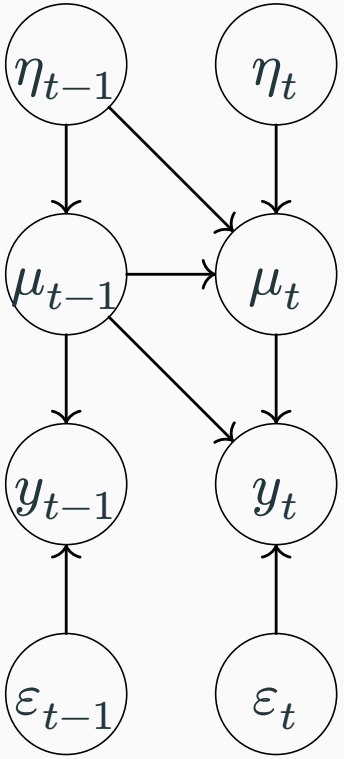
- Observation is a noisy version of the state:

$$y_t = \overbrace{Z_t}^{\text{design matrix}} \mu_t + \varepsilon_t$$



Why showing the state space model formulation?

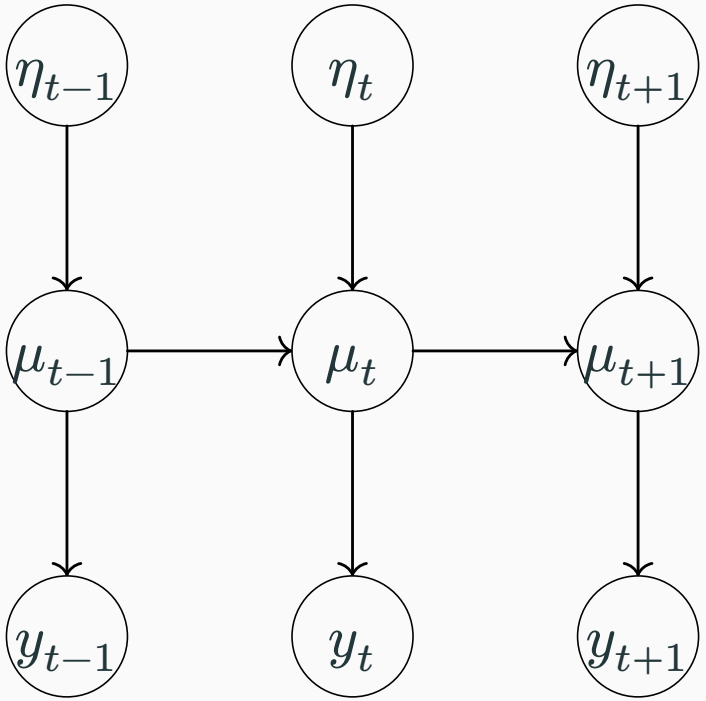
- I better understand ARIMA formulated as state space models.
- SSM are more general than ARIMA models.
- ARIMA are (often) fitted with SSM optimization algorithms.

Good reference

(Murphy, 2022, book 2, chap 29) s

AR(1)

DAG



Formalization

Latent: $\mu_t = \rho\mu_{t-1} + \eta_t$

Observation: $y_t = \mu_t$

with $\eta_t \sim N(0, \sigma^2)$

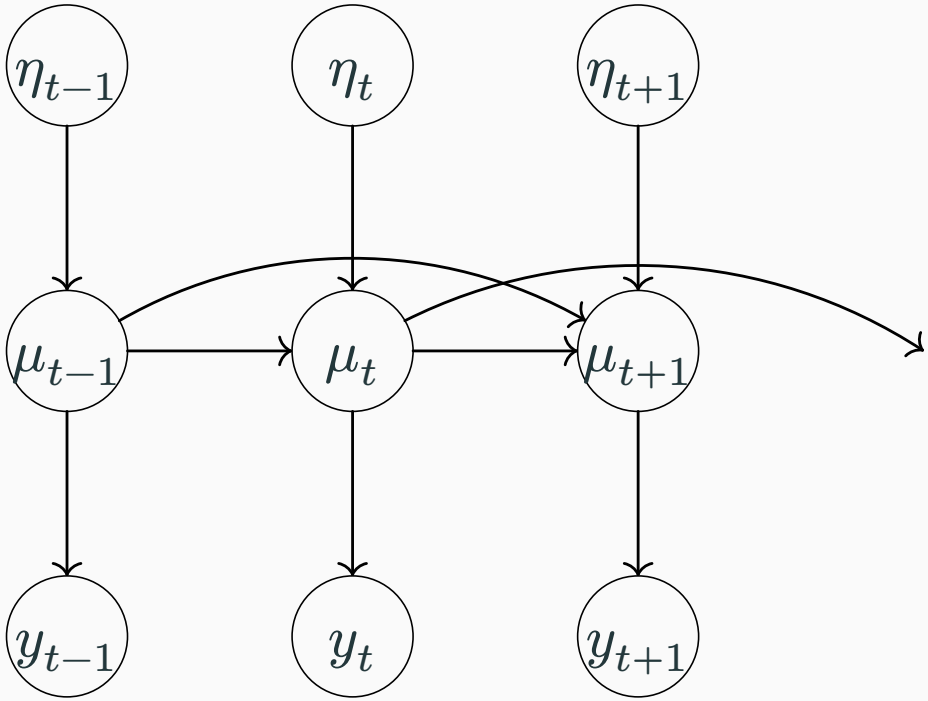
$$|\rho| < 1$$

State space models: AR(1) model example

Auto-regression time series model an outcome as a linear regression of its prior values.

AR(2)

DAG



Formalization

Latent: $\mu_t = \begin{pmatrix} \rho_1 & \rho_2 \\ 1 & 0 \end{pmatrix} \mu_{t-1} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} \eta_t$

Observation: $y_t = [1, 0]\mu_t$

with $\eta_t \sim N(0, \sigma^2)$

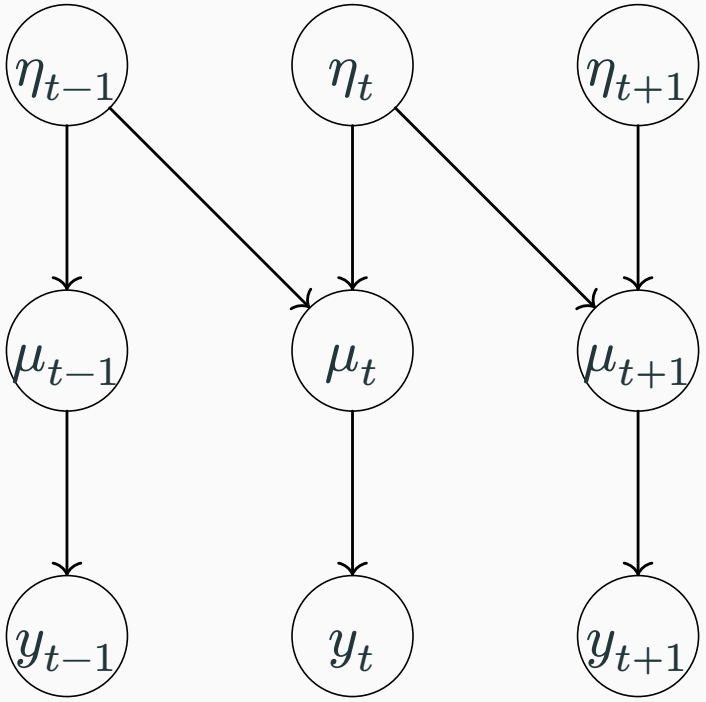
$$|\rho_1| < 1, |\rho_2| < 1$$

Observation unrolled:

$y_t = \rho_1 y_{t-1} + \rho_2 y_{t-2} + \eta_t$

State space models: MA(1) i.e. ARIMA(0,0,1) model example

DAG



Formalization

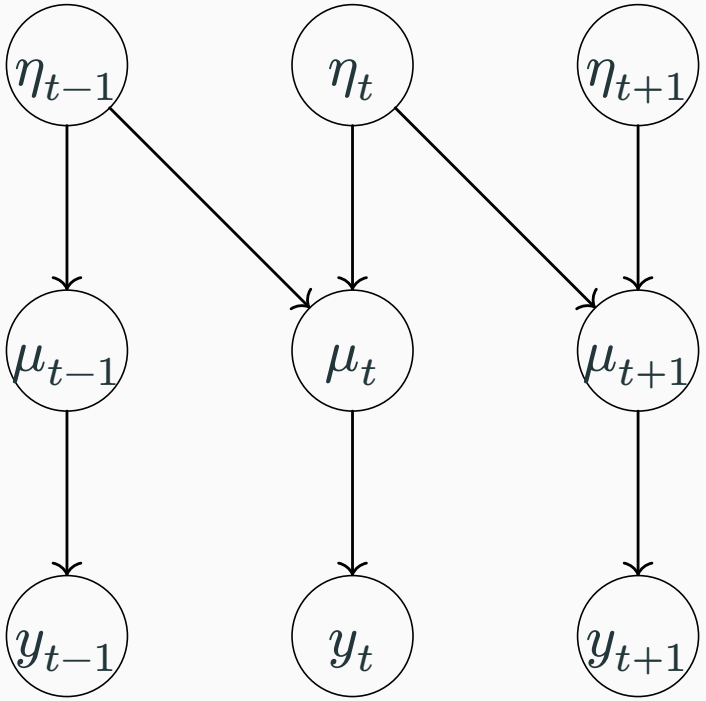
Latent: $\mu_t = [1, \theta] \begin{pmatrix} \eta_t \\ \eta_{t-1} \end{pmatrix}$

Observation: $y_t = \mu_t$

with $\eta_t \sim N(0, \sigma^2)$

State space models: MA(1) i.e. ARIMA(0,0,1) model example

DAG



Formalization

Latent: $\mu_t = [1, \theta] \begin{pmatrix} \eta_t \\ \eta_{t-1} \end{pmatrix}$

Observation: $y_t = \mu_t$

with $\eta_t \sim N(0, \sigma^2)$

The MA time series models the residual of the regression of y_t on its previous values as a linear combination of the previous residuals : i.e. vanishing shocks.

Formalization (Hamilton form)

Let $r = \max(p, q+1)$

Observation: $y_t = (1, \theta_1, \theta_2, ..., \theta_{r-1})\ \mu_t$

Latent: $\mu_t = \begin{pmatrix} 1 & \rho_1 & \rho_2 & ... & \rho_{r-1} \\ 1 & 0 & 0 & ... & 0 \\ 0 & 1 & 0 & ... & 0 \\ \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & ... & 0 & 1 & 0 \end{pmatrix} \mu_{t-1} + \begin{pmatrix} \varepsilon_t \\ 0 \\ ... \\ 0 \end{pmatrix}$ with $\varepsilon_t \sim N(0, \sigma^2)$

Unfolding the state space equations

$y_t = \sum_{i=1}^{p} \rho_i y_{t-i} + \sum_{j=1}^{q} \theta_j \varepsilon_{t-j}$

Formalization

Observation with covariates and seasonality:

$$y_t = \beta x_t + s_t + \underbrace{\rho \mu_{t-1}}_{\text{AR}(1)} + \underbrace{\theta \eta_{t-1} + \eta_t}_{\text{MA}(1)}$$

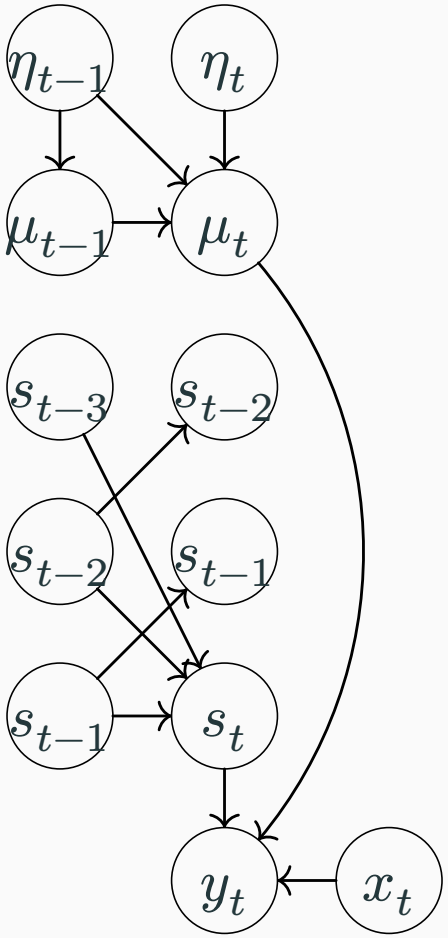
Where seasonality:

$$s_t = -\sum_{k=1}^{S-1} s_{t-k} + \varepsilon_{s,t}$$

$$\text{with } \varepsilon_s, t \sim N(0, \sigma_s^2)$$

State space models: Adding a seasonnality and a covariate component

DAG



State space models: General formulation

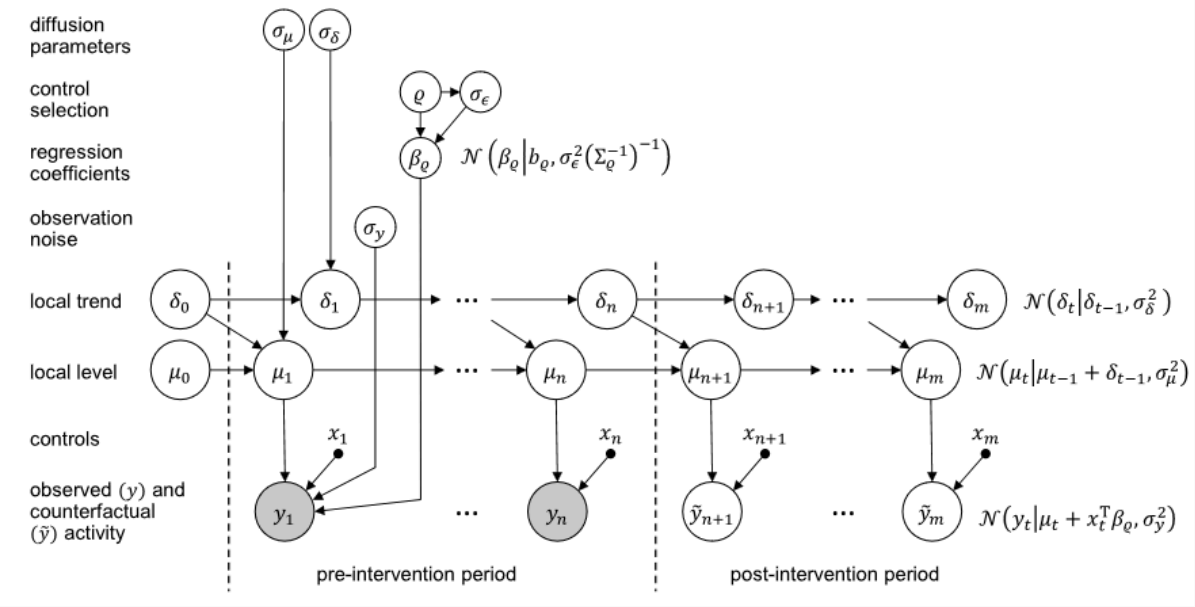
Latent:

$$\mu_t = T_t \mu_{t-1} + R_t \eta_t$$

Observation:

$$y_t = Z_t \mu_t + \beta^T x_t + \varepsilon_t$$

With η_t and ε_t mean zero gaussian noise, sometimes with a specific covariance structure.



Complex SSM DAG from the Causal Impact paper (Brodersen et al., 2015).

State space models: a brief word on fitting (i.e. learning the parameters)

When the error terms are gaussians

These modeles are called linear Gaussian state space model (LG-SSM) or linear dynamical system (LDS).

The likelihood is jointly gaussian

Closed form formula for the likelihood of the data under the model.

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Expectation-Minimization: a widespread algorithm for fitting

- Expectaction: Compute the joint likelihood of the data and the parameters (observed outcome, unknown state) given the parameters.
- Maximization: find parameters maximizing the likelihood: analytically since gaussian.
- Iter until convergence to a (local) maximum of likelihood.

Modern state space models

Long Short Term Memory (LSTM) networks

A type of Recurrent Neural Network (RNN) that can learn long-term dependencies (Graves & Graves, 2012).

It was state of the art for language tasks before transformers.

It is notably hard to train due to vanishing gradient through the time dimension.

Mamba

A recent proposition to mitigate one of the main limitations of the transformer architecture: high complexity relative to the length of the sequence (Gu & Dao, 2023).

Good blog-style introduction in (Ayonrinde, 2024).

Example of ITS with ARIMA: the French antibiotics campaign of 2002-2007

Context

In 2001, compared to the European Union countries, France was a country where:

- the population consumed the most antibiotics in town
- the resistance of Streptococcus pneumoniae to β-lactams was the highest (53%)
- a significant number of antibiotic prescriptions would be unnecessary (viral infections)

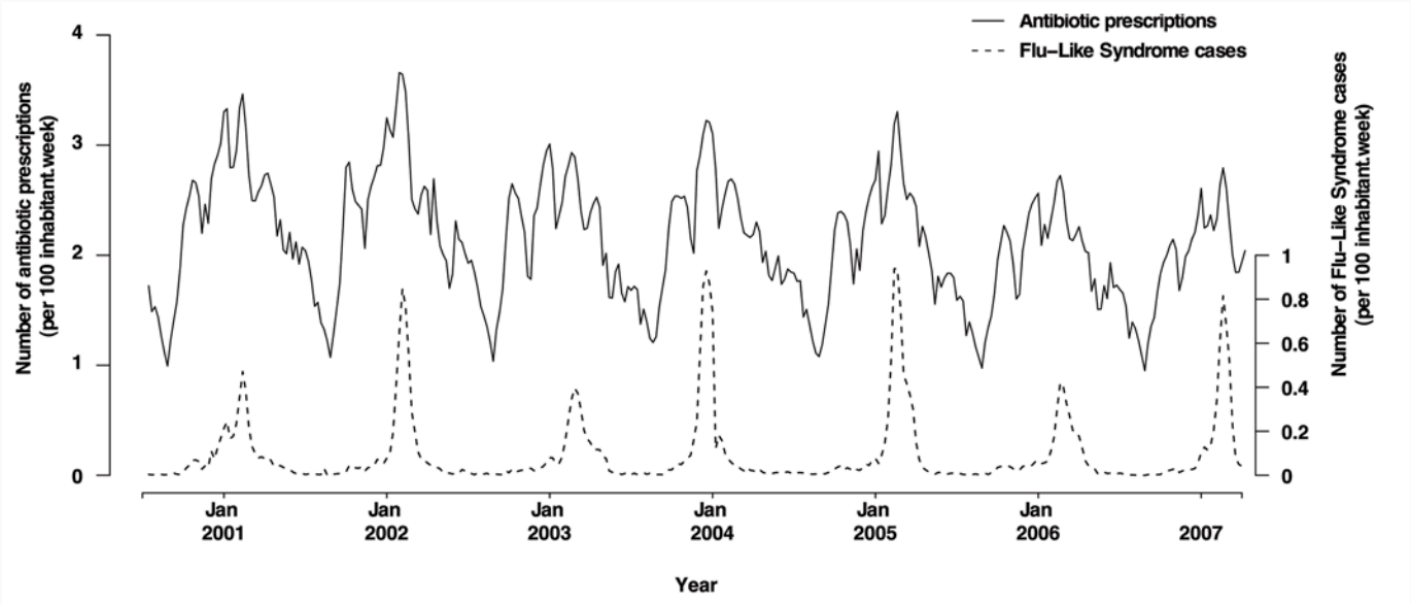
Campaign (october 2002, then every year october to march)

France implemented a national plan to "preserve the effectiveness of antibiotics and improve their use" with the main action undertaken by the National Health Insurance.

Question

What has been the effect of the campaign on the consumption of antibiotics? (Sabuncu et al., 2009)

Weekly reimbursed prescription of antibiotics in town



Interventions during the months of october to march: $\text{month}(t) \in M_0$.

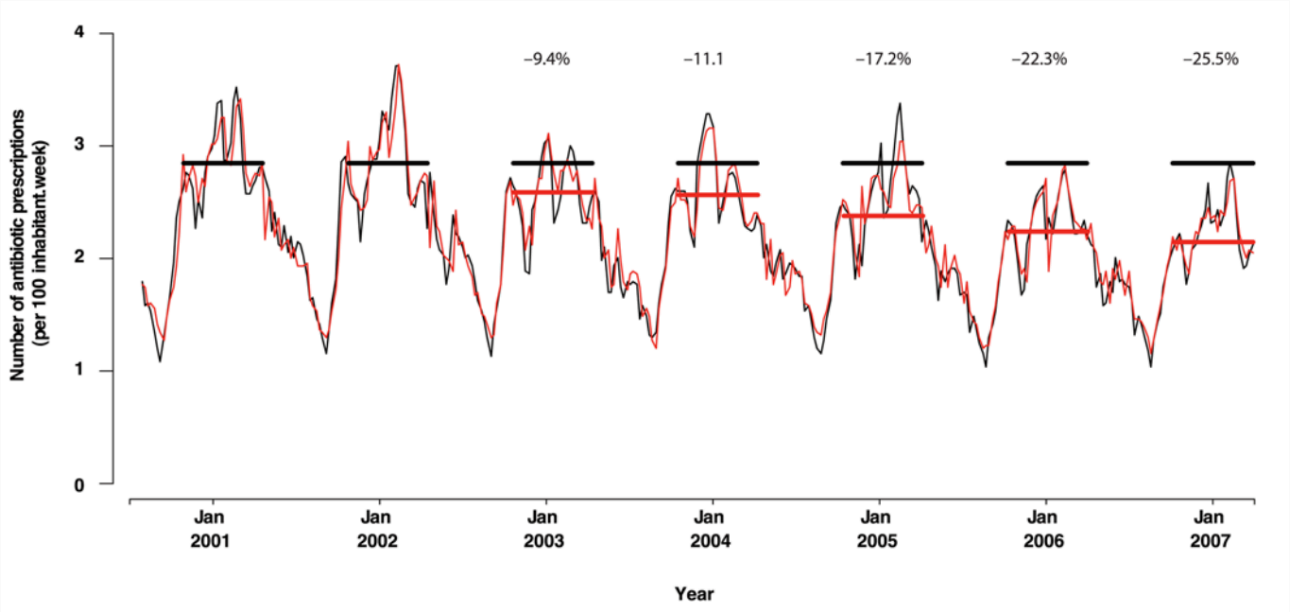
Estimation

- Fit an ARIMA model on the pre-treatment trend
- Introduce an additive term for the intervention:

$$Y_t = c + \sum_i \hat{\tau}_i \mathbb{1}[\text{month}(t) \in M_0 \wedge \text{year}(t) == i] + \underbrace{[a(B)^{-1} - b(B)\varepsilon_s]}_{\text{ARIMA term fitted on pre-treatment}}$$

- Assess if the additive term and other parameters are significantly different pre-treatment and post-treatment.

Example of ITS with ARIMA: the French antibiotics campaign of 2002-2007



- Red curve: arima fitted with intervention
- Red Horizontal line: intervention effect fitted during intervention
- Black curve: arima fitted without intervention
- Black horizontal line: intervention effect fitted pre-intervention

A word on model families for ITS

We saw ARIMA models and the more general class of state space models.

However, we could any model that we want to fit the pre-treatment trend !

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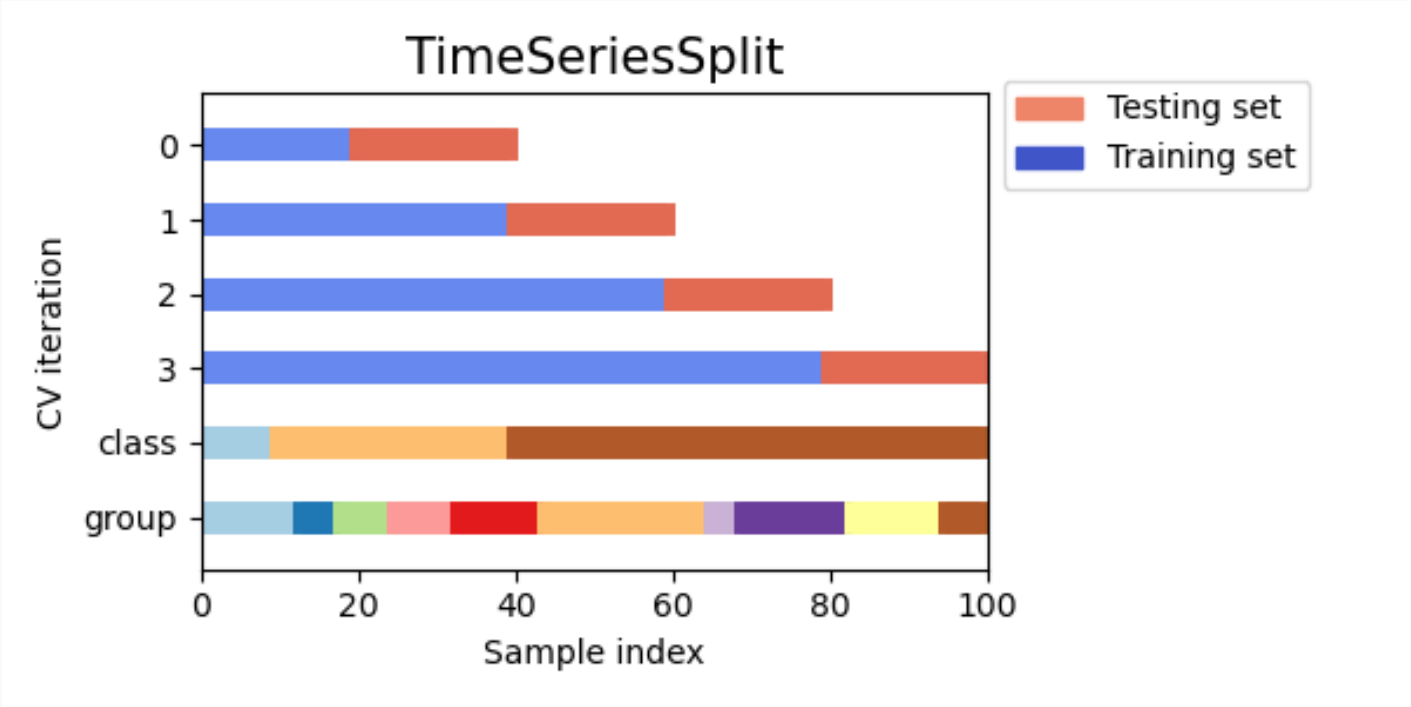
Any sklearn estimator could do the trick: Linear regression, Random Forest, Gradient Boosting...

⚠️ You should pay attention to appropriate train/test split when cross-validating a time-series model not to use the future to predict the past.

Relevant remark for all time series models (even ARIMA or state space models).

Cross-validation for time-series models

```python
from sklearn.model_selection import TimeSeriesSplit
```

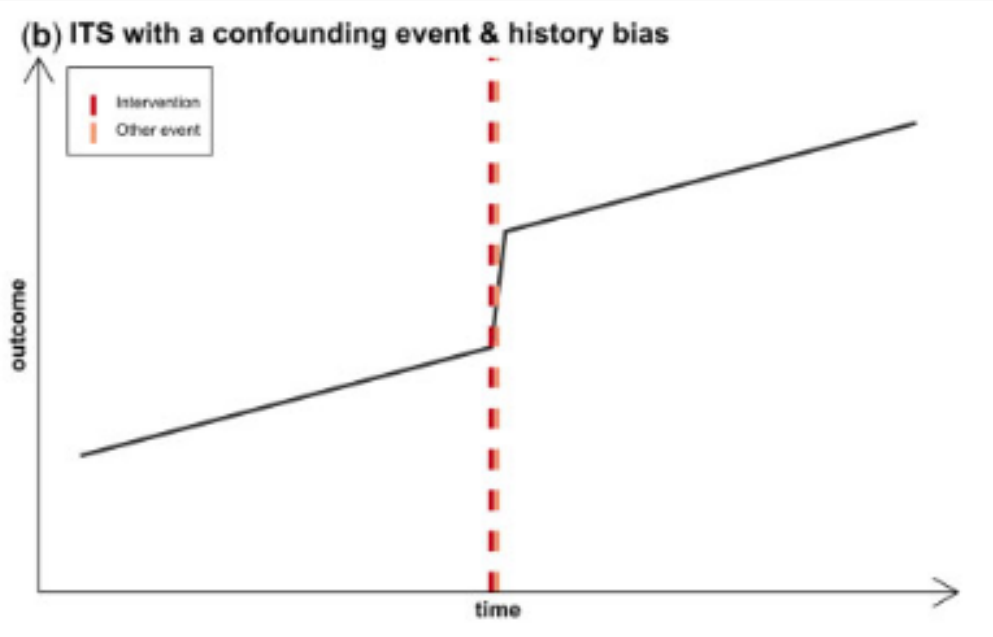


This avoids to use the future to predict the past.

Main threat to validity for an ITS: historical bias

(Degli Esposti et al., 2020, Fig. 1)

⚠️ If there is a co-intervention, it will impact the outcome trend and bias the treatment effect estimation.



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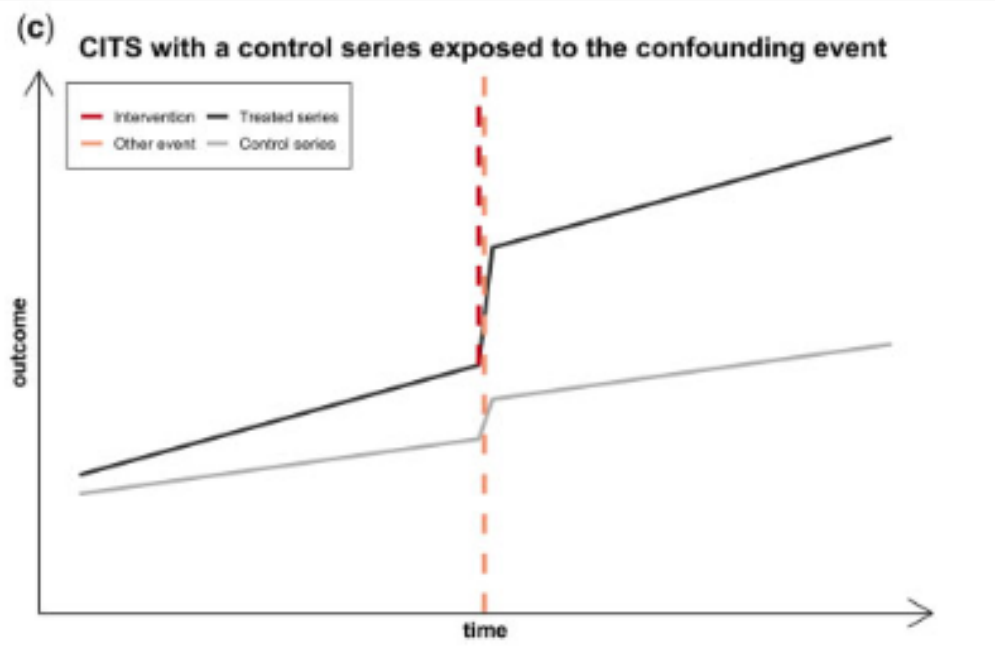
💡 Adding a control series of predictors can help to mitigate this bias.



Take-away on ITS

Pros

- Suitable when no control unit is available. The pre-treatment trend is the control.
- Handles multiple time periods.
- A lot of software available (eg. ARIMA models).
- Simple: few parameters to tune.

Cons

- Prone to bias by other events happening around the treatment time and impacting the outcome trend.
- Prone to overfitting of the pre-treatment trend.

An attempt to map event study methods

Methods	Characteristics	Hypotheses	Community	Introduction
DID/TWFE	Treated/control units, few time periods, no predictors	Parallel trends, no anticipation, homogeneous effect	Economics	Causal Inference for the Brave and True, chapter 13
ARIMA, ITS	No controls, no/few predictors, seasonality	Stationnarity , no anticipation, prone to overfitting	Epidemiology, Economics	Forecasting: Principles and Practice
State space models	Multiple time periods, control units or predictors, generalization of ARIMA	Contional ignorability on predictors, goodness of fit pre-treatment	Machine learning, bayesian methods	Introduction to Time Series and Forecasting, chapter 9
Synthetic control	Treated/control units, multiple time periods	Conditional parallel trend on controls, goodness of fit pre-treatment	Economics	Causal Inference for the Brave and True, chapter 15

A summary on R packages for event studies

Package name	Methods	Predictors	Control units	Multiple time periods
did	Difference-in-differences	❌	✅	❌
fixest	Fixed effects	✅	✅	✅
forecast	ARIMA, ITS	✅	❌	✅
Synth	Synthetic control	❌	✅	✅
Causal impact	Bayesian state space models	✅	❌	✅

A summary on Python packages for event studies

Package name	Methods	Predictors	Control units	Multiple time periods
❤️ pyfixest	Fixed effects	✅	✅	✅
statsmodels	ARIMA(X), ITS, bayesian state space models, DID	✅	❌	✅
pmdarima	ARIMA(X), ITS	✅	❌	✅
❤️ DoubleML	DID with double ML debiasing	✅	✅	✅
pysyncon	Synthetic control	❌	✅	✅
❤️ CausalPy	Synthetic control, interrupted time series, DID	✅	✅	✅

Final word -- What methods to chose: some guides

DID-family methods

- Control units available (at least one)
- Few time periods
- Parallel trend is credible (if necessary by adjusting the model on predictors).

Synthetic Control Methods

- Mutiple and different controls as well as multiple time periods
- Pre-treatment outcomes of the control units predict well the treated unit outcome.
- No-spill over from the treatment to the control units.

ITS: SARIMA or state space models

- No evident control units
- Pre-treatment outcome of the treated unit seems a good control
- Control predictors not impacted by the treatment availables
- No co-intervention that could impact the treated outcome.

Python hands-on

To your notebooks 👩‍💻 !

- url: https://github.com/strayMat/causal-ml-course/tree/main/notebooks

Supplementary materials

Examples of application of synthetic controls to epidemiology

- Literature review of the usage of SCM in healthcare (up to 2016): (Bouttell et al., 2018)

Some use cases

- What is the effect of UK pay-for-performance program in primary care on mortality? (Ryan et al., 2016)
- What is the effect of soda taxes on sugar-based product consumption? (Puig-Codina et al., 2021)
- What is the effect of Ohio vaccine lottery on covid-19 vaccination? (Brehm et al., 2022)
- What is the effect of wildfire storm on respiratory hospitalizations? (Sheridan et al., 2022)

Synthetic controls: conformal prediction inference

Introduced by (Chernozhukov et al., 2021)

- Recast the problem as counterfactual inference, i.e. predict: $Y_{it}(0)$ for $t > T_0$
- Test hypothesis: H_0 eg. $H_0 = (0, 0, .., 0)$ ie no effect for $t > T_0$
- This imply the generation of a hypothesis counterfactual trajectory $Y_t(0)$

Question

Are the post-treatment residuals of a model fitted on the hypothesis counterfactual trajectory an outlier of the distribution of the residuals pre-treatment?

Why does this works?

Syntehtic controls estimation are invariant under the time series dimension so we can resample under this dimension to introduce data variability.

Conformal inference: hypothesis generation

- Test a hypothesis : H_0 eg. $H_0 = (0, 0, .., 0)$ ie no effect for $t > T_0$
- Gerenate a counterfactual trajectory $Y_t(0)$ under this null

Conformal inference: Fit a model and compute residuals

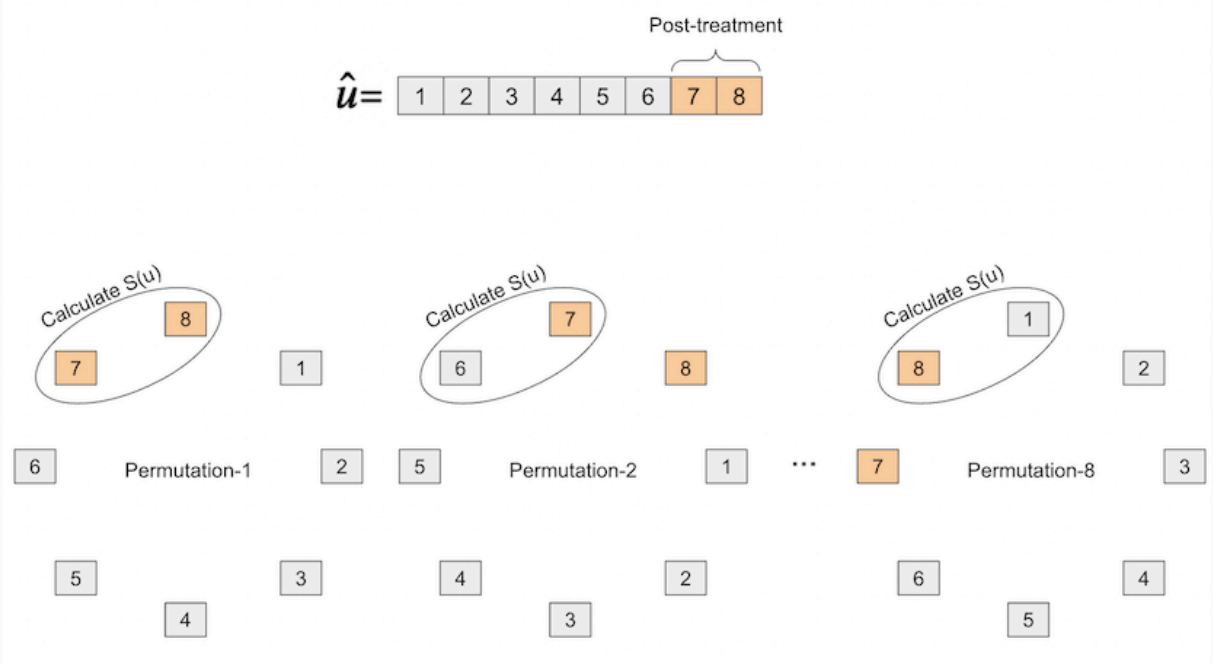
- Fit a counterfactual model on the full generated trajectory: $\hat{Y}_t$
- Compute the residuals: $\hat{u}_t = Y_t(0) - \hat{Y}_t$

Conformal inference: test statistic and resampling

Summarize the residuals in a statistic: $S(\hat{u}) = \left(\frac{1}{\sqrt{T-T_0+1}} \sum_{t=T_0+1}^{T} |\hat{u}_t|^q \right)^{\frac{1}{q}}$

Resample this statistic by block permutation π of the time periods

Same as permutting the data since SCM are invariant under the time series dimension.



Conformal inference: resampling

Image from: Causal Inference for the Brave and True

Conformal inference: P-value

- Assess if the post-treatment statistics is an outlier of this distribution.
- P-value: $\hat{F}(x) = \frac{1}{|\Pi|} \sum_{\pi \in \Pi} \mathbb{1}\left[S\left(\hat{u}_{\pi_0}\right) \leq S(\hat{u}_{\pi})\right]$ where π_0 is the original data.

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