# Machine Learning for econometrics

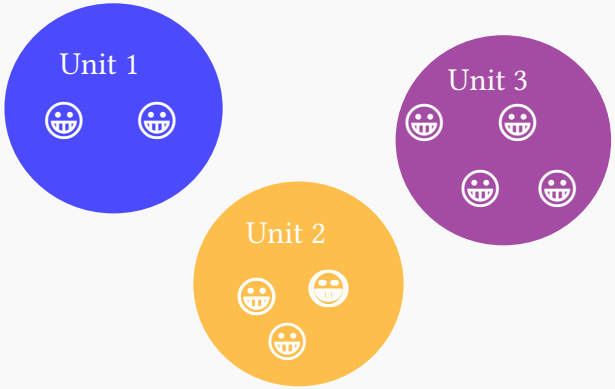
## Event studies: Causal methods for pannel data

Matthieu Doutreligne March, 11th, 2025

# “Motivation”

## Estimation of the effect of a treatment when data is:

- Aggregated: country-level data such as employment rate, GDP...



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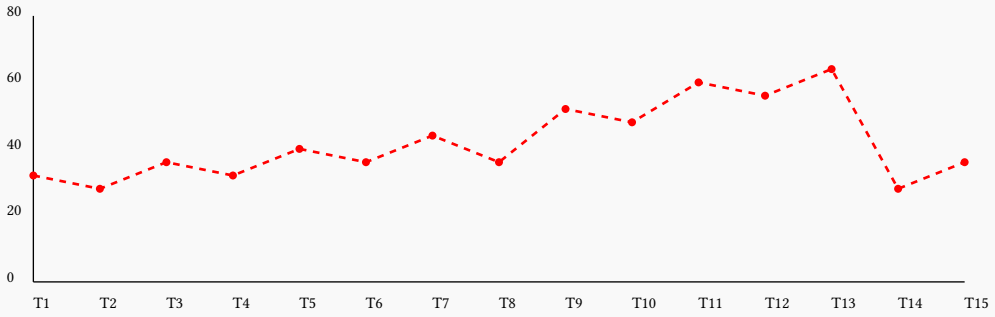

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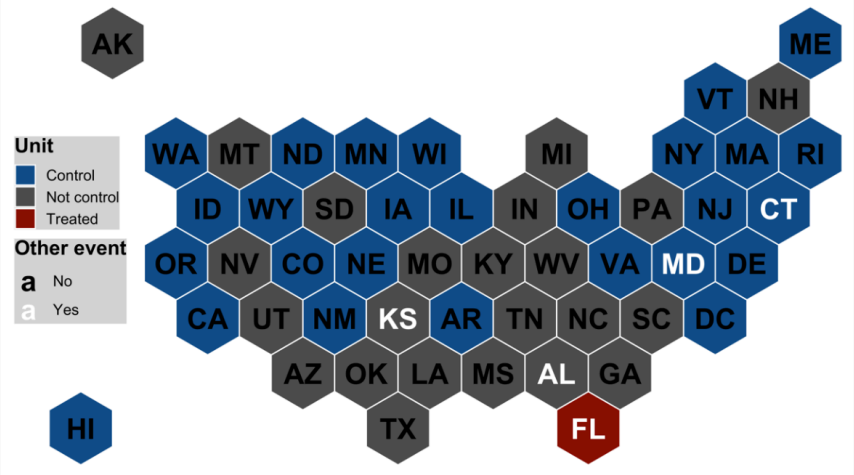


Figure from (Degli Esposti et al., 2020)

# Setup

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- Aggregated: country-level data such as employment rate, GDP…
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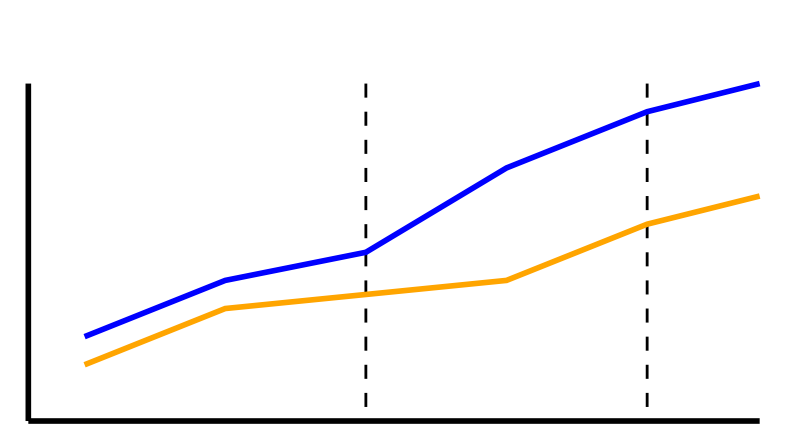
- Staggered adoption of the treatment: units adopt the policy/treatment at different times…



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This setup is known as:

**Panel data, event studies, longitudinal data, time-series data.**

# Examples of event studies

**Archetypal questions**

- Did the new marketing campaign had an effect on the sales of a product?
- Did the new tax policy had an effect on the consumption of a specific product?
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**Modern examples**

- What is the effect of the extension of Medicaid on mortality? (Miller et al., 2019)
- What is the effect of Europe's protected area policies (Natura 2000) on vegetation cover and on economic activity? (Grupp et al., 2023)
- Which policies achieved major carbon emission reductions? (Stechemesser et al., 2024)

# Setup: event studies are quasi-experiment

**Quasi-experiment**

A situation where the treatment is not randomly assigned by the researcher but by nature or society.
It should introduce some randomness in the treatment assignment: enforcing treatment exogeneity, i.e. ignorability (i.e. unconfoundedness).

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## Other quasi-experiment designs

- Instrumental variables: a variable that is correlated with the treatment but not with the outcome.
- Regression discontinuity design: the treatment is assigned based on a threshold of a continuous variable.

# “Reminder on difference-in-differences”

## History

- First documented example (though not formalized): John Snow showing how cholera spread through the water in London (Snow, 1855)[1]
- Modern usage introduced formally by (Ashenfelter, 1978), applied to labor economics

[1] Good description: https://mixtape.scunning.com/09-difference_in_differences#john-snows-cholera-hypothesis

## History

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## Idea

- Contrast the temporal effect of the treated unit with the control unit temporal effect.
- The difference between the two differences is the treatment effect.

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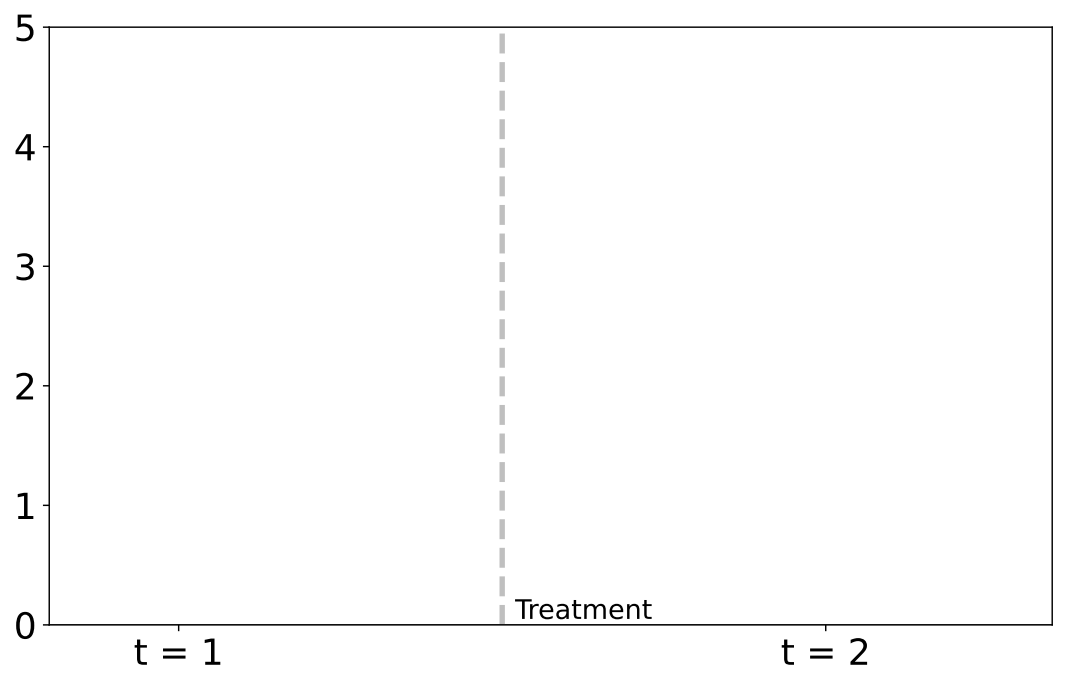
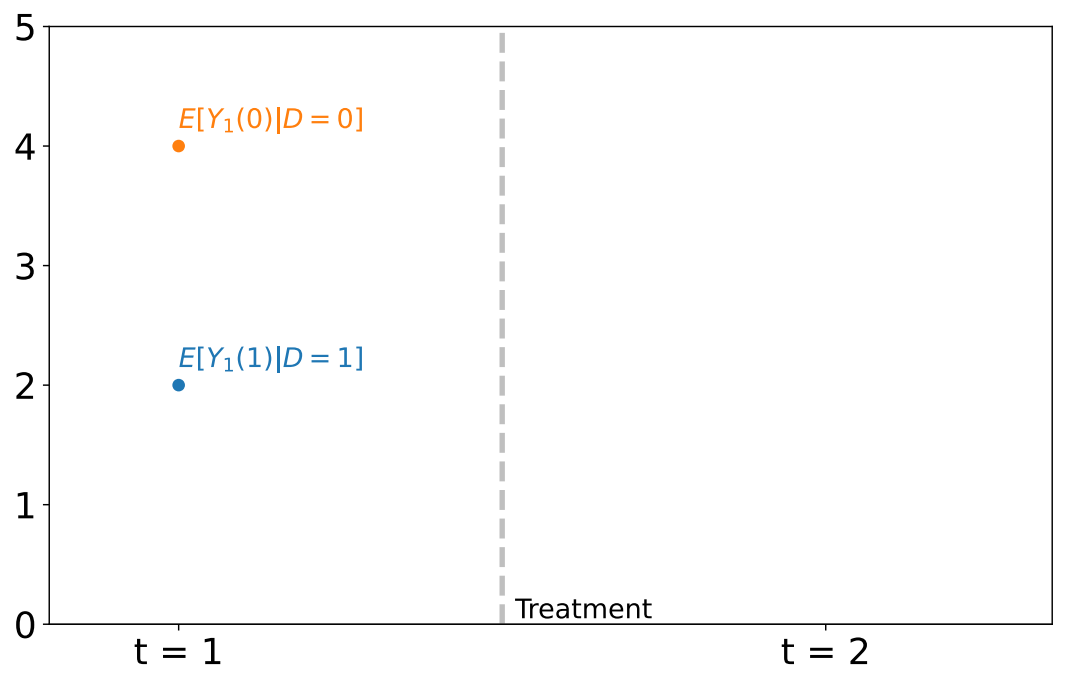
## Two period of times: t=1, t=2



# Difference-in-differences

**Potential outcomes: $Y_t(d)$ where $d = \{0, 1\}$ is the treatment at period 2**



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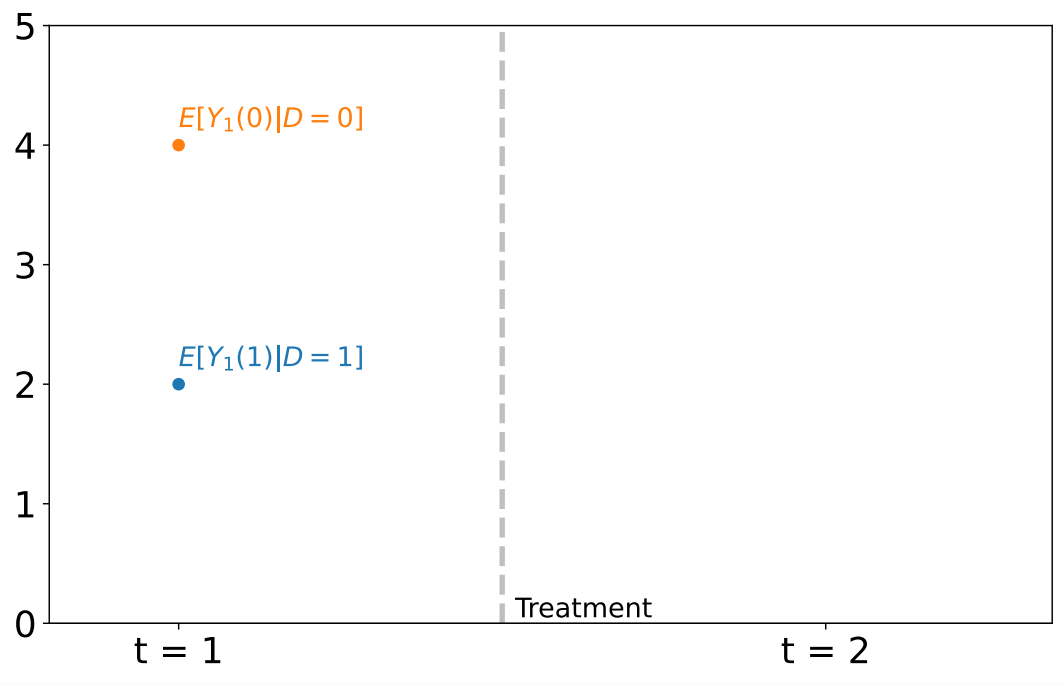
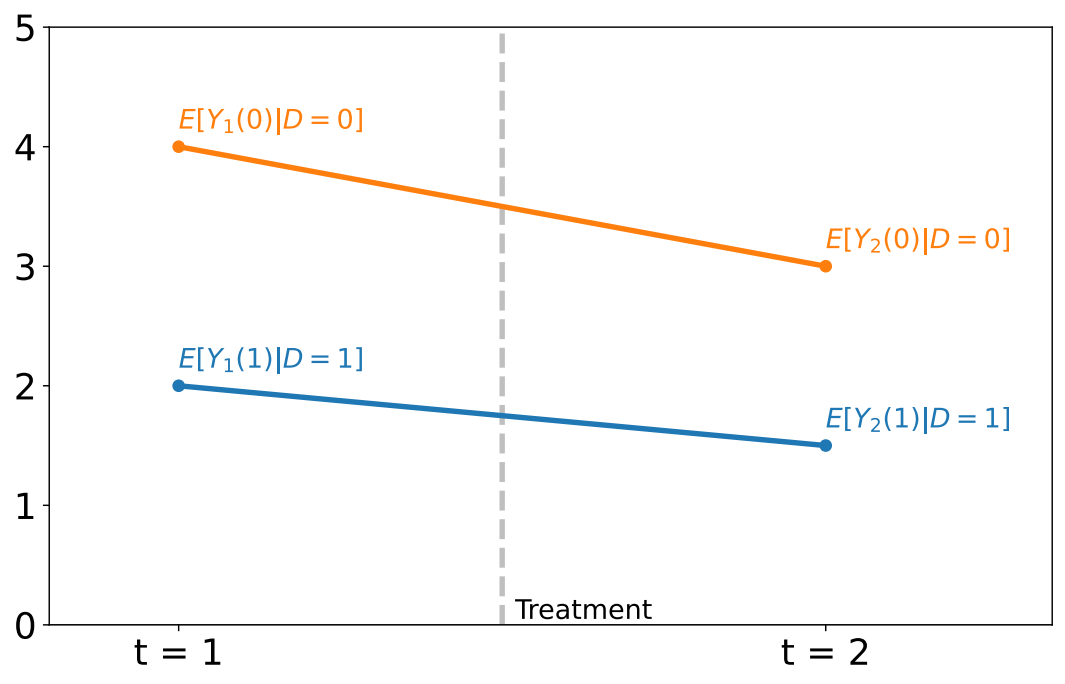



⚠️ $\mathbb{E}[Y_1(1)] = \underbrace{[\mathbb{E}[Y_1(1) \mid D = 0]]}_{\text{counterfactural}}\mathbb{P}(D = 0) + \underbrace{[Y_1(1) \mid D = 1]}_{\text{observed}}\mathbb{P}(D = 1)$

# Difference-in-differences

**Our target is the average treatment effect on the treated (ATT)**




$$\tau_{\text{ATT}} = \mathbb{E}[Y_2(1)|\ D=1] - \mathbb{E}[Y_2(0)|\ D=1]$$

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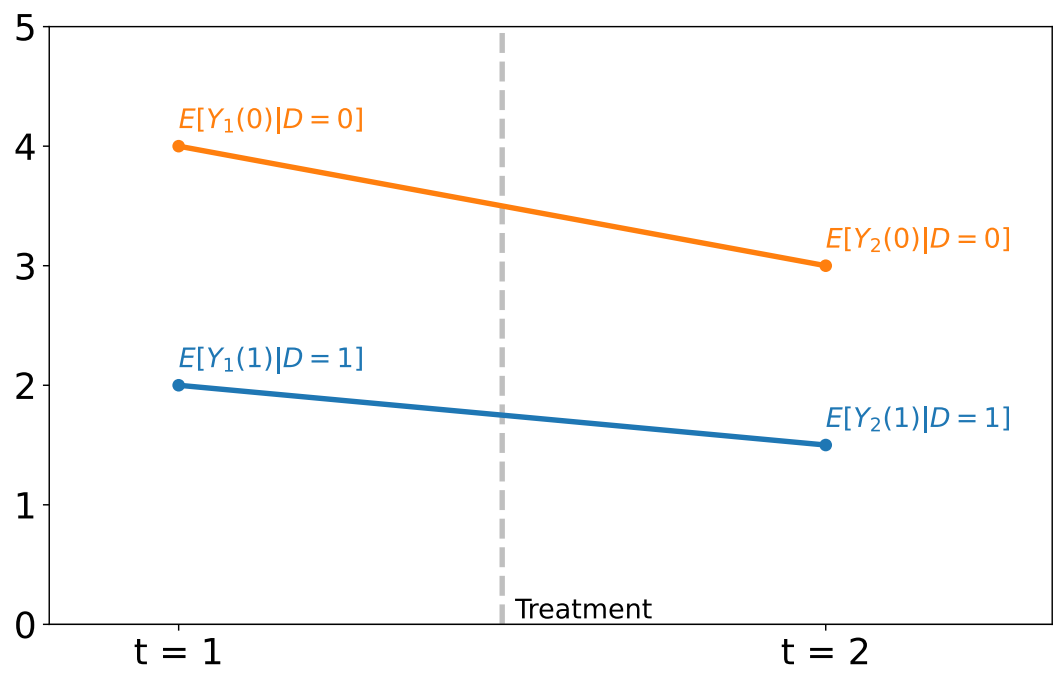
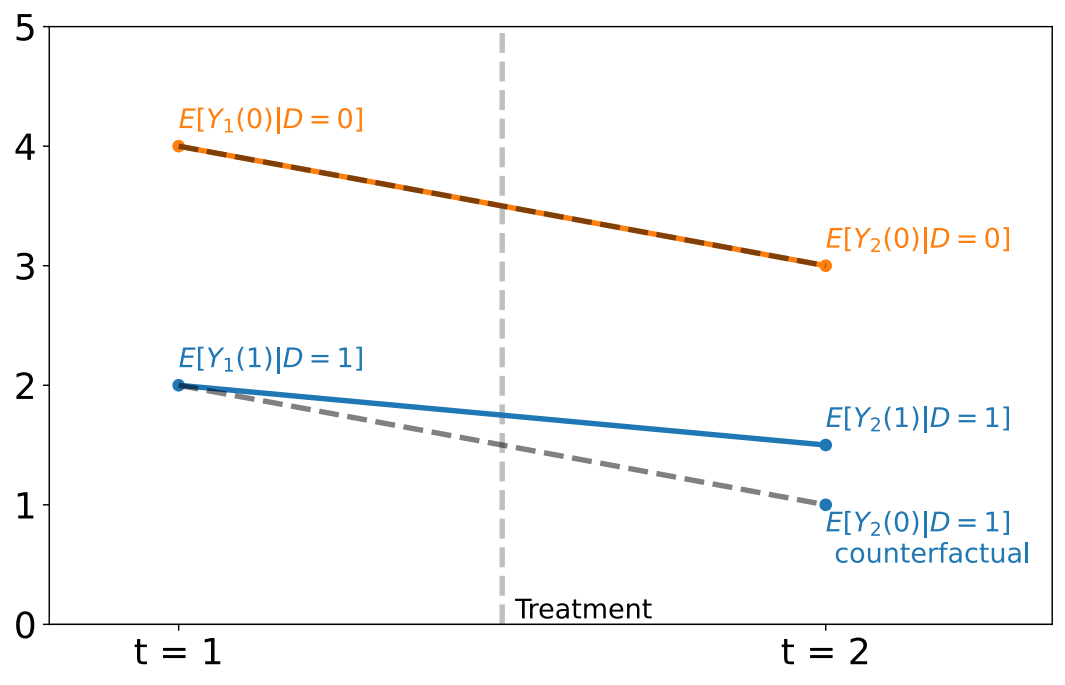



$$\tau_{\text{ATT}} = \underbrace{[Y_2(1)|\ D=1]}_{\text{treated outcome for t=2}} - \underbrace{\mathbb{E}[Y_2(0)|\ D=1]}_{\text{unobserved counterfactual}}$$

# Difference-in-differences

**First assumption, parallel trends**

$$\mathbb{E}[Y_2(0) - Y_1(0) \mid D = 1] = \mathbb{E}[Y_2(0) - Y_1(0) \mid D = 0]$$



# Difference-in-differences

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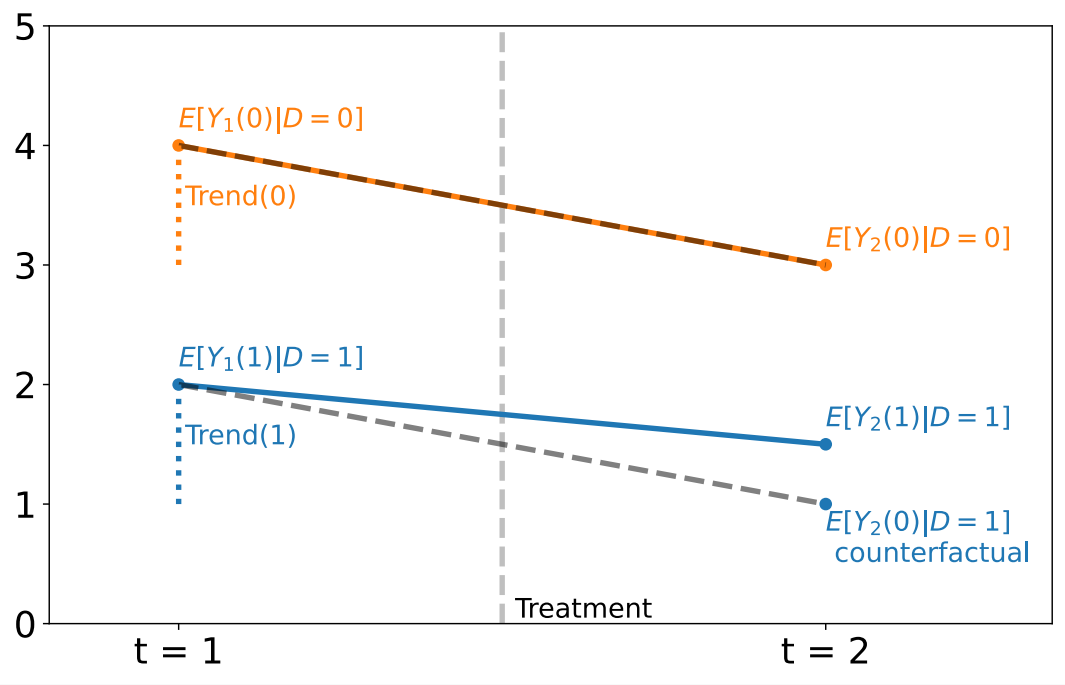
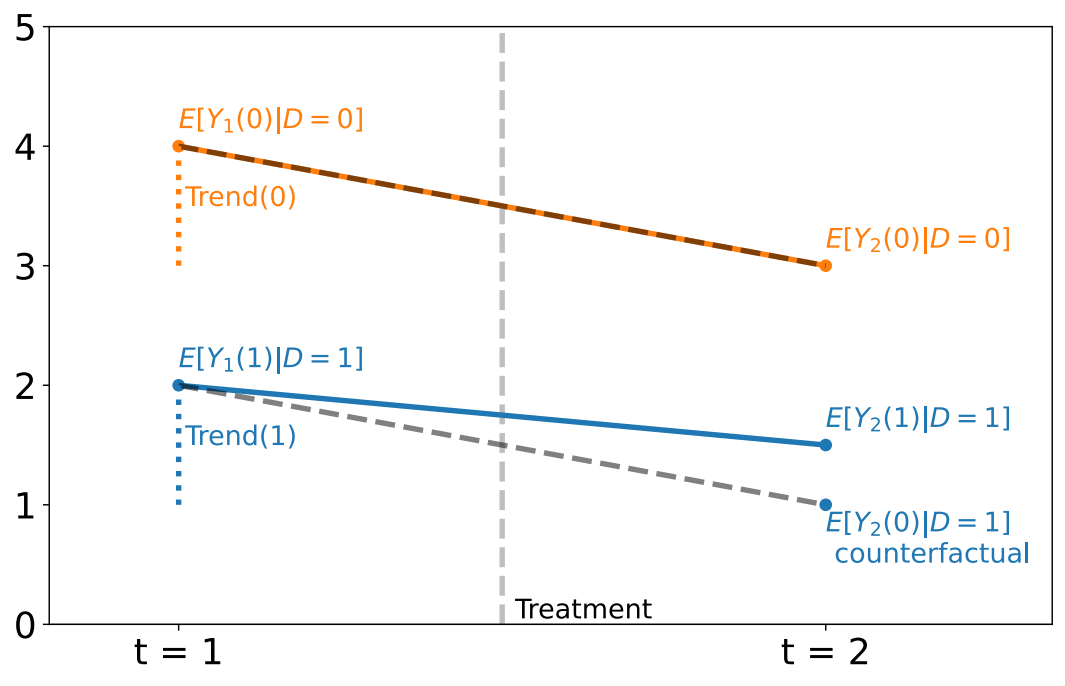
$$\underbrace{[Y_2(0) - Y_1(0) \mid D = 1]}_{\text{Trend}(1)} = \underbrace{\mathbb{E}[Y_2(0) - Y_1(0) \mid D = 0]}_{\text{Trend}(0)}$$



# Difference-in-differences

**First assumption, parallel trends**

$$\mathbb{E}[Y_2(0) \mid D = 1] = \mathbb{E}[Y_1(0) \mid D = 1] + \mathbb{E}[Y_2(0) - Y_1(0) \mid D = 0]$$



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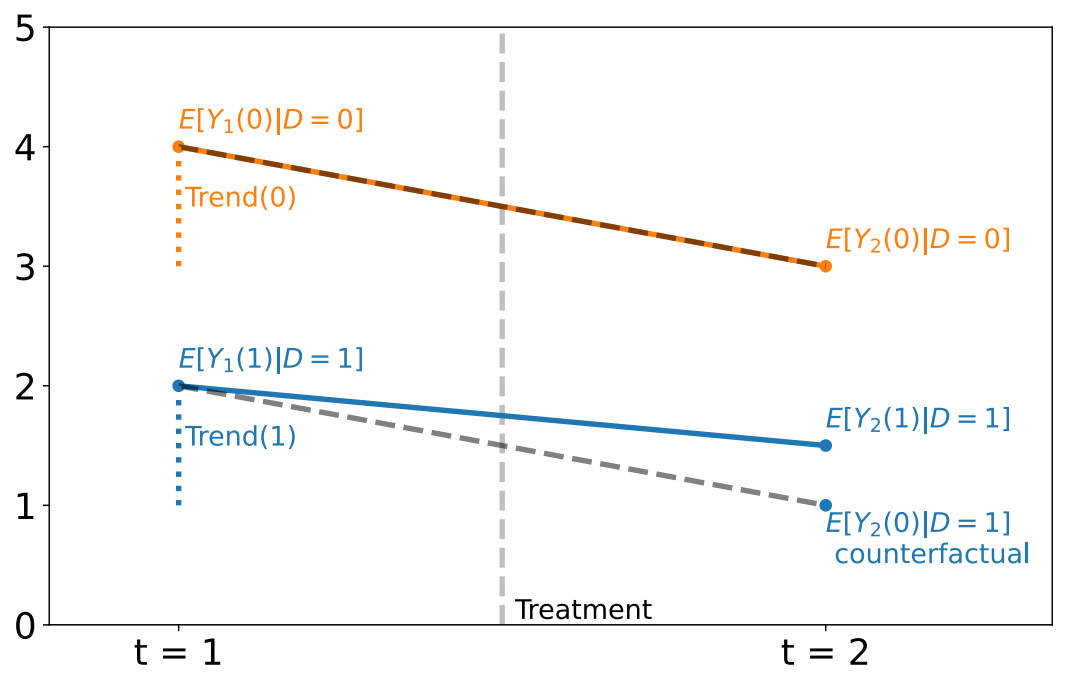
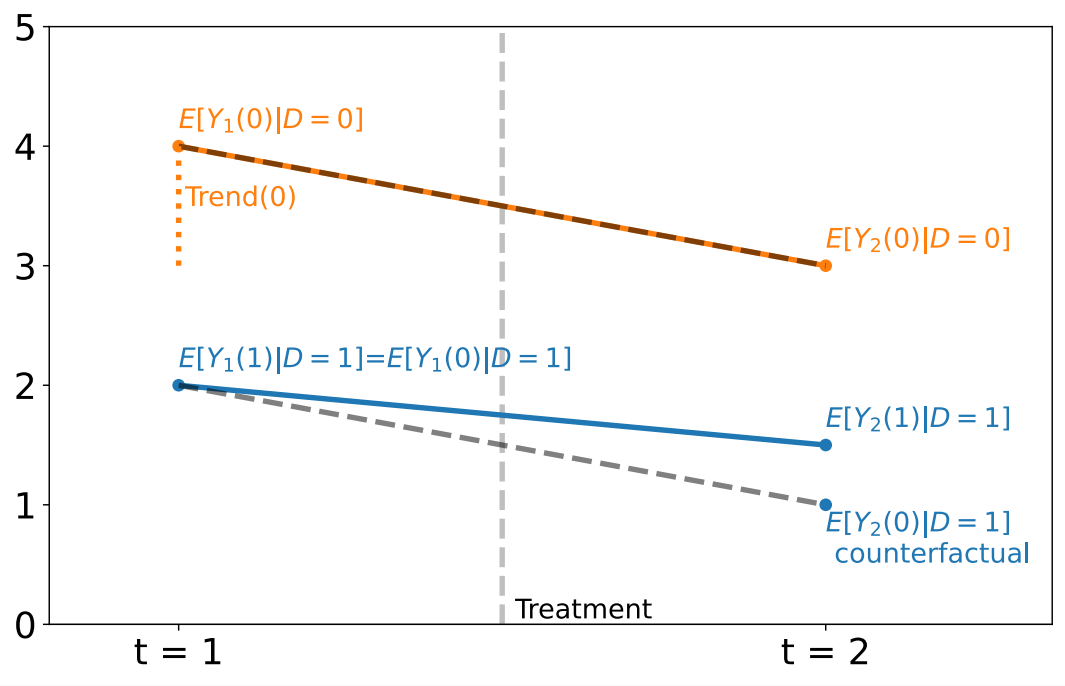
$$\mathbb{E}[Y_2(0) \mid D=1] = \underbrace{\mathbb{E}[Y_1(0) \mid D=1]}_{\text{unobserved counterfactual}} + \mathbb{E}[Y_2(0) - Y_1(0) \mid D=0]$$



# Difference-in-differences

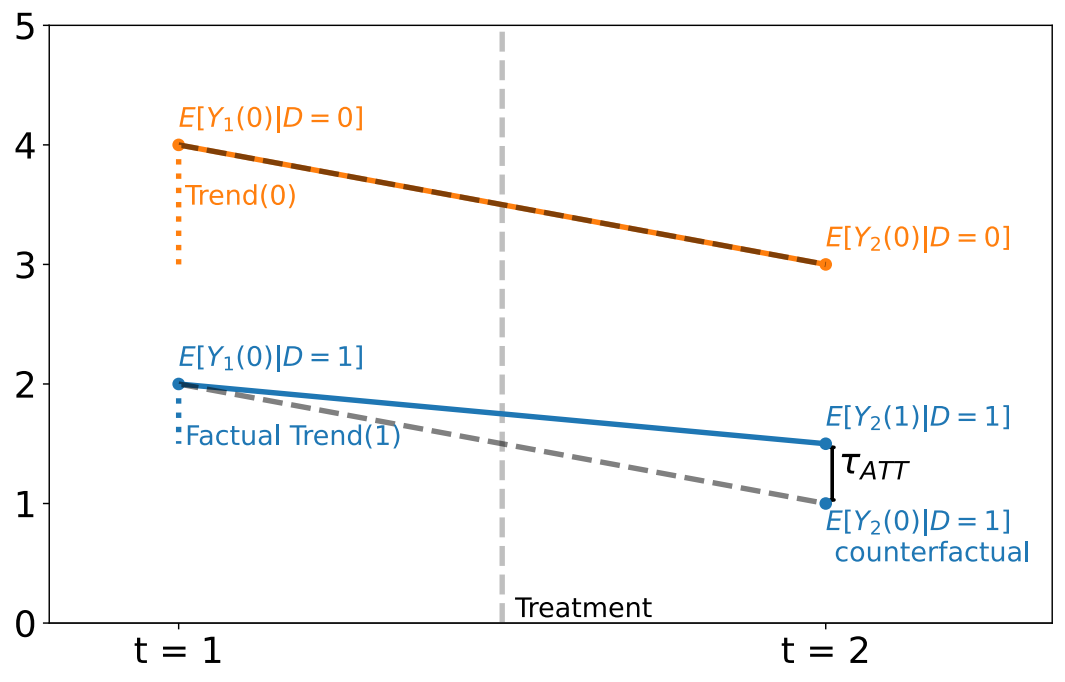
**Second assumption, no anticipation of the treatment**

$$\mathbb{E}[Y_1(1)|D=1] = \mathbb{E}[Y_1(0)|D=1]$$



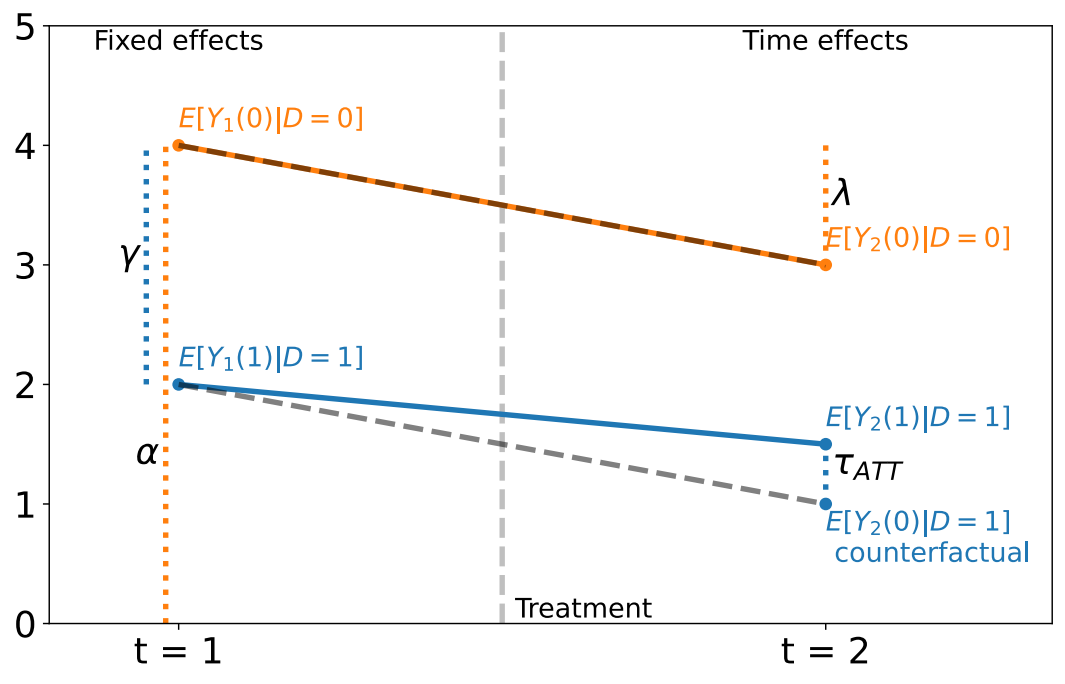
# Identification of ATT

$$\tau_{\text{ATT}} = \mathbb{E}[Y_2(1)|\ D=1] - \mathbb{E}[Y_2(0)|\ D=1]$$
$$= \underbrace{\mathbb{E}[Y_2(1)|\ D=1] - \mathbb{E}[Y_1(0)|D=1]}_{\text{Factual Trend}(1)} - \underbrace{\mathbb{E}[Y_2(0)|D=0] - \mathbb{E}[Y_1(0)|D=0]}_{\text{Trend}(0)}$$



# Estimation: link with two way fixed effect (TWFE)

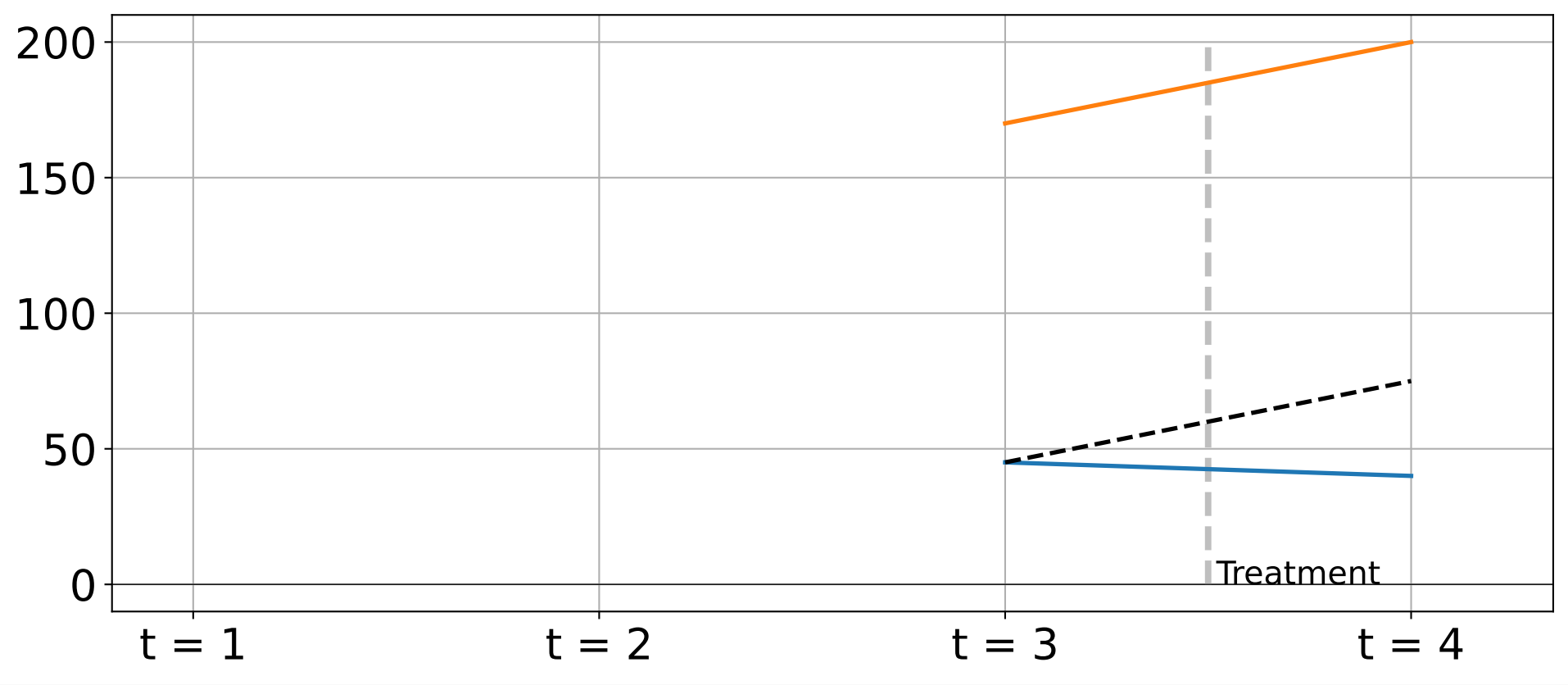
$$Y = \alpha + \gamma D + \lambda \mathbb{1}(t = 2) + \tau_{\text{ATT}} D \mathbb{1}(t = 2)$$




⚠️ Mechanic link: works only under parallel trends and no anticipation assumptions.

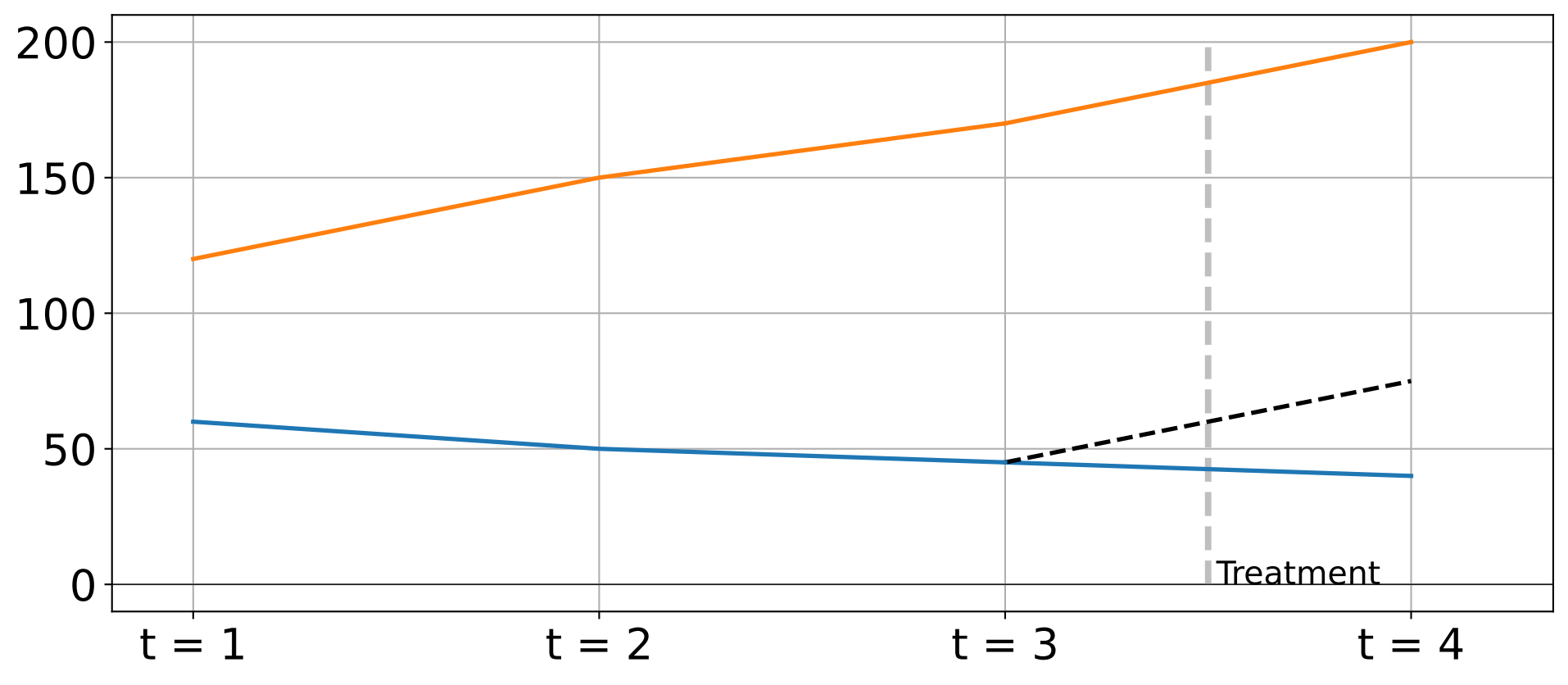
# Failure of the parallel trend assumption

**Seems like the treatment decreases the outcome!**



# Failure of the parallel trend assumption

## Oups...



# DID estimator for more than two time units

**Target estimand: sample average treatment effect on the treated (SATT)**

$$\tau_{\text{SATT}} = \frac{1}{|\{i:D_i=1\}|} \sum_{i:D_i=1} \frac{1}{T-H} \sum_{t=H+1}^{T} Y_{it}(1) - Y_{it}(0)$$

**DID estimator**

$$\widehat{\tau_{\text{DID}}} = \frac{1}{|\{i:D_i=1\}|} \sum_{i:D_i=1} \left[ \frac{1}{T-H} \sum_{t=H+1}^{T} Y_{it} - \frac{1}{H} \sum_{t=1}^{H} Y_{it} \right] - \frac{1}{|\{i:D_i=0\}|} \sum_{i:D_i=0} \left[ \frac{1}{T-H} \sum_{t=H+1}^{T} Y_{it} - \frac{1}{H} \sum_{t=1}^{H} Y_{it} \right]$$

**Assumption**

**No anticipation of the treatment:** $Y_{it}(0) = Y_{it}(1) \forall t = 1, ..., H.$

**Parallel trend:** $\mathbb{E}[Y_{it}(0, \infty) - Y_{i1}(0, \infty)] = \beta_t, t = 2, ..., T.$

See (Wager, 2024) for a clear proof of consistancy.

# DID: Take-away

## Pros

- Extremely common in economics and quite simple to implement.
- Can be extended to (Wager, 2024)
  - more than two time periods: exact same formulation
  - staggered adoption of the treatment: a bit more complex

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## Can we do better: i.e. robust to the parallel trend assumption?

# “Synthetic controls”

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**Idea**

Find a weighted average of controls that predicts well the treated unit outcome before treatment.

## Example

What is the effect of tobacco tax on cigarettes sales? (Abadie et al., 2010)

# Examples of application of synthetic controls to epidemiology

- Literature review of the usage of SCM in healthcare (up to 2016): (Bouttell et al., 2018)

## Some use cases

- What is the effect of UK pay-for-performance program in primary care on mortality? (Ryan et al., 2016)
- What is the effect of soda taxes on sugar-based product consumption? (Puig-Codina et al., 2021)
- What is the effect of Ohio vaccine lottery on covid-19 vaccination? (Brehm et al., 2022)
- What is the effect of wildfire storm on respiratory hospitalizations? (Sheridan et al., 2022)

# Synthetic control example: California's Proposition 99 (Abadie et al., 2010)

## Context

1988: 25-cent tax per pack of cigarettes, ban of on cigarette vending machines in public areas accessible by juveniles, and a ban on the individual sale of single cigarettes.

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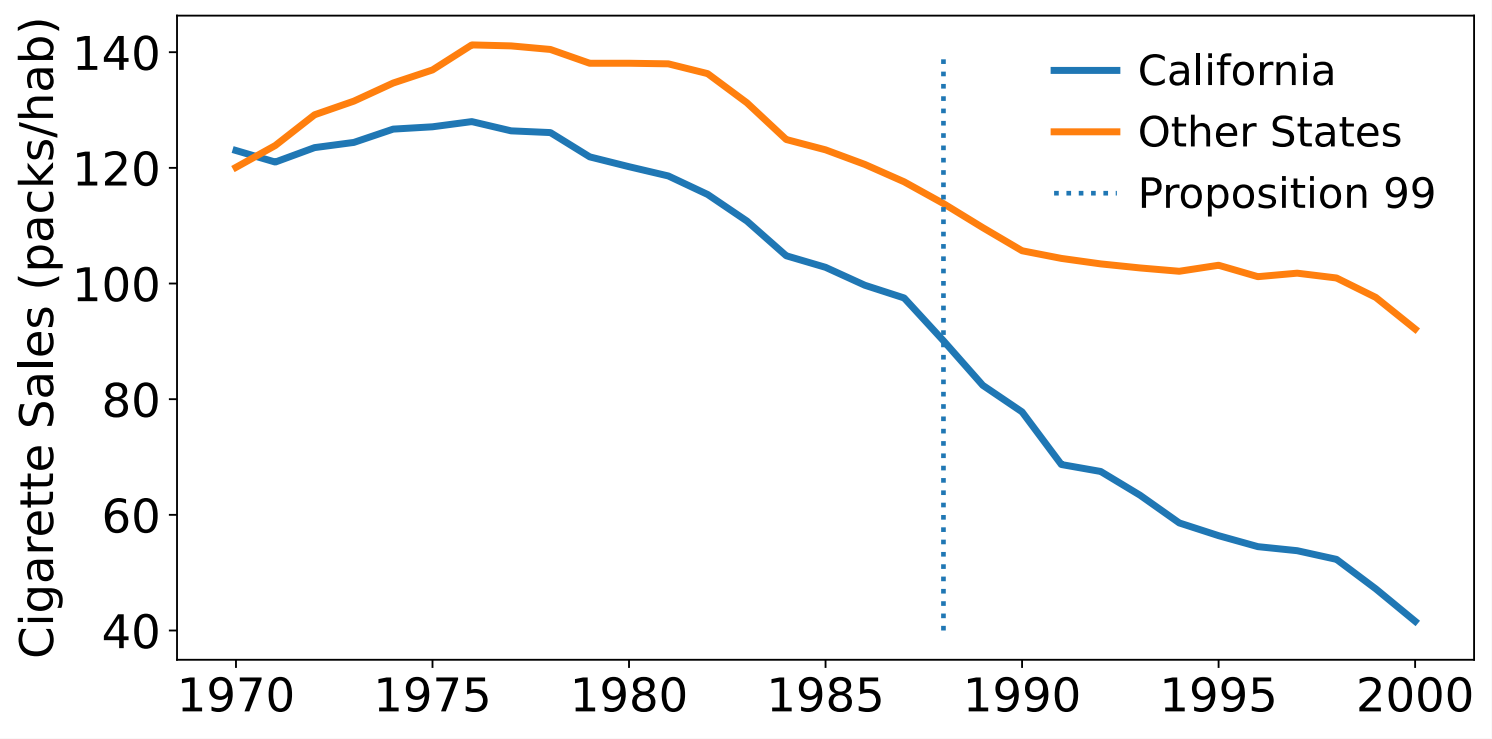
**Covariates $X_{j,t}$: cigarette price, previous cigarette sales.**

# Synthetic control example: plot the data



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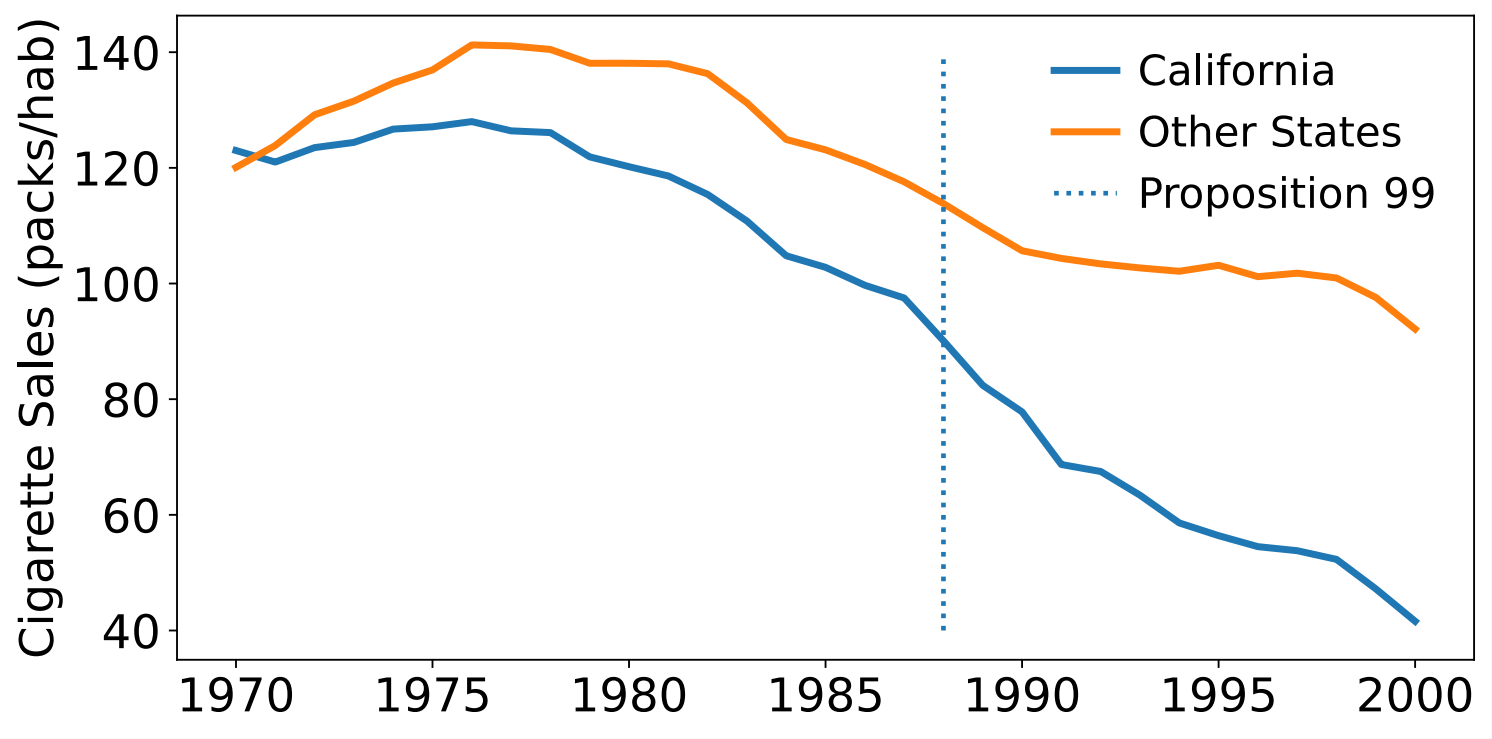


😲 Decrease in cigarette sales in California.

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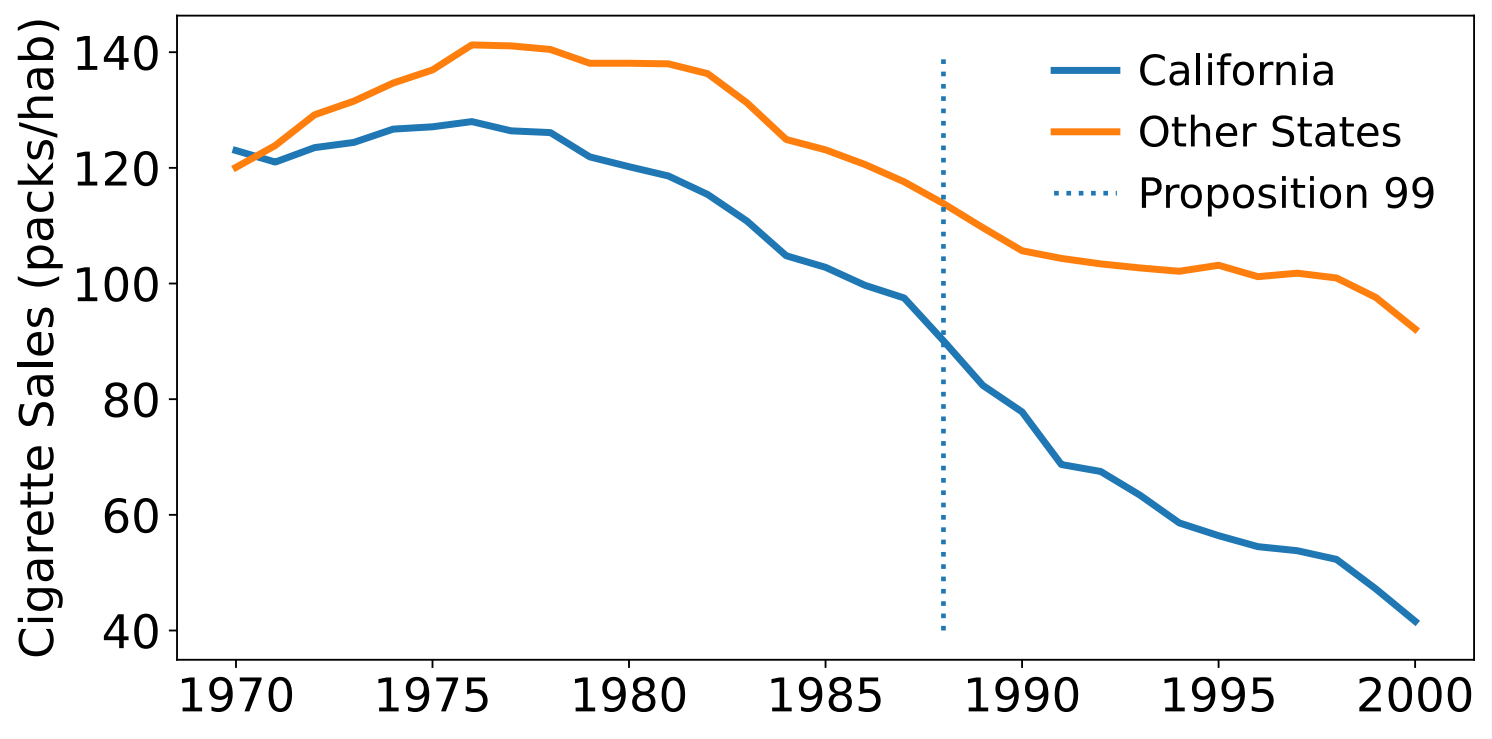


😲 Decrease in cigarette sales in California.

🤔 Decrease began before the treatment and occured also for other states.

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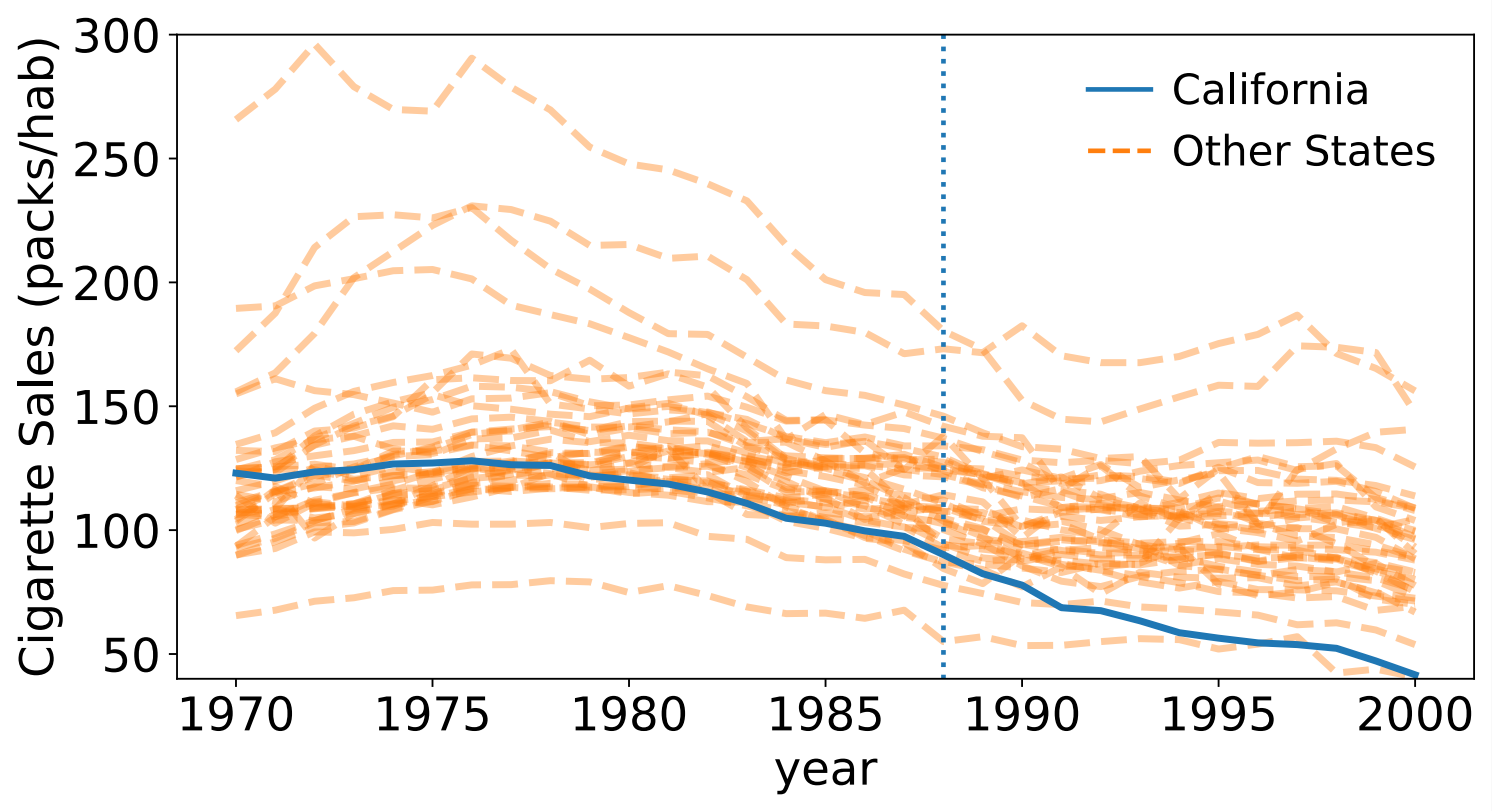

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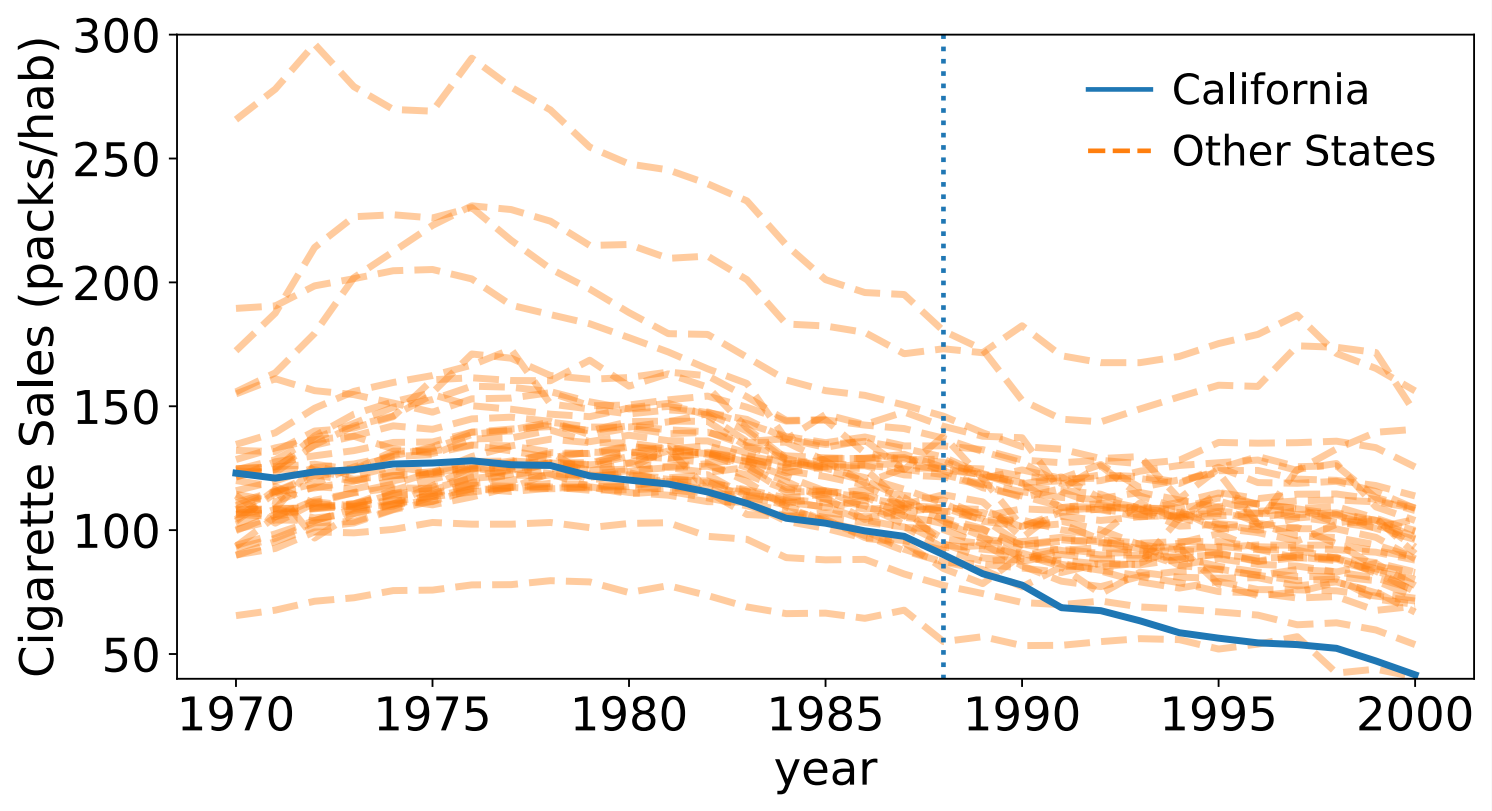
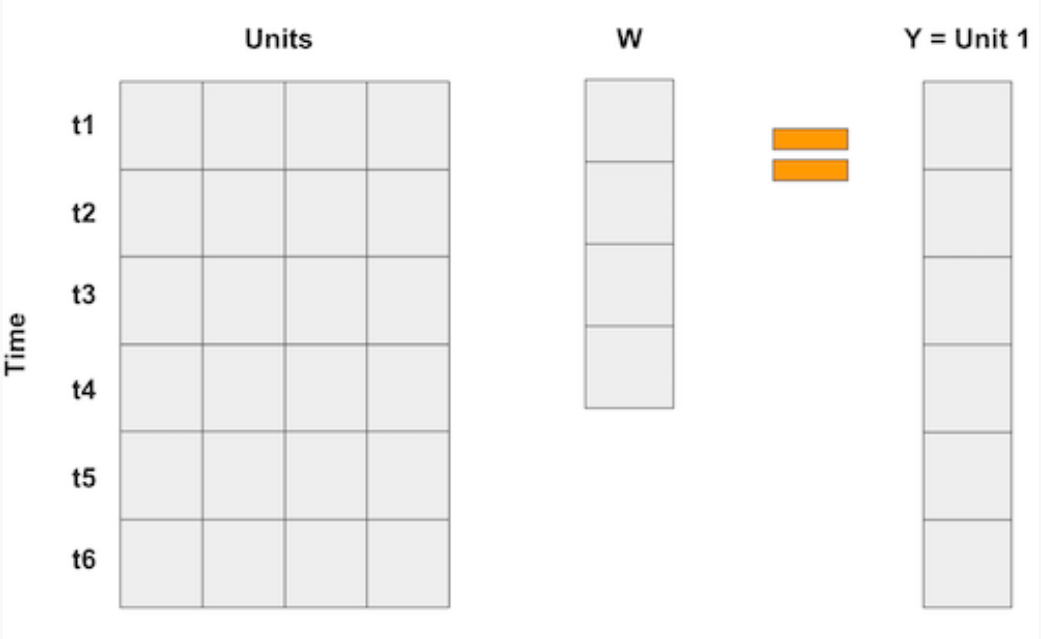



💡 Force parallel trends: Find a weighted average of other states that predicts well the pre-treatment trend of California (before $T_0 = 1988$).

# Synthetic control as weighted average of control outcomes

Build a predictor for $Y_{1,t}$ (California):

$$\hat{Y}_{1,t} = \sum_{j=2}^{n_0+1} \hat{w}_j Y_{j,t}$$



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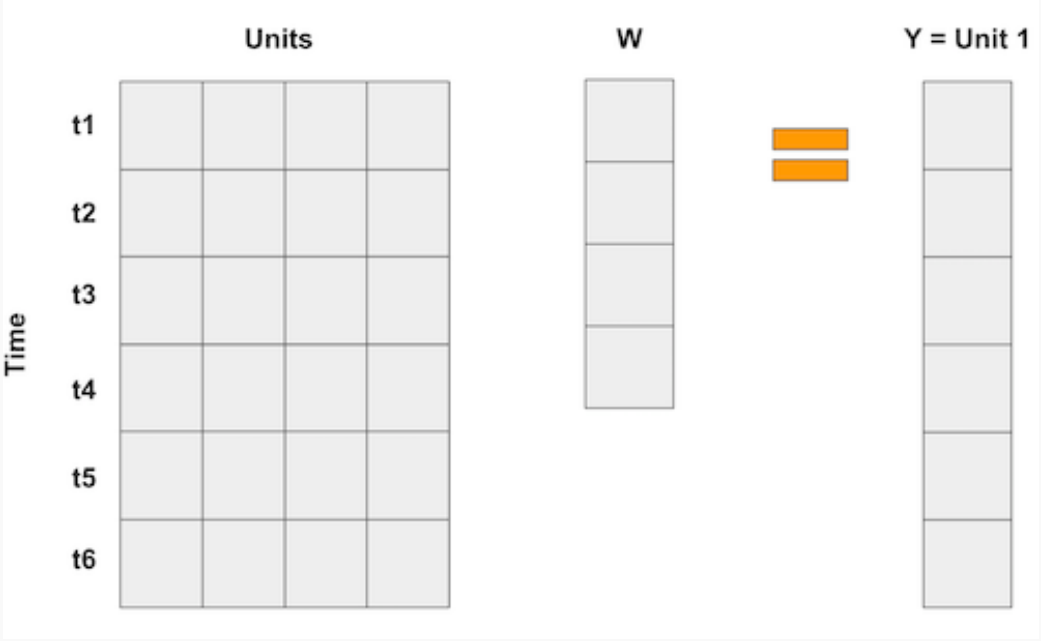
🤔 How to choose the weights?

Minimize some distance between the treated and the controls.



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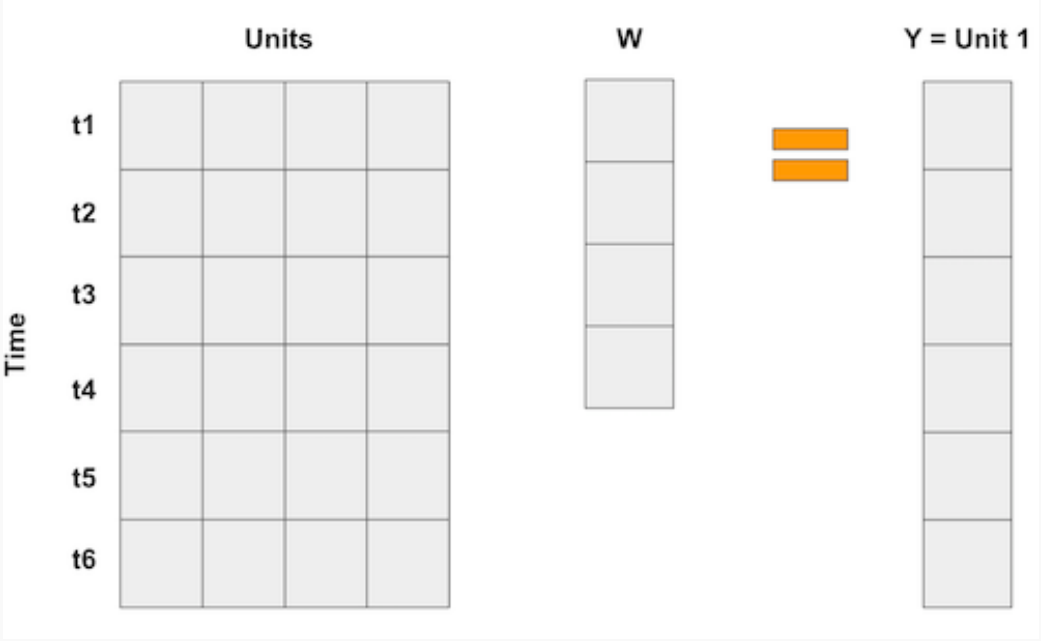

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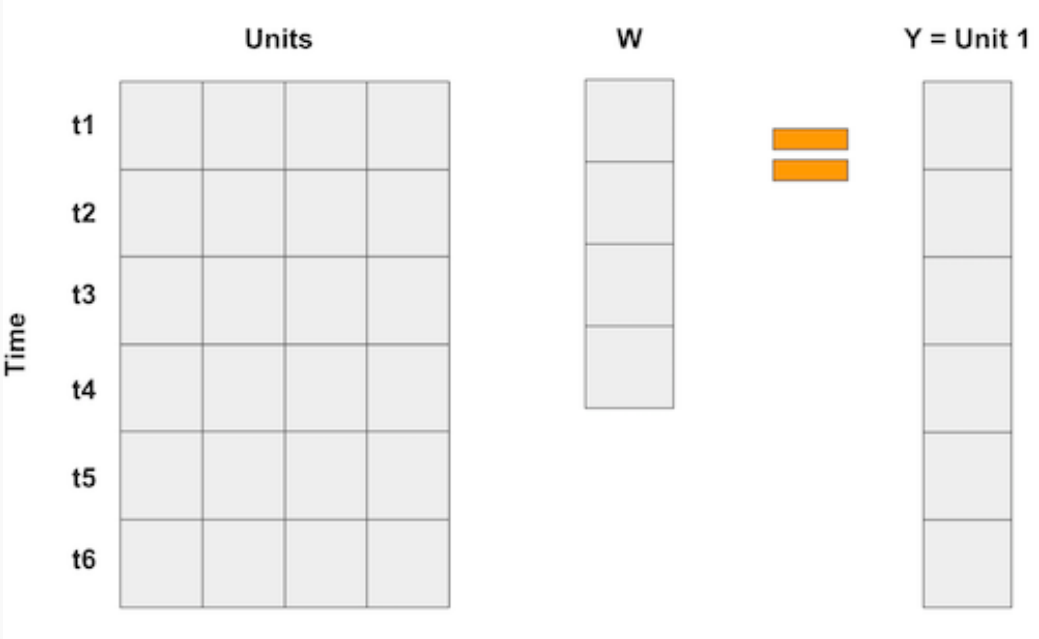
🤓 This is called a balancing estimator: kind of Inverse Probability Weighting.

Cf. (Wager, 2024, chapter 7) for details on balancing estimators.



# Synthetic controls: minimization problem

**Characteristics**

Pre-treatment characteristics concatenate pre-treatment outcomes and other pre-treatment predictors $Z_1$ eg. cigarette prices:

$$X_{\text{treat}} = X_1 = \begin{pmatrix} Y_{1,1} \\ Y_{1,2} \\ \dots \\ Y_{1,T_0} \\ Z_1 \end{pmatrix} \in R^{p \times 1} \qquad X_{\text{control}} = \left( X_2, .., X_{n_0+1} \right) \in R^{p \times n_0}$$

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**Minimization problem with constraints**

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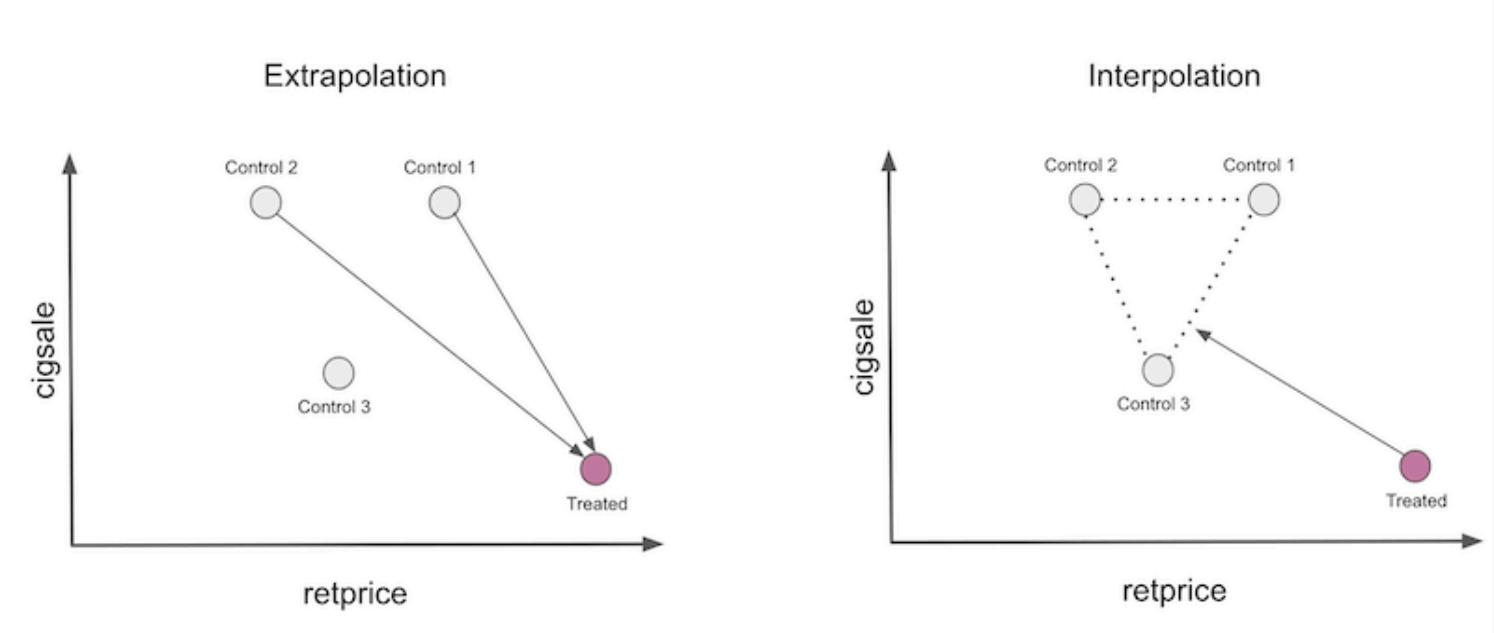
# Synthetic controls: Why choose positive weights summing to one?

## This is called interpolation (vs extrapolation)



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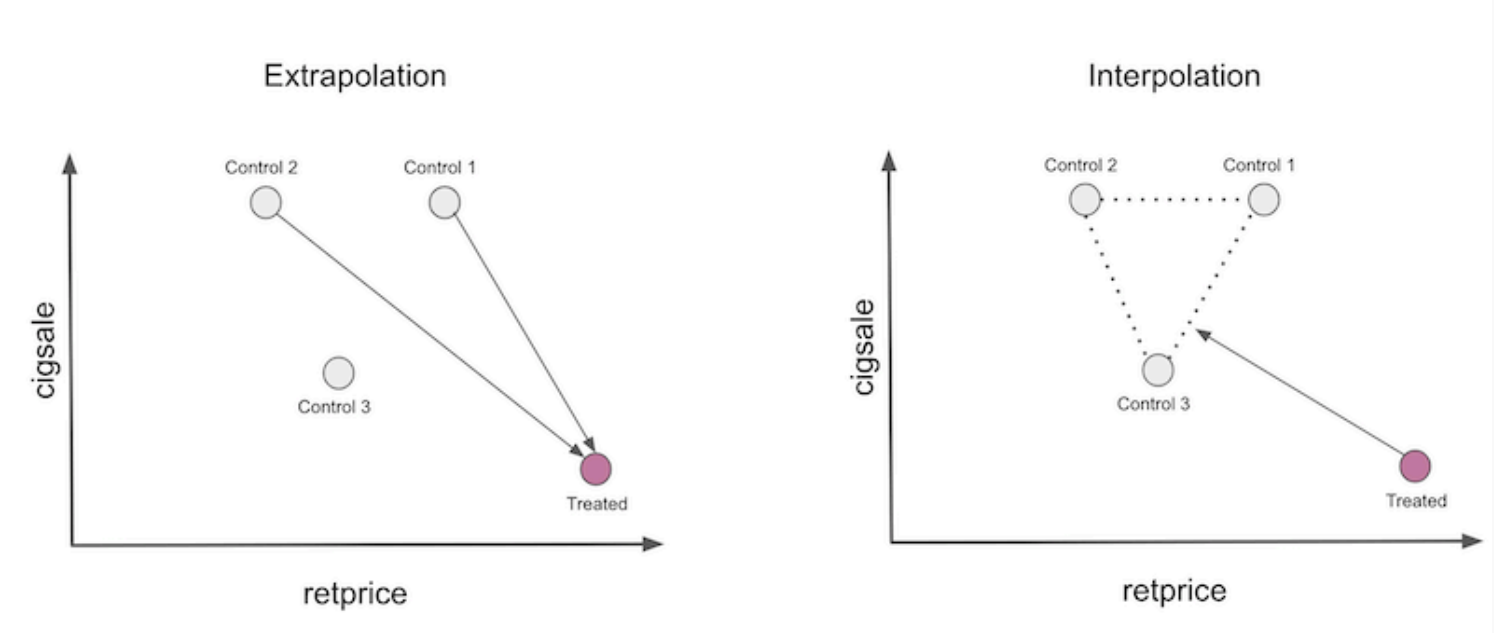


## Interpolation enforces regularization, thus limits overfitting

Same kind of regularization than L1 norm in Lasso: forces some coefficient to be zero.

# Synthetic controls: Extrapolation failure with unconstrained weights

$p = 2T_0$ covariates:

$$X_j = \begin{pmatrix} Y_{j,1} \\ \dots \\ Y_{j,T_0} \\ Z_{j,1} \\ \dots \\ Z_{j,T_0} \end{pmatrix}^T \in R^{2T_0}$$

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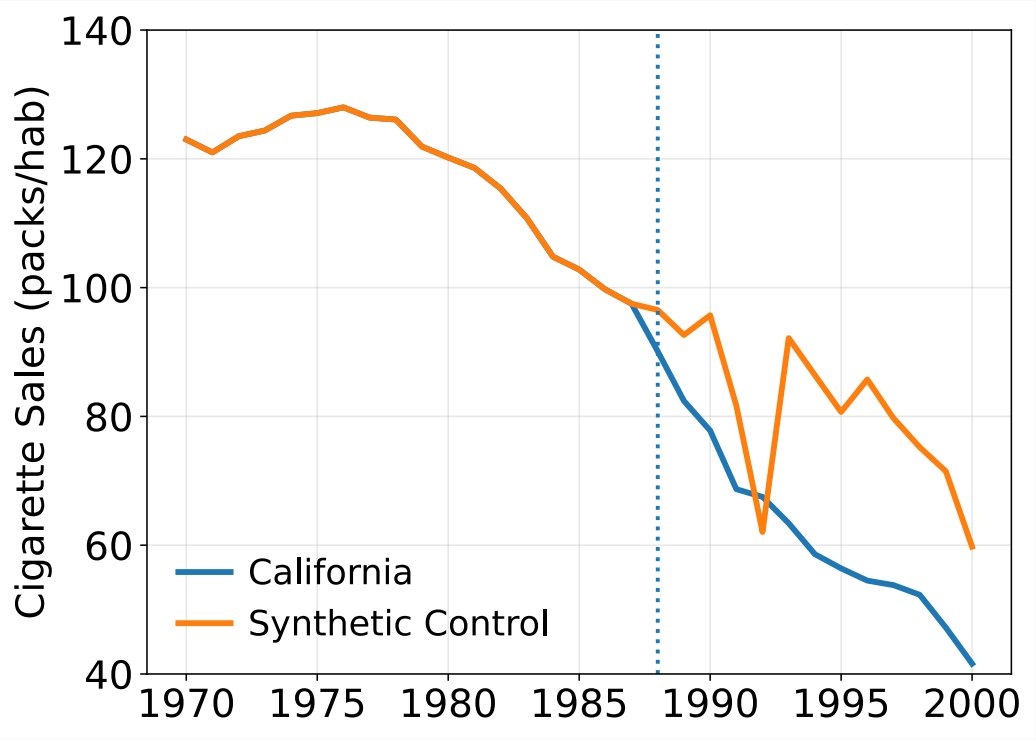
Model: $\underbrace{X_{\text{treat}}}_{p\times 1} \sim \underbrace{X_{\text{control}}}_{p\times n_0}\underbrace{w}_{n_0}$



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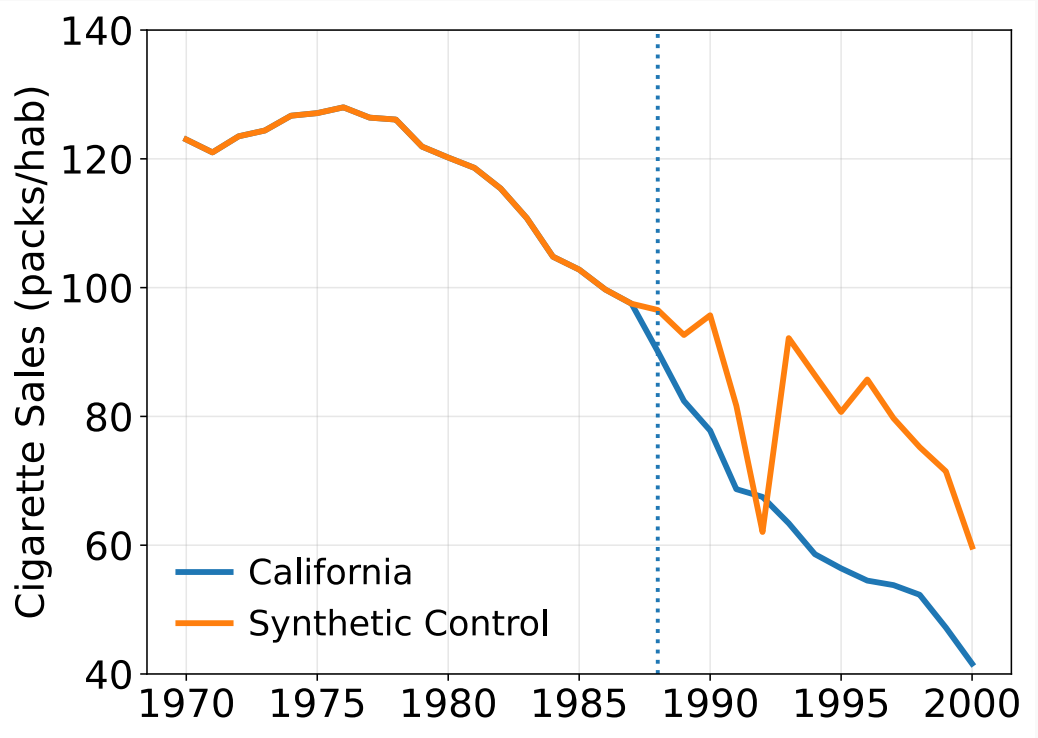
-> Simple linear regression estimated by OLS

Prediction: $\hat{Y}_{\text{synth}} = (Y_{t,j})_{\substack{t=1..T \\ j=2..n_0+1}} w$



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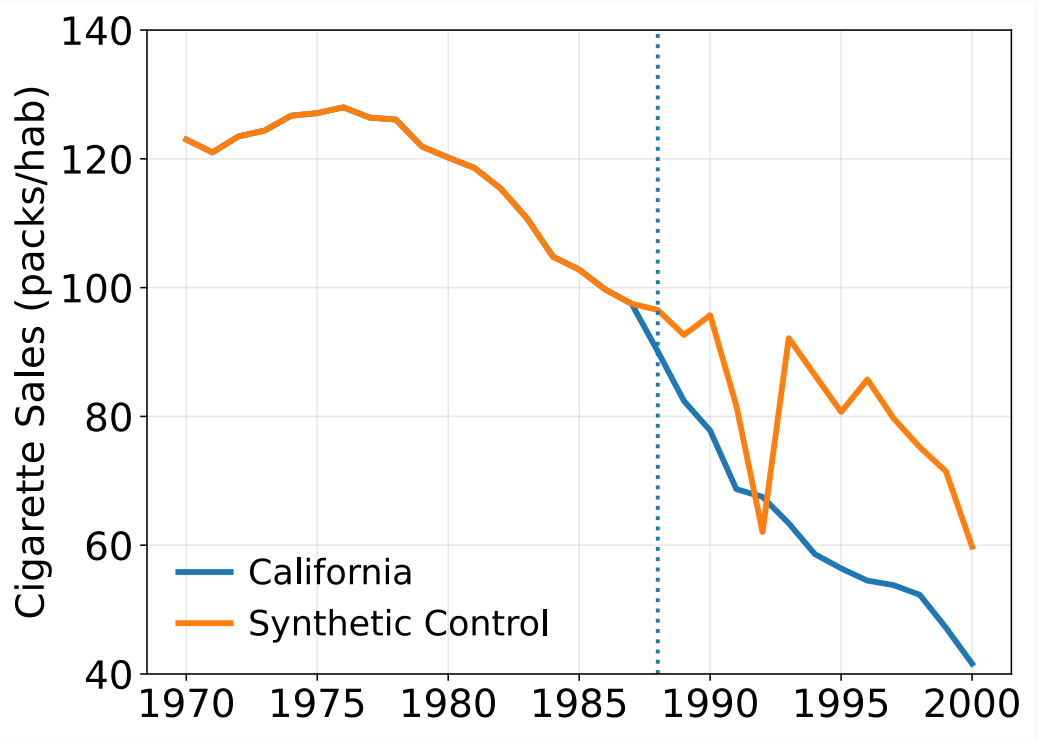

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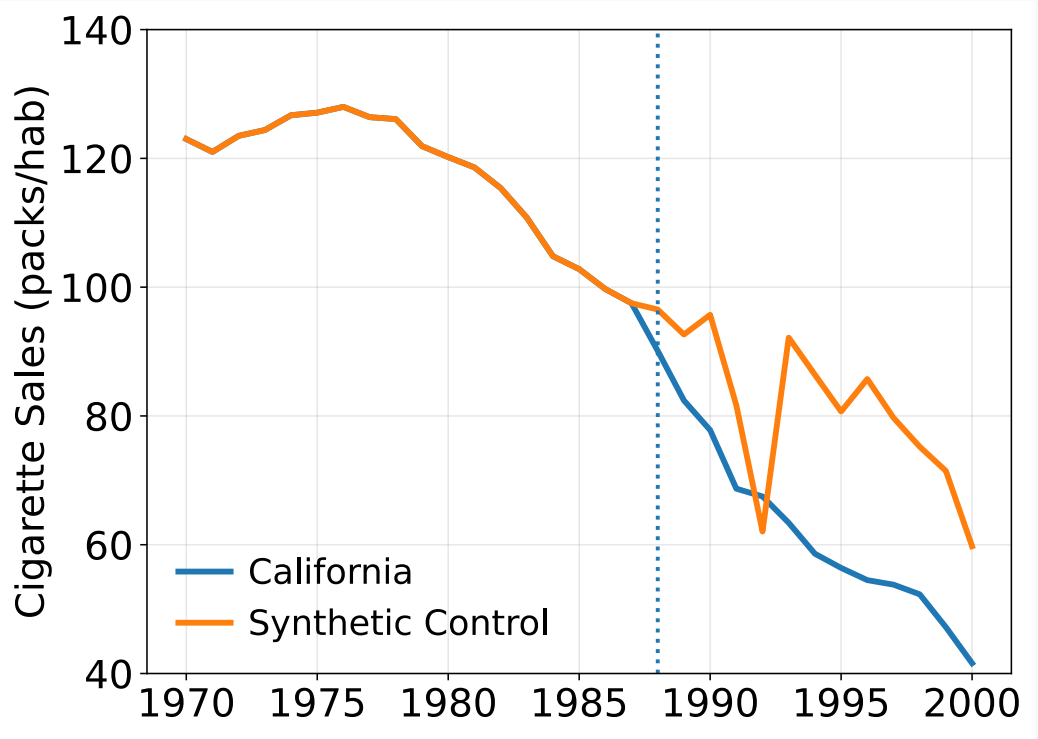
😭 **Overfitting**



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3. Minimize the pre-treatment mean squared prediction error (MSPE) of the treated unit:

$$\begin{aligned}\text{MSPE}(V) &= \sum_{t=1}^{T_0}\left[Y_{1,t} - \sum_{j=2}^{n_0+1} w_j^*(V) Y_{j,t}\right]^2 \\ &= \| \left(Y_{1,t}\right)_{t=1..T_0} - \left(Y_{j,t}\right)_{\substack{j=2..n_0+1 \\ t=1..T_0}}^{T} \hat{w} \|_2^2\end{aligned}$$

This solution is solved by running two optimization problems:

- Inner loop solving $w^*(V) = \text{argmin}_w \|X_{\text{treat}} - X_{\text{control}} w\|_V^2$
- Outer loop solving $V^* = \text{argmin}_V \text{MSPE}(V)$

# Synthetic controls: estimation without the outer optimization problem

Same coviarates: $X_j = \begin{pmatrix} Y_{j,1} \\ \dots \\ Y_{j,T_0} \\ Z_{j,1} \\ \dots \\ Z_{j,T_0} \end{pmatrix}^T$

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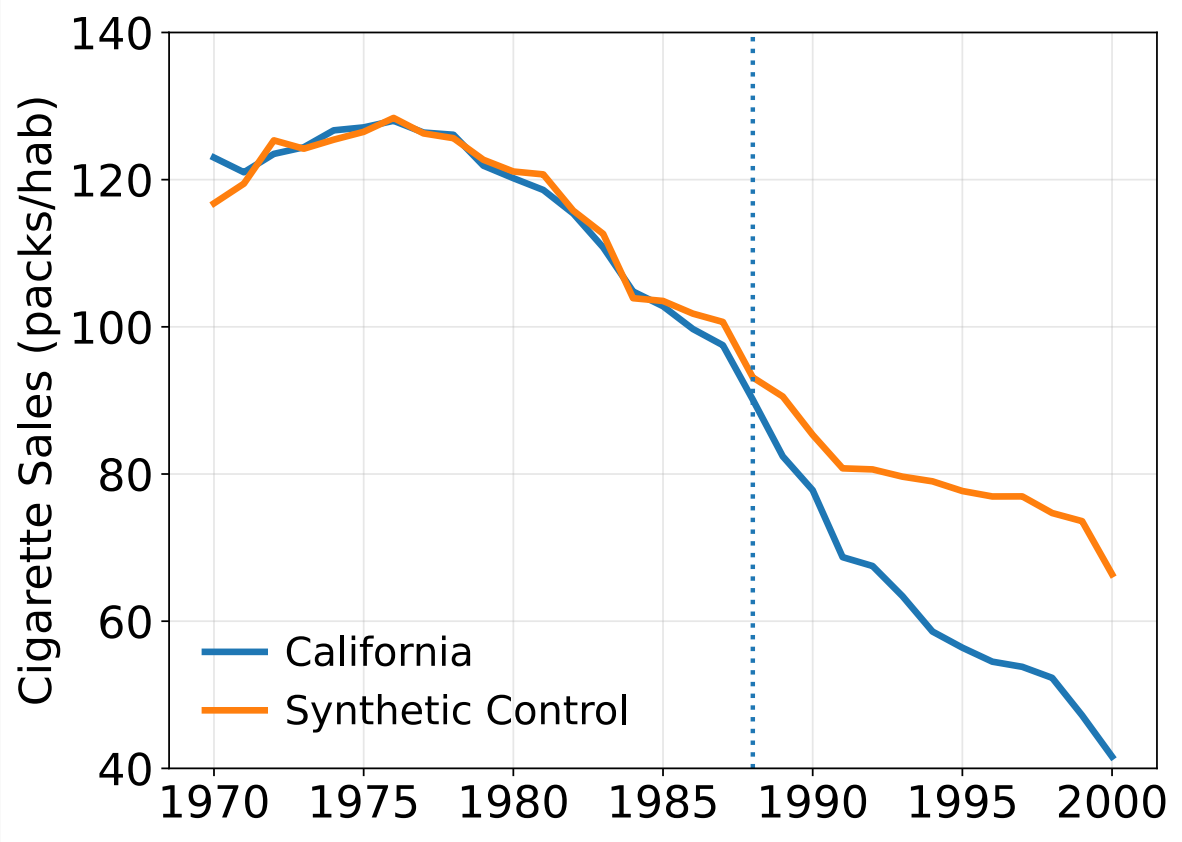

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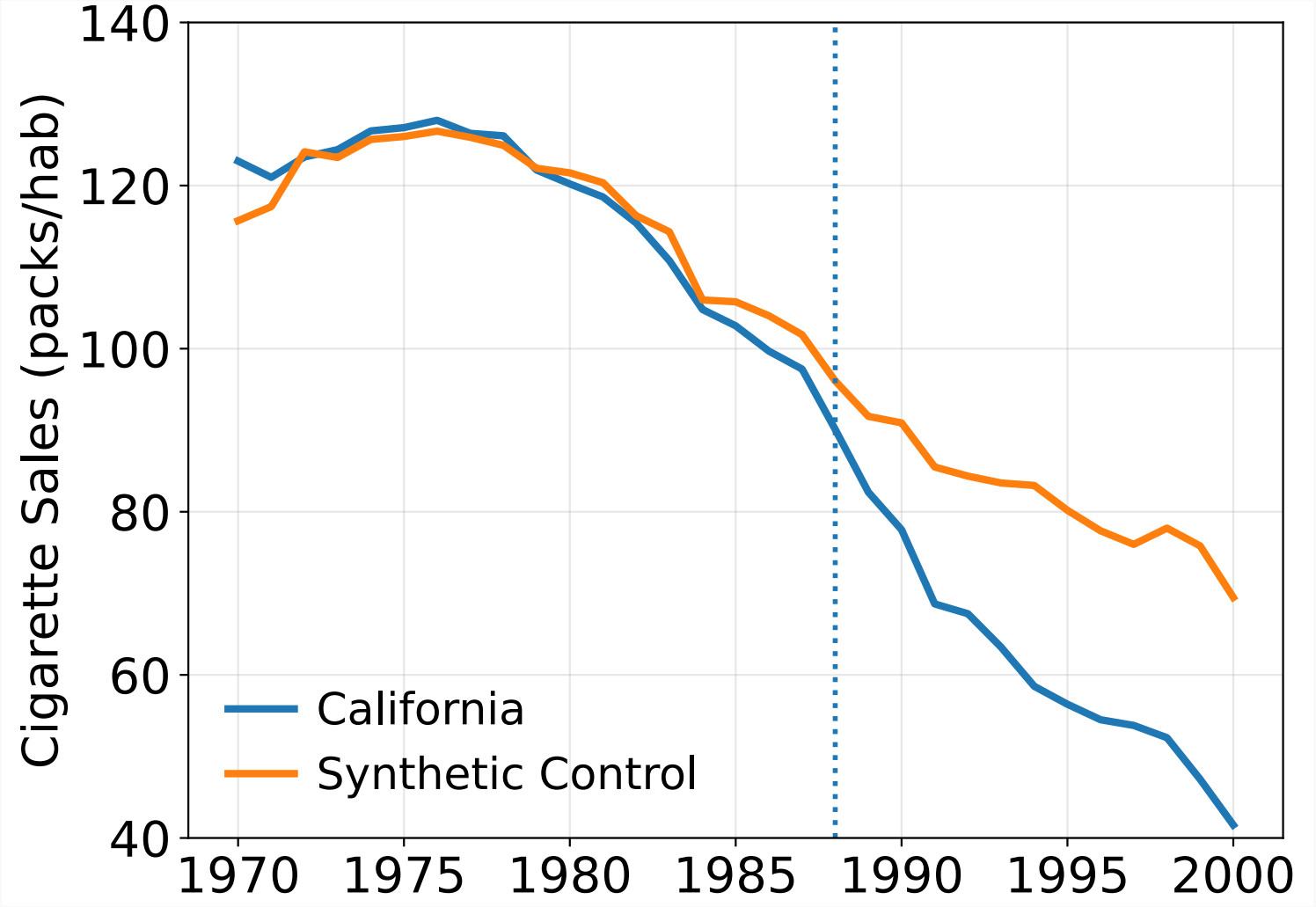

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Treatment assignment introduces more noise than outcome variability.

# Synthetic controls: inference with Placebo tests

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**... but from the variability of the chosen control units**

Treatment assignment introduces more noise than outcome variability.

(Abadie et al., 2010) introduced the placebo test to assess the variability of the synthetic control.

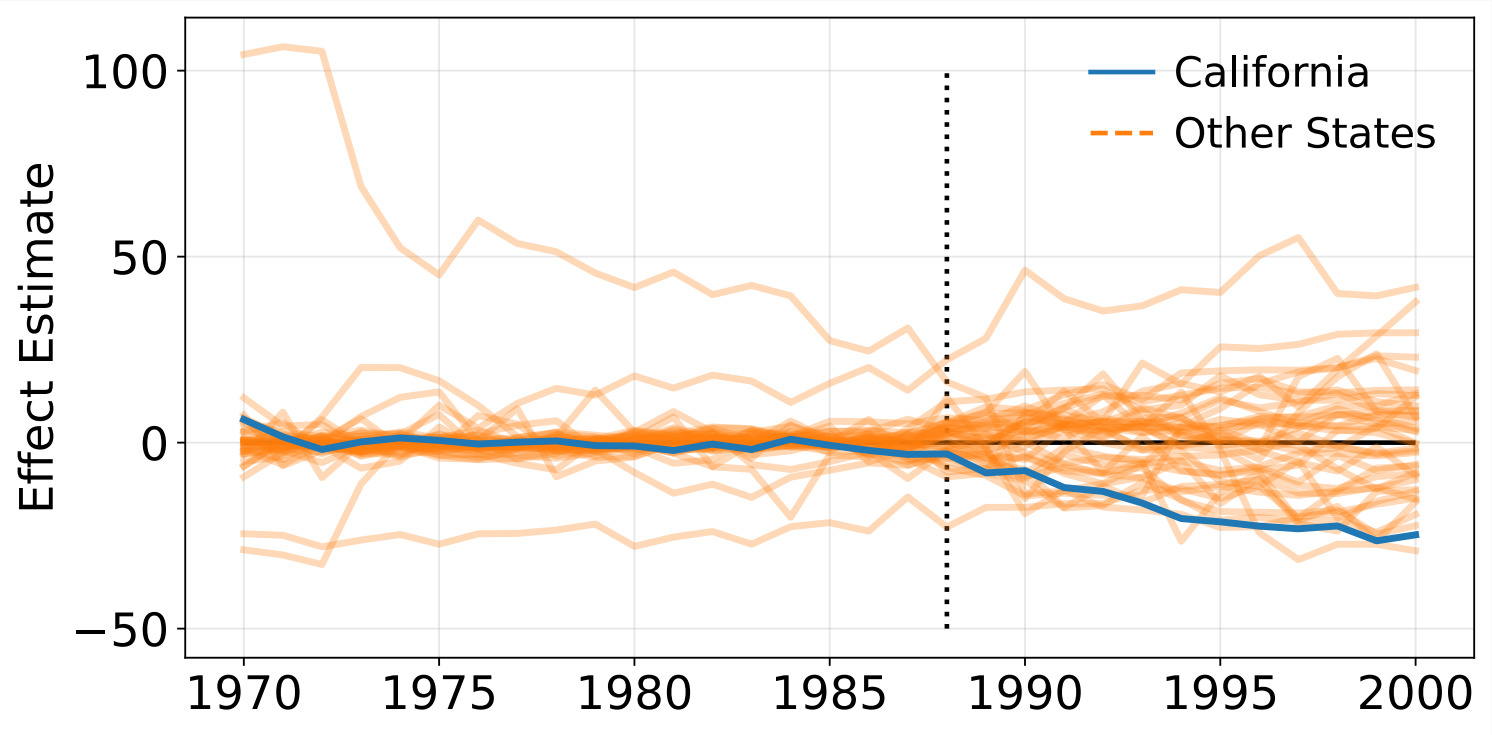
There is also a modern approach on inference for SCM based on Conformal prediction (Chernozhukov et al., 2021) (see end of the slides for intuition).

# Example

**Idea of placebo tests, also called Fisher's Exact tests**

- Permute the treated and control exhaustively.
- For each unit, we pretend it is the treated while the others are the control: we call it a placebo
- Compute the synthetic control for each placebo: it should be close to zero.

## Placebo estimation for all 38 control states



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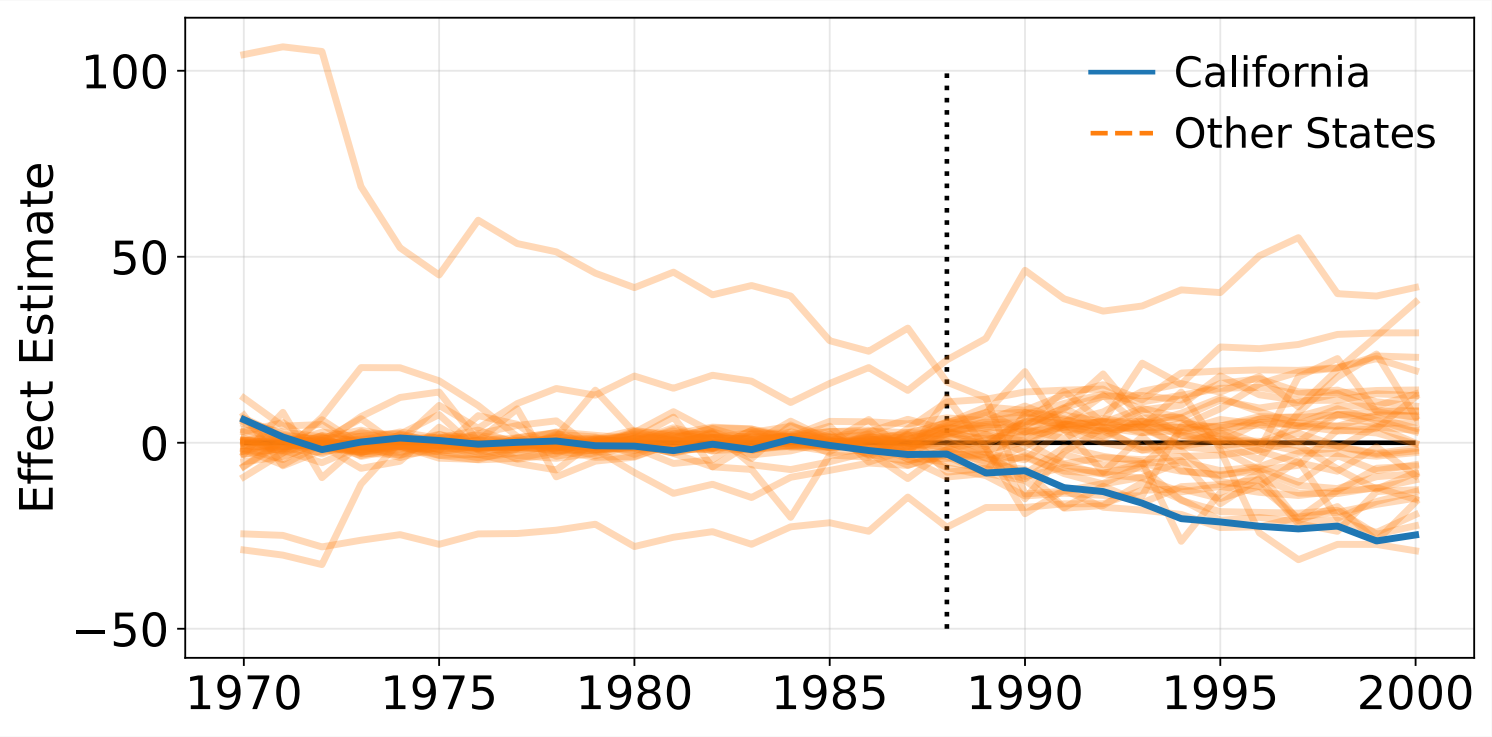
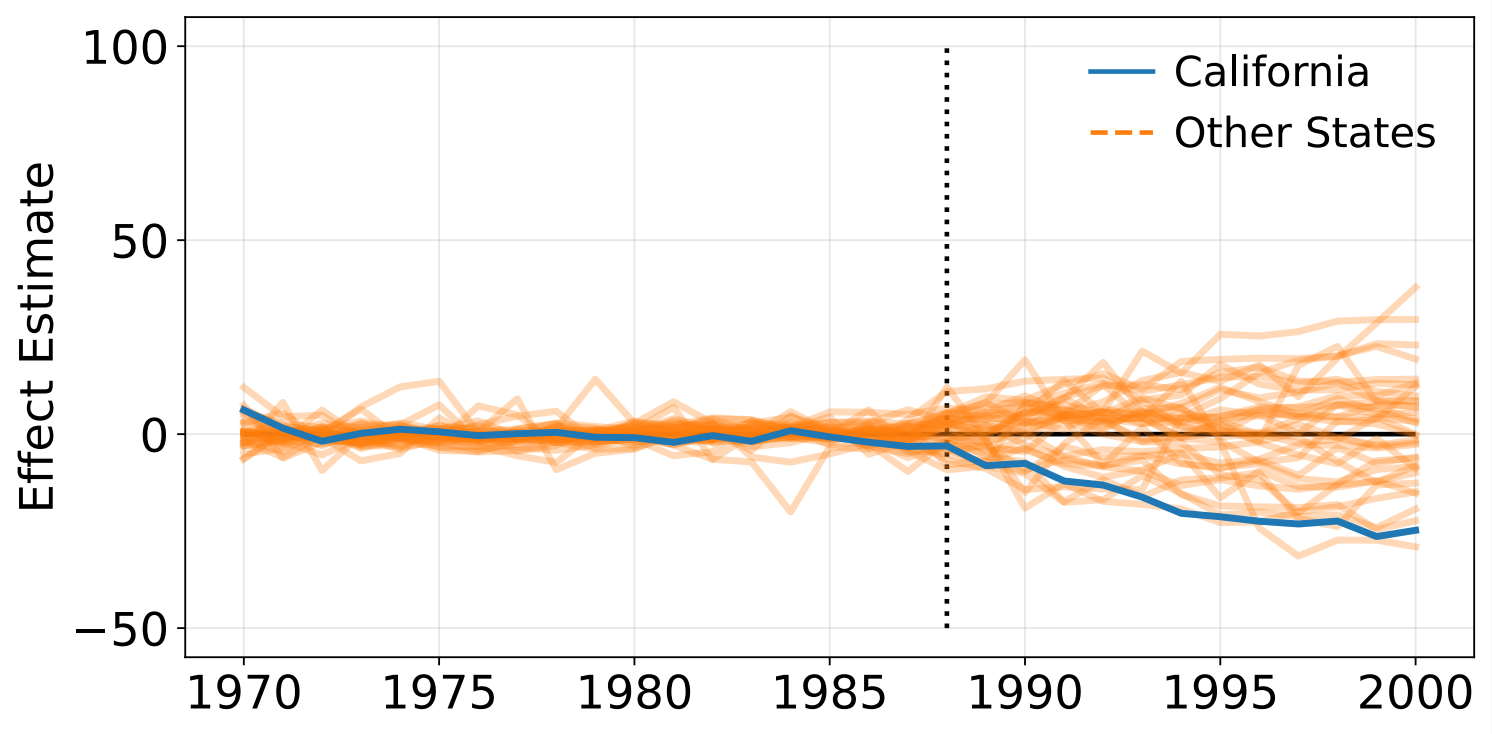



- More variance after the treatment for California than before.
- Some states have pre-treatment trends which are hard to predict.

# Example

**Placebo estimation for 34 control states with "good" pre-treatment fit**




I removed the states above the 90 percentiles of the distribution of the pre-treatment fit.

# Example

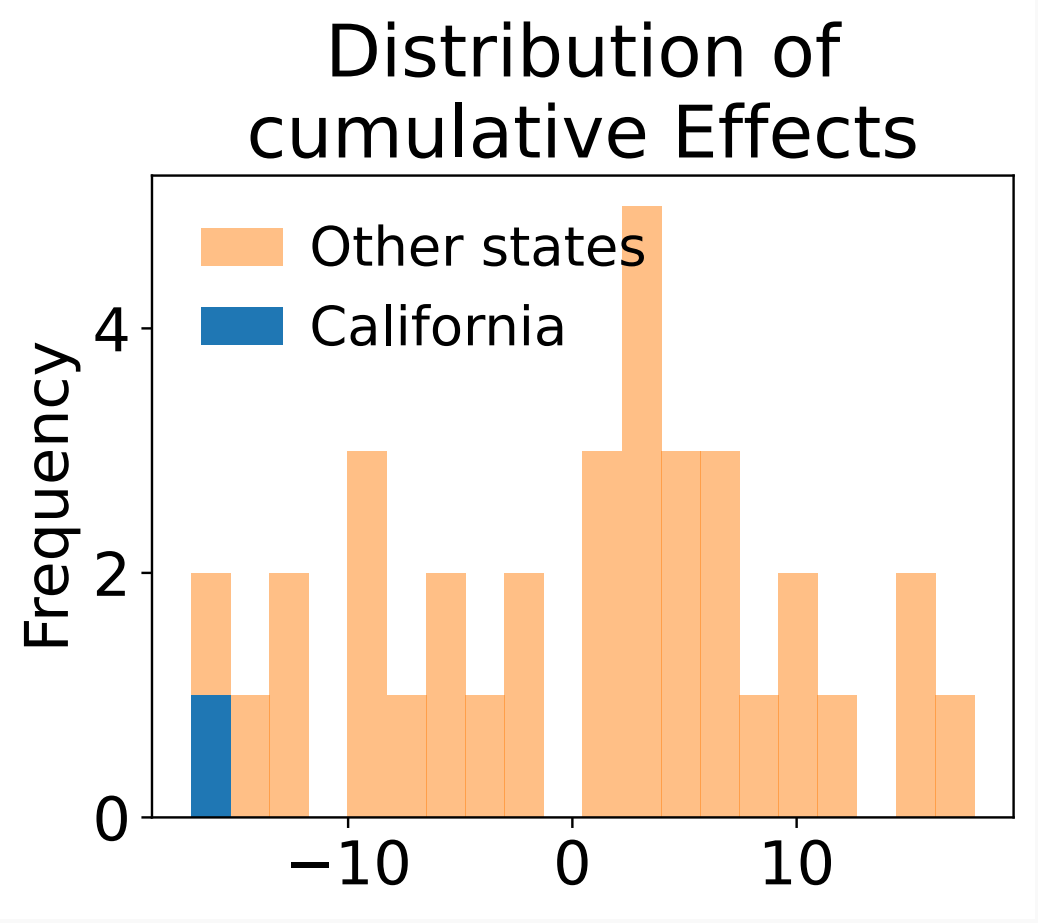
**California absolute cumulative effect**

$$\hat{\tau}_{\text{scm, california}} = -17.00$$

**Get a p-value**

$$\text{PV} = \frac{1}{n_0} \sum_{j=2}^{n_0} \mathbb{1}( \, |\hat{\tau}_{\text{scm, california}}| > |\hat{\tau}_{\text{scm},j}|$$

$$= 0.029$$



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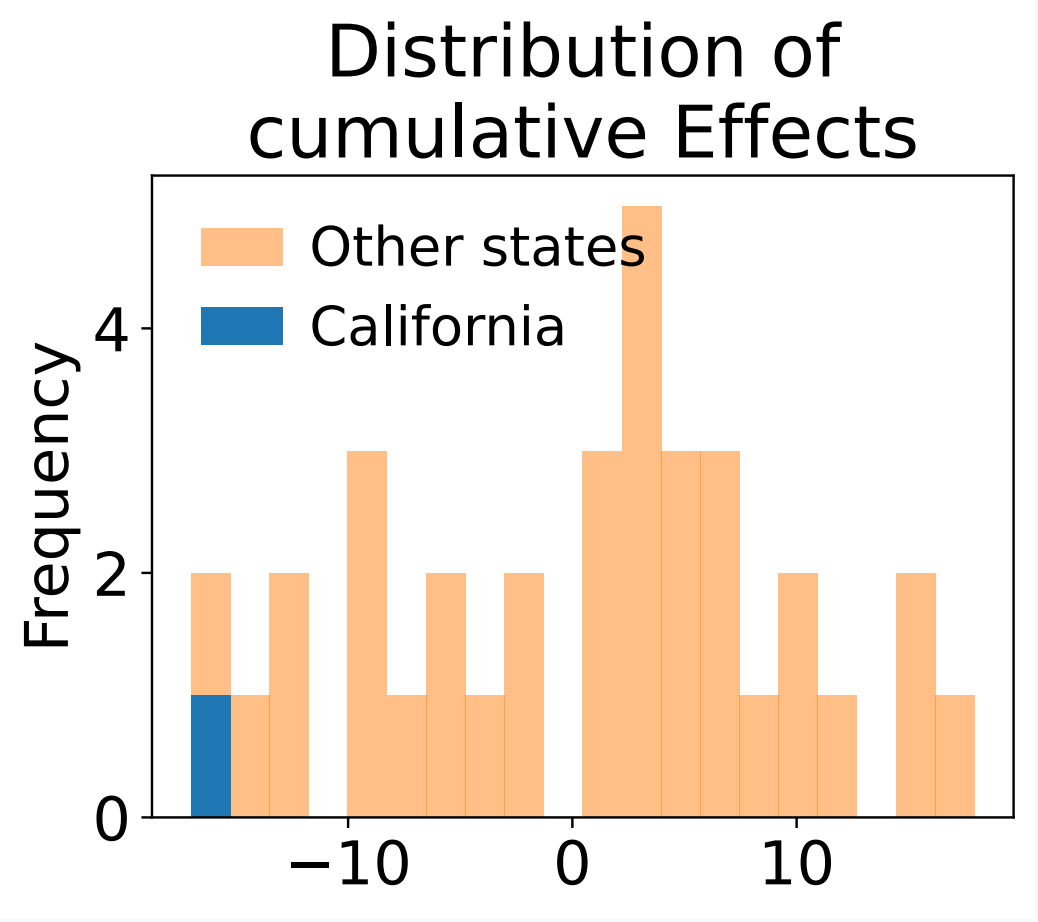
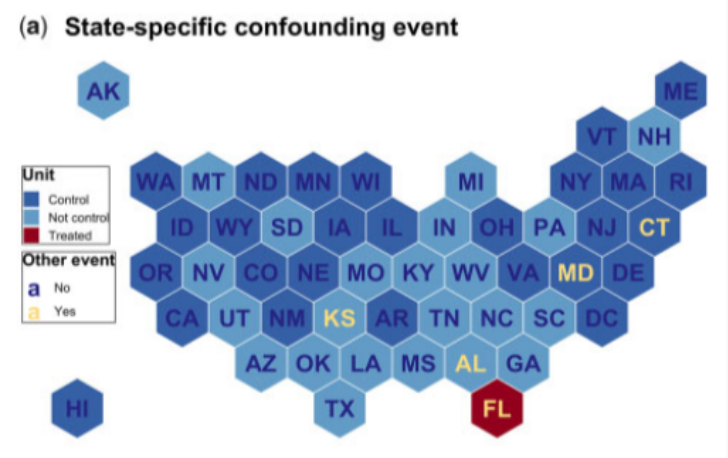


## An event affecting the outcome for the treated unit and only part of the controls

Setup (Degli Esposti et al., 2020):

- Population: US states
- Intervention: Stand Your Ground law in Florida (october 2005)
- Comparator: Other states without SYG laws
- Outcome: homicide rate



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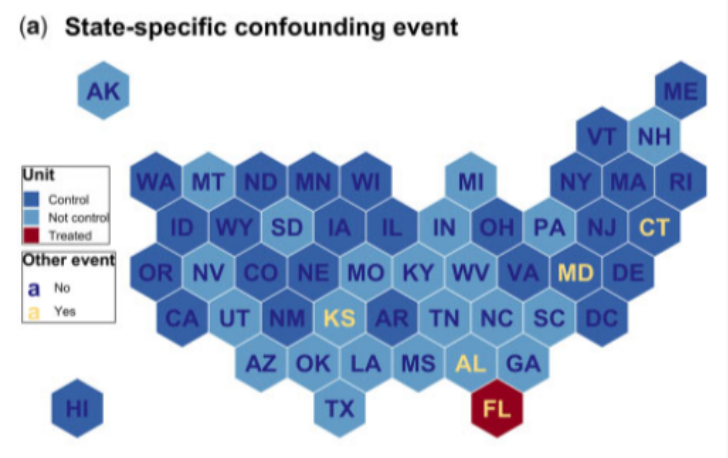
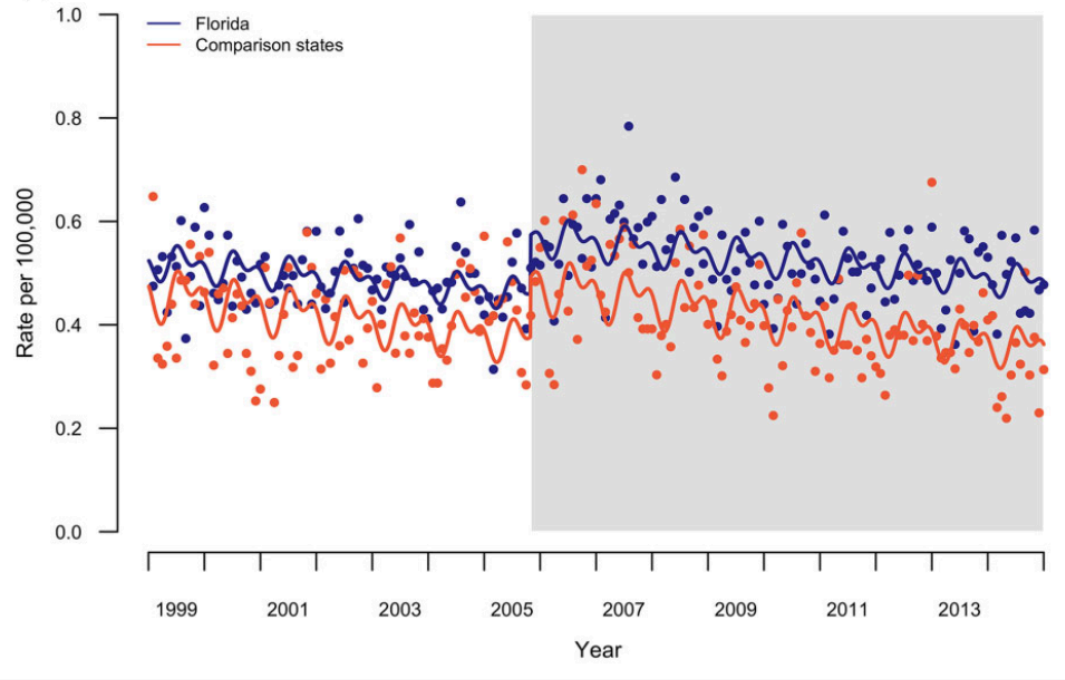



If it has an impact on the outcome after the treatment: For state in [KS, MD, AL, CT, FL], there is a step change in the outcome after the treatment: $\mathbb{1}[t > T_0]$

# Synthetic controls failure: data-driven controls

**Focus only on states affected by the confounding events**

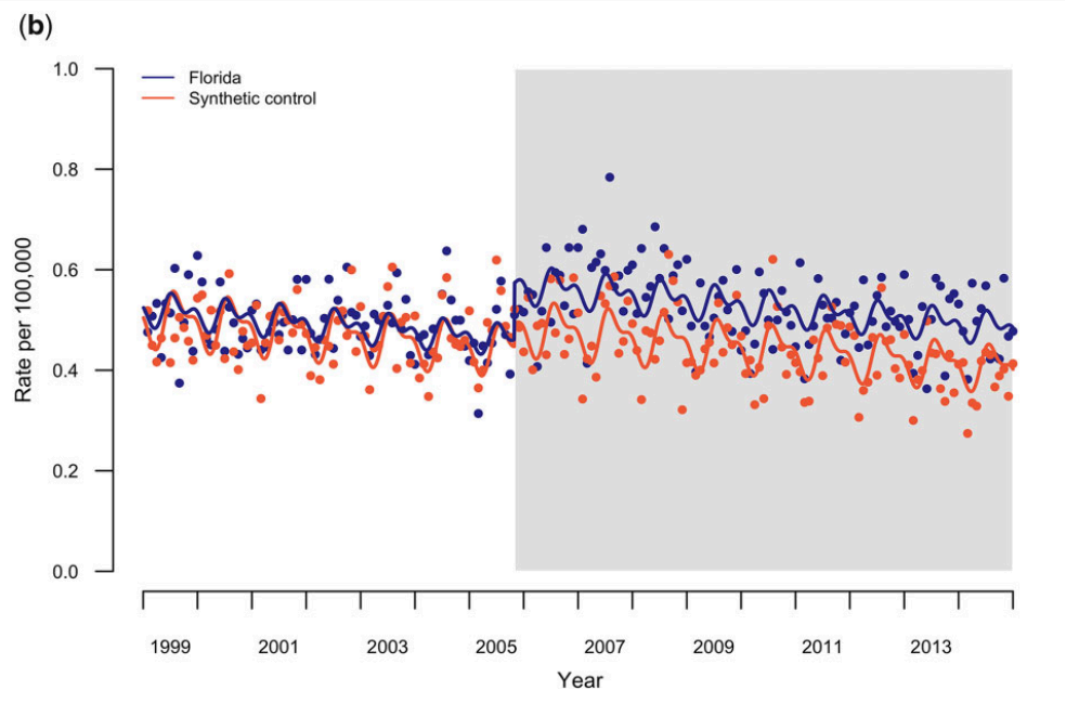



Comparison states: KS, MD, AL, CT -> also affected by the counfounding event.

🤩 We would conclude to no effect of the treatment.

# Synthetic controls: Take-away

## Focus on all comparison states




SCM matches pre-treatment trends, without taking into account the confounding event.

😢 We would falsely conclude to a positive treatment effect.

# Synthetic controls: Take-away

## Pros

- More convincing for parallel trends assumption.
- Handle multiple time periods.
- Data driven.
- Gives confidence intervals thanks to placebo test.

## Cons

- Many controls needed for good pre-treatment fits.
- Prone to overfitting during the pre-treatment period.
- Strong assumption: weights should balance the post-treatment unexposed outcomes i.e. conditional ignorability.
- Still requires the no-anticipation assumption.

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# Interrupted Time Series: intuition

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See (Arkhangelsky et al., 2021) for discussions.

# “Interrupted time-series: methods without a control group”

# Interrupted Time Series: intuition

## Setup

- One treated unit, no control unit.
- Multiple time periods.
- Sometimes, predictors are availables: there are called exogeneous covariates.

# Interrupted Time Series: illustration from (Schaffer et al., 2021)

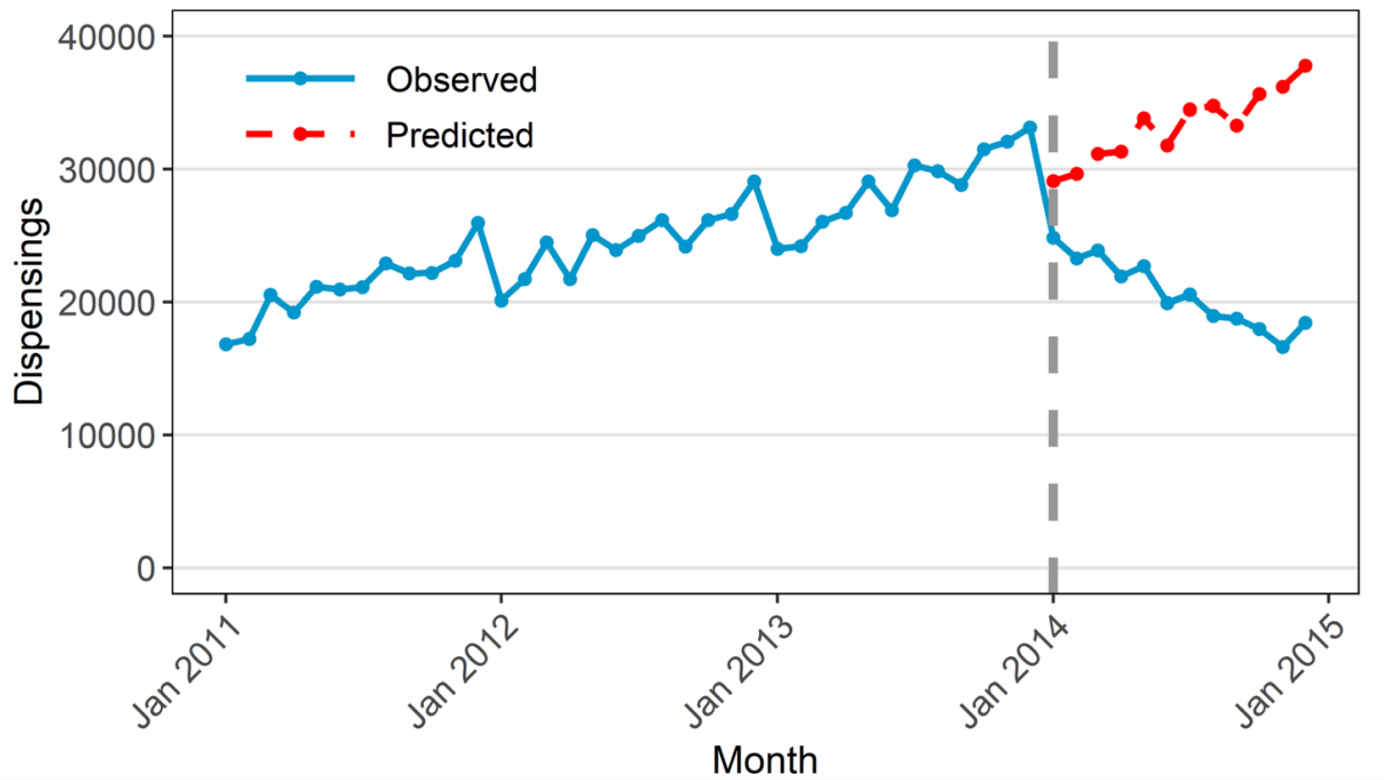
## Setup

- One treated unit, no control unit.
- Multiple time periods.
- Sometimes, predictors are availables: there are called exogeneous covariates.

## Intuition

- Model the pre-treatment trend: $Y_{t(1)}$ for $t < T_0$
- Predict post-treatment trend as the control: $\widehat{Y}_t(0)$ for $t > T_0$
- Obtain treatment effect by taking the difference between observed and predicted post-treatment observations: $Y_t(1) - \widehat{Y}_t(0)$

# Modelization of a time-series




$Y_t$: Dispensations of quetiapine, an anti-psychotic medicine.

Treatment: Restriction of the conditions under which quetiapine could be subsidised.

# ARIMA are State Space Models (SSM) says the machine learning community

## Tools

- ARIMA models: AutoRegressive Integrated Moving Average

## Motivation of ARIMA

- Structure of autodependance between observation (auto-regression, moving average),
- Linear trends,
- Seasonality.

### Good reference

Forecasting (fpp3): Principles and Practice, chapter 8

## What is a (linear) state space model?

- Two (sometimes multi-dimensional) components: the state $\mu_t$ and the observation $y_t$.
- State, ie. latent (unobserved) variable:

$$\mu_t = \overbrace{T_t}^{\text{Transition matrix}} \mu_{t-1} + \overbrace{R_t}^{\text{Transition matrix}} \underbrace{\eta_t}_{\text{gaussian white noise}}$$

- Observation is a noisy version of the state:

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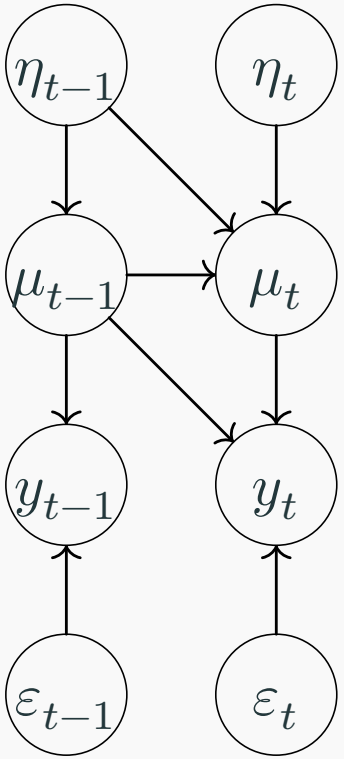

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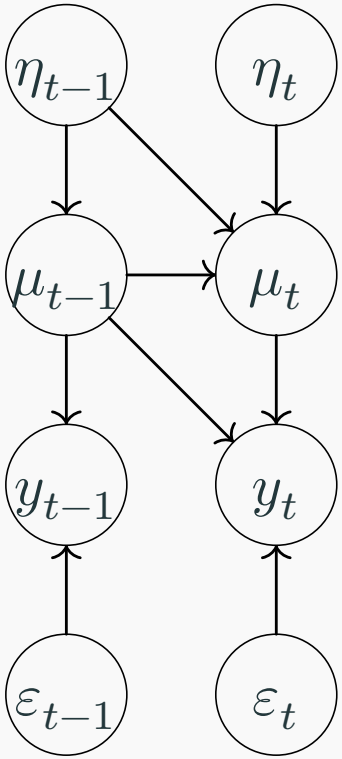

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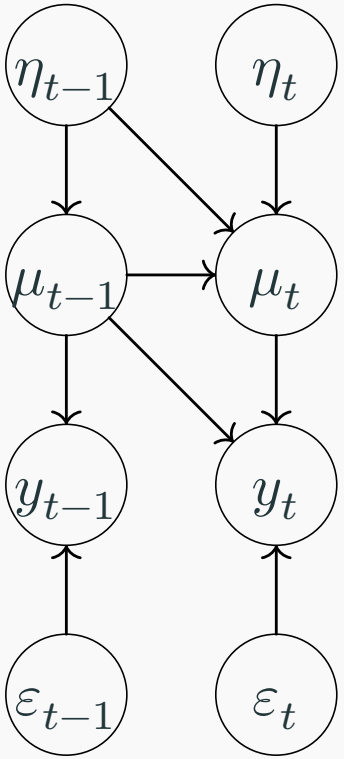

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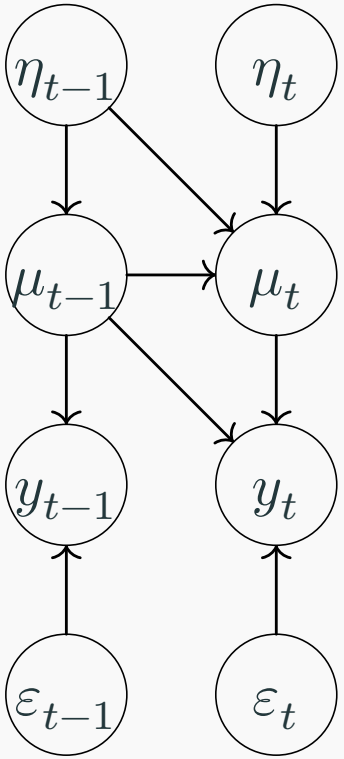
- Observation is a noisy version of the state:

$$y_t = \overbrace{Z_t}^{\text{design matrix}} \mu_t + \varepsilon_t$$



# State space models: MA(1) i.e. ARIMA(0,0,1) model example

- I better understand ARIMA formulated as state space models.
- SSM are more general than ARIMA models.
- ARIMA are (often) fitted with SSM optimization algorithms.

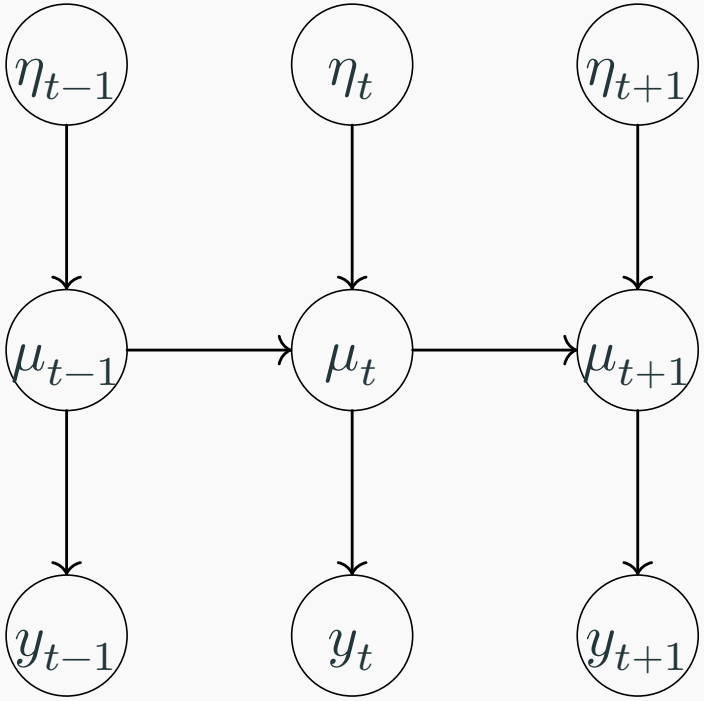
**Good reference**

(Murphy, 2022, book 2, chap 29) s

# State space models: ARMA(p, q) i.e. ARIMA(p,0,q) model example

## AR(1)

### DAG




### Formalization

Latent: $\mu_t = \rho\mu_{t-1} + \eta_t$

Observation: $y_t = \mu_t$

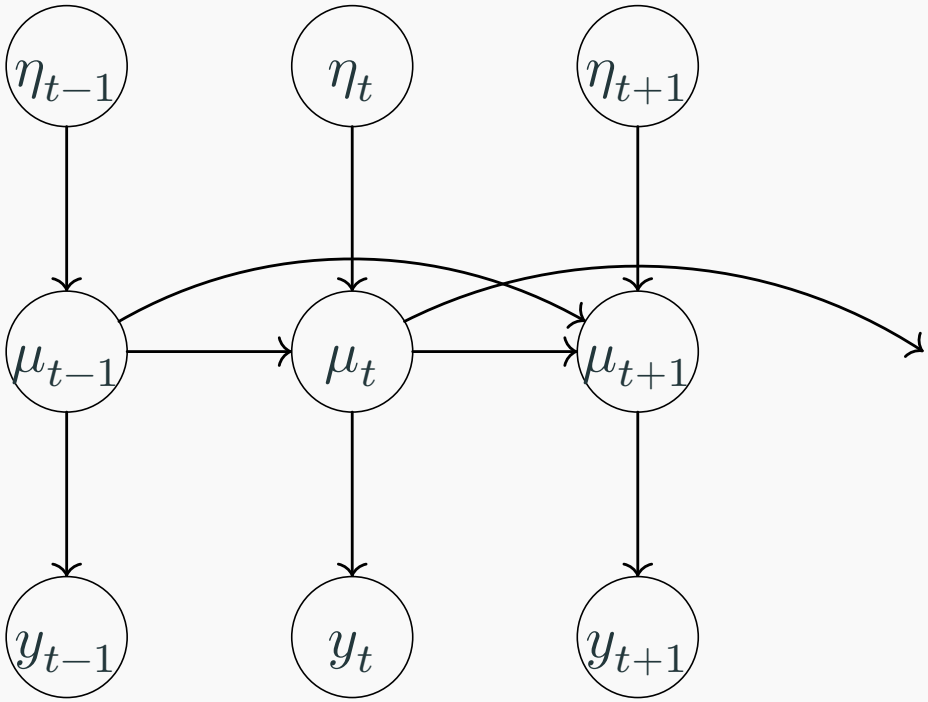
with $\eta_t \sim N(0, \sigma^2)$

$|\rho| < 1$

Auto-regression time series model an outcome as a linear regression of its prior values.

## AR(2)

### DAG




### Formalization

Latent: $\mu_t = \begin{pmatrix} \rho_1 & \rho_2 \\ 1 & 0 \end{pmatrix} \mu_{t-1} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} \eta_t$

Observation: $y_t = [1, 0]\mu_t$

with $\eta_t \sim N(0, \sigma^2)$

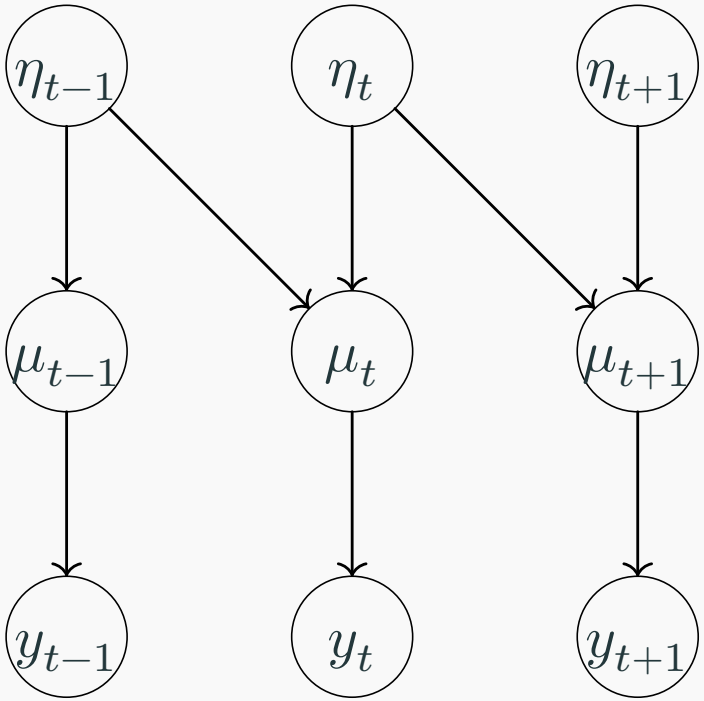
$$|\rho_1| < 1, |\rho_2| < 1$$

Observation unrolled:

$$y_t = \rho_1 y_{t-1} + \rho_2 y_{t-2} + \eta_t$$

# State space models: General formulation

**DAG**




## Formalization

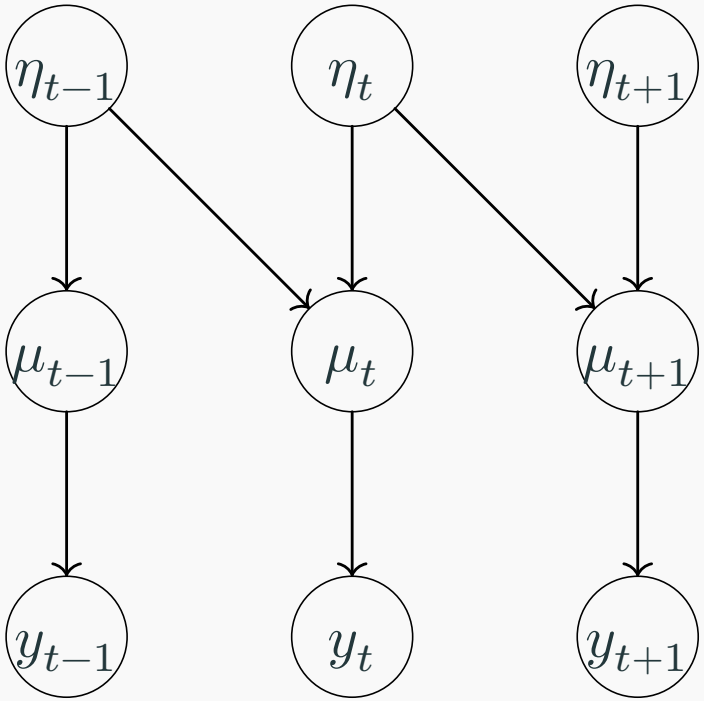
Latent: $\mu_t = [1, \theta] \begin{pmatrix} \eta_t \\ \eta_{t-1} \end{pmatrix}$

Observation: $y_t = \mu_t$

with $\eta_t \sim N(0, \sigma^2)$

# State space models: a brief word on fitting (i.e. learning the parameters)

**DAG**




**Formalization**

Latent: $\mu_t = [1, \theta] \begin{pmatrix} \eta_t \\ \eta_{t-1} \end{pmatrix}$

Observation: $y_t = \mu_t$

with $\eta_t \sim N(0, \sigma^2)$

The MA time series models the residual of the regression of $y_t$ on its previous values as a linear combination of the previous residuals : i.e. vanishing shocks.

## Formalization (Hamilton form)

Let $r = \max(p, q+1)$

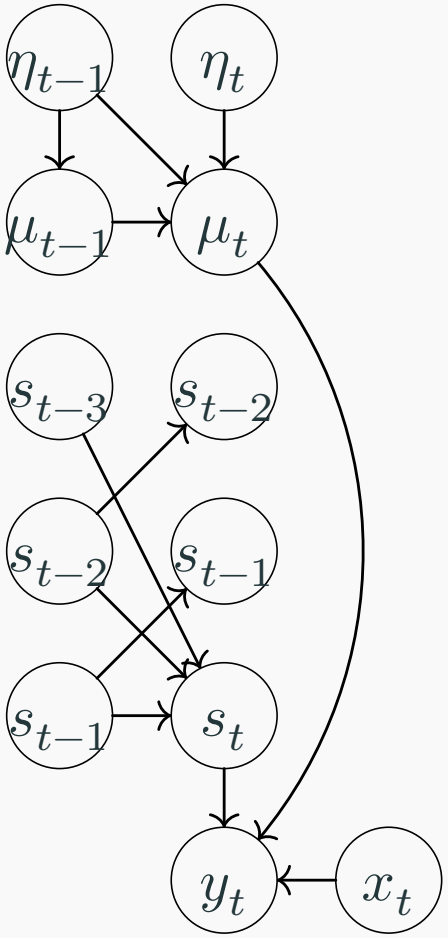
Observation: $y_t = (1, \theta_1, \theta_2, ..., \theta_{r-1})\ \mu_t$

Latent: $\mu_t = \begin{pmatrix} 1 & \rho_1 & \rho_2 & ... & \rho_{r-1} \\ 1 & 0 & 0 & ... & 0 \\ 0 & 1 & 0 & ... & 0 \\ \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & ... & 0 & 1 & 0 \end{pmatrix} \mu_{t-1} + \begin{pmatrix} \varepsilon_t \\ 0 \\ ... \\ 0 \end{pmatrix}$ with $\varepsilon_t \sim N(0, \sigma^2)$

## Unfolding the state space equations

$y_t = \sum_{i=1}^{p} \rho_i y_{t-i} + \sum_{j=1}^{q} \theta_j \varepsilon_{t-j}$

**DAG**




## Formalization

Observation with covariates and seasonality:

$$y_t = \beta x_t + s_t + \underbrace{\rho \mu_{t-1}}_{\text{AR}(1)} + \underbrace{\theta \eta_{t-1} + \eta_t}_{\text{MA}(1)}$$

Where seasonality:

$$s_t = -\sum_{k=1}^{S-1} s_{t-k} + \varepsilon_{s,t}$$

$$\text{with } \varepsilon_s, t \sim N(0, \sigma_s^2)$$

# Example of ITS with ARIMA: the French antibiotics campaign of 2002-2007

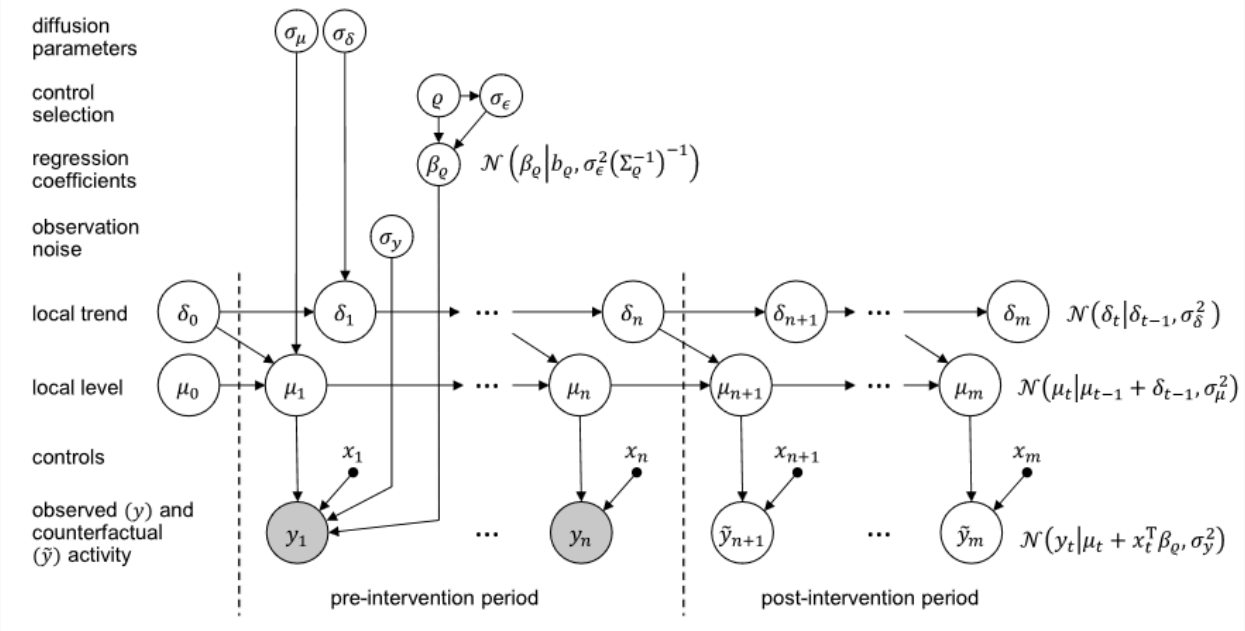
Latent:

$$\mu_t = T_t \mu_{t-1} + R_t \eta_t$$

Observation:

$$y_t = Z_t \mu_t + \beta^T x_t + \varepsilon_t$$

With $\eta_t$ and $\varepsilon_t$ mean zero gaussian noise, sometimes with a specific covariance structure.




Complex SSM DAG from the Causal Impact paper (Brodersen et al., 2015).

# Example of ITS with ARIMA: the French antibiotics campaign of 2002-2007

**When the error terms are gaussians**

These modeles are called linear Gaussian state space model (LG-SSM) or linear dynamical system (LDS).

**The likelihood is jointly gaussian**

Closed form formula for the likelihood of the data under the model.

# Example of ITS with ARIMA: the French antibiotics campaign of 2002-2007

**When the error terms are gaussians**

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**The likelihood is jointly gaussian**

Closed form formula for the likelihood of the data under the model.

## Expectation-Minimization: a widespread algorithm for fitting

- Expectaction: Compute the joint likelihood of the data and the parameters (observed outcome, unknown state) given the parameters.
- Maximization: find parameters maximizing the likelihood: analytically since gaussian.
- Iter until convergence to a (local) maximum of likelihood.

**Long Short Term Memory (LSTM) networks**

A type of Recurrent Neural Network (RNN) that can learn long-term dependencies (Graves & Graves, 2012).

It was state of the art for language tasks before transformers.

It is notably hard to train due to vanishing gradient through the time dimension.

**Mamba**

A recent proposition to mitigate one of the main limitations of the transformer architecture: high complexity relative to the length of the sequence (Gu & Dao, 2023).

Good blog-style introduction in (Ayonrinde, 2024).

# A word on model families for ITS

## Context

In 2001, compared to the European Union countries, France was a country where:

- the population consumed the most antibiotics in town
- the resistance of Streptococcus pneumoniae to β-lactams was the highest (53%)
- a significant number of antibiotic prescriptions would be unnecessary (viral infections)

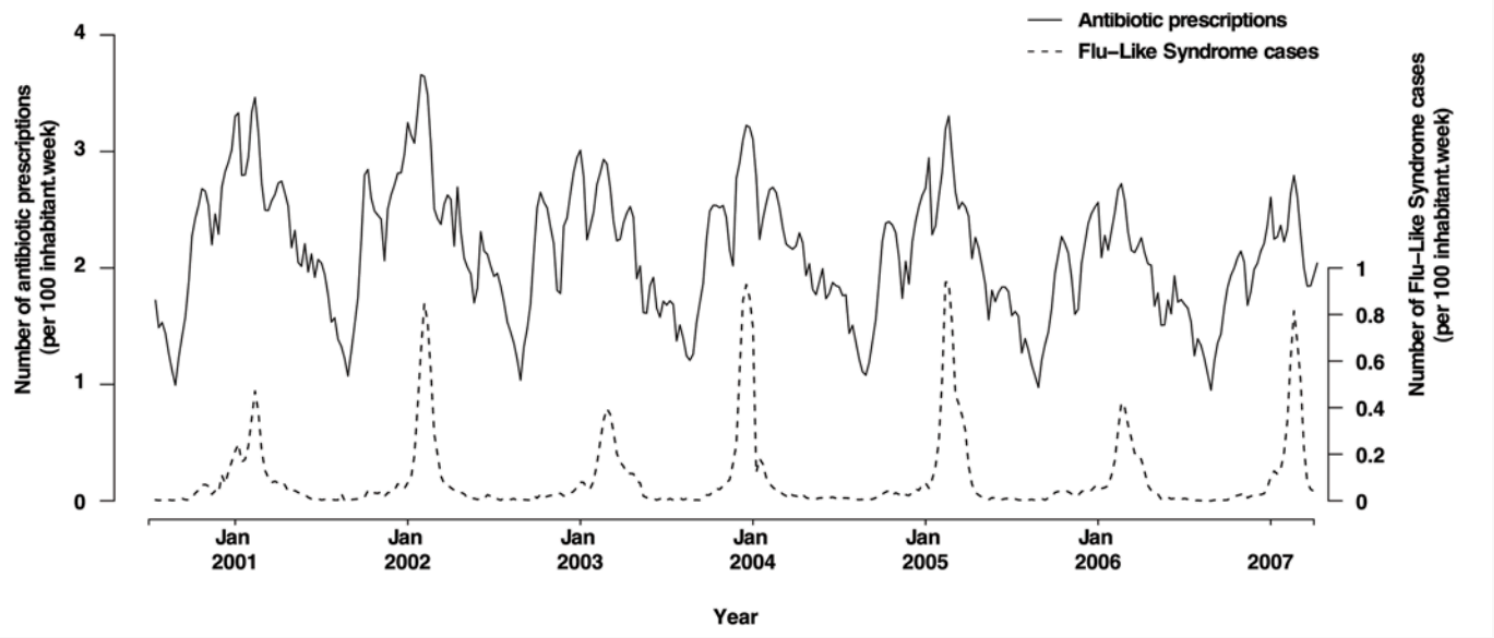
## Campaign (october 2002, then every year october to march)

France implemented a national plan to "preserve the effectiveness of antibiotics and improve their use" with the main action undertaken by the National Health Insurance.

## Question

What has been the effect of the campaign on the consumption of antibiotics? (Sabuncu et al., 2009)

## Weekly reimbursed prescription of antibiotics in town




Interventions during the months of october to march: $\text{month}(t) \in M_0$.

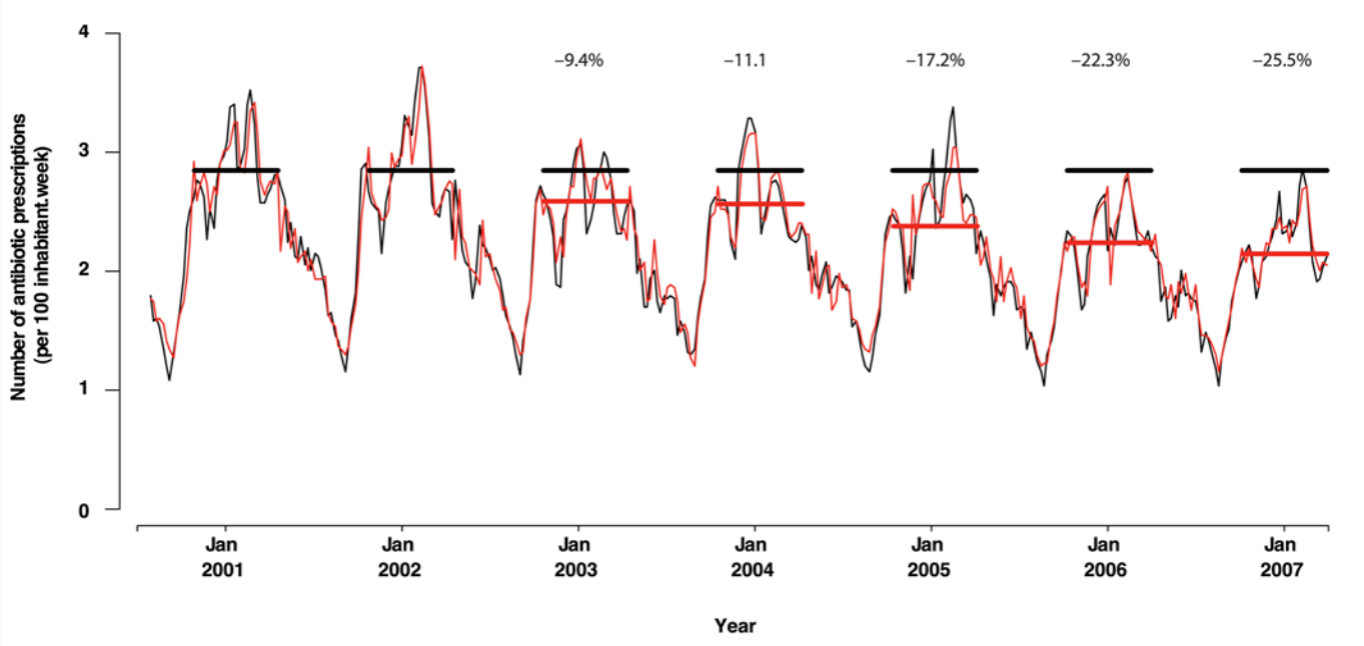
## Estimation

- Fit an ARIMA model on the pre-treatment trend
- Introduce an additive term for the intervention:

$$Y_t = c + \sum_i \hat{\tau}_i \mathbb{1}[\text{month}(t) \in M_0 \wedge \text{year}(t) == i] + \underbrace{[a(B)^{-1} - b(B)\varepsilon_s]}_{\text{ARIMA term fitted on pre-treatment}}$$

- Assess if the additive term and other parameters are significantly different pre-treatment and post-treatment.

# A word on model families for ITS




- Red curve: arima fitted with intervention
- Red Horizontal line: intervention effect fitted during intervention
- Black curve: arima fitted without intervention
- Black horizontal line: intervention effect fitted pre-intervention

# Cross-validation for time-series models

We saw ARIMA models and the more general class of state space models.

However, we could any model that we want to fit the pre-treatment trend !

# Main threat to validity for an ITS: historical bias

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However, we could any model that we want to fit the pre-treatment trend !

Facebook prophet model (Taylor & Letham, 2018) uses Generalized Additive Models (GAM).

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Any sklearn estimator could do the trick: Linear regression, Random Forest, Gradient Boosting...

# Take-away on ITS

We saw ARIMA models and the more general class of state space models.

However, we could any model that we want to fit the pre-treatment trend !

Facebook prophet model (Taylor & Letham, 2018) uses Generalized Additive Models (GAM).

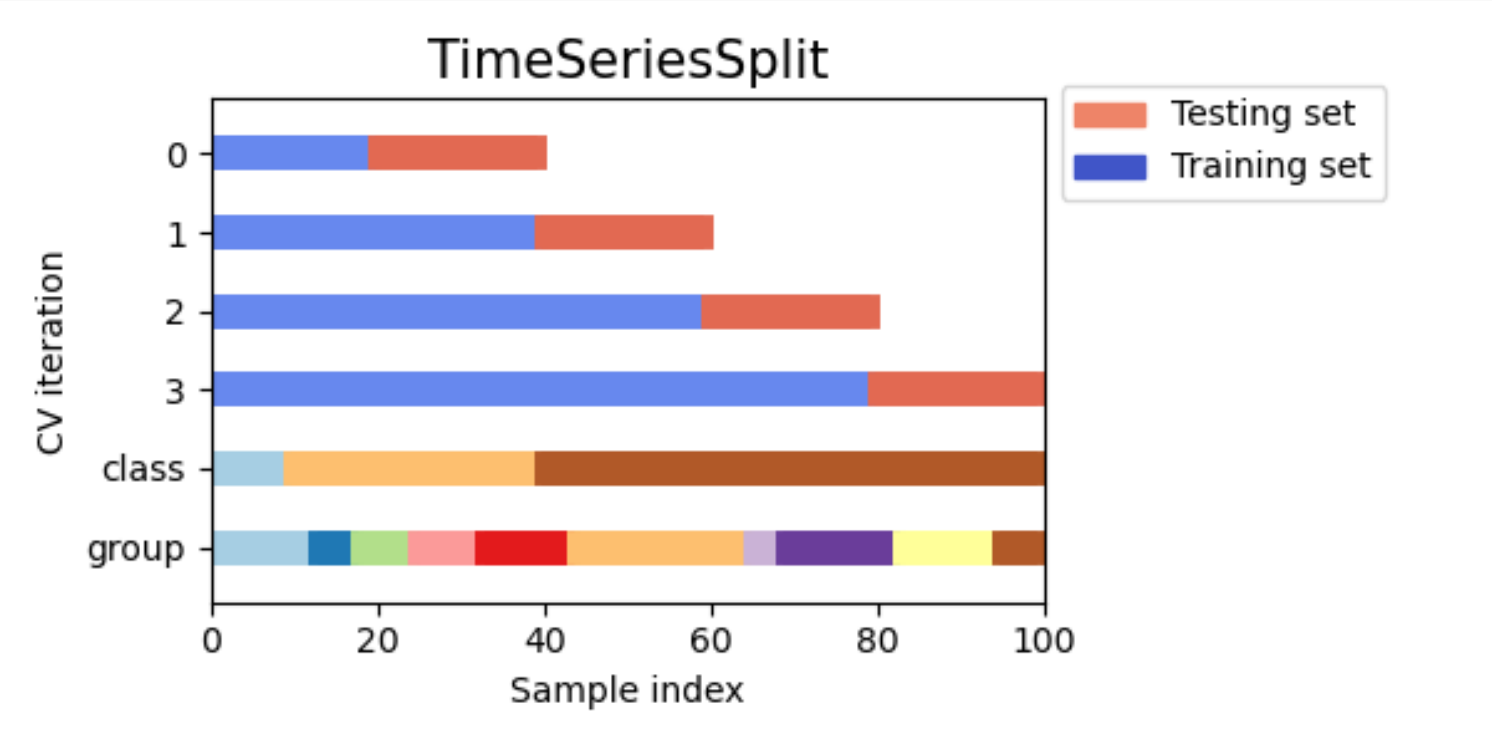
Any sklearn estimator could do the trick: Linear regression, Random Forest, Gradient Boosting...

⚠️ You should pay attention to appropriate train/test split when cross-validating a time-series model not to use the future to predict the past.

Relevant remark for all time series models (even ARIMA or state space models).

# An attempt to map event study methods

```python
from sklearn.model_selection import TimeSeriesSplit
```

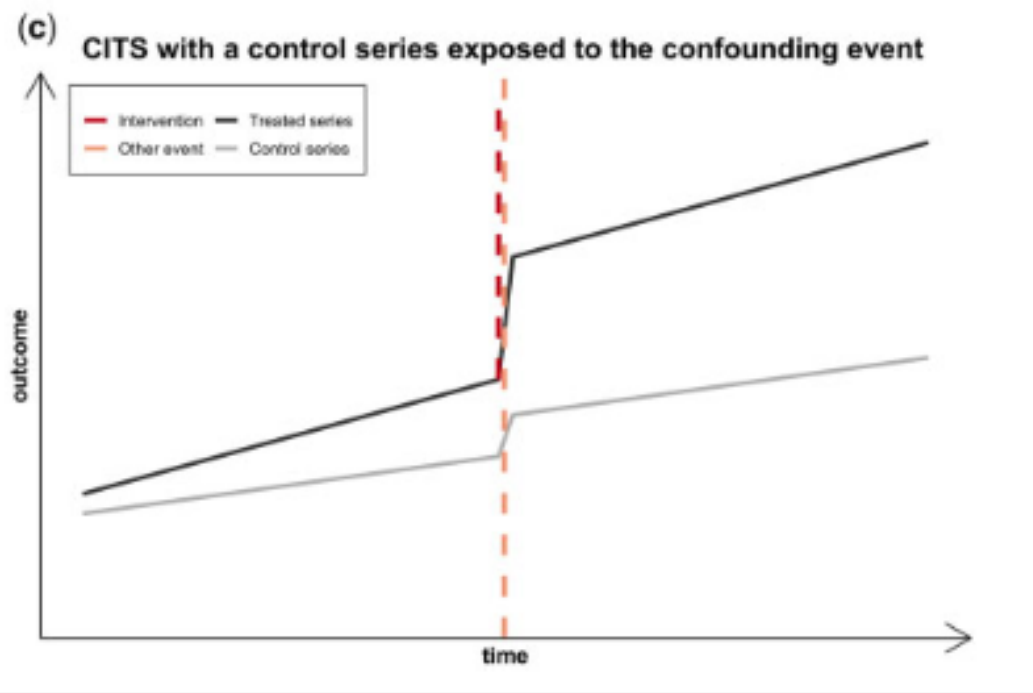



This avoids to use the future to predict the past.

(Degli Esposti et al., 2020, Fig. 1)

⚠️ If there is a co-intervention, it will impact the outcome trend and bias the treatment effect estimation.

💡 Adding a control series of predictors can help to mitigate this bias.



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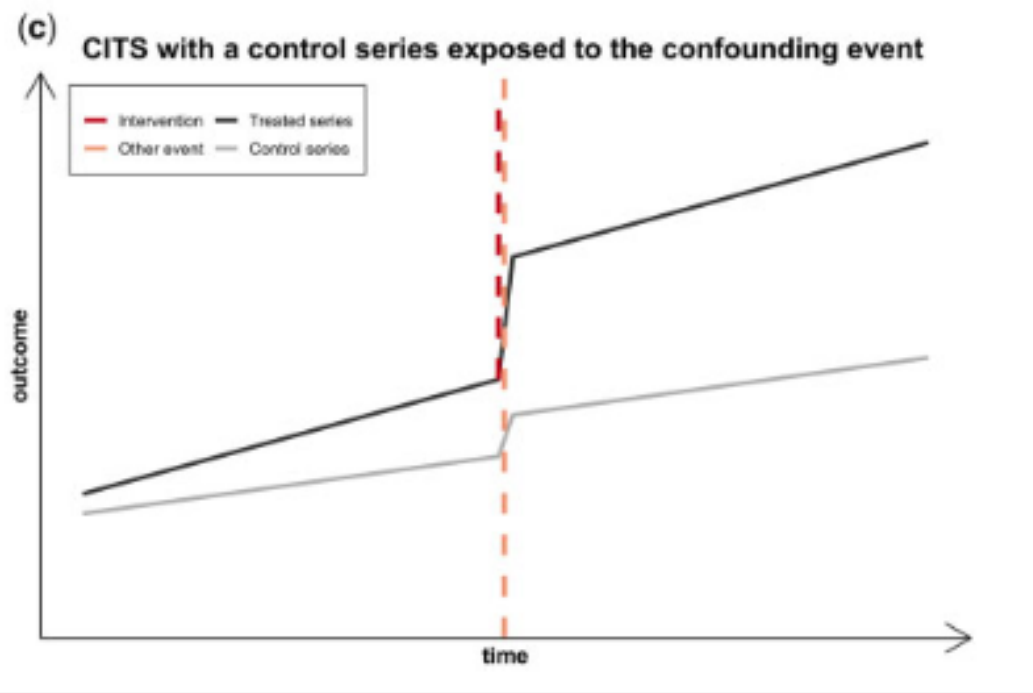

# Final word – What methods to chose: some guides

## Pros

- Suitable when no control unit is available. The pre-treatment trend is the control.
- Handles multiple time periods.
- A lot of software available (eg. ARIMA models).
- Simple: few parameters to tune.

## Cons

- Prone to bias by other events happening around the treatment time and impacting the outcome trend.
- Prone to overfitting of the pre-treatment trend.

# To your notebooks 👩‍💻 !

| **Methods** | Characteristics | Hypotheses | Community | Introduction |
|---|---|---|---|---|
| DID/TWFE | Treated/control units, few time periods, no predictors | Parallel trends, no anticipation, prone to overfitting | Economics | Causal Inference for the Brave and True, chapter 13 |
| ARIMA, ITS | No controls, no/few predictors, seasonality | Stationnarity , no anticipation, prone to overfitting | Epidemiology, Economics | Forecasting: Principles and Practice |
| State space models | Multiple time periods, control units or predictors, generalization of ARIMA | Contional ignorability on predictors, goodness of fit pre-treatment | Machine learning, bayesian methods | Introduction to Time Series and Forecasting, chapter 9 |
| Synthetic control | Treated/control units, multiple time periods | Conditional parallel trend on controls, goodness of fit pre-treatment | Economics | Causal Inference for the Brave and True, chapter 15 |

# Synthetic controls: conformal prediction inference

| Package name | Methods | Predictors | Control units | Multiple time periods |
|---|---|---|---|---|
| did | Difference-in-differences | ❌ | ❌ | ❌ |
| forecast | ARIMA, ITS | ✅ | ❌ | ✅ |
| Synth | Synthetic control | ❌ | ✅ | ✅ |
| Causal impact | Bayesian state space models | ✅ | ❌ | ✅ |

# Conformal inference: hypothesis generation

| Package name | Methods | Predictors | Control units | Multiple time periods |
|---|---|---|---|---|
| statsmodels.OLS | Difference-in-differences, TWFE | ❌ | ❌ | ❌ |
| statsmodels | ARIMA(X), ITS, bayesian state space models | ✅ | ❌ | ✅ |
| pmdarima | ARIMA(X), ITS | ✅ | ❌ | ✅ |
| SyntheticControlMethods | Synthetic control | ❌ | ✅ | ✅ |
| pysyncon | Synthetic control | ❌ | ✅ | ✅ |
| causalimpact (pymc implementation) | Bayesian state space models | ✅ | ❌ | ✅ |
| causal-impact (statsmodels implementation) | Bayesian state space models | ✅ | ❌ | ✅ |

# Conformal inference: Fit a model and compute residuals

## DID-family methods

- Control units available (at least one)
- Few time periods
- Parallel trend is credible (if necessary by adjusting the model on predictors).

## Synthetic Control Methods

- Mutiple and different controls as well as multiple time periods
- Pre-treatment outcomes of the control units predict well the treated unit outcome.
- No-spill over from the treatment to the control units.

## ITS: SARIMA or state space models

- No evident control units
- Pre-treatment outcome of the treated unit seems a good control
- Control predictors not impacted by the treatment availables
- No co-intervention that could impact the treated outcome.

# “Python hands-on”

# Conformal inference: test statistic and resampling

- url: https://github.com/strayMat/causal-ml-course/tree/main/notebooks

# “Supplementary materials”

# Conformal inference: resampling

Introduced by (Chernozhukov et al., 2021)

- Recast the problem as counterfactual inference, i.e. predict: $Y_{it}(0)$ for $t > T_0$
- Test hypothesis: $H_0$ eg. $H_0 = (0, 0, .., 0)$ ie no effect for $t > T_0$
- This imply the generation of a hypothesis counterfactual trajectory $Y_t(0)$

## Question

Are the post-treatment residuals of a model fitted on the hypothesis counterfactual trajectory an outlier of the distribution of the residuals pre-treatment?

## Why does this works?

Syntehtic controls estimation are invariant under the time series dimension so we can resample under this dimension to introduce data variability.

# Conformal inference: P-value

- Test a hypothesis : $H_0$ eg. $H_0 = (0, 0, .., 0)$ ie no effect for $t > T_0$
- Gerenate a counterfactual trajectory $Y_t(0)$ under this null

# Bibliography

- Fit a counterfactual model on the full generated trajectory: $\widehat{Y_t}$
- Compute the residuals: $\hat{u}_t = Y_t(0) - \hat{Y}_t$

Summarize the residuals in a statistic: $S(\hat{u}) = \left(\frac{1}{\sqrt{T-T_0+1}} \sum_{t=T_0+1}^{T} |\hat{u}_t|^q \right)^{\frac{1}{q}}$

# Resample this statistic by block permutation $\pi$ of the time periods

Same as permutting the data since SCM are invariant under the time series dimension.

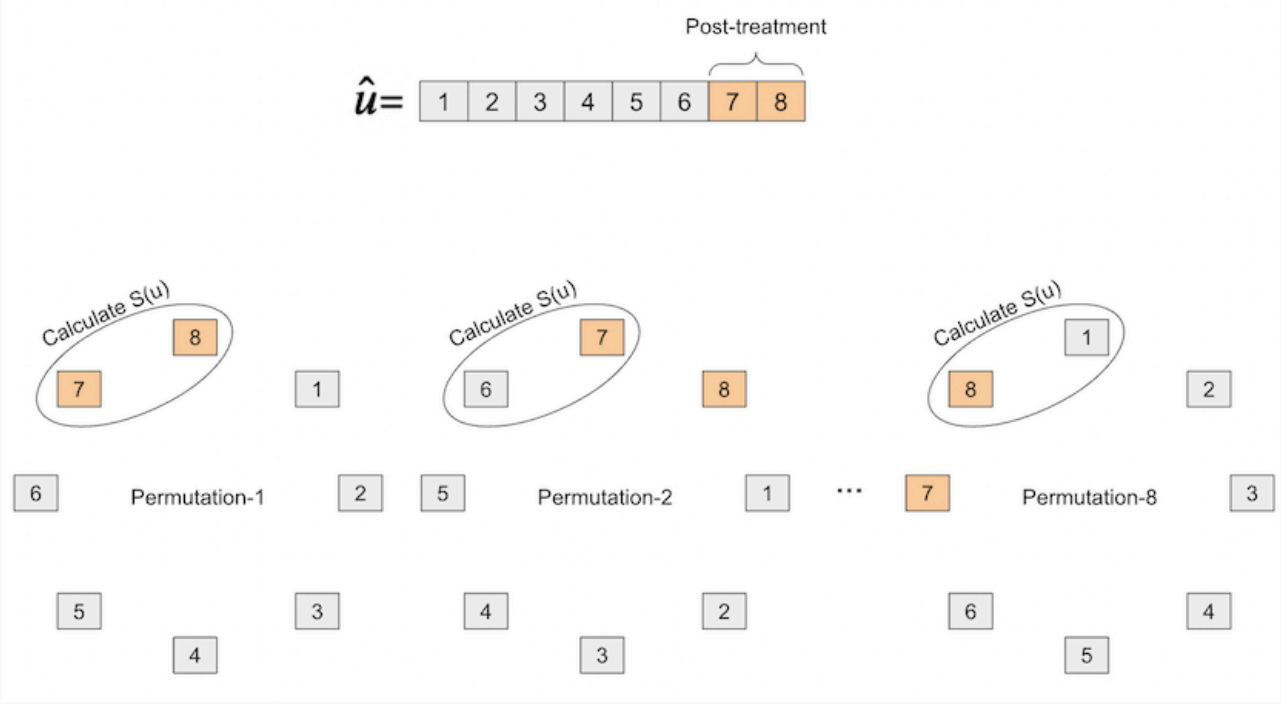


Image from: Causal Inference for the Brave and True

- Assess if the post-treatment statistics is an outlier of this distribution.
- P-value: $\hat{F}(x) = \frac{1}{|\Pi|} \sum_{\pi \in \Pi} \mathbb{1}\left[S\left(\hat{u}_{\pi_0}\right) \leq S(\hat{u}_{\pi})\right]$ where $\pi_0$ is the original data.

*Abadie, A. (2021). Using synthetic controls: Feasibility, data requirements, and methodological aspects. Journal of Economic Literature, 59(2), 391–425.*

*Abadie, A., & Gardeazabal, J. (2003). The economic costs of conflict: A case study of the Basque Country. American Economic Review, 93(1), 113–132.*

*Abadie, A., Diamond, A., & Hainmueller, J. (2010). Synthetic control methods for comparative case studies: Estimating the effect of California's tobacco control program. Journal of the American Statistical Association, 105(490), 493–505.*

*Arkhangelsky, D., Athey, S., Hirshberg, D. A., Imbens, G. W., & Wager, S. (2021). Synthetic difference-in-differences. American Economic Review, 111(12), 4088–4118.*

*Ashenfelter, O. (1978). Estimating the effect of training programs on earnings. The Review of Economics and Statistics, 47–57.*

*Athey, S., & Imbens, G. W. (2017). The state of applied econometrics: Causality and policy evaluation. Journal of Economic Perspectives, 31(2), 3–32.*

*Ayonrinde, K. (2024). Mamba Explained. The Gradient.*

*Bonander, C., Humphreys, D., & Degli Esposti, M. (2021). Synthetic control methods for the evaluation of single-unit interventions in epidemiology: a tutorial. American Journal of Epidemiology, 190(12), 2700–2711.*

*Bouttell, J., Craig, P., Lewsey, J., Robinson, M., & Popham, F. (2018). Synthetic control methodology as a tool for evaluating population-level health interventions. J Epidemiol Community Health, 72(8), 673–678.*

*Brehm, M. E., Brehm, P. A., & Saavedra, M. (2022). The Ohio vaccine lottery and starting vaccination rates. American Journal of Health Economics, 8(3), 387–411.*

*Brodersen, K. H., Gallusser, F., Koehler, J., Remy, N., & Scott, S. L. (2015). Inferring causal impact using Bayesian structural time-series models.*

*Chernozhukov, V., Wüthrich, K., & Zhu, Y. (2021). An exact and robust conformal inference method for counterfactual and synthetic controls. Journal of the American Statistical Association, 116(536), 1849–1864.*

*De Chaisemartin, C., & d'Haultfoeuille, X. (2020). Two-way fixed effects estimators with heterogeneous treatment effects. American Economic Review, 110(9), 2964–2996.*

*Degli Esposti, M., Spreckelsen, T., Gasparrini, A., Wiebe, D. J., Bonander, C., Yakubovich, A. R., & Humphreys, D. K. (2020). Can synthetic controls improve causal inference in interrupted time series evaluations of public health interventions?. International Journal of Epidemiology, 49(6), 2010–2020.*

*Graves, A., & Graves, A. (2012). Long short-term memory. Supervised Sequence Labelling with Recurrent Neural Networks, 37–45.*

*Grupp, T., Mishra, P., Reynaert, M., & Benthem, A. A. van. (2023). An evaluation of protected area policies in the european union.*

*Gu, A., & Dao, T. (2023). Mamba: Linear-time sequence modeling with selective state spaces. Arxiv Preprint Arxiv:2312.00752.*

*Miller, S., Johnson, N., & Wherry, L. R. (2019). Medicaid and mortality: new evidence from linked survey and administrative data.*

*Murphy, K. P. (2022). Probabilistic machine learning: an introduction. MIT press.*

*Puig-Codina, L., Pinilla, J., & Puig-Junoy, J. (2021). The impact of taxing sugar-sweetened beverages on cola purchasing in Catalonia: an approach to causal inference with time series cross-sectional data. The European Journal of Health Economics, 22(1), 155–168.*

*Ryan, A. M., Krinsky, S., Kontopantelis, E., & Doran, T. (2016). Long-term evidence for the effect of pay-for-performance in primary care on mortality in the UK: a population study. The Lancet, 388(10041), 268–274.*

*Sabuncu, E., David, J., Bernède-Bauduin, C., Pépin, S., Leroy, M., Boëlle, P.-Y., Watier, L., & Guillemot, D. (2009). Significant reduction of antibiotic use in the community after a nationwide campaign in France, 2002–2007. Plos Medicine, 6(6), e1000084.*

*Schaffer, A. L., Dobbins, T. A., & Pearson, S.-A. (2021). Interrupted time series analysis using autoregressive integrated moving average (ARIMA) models: a guide for evaluating large-scale health interventions. BMC Medical Research Methodology, 21, 1–12.*

*Sheridan, P., McElroy, S., Casey, J., & Benmarhnia, T. (2022). Using the generalized synthetic control method to estimate the impact of extreme weather events on population health. Epidemiology, 33(6), 788–796.*

*Snow, J. (1855). On the mode of communication of cholera. John Churchill.*

*Stechemesser, A., Koch, N., Mark, E., Dilger, E., Klösel, P., Menicacci, L., Nachtigall, D., Pretis, F., Ritter, N., Schwarz, M., & others. (2024). Climate policies that achieved major emission reductions: Global evidence from two decades. Science, 385(6711), 884–892.*

*Taylor, S. J., & Letham, B. (2018). Forecasting at scale. The American Statistician, 72(1), 37–45.*

*Wager, S. (2024, ). Causal inference: A statistical learning approach. preparation.*