

# Machine Learning for econometrics

Reminders of potential outcomes and Directed Acyclic Graphs

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Matthieu Doutreligne

Thanks to Judith Abecassis for the slides on DAGs

## Contents

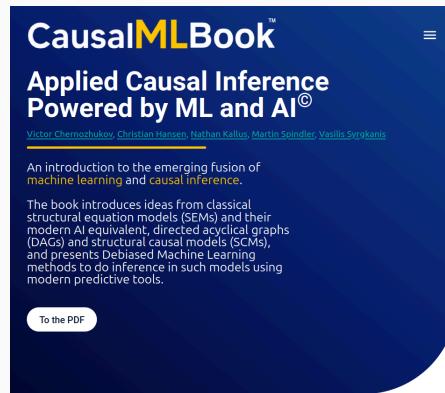
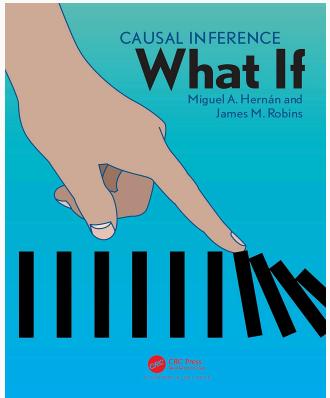
Observational data .....	16
$Y(1), Y(0) \perp\!\!\!\perp A$ .....	16
RCT data .....	16
$Y(1), Y(0) \perp\!\!\!\perp A$ .....	16
Bibliography .....	70

# Contents

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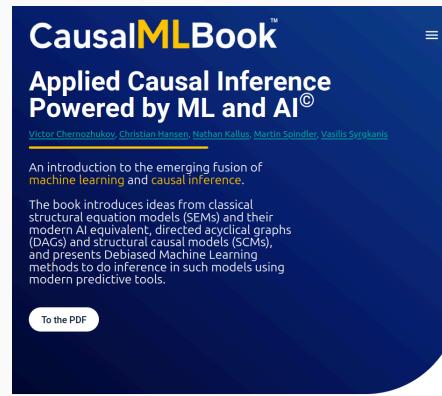
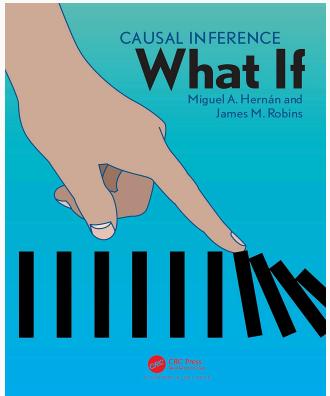
## Introduction

# Causal inference: subfield of statistics dealing with "why questions"



At the center of epidemiology (Hernan & Robins, 2020), econometrics (Chernozhukov et al., 2024), social sciences,

# Causal inference: subfield of statistics dealing with "why questions"



At the center of epidemiology (Hernan & Robins, 2020), econometrics (Chernozhukov et al., 2024), social sciences, machine learning...

Now, bridging with machine learning (Kaddour et al., 2022) : Fairness, reinforcement learning, causal discovery, causal inference for LLM, causal representations...

What is a "why question"?

**Economics: How does supply and demand (causally) depend on price?**

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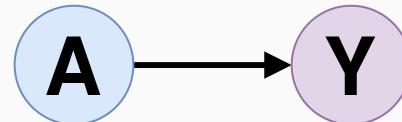
**Public health :** Is this prevention campaign effective?

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**Sociology:** What is the effect of social media on political opinions?

This is different from predictive questions

**Prediction (ML): What usually happens in a given situation?**



Prediction models ( $X, Y$ )

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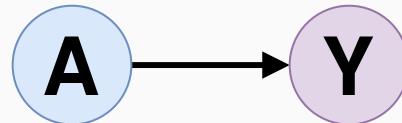


Prediction models  $(X, Y)$

**What will be the weather tomorrow?**

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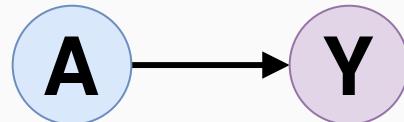
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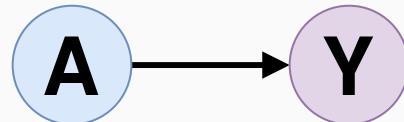
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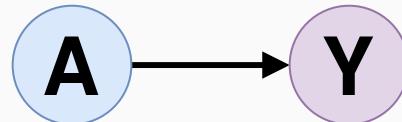
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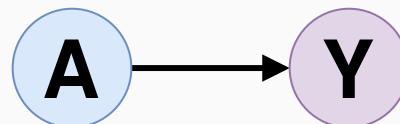
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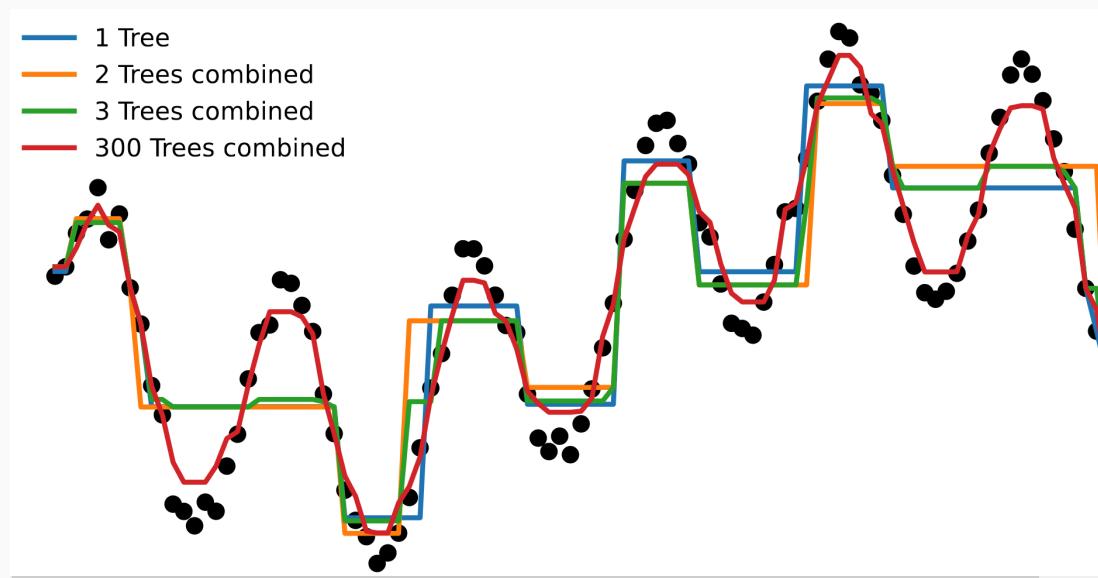
**What is the cardio-vacular risk of this patient?**

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**Assumption** Train and test data are drawn from the same distribution.

# Machine learning is pattern matching

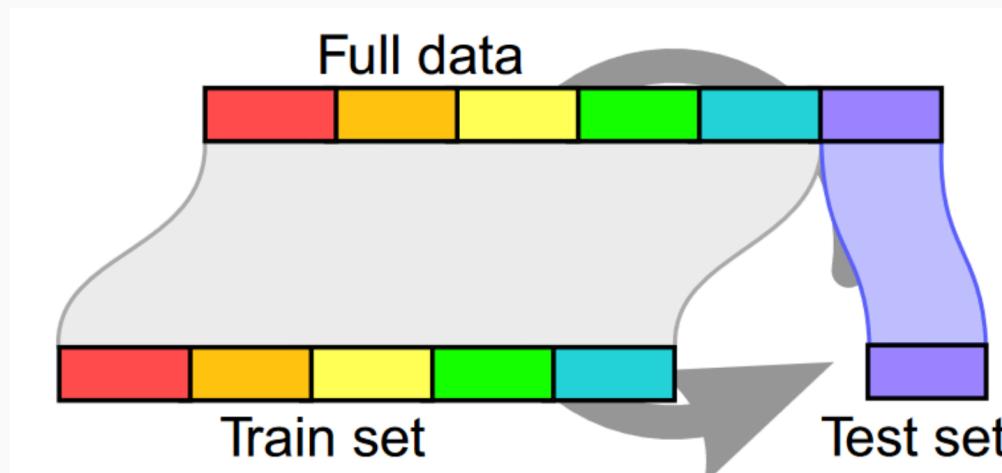
Find an estimator  $f : x \rightarrow y$  that approximates the true value of  $y$  so that  $f(x) \approx y$



Boosted trees : iterative ensemble of decision trees

# Machine learning is pattern matching that generalizes to new data

Select models based on their ability to generalize to new data : (train, test) splits and cross validation (Stone, 1974).



“Cross validation” (Varoquaux et al., 2017)

Machine learning is great for prediction on complex data

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(Krizhevsky et al., 2012)**

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**Speech-to-text: Towards end-to-end  
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# Machine learning is great for prediction on complex data

**Images: Image classification with deep convolutional neural networks**  
(Krizhevsky et al., 2012)

**Speech-to-text: Towards end-to-end speech recognition with recurrent neural networks** (Graves & Jaitly, 2014)

**Text: Attention is all you need**  
(Vaswani, 2017)



ImageNet 1K: 1.5 million images, 1000 classes

# Machine learning is great for prediction on complex data

Motif :

Le patient est admis le 29 août date pour des difficultés respiratoires custom .

Antécédents familiaux :

Le père du patient n'est pas asthmatique custom .

HISTOIRE DE LA MALADIE

Le patient dit avoir de la toux cim10 R05 depuis trois jours date . Elle a empiré jusqu'à nécessiter un passage aux urgences.

Named entity recognition

Machine learning might be less successful for what if questions

## **Machine learning is not driven by causal mechanisms**

- For example people that go to the hospital die more than people who do not<sup>1</sup>:
- Naive data analysis might conclude that hospitals are bad for health.

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<sup>1</sup>Example from [https://inria.github.io/scikit-learn-mooc/concluding\\_remarks.html?highlight=causality](https://inria.github.io/scikit-learn-mooc/concluding_remarks.html?highlight=causality)

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- For example people that go to the hospital die more than people who do not<sup>1</sup>:
- Naive data analysis might conclude that hospitals are bad for health.
- The fallacy is that we are comparing different populations: people who go to the hospital typically have a worse baseline health than people who do not.

### Definition

This is a confounding factor: A variable that influences both the treatment and the outcome.

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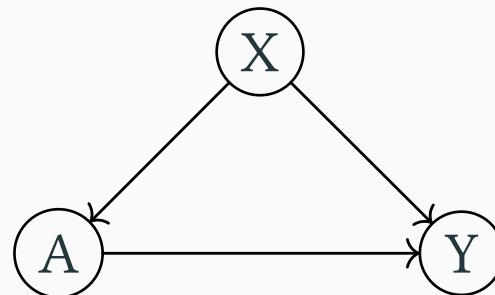
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## Why is prediction different from causation? (2/2)

**Causal inference (most part of economists) : What would happen if we changed the system ie. under an intervention?**

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**Causal inference (most part of economists) : What would happen if we changed the system ie. under an intervention?**



Causal inference models  $(X, A, Y(A = 1), Y(A = 0))$   
ie. the covariate shift between treated and control units.

Assumption

No unmeasured variables influencing both treatment and outcome  
ie. no confounders.

# Illustration of the fundamental problem of causal inference (epidemiology)

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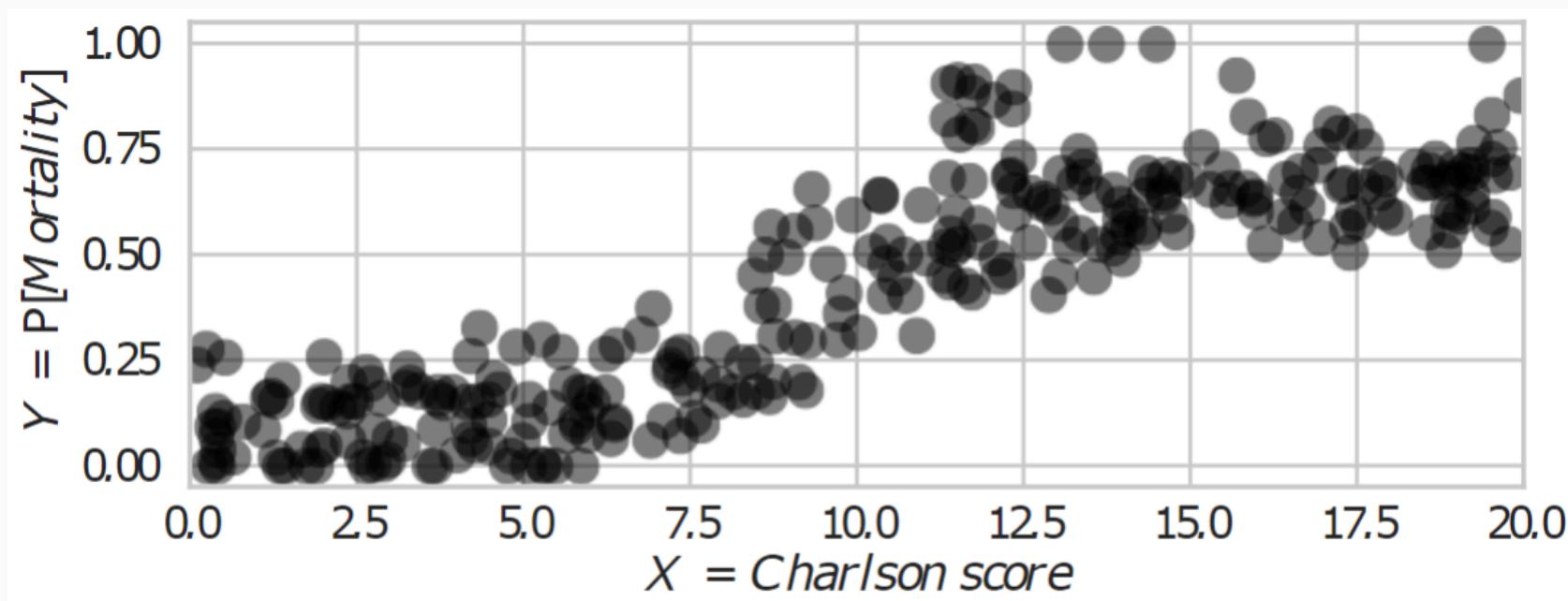
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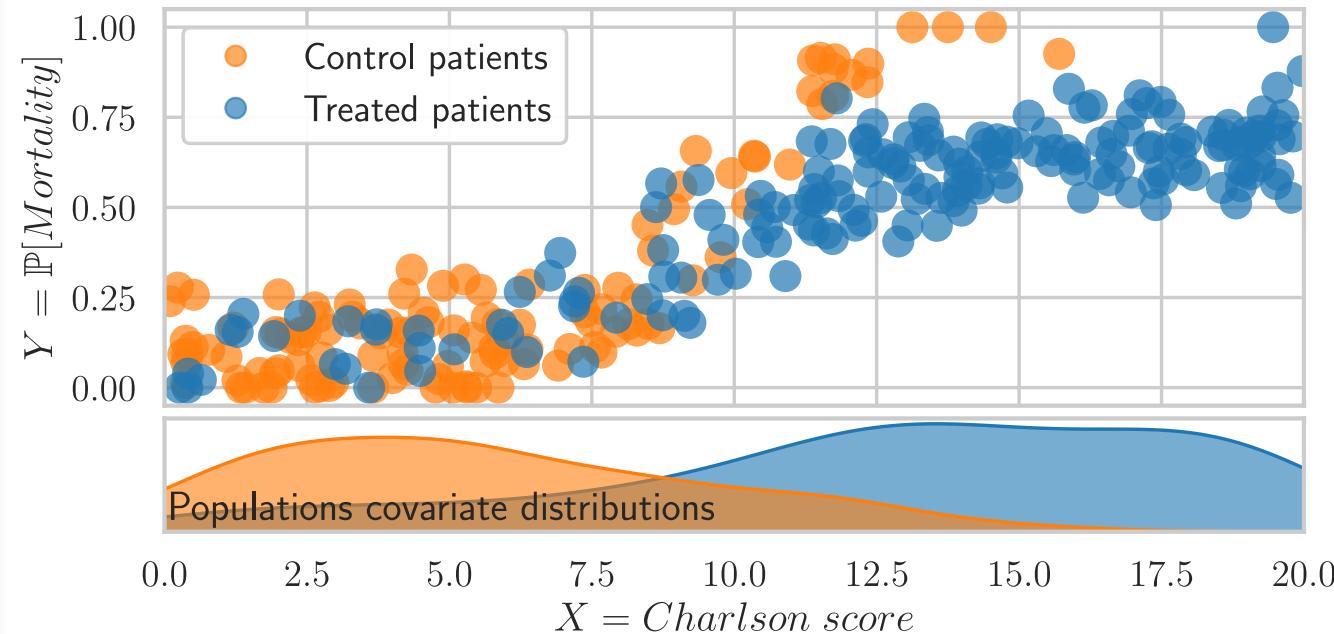
**?** What is the effect of early access to MRI on the mortality at 7 days for stroke patients?

## Illustration: observational data

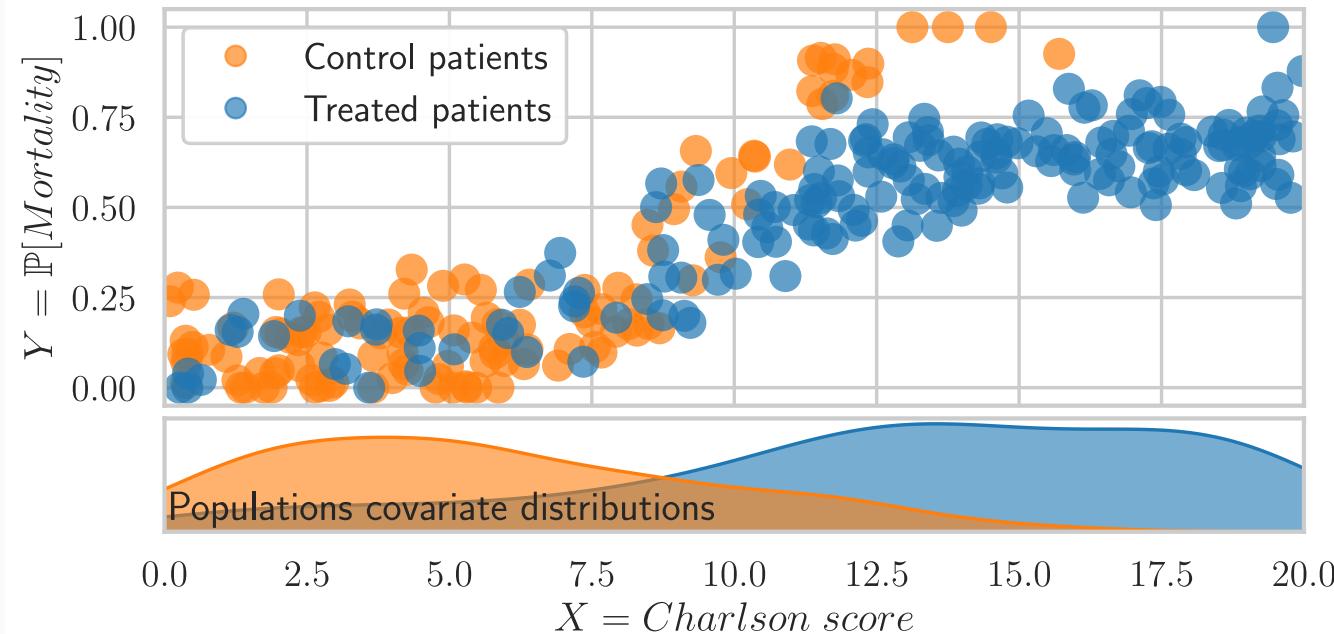
Draw a population sample **without treatment status**



## Draw a population sample **with treatment status**



## Draw a population sample **with treatment status**



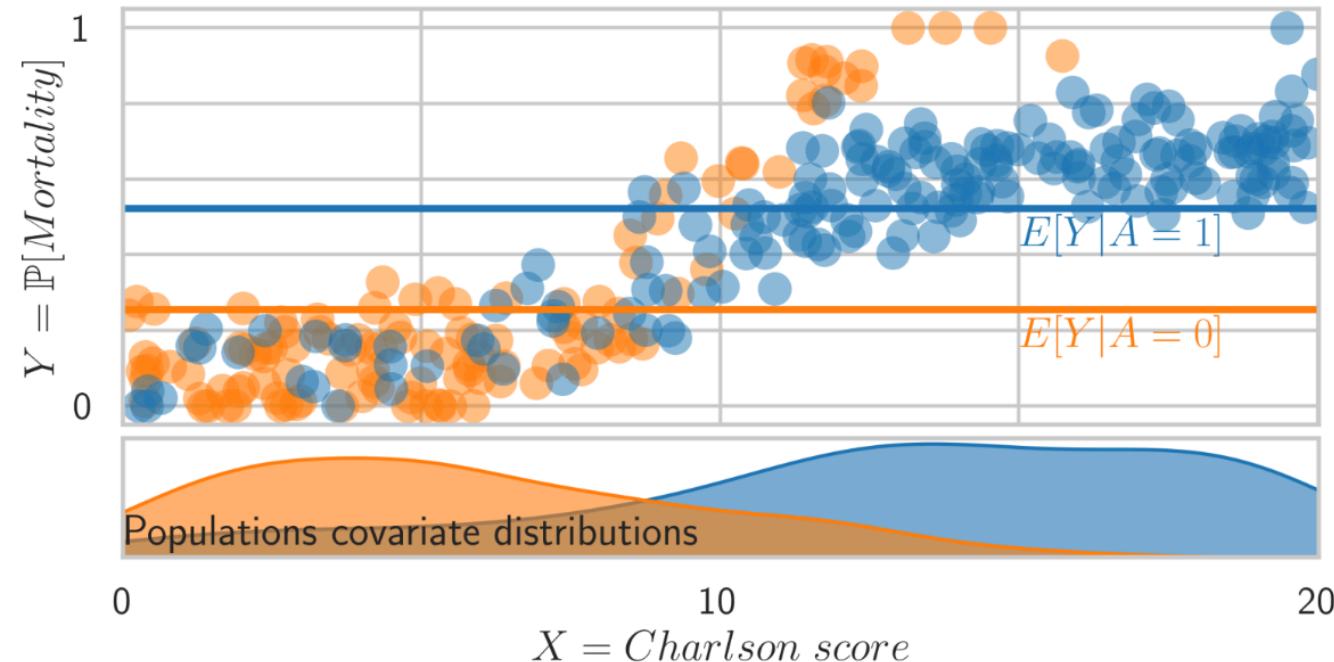
⌚ Patient with higher risks have early access to MRI.

## Illustration: observational data, a naive solution

**Compute the difference in mean (DM):**  $\tau_{\text{DM}} = \mathbb{E}[Y(1)] - \mathbb{E}[Y(0)]$

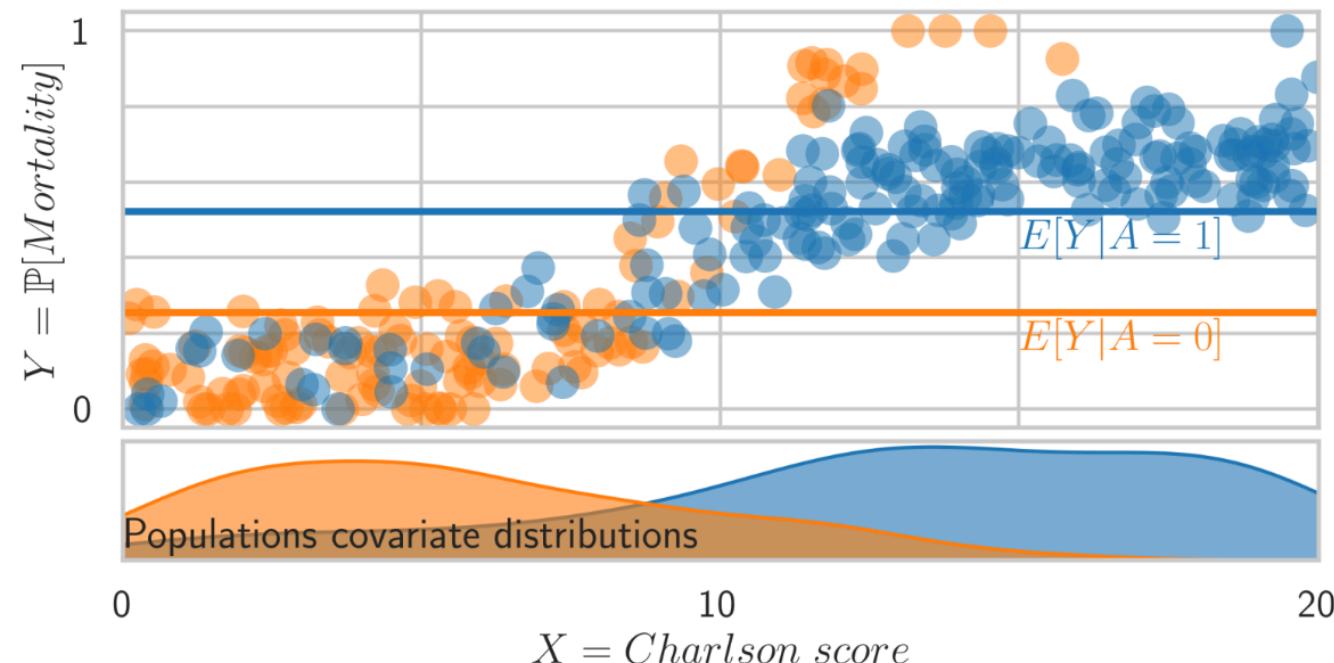
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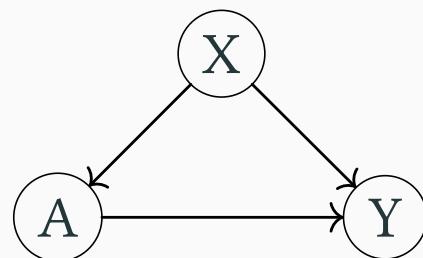
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(False) conclusion: early access to MRI is associated with a higher mortality at 7 days.

# RCT case: No problem of confounding

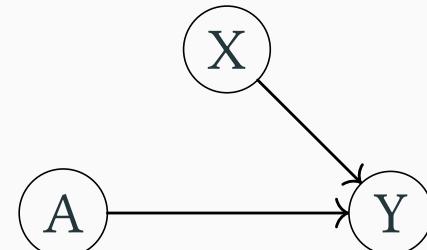
## Observational data



$$Y(1), Y(0) \perp\!\!\!\perp A$$

**Intervention is not random**  
(with respect to the confounders)

## RCT data



$$Y(1), Y(0) \perp\!\!\!\perp A$$

**Force random assignment of  
the intervention**

## Illustration: RCT data

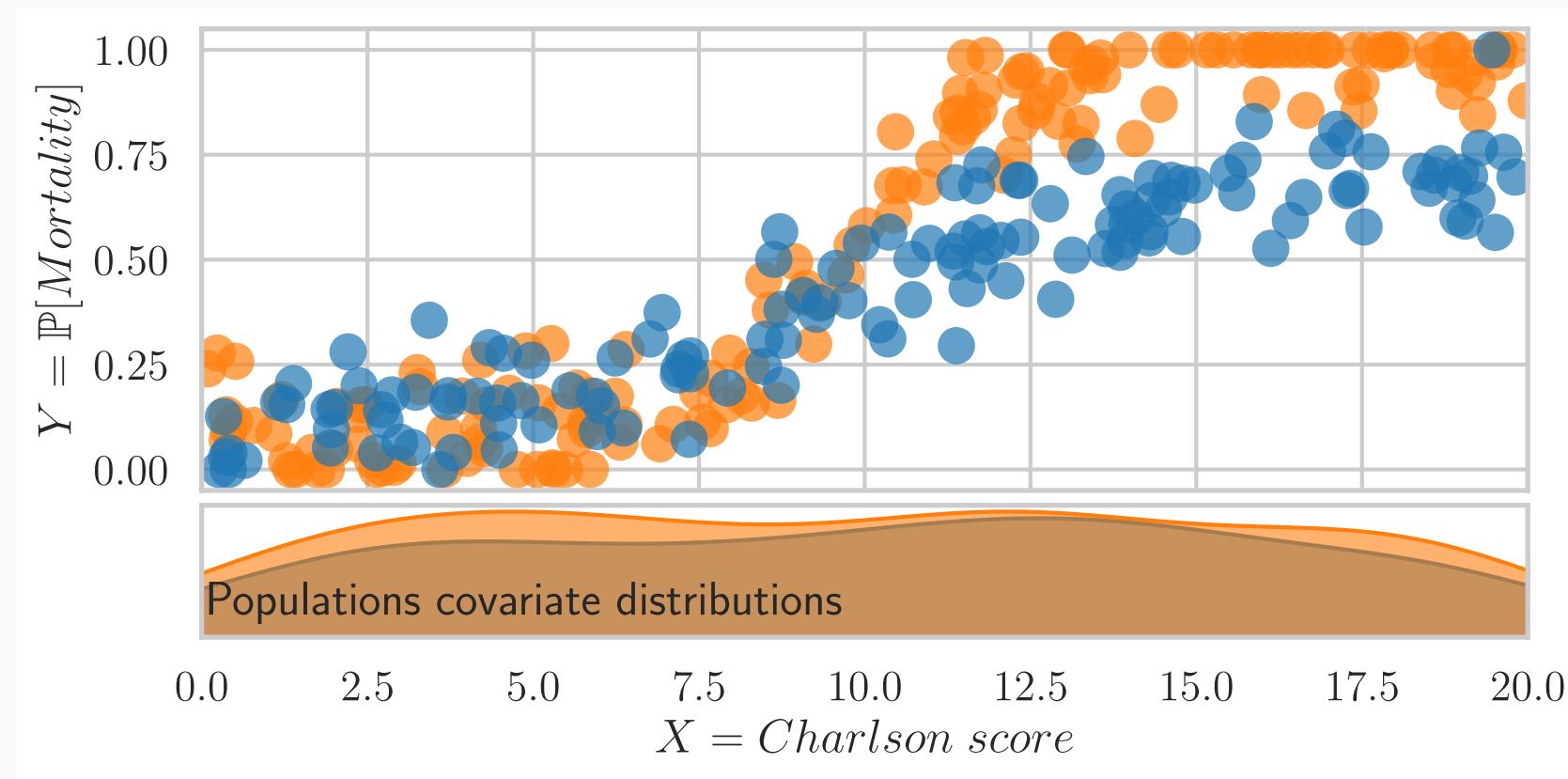
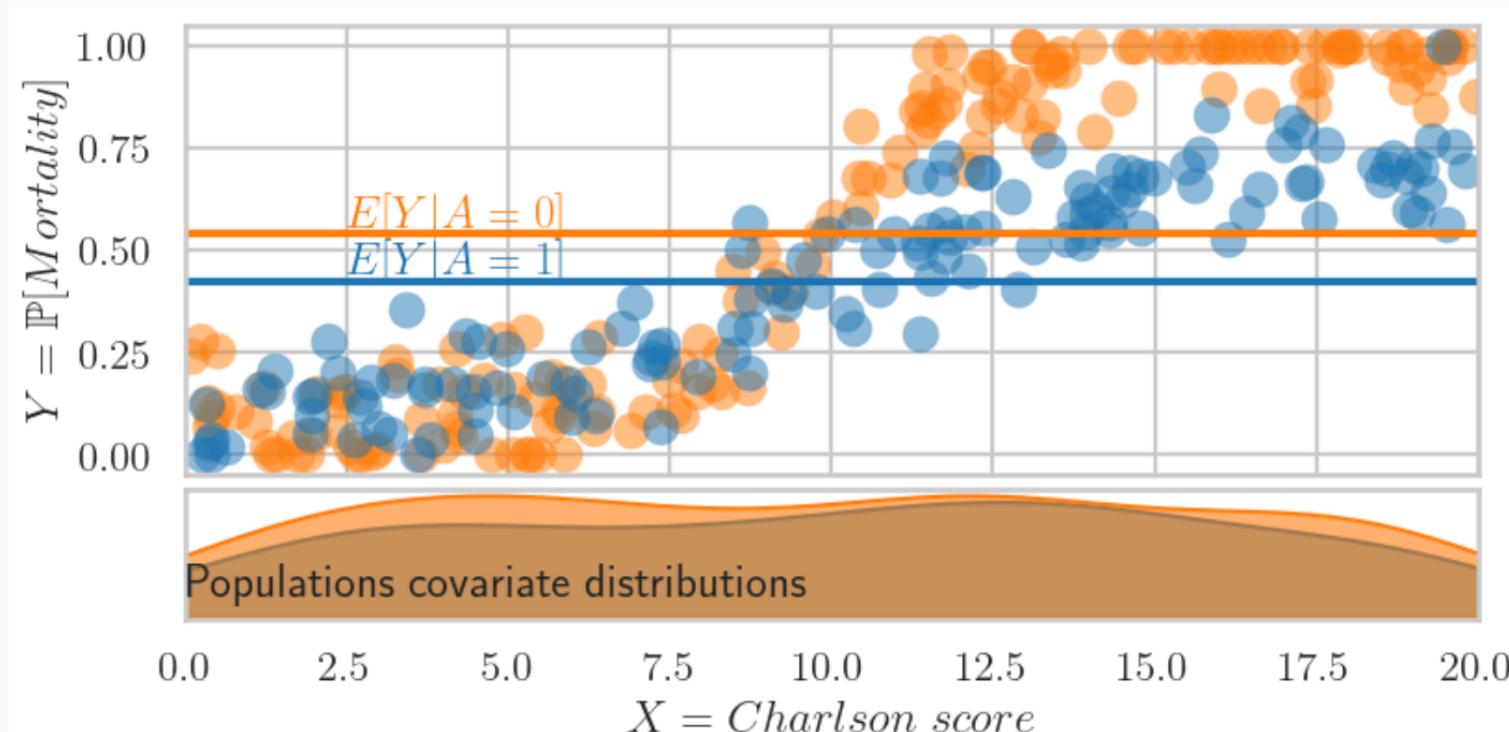
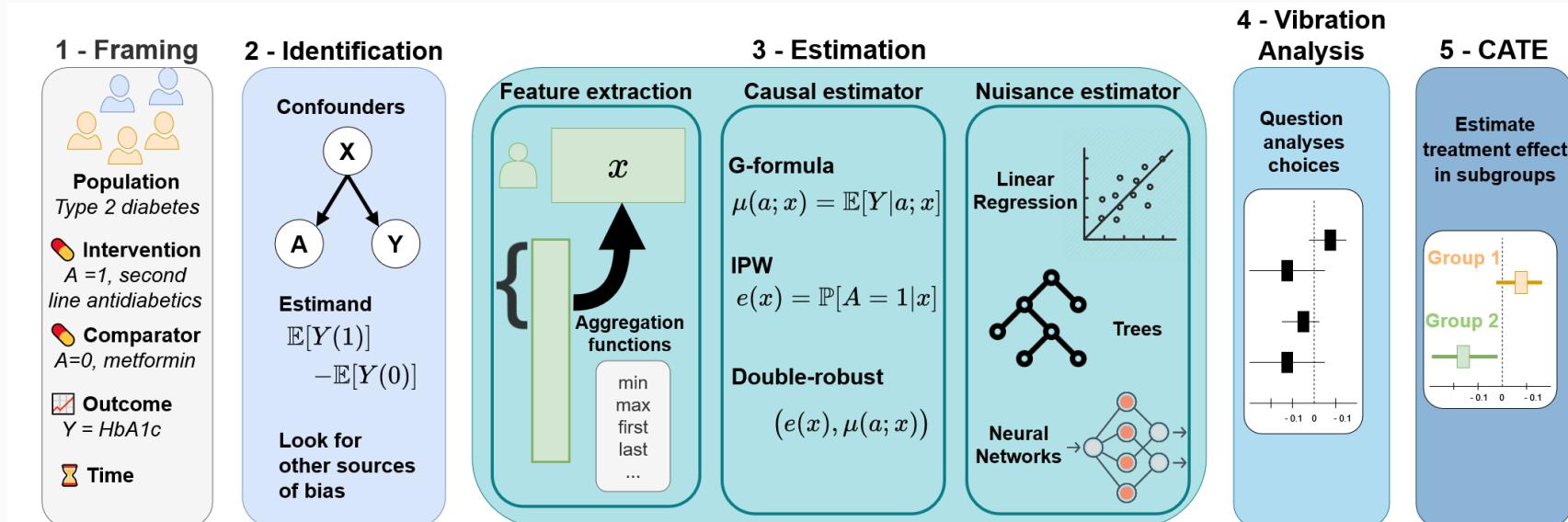


Illustration: RCT data, a naive solution that works!

**Compute the difference in mean (DM):**  $\tau_{\text{DM}} = \mathbb{E}[Y(1)] - \mathbb{E}[Y(0)]$

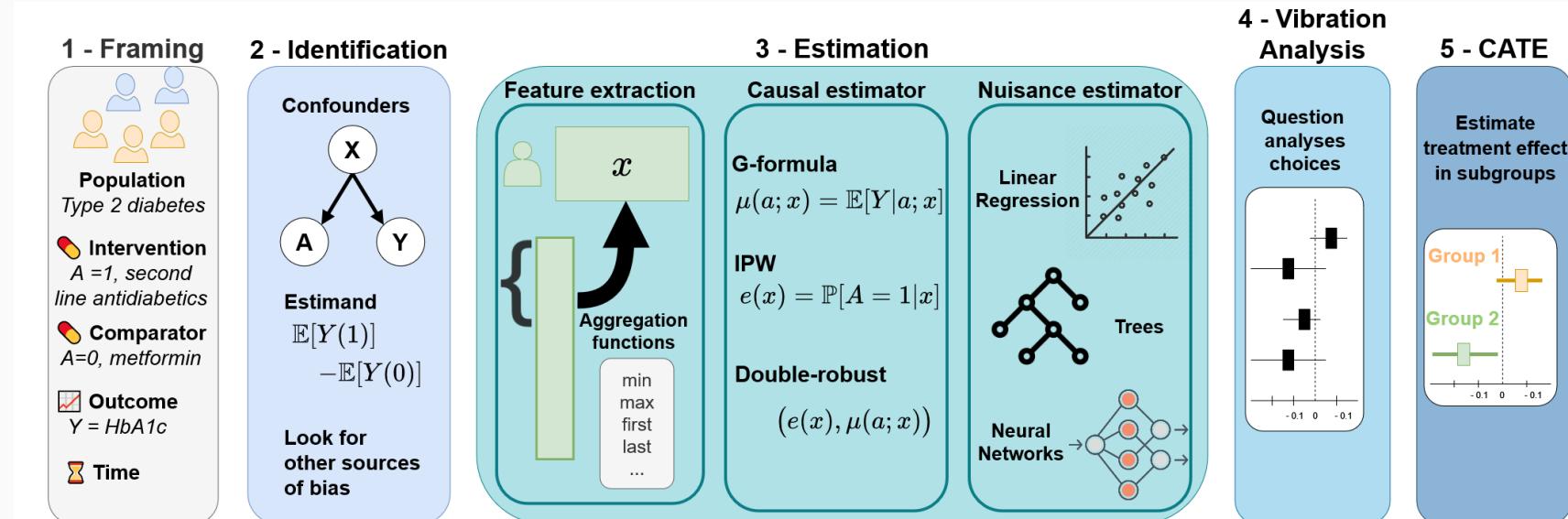


# Causal inference: framing with PICO and identification with DAGs



Five steps for causal inference: an applied tutorial (Doutreligne et al., 2025, Figure 1)

# Causal inference: framing with PICO and identification with DAGs



Five steps for causal inference: an applied tutorial (Doutreligne et al., 2025, Figure 1)

Further references:

- Gentle introduction from ML and epidemiologists (Abécassis et al., 2024)
- Advanced statistical point of view (Wager, 2024)

$$Y(1), Y(0) \perp\!\!\!\perp A$$

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Framing: How to ask a sound causal question

# PICO framework (Richardson et al., 1995)

Originally designed for clinical research. It is a structured approach to formulate a research question. Critical for health technology assessment (eg. Haute Autorité de santé).

## **PICO stands for**

- Population : Who are we interested in?
- Intervention : What treatment/intervention do we study?
- Comparison : What are we comparing it to?
- Outcome : What are we interested in?

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# PICO framework (Richardson et al., 1995)

## **Example with the job dataset (LaLonde, 1986)**

Built to evaluate the impact of the National Supported Work (NSW) program. The NSW is a transitional, subsidized work experience program targeted towards people with longstanding employment problems.

# The PICO framework

Component	Description	Example
Population	What is the target population of interest?	People with longstanding employment problems
Intervention	What is the intervention?	On-the-job training lasting between nine months and a year
Control	What is the relevant comparator?	No training
Outcome	What are the outcomes?	Earnings in 1978
Time	Is the start of follow-up aligned with intervention assignment?	The period of follow-up for the earning is the year after the intervention.

## PICO: other examples in econometrics

The Oregon Health Insurance Experiment (Finkelstein et al., 2012) : A randomized experiment by lottery assessing the impact of Medicaid on low-income adults in Oregon.

- P: Low-income adults in Oregon
- I: Medicaid
- C: No insurance
- O: Healthcare uses and expenditures, health outcomes

# PICO: other examples in econometrics

The economic impact of climate change on US agricultural land. (Deschênes & Greenstone, 2007): difference-in-differences design assessing the impact of climate change on agricultural profits.

- P: US agricultural land
- I: Climate change
- C: No climate change
- O: Agricultural profits

# PICO: other examples in econometrics

The impact of class size on test scores. (Angrist & Lavy, 1999): regression discontinuity design.

- P: Fourth and fifth grades school in Israel
- I: Class size increases by one unit
- C: No class size increase
- O: Test scores (math and reading)

$$Y(1), Y(0) \perp\!\!\!\perp A$$

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Identification: List necessary information to answer the causal question

# Identification: Build the causal model

A causal effect is said to be identified if it is possible, with ideal data (infinite sample size and no measurement error), to purge an observed association of all noncausal components such that only the causal effect of interest remains.

— (Elwert & Winship, 2014)

## Steps

- Potential outcome framework : mathematical tool to reason about causality
- Directed acyclic graphs (DAG) : graphical tool to reason about causality
- Causal estimand : what is the targeted quantity?

## Potential outcomes, (Neyman, 1923; Rubin, 1974)

The Neyman-Rubin model, let:

- $Y$  be the outcome,
- $A$  the (binary) treatment

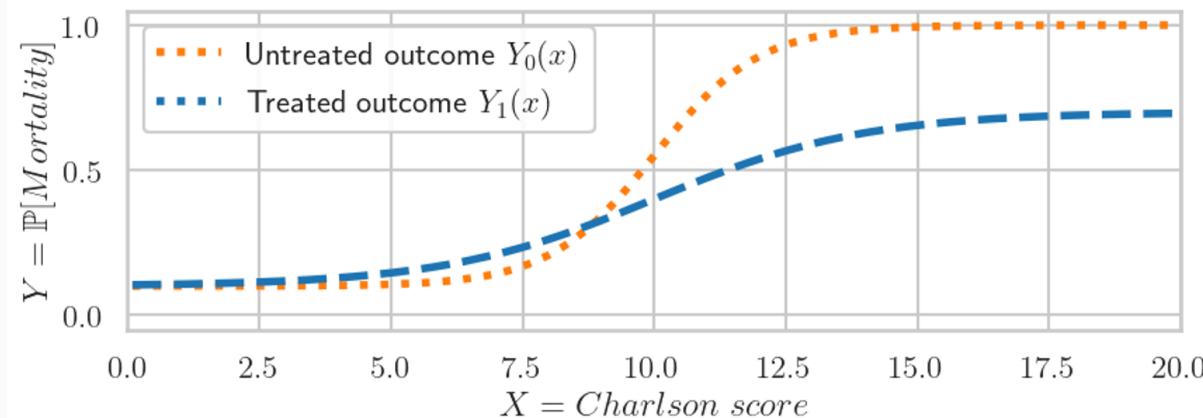
For each individual, we have two potential outcomes:  $Y(1)$  and  $Y(0)$ . But only one is observed, depending on the treatment assignment:  $Y(A)$ .

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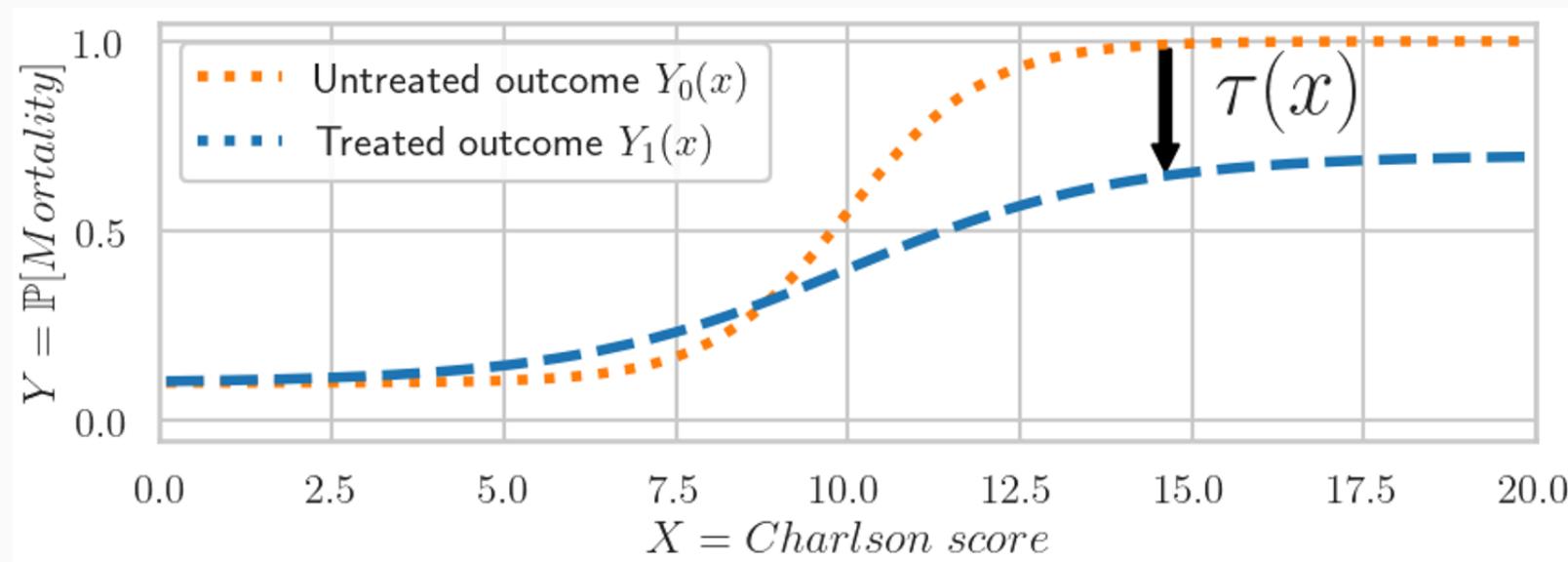


## Causal estimand: What is the targeted quantity (with potential outcomes)?

- Average treatment effect (ATE):  $\mathbb{E}[Y(1) - Y(0)]$
- Conditional average treatment effect (CATE):  $\mathbb{E}[Y(1) - Y(0) \mid X]$

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Causal estimand: What is the targeted quantity (with potential outcomes)?

## Other estimands

- Average treatment effect on the treated (ATT):  $\mathbb{E}[Y(1) - Y(0) \mid A = 1]$
- Conditional average treatment effect on the treated (CATT):

$$\mathbb{E}[Y(1) - Y(0) \mid A = 1, X]$$

Causal estimand: What is the targeted quantity (with potential outcomes)?

## Other estimands

- Average treatment effect on the treated (ATT):  $\mathbb{E}[Y(1) - Y(0) \mid A = 1]$
- Conditional average treatment effect on the treated (CATT):

$$\mathbb{E}[Y(1) - Y(0) \mid A = 1, X]$$

## Other estimands more used in epidemiology

- Risk ratio (RR):  $\frac{\mathbb{E}[Y(1)]}{\mathbb{E}[Y(0)]}$
- Odd ratio (OR) for binary outcome:  $\left(\frac{\mathbb{P}[Y(1)=1]}{\mathbb{P}[Y(1)=0]}\right) / \left(\frac{\mathbb{P}[Y(0)=1]}{\mathbb{P}[Y(0)=0]}\right)$

See (Colnet et al., 2023) for a review of the different estimands and the impact on generalization.

# PICO framework, link to the potential outcomes

Component	Description	Notation	Example
Population	What is the target population of interest?	$X \sim P(X)$	People with longstanding employment problems
Intervention	What is the intervention?	$A \sim P(A = 1) = p_A$	On-the-job training lasting between nine months and a year
Control	What is the relevant comparator?	$1 - A \sim 1 - p_A$	No training
Outcome	What are the outcomes?	$Y(1), Y(0) \sim P(Y(1), Y(0))$	Earnings in 1978
Time	Is the start of follow-up aligned with intervention assignment?	N/A	The period of follow-up for the earning is the year after the intervention

**What can we learn from the data?**

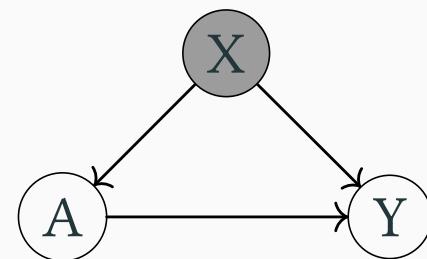
**Four assumptions, referred as strong ignorability**

**Required to assure identifiability of the causal estimands with observational data (Rubin, 2005)**

# Assumption 1: Unconfoundedness, also called ignorability

**Treatment assignment is as good as random given the covariates  $X$**

$$\{Y(1), Y(0)\} \perp\!\!\!\perp A \mid X$$



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**Treatment assignment is as good as random given the covariates  $X$**

$$\{Y(1), Y(0)\} \perp\!\!\!\perp A \mid X$$

- Equivalent to conditional independence on the propensity score:  
 $e(X) \stackrel{\text{def}}{=} \mathbb{P}(A = 1|X)$  (Rosenbaum & Rubin, 1983):

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- **Knowledge based** ie. cannot be validated with data
  - ▶ Because of possibly unmeasured confounders

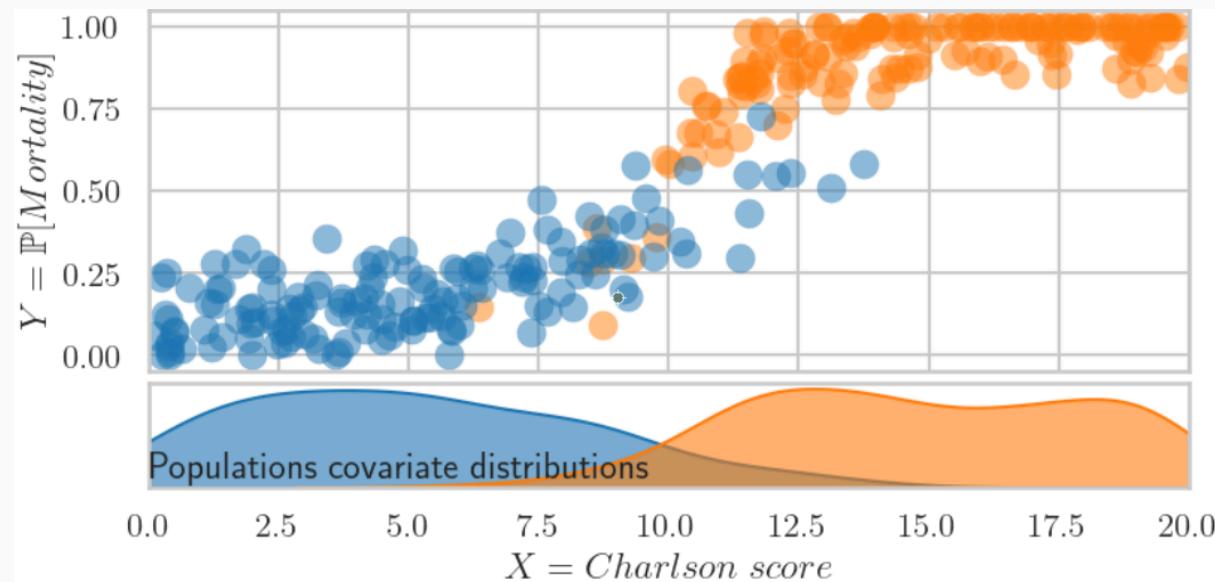
## Assumption 1: Unconfoundedness, also called ignorability

- ▶ In practice : ask yourself if you have measured all the relevant variables that could influence both the treatment and the outcome.

## Assumption 2: Overlap, also known as positivity

The treatment is not deterministic given  $X$

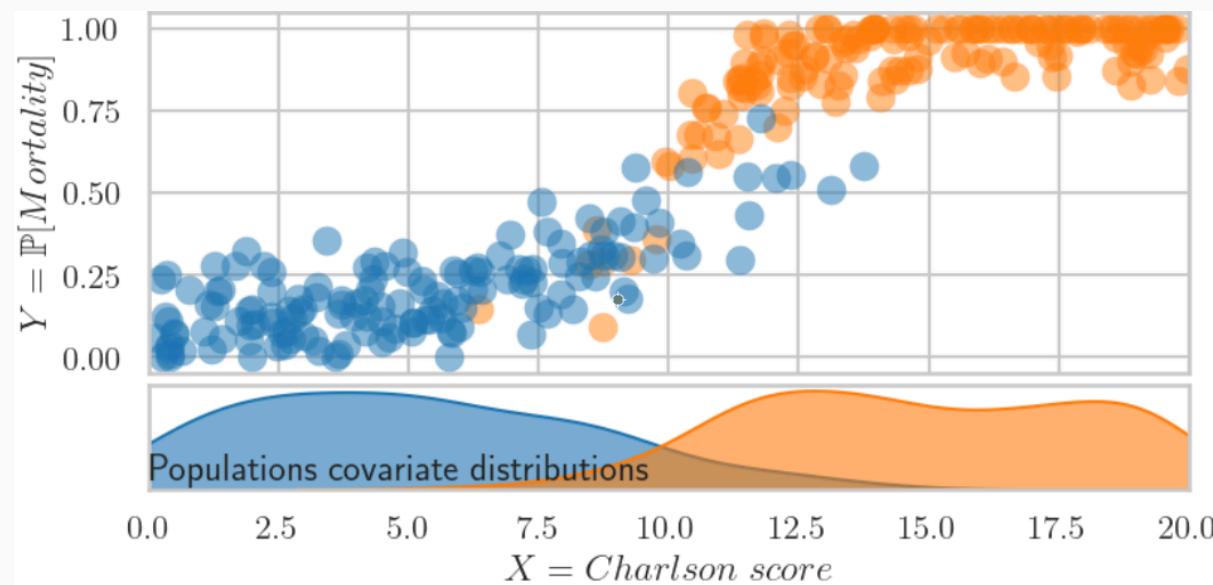
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Choice of covariates  $X$ : trade-off between ignorability and overlap (D'Amour et al., 2021)

## Assumption 3 and 4: Consistency and generalization

### **Consistency, also called Stable Unit Treatment Values (SUTVA)**

The observed outcome is the potential outcome of the assigned treatment for each unit i.

$$Y_i = A_i Y_i(1) + (1 - A_i) Y_i(0)$$

## Assumption 3 and 4: Consistency and generalization

### **Consistency, also called Stable Unit Treatment Values (SUTVA)**

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$$Y_i = A_i Y_i(1) + (1 - A_i) Y_i(0)$$

- The intervention A is well defined (Hernan & Robins, 2020)
- There is no interference ie. network effect

## Assumption 3 and 4: Consistency and generalization

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- There is no interference ie. network effect

### **Generalization, also called no-covariate shift**

Training and test data are drawn from the same distribution

$$Y(1), Y(0) \perp\!\!\!\perp A$$

---

Directed acyclic graphs (DAGs)

# Directed acyclic graphs (DAG), a tool to reason about causality

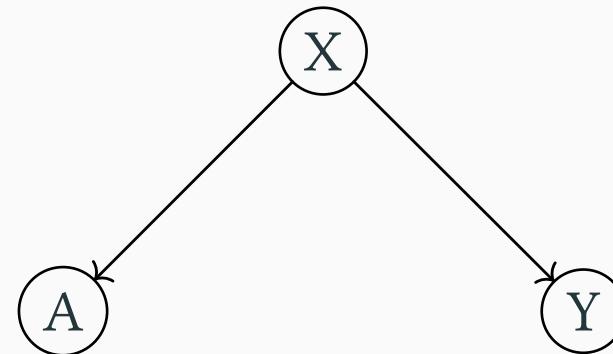
**DAGs encode the causal structure of the data generating process**

Introduced by (Pearl, 1995), (Pearl & others, 2000). Good practical overview in (VanderWeele, 2019).

## Motivation

- Reason about the relation between variables.
- Help identify for which (minimal) set of variables, the ATE is identifiable.

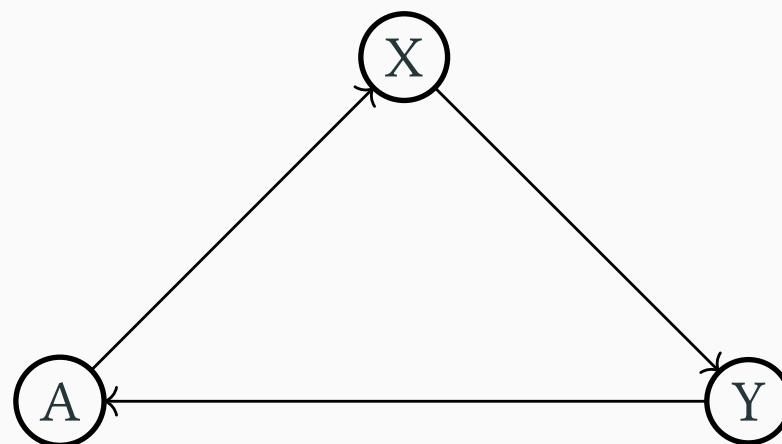
# Directed acyclic graphs (DAG), definitions



- **Graph:** A set of relations between nodes described by edges between those nodes.
- **Directed:** Edges between nodes have direction. The direction of the arrow represents a cause-effect relationship.
- **Acyclic:** : There are no cycles or loops in the causal structure. A variable can't be a cause of itself.

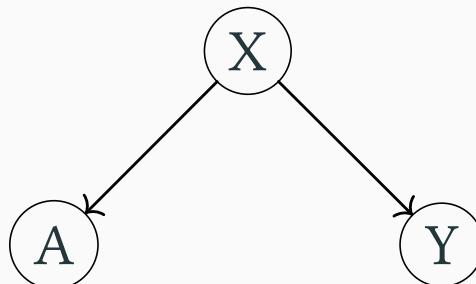
# A cyclic graph

**This is not a DAG**



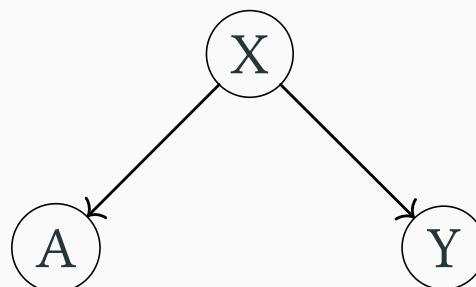
## DAGs: nodes

- **Nodes** represent random variables.
- **Edges** between nodes symbolize causal effects (i.e. difference in potential outcomes). Here,  $Y_i(a) \neq Y_i(a')$  for two different levels of  $A_i$  because of the arrow from A to Y.
- **Lack of edges** between nodes denotes the absence of a causal relationships.



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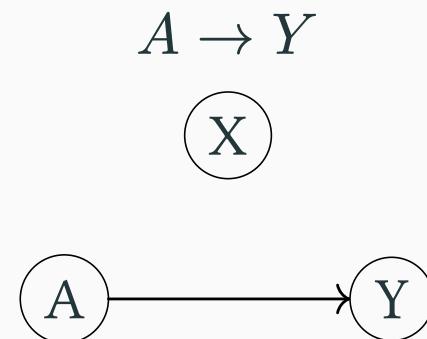
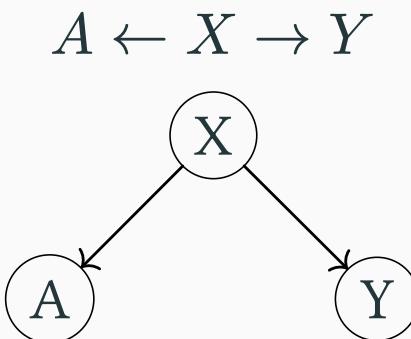


Important

Not drawing an arrow makes a stronger assumption about the relationship between those two variables than drawing an arrow.

## DAGs: paths

- A path between two nodes is a route that connects the two nodes following non-intersecting edges.
- A path exists even if the arrows are not pointing in the good direction.
- Two examples of paths between A and Y:



## DAGs: causal paths

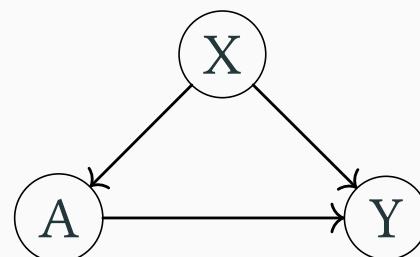
- Paths encode dependencies between random variables: not necessarily causal dependencies, it can be mere associations.

# DAGs: causal paths

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- We distinguish:
  - **Causal** paths: arrows are all in the same direction.
  - From **Non-causal** paths: arrows pointing in different directions
- When there is a causal path between two variables A and B, we say that B is a **descendant** of A (it is causally impacted by A)

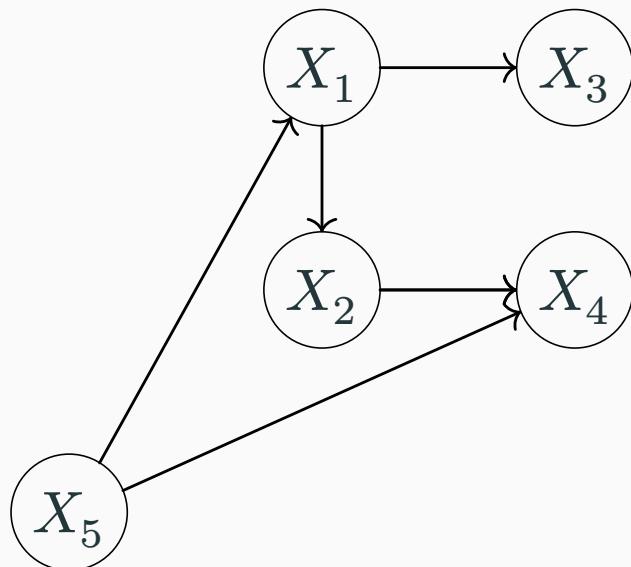
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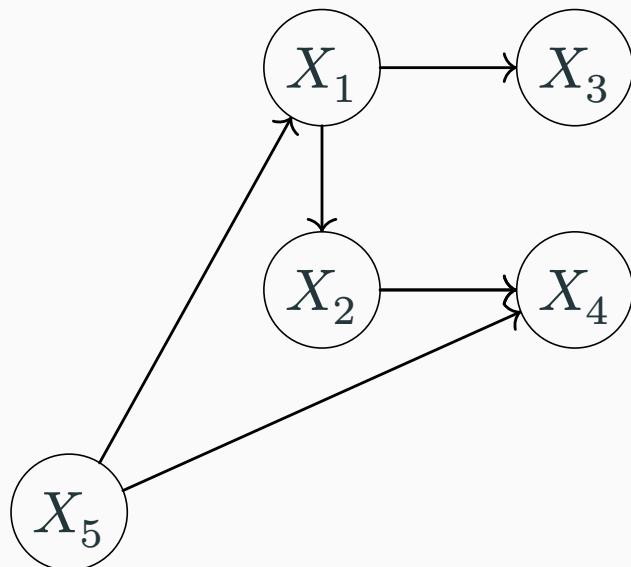


$A \rightarrow Y$  is **causal**  
 $Y \rightarrow A$  is **non-causal**

Your turn: Paths between  $X_1$  and  $X_4$ ? Which of them are causal?



Your turn: Paths between  $X_1$  and  $X_4$ ? Which of them are causal?



- $X_5 \rightarrow X_1 \rightarrow X_2 \rightarrow X_4$
- $X_5 \rightarrow X_4$
- $X_1 \leftarrow X_5 \rightarrow X_4$  : non causal
- $X_3 \leftarrow X_1 \rightarrow X_2 \rightarrow X_4$  : non causal

## Three types of directed edges: path

There are three kinds of “triples” or paths with three nodes: These constitute the most basic building blocks for causal DAGs.

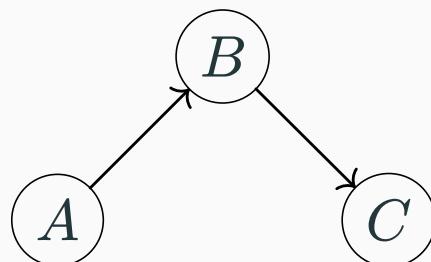
**First, a causal path (or chain):  $A \rightarrow B \rightarrow C$**

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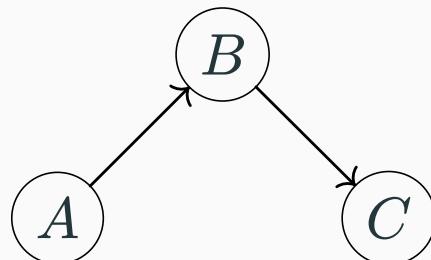


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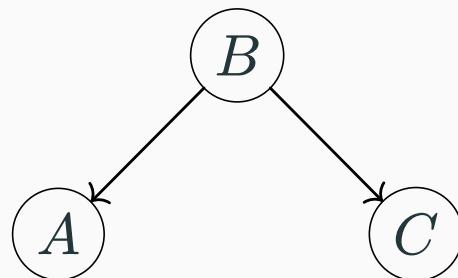
### Example

An individual receiving a message (A) encouraging them to vote causes that individual to actually vote (C) only if the individual actually reads (B) the message.

## Three types of directed edges: mutual dependence

### Second, mutual dependence or fork or confounder: $A \leftarrow B \rightarrow C$

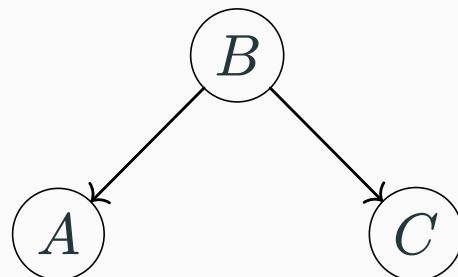
- A and C are not causally related but B is a common cause of both.
- A and C are not independent, but are independent conditional on B.
- Not conditioning on X introduces bias.



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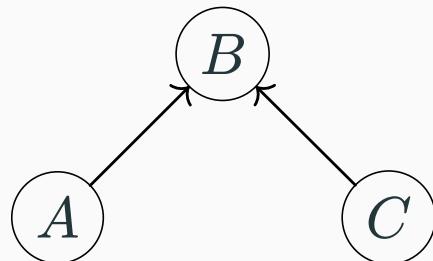
### Examples

- Rise in temperature (B) causes both the thermometer (A) to change, and ice to melt (C), but the thermometer changing does not cause ice to melt.
- A is prostate cancer; B is age; and C is Alzheimer's disease.

## Three types of directed edges: collider

### Third, **collider**: $A \rightarrow B \leftarrow C$

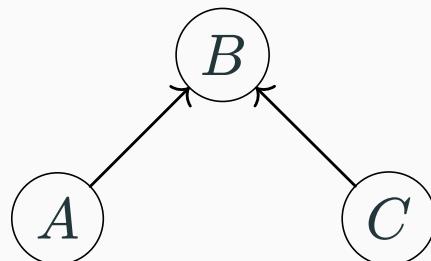
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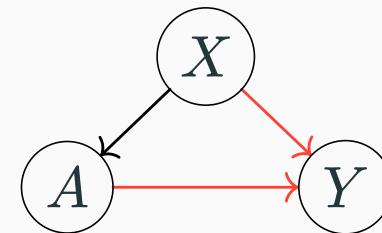
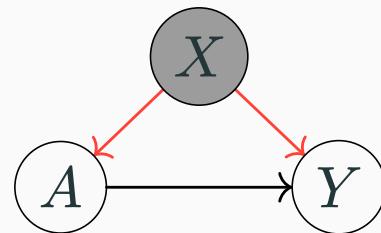
### Examples

- A is result from dice 1, C is results from dice 2, B is sum of dice 1 and dice 2.
- A is height, C is speed, B is whether an athlete plays in the NBA.

# Open and blocked paths by conditionning

A path is **blocked** (or d-separated) if:

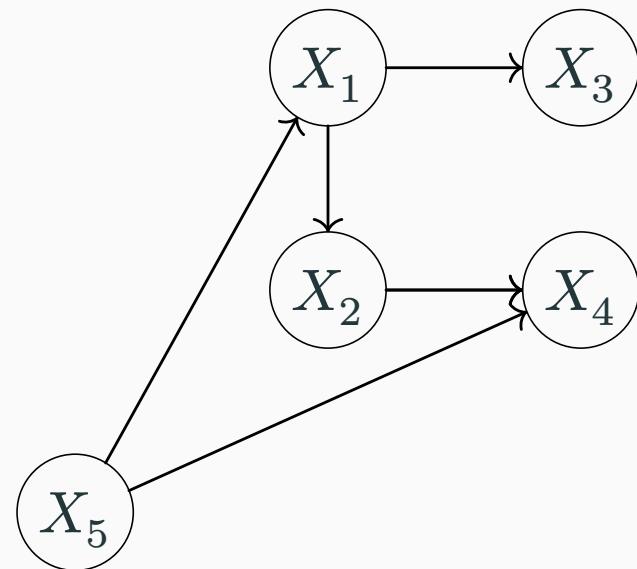
- the path contains a non-collider that has been conditioned on.
- or the path contains a collider that has not been conditioned on (and has no descendants that have been conditioned on).



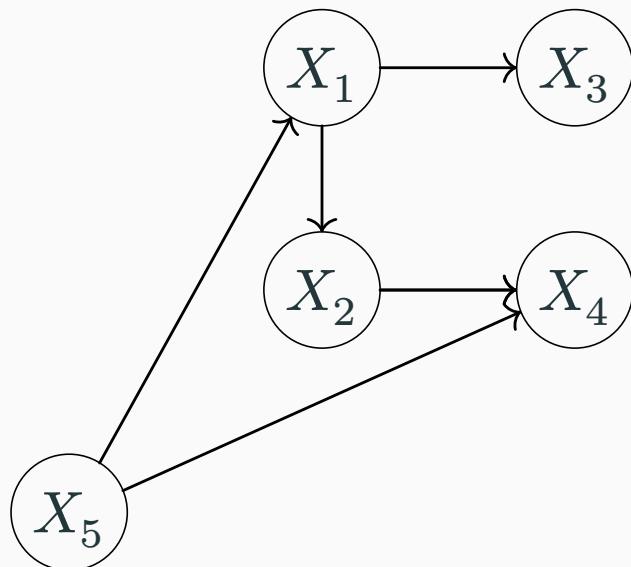
Conditioning on a variable:

- **Blocks** a path if that variable is **not a collider** on that path.
- **Opens** a path if that variable is a **collider** on that path.

Your turn: Paths from  $X_5$  to  $X_2$ ? Which of them are opened/blocked?



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$X_5 \rightarrow X_1 \rightarrow X_2$  (blocked by conditionning on  $X_1$ )

$X_5 \rightarrow X_1 \rightarrow X_2 \rightarrow X_4$  (opened by conditionning  $X_4$ )

# Backdoor paths: a special type of paths

## Def. Backdoor path

A Backdoor path from a variable A to another Y, is any non-causal path between A and Y that does not include descendants of A.

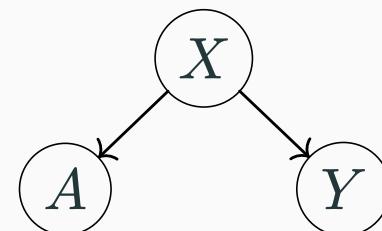
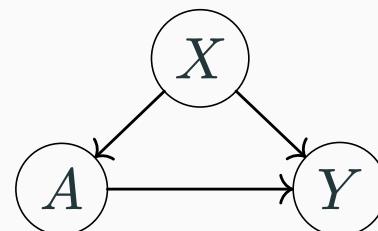
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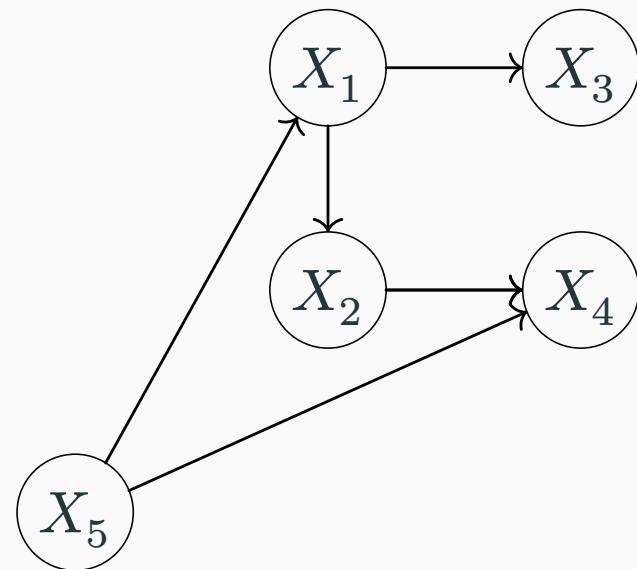
A Backdoor path from a variable A to another Y, is any non-causal path between A and Y that does not include descendants of A.

**Identifying backdoor paths:** Backdoor paths from A to Y are all those paths that remain between A and Y after removing all arrows coming out of A.

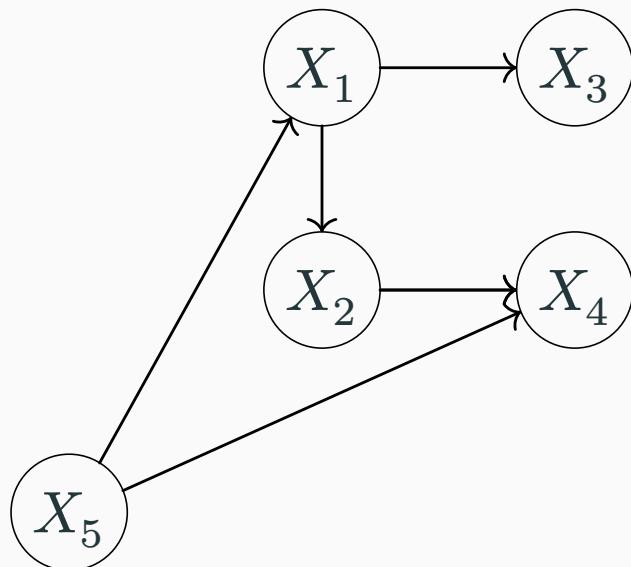
These paths are responsible for confounding bias: they imply association not causation.



Your turn: What are the backdoor paths from  $X_1$  to  $X_4$ ? from  $X_2$  tot  $X_4$ ?



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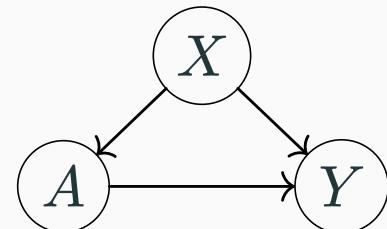


$$X_1 \leftarrow X_5 \rightarrow X_4$$

$$X_2 \leftarrow\leftarrow X_5 \rightarrow X_4$$

# Graphical identification

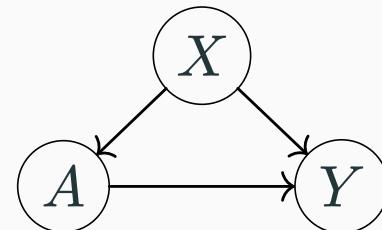
DAGs help us know whether observed covariates are enough to identify a treatment effect.



In other words, how can we make it so that there are no non-causal dependencies between treatment and outcome?

# Graphical identification

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In other words, how can we make it so that there are no non-causal dependencies between treatment and outcome?

Graphical identification (Pearl & others, 2000)

The effect of A on Y is identified if all backdoor paths from A to B are blocked, and no descendant of A is conditioned on.

# On which variables should we condition? General rules

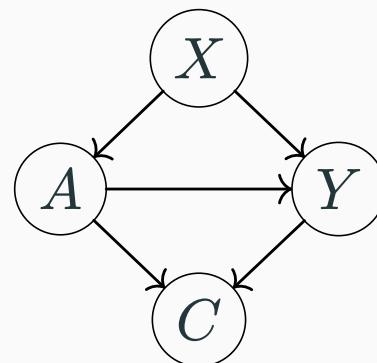
- Do not condition for variables on causal paths from treatment to outcome
- Condition on variables that block non-causal backdoor paths
- Don't condition on colliders! Eg. don't condition on post-treatment variables.

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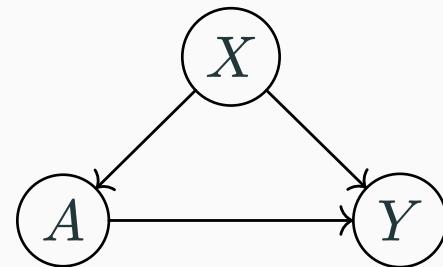
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- Don't condition on colliders! Eg. don't condition on post-treatment variables.

In the following example, to estimate the effect of T on Y, we should:

- Condition on X
- NOT condition on M because it is a descendant of T



# Famous examples of confounders

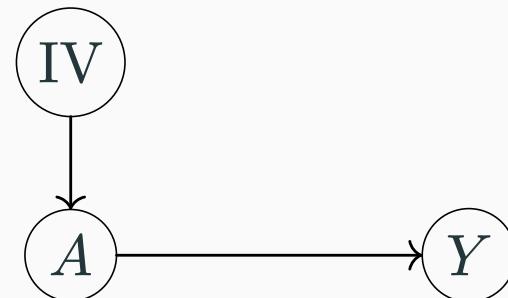


Effect of education on earnings

The family background can act as a confounder: Wealthier families may provide better education opportunities AND influence earnings independently of the education itself.

# Famous examples of instrumental variables

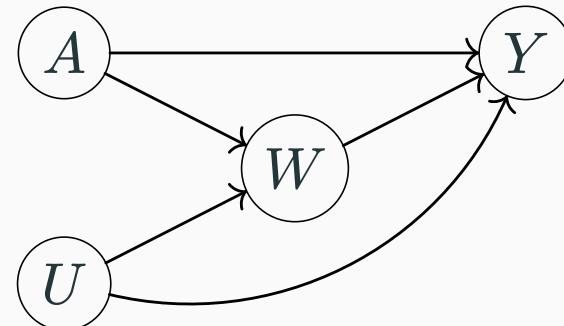
**Instrumental variable (IV): influences only the treatment.**



Effect of education on earnings (Angrist & Krueger, 1991)

Quarter of birth are randomly assigned but influence the lengths of schooling due to school entry laws.

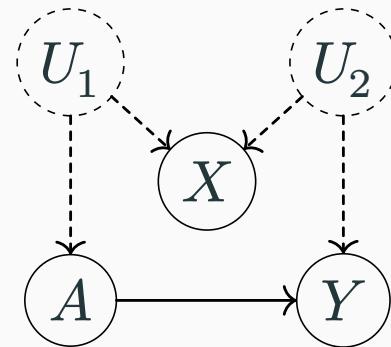
## Famous examples of colliders: consequence of both the treatment and the outcome



Effect of smoking on mortality (Hernández-Díaz et al., 2006)

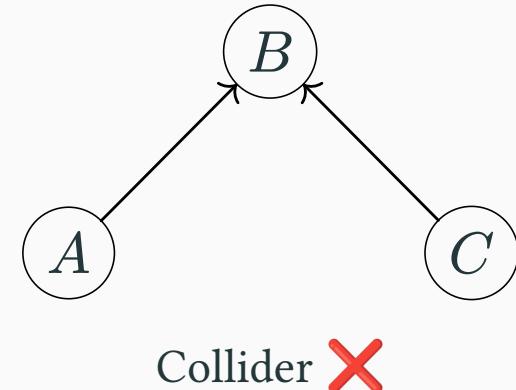
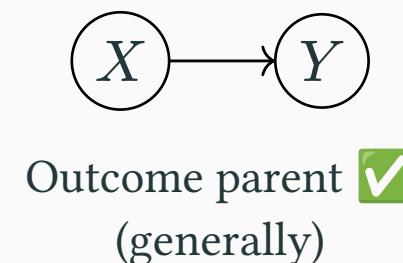
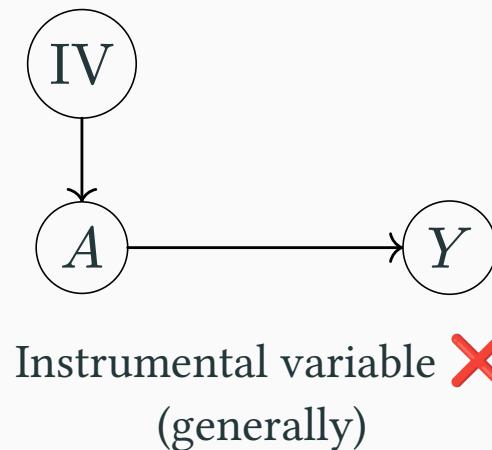
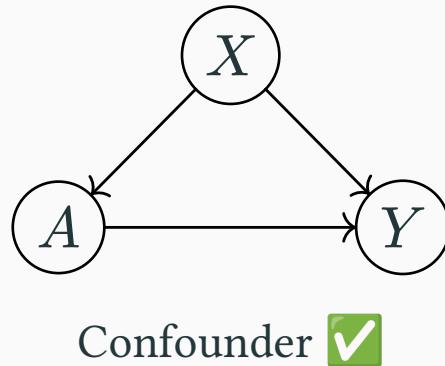
Birth weight is influenced by smoking and other factors. Conditioning on birth weight (a collider) creates a spurious negative correlation between smoking and other risk factors, leading to the paradoxical conclusion that smoking reduces infant mortality, even though it harms overall health.

## More colliders: M-bias

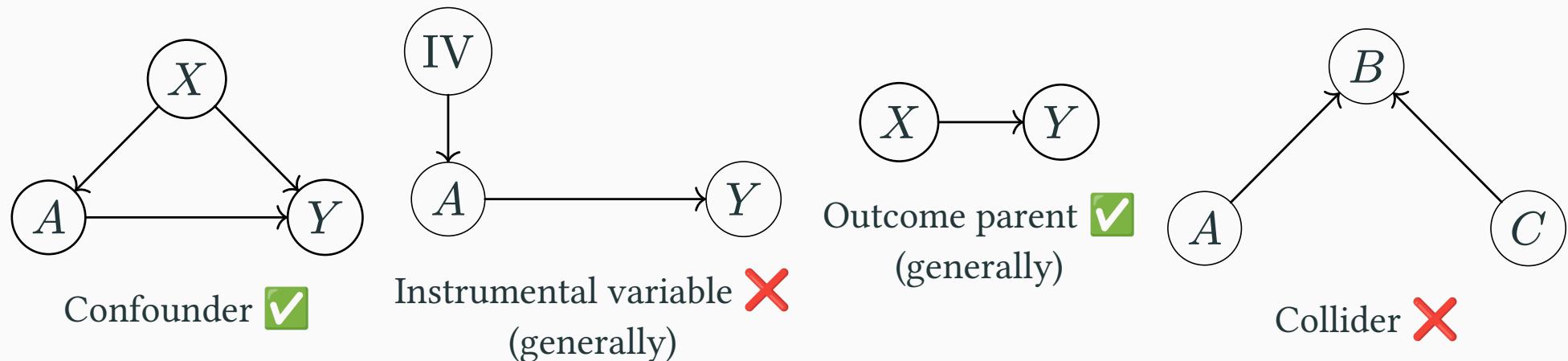


- Do not condition on any pre-exposure variable that you have at disposal!
- Should we condition on X in trying to estimate the effect of A on Y ?
- There is a backdoor path through two unobserved variables  $(U_1, U_2)$ . But it is blocked because X is a collider along that path.
- Conditioning on X opens up that path, inducing a non-causal association between T and Y.

# Which variable to include into your analysis?



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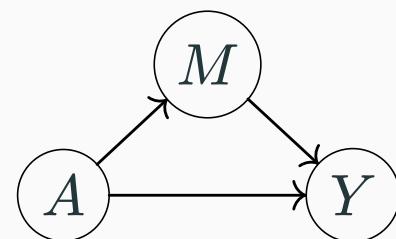


## High-level strategy

Control solely for pre-treatment variables that influences both the outcomes, the treatment or both.

# Special types of variables: mediators

A **mediator** blocks the path from the treatment to the outcome.



Here, two causal paths from A:

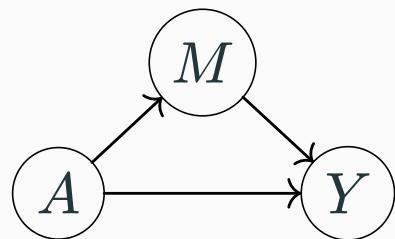
- $A \rightarrow Y$  - a “direct effect”
- $A \rightarrow M \rightarrow Y$  - an “indirect effect” through M

Effect of children poverty on economic outcomes (Bellani & Bia, 2019)

- Y is economic outcomes in adulthood, A is child poverty, M is education.
- What part of the effect of poverty on outcome is mediated by education?

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Here, two causal paths from A:

- $A \rightarrow Y$  - a “direct effect”
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- All causal paths from a treatment capture its overall treatment effect.
- The average treatment effect of T combines both the “direct effect” and the “indirect effect”.

# Take aways on DAGs

## On DAGs

- DAGs are a powerful tool to reason about causality
- Useful to identify the variables to condition on / to include into the analysis
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## On covariate selection: What is important part to insure validity?

- The covariate included (an appropriate DAG)?
- The design of the study?
- The causal estimator (IPW, G-formula, AIPW...)
- The statistical estimator (Linear regression, Logistic regression...)

# Illustration: Causal analysis of EHRs (Doutreligne et al., 2025)

**Data: electronic health records from an hospital in Boston**

**Population: patients with sepsis in the intense care unit**

**Intervention: combination of crystalloids and albumin for fluid resuscitation**

**Control: crystalloids only**

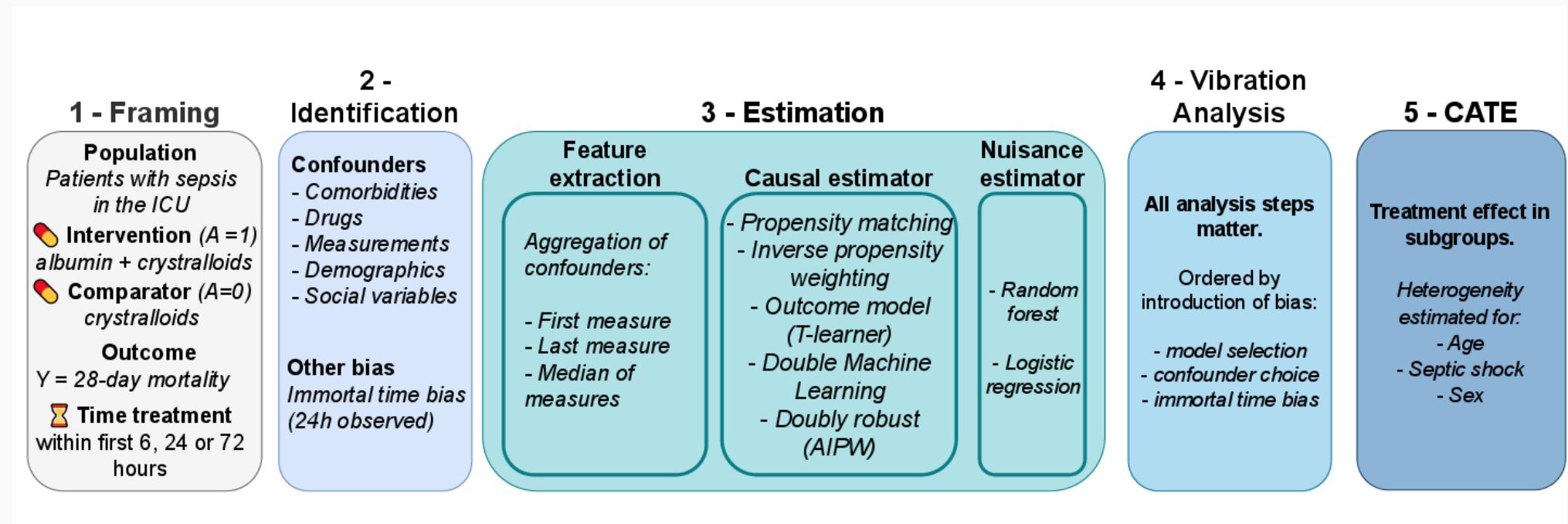
**Outcome: 28-day mortality**

**Time: Intervention within the first day**

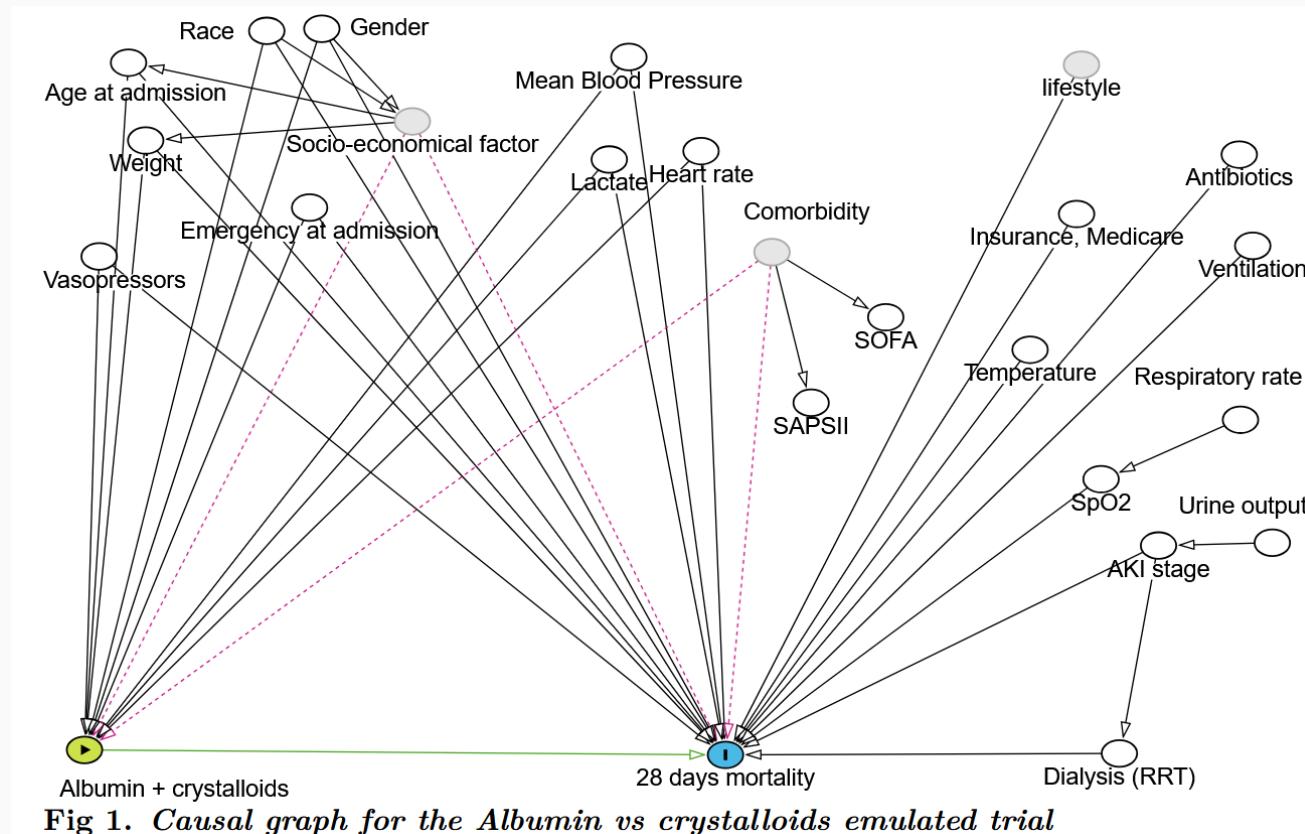
 Question already answer with RCTs: we have a gold standard for the treatment effect ( $\tau = 0$ )

# Studying the consequences of various design choices

## Full pipeline

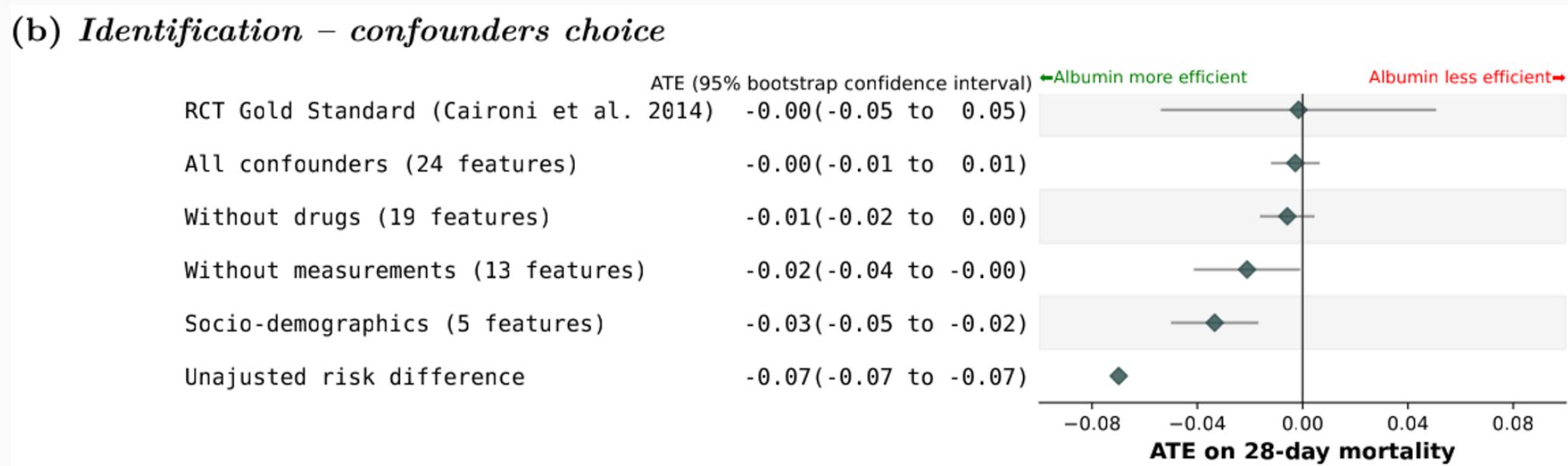


# Focus: studying the effect of incomplete DAGs



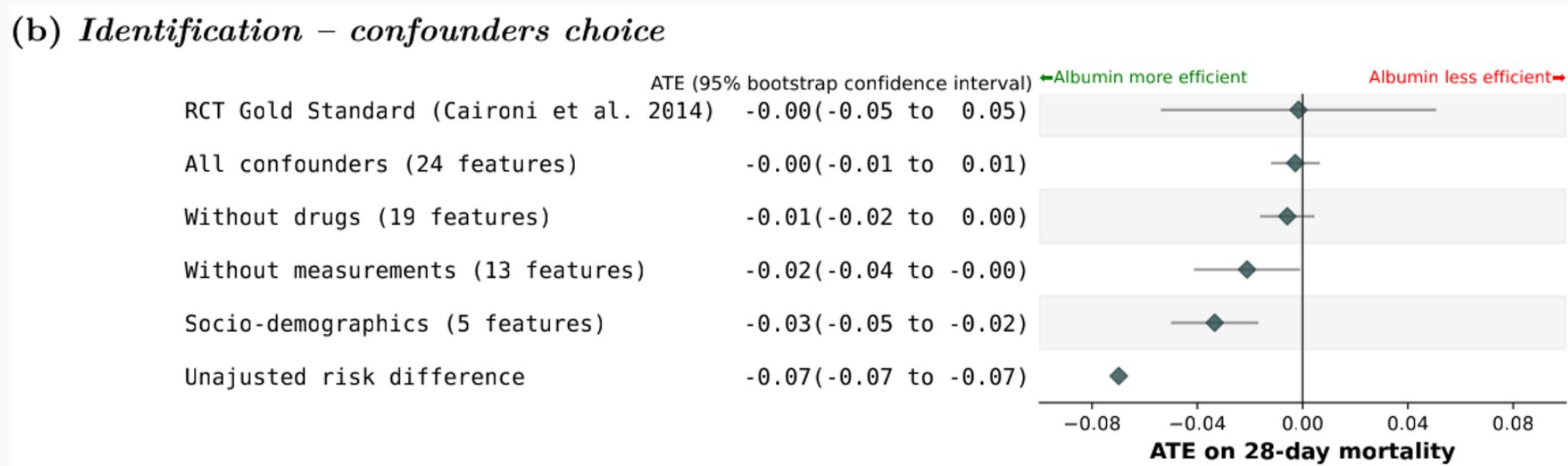
# Results of an increasingly complete confounder set

## (b) Identification – confounders choice



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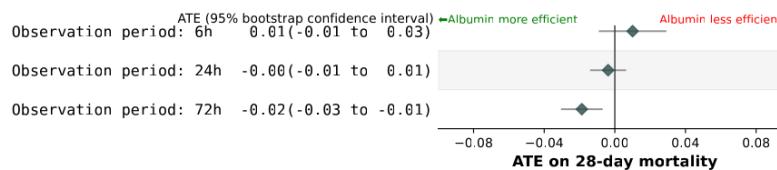


- Missing important confounders lead to bias estimates.
- Missing less important confounders lead to less precise estimates: a perfect DAG might not be needed.

# Full results of the sensitivity analysis

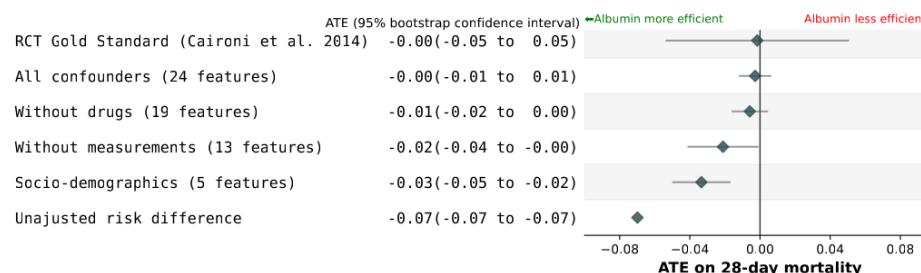
(a)

Framing – Immortal Time Bias



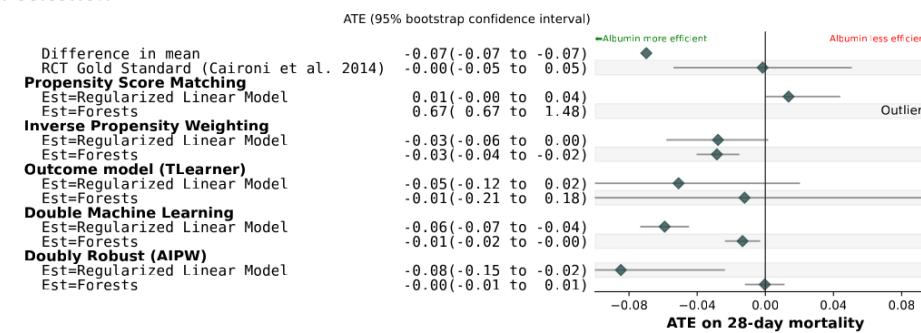
(b)

Identification – confounders choice



(c)

Model selection



$$Y(1), Y(0) \perp\!\!\!\perp A$$

---

Practical session

To your notebooks!



- url: [https://straymat.github.io/causal-ml-course/practical\\_sessions.html](https://straymat.github.io/causal-ml-course/practical_sessions.html)

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## *Bibliography*

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*Supplementary material*

# DAG: Effect modifier

*Effect modifier: influences the treatment effect on the outcome.*

