Machine Learning for econometrics

Statistical learning and penalized regression

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Today's program

- Last session: importance of causal variable status
- Today: predictive inference in high dimensions
  - Statistical learning basics
  - Penalized regression for predictive inference

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- Last session: importance of causal variable status
- Today: predictive inference in high dimensions
  - Statistical learning basics
  - Penalized regression for predictive inference
- Next session:
  - Flexible models: Trees, Random Forests, Gradient Boosting
  - Practical scikit-learn

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Settings: statistical learning

Statistical learning, ie. predictive inference

Goal

- Predict the value of an outcome based on one or more input variables.

Setting

- Data: n pairs of (features, outcome), $(x_i, y_i) \in \mathcal{X} \times \mathcal{Y}$ identically and independently distributed (i.i.d.) from an unknown distribution P.
- Goal: find a function $\hat{f} : \mathcal{X} \to \mathcal{Y}$ that approximates the true value of y ie. for a new pair (x, y), we should have:

$$\hat{y} = \hat{f}(x) \approx y$$

Vocabulary

Finding the appropriate model $\hat{f}$ is called learning, training or fitting the model.

Statistical learning, two types of problems

Regression

- The outcome is continuous: eg. wage prediction
- The error is often measured by the mean squared error (MSE):

$$\text{MSE} = \mathbb{E}\left[\left(Y - \hat{f}(X)\right)^2\right]$$

Classification

- The outcome is categorical: eg. diagnosis, loan default, ...
- The error is often measured by the accuracy:

$$\text{Misclassification rate} = \mathbb{E}\left[\mathbb{1}\left(Y \neq \hat{f}(X)\right)\right]$$

Motivation: Why prediction?

Why do we need prediction for ?

Statistical inference

- Goal: infer some intervention effect with a causal interpretation
- Require to regress "away" the relationship between the treatment or the outcome and the confounders -> more on this in sessions on Double machine learning.

Predictive inference

- Some problems in economics requires accurate prediction (Kleinberg et al., 2015) without a causal interpretation
- Eg. Stratisfying on a risk score (loan, preventive care, ...)

Do we need more than linear models?

Let:

- p is the number of features
- n is the number of observations

Maybe no

- Low-dimensional data: $n \gg p$
- High predictive performances

Maybe yes

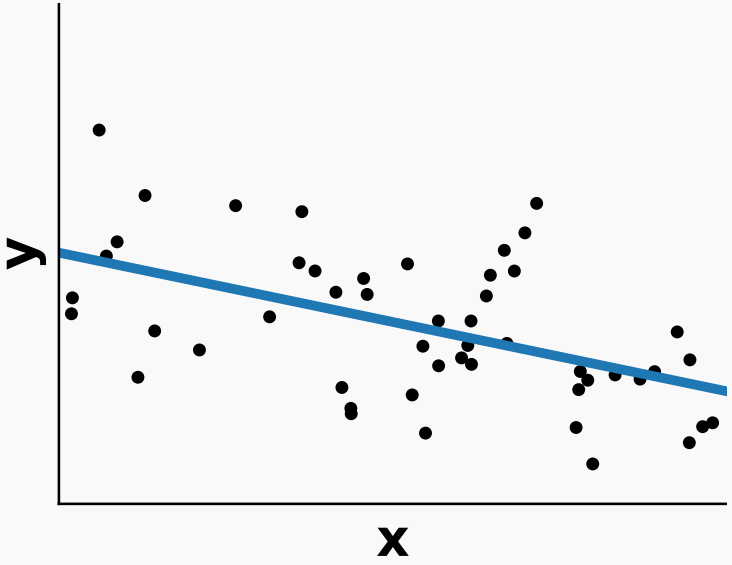
- High-dimensional data: ie. $p \gg n$
- Poor predictive performances

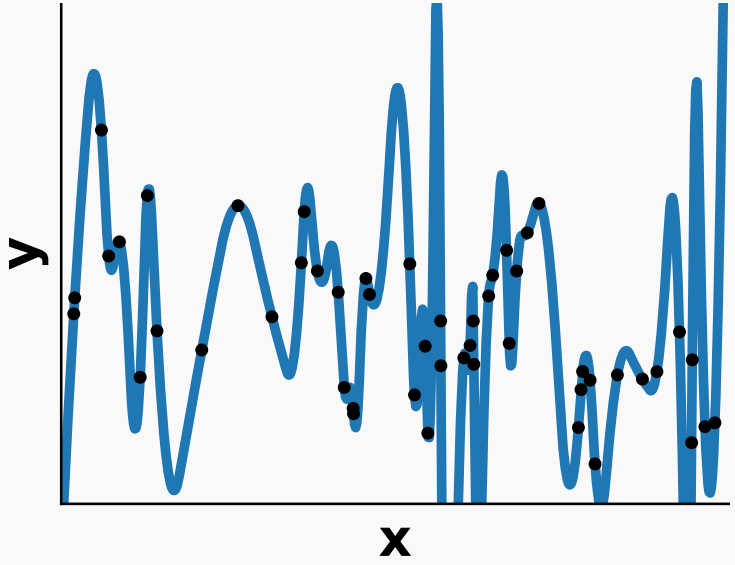
When do we have "high-dimension"?

- $p \gg n$ is a common setting in economics

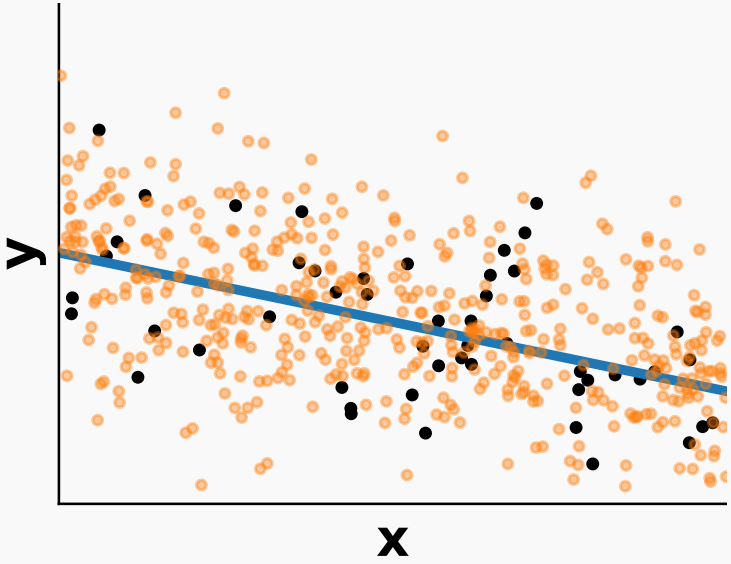
Statistical learning theory

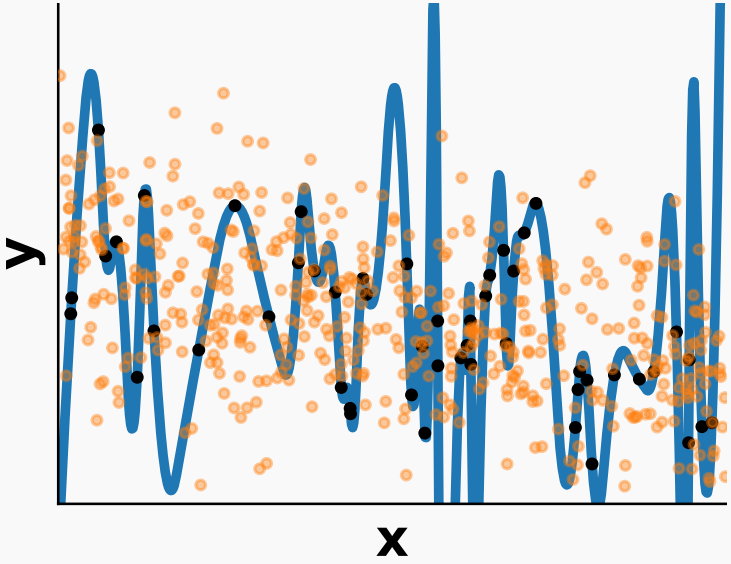
Which data fit do you prefer?





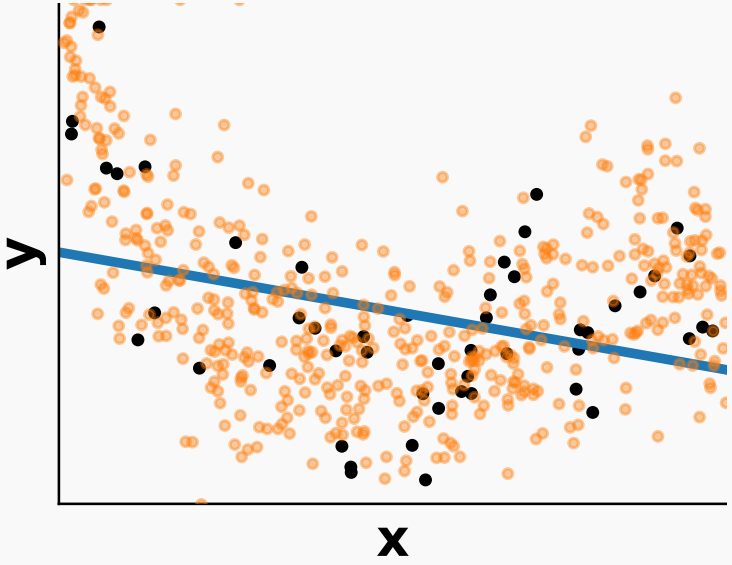
Which data fit do you prefer? (new data incoming)

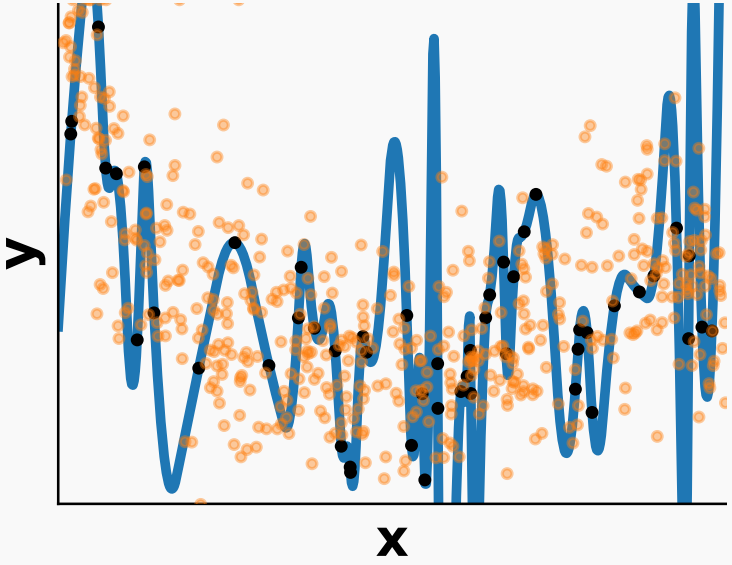




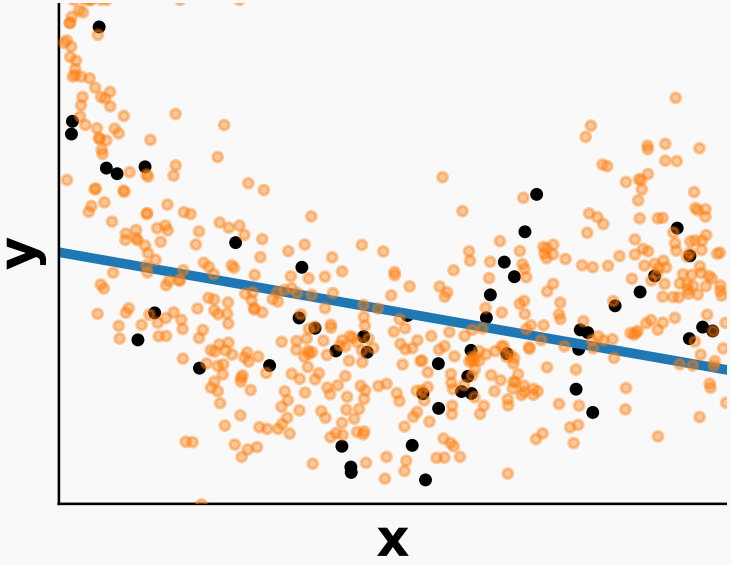
- Answering this question might be hard.
- Goal: create models that generalize.
- The good way of framing the question is: how will the model perform on new data?

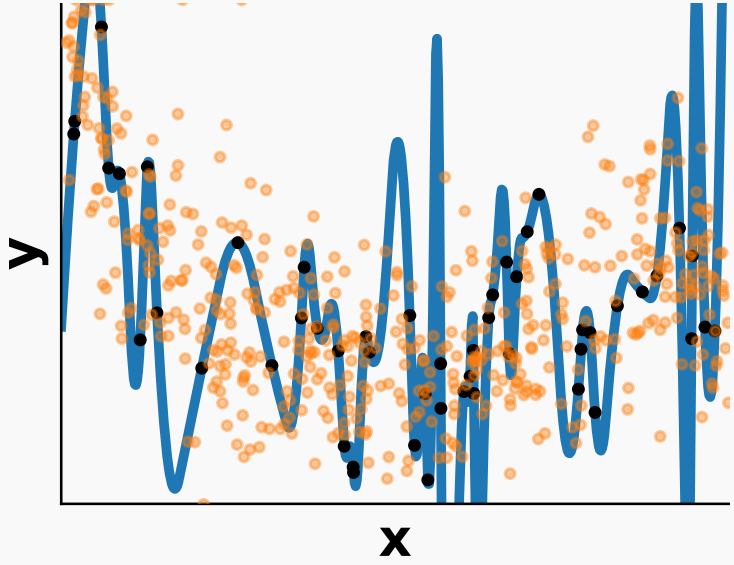
Which data fit do you prefer? New example!





Which data fit do you prefer? New example!





This trade-off is is called Bias variance trade-off.

- Let's recover this trade-off in the context of statistical learning theory.

Empirical Risk Minimization

- Define a loss function ℓ that defines proximity between the predicted value $\hat{y} = f(x)$ and the true value y: $\ell(f(x), y)$
- Usually, for continuous outcomes, the squared loss is used: $\ell(f(x), y) = (f(x) - y)^2$
- We choose among a (finite) family of functions $f \in \mathcal{F}$, the best possible function $f^\star$ minimizes the risk or expected loss $R = \mathbb{E}(\ell)$:

$$f^\star = \text{argmin}_{f \in \mathcal{F}} \mathbb{E}\left[(y - y)^2\right]$$

- In finite sample regimes, the expectation is not accessible since we only have access to a finite number of data pairs
- In practice, we minimize the empirical risk or average loss $R_{\text{emp}} = \sum_{i=1}^{n} \left(f(x_i) - y_i\right)^2$:

$$\hat{f} = \operatorname{argmin}_{f \in \mathcal{F}} \sum_{i=1}^{n} \left(f(x_i) - y_i\right)^2$$

Bayes error rate: Randomness of the problem

In most interesting problems, there is some randomness: ie. $y = g(x) + e$ with $E(e|x) = 0$ and $\text{Var}(e|x) = \sigma^2$

Bias variance trade-off

In most interesting problems, there is some random

Lasso for predictive inference

Bias-variance trade-off, take home messages

High bias == underfitting

- systematic prediction errors
- the model prefers to ignore some aspects of the data
- mispecified models

High variance == overfitting:

- prediction errors without obvious structure
- small change in the training set, large change in model
- unstable models

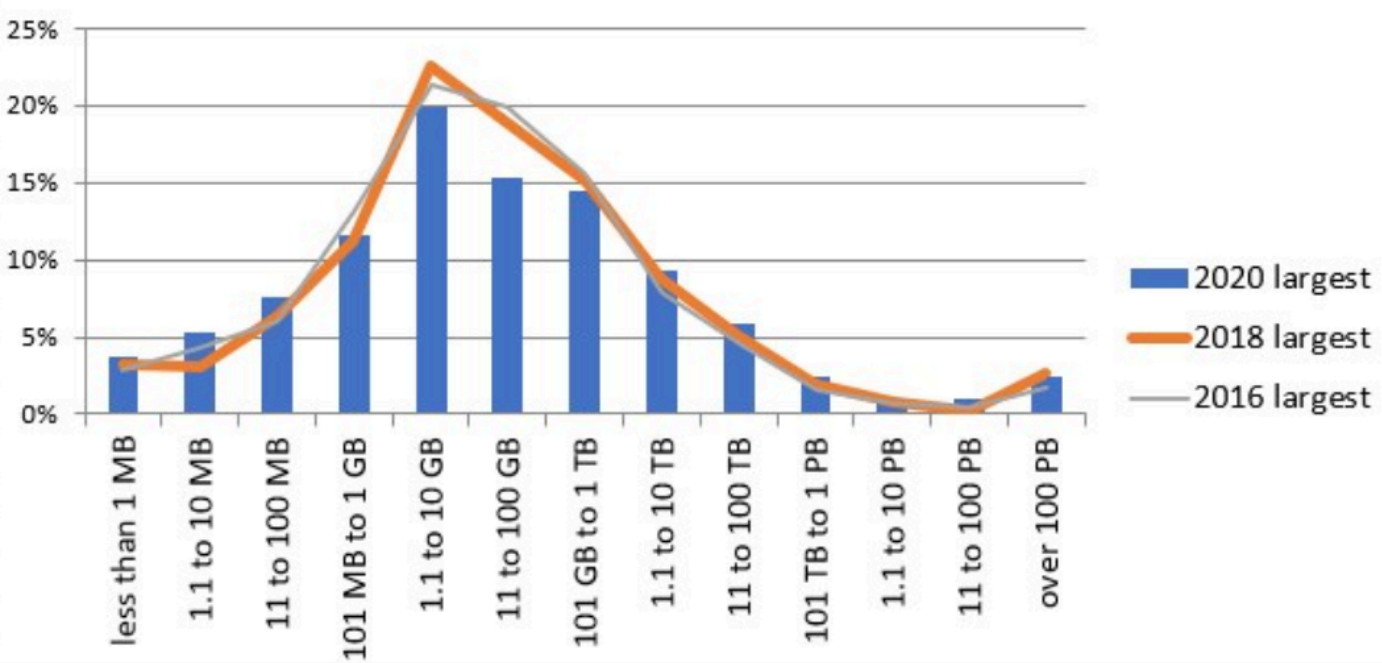
A word on deep learning

Why not use deep learning everywhere?

- Success of deep learning in image, speech recognition and text
- Why not so used in economics?

Limited data settings

- Typically in economics everywhere, we have a limited number of observations

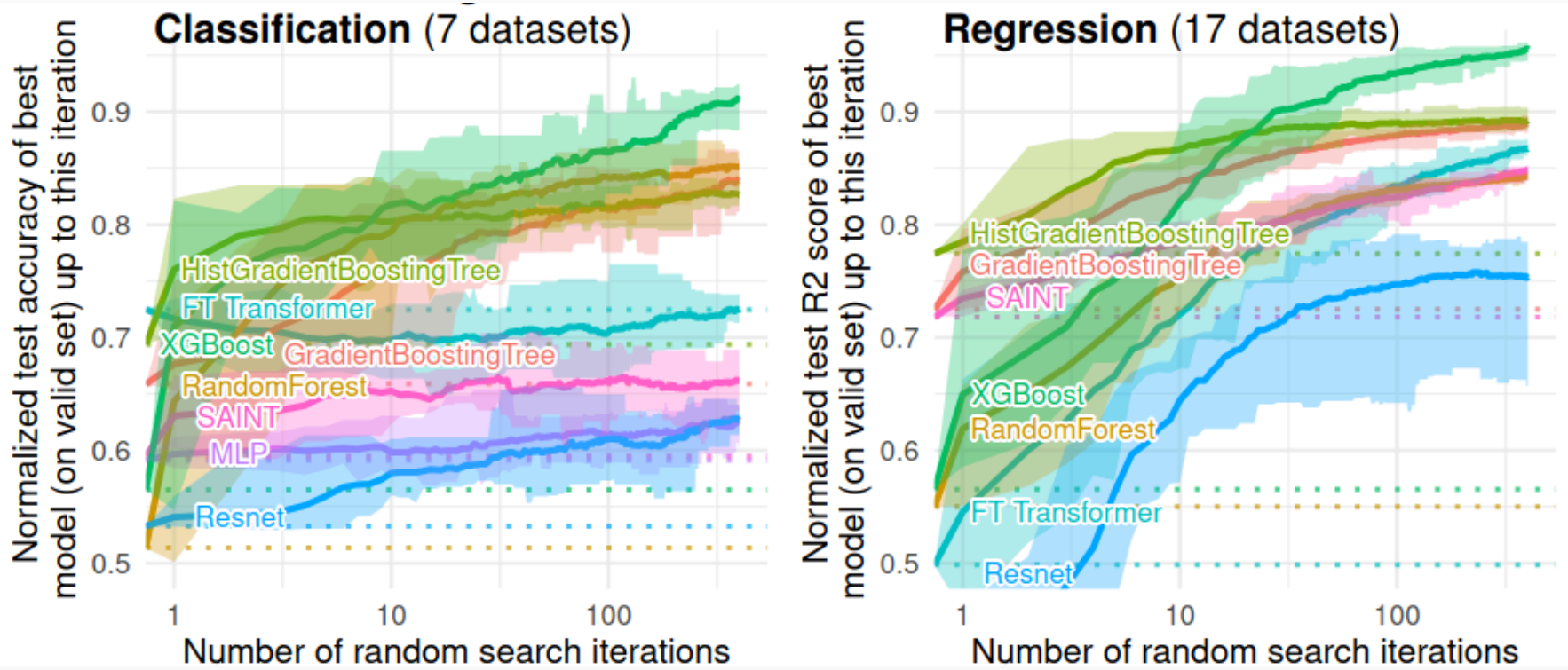


Typical dataset are mid-sized. This does not change with time.[1]

[1] https://www.kdnuggets.com/2020/07/poll-largest-dataset-analyzed-results.html

Deep learning underperforms on data tables

Tree-based methods outperform tailored deep learning architectures (Grinsztajn et al., 2022)



DAG for a RCT: the treatment is independent of the confounders

Resources

- https://web.stanford.edu/~swager/stats361.pdf
- https://www.mixtapesessions.io/
- https://alejandroschuler.github.io/mci/
- https://theeffectbook.net/index.html

Bibliography

Grinsztajn, L., Oyallon, E., & Varoquaux, G. (2022). Why do tree-based models still outperform deep learning on typical tabular data?. Advances in Neural Information Processing Systems, 35, 507–520.

Kleinberg, J., Ludwig, J., Mullainathan, S., & Obermeyer, Z. (2015). Prediction policy problems. American Economic Review, 105(5), 491–495.