Machine Learning for econometrics

Event studies: Causal methods for pannel data

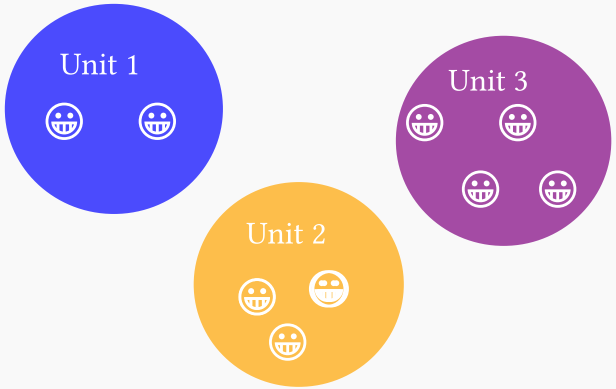
Authors

February, 11th, 2025

Motivation

Estimation of the effect of a treatment when data is:

Aggregated: country-level data such as employment rate, GDP...



Setup: event studies

Estimation of the effect of a treatment when data is:

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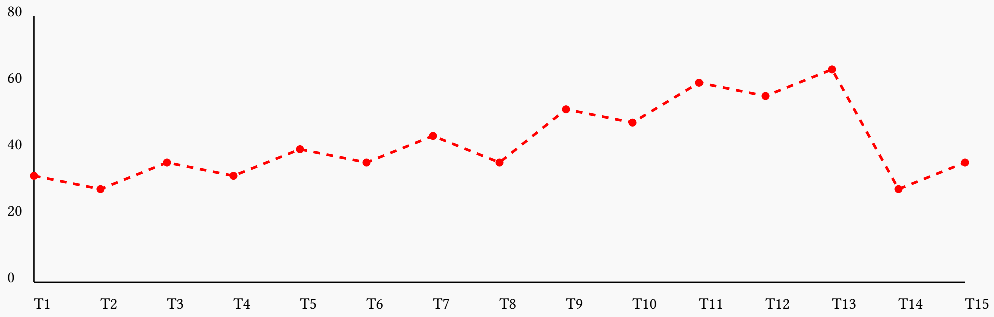
Longitudinal: multiple time periods (or repeated cross-sections)…



Estimation of the effect of a treatment when data is:

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With multiple aggregated units: countries, firms, geographical regions…

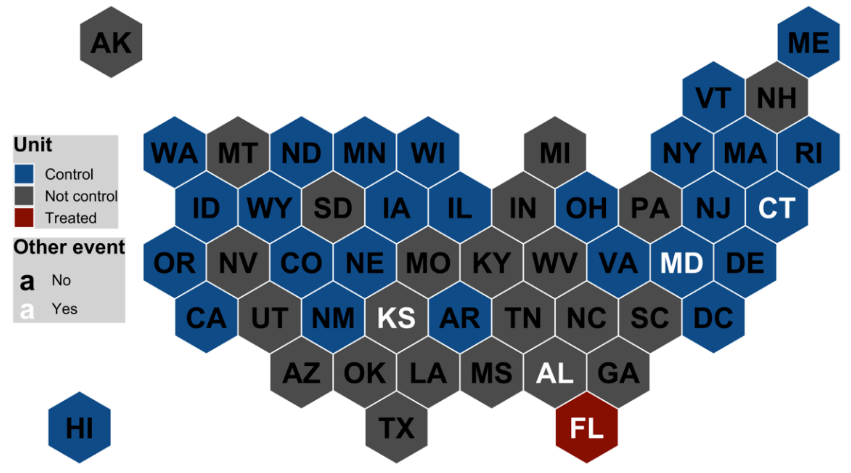


Figure from (Degli Esposti et al., 2020)

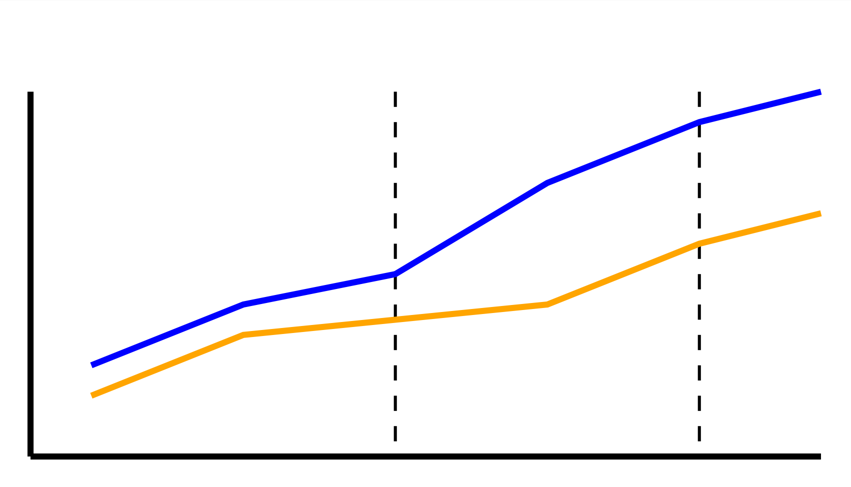
Estimation of the effect of a treatment when data is:

Aggregated: country-level data such as employment rate, GDP…

Longitudinal: multiple time periods (or repeated cross-sections)…

With multiple aggregated units: countries, firms, geographical regions…

Staggered adoption of the treatment: units adopt the policy/treatment at different times…



Setup: event studies

Estimation of the effect of a treatment when data is:

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Longitudinal: multiple time periods (or repeated cross-sections)...

With multiple aggregated units: countries, firms, geographical regions...

Staggered adoption of the treatment: units adopt the policy/treatment at different times...

This setup is known as

Panel data, event studies, longitudinal data, time-series data.

Examples of event studies

Archetypal questions

- Did the new marketing campaign had an effect on the sales of a product?
- Did the new tax policy had an effect on the consumption of a specific product?
- Did the guidelines on the prescription of a specific drug had an effect on the practices?

Examples of event studies

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Modern examples

- What is the effect of the extension of Medicaid on mortality? (Miller et al., 2019)
- What is the effect of Europe's protected area policies (*Natura 2000*) on vegetation cover and on economic activity? (Grupp et al., 2023)
- Which policies achieved major carbon emission reductions? (Stechemesser et al., 2024)

Setup: event studies are quasi-experiment

Quasi-experiment

A situation where the treatment is not randomly assigned by the researcher but by nature or society.
It should introduce *some* randomness in the treatment assignment: enforcing treatment exogeneity, ie. ignorability (ie. unconfoundedness).

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Other quasi-experiment designs

- Instrumental variables: a variable that is correlated with the treatment but not with the outcome.
- Regression discontinuity design: the treatment is assigned based on a threshold of a continuous variable.

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Reminder on difference-in-differences

History

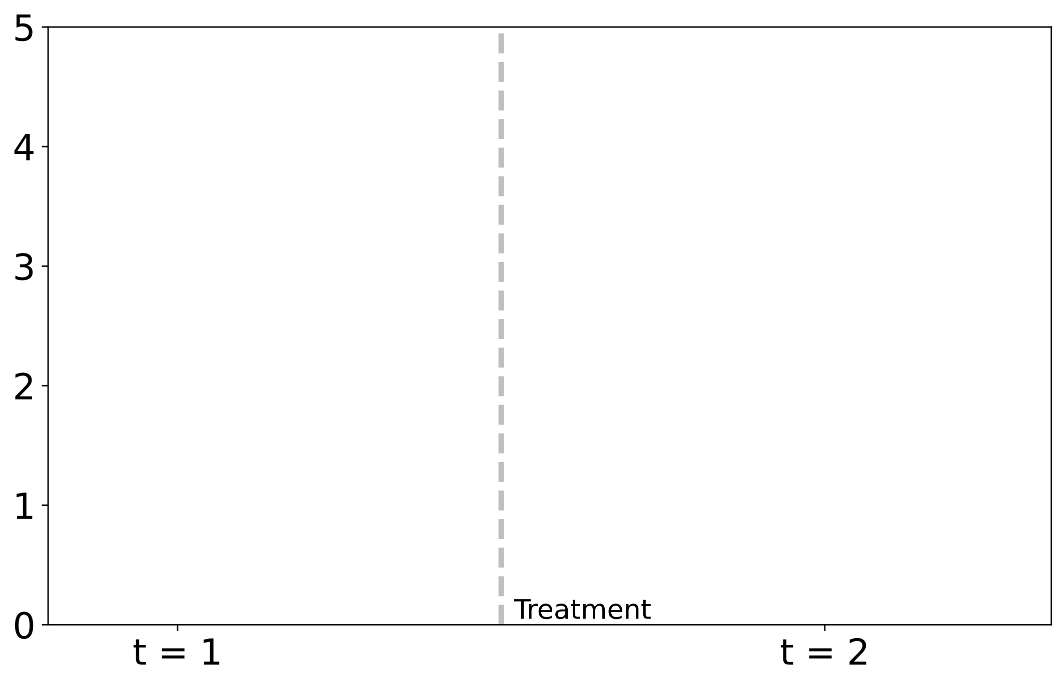
- First documented example (though not formalized): John Snow showing how cholera spread through the water in London (Snow, 1855)[1]
- Modern usage introduced formally by (Ashenfelter, 1978), applied to labor economics

Idea

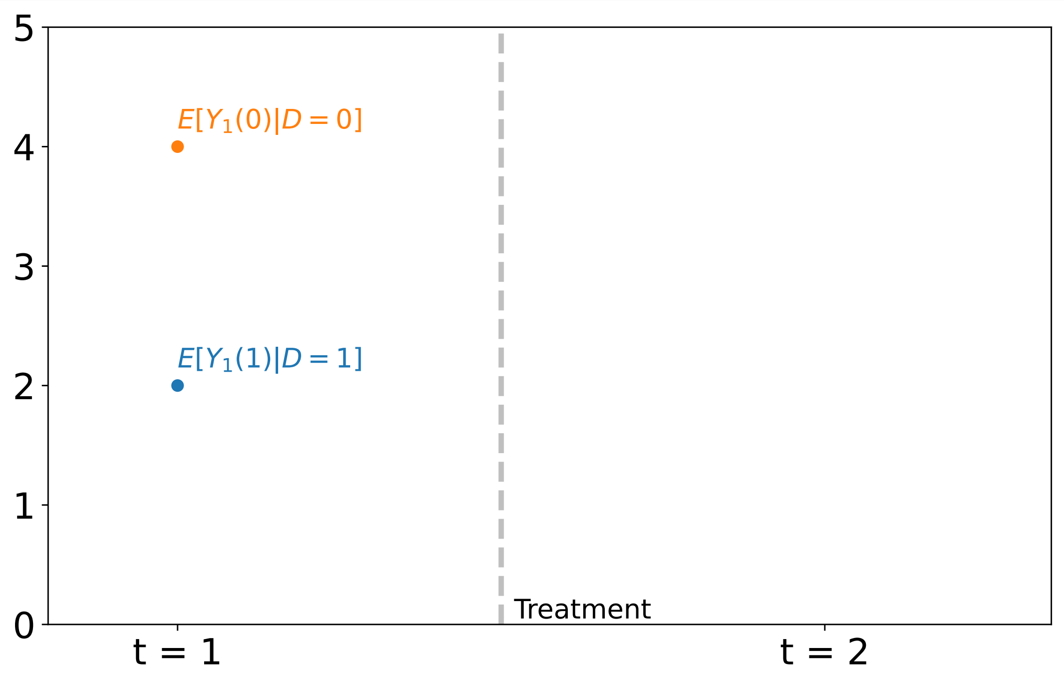
- Contrast the temporal effect of the treated unit with the control unit temporal effect:
- The difference between the two differences is the treatment effect

[1] Good description: https://mixtape.scunning.com/09-difference_in_differences#john-snows-cholera-hypothesis

Two period of times: t=1, t=2

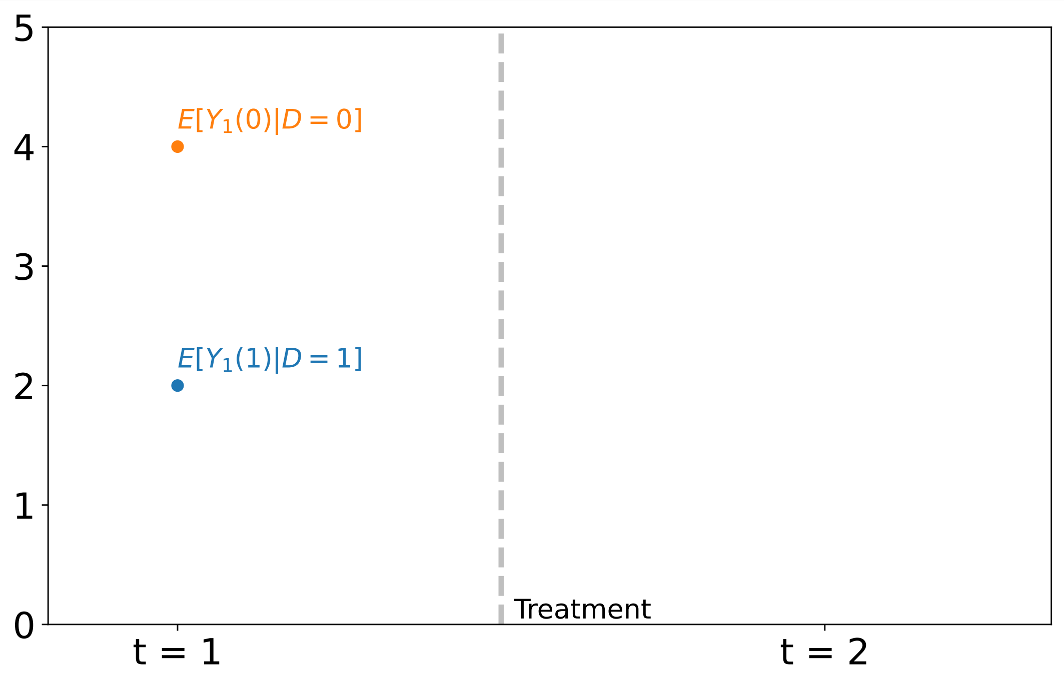


Potential outcomes: $Y_t(d)$ where $d = \{0, 1\}$ is the treatment at period 2



Difference-in-differences framework

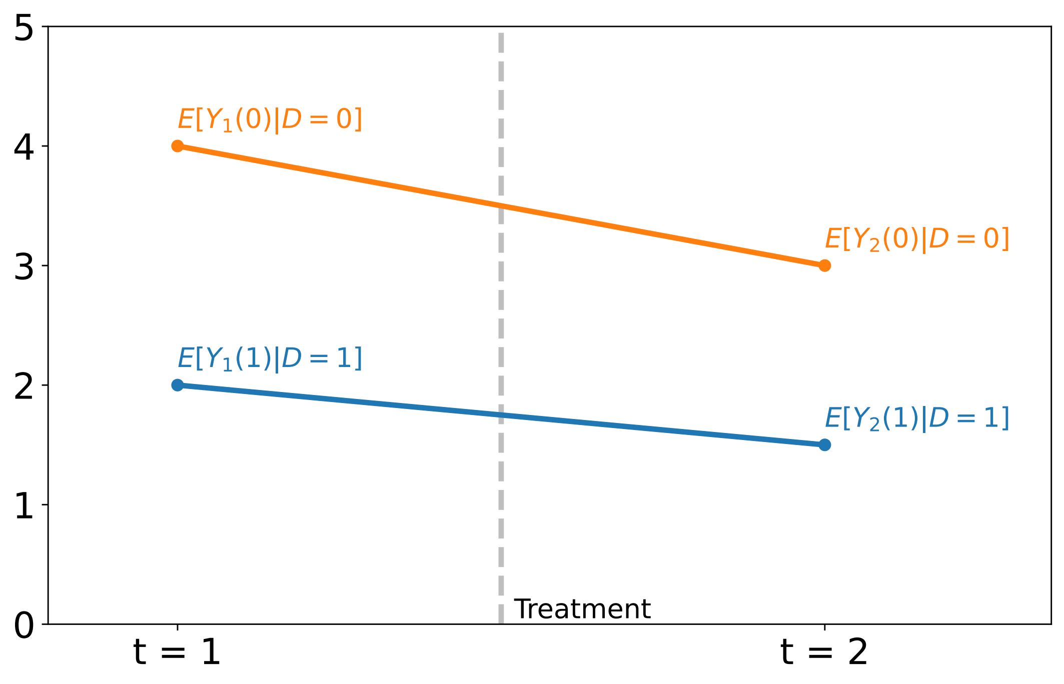
Potential outcomes: $Y_t(d)$ **where** $d = \{0, 1\}$ **is the treatment at period 2**



⚠️ $\mathbb{E}[Y_1(1)] = \mathbb{E}[Y_1(1) \ |D = 1]\mathbb{P}(D = 1) + \mathbb{E}[Y_1(1) \ |D = 0]\mathbb{P}(D = 0)$
but we only observe $\mathbb{E}[Y_1(1) \ |D = 1]$

Difference-in-differences framework

Our target is the average treatment effect on the treated (ATT)



$$\tau_{\text{ATT}} = \mathbb{E}[Y_2(1)|\ D = 1] - \mathbb{E}[Y_2(0)|\ D = 1]$$

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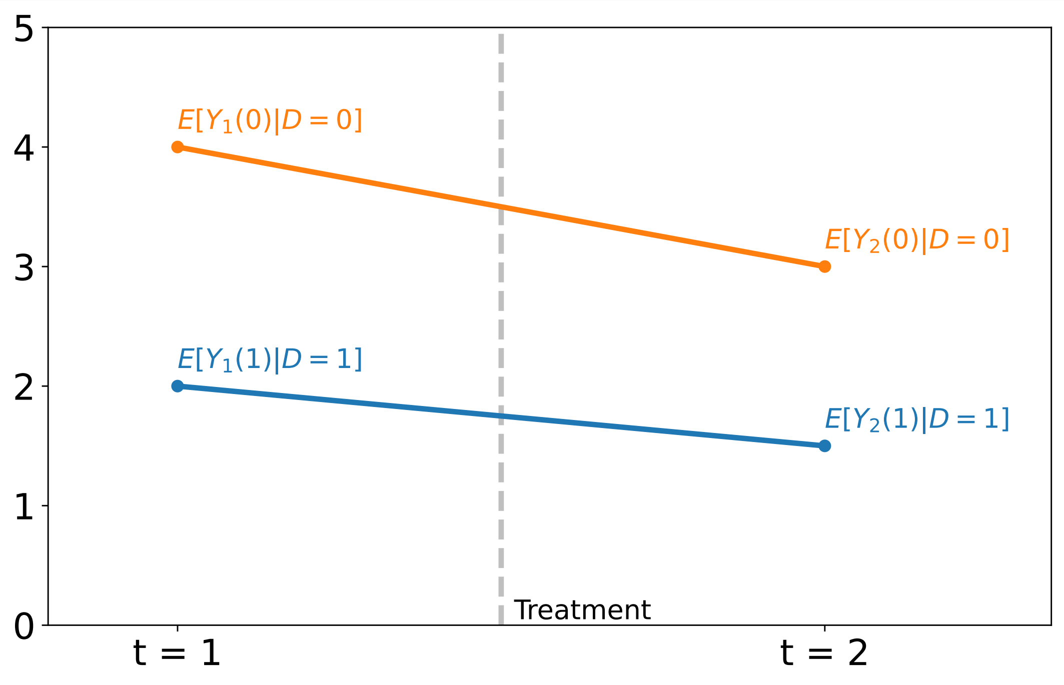
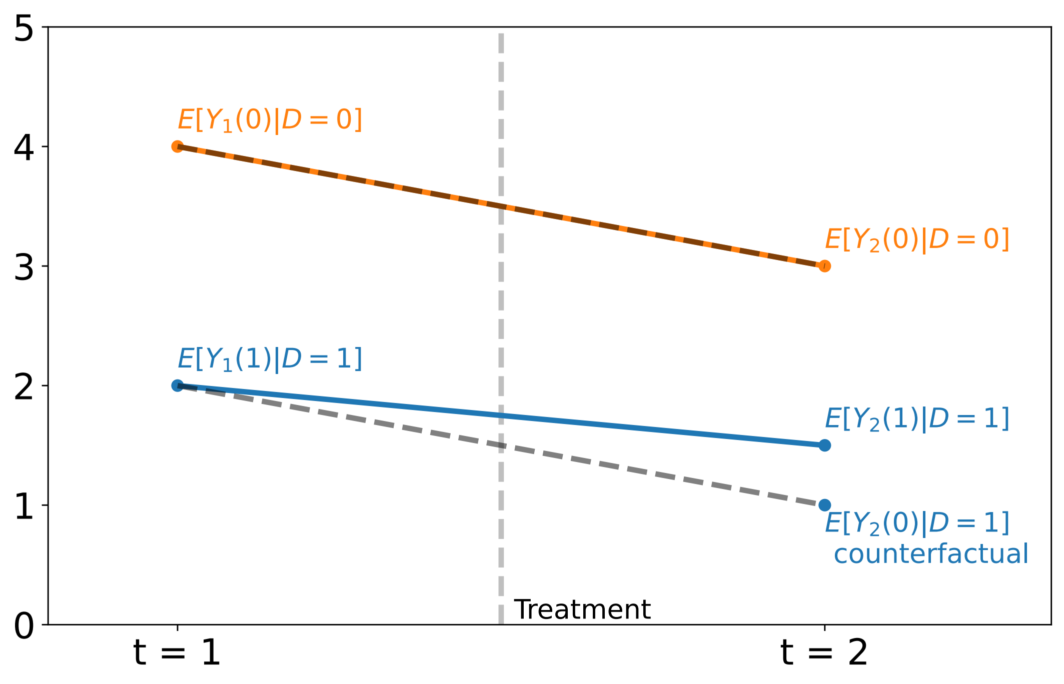


$$\tau_{\mathrm{ATT}} = \underbrace{[Y_2(1)|\ D=1]}_{\text{treated outcome for t=2}} - \underbrace{\mathbb{E}[Y_2(0)|\ D=1]}_{\text{unobserved counterfactual}}$$

Difference-in-differences framework

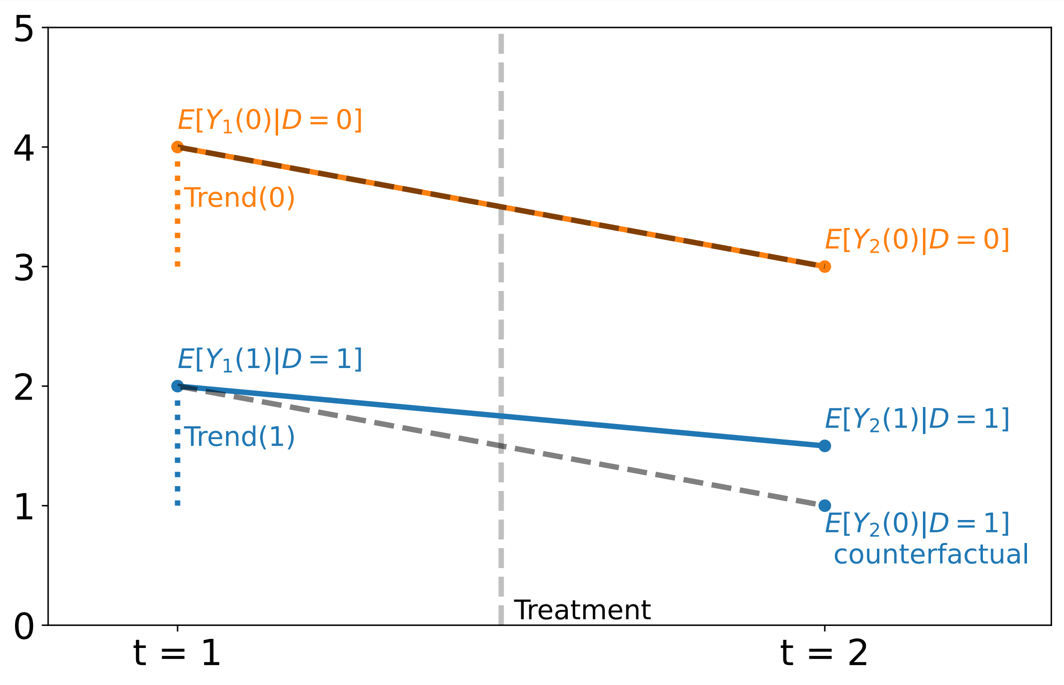
First assumption, parallel trends

$$\mathbb{E}[Y_2(0) - Y_1(0) \mid D = 1] = \mathbb{E}[Y_2(0) - Y_1(0) \mid D = 0]$$



First assumption, parallel trends

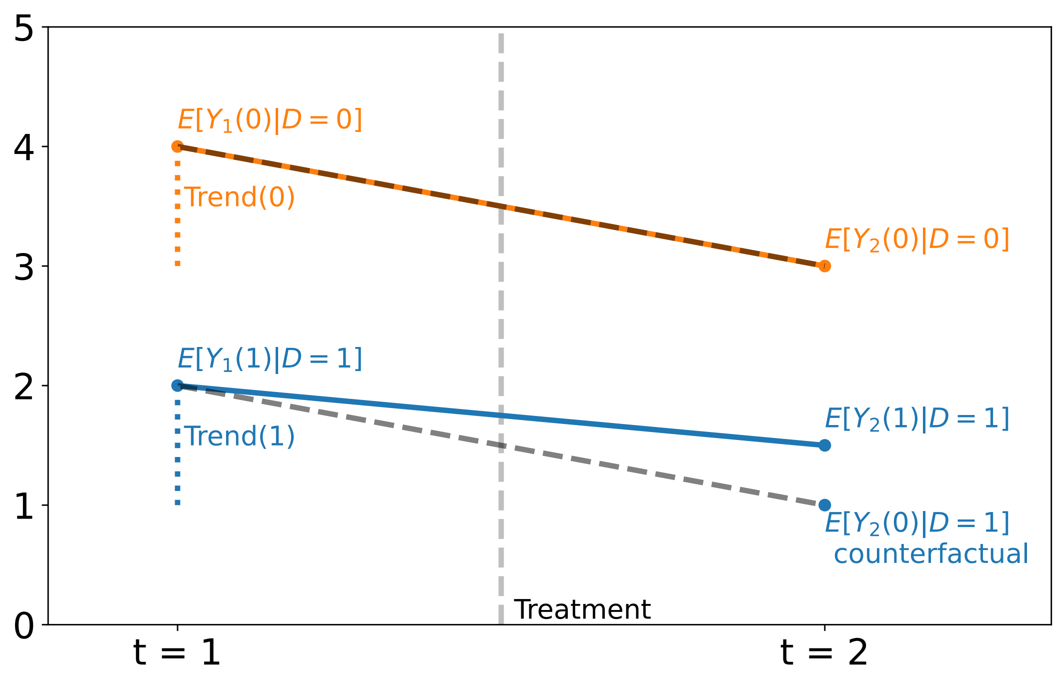
$$\underbrace{[Y_2(0) - Y_1(0) \mid D = 1]}_{\color{blue}{\text{Trend}(1)}} = \underbrace{\mathbb{E}[Y_2(0) - Y_1(0) \mid D = 0]}_{\color{orange}{\text{Trend}(0)}}$$



Difference-in-differences framework

First assumption, parallel trends

$$\mathbb{E}[Y_2(0) \mid D = 1] = \mathbb{E}[Y_1(0) \mid D = 1] + \mathbb{E}[Y_2(0) - Y_1(0) \mid D = 0]$$



Difference-in-differences framework

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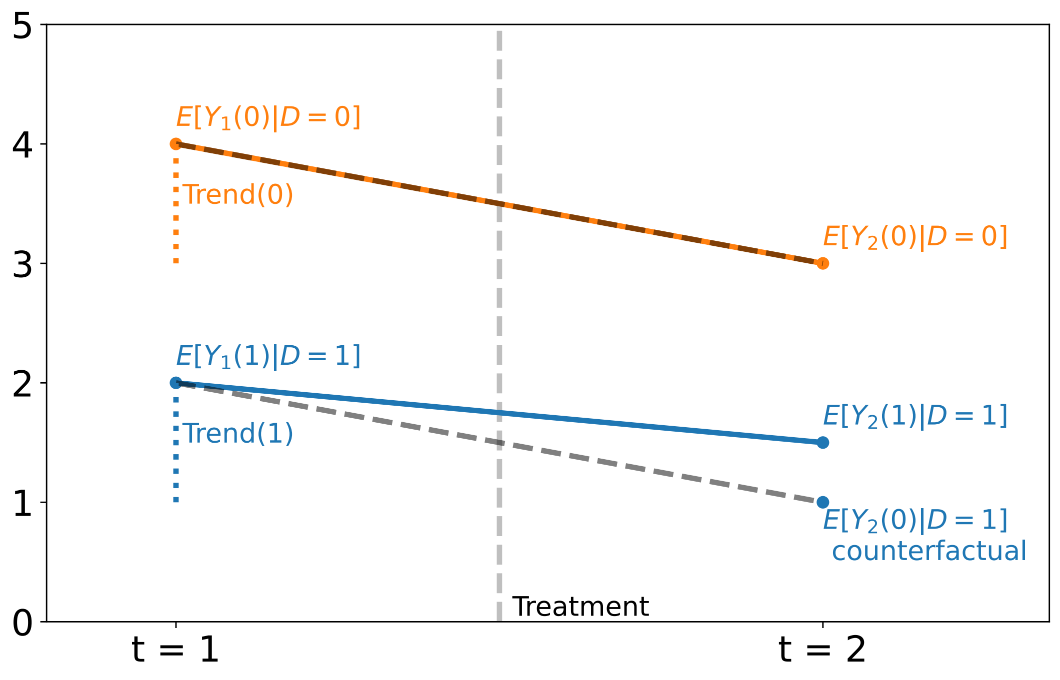
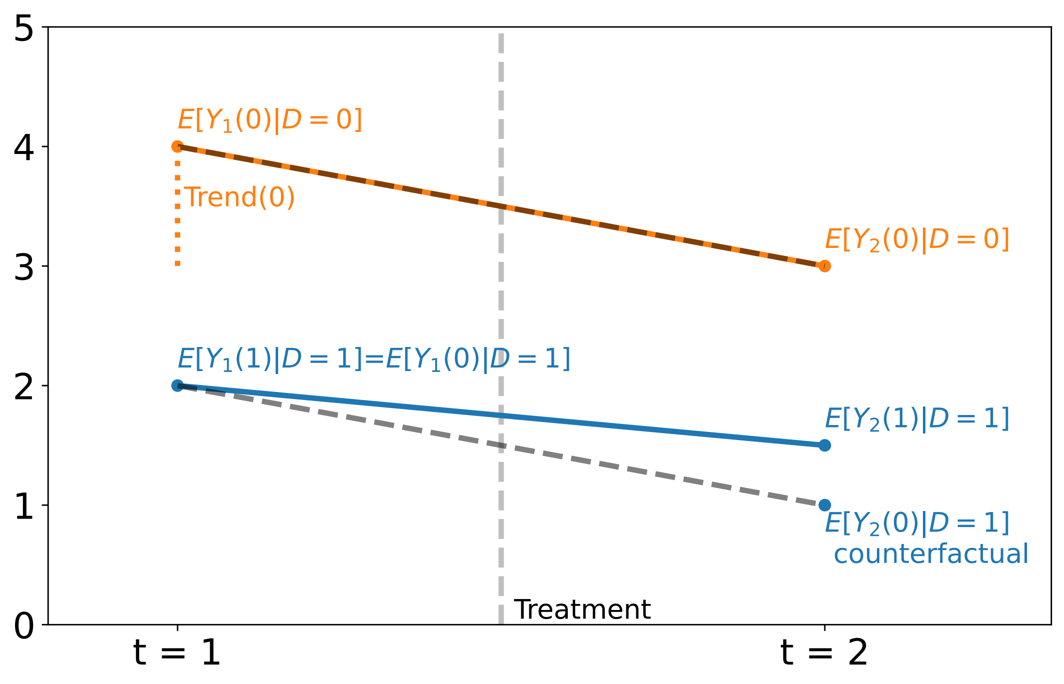
$$\mathbb{E}[Y_2(0) \mid D=1] = \underbrace{[Y_1(0) \mid D=1]}_{\text{unobserved counterfactual}} + \mathbb{E}[Y_2(0) - Y_1(0) \mid D=0]$$



Second assumption, no anticipation of the treatment

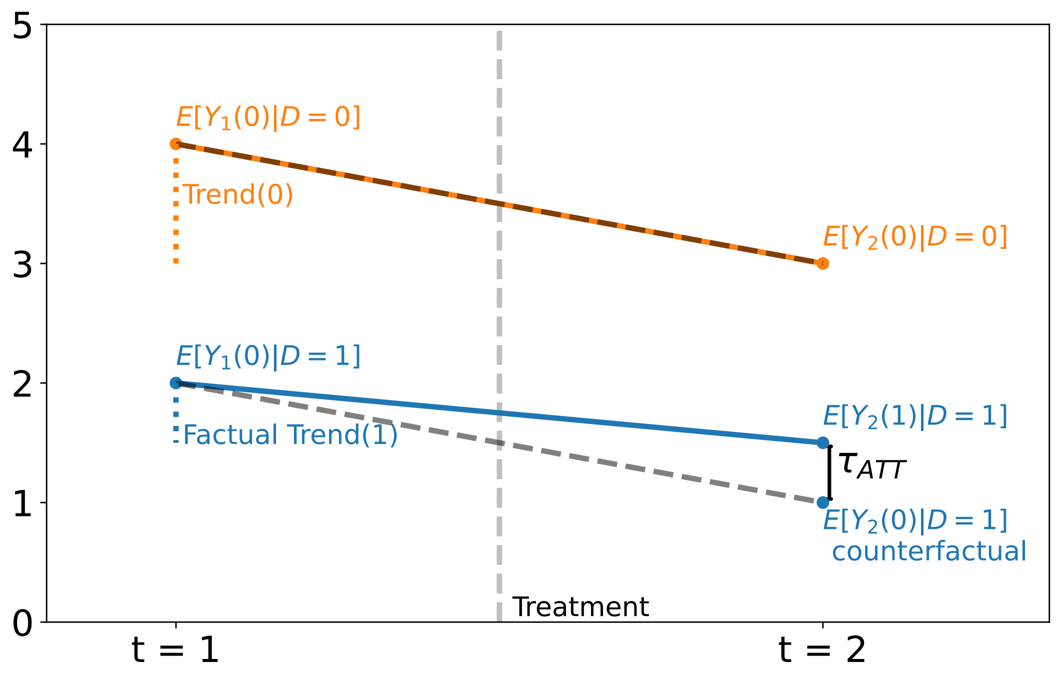
$$E[Y_1(1)|D=1] = E[Y_1(0)|D=1]$$



Difference-in-differences framework: identification of ATT

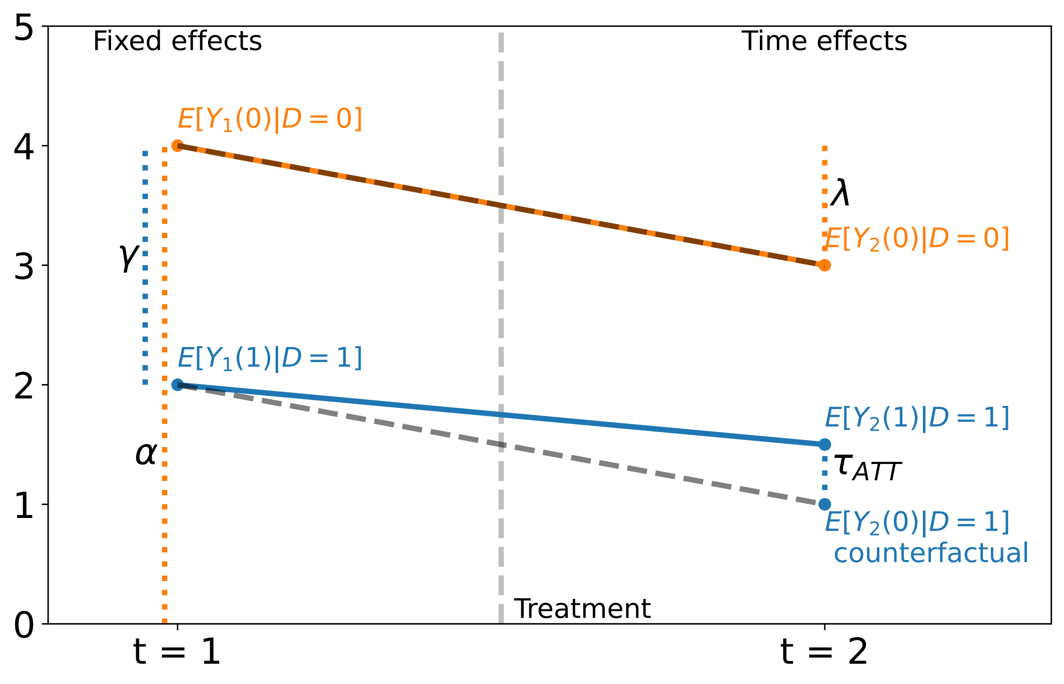
$$\tau_{\text{ATT}} = \mathbb{E}[Y_2(1)|\ D=1] - \mathbb{E}[Y_2(0)|\ D=1]$$

$$= \underbrace{\mathbb{E}[Y_2(1)|\ D=1] - \mathbb{E}[Y_1(0)|D=1]}_{\text{Factual Trend}(1)} - \underbrace{\mathbb{E}[Y_2(0)|D=0] - \mathbb{E}[Y_1(0)|D=0]}_{\text{Trend}(0)}$$



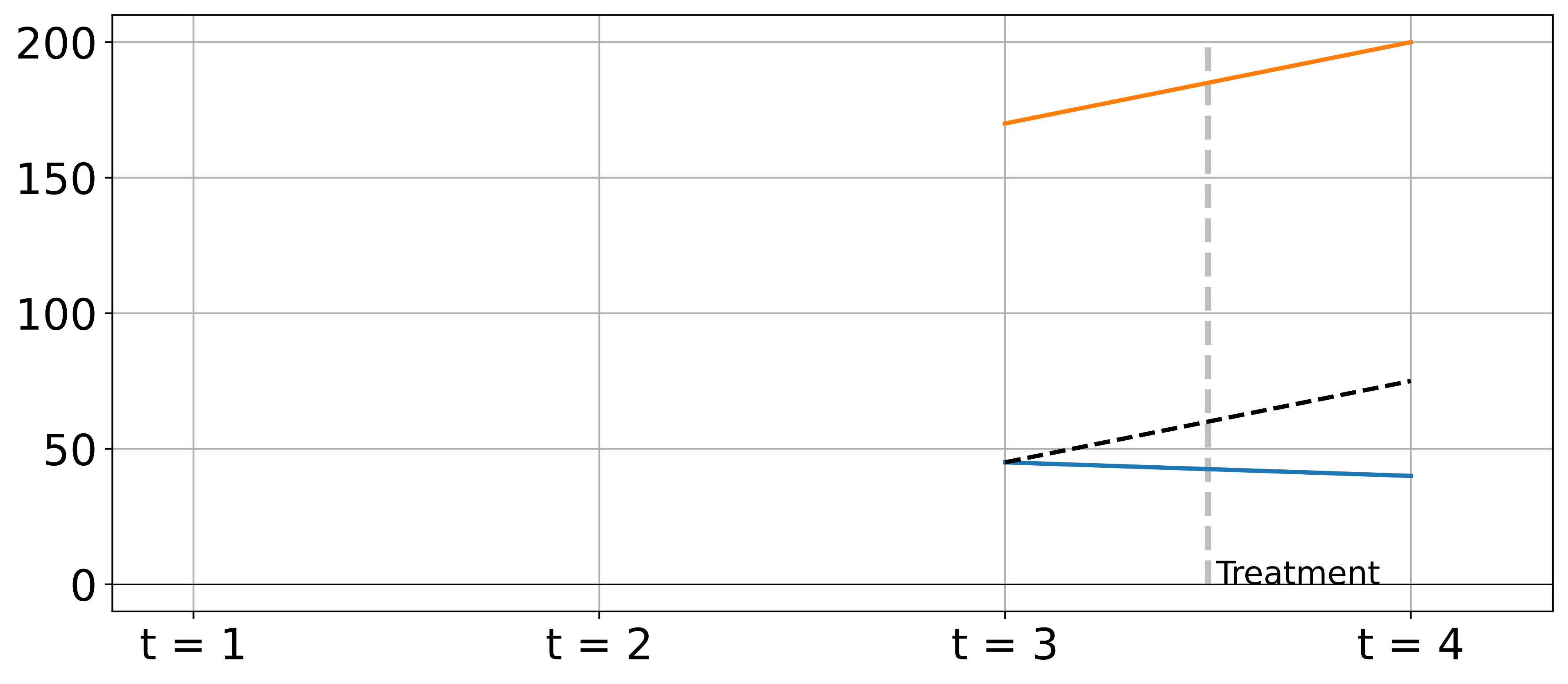
Estimation: link with two way fixed effect (TWFE)

$$Y = \alpha + \gamma D + \lambda \mathbb{1}(t=2) + \tau_{\text{ATT}} D \mathbb{1}(t=2)$$



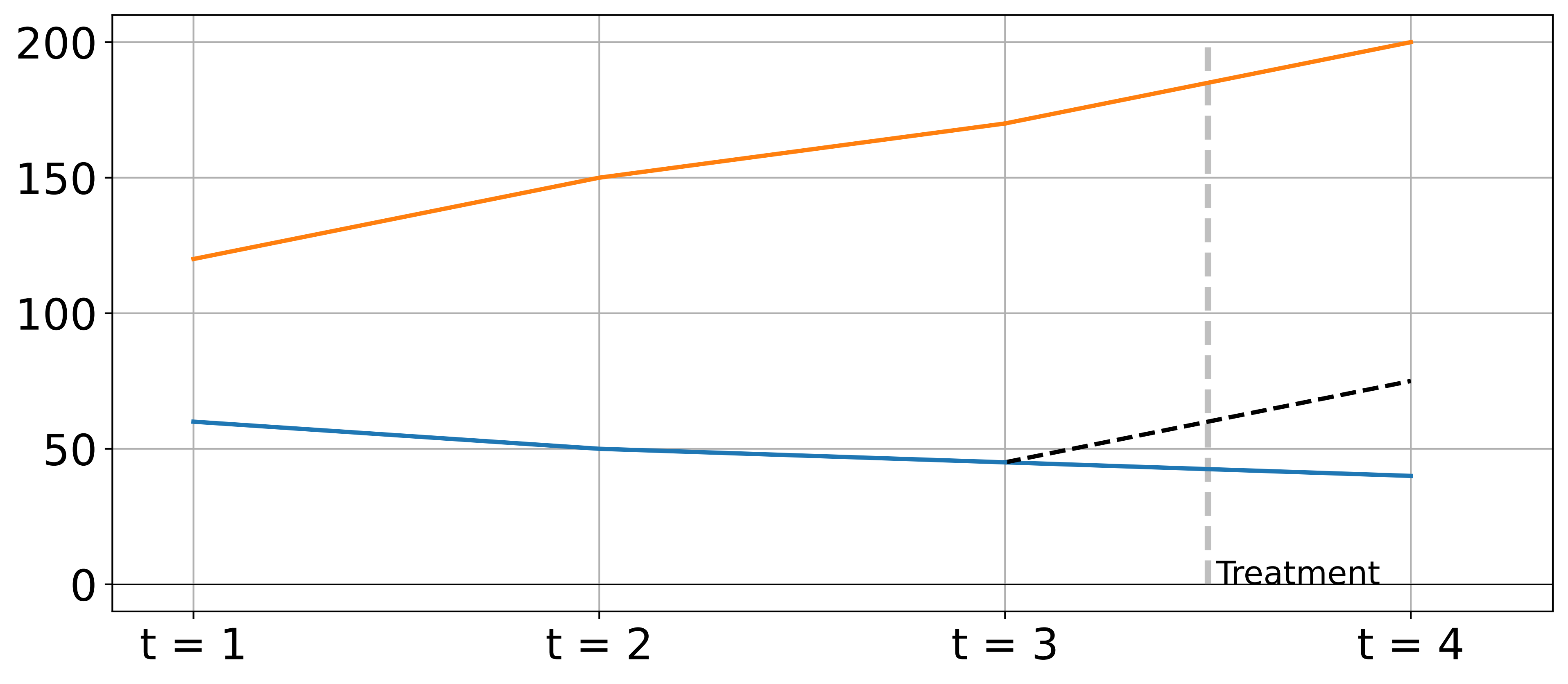
⚠️ Mechanic link: works only under parallel trends and no anticipation assumptions.

Seems like the treatment decreases the outcome!



Failure of the parallel trend assumption

Oups...



DID estimator for more than two time units

Target estimand: sample average treatment effect on the treated (SATT)

$$\tau_{\text{SATT}} = \frac{1}{|\{i:D_i=1\}|} \sum_{i:D_i=1} \frac{1}{T-H} \sum_{t=H+1}^{T} Y_{it}(1) - Y_{it}(0)$$

DID estimator

$$\widehat{\tau_{\text{DID}}} = \frac{1}{|\{i:D_i=1\}|} \sum_{i:D_i=1} \left[\frac{1}{T-H} \sum_{t=H+1}^{T} Y_{it} - \frac{1}{H} \sum_{t=1}^{H} Y_{it}\right] - \frac{1}{|\{i:D_i=0\}|} \sum_{i:D_i=0} \left[\frac{1}{T-H} \sum_{t=H+1}^{T} Y_{it} - \frac{1}{H} \sum_{t=1}^{H} Y_{it}\right]$$

Assumption

No anticipation of the treatment: $Y_{it}(0) = Y_{it}(1) \forall t = 1, ..., H$.

Parallel trend: $\mathbb{E}[Y_{it}(0, \infty) - Y_{i1}(0, \infty)] = \beta_t, t = 2, ..., T$.

See (Wager, 2024) for a clear proof of consistancy.

DID: Take-away

Pros

- Extremely common in economics and quite simple to implement.
- Can be extended to (Wager, 2024)
  - more than two time periods: exact same formulation
  - staggered adoption of the treatment: a bit more complex

Cons

- Very strong assumptions: parallel trends and no anticipation.
- Does not account for heterogeneity of treatment effect over time (De Chaisemartin & d'Haultfoeuille, 2020).

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Can we do better: ie. robust to the parallel trend assumption?

Synthetic controls

Synthetic controls

References

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Idea

Find a weighted average of controls that predicts well the treated unit outcome before treatment.

Example

What is the effect of tobacco tax on cigarettes sales? (Abadie et al., 2010)

Examples of application of synthetic controls to epidemiology

- What is the effect of taxes on sugar-based product consumption (Puig-Codina et al., 2021)

Synthetic control example: California's Proposition 99 (Abadie et al., 2010)

Context

1988: 25-cent tax per pack of cigarettes, ban of on cigarette vending machines in public areas accessible by juveniles, and a ban on the individual sale of single cigarettes.

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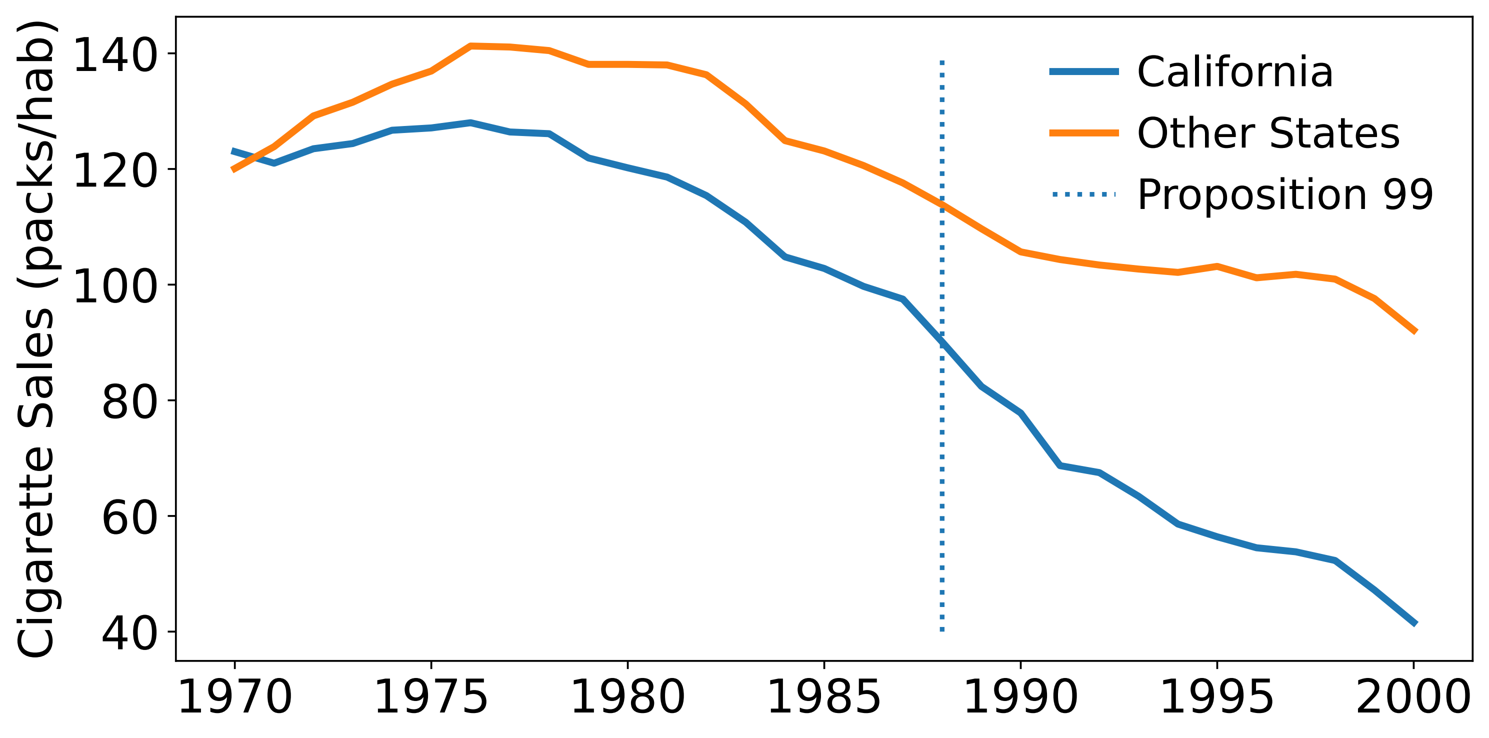
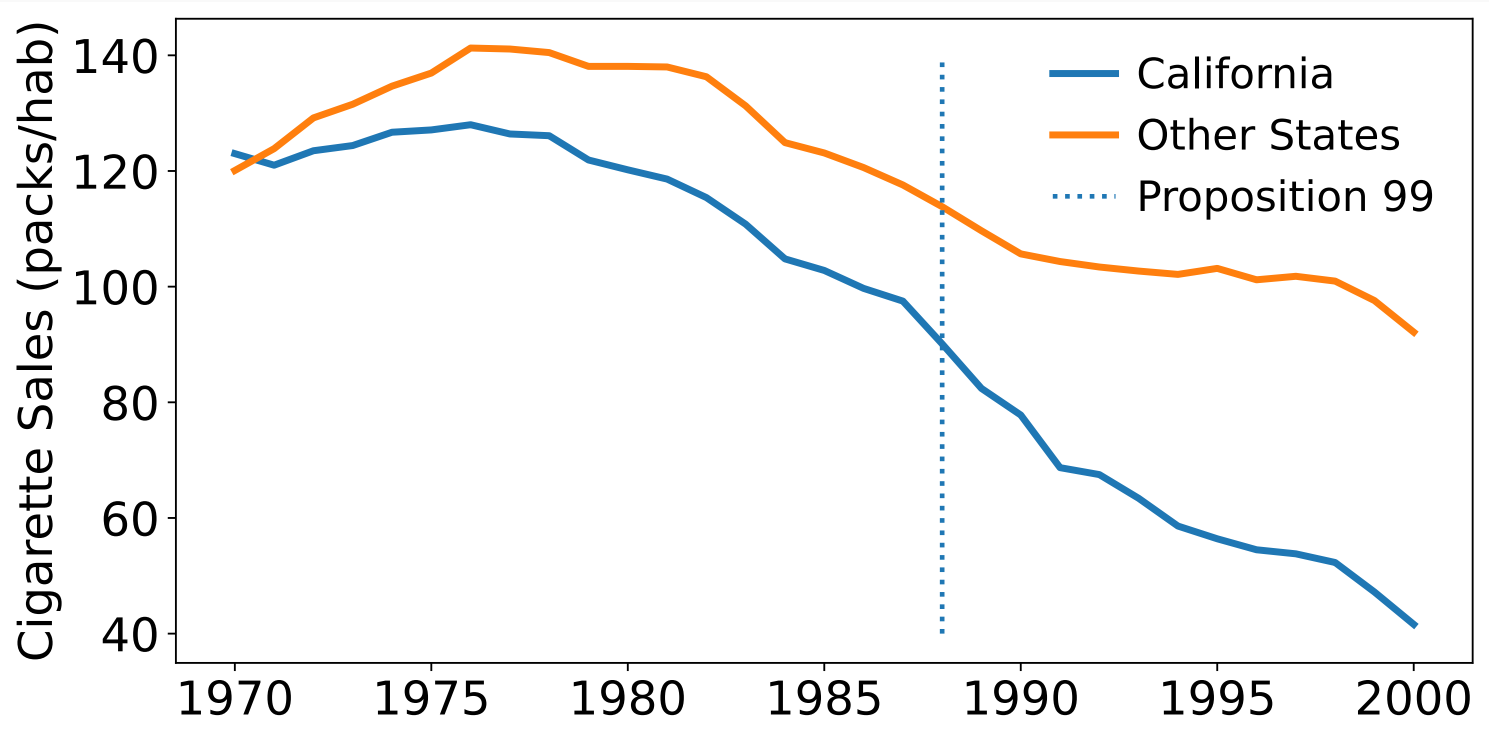
Covariates $X_{j,t}$: cigarette price, previous cigarette sales.

Synthetic control example: plot the data



Synthetic control example: plot the data



😯 Decrease in cigarette sales in California.

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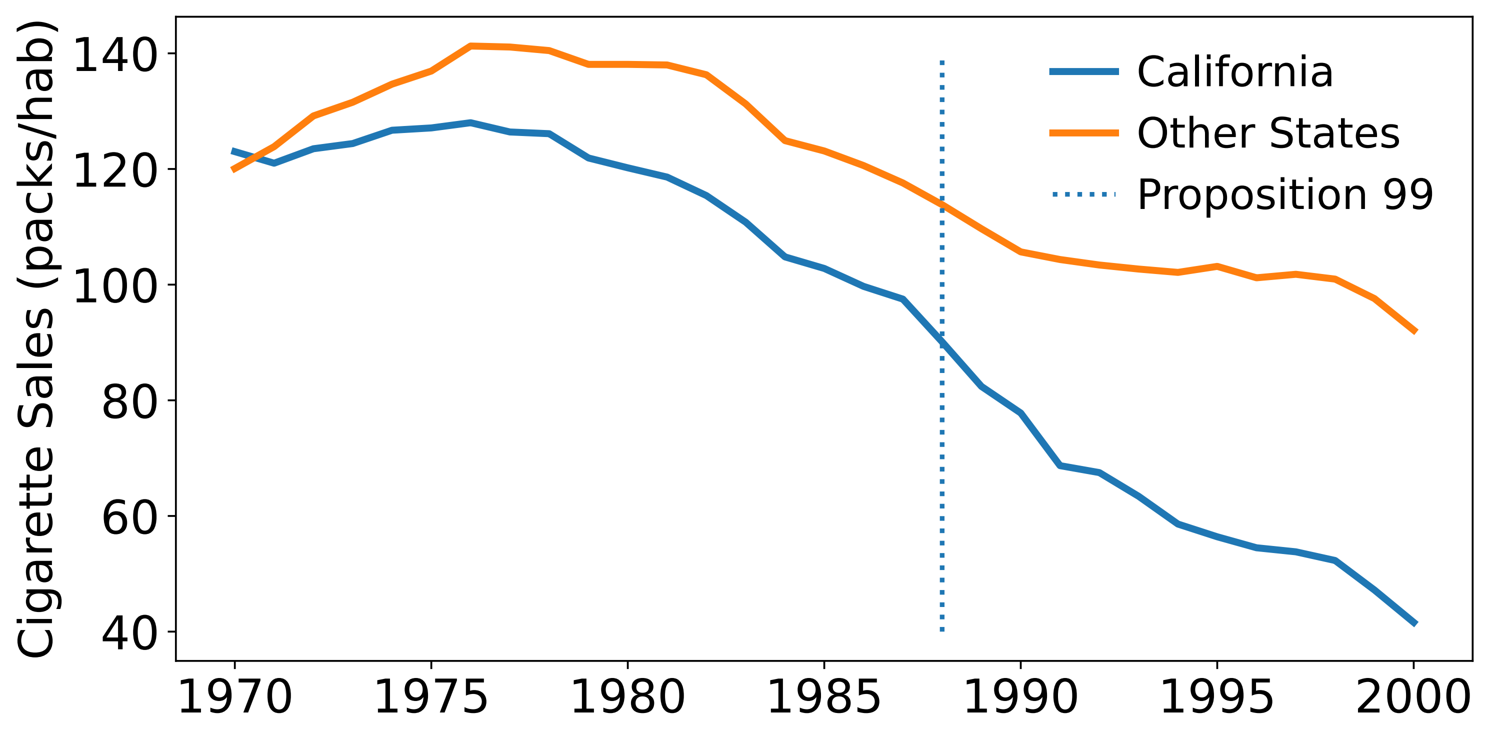

😯 Decrease in cigarette sales in California.

🤔 Decrease began before the treatment and occured also for other states.

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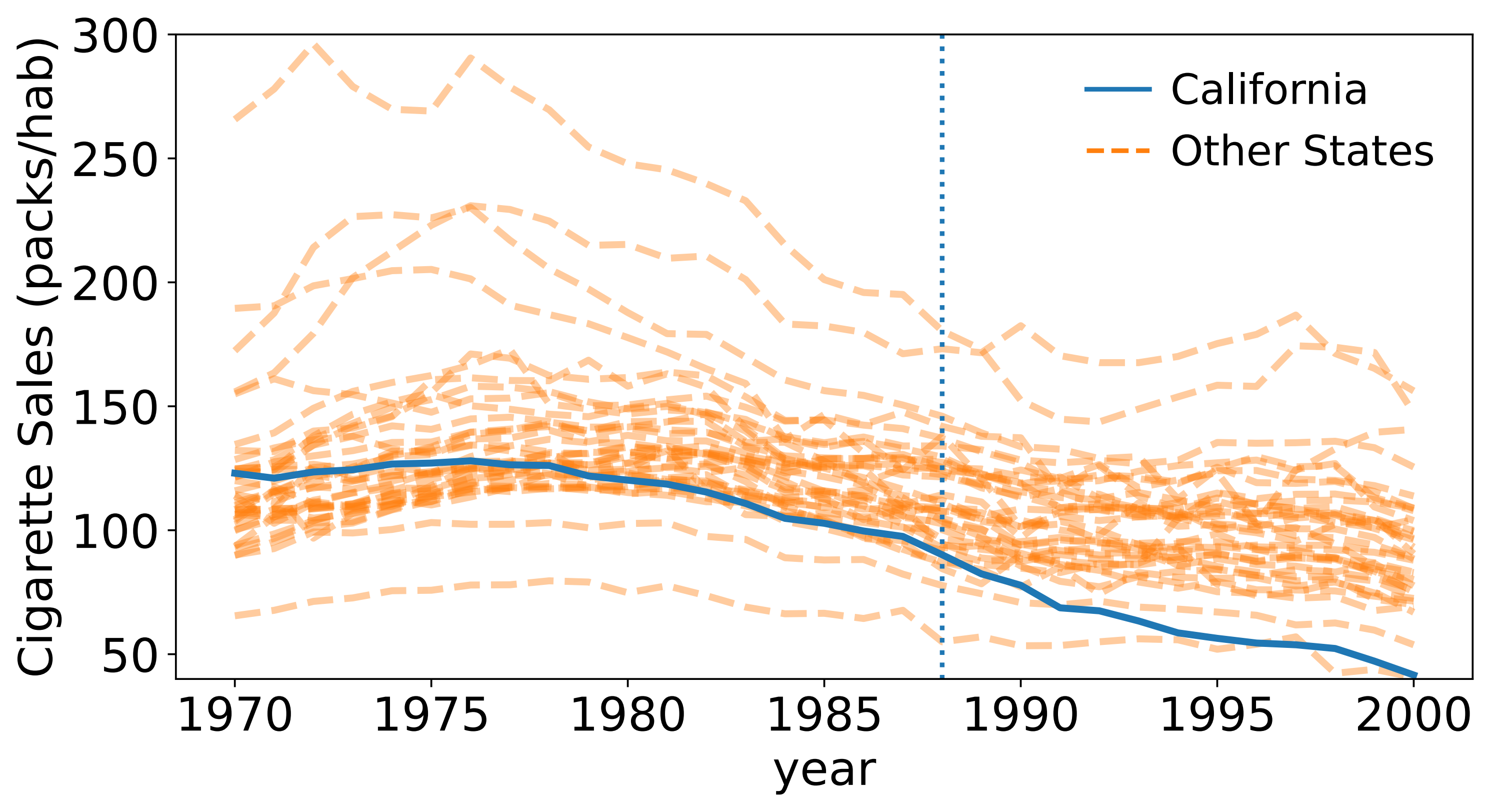

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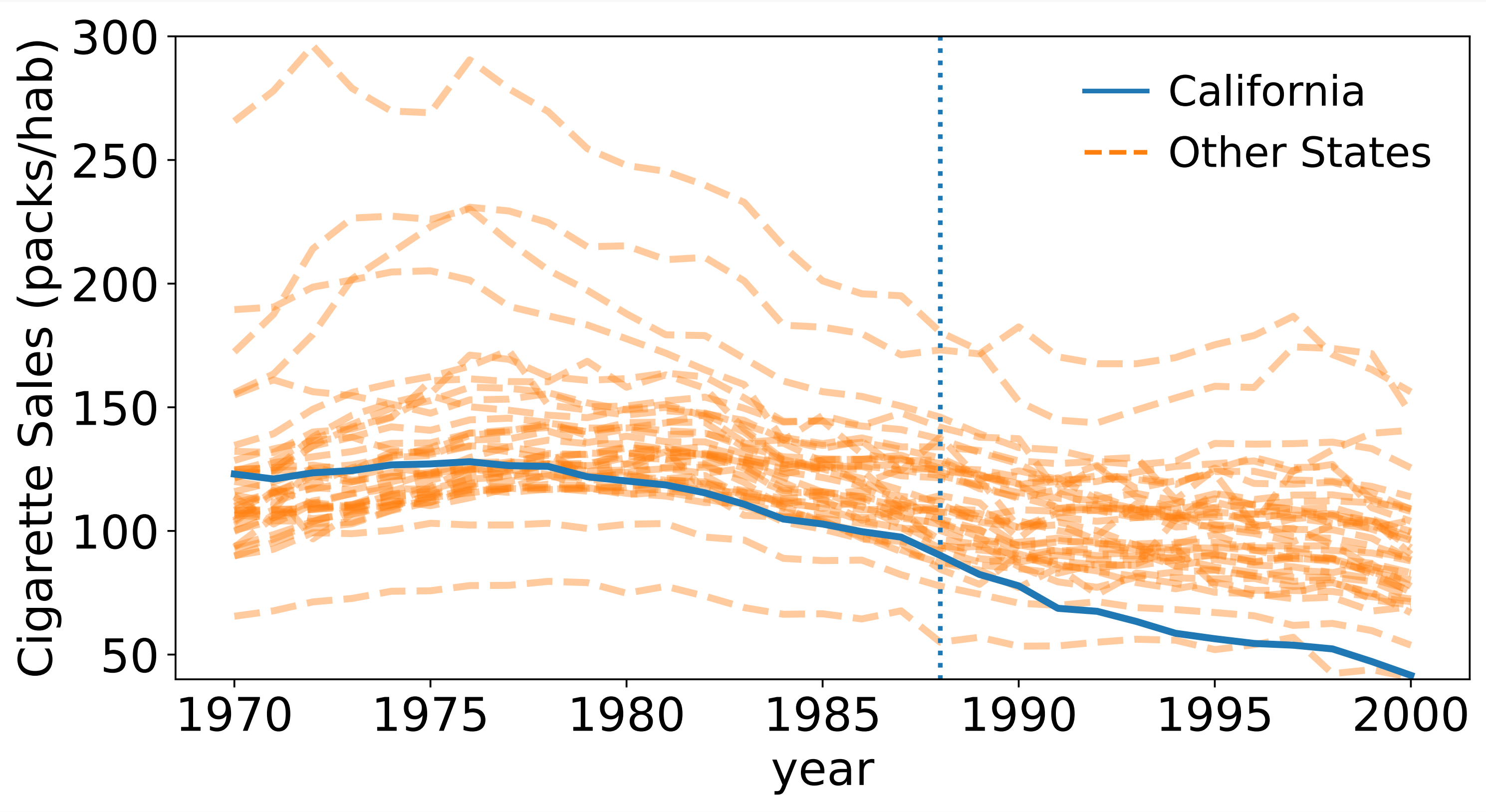
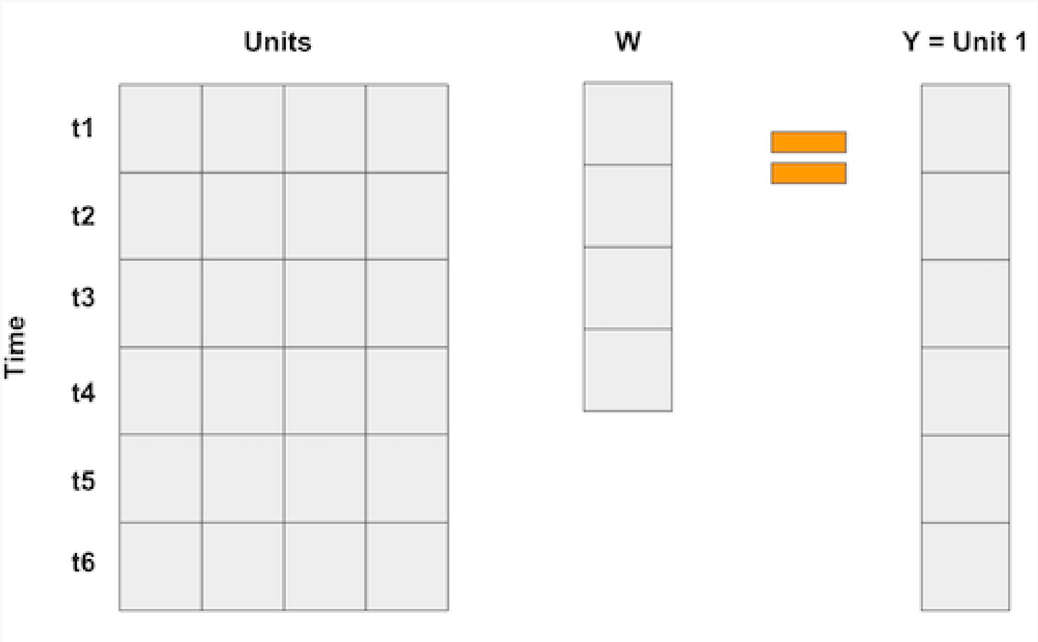


💡 Force parallel trends: Find a weighted average of other states that predicts well the pre-treatment trend of California (before $T_0 = 1988$).

Synthetic control as weighted average of control outcomes

Build a predictor for $Y_{1,t}$ (California):

$\hat{Y}_{1,t} = \sum_{j=2}^{n_0+1} \hat{w}_j Y_{j,t}$



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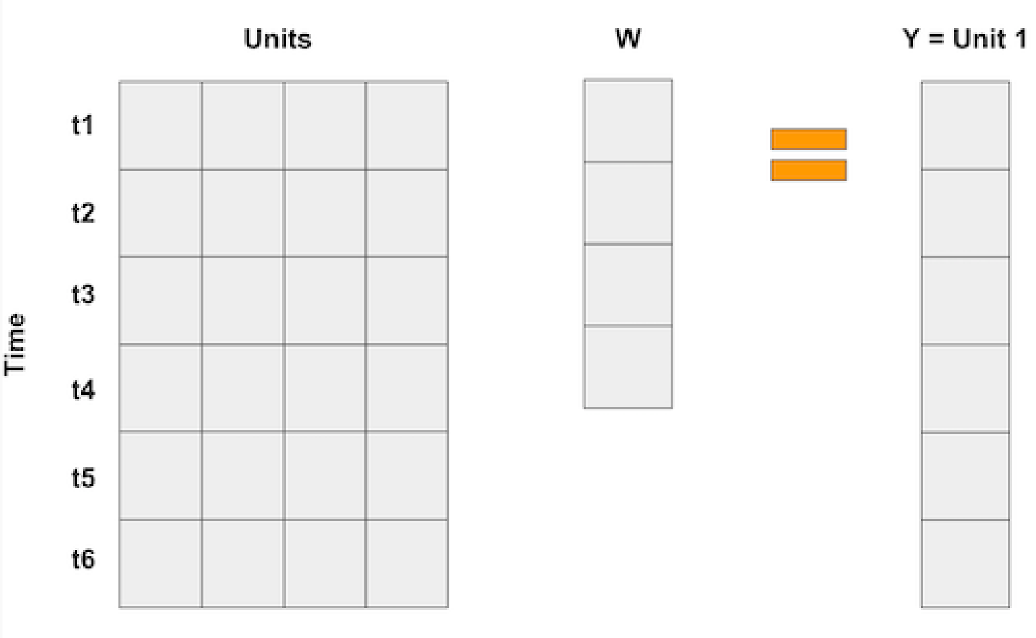
🤔 How to choose the weights?

Minimize some distance between the treated and the controls.



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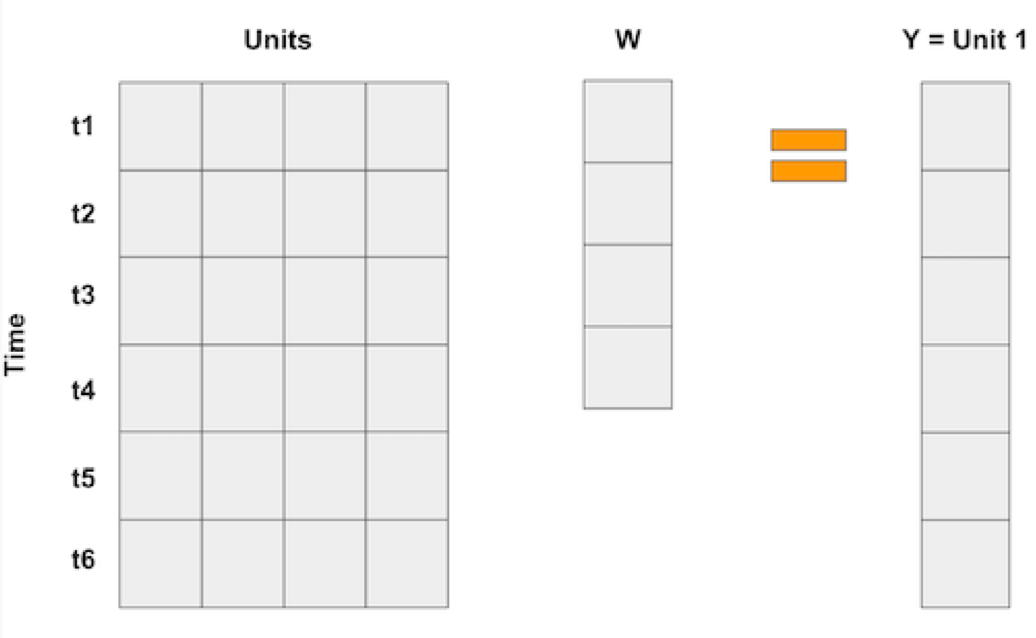

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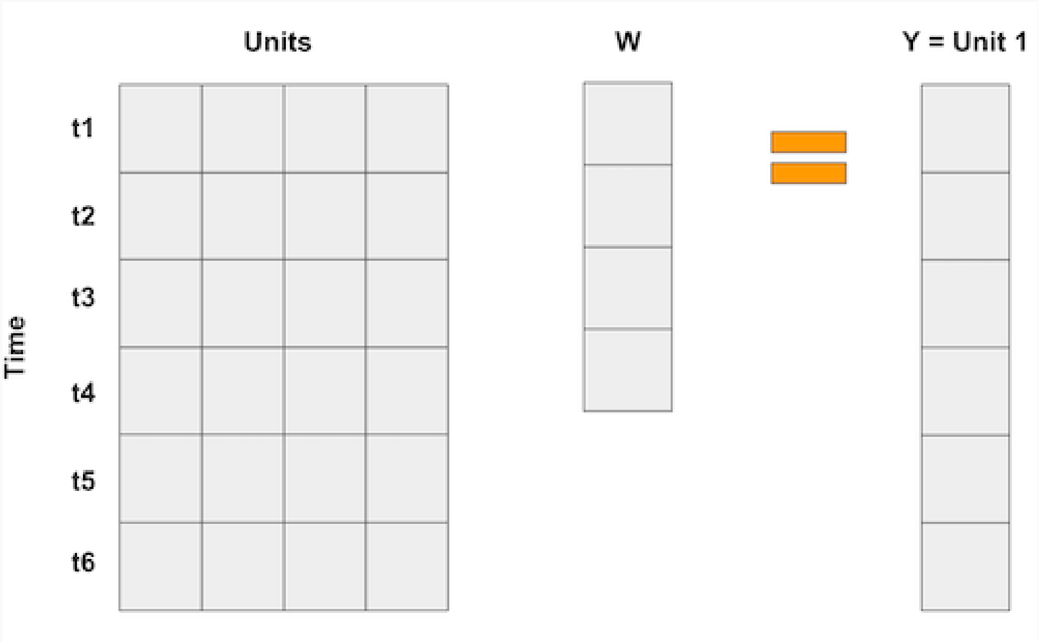
🤓 This is called a balancing estimator: kind of Inverse Probability Weighting (Wager, 2024, chapter 7)



Synthetic controls: minimization problem

Characteristics

Pre-treatment characteristics concatenate pre-treatment outcomes and other pre-treatment predictors Z_1 eg. cigarette prices:

$$X_{\text{treat}} = X_1 = \begin{pmatrix} Y_{1,1} \\ Y_{1,2} \\ .. \\ Y_{1,T_0} \\ Z_1 \end{pmatrix} \in R^{p \times 1}$$

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Let the control pre-treatment characteristics be: $X_{\text{control}} = \left(X_2, .., X_{n_0+1}\right) \in R^{p\times n_0}$

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$$w^* = \text{argmin}_w \, \|X_{\text{treat}} - X_{\text{control}} w\|_V^2$$

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$$\text{where } \|X\|_V = \sqrt{X^T V X} \ \text{ with } \ V \in \text{diag}(R^p)$$

This gives more importance to some features than others.

Synthetic controls: minimization problem

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Let the control pre-treatment characteristics be: $X_{\text{control}} = \left(X_2, .., X_{n_0+1}\right) \in R^{p \times n_0}$

Minimization problem with constraints

$$\begin{aligned} w^* = \text{argmin}_w \; & \|X_{\text{treat}} - X_{\text{control}} w\|_V^2 \\ & s.t.\ w_j \geq 0, \\ & \textstyle\sum_{j=2}^{n_0+1} w_j = 1 \end{aligned}$$

Synthetic controls: Why choose positive weights summing to one?

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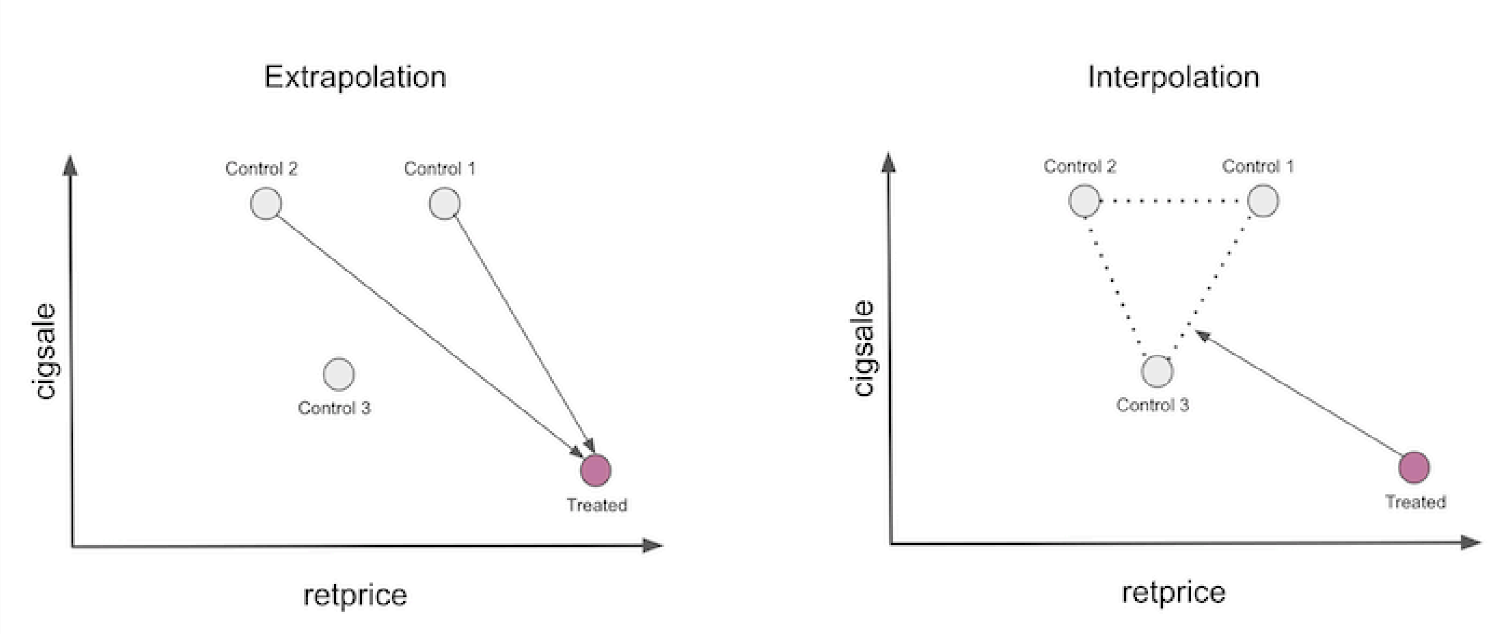
This is called interpolation (vs extrapolation)



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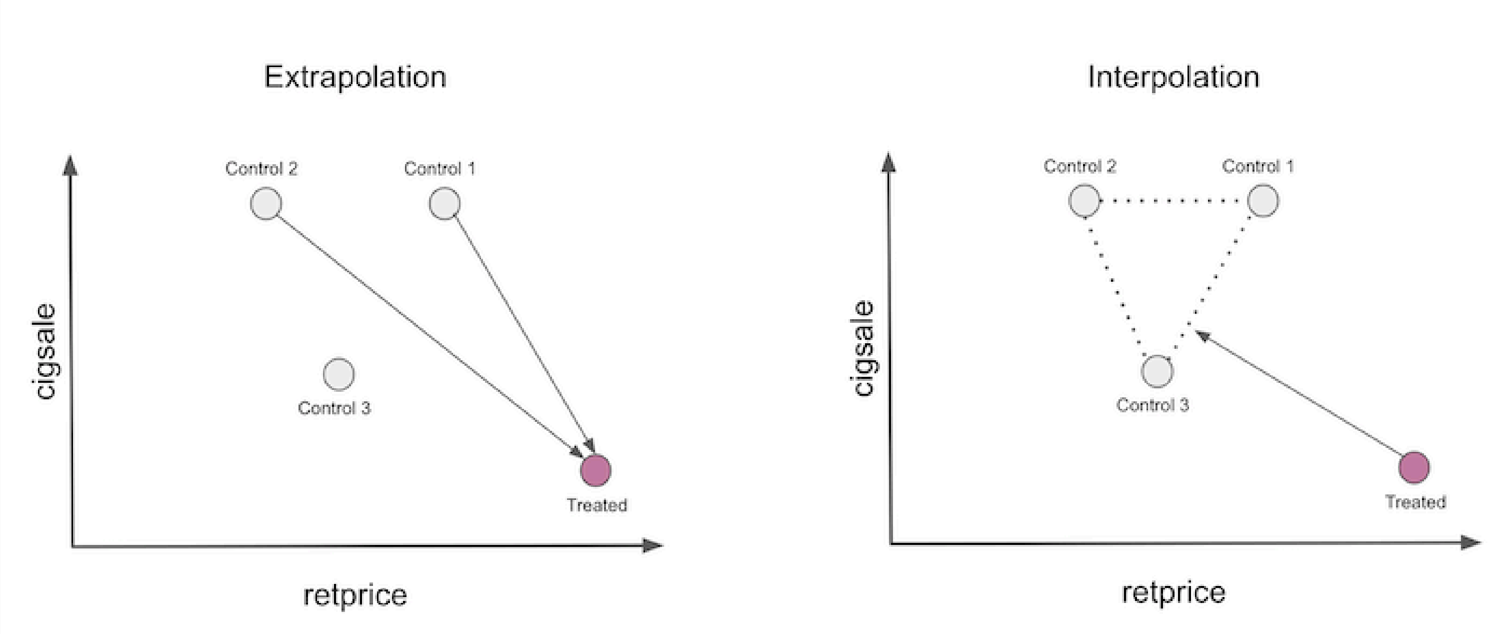

Interpolation enforces regularization, thus limits overfitting

Same kind of regularization than L1 norm in Lasso: forces some coefficient to be zero.

Synthetic controls: Extrapolation failure with unconstrained weight

$p = 2T_0$ covariates:

$$X_j = \begin{pmatrix} Y_{j,1} \\ .. \\ Y_{j,T_0} \\ Z_{j,1} \\ .. \\ Z_{j,T_0} \end{pmatrix}^T \in R^{2T_0}$$

Y cigarette sales, Z cigarette prices.

Synthetic controls: Extrapolation failure with unconstrained weight

$p = 2T_0$ covariates:

$$X_j = \begin{pmatrix} Y_{j,1} \\ .. \\ Y_{j,T_0} \\ Z_{j,1} \\ .. \\ Z_{j,T_0} \end{pmatrix}^T \in R^{2T_0}$$

Y cigarette sales, Z cigarette prices.

Model: $\underbrace{X_{\text{treat}}}_{p\times 1} \sim \underbrace{X_{\text{control}}}_{p\times n_0} \underbrace{w}_{n_0}$

-> simple linear regression estimated by OLS

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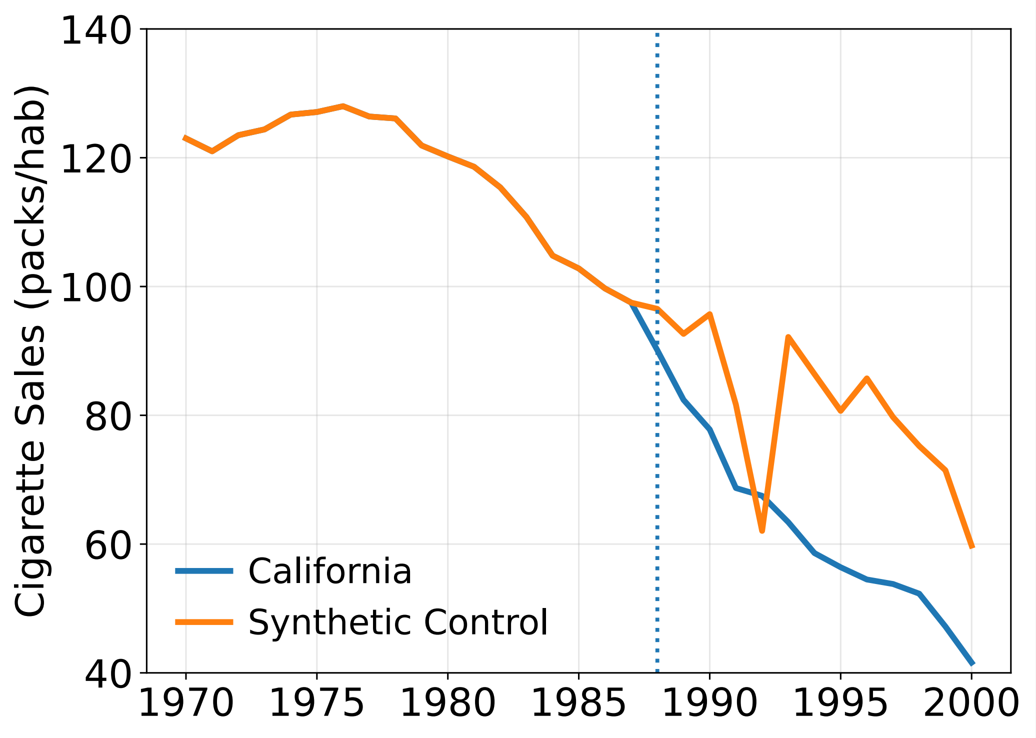

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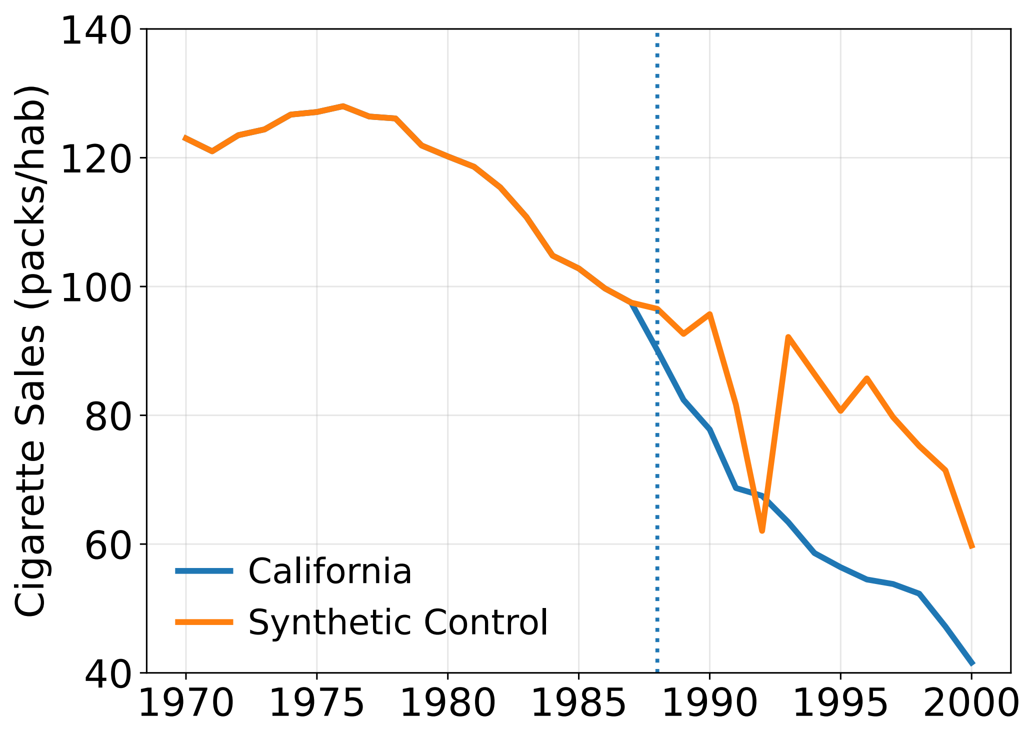
😭 **Overfitting**



Synthetic controls: How to choose the predictor weights V?

1. Don't choose: set $V = I_p$, ie. $\|X\|_V = \|X\|_2$.
2. Rescale by the variance of the predictors:
   $V = \operatorname{diag}\left(\operatorname{var}\left(Y_{j,1}\right)^{-1}, .., \operatorname{var}\left(Y_{j,T_0}\right)^{-1}, \operatorname{var}\left(Z_{j,1}\right)^{-1}, .., \operatorname{var}\left(Z_{j,T_0}\right)^{-1}\right)$.
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3. Minimize the pre-treatment mean squared prediction error (MSPE) of the treated unit:

$$\text{MSPE}(V) = \sum_{t=1}^{T_0} \left[Y_{1,t} - \sum_{j=2}^{n_0+1} w_j^*(V) Y_{j,t}\right]^2$$

$$= \| \left(Y_{1,t}\right)_{t=1..T_0} - \left(Y_{j,t}\right)_{\substack{j=2..n_0+1 \\ t=1..T_0}}^{T} \hat{w} \|_2^2$$

This solution is solved by running two optimization problems:

- Inner loop solving $w^*(V) = \text{argmin}_w \|X_{\text{treat}} - X_{\text{control}} w\|_V^2$
- Outer loop solving $V^* = \text{argmin}_V \text{ MSPE}(V)$

Synthetic controls: estimation without the outer optimization problem

Same coviarates: $X_j = \begin{pmatrix} Y_{j,1} \\ .. \\ Y_{j,T_0} \\ Z_{j,1} \\ .. \\ Z_{j,T_0} \end{pmatrix}^T$

Y cigarette sales, Z cigarette prices.

SCM minization with $V = I_p$, hence, $\|X\|_V = \|X\|_2$.

$$w^* = \text{argmin}_w \ \|X_{\text{treat}} - X_{\text{control}} w\|_2^2$$
$$s.t. \ w_j \geq 0,$$
$$\textstyle\sum_{j=2}^{n_0+1} w_j = 1$$

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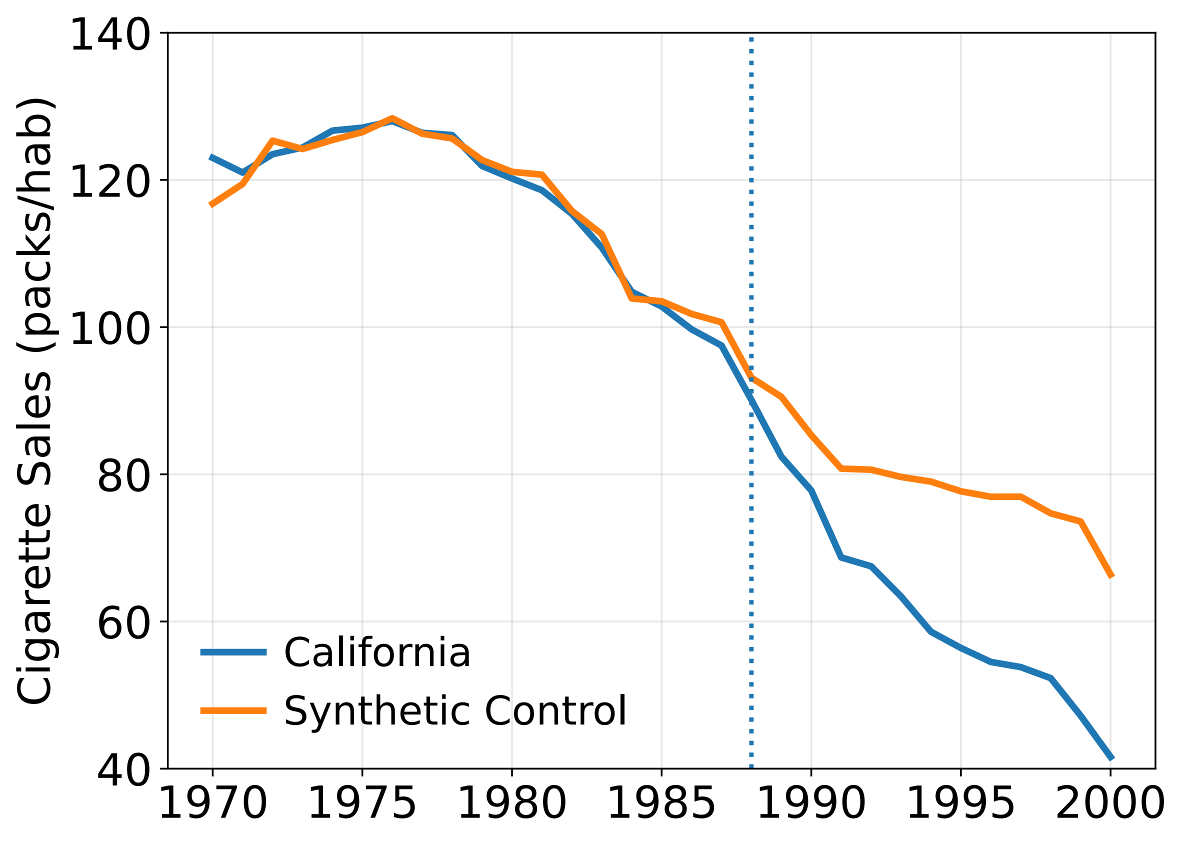
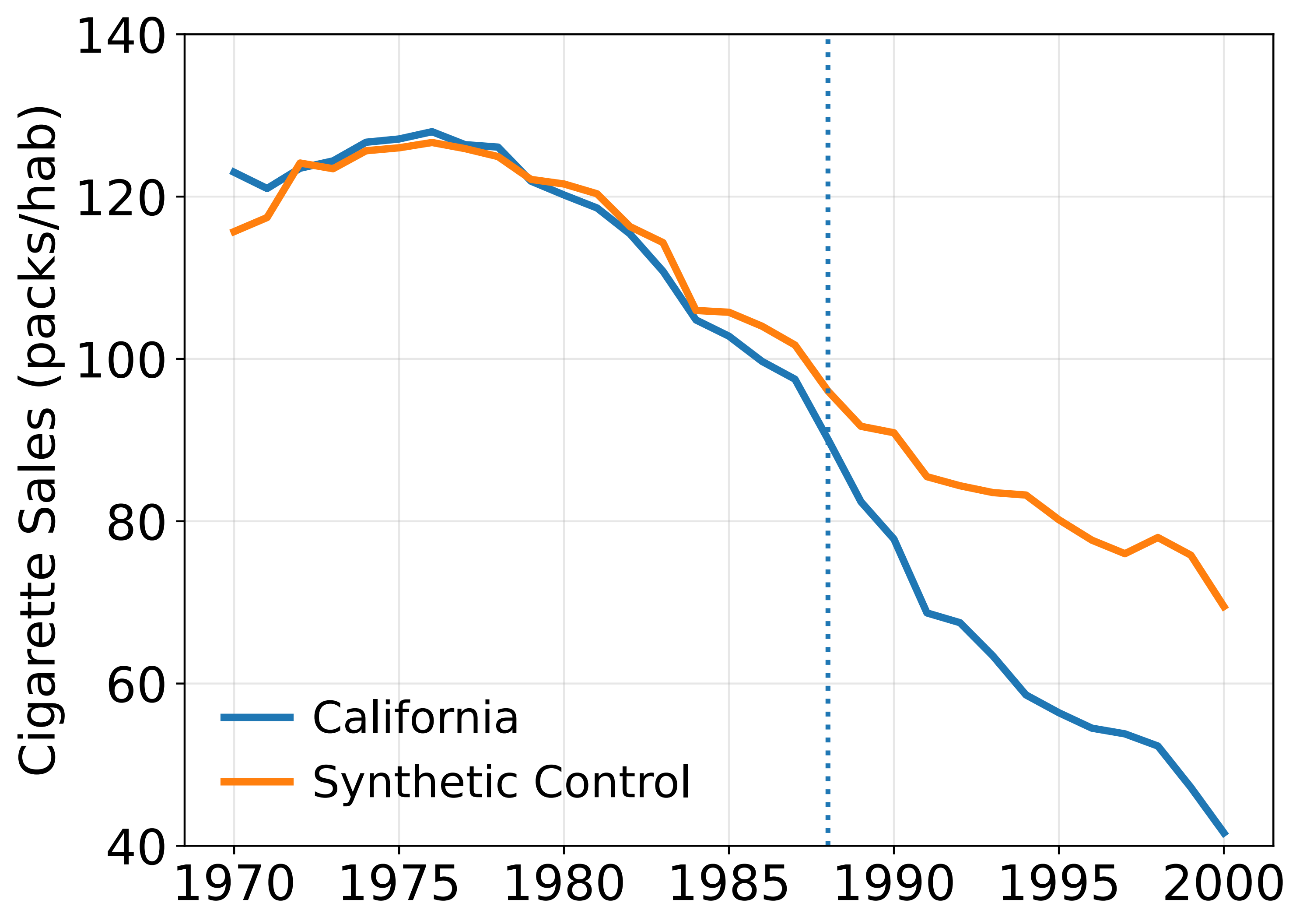


Synthetic controls: estimation with the outer optimization problem



Synthetic controls: inference

Variability does not come from the variability of the outcomes

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… but from the variability of the chosen control units

Treatment assignment introduces more noise than outcome variability.

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... but from the variability of the chosen control units

Treatment assignment introduces more noise than outcome variability.

(Abadie et al., 2010) introduced the placebo test to assess the variability of the synthetic control.

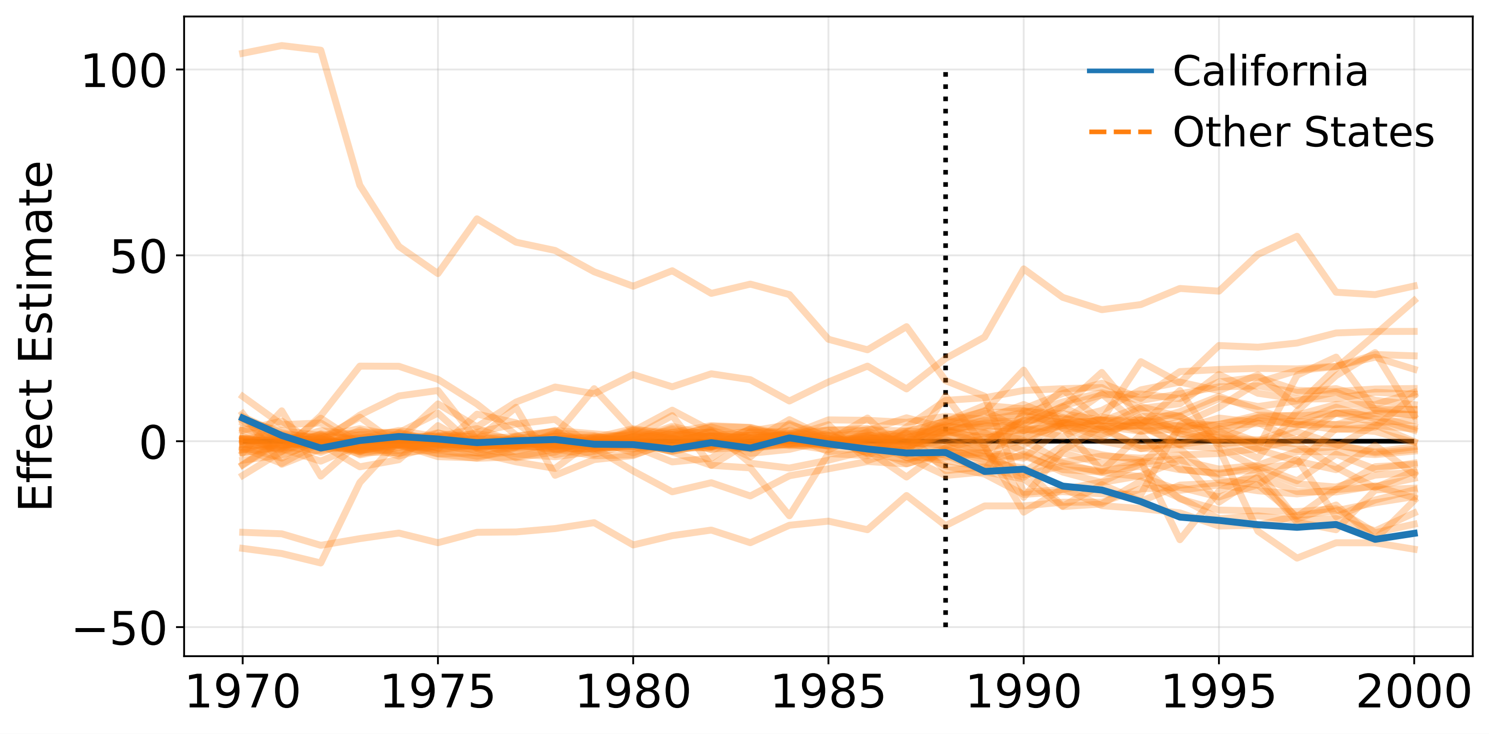
Synthetic controls: inference with Placebo tests

Idea of Fisher's Exact tests

- Permute the treated and control exhaustively.
- For each unit, we pretend it is the treated while the others are the control: we call it a placebo
- Compute the synthetic control for each placebo: it should be close to zero.

Synthetic controls: inference with Placebo tests, example

Placebo estimation for all 38 control states



Synthetic controls: inference with Placebo tests, example

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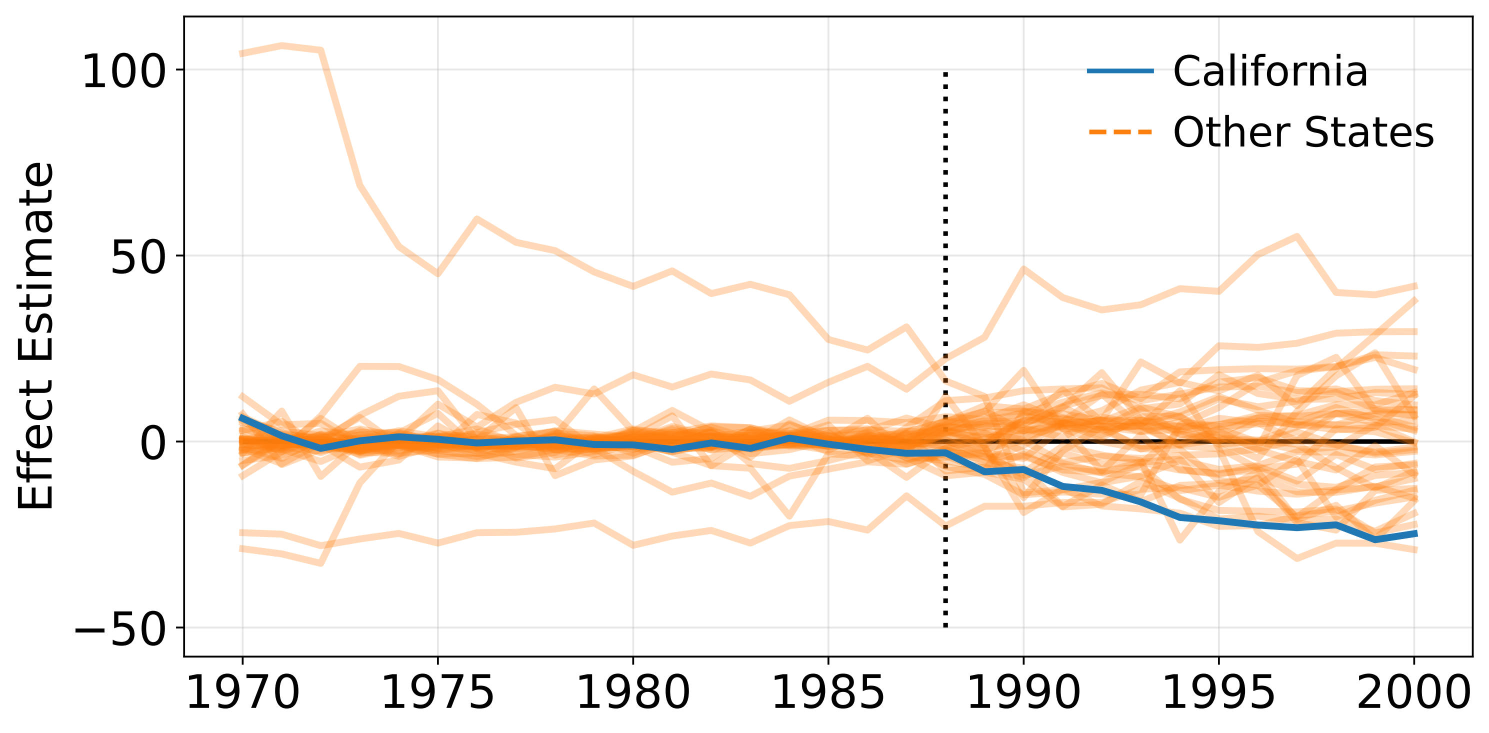
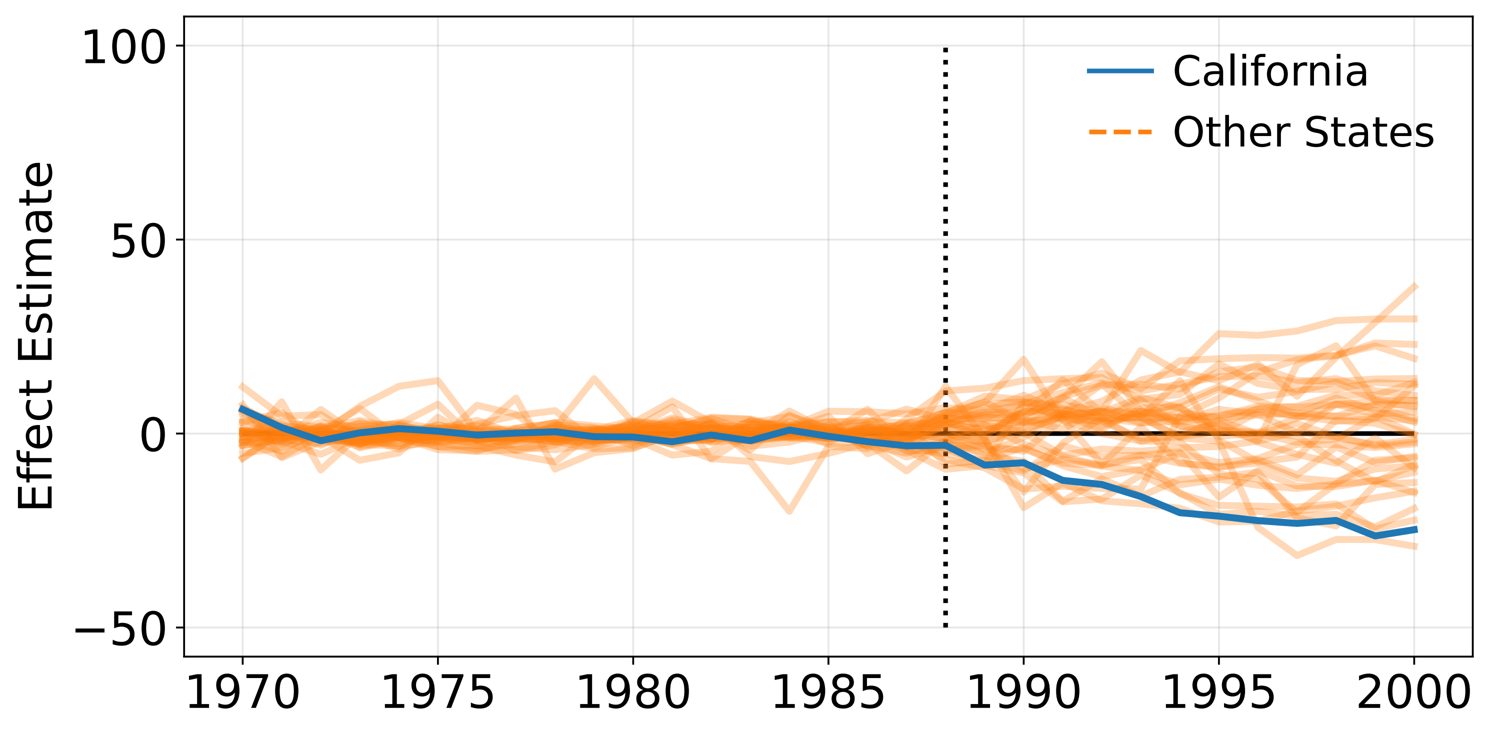


- More variance after the treatment for California than before.
- Some states have pre-treatment trends which are hard to predict.

Synthetic controls: inference with Placebo tests, example

Placebo estimation for 34 control states with "good" pre-treatment fit

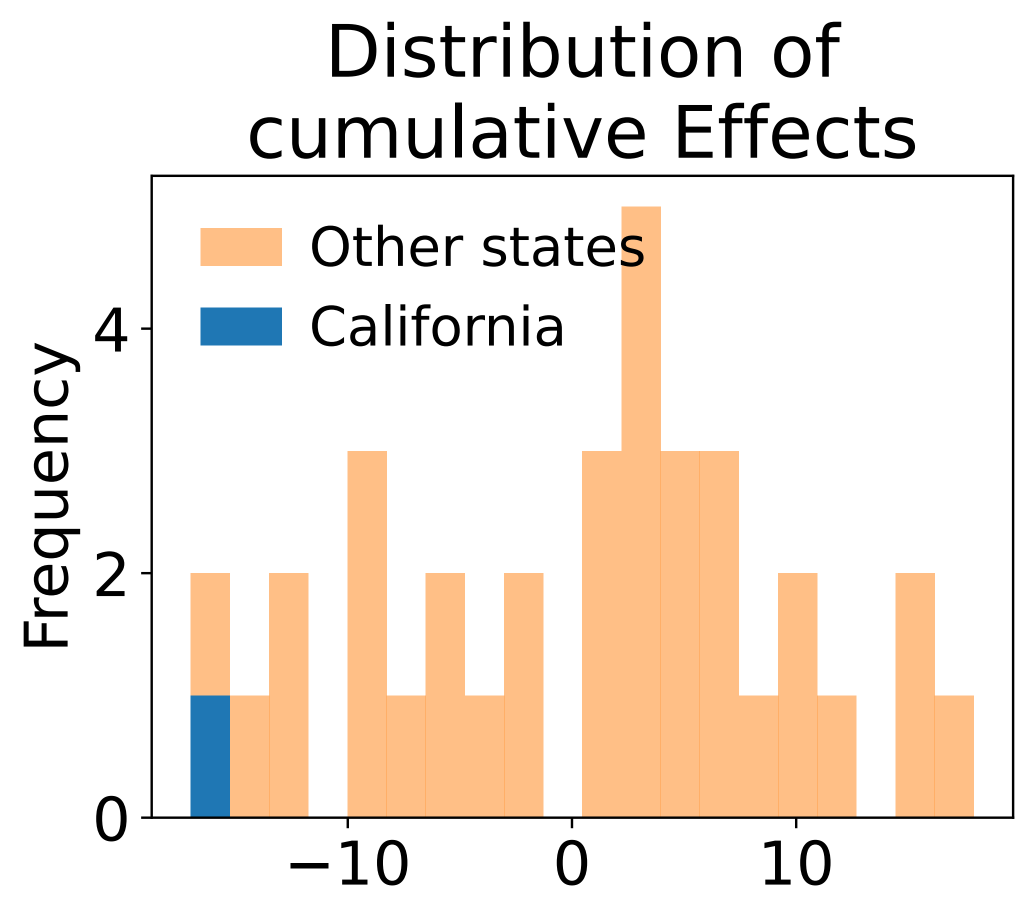


I removed the states above the 90 percentiles of the distribution of the pre-treatment fit.

Synthetic controls: inference with Placebo tests, example

California absolute cumulative effect

$\hat{\tau}_{\text{scm, california}} = -17.00$

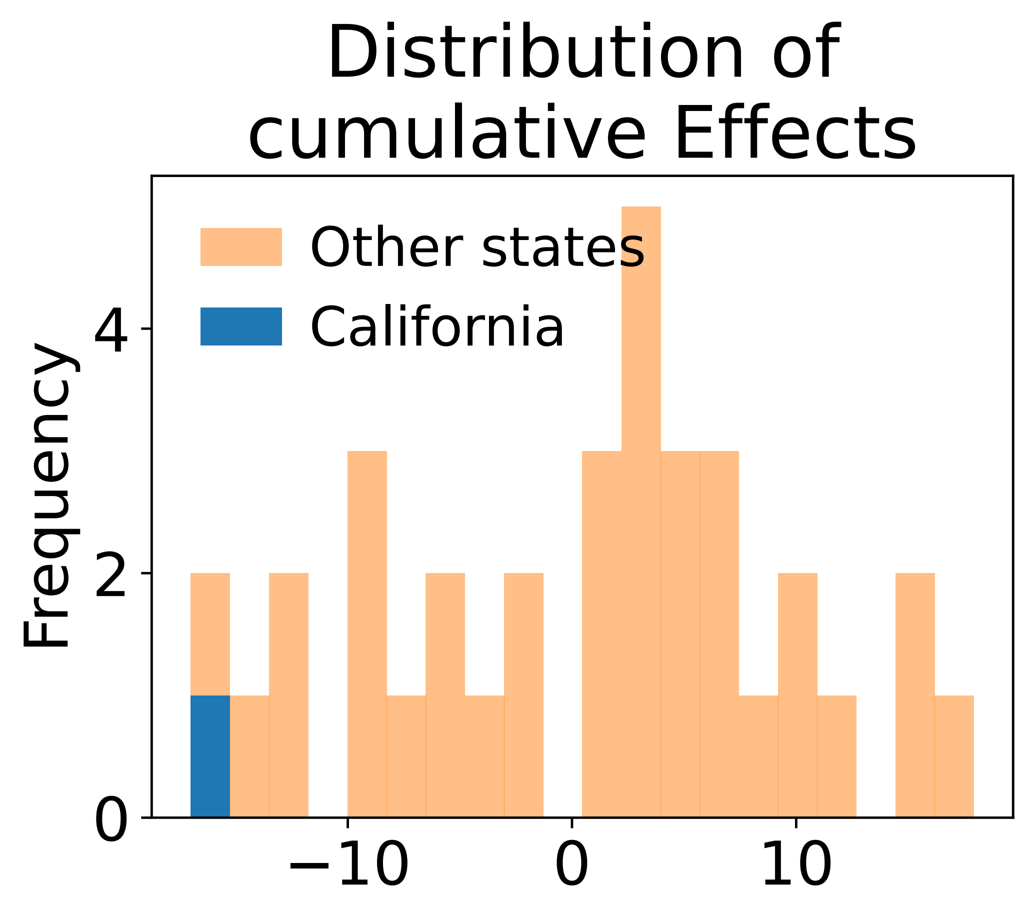


California absolute cumulative effect

$\hat{\tau}_{\text{scm, california}} = -17.00$

Get a p-value

$$\text{PV} = \frac{1}{n_0} \sum_{j=2}^{n_0} \mathbb{1}\left(\left|\hat{\tau}_{\text{scm, california}}\right| > \left|\hat{\tau}_{\text{scm},j}\right|\right)$$
$$= 0.029$$



Synthetic controls: inference with conformal prediction

Synthetic controls: Take-away

Pros

- More convincing for parallel trends assumption.
- Simple for multiple time periods.
- Gives confidence intervals.

Cons

- Requires many control units to yield good pre-treatment fits.
- Might be prone to overfitting during the pre-treatment period.
- Still requires a strong assumption: the weights should also balance the post-treatment unexposed outcomes. See (Arkhangelsky et al., 2021) for discussions.
- Still requires the no-anticipation assumption.

Conditional difference-in-differences

Time-series modelisation: methods without a control group

Interrupted Time Series: intuition

Setup

- One treated unit, no control unit.
- Multiple time periods.
- Sometimes, predictors are availables: there are called exogeneous covariates.

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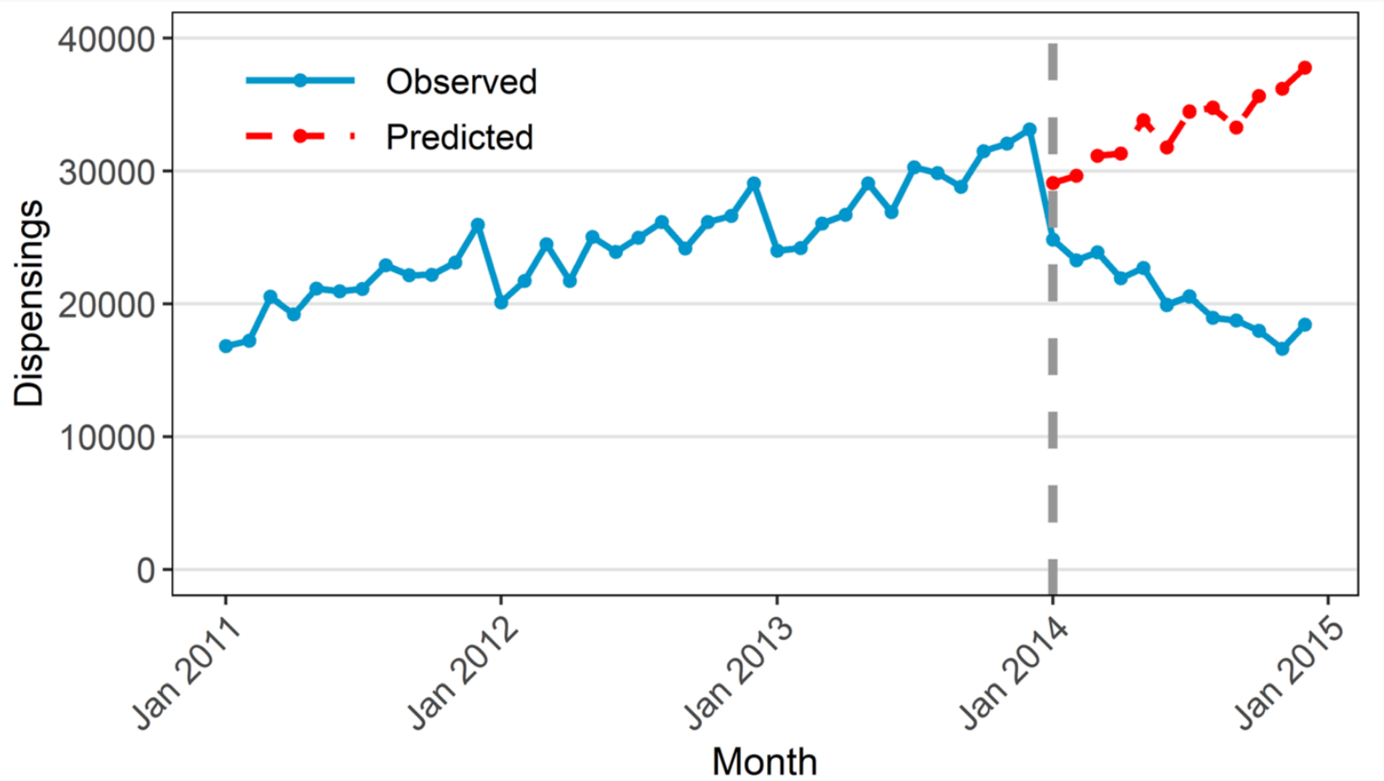
Intuition

- Model the pre-treatment trend: $Y_{t(1)}$ for $t < T_0$
- Predict post-treatment trend as the control: $\hat{Y}_t(0)$ for $t > T_0$
- Obtain treatment effect by taking the difference between observed and predicted post-treatment observations: $Y_t(1) - \hat{Y}_t(0)$

Interrupted Time Series: illustration from (Schaffer et al., 2021)



Y_t: Dispensations of quetiapine, an anti-psychotic medicine.

Treatment: Restriction of the conditions under which quetiapine could be subsidised.

Tools

- ARIMA models: AutoRegressive Integrated Moving Average

Motivation of ARIMA

Take into account:

- the structure of autodependance between observation,
- linear trends,
- seasonality.

A good reference for ARIMA: Forecasting: Principles and Practice, chapter 8

ARIMA are State Space Models (SSM) says the machine learning community

Why showing this formulation ?

- I better understand ARIMA formulated as state space models.
- SSM are more general than ARIMA models.
- ARIMA are (almost always) fitted with SSM optimization algorithms.

What is a state space model?

A word on model families for ITS

We saw ARIMA models and the more general class of state space models.

However, we could any model that we want to fit the pre-treatment trend !

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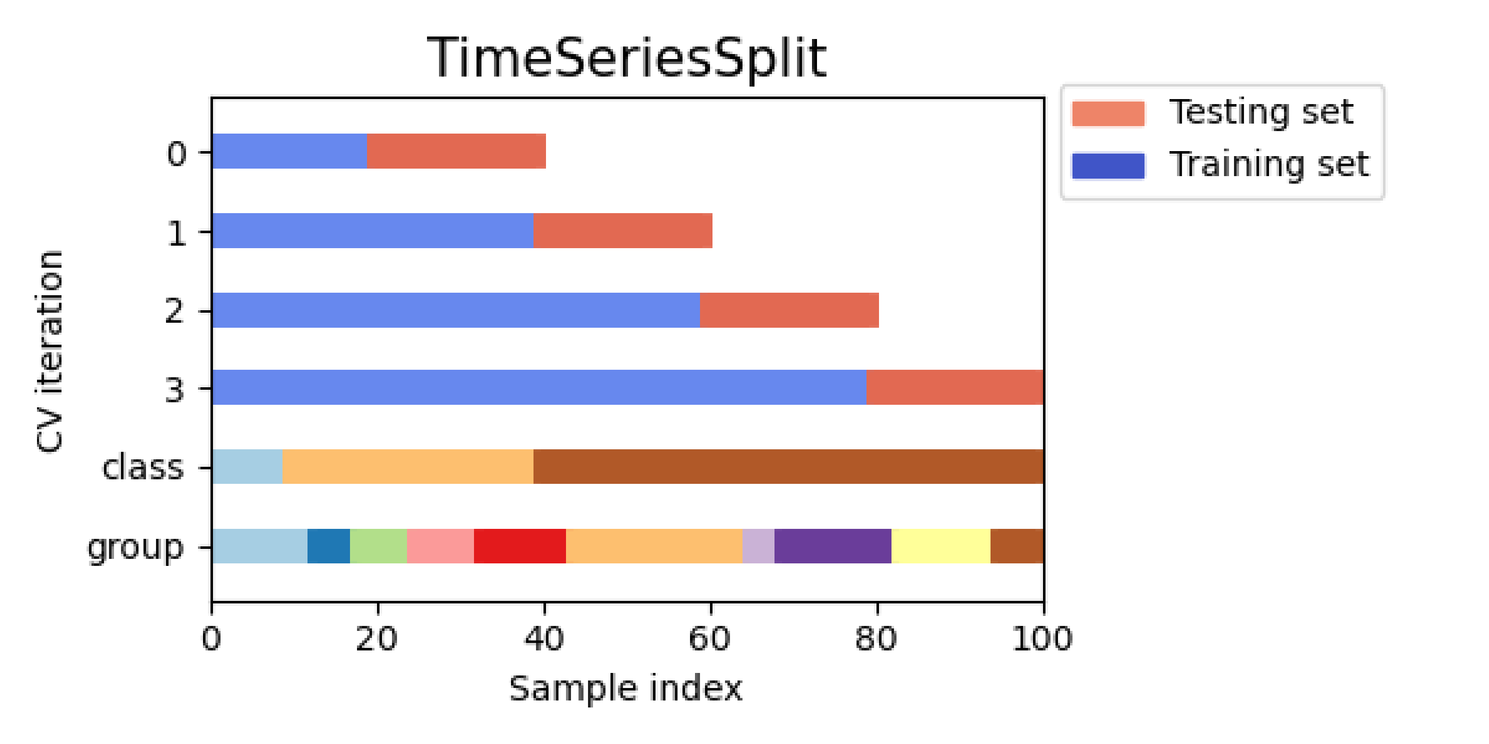
⚠️ You should pay attention to appropriate train/test split when cross-validating a time-series model not to use the future to predict the past.

Relevant remark for all time series models (even ARIMA or state space models).

Cross-validation for time-series models

```python
from sklearn.model_selection import TimeSeriesSplit
```



This avoids to use the future to predict the past.

Take-away on ITS

Pros

- Suitable when no control unit is available. The pre-treatment trend is the control.
- Handles multiple time periods.
- A lot of software available (eg. ARIMA models).
- Simple: few parameters to tune.

Cons

- Prone to bias by other events happening around the treatment time and impacting the outcome trend.
- Prone to overfitting of the pre-treatment trend.

An attempt to map event study methods

Methods	Characteristics	Hypotheses	Community	Introduction	Good reference
DID/TWFE	Treated/control units, few time periods, no predictors	Parallel trends, no anticipation, prone to overfitting	Economics	Causal Inference for the Brave and True, chapter 13	(Arkhangelsky & Imbens, 2024)
ARIMA, ITS	No controls, no/few predictors, seasonality	Stationnarity , no anticipation, prone to overfitting	Epidemiology, Economics	Forecasting: Principles and Practice	(Schaffer et al., 2021)
State space models	Multiple time periods, control units or predictors, generalization of ARIMA	Contional ignorability on predictors, goodness of fit pre-treatment	Machine learning, bayesian methods	(Brockwell & Davis, 2016, chapter 9)	(Murphy, 2022, chapter 18)
Synthetic control	Treated/control units, multiple time periods	Conditional parallel trend on controls, goodness of fit pre-treatment	Economics	Causal Inference for the Brave and True	(Abadie, 2021)

A summary on R packages for event studies

Package name	Methods	Predictors	Control units	Multiple time periods
did	Difference-in-differences	❌	❌	❌
forecast	ARIMA, ITS	✅	❌	✅
Synth	Synthetic control	❌	✅	✅
Causal impact	Bayesian state space models	✅	❌	✅

A summary on Python packages for event studies

Package name	Methods	Predictors	Control units	Multiple time periods
statsmodels.OLS	Difference-in-differences, TWFE	❌	❌	❌
statsmodels	ARIMA(X), ITS, bayesian state space models	✅	❌	✅
pmdarima	ARIMA(X), ITS	✅	❌	✅
SyntheticControlMethods	Synthetic control	❌	✅	✅
pysyncon	Synthetic control	❌	✅	✅
causalimpact (pymc implementation)	Bayesian state space models	✅	❌	✅
causal-impact (statsmodels implementation)	Bayesian state space models	✅	❌	✅

Python hands-on

To your notebooks 🧑‍💻 !

- url: https://github.com/strayMat/causal-ml-course/tree/main/notebooks

Bibliography

Abadie, A. (2021). Using synthetic controls: Feasibility, data requirements, and methodological aspects. Journal of Economic Literature, 59(2), 391–425.

Abadie, A., & Gardeazabal, J. (2003). The economic costs of conflict: A case study of the Basque Country. American Economic Review, 93(1), 113–132.

Abadie, A., Diamond, A., & Hainmueller, J. (2010). Synthetic control methods for comparative case studies: Estimating the effect of California's tobacco control program. Journal of the American Statistical Association, 105(490), 493–505.

Arkhangelsky, D., & Imbens, G. (2024). Causal models for longitudinal and panel data: A survey. The Econometrics Journal, 27(3), C1–C61.

Arkhangelsky, D., Athey, S., Hirshberg, D. A., Imbens, G. W., & Wager, S. (2021). Synthetic difference-in-differences. American Economic Review, 111(12), 4088–4118.

Bibliography

Ashenfelter, O. (1978). Estimating the effect of training programs on earnings. The Review of Economics and Statistics, 47–57.

Athey, S., & Imbens, G. W. (2017). The state of applied econometrics: Causality and policy evaluation. Journal of Economic Perspectives, 31(2), 3–32.

Bonander, C., Humphreys, D., & Degli Esposti, M. (2021). Synthetic control methods for the evaluation of single-unit interventions in epidemiology: a tutorial. American Journal of Epidemiology, 190(12), 2700–2711.

Brockwell, P. J., & Davis, R. A. (2016). Introduction to time series and forecasting, Third edition. Springer. http://repository.cinec.edu/bitstream/cinec20/1109/1/2016_Book_IntroductionToTimeSeriesAndFor.pdf

De Chaisemartin, C., & d'Haultfoeuille, X. (2020). Two-way fixed effects estimators with heterogeneous treatment effects. American Economic Review, 110(9), 2964–2996.

Bibliography

Degli Esposti, M., Spreckelsen, T., Gasparrini, A., Wiebe, D. J., Bonander, C., Yakubovich, A. R., & Humphreys, D. K. (2020). Can synthetic controls improve causal inference in interrupted time series evaluations of public health interventions?. International Journal of Epidemiology, 49(6), 2010–2020.

Grupp, T., Mishra, P., Reynaert, M., & Benthem, A. A. van. (2023). An evaluation of protected area policies in the european union.

Miller, S., Johnson, N., & Wherry, L. R. (2019). Medicaid and mortality: new evidence from linked survey and administrative data.

Murphy, K. P. (2022). Probabilistic machine learning: an introduction. MIT press.

Puig-Codina, L., Pinilla, J., & Puig-Junoy, J. (2021). The impact of taxing sugar-sweetened beverages on cola purchasing in Catalonia: an approach to causal inference with time series cross-sectional data. The European Journal of Health Economics, 22(1), 155–168.

Bibliography

Schaffer, A. L., Dobbins, T. A., & Pearson, S.-A. (2021). Interrupted time series analysis using autoregressive integrated moving average (ARIMA) models: a guide for evaluating large-scale health interventions. BMC Medical Research Methodology, 21, 1–12.

Snow, J. (1855). On the mode of communication of cholera. John Churchill.

Stechemesser, A., Koch, N., Mark, E., Dilger, E., Klösel, P., Menicacci, L., Nachtigall, D., Pretis, F., Ritter, N., Schwarz, M., & others. (2024). Climate policies that achieved major emission reductions: Global evidence from two decades. Science, 385(6711), 884–892.

Taylor, S. J., & Letham, B. (2018). Forecasting at scale. The American Statistician, 72(1), 37–45.

Wager, S. (2024,). Causal inference: A statistical learning approach. preparation.