Internal-External Liquidity Feedbacks*

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Abstract

I develop a model that investigates the relation between corporate policies and secondary stock market liquidity. I show that secondary market illiquidity limits a firm's ability to hold precautionary liquidity, exacerbates financial constraints, reduces investment, and decreases corporate value. I also show that this drop in firm value feeds back into the secondary market by deterring the participation of liquidity providers, thereby making the market more illiquid. The self-reinforcing nature of this relation gives rise to an internal-external liquidity loop, whereby corporate liquidity stimulates market liquidity, and vice-versa. The loop singles out a propagation mechanism between financial markets and the corporate sector.

Keywords Feedback effect, liquidity, cash holdings, transaction costs, real effects of financial markets, market participation

JEL Classification Numbers: D83; G12; G32; G35

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1 Introduction

Capital movements are costly and, at times, difficult.[1] Since the work of Amihud and Mendelson (1986), the effects of trading frictions in secondary markets, such as transaction costs and delays, have been the subject of considerable research in asset pricing. However, to this date little is known about how these frictions relate to corporate decision-making. The objective of this paper is to fill this void.

By definition, secondary markets are exchanges where investors trade stocks without the involvement of the issuing firm. Yet, a growing body of evidence documents that secondary market activity affects corporate outcomes.[2] Within this literature, several studies report that firms with illiquid stocks are more constrained in their ability to raise fresh funds (e.g., Butler, Grullon, and Weston, 2005; Stulz, Vagias, and VanDijk, 2013; or Campello, Ribas, and Wang, 2011). Other things equal, precautionary motives should prompt these firms to keep more cash than their more liquid peers. In contrast, the data suggests the opposite. Figure 1 focuses on Compustat firms traded on NYSE, NASDAQ, and AMEX markets and shows that firms with liquid stocks hold more cash on average. It also illustrates that the stocks of cash-rich firms are more liquid.[3] Supporting this evidence, Nyborg and Wang (2014) document a positive relation between stock market liquidity and corporate cash holdings, which is consistent with the observation that very liquid stocks (e.g., Microsoft or Apple) hoard huge cash reserves.

This counterintuitive pattern provides a clear illustration that the interactions between market liquidity and corporate policies are not trivial and deserve a thorough investigation. To this end, I develop a dynamic model that studies the joint dynamics of corporate decisions and stock market liquidity. In so doing, I place a special focus on the relation between *external* market liquidity — i.e., the ease with which a stock is

[1] The presidential address of Stoll (2000) is a landmark in the characterization of trading frictions. Stoll distinguishes between real frictions (arising from order processing) and informational frictions (arising from information asymmetries). Using NYSE, AMEX, and NASDAQ data, he finds that *"Real friction [...] is important, and evidence of informational friction is also in the data."* Several works document illiquidity for investors in major markets, e.g., Huang and Stoll (1996), Chordia, Roll, Subrahmanyam (2001), Chordia, Sarkar, Subrahmanyam (2005), Coval and Stafford (2007). In addition, Comerton-Forde et al. (2010), Aragon and Strahan (2012), and Hameed, Kang, Viswanathan (2010) report important constraints in the market-making sector even for U.S. equities, potentially translating in liquidity dry-ups.

[2] Fang, Noe and Tice (2009), Edmans, Goldstein, and Jiang (2012), Derrien and Kecksés (2013), Stulz, Vagias, and van Dijk, (2013), and Fang, Tian, and Tice (2014), among others.

[3] Appendix C discusses the evidence relating internal and external liquidity.

traded — and *internal* corporate liquidity — i.e., the cash reserves of the issuing firm. The link between market illiquidity and corporate decisions arises from the costs that shareholders face when occasionally liquidity-shocked, which are reflected in the cost of capital and, hence, in firm value. The magnitude of these costs, however, depends on the extent to which intermediaries provide liquidity in the market, which is in turn related to firm value. The paper makes three main contributions. First, it shows how market illiquidity constrains corporate decisions and reduces corporate value. Second, it demonstrates that the firms' response to market illiquidity may feed back into market illiquidity by affecting the participation of liquidity providers. Third, the analysis reveals that this relation is self-reinforcing and gives rise to an internal-external liquidity loop. This loop singles out a novel mechanism through which shocks propagate between financial markets and corporations.

To illustrate these patterns, I start by examining how market illiquidity affects corporate decisions when the liquidity provision by intermediaries is perfect. In this setting, market illiquidity is represented by the cost that liquidity-shocked investors face to unwind their asset holdings. I show that, to compensate for this cost, investors require a higher rate of return to invest in the stock. That is, the issuing firm needs to promise an illiquidity premium to its investors. The premium raises the opportunity cost of holding cash, which calls for a decrease in the firm's target cash level and for an increase in the firm's payout rate. The larger payout rate, however, makes the firm more constrained. Inefficient liquidations then become more likely, outside funding becomes relatively more difficult, and the firm's investment opportunity set narrows, leading to underinvestment at the firm level. As a result, firm value decreases.

In practice, however, illiquidity — and thus the cost of a liquidity shock — depends on the extent to which intermediaries provide immediacy in the market for the stock. To account for this feature, I bring more structure into the model and endogenize the measure of intermediaries who actively follow the stock. In this richer environment, the detrimental effects of market illiquidity on firm value feed back into market illiquidity. Specifically, the illiquidity-driven drop in firm value decreases the expected rents accruing to the financial sector, and some intermediaries stay away from the market of the stock. Therefore, secondary market transactions become more costly for investors. Firms then need to increase further the illiquidity premium, which leads to an additional decrease in corporate cash reserves and firm value. Again, the drop in firm value feeds back into the secondary market by crowding out other intermediaries, and so on. As a result, a self-reinforcing relation arises, which exacerbates the detrimental effects of market illiquidity on firm value.

The model delivers a rich set of testable predictions. First, it characterizes a firm's policies in relation to the liquidity of its stock. Specifically, the model predicts that firms with illiquid stocks, ceteris paribus, should display larger payouts and smaller cash holdings, limit the size of equity issues, and access primary market less often. In the cross-section, these firms should have a larger default risk and face underinvestment problems. Second, the model relates intermediaries' participation to firm characteristics. In particular, the model predicts that firms that are less profitable, whose cash flows are more volatile, that are more financially constrained, or that have more intangible assets should be less followed by intermediaries and, therefore, less liquid.

Importantly, this self-reinforcing relation singles out a novel mechanism through which shocks propagate between financial markets and corporations. In fact, shocks that affect the participation of liquidity providers (e.g., market presence becomes more costly) become more severe when reflected into firm value. While increasing the cost of trading for investors, these shocks lead firms to pay a larger illiquidity premium that decreases firm value and crowds out other intermediaries from the market of the stock. Notably, this amplification mechanism provides a complementary channel that may help explain the low-participation puzzle (see Guiso and Jappelli, 2005, Guiso, Sapienza, and Zingales, 2008). In turn, corporate cash flow shocks become more persistent for firms with illiquid stocks. In fact, market illiquidity reduces corporate cash holdings and makes it more difficult and costly to raise fresh funds, then leaving firms relatively more financially constrained. The model then reveals sizeable benefits from policies aimed at enhancing market liquidity. By relieving trading frictions to investors, improving market liquidity helps attain a more efficient allocation of resources at the firm level.

One may argue that informational frictions can provide a more convincing channel to explain the feedback effect between financial markets and corporate outcomes. While information asymmetry is certainly a central issue in this context, several empirical and theoretical contributions show that costs of order processing are at least as crucial, through the impact on investors' participation and thereby on the firm's cost of capital, see e.g., Stoll (2000). Most importantly, adverse selection can hardly match the observed positive relation between internal and external liquidity. Indeed, fear of adverse selection alone would boost a firm's incentive to keep more cash ex ante, then leading to a negative relation between internal and external liquidity.

Related literature. This paper contributes to the literature studying the impact of financial market frictions on corporate decisions, with a particular focus on the effect of impediments in secondary market transactions. Within this strand of literature, recent contributions focus on bond markets and examine the relation between market liquidity and default risk (He and Xiong, 2012, He and Milbradt, 2014, or Chen, Cui, He, and Milbradt, 2014) or market liquidity and corporate debt maturity dynamics (Chen, Xu, and Yang, 2013). Looking instead at stock markets, previous contributions primarily focus on information asymmetries and the learning and incentive effects thereof, see Bond, Edmans, and Goldstein (2012) for a survey.[4] This paper focuses instead on the real costs of transactions borne by investors and intermediaries and investigates how firms need to adjust their policies when shareholders face such transaction costs.

This paper also relates to the literature examining the effects of participation shocks in financial markets. Although these shocks are technically independent of cash flow shocks, theoretical contributions have proposed different channels through which they may affect corporate outcomes. Bencivenga, Smith, and Starr (1995) were among the first to study how transaction costs in equity markets impact the investors' willingness to commit funds for long horizons, thereby affecting the firm's cost of capital. Pagano (1989) highlights a feedback effect between equity market thinness and volatility, showing that transaction costs may trap the market in a low-trades/high-volatility equilibrium, preventing the firm from issuing equity. More recently, Titman (2013) and Subrahmanyam and Titman (2013) show that shocks that stimulate equity participation can spur the entry of new firms, in turn affecting incumbents' cash flows. Instead of studying exogenous costs of investors' participation, the present paper makes them depend on the supply of liquidity. It does so by analyzing the following feedback loop: Shareholder value increases in intermediaries' participation, and the rents accruing to the intermediary sector increase in shareholder

[4]See, for instance, Subrahmanyam and Titman (2001), Goldstein, Ozdenoren, and Yuan (2014), or Collin-Dufresne and Fos (2014).

value. Thus, the endogenous decision of intermediaries to participate generates a shift in market liquidity and firm value by affecting the firm's cost of capital.

Moreover, the paper contributes to the literature studying feedback effects between financial markets and corporate outcomes. In particular, it relates to the work of He and Milbradt (2014) and Malherbe (2014). He and Milbradt (2014) show that bond market illiquidity influences shareholders' endogenous default decision, which in turn feeds back into market illiquidity. Malherbe (2014) demonstrates that hoarding behavior and adverse selection may reinforce each other. He shows that future adverse selection makes current cash holdings more appealing; this effect may result in hoarding behavior and market breakdown. In the present model, illiquidity calls for an additional component in the firm's cost of capital, which significantly distorts corporate policies and depresses firm value. I also highlight how these effects can affect market illiquidity by exacerbating it.

Lastly, the paper relates to the literature studying the determinants of corporate cash holdings using dynamic models, e.g. Décamps, Mariotti, Rochet, and Villeneuve (2011), Bolton, Chen, and Wang (2011, 2013), and Hugonnier, Malamud, and Morellec (henceforth HMM, 2014). These papers show that financing frictions, such as costs or uncertainty in raising funds in primary markets, should increase a firm's propensity to keep a precautionary buffer. In these models, corporate liquidity is a strategic substitute to primary market liquidity. I contribute to this literature by showing that, other things equal, secondary market frictions effectively reduce the availability of internal liquidity as a risk-management tool. The model *does not* claim that secondary market liquidity boosts the firm's demand of cash reserves. Instead, it claims that market liquidity *allows* firms to maintain their optimal level of cash reserves, whose demand is driven by primary market frictions. Importantly, while the extant cash holdings models take the opportunity cost of cash as exogenous and impute such a cost to a free-cash-flow problem, the current model endogenizes this cost via stock market illiquidity.

The rest of this paper is organized as follows. Section 2 describes the model. Section 3 analyzes the effects of market illiquidity on corporate policies. Section 4 endogenizes market illiquidity and derives the internal-external liquidity loop. Section 5 examines the propagation effects generated by the liquidity loop. Section 6 concludes. All the proofs, along with motivating evidence, are gathered in the Appendix.

2 The Model

Time is continuous and uncertainty is modeled by a probability space $(\Omega, \mathcal{F}, P)$ equipped with a filtration $(\mathcal{F}_t)_{t\geq 0}$. The filtration represents the information available at time t and satisfies the usual conditions (Protter, 1990). Agents are risk-neutral and discount cash flows at a constant rate $\rho > 0$.

The firm. I consider a firm owned by an atomless continuum of identical shareholders. The firm operates a set of assets in place that generate a continuous and stochastic flow of revenues as long as the firm is in operation. The flow of revenues is modeled as an arithmetic Brownian motion $(Y_t)_{t\geq 0}$, whose dynamics evolve as

$$dY_t = \mu dt + \sigma dZ_t. \tag{1}$$

In this equation, the parameters μ and σ are strictly positive and represent the mean and volatility of the firm's cash flows, respectively. $(Z_t)_{t\geq 0}$ is a standard Brownian motion that represents random shocks to corporate cash flows.

The cash flow process in equation (1) implies that the firm is exposed to operating losses. If capital supply was perfectly elastic at the correct price, as in the Modigliani-Miller benchmark, operating shortfalls could be immediately covered by raising fresh funds on the capital market. In practice, however, firms often face capital supply frictions in the form of costs or uncertainty when raising new funds. When this is the case, corporate management may find it optimal to limit the risk of incurring refinancing costs or inefficient liquidations by retaining earnings in a cash reserve.

I account for this real-world feature and assume that the firm's access to outside funds is uncertain, as also in HMM and Bolton, Chen, and Wang (2013). More specifically, I model capital supply uncertainty as in HMM in that it takes time to secure outside financing in primary markets. Upon searching, the firm meets outside investors at the jump times of a Poisson process $(N_t)_{t\geq 0}$ with intensity λ.[5] This means that, if the firm

[5] As formally proved by HMM (Appendix A), bargaining between management and new investors over the terms of the issue simply results in a reduction of the effective rate of arrival of financing opportunities. I abstract from this feature, as it would affect the interpretation of the results only slightly.

decides to raise outside funds, the expected financing lag is $1/\lambda$ periods.[6] In Section 3.2.1, I extend the external financing choices available to the firm to bank credit to illustrate that the main predictions of this baseline setup are robust to debt financing.

I denote by $(C_t)_{t\geq 0}$ the firm's cash reserves at any $t \geq 0$. Throughout the paper, I will use internal liquidity, balance-sheet liquidity, or cash holdings interchangeably to indicate such reserves. Cash reserves earn a constant rate $r \leq \rho$ inside the firm. Whenever $r < \rho$, keeping cash entails an opportunity cost. Such a cost can be interpreted as a free-cash flow problem à la Jensen (1986) or as the tax disadvantages documented by Graham (2000). In contrast with extant cash holdings models — in which the strict inequality $r < \rho$ is needed to depart from the corner solution in which firms seek to pile an infinite cash buffer — I allow for the case in which the return on cash equals the market interest rate. In particular, the cash holdings process $(C_t)_{t\geq 0}$ evolves as

$$dC_t = \ rC_t dt + \mu dt + \sigma dZ_t - dD_t + f_t dN_t.$$

In this equation, $dD_t \geq 0$ represents the instantaneous flow of payouts at time t. The process $(D_t)_{t\geq 0}$ is non-decreasing, reflecting shareholders' limited liability. As a result, cash holdings decrease with payout payments as well as with operating losses. Moreover, $f_t \geq 0$ denotes the instantaneous flow of outside funds when financing opportunity arise, in which case management stores the proceeds in the cash reserves. The cash reserves then increase with external financing, and also with positive retained earnings and with the interest earned on cash holdings. D and f are endogenously determined in the following.

Management can distribute cash and liquidate the firm's assets at any time. Nonetheless, liquidation is inefficient as the cash flow that shareholders recover in liquidation, denoted by ℓ, is smaller than the firm's first best, μ/ρ. The cost of liquidation may be due to the low marketability of intangible assets or to fire sale prices related to unfavourable market conditions. For simplicity, I rule out partial liquidations. Because liquidation is inefficient, management shuts down operations when operating losses cannot be covered

[6] Alternatively, primary market frictions could be modeled as refinancing costs. These costs could be time-varying in a spirit analogous to Bolton, Chen, and Wang (2013). Specifically, the firm could be in one of two observable states of the world: A good state in which external financing is accessible and relatively cheap, and a bad state in which external financing is excessively expensive. The results in this environment would be qualitatively similar to those of the current model. In the present model, issuance costs are normalized to zero to keep the analysis tractable. This is without loss of generality: I solved a version of the model featuring refinancing costs and the results are analogous.

by drawing funds from the cash reserves or by issuing stocks on the primary market. Thus, the time of liquidation is given by

$$\tau = \inf \{t \geq 0 : C_t \leq 0\}, \tag{2}$$

and represents the first time that the cash buffer is depleted.

Secondary market transactions. Shareholders are ex ante identical and infinitely lived. Each of them has measure zero and cannot short-sell. As in Duffie, Gârleanu, and Pedersen (henceforth DGP, 2005), shareholders are exposed to private liquidity shocks that reduce their willingness to hold the asset. Specifically, shareholders keep their claim to cash flows for a random period that ends upon the arrival of Poisson (liquidity) shocks. These shocks are independent across investors and are described by the process $(M_t)_{t\geq 0}$, with intensity $\delta > 0$. In aggregate, this implies that the firm has a fixed fraction δdt of shocked investors over any interval $[t, t + dt]$.

Liquidity shocks are idiosyncratic and short-lived. Following DGP, they can be interpreted as take-it-or-leave-it outside investment opportunities, unpredictable financing needs, or unpredictable changes in hedging needs. That is, liquidity shocks lead to a sudden need for liquidity, which is modeled as a holding cost X_h. The holding cost is proportional to the value of the claim and can be interpreted as the opportunity cost of being locked into an undesirable position. The liquidity shock vanishes once the holding cost is borne. To avoid bearing this cost, liquidity-shocked investors can sell their claim in the secondary market. I assume that non-intermediated trading is costly to arrange on the spot, see e.g. Duffie (2010). Specifically, non-intermediated trading costs a proportion X_d of the value of the claim due to the costs of retrieving and processing the information on the stock, identifying counterparties, and routing the trade. In the following, I denote the cost associated with the best outside option available to shocked investors by:

$$\chi = \min (X_h, X_d).$$

Alternatively, shocked shareholders can sell their stock through an active intermediary. The sequence of events for a shocked investor is illustrated by Diagram 1.

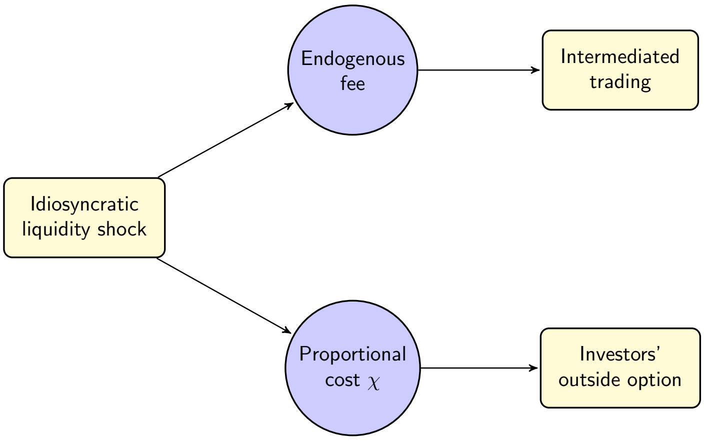

Diagram 1: Resolution of a liquidity shock.

In the following, the term "intermediaries" will denote liquidity providers that are actively ready to trade, then representing designated market makers, dealers, brokers, trading desks, or floor traders. As in previous contributions, I assume that the market for liquidity provisions is competitive and that liquidity providers are pure pass-through intermediaries with no inventories. Intermediaries' constant market presence allows them to get costlessly in contact with non-liquidity-shocked investors who are indifferent between staying out of the market or buying shares at their indifference price. As in He and Milbradt (2014), this mass of non-liquidity-shocked potential buyers is larger than the mass of liquidity-shocked shareholders.[7] In the baseline version of the model, I assume that the intermediation sector is frictionless and composed of a continuum of intermediaries. I relax this assumption in Section 4.

Intermediaries are active on both the ask and the bid side of transactions. On the bid side, intermediaries trade with the flow of shocked investors. Shocked investors are willing to trade with the intermediaries if the corresponding transaction price is more attractive than their best outside option. As in DGP, I model the surplus sharing from

[7]Extending their baseline setup, He and Milbradt (2014) microfound the search setting and provide conditions under which this assumption holds. They find that this is the case when there is an almost negligible recovery intensity from a liquidity shock (as in the current model, in which investors recover from liquidity shocks by bearing the holding cost or by selling the asset) and a very large mass of investors.

the meetings between shocked investors and intermediaries via Nash bargaining. In the following, I denote the bargaining power of financial intermediaries by $\eta \in [0,1]$. After completing the transaction, designated intermediaries and shocked investors part ways. The intermediaries can then re-sell the claim bought from shocked investors to non-liquidity-shocked investors on the sideline or to other intermediaries that are themselves in contact with non-liquidity-shocked investors.

Corporate decision-making. Throughout the paper, corporate management maximizes shareholder value. When maximizing shareholders' wealth, management takes into account secondary market trading frictions and solves:

$$\sup_{(D,f,\tau)} \mathbb{E}\left[\int_0^\tau e^{-\rho t}\left(dD_t - f_t\, dN_t - \Omega_t dt\right) + e^{-\rho\tau}\ell\right] \tag{3}$$

by choosing payouts (D), financing (f), and liquidation (τ) policies. As customary in dynamic inventory models, management maximizes the expected present value of payouts net of the claim to new investors until liquidation (the first and second term in the integral of 3), plus the present value of the liquidation value (the last term in 3). The novelty of the model is then the third term in the integral, Ω_t. This term represents the loss borne by liquidity-shocked investors and is endogenously determined in the following.

3 Market illiquidity and corporate policies

In this section, I analyze the effects of secondary market illiquidity on corporate policies by assuming that investors always have access to intermediaries' liquidity provision. I denote the value of the firm's equity in this framework by $V(c)$.

3.1 Trading frictions and shareholder value

On any time interval $[t, t+dt]$, a fraction δdt of shareholders is liquidity-shocked. Since non-liquidity-shocked investors on the sideline are willing to buy the asset at fair value, the intermediaries' gain on the ask side of the transaction is null.[8] In order to buy the asset from shocked investors, intermediaries thus pay at most the full value of the claim, i.e., the intermediaries' reservation price on the aggregate claim of shocked shareholders is $\delta V(c)$. In turn, shocked shareholders are willing to sell the stock to the intermediaries at a discount that is at most as large as the cost of their best outside option. That is, shocked shareholders require a price at least equal to $\delta V(c)(1-\chi)$ on aggregate. The surplus that intermediaries and shocked investors share is thus given by $\delta\chi V(c)$. Nash bargaining over this surplus delivers the rents that financial intermediaries extract from the shocked investors, given by $\eta\chi\delta V(c)$. Equivalently, the transaction price $\delta T(c)$ of the aggregate claim of the shocked investors is given by

$$\delta T(c) \equiv \ \eta\delta V(c)\,(1-\chi) + (1-\eta)\delta V(c) = \ (1-\chi\eta)\delta V(c). \tag{4}$$

This transaction price guarantees positive gains from trade to the intermediaries, as $\delta T(c) < \delta V(c)$. It also reduces the expected loss to shareholders when a liquidity shock hits, as $(1-\chi)\delta V(c) < \delta T(c)$. Therefore, investors favor the immediacy provided by intermediaries over their outside options. If investors' outside options improve (lower χ) or intermediaries' bargaining power gets weaker (lower η), the price at which intermediaries buy the asset is larger, making investors better off. In these cases, intermediaries can extract fewer rents, as supported by the experimental evidence in Lamoureaux and Schnitzlein (1997). Diagram 2 depicts the possible transactions taking place in the secondary market of the firm's stock.

Consider next the effects of secondary market transactions on corporate policies. Management selects the firm's payout, savings, liquidation, and financing policies to maximize the claim of the shareholders. Since the cost of holding cash is constant (as derived below) whereas the related benefits are decreasing, there exists some target level C_V above which it is optimal to distribute all excess cash to shareholders. Below C_V, management retains earnings in the cash reserves as a hedge for the times of need. Applying Itô's lemma, it

[8]As in He and Milbradt (2014), potential buyers are indifferent to staying out of the market or buying stocks at their fair market value. Under the assumption that the measure of non-liquidity-shocked investors is larger than the measure of shocked investors, Bertrand competition drives the surplus extracted by intermediaries on this side of the transaction to zero.

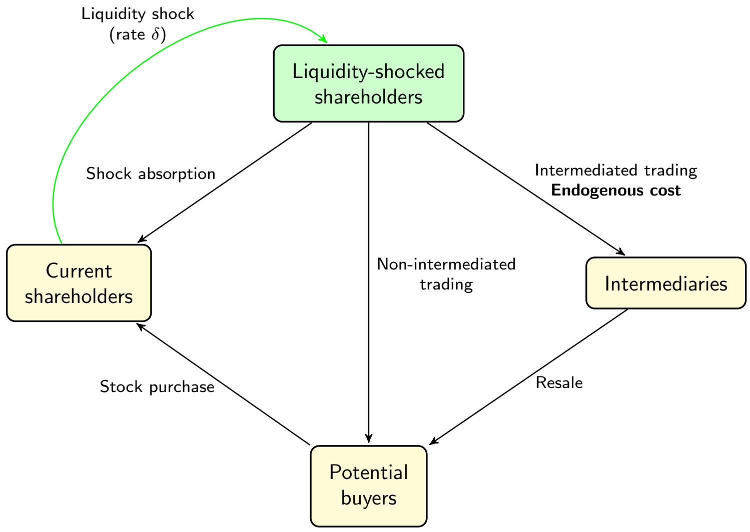

DIAGRAM 2: The secondary market for the firm's stock.

follows that shareholder value satisfies the following ODE

$$\rho V(c) = (rc + \mu) V'(c) + \frac{\sigma^2}{2} V''(c) + \lambda [V(C_V) - V(c) - C_V + c] + \delta [T(c) - V(c)] \quad (5)$$

for any $c < C_V$. The left-hand side of this equation is the return that investors require to invest in the stock notwithstanding the secondary market frictions. The first and the second terms on the right-hand side represent the effect of cash holdings and cash flow volatility on shareholder value. The third term represents the effect of primary issues on firm value. This term is the probability of raising funds times the surplus accruing to incumbent shareholders. The last term reflects the effects of secondary market frictions and provides the explicit expression for the term Ω_t in equation (3).

Substituting the transaction price (4) into equation (5), it follows that shareholder value satisfies

$$(\rho + \delta\chi\eta) V(c) = (rc + \mu) V'(c) + \frac{\sigma^2}{2} V''(c) + \lambda [V(C_V) - V(c) - C_V + c] . \quad (6)$$

The left-hand side of equation (6) reveals that, when the secondary market is illiquid, investors' shocks translate into a higher cost of capital. That is, investors require a larger rate of return to invest in the firm, which depends on the likelihood of a shock as well as on the associated cost. This additional compensation represents the premium that investors require ex ante to bear trading frictions when selling the stock. This premium, in turn, increases the opportunity cost of cash from $\rho - r$ to $\rho + \delta\chi\eta - r$.

The value of equity is solved subject to the following boundary conditions. Management does not liquidate the firm until cash is exhausted because liquidation is inefficient. Therefore, the condition

$$V(0) = \ell$$

holds as soon as the firm runs out of cash at $c = 0$. Moreover, it is optimal to distribute cash holdings above C_V with a specially designated dividend or a share repurchase.[9] This results in

$$V(c) = V(C_V) + c - C_V \quad \forall c \in [C_V, \infty), \tag{7}$$

where $V(C_V)$ represents the shareholders value at C_V, whose expression is reported in Proposition 1 below. That is, it is optimal to start paying out cash when the marginal value of one dollar inside the firm equals the value of a dollar if paid out to shareholders. Subtracting $V(c)$ from both sides of (7), dividing by $c - C_V$, and taking the limit as c tends to C_V delivers

$$\lim_{c \uparrow C_V} V'(c) = 1. \tag{8}$$

The target cash level C_V that maximizes shareholder value is determined by the super-contact condition

$$\lim_{c \uparrow C_V} V''(c) = 0, \tag{9}$$

see HMM. Solving for $V(c)$ leads to the following result.

Proposition 1 *When investors face an illiquid secondary market, shareholder value is*

[9] As investigated in Appendix A.1.1, it is never optimal for management to buy back the shares of the shocked investors for $c < C_V$.

given by

$$V(c) = \begin{cases} \Pi_V(c) + \left[\ell - \Pi_V(0)\right] L_V(c) + \left[V(C_V) - \Pi_V(C_V)\right] H_V(c) & 0 \leq c < C_V \\ c - C_V + V(C_V) & c \geq C_V \end{cases}$$

where the functions $L_V(c)$, $H_V(c)$ and $\Pi_V(c)$ are solved in closed-form in Appendix A. Management liquidates the firm when the cash reserves are depleted, raises the cash reserves up to C_V whenever a financing opportunity arises, and pays out all the cash in excess to C_V as dividend payments to shareholders. In particular, firm value at the target cash level C_V satisfies

$$V(C_V) = \frac{rC_V + \mu}{\rho + \delta\chi\eta}. \tag{10}$$

The expression for $V(c)$ in Proposition 1 admits the following interpretation. Over the region where management retains all earnings ($0 \leq c < C_V$), firm value is the sum of the present value of the claim that shareholders receive if the firm has an opportunity to raise external financing (the first term) plus the present value of payments that shareholders obtain if cash reserves reach either the liquidation threshold (the second term) or the payout threshold (the third term). These expressions are calculated by taking into account that, in the presence of secondary market frictions, the firm's discount rate has an additional component, i.e. an illiquidity premium. This premium increases when investors' outside options deteriorate (χ increases), when the demand for liquidity is larger (equivalently, when liquidity shocks are more frequent, δ increases), or when the bargaining power of financial intermediaries is larger (η increases). In these cases, intermediaries can extract more rents from shocked shareholders, transaction costs are larger, and shareholders' value decreases. Conversely, when the secondary market is perfectly liquid as when $\chi = 0$ or $\eta = 0$ (or investors do not face liquidity shocks, $\delta = 0$), liquidity shocks do not impact corporate policies because shareholders can sell their claims immediately and at no cost. In this case, optimal policies and firm value are as in HMM (Proposition 1).

3.2 The effects of market illiquidity

Section 3.1 illustrates that stock market illiquidity affects firm value. I now investigate the channels through which this relation works by examining the implications for internal and external financing, payout, and capital allocation decisions.

Internal and external liquidity. Extant cash holdings models show that internal liquidity is a strategic substitute to primary market liquidity (see Bolton et al., 2011, Décamps et al. 2011, and HMM). If external financing is costly or uncertain, firms hold cash reserves as a precautionary hedge. The present model additionally demonstrates that internal liquidity is a strategic complement to secondary market liquidity. That is, when the secondary market for a stock is illiquid, the issuing firm reduces its cash reserves because the opportunity cost of holding cash is larger. Denoting by C^* the target level of cash holdings when the secondary market is perfectly liquid, the following result holds.

Proposition 2 *When the access to external financing tightens (lower λ), a firm increases its target level of cash holdings. However, secondary market frictions reduce the firm's ability to engage in this precautionary policy, and the target level of cash holdings C_V is below C^*. The target level C_V decreases with the severity of trading frictions (η and χ) and with the frequency of liquidity shocks (larger δ).*

When making savings and payout decisions, management trades off precautionary concerns — driven by primary market frictions — against the need to make the stock attractive to investors — driven by secondary market frictions. When the cost of transacting the security rises (χ or η increase), the illiquidity premium to the investors increases, and so does the opportunity cost of holding cash. Firms then decrease their target level of cash holdings and increase their payout rate. In so doing, they compensate ex ante the frictions that investors bear ex post when selling the stock. This result can help rationalize the positive relation between internal and external liquidity discussed in the introduction. It is also consistent with the findings of Bekaert, Harvey, and Lundblad (2007), who document a negative relation between market liquidity and dividend yield shocks. Furthermore, the model gives theoretical ground to the evidence that firms whose investors have longer horizons hold more cash, as documented by Harford, Kecskés, and Mansi (2012). In the model, the horizon of investors is represented by $1/\delta$. As illustrated in Proposition 2, this measure is positively related to target cash holdings.

While secondary market frictions have been the subject of considerable attention since Amihud and Mendelson (1986), the present paper is the first to highlight that these frictions lead to an increase in the opportunity cost of cash. This feature marks a stark difference between the current model and the extant cash holdings models. To rationalize the optimality of non-infinite target cash levels, previous cash holdings models assume that the rate of return on cash r is strictly lower than the discount rate ρ. In these models, the opportunity cost of cash is $\rho - r$. Conversely, the current model delivers finite target cash levels even when r and ρ coincide. The reason is that market illiquidity calls for an additional component in the firm's cost of capital, and the opportunity cost of cash is $\rho + \delta\eta\chi - r$. Under the baseline parametrization reported in Table 1 — whereby the illiquidity premium amounts to 1.35% — the target level of cash holdings is about 0.6346 if the market is illiquid, whereas it is 0.7156 if the market is perfectly liquid. Under the same parametrization but setting $r = \rho = 5\%$, the target cash level is about 0.7083 if the market is illiquid whereas it is infinite if the market is perfectly liquid.

The model *does not* claim that secondary market liquidity boosts the firm's demand of cash. Instead, the model claims that market liquidity *allows* firms to keep their optimal level of cash reserves, whose demand is driven by capital supply frictions on primary markets. In the limit case in which the secondary market is perfectly liquid (that is, $\chi = 0$ or $\eta = 0$) the firm keeps the amount of cash that is solely driven by precautionary considerations. When the secondary market is illiquid, firms "waste" cash by making larger payouts, and they maintain a suboptimal level of cash reserves. In other words, the model predicts that secondary market frictions affect the availability of internal liquidity as a risk-management tool. This may be detrimental for firm value, since risk management directly affects financial flexibility and increases the value of constrained firms (see Bonaimé, Hanskin, and Harford, 2014, and Pérez-González and Yun, 2013). These aspects are investigated in the next paragraphs.

Market illiquidity and financial constraints. I now investigate the effect of market illiquidity on *external* financing. Namely, I examine how the size and likelihood of equity issues respond to market illiquidity. Corollary 3 is a direct implication of Proposition 2.

Corollary 3 *Ceteris paribus, secondary market frictions reduce the size of equity issues, as the inequality $C^* - c > C_V - c$ holds for any given cash level c.*

Corollary 3 illustrates that the illiquidity-driven reduction in target cash holdings leads to a decrease in the *size* of equity issues. This result is consistent with the evidence that savings are the primary use of issuance proceeds (McLean, 2010, and Eisfeldt and Muir, 2014). To investigate whether the *likelihood* of issuing equity is also affected, I define the probability of raising external financing $P^f(c, C_V)$

$$P^f(c, C_V) = E_c\left[1 - e^{-\lambda\tau(C_V)}\right],$$

and complementarily the probability of foreclosing while searching for outside funds, $P^l(c, C_V) = E_c\left[e^{-\lambda\tau(C_V)}\right]$. Secondary market frictions enter these probabilities through $\tau(C_V)$, representing the first time that the cash process, reflected from above at C_V, is absorbed at zero. I establish the following result.

Proposition 4 *In the presence of market illiquidity, the probability of external financing decreases, $P^f(c, C_V) < P^f(c, C^*)$, whereas the probability of foreclosing increases, $P^l(c, C_V) > P^l(c, C^*)$.*

Proposition 4 implies that secondary market frictions have an adverse impact on the firm's access to outside equity, consistent with the evidence in Campello, Ribas, and Wang (2011), Butler, Grullon, and Weston (2005), and Stulz, Vagias, and van Dijk (2013). Complementarily, firms whose stocks are illiquid are more exposed to inefficient liquidations, as documented by Li and Xia (2014). The model then suggests that secondary market illiquidity exacerbates firm's financial constraints; in particular it deliver three testable implications. First, firms issuing illiquid stocks keep less precautionary liquidity. Second, the access to the primary market is relatively more uncertain for these firms. Third, their equity issues are smaller.

These effects are significant even for small bid-ask spreads, i.e. for those stocks that are traded on the most liquid markets. For instance, a bid-ask spread equal to 0.26% (i.e., the most recent average for NYSE stocks reported by Hameed, Kang, and Vishwanathan, 2011) leads to a 2.4% decrease in the target level of cash holdings. In a cross-section of firms with cash reserves uniformly distributed between zero and the target level, the average probability of liquidation is 2.5% larger if the secondary market is illiquid. For these firms, market illiquidity leads to a 4.9% drop in firm value.

Market illiquidity and capital allocation. I now analyze the effects of market illiquidity on firm's capital allocation decisions. To keep the analysis tractable, I assume that the firm has monopolistic access to an investment opportunity. Specifically, the firm can pay a lump sum $I > 0$ to increase its income stream from dY_t to dY_t^+, defined as

$$dY_t^+ = dY_t + (\mu_+ - \mu)dt\,.$$

In this equation, μ_+ is strictly larger than μ, i.e., the option increases the firm's average profitability. Following Décamps and Villeneuve (2007) and HMM, I derive the zero-NPV cost, representing the maximum amount that the firm is willing to pay for the option.

Proposition 5 *When the secondary market is illiquid, the zero-NPV cost is given by*

$$I_V = \frac{\mu_+ - \mu}{\rho + \delta\chi\eta} - (C_{V+} - C_V)\left[1 - \frac{r}{\rho + \delta\chi\eta}\right], \tag{11}$$

where C_{V+} denotes the target cash level after the option is exercised.

Using similar arguments, the zero-NPV cost with perfectly-liquid secondary market is given by

$$I^* = \frac{\mu_+ - \mu}{\rho} - (C_+^* - C^*)\left(1 - \frac{r}{\rho}\right), \tag{12}$$

where C_+^* denotes the post-investment target cash level. The comparison between (11) and (12) illustrates that secondary market frictions decrease the investment reservation price, then leading to underinvestment. Specifically, the severity of the under-investment problem is given by[10]

$$\Delta I_V = I_V - I^* \approx -\frac{\mu^+ - \mu}{\rho}\frac{\delta\chi\eta}{\rho + \delta\chi\eta} < 0. \tag{13}$$

For $I \in [I_V, I^*]$, the growth option has a negative NPV if the secondary market is illiquid but it has a positive NPV if the market is perfectly liquid. Because market illiquidity increases the firm's cost of capital, growth opportunities are less profitable and the maximum investment cost decreases.

Considering again an average bid-ask spread equal to 0.26%, and assuming that the option leads to a 20% increase in operating profitability as $\mu_+ = 1.2\,\mu$, ΔI_V represents a 5% decrease in the zero-NPV cost. The gap increases in η and χ; in other words, the

[10]In fact, the difference of the target cash level plays a second order effect.

under-investment problem is more severe when transacting the security is more costly. In addition, the model suggests that a decrease in the investors' horizon (a decrease in $1/\delta$) can effectively cause myopic decisions at the firm level if the market is illiquid. In these cases, management foregoes profitable investment opportunities to favor payouts to current shareholders.

3.2.1 Bank credit as an alternative source of liquidity

The analysis so far has considered internal cash as the only source of immediate liquidity available to firms. In practice, firms often access bank credit; in particular they can secure credit lines that may be used in times of need (see Sufi, 2009). In this section, I assess the robustness of the results delivered so far to this additional source of liquidity.

A credit line is a source of funding that the firm can access at any time up to a pre-established limit that I denote by L. I follow Bolton, Chen, and Wang (2011) by assuming that the firm pays a constant spread β over the risk-free rate on the amount of credit used, whenever it effectively draws funds from the credit line. Because of this cost, it is optimal for the firm to access the credit line only when cash holdings are exhausted. The firm then uses internal cash as the marginal source of financing if $c \in [0, C_V(L)]$ (in the following, the cash region), where $C_V(L)$ denotes the target cash level in this environment. Conversely, the firm draws funds from the credit line when $c \in [-L, 0]$ (the credit line region). Firm value satisfies (6) in the cash region, whereas it satisfies

$$(\rho + \delta\chi\eta)\, V(c) = [(\rho + \beta)c + \mu]\, V'(c) + \frac{\sigma^2}{2} V''(c) + \lambda\, [V(C_V) - V(c) - C_V + c] \qquad (14)$$

in the credit line region. On top of the smooth-pasting and super-contact conditions at $C_V(L)$ similar to (8) and (9), the system of ODEs (6)-(14) is solved subject to

$$\begin{aligned} V(-L) &= \max\,[\ell - L, 0]\,, \\ \lim_{c\uparrow 0} V(0) &= \lim_{c\downarrow 0} V(0), \\ \lim_{c\uparrow 0} V'(0) &= \lim_{c\downarrow 0} V'(0). \end{aligned}$$

The first condition means that, if $\ell \geq L$, the credit line is fully secured and shareholders are residual claimants in liquidation. The second and the third conditions guarantee continuity and smoothness at the point where the cash and the credit line regions are pieced together.

Figure 3 compares the effects of market liquidity on corporate policies, when a firm does and does not access bank credit. Benchmark parameters are as in Table 1; in addition I set $L = 0.3$ (the credit line is fully collateralized) and $\beta = 1.5\%$ consistently with Sufi (2009). The figure shows that the patterns of a firm exploiting bank credit closely follow those of a firm managing liquidity only with internal cash. Access to bank credit decreases the target cash level in absolute magnitude, but market illiquidity still lowers this target level below the benchmark with perfect market liquidity. Financing, liquidation, and capital allocation decisions are almost unaffected. In the following, I therefore abstract from bank debt, without loss of generality.

4 Endogenous liquidity and feedback loops

I now relax the assumption that investors can always access the intermediaries' liquidity provision. Specifically, I endogenize the intermediaries' decision to participate in the market for the stock. This allows me to shed light on the two-way connection between market illiquidity and corporate value and to introduce the notion of feedback loop. I denote the value of equity in this framework by $S(c)$.

4.1 The two-sided link between trading frictions and firm value

Following previous contributions, I assume that intermediaries acting as liquidity providers operate in a perfectly competitive market (for consistency with real-world evidence see Wahal, 1997, and Weston, 2000). Intermediaries who are active in the market provide liquidity to shocked investors. When meeting with intermediaries, shocked shareholders unload their holdings in the intermediary sector, and the intermediaries earn the transaction cost determined in Section 3.1. Intermediaries' constant market presence, however, entails a flow cost γ that is paid as long as the intermediary is active in the market. This participation fee can be interpreted as the cost of monitoring market movements and handling the orders, e.g., order routing, execution, and clearing (see Stoll, 2003), as well as the cost of funding (see Brunnermeier and Pedersen, 2009).

Intermediaries are an infinite and atomless mass, but free-entry means that only a finite measure θ_t is active. The measure of active intermediaries determines the instantaneous probability with which shareholders find a liquidity provider when a liquidity shock hits. I define this probability as

$$\pi_t \equiv \frac{\theta_t}{\alpha + \theta_t}, \tag{15}$$

where $\alpha > 0$ captures inefficiencies that hamper the ease with which such meetings occur, e.g., rigidities in trading protocols, or technological deficiencies. A higher value for α implies a lower probability of a meeting. Conversely, if θ_t tends to infinity, investors find an intermediary with probability one (as in Section 3). The model captures the notion of competition by order flow since the probability with which intermediaries contact investors, given by $\frac{\pi_t}{\theta_t}$, is decreasing in θ_t (see also Lagos and Rocheteau, 2007).

The competitiveness of the market for liquidity services implies that the equilibrium measure of active intermediaries is determined by the zero-profit condition, given by

$$\frac{\pi(\theta)}{\theta}\, \chi\eta\, \delta S(c) - \gamma = 0. \tag{16}$$

The first term in this equation is the expected flow accruing to an active intermediary. This term is given by the transaction fee resulting from equation (4), multiplied by the probability that such a trade occurs. The second term denotes the cost of being active on the market. Plugging (15) into (16) delivers the equilibrium measure of active intermediaries $\theta(c)$ and the probability of intermediated trading $\pi(c)$:[11]

$$\theta(c) = \left(\frac{\delta\chi\eta}{\gamma} S(c) - \alpha\right)^+ \quad \text{and} \quad \pi(c) = \left(1 - \frac{\alpha\gamma}{\delta\chi\eta S(c)}\right)^+.$$

Since these quantities must be non-negative, it follows that intermediaries participate in the market for the stock if the value of equity is larger than a critical value $\underline{S}$ uniquely defined by

$$\underline{S} \equiv \frac{\alpha\gamma}{\chi\delta\eta}.$$

[11] Alternatively, market liquidity could be endogenized via the intermediaries' bargaining power η. However, the effects on market liquidity (and thus on firm value) would be analogous: frictions in intermediaries' participation would lead to an increase in the intermediaries bargaining power η, which in turn would lead to larger cost borne by shocked investors and to a larger illiquidity premium at the firm level.

Shareholders thus have a positive probability to exploit the liquidity provision of intermediaries *if* the value of equity is sufficiently high, namely if $S(c) > \underline{S}$. If instead the value of equity is lower than $\underline{S}$, intermediaries stay away from the market of the stock and investors turn to their outside options. They directly look for a trading counterparty if $X_h > X_d$. Otherwise they keep the asset. The critical value $\underline{S}$ that stimulates the provision of liquidity in the market for the stock increases in the frictions faced by intermediaries, i.e., in the participation cost γ and in the market inefficiencies α. Conversely, $\underline{S}$ decreases if intermediaries' bargaining power is stronger (η is larger), if the investors' outside options deteriorate (higher χ), or if the demand for liquidity is larger (δ is higher) — which all increase the expected flow of rents to the intermediary sector.

Intermediaries' participation and shareholder value. When no intermediaries participate in the market of the stock, investors' liquidity shocks entail larger costs. The anticipation of this outcome implies that the firm needs to promise a higher premium to its investors. As before, there exists a target level of the cash buffer, denoted by C_S, such that management retains operating cash flows when $c < C_S$. In addition, shareholder value in the retention region depends on secondary market liquidity that, in turn, depends on the participation of financial intermediaries. The following result holds.

Proposition 6 *There exists at most one threshold $\underline{C} \in [0, C_S]$, such that $S(\underline{C}) = \underline{S}$. Shareholders meet intermediaries with positive probability if $c > \underline{C}$.*

Proposition 6 singles out an interval $\underline{C} < c < C_S$ featuring a strictly positive measure of intermediaries that are active in the market for the stock. When this is the case, shareholders can exploit intermediaries' liquidity provision with probability $\pi(c) > 0$. Conversely, intermediaries do not participate in the market of the stock if $c < \underline{C}$; in this case, investors need to turn to their outside options. It is worth noting that two corner cases may arise. If $\ell > \underline{S}$, the probability of intermediated trading $\pi(c)$ is strictly positive for any $c \in [0, C_S]$. If, on the other hand, the inequality $\underline{S} > S(C_S)$ holds, the probability of intermediated trading is always zero. In the following, I derive shareholder value by considering the more general case in which $\ell < \underline{S} < S(C_S)$, whereby $\underline{C}$ lies between the liquidation and the target cash level (the corner cases are reported in Appendix B.2).

For $c < \underline{C}$, liquidity-shocked shareholders exploit their best outside option and bear the cost $\delta\chi S(c)$ on aggregate. Shareholder value then satisfies the following ODE

$$(\rho + \delta\chi)\, S(c) = \ (rc + \mu)\, S'(c) + \frac{\sigma^2}{2} S''(c) + \lambda\, [S(C_S) - C_S + c - S(c)]\, . \tag{17}$$

This equation admits an analogous interpretation to (6), but the return required by the investors (the left-hand side) is larger. The reason is that the inability to exploit intermediaries' liquidity provision increases the investors' transaction cost, the illiquidity premium, and, hence, the firm's cost of capital.

For $c > \underline{C}$, investors have a positive probability of meeting an active intermediary; this probability increases in c. In these instances, the expected loss associated with a liquidity shock (Ω_t in equation (3)) is

$$\big[\pi(c)\chi\eta\ \delta S(c) + \big(1 - \pi(c)\big)\chi\delta S(c)\big]\, dt = \ \chi\delta S(c)\ \big[1 - \pi(c)(1-\eta)\big]\, dt.$$

The first term on the left-hand side represents the expected cost of intermediated trading to the shocked shareholders, represented by the transaction cost times the probability of meeting an intermediary. The second term represents the loss associated with their best outside option, which is tapped if investors do not find an intermediary when a shock hits (an event with probability $1 - \pi(c)$). For any $c \geq \underline{C}$, shareholder value then satisfies

$$(\rho + \delta\chi\eta)\, S(c) = (rc + \mu)\, S'(c) + \frac{\sigma^2}{2} S''(c) + \lambda\, [S(C_S) - C_S + c - S(c)] - \frac{\alpha\gamma(1-\eta)}{\eta}. \tag{18}$$

Equation (18) illustrates that, for any $\underline{C} < c < C_S$, the return required by the investors (the left hand side) is the same as in the case in which intermediaries' liquidity provision is infinitely elastic. The last term on the right-hand side, in addition, captures the idea that frictions that are borne by intermediaries also matter for investors. In fact, these frictions increase the expected cost of trading by decreasing the investors' ability to unwind the stock in the intermediary sector. This term is akin a flow cost that is larger if intermediaries' participation frictions, α and γ, increase. Note that, when intermediaries' participation is endogenous, the bargaining constant η carries two separate effects. An increase in η makes it more attractive for intermediaries to participate, but it also increases the cost of capital at the firm level.

Continuity and smoothness at the trading threshold mean that the system of ODEs (17)-(18) satisfies the following conditions,

$$\lim_{c\uparrow \underline{C}} S(c) = \lim_{c\downarrow \underline{C}} S(c) \quad \text{and} \quad \lim_{c\uparrow \underline{C}} S'(c) = \lim_{c\downarrow \underline{C}} S'(c)\,.$$

In addition, $S(c)$ is subject to the boundary conditions at the liquidation threshold and at the target cash level analogous to those in Section 3.1:

$$S(0) - \ell = \lim_{c\uparrow C_S} S'(c) - 1 = 0.$$

Again, the target cash level is identified by the super-contact condition, $\lim_{c\uparrow C_S} S''(c) = 0$. Solving for $S(c)$ leads to the following result.

Proposition 7 *When liquidity provision by intermediaries is endogenous, and $\underline{C} \in (0, C_S)$, shareholder value is characterized by three regions*

$$S(c) = \begin{cases} \Pi_d(c) + \left[\ell - \Pi_d(0)\right] L_d(c) + \left[\underline{S} - \Pi_d(\underline{C})\right] H_d(c) & 0 \le c < \underline{C} \\ \Pi_u(c) + \left[\underline{S} - \Pi_u(\underline{C})\right] L_u(c) + \left[S(C_S) - \Pi_u(C_S)\right] H_u(c) & \underline{C} \le c < C_S \\ c - C_S + S(C_S) & c \ge C_S \end{cases}$$

where the functions $L_i(c)$, $H_i(c)$, and $\Pi_i(c)$, $i \in \{d, u\}$ are defined in the Appendix. In addition, the value of equity at the target cash level C_S satisfies

$$S(C_S) = \frac{rC_S + \mu - \alpha\gamma(1-\eta)\eta^{-1}}{\rho + \delta\chi\eta}.$$

Endogenous participation implies that liquidity provision may dry-up when the frictions faced by intermediaries are too severe (α and γ are large), or shareholder value is too low (see also Bolton and von Thadden, 1998). In these instances, intermediated transactions are frozen, and the expected loss to shocked shareholders is the largest. In the following, I investigate the implications of this result.

4.2 Self-reinforcing effects

The analysis in Section 3 shows that transaction costs in secondary markets increase the return required by investors, then constraining corporate policies and decreasing firm value. When the liquidity provision by financial intermediaries is endogenous, this illiquidity-driven drop in firm value feeds back into stock market illiquidity. That is, a self-reinforcing feedback effect arises, generating an internal-external liquidity loop.

4.2.1 The internal-external liquidity loop

As in Section 3, in which intermediaries' participation is frictionless, market illiquidity leads investors to require an additional compensation to invest in the firm. This premium constrains corporate policies — in particular it leads to an increase in the firm's payout rate and to a decrease in corporate cash holdings — and decreases corporate value. When intermediaries' participation is costly, this illiquidity-driven drop in firm value feeds back into market liquidity. In fact, it shrinks the expected rents flowing to the intermediary sector and affects the participation constraint of a single intermediary. A strategic complementarity thus arises that makes market liquidity endogenous.

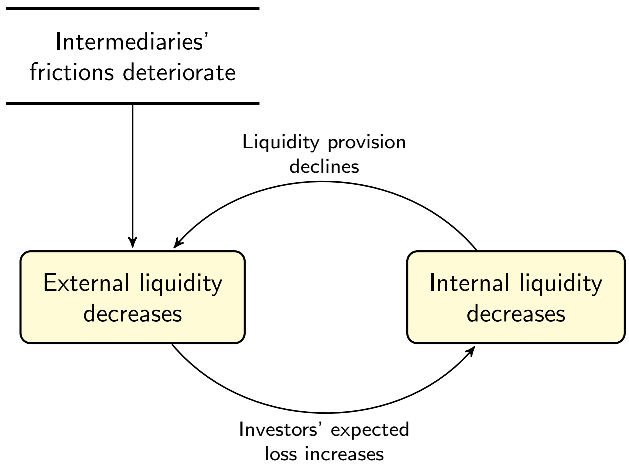

Diagram 3: The internal-external liquidity loop.

As the expected flow of rents to the intermediary sector decreases, some intermediaries stay away from the market. Intermediated trading is less likely, and the expected loss when a liquidity shock hits becomes more severe. This effect leads to a further increase in the illiquidity premium, which further decreases the target cash level and the value of equity. Effectively, the firm's payout rate increases to offset the tighter frictions faced by investors when trading. The further decrease in the value of equity feeds back again into the measure of active intermediaries by reducing it, and so on. A loop between internal and external liquidity then arises, whose outcome delivers a unique target cash level (C_S). The target level C_S is suboptimal from a precautionary perspective.

Proposition 8 *Frictions affecting the liquidity provision of intermediaries lead to a decrease in the firm's target level of cash holdings in that $C_S < C_V < C^*$. The decrease in C_S is more severe if these frictions, α and γ, are larger.*

As intermediaries' liquidity provision may dry up, the anticipation of this effect increases the illiquidity premium at the firm level, then decreasing ex ante the target level of corporate liquidity. In the following, I illustrate the implications of this loop on firm value as well as on market liquidity.

4.2.2 The firm response to the imperfect liquidity provision

If shareholder value increases in intermediaries' participation and the rents accruing to the intermediary sector increase in shareholder value, this complementarity should magnify the detrimental effects of market illiquidity on firm value highlighted in Section 3.2. To confirm this intuition, Figure 4 illustrates the target level of cash reserves, the average probabilities of external financing and liquidation,[12] and capital allocation decisions, when market liquidity is perfect (solid line), when frictions only affect the investors' side (as in Section 3, dashed line), and when frictions also affect intermediaries' participation (in the presence of the feedback loop, dotted line).

Confirming Proposition 8, internal liquidity is lowest when the feedback loop is at work, i.e., when participation is endogenous. The wedge between C_S and C^* (and between C_S and C_V) is larger when frictions faced by financial intermediaries — i.e., participation costs γ and market rigidities α — are more severe. To counteract market illiquidity, the firm needs to pay a larger illiquidity premium. The opportunity cost of cash increases even further, so the firm reduces its liquidity buffer and its payout rate increases. Yet, this comes at the cost of being more exposed to inefficient liquidations. In each graph of Figure 4, the dotted line becomes flat at the critical level of $\bar{\gamma}$ and $\bar{\alpha}$ where the inequality

[12] As in Section 3.2, these average probabilities are computed for a cross-section of firms with cash reserves uniformly distributed between zero and the target level.

$\underline{S} \geq S(C_S)$ binds. For any $\gamma > \bar{\gamma}$ and $\alpha > \bar{\alpha}$, intermediaries stay away from the market of the stock for any c and investors can never exploit their liquidity provision. Hence, a further increase in α and γ above such critical levels has no effect on corporate decisions.

In addition, the feedback loop may exacerbate the under-investment problem in Proposition 5. Suppose that the severity of the frictions in the intermediary sector, and the adjustment in firm value thereof, are such that no intermediaries are active in the market for the stock (i.e., $\underline{S}$ is larger than $S(C_S^+)$ and $S(C_S)$). As the firm needs to pay a larger illiquidity premium to its investors, the under-investment problem becomes more severe and is approximately given by

$$\Delta I_S \;\approx\; -\frac{\mu^+ - \mu}{\rho}\,\frac{\delta\chi}{\rho + \delta\chi} < \Delta I_V \; < 0,$$

where $\Delta I_S = I_S - I^*$ and ΔI_V is defined in (13).[13] By increasing firm profitability, however, investment may stimulate liquidity by attracting the participation of liquidity providers. Indeed, there exist parametric regions (i.e., $\bar{\alpha} < \alpha < \bar{\alpha}^+$ and $\bar{\gamma} < \gamma < \bar{\gamma}^+$, where $\bar{\gamma}^+$ and $\bar{\alpha}^+$ bind $\underline{S} = S(C^+)$ and $\bar{\gamma}$ and $\bar{\alpha}$ are as above) such that liquidity providers participate in the market for the stock only *after* the firm has invested. In these cases (corresponding to the interval in which the dotted line is steeper), a decrease in α or γ leads to a larger reduction in the underinvestment problem.[14]

Overall, the decrease in equity value is sharper in the presence of the feedback loop (see Figure 6). For a bid-ask spread of 0.26%, the loop leads to an average 7.7% drop in firm value, being roughly 3% larger than when the feedback effect is not accounted for (i.e. as in Section 3.2). An increase in the participation cost γ or in the inefficiency α shifts $S(c)$ downwards, decreasing equity value for any $c > 0$. An increase in χ shifts both $S(c)$ and $V(c)$ downwards, but the relative gap between $V(c)$ and $S(c)$ narrows. While increasing the transaction costs to investors, an increase in χ increases the proportional fee accruing to the intermediary sector, then crowding new intermediaries in. An increase in η has a similar effect (Figure 6 reports this static): While directly increasing the discount rate

[13] Using arguments analogous to those in Proposition 5, the zero-NPV cost when the participation of intermediaries is endogenous is $I_S = \frac{\mu_+ - \mu}{\rho + \delta\chi} - (C_{S+} - C_S)\left[1 - \frac{r}{\rho + \delta\chi}\right]$.

[14] Note that, for $\alpha < \bar{\alpha}$ and $\gamma < \bar{\gamma}$ (corresponding to the interval where the dotted line gets flatter and closer to the dashed line), the severity of the underinvestment problem is close to ΔI_V, because the illiquidity premium at the target cash level is $\delta\chi\eta$ both before and after the exercise of the growth option, and the difference in the target cash levels plays a second-order role.

in the intermediated-trading region, it decreases the flow cost of investor's participation that firms need to pay in that region (the last term in equation (18)).

4.2.3 Microfoundations for the supply of liquidity

I now analyze the implications of the model for secondary market transactions, in particular for the intermediaries' decision to provide liquidity in the market for the stock. Firstly, the model predicts that secondary market illiquidity is self-fulfilling, as shareholders' fear of future illiquidity generates current illiquidity. In the model, fear of illiquidity brings an additional component to the cost of capital. By decreasing corporate value, this premium causes an effective reduction in the liquidity provision, which exacerbates trading frictions for investors and makes the market more illiquid.

Secondly, the model provides a microfoundation for the observation that intermediaries sometimes withdraw from providing liquidity, as documented by Hameed, Kang, and Viswanathan (2010). The model predicts that this effect is asymmetric, that is, negative shocks to firm value have larger impact on market liquidity than positive shocks (see also Chordia, Roll, and Subrahmanyam, 2001) as shown in Proposition 9.

Proposition 9 *For any $c > \underline{C}$, the measure of active intermediaries $\theta(c)$ and the probability of intermediated trading $\pi(c)$ are non-decreasing and concave functions of c. For any $c \leq \underline{C}$, intermediated transactions are frozen as there are no active intermediaries.*

Proposition 9 shows that, above $\underline{C}$, the probability of intermediated trading increases with internal liquidity c. While making the firm more resilient, an increase in c stimulates the participation of liquidity providers that, in turn, improves the liquidity of the secondary market. Internal and external liquidity are then positively related. The increase is less important when c is high, because the firm is less constrained and firm value is less sensitive to internal liquidity ($S(c)$ is increasing and concave in c, see Appendix B).

Below the threshold $\underline{C}$ (corresponding to the light-shaded area in the left plots of Figure 6), the expected gain from providing liquidity in the market does not compensate for the cost of constant market presence. In these cases, intermediated transactions are frozen, an event that is more likely to occur when the trading threshold $\underline{C}$ is larger. In the limit case in which $\underline{C}$ is greater than C_S, intermediaries never step into the market for the stock. This is the case for the parameter values in the dark-shaded area of the left plots of Figure 6, which is delimited by the critical values $\bar{\gamma}$, $\bar{\alpha}$, $\bar{\eta}$, and $\bar{\chi}$ that bind the inequality $\underline{S} \geq S(C_S)$. The model is then flexible enough to depict those stocks having a large number of market makers, those followed by a limited number of dealers, as well as OTC stocks (see e.g., Ang, Shtauber, and Tetlock, 2013).

In line with the extant literature documenting the effect of intermediaries' constraints on market liquidity (e.g., Comerton-Forde et al., 2010, Aragon and Strahan, 2012, Anand et al., 2013, or Getmansky et al., 2014), Figure 6 shows that an increase in γ and α delays intermediaries' participation, i.e., the trading threshold $\underline{C}$ increases in these parameters. As a result, the average probability of intermediated trading decreases dramatically in the severity of the frictions α and γ, and becomes insensitive at the critical levels $\bar{\gamma}$ and $\bar{\alpha}$. If investors' outside options improve (lower χ), or intermediaries' bargaining power decreases (lower η), participation in the intermediary sector decreases, but the firm's cost of capital also decreases. The figure shows that the former effect dominates, and the average probability of intermediated trading increases in η and χ (above $\bar{\eta}$ and $\bar{\chi}$).

Moreover, the model relates liquidity provision to firm characteristics. It predicts that less profitable firms should be followed by fewer intermediaries, as documented by Næs, Skjelorp, and Ødegaard (2011). In fact, a decrease in μ leads to an increase in the trading threshold $\underline{C}$ and to a drop in the probability of intermediated trading. This effect is consistent with the sharp deterioration in market liquidity for delisted firms due to bankruptcy violations (see Harris, Panchapagesan, and Werner, 2008), as well as with the market freezes observed during the recent crisis during which professional traders did not participate in markets when insolvency had become a concern. Moreover, stocks issued by firms with a more uncertain access to the capital market (smaller λ) should be less liquid. Being more constrained and less financially resilient, these firms are less attractive to intermediaries. Conversely, stocks issued by firms whose assets are more tangible should be more liquid;[15] this is consistent with Gopalan, Kadan, and Pevzner (2012). Finally, cash flow volatility decreases the participation of financial intermediaries. These results are consistent with the evidence in Anand, Irvine, Puckett, and Venkataraman (2013) documenting that intermediaries reduce trading activity in small and volatile stocks.

[15] I assume asset tangibility ϕ to be inversely related to the gap between the liquidation value ℓ and the firm's first best μ/ρ, as in HMM. In particular, I set $\ell = \phi \frac{\lambda}{\lambda+1} \frac{\mu}{\rho}$ to also capture low marketability or fire sale prices related to unfavourable market conditions.

5 Propagation effects

The analysis so far has shown that shocks affecting the participation of intermediaries, though orthogonal to fundamentals, can affect firm value. This effect is stirred by the existence of spillovers in pay-offs among agents in the marketplace, which cause shocks to propagate across the financial sector and the corporate sector.

Propagation and amplification effects. I start by analyzing the effect of participation shocks to intermediaries, due to an increase in γ — making market presence more costly — or an increase in α — making transactions more difficult, e.g., due to more demanding trading protocols. An increase in these quantities directly decreases the supply of liquidity by making the free-entry condition binding with fewer intermediaries. If corporate value was independent of market liquidity, the impact of these shocks would be limited to this direct effect. This effect, however, makes transactions more costly to investors, which is the channel through which shocks to the supply of liquidity hit corporations. In fact, intermediaries' exit generates a negative externality by increasing the cost of illiquidity borne by corporations and then decreasing firm value. As a result, the decrease in intermediaries' participation gets amplified because the free-entry condition becomes binding with even fewer intermediaries. This mechanism is then able to rationalize episodes in which *"liquidity begets liquidity"*, see Chordia et al. (2001).[16]

To disentangle and quantify the magnitude of the direct effects and the amplification effects, I use a numerical example. Consider an exogenous increase of 5% in γ (respectively, α). If the firm did not need to adjust the illiquidity premium to investors, such an increase would lead to a decrease of 14% (9.7%) in the measure of active intermediaries.[17] However, investors anticipate that trading frictions will tighten; hence, the firm needs to pay a larger premium. This reaction eventually leads to a decrease of 15.3% (11.1%) in the measure of active intermediaries, magnifying the impact of the initial exogenous shock. Figure 8 displays the effect of a change in α (up to 0.45 and down to 0.25) and γ (up to 0.1 and down to 0.06). It illustrates that amplification can also be beneficial. A decrease in α (e.g., the introduction of technological innovations or trading protocols easing transactions) or in γ (e.g., policies easing lending to liquidity providers) increases

[16] They report that liquidity anomalies are self-perpetuating: "*If agents have found out that liquidity has decreased, this crowds out other agents, which will further reduce liquidity in those periods.*"

[17] To fix ideas, I look at the measure of intermediaries at the target cash level.

the measure of active intermediaries beyond the direct effect, via the reduction in the illiquidity premium.

Similarly, firm operating shocks spill over to intermediaries' participation, and feed back into corporate valuation. When external financing is uncertain, cash represents a resource that is immediately available to buffer operating losses. As firms draw down their cash reserves, liquidity constraints worsen, liquidation gets more likely, and shareholder value decreases. As explained, market liquidity worsens. The anticipation of this deterioration in market liquidity increases the illiquidity premium and, hence, the opportunity cost of cash holdings. As a result, the firm reduces its target level of cash holdings. This decision in turn negatively impacts the firms' access to outside funding. *Ceteris paribus*, adverse operating shocks then become more persistent for firms with illiquid stocks. In fact, it is more difficult for these firms to recover from losses by raising fresh funds.

Two points are worth making. First, firms with illiquid stocks are less financially resilient since they hold less cash to absorb losses. Second, for a cumulative loss of a given size, liquidation becomes more likely when the liquidity loop is at work. Absent market frictions, a series of shocks reducing the cash buffer from $C^*/2$ to $C^*/4$ would increase the probability of liquidation from 1% to 10.1%. If market liquidity were independent of firm value, the reduction from $C_V/2$ to $C_V/4$ would be caused by a cumulative loss 11.3% smaller than in the previous case, but it would increase the probability of liquidation from 1.7% to 13.2%. Finally, if intermediaries' participation is endogenous, a cumulative loss reducing the buffer from $C_S/2$ to $C_S/4$ is 14.1% smaller than in the first-best case but increases the probability of liquidation from 1.9% to 14%.

Consequently, shocks that originate in the intermediary sector propagate to the corporate sector and eventually get amplified. Analogously, corporate operating shocks propagate to the intermediary sector and eventually become more persistent. But what is the effect of shocks affecting the investors' demand of liquidity? These shocks directly and simultaneously affect both intermediaries and corporations. More specifically, an increase in χ (resulting in more costly outside options) or in δ (resulting in more frequent shocks) are beneficial for intermediaries but detrimental to firms. These effects feed back into each other. As a result, the decrease in firm value is partially offset by a mild improvement in liquidity provisions, and the increase in intermediary participation is in part offset by the decrease in equity value.

Amplification effects and policy implications. To better understand their relevance, I illustrate how these propagation effects would operate during a period of financial turmoil. In these instances, two distinct triggers may ignite the liquidity loop: (1) An increase in the intermediaries' cost of being active (larger γ), or (2) A deterioration in firms' ability to access primary markets (smaller λ). The first trigger would cause a direct decrease in the number of intermediaries following the stock. As explained previously, such a direct effect would generate a negative externality on firm value as well as on the participation of other intermediaries. The firm would need to reduce its liquidity buffer and, for any given λ, would become more constrained. The corresponding decrease in firm value would dry up intermediaries' participation and market liquidity further. Turning to the second trigger, it would directly make the firm more constrained and cause a decrease in firm value. This decrease, however, would crowd intermediaries out of the market for the stock, which would make firm financial constraints even more severe.

The joint consideration of these effects would compound their single outcomes, then representing a complementary mechanism to explain the severe dry-ups in market liquidity documented during the recent financial crisis. In a richer environment, the detrimental effect of negative participation shocks amplified by the loop could be exacerbated by demand effects, i.e., panic selling or informational cascades. Although some trading patterns are surely fraught with asymmetric information, it seems useful to better understand the effects of market liquidity in the more parsimonious world of symmetric information. By singling out a plain-vanilla mechanism with risk-neutral agents and perfect information, the model opens the way to a number of levers that may be deepened by future research.

The analysis then reveals that the architecture of secondary market transactions is key to improve the efficiency of the corporate sector. The fact that participating agents cannot fully internalize the benefit from their liquidity provision leads to suboptimal market outcomes, characterized by low market liquidity and high illiquidity premia. An intermediary has no incentive to enter the market for the stock when the free-entry condition is binding. The planner, differently, takes into account that the entry of new intermediaries can make investors better off, by reducing firms' cost of capital and stimulating intermediaries' participation further (see also Lagos, Rocheteau, and Weill, 2011).

The model then supports interventions aimed at sustaining liquidity provisions in equity markets, e.g., policies aimed at (1) subsidising the provision of liquidity (in the model, lowering γ),[18] or (2) achieving technological improvements in the trading process (lowering α). Concerning policy (1), the subsidy may not necessarily come from a public authority. Suppose a firm reaches an agreement with the lead manager of a stock issue, whereby the intermediary agrees on making a market for the issue in exchange for a periodic rent. By giving up a fraction of its cash flows, the firm could subsidize the intermediaries dealing in the stock. As long as the decrease in firm value due to the rents paid to the intermediary is lower than the increase in firm value due to improved market liquidity, a coordination problem has been solved: Liquidity providers participate more, the cost of trading for investors decreases, and firm value is enhanced. In this context, Marès (2001) also suggests policies qualifying public authorities as a market maker of last resort; this is equivalent to setting a floor to the amount of liquidity produced in a given market. From the partial equilibrium perspective of this paper, this policy may benefit both a firm and its investors, but it is unlikely to be implemented for a single corporate issuer. A public authority would be more likely to implement policy (2), as technological improvements would effectively benefit a larger number of agents.

6 Concluding remarks

This paper develops a dynamic model that sheds light on the interactions between stock market liquidity and corporate policies. Market illiquidity makes investors' participation more costly and increases the firm's cost of capital, which in turn constrains corporate policies and decreases firm value. The model shows that market illiquidity reduces the availability of internal liquidity as a risk management tool, leads to an increase in firms' payout, makes the access to external financing more difficult, leads to under-investment, and decreases firm value. The analysis also reveals that this illiquidity-driven decrease in firm value deters intermediaries from participating in the market of the stock, then making the market more illiquid. As such, the strategic complementarity in the actions of intermediaries and corporate management gives rise to an internal-external liquidity

[18] This is in line with Weill (2007), pointing out that subsidizing loans to market makers may be welfare-increasing. While it has been enacted in times of financial disruptions (e.g., the Federal Reserve lowered the fund rate and encouraged commercial banks to lend generously to security dealers during the October 1987 crash, see Huang and Wang, 2010), the paper points out that this policy could be desirable in normal times, for all those market-makers providing immediacy for less traded stocks.

loop, whereby market liquidity and balance-sheet liquidity stimulate each other.

The paper supports the idea that financial markets are not a sideshow for the corporate sector. The model is capable of reproducing the observed characteristics that are common among firms whose stocks are illiquid. Specifically, it captures the positive relation between internal and external liquidity as well as the tighter financing constraints and the larger default risk faced by these firms. At the same time, the model relates intermediaries' participation to firm characteristics. In particular, it predicts that firms that are less profitable, whose cash flows are more volatile, that are more financially constrained, or that have more intangible assets should be less followed by intermediaries and, therefore, less liquid. Moreover, the model captures the self-perpetuating nature of market liquidity and the asymmetric adjustment of liquidity provisions following changes in asset value. Importantly, the model delivers a propagation mechanism that amplifies shocks arising in the intermediary or in the corporate sector. The model then supports policies aimed at enhancing liquidity provisions in the stock market: These policies could foster a more efficient allocation of corporate resources.

Appendices

A Proof of the results in Section 3

A.1 Proof of Proposition 1

Throughout the Appendix, I define

$$\Phi \equiv \delta\chi\eta$$

to ease the notation. I consider the homogeneous ordinary differential equation (ODE)

$$(\rho+\lambda+\Phi)\ V(c) = V'(c)\,(rc+\mu) + \frac{\sigma^2}{2}V''(c) + \lambda\Big[V(C_V) - C_V + c\Big] \tag{19}$$

This equation is solvable in closed form by the change of variable $V(c) = g\left(-\frac{(rc+\mu)^2}{r\sigma^2}\right)$, that transforms the ODE into the following Kummer's equation with parameters

$$a = -\frac{\rho+\lambda+\Phi}{2r},\ b = \frac{1}{2},\ z = -\frac{(rc+\mu)^2}{r\sigma^2}.$$

The solution of the homogenous ODE can be found employing the two following linearly independent solutions (as in HMM),

$$F(c) = M(a,b,z) \quad \text{and} \quad G(c) = z^{1-b}M(1+a-b, 2-b, z)$$

where $M(.)$ is the confluent hypergeometric function. Therefore, the general solution takes the form

$$V(c) = \ a_1F(c) \ + \ a_2G(c) + \Pi_V(c) \tag{20}$$

where a_1 and a_2 are constants (derived in the following), while $\Pi_V(c)$ is the particular solution of the ODE. I conjecture $\Pi_V(c) = Bc + A$. By straightforward calculations, it follows that

$$\Pi_V(c) = \frac{\lambda}{\lambda+\rho+\Phi-r}\,c + \frac{\lambda}{\lambda+\rho+\Phi}\left[\frac{(r-\rho-\Phi)C_V+\mu}{\rho\ +\ \Phi} + \frac{\mu}{\rho+\lambda+\Phi-r}\right]. \tag{21}$$

Now, I turn to a_1 and a_2. I exploit Abel's identity

$$F'(c)G(c) - F(c)G'(c) = -\frac{\sqrt{r}}{\sigma}e^{-(rc+\mu)^2(r\sigma^2)^{-1}}$$

Differentiating, I get

$$F''(c)G(c) - G''(c)F(c) = \frac{2}{\sigma^3}\sqrt{r}\,[rc+\mu]\,e^{-(rc+\mu)^2(r\sigma^2)^{-1}}$$

Finally, recalling that $F(c)$ and $G(c)$ are solution of the homogenous ODE, it follows that

$$F''(c)G'(c) - F'(c)G''(c) = \frac{2(\rho+\Phi+\lambda)}{\sigma^3}\sqrt{r}e^{-(rc+\mu)^2(r\sigma^2)^{-1}}$$

Using the three identity above, jointly with the smooth-pasting and super-contact conditions at C_V, I get by calculations

$$
\begin{aligned}
a_1 &= -\frac{\sigma^3 \; e^{(rC_V+\mu)^2(r\sigma^2)^{-1}}}{2\sqrt{r}(\rho+\lambda+\Phi)} \frac{\rho+\Phi-r}{\rho+\Phi+\lambda-r} G''(C_V), \\
a_2 &= \frac{\sigma^3 \; e^{(rC_V+\mu)^2(r\sigma^2)^{-1}}}{2\sqrt{r}(\rho+\lambda+\Phi)} \frac{\rho+\Phi-r}{\rho+\Phi+\lambda-r} F''(C_V).
\end{aligned}
$$

Finally, to express the solution in the more intuitive guise involving the discount factors, I just need to define and solve the functions

$$
\begin{aligned}
L_V(c) &= E\left[e^{-(\rho+\Phi+\lambda)\tau} 1_{\tau \le \tau_V}\right], \\
H_V(c) &= E\left[e^{-(\rho+\Phi+\lambda)\tau_V} 1_{\tau \ge \tau_V}\right]
\end{aligned}
$$

where $\tau = \inf\{t \ge 0 : C_t \le 0\}$, and $\tau_V = \inf\{t \ge 0 : C_t = \; C_V\}$. Then, the functions $L_V(c)$ and $H_V(c)$ denote respectively the first time that the cash reserve process hits zero or the payout boundary. The following result holds.

Lemma 10 *The functions $L_V(c)$ and $H_V(c)$ solve*

$$
\begin{aligned}
L_V(c) &= \frac{G(C_V)F(c) - F(C_V)G(c)}{G(C_V)F(0) - F(C_V)G(0)}, \\
H_V(c) &= \frac{G(c)F(0) - F(c)G(0)}{G(C_V)F(0) - F(C_V)G(0)}
\end{aligned}
$$

Proof. The functions $L_V(c)$ and $H_V(c)$ satisfy the homogeneous ODE associated to the dynamics of $V(c)$, so their general solution is $K_i\; F(c) \; + \; K_j\; G(c)$. In addition, they are solved subject to $L(0) = 1$, $L(C_V) = 0$, and $H(0) = 0$, $H(C_V) = 1$. Therefore, standard calculations deliver the result. ■

Now, I prove that the value function $V(c)$ is increasing and concave for any $c < C_V$.

Lemma 11 *$V'(c) > 1$ and $V''(c) < 0$ for any $c \in [0, C_V)$.*

Proof. Simply differentiating equation (19), one gets

$$
(\rho+\lambda+\Phi-r)\, V'(c) = V''(c)\,(rc+\mu) + \frac{\sigma^2}{2} V''(c) + \lambda.
$$

By the conditions $V'(C_V) = 1$ and $V''(C_V) = 0$, it follows that $V'''(C_V) = \frac{2}{\sigma^2}(\rho+\Phi-r) > 0$ as $r < \rho$, meaning that there exists a left neighbourhood of C_V such that for any $c \in (C_V - \epsilon, C_V)$, with $\epsilon > 0$, the inequalities $V'(c) > 1$ and $V''(c) < 0$ hold. Toward a contradiction, I assume that $V'(c) < 1$ for some $c \in [0, C_V - \epsilon]$. Then there should exists a point $C_c \in [0, C_V - \epsilon]$ such that $V'(C_c) = 1$ and $V'(c) > 1$ over (C_c, C_V), so

$$
V(C_V) - V(c) > C_V - c \tag{22}
$$

for any $c \in (C_c, C_V)$. For any $c \in (C_c, C_V)$ it must be also that

$$V''(c) = \frac{2}{\sigma^2} \left\{ (\rho + \lambda + \Phi) V(c) - [rc + \mu] V'(c) - \lambda(V(C_V) + c - C_V) \right\}$$

Using (22), jointly with $V(C_V) = \frac{rC_V + \mu}{\rho + \Phi}$, it follows that

$$V''(c) < \frac{2}{\sigma^2} \left\{ (\rho + \Phi)(V(C_V) + c - C_V) - rc - \mu \right\} = \frac{2}{\sigma^2}(c - C_V)(\rho + \Phi - r) < 0.$$

This means that $V'(c)$ is decreasing for any $c \in (C_c, C_V)$, which contradicts $V'(C_c) = V'(C_V) = 1$. It follows that C_c cannot exists. So, $V'(c) > 1$ and $V''(c) < 0$ for any $c \in [0, C_V)$, and the claim follows. ■

A.1.1 Considering share repurchases

As the analysis in Section 3.1 shows, market illiquidity translates into an illiquidity premium that is required by the investors to invest in the firm's stock. One natural question that may arise is then why the firm does not make a commitment to repurchase the shares of liquidity shocked investors to decrease the cost of investor's illiquidity. When doing so, the firm would act as a liquidity provider.

Suppose that management follows this policy, that is, on any time interval it repurchases the shares of the liquidity-shocked investors. If management bought the value of their claim, the firm would have a constant outflow $\delta V(c)$. In this case, shareholder value would satisfy

$$\rho V(c) = [rc + \mu - \delta V(c)] \, V'(c) + \frac{\sigma^2}{2} V''(c) + \lambda \left[V(C_V) - C_V + c - V(c) \right] . \tag{23}$$

This equation differs from (5) in that there is no loss borne by liquidity-shocked investors thanks to the firm's commitment to buy back their shares. By comparing equation (23) with equation (6), however, it is clear that the firm's decision to buy back shares for $c < C_V$ is suboptimal. In fact, $\delta V(c) V'(c) > \delta V(c) \chi\eta$ due to the fact that $V'(c) \geq 1 > \chi\eta > 0$.

Management may then have the incentive to buy back shares at a price that is smaller than $\delta V(c)$. Yet, this price cannot be smaller than $\delta T(c)$, i.e. the price stemming from the bargaining game with the intermediaries. Thus, management has to buy back the shares at a price that is at least equal to $\delta T(c)$. When following this policy, shareholder value would satisfy

$$\begin{aligned} \rho V(c) = & \, [rc + \mu - (1 - \eta\chi)\delta V(c)] \, V'(c) + \frac{\sigma^2}{2} V''(c) + \lambda \left[V(C_V) - C_V + c - V(c) \right] \\ & + (1 - \eta\chi - 1)\delta V(c) \, . \end{aligned} \tag{24}$$

where the last term on the right-hand side denotes the loss borne by shocked shareholders. Again, note that $(1 - \eta\chi)\delta V(c) V'(c) + \eta\chi\delta V(c) > \eta\chi\delta V(c)$, meaning that the this policy is also suboptimal. The suboptimality of this policy can be generalizes to all the transaction price in $[\delta T(c), \delta V(c)]$. Then, lumpy payouts in the form of share repurchases are suboptimal for $c < C_V$.

A.2 Proof of Proposition 2

In this section, I express the function $V(c)$ as a function of X, denoting the threshold satisfying $V'(X,X) - 1 = V''(X,X) = 0$. To prove the claim, I exploit the following auxiliary results.[19]

Lemma 12 *The function $V(c, X)$ is decreasing in the payout threshold X.*

Proof. To prove the claim, I take $X_1 < X_2$, and I define the auxiliary function $k(c) = V(c, X_1) - V(c, X_2)$, that satisfies

$$(\rho + \Phi + \lambda)k(c) = (rc + \mu)k'(c) + 0.5\sigma^2 k''(c) + \lambda(X_1 - X_2)[r/(\rho + \Phi) - 1] \tag{25}$$

for any $c \in [0, X_1]$. By previous result and straightforward calculations, the function is positive at X_2 as $k(X_2) = (X_1 - X_2)[r/(\rho+\Phi)-1] > 0$. By the definition of X_1 and X_2, the function $k(c)$ is decreasing and convex for $c \in [X_1, X_2)$. Therefore, $k(X_1) > 0$. Note that the function cannot have a negative local minimum on $[0, X_1]$ because the last term on the right hand side of (25) is positive. In addition, the function $k'(c)$ does not have neither a positive local maximum nor a negative local minimum, otherwise the equation $(\rho+\Phi+\lambda-r)k'(c) = (rc+\mu)k''(c)+0.5\sigma^2 k'''(c)$ would not hold (respectively $k'(c) > 0 = k''(c) > k'''(c)$ and $k'(c) < 0 = k''(c) < k'''(c)$ at a positive maximum and at a negative minimum). As k is convex at X_1, this means that k' is increasing at X_1, and therefore it must be negative for any $c \in [0, X_1]$. Jointly with $k(X_1) > 0$, this means that $k(c) > 0$ for any $c \in [0, X_2]$. The claim follows. ■

Lemma 13 *For a given payout threshold X and two given $\chi_1 > \chi_2$, $V(c, X, \chi_2) > V(c, X; \chi_1)$ for any $c \in [0, X]$.*

Proof. As in the previous section, I define $\Phi_i = \delta\chi_i\eta$ with $i = 1, 2$, and the auxiliary function $h(c) = V(c, X; \chi_2) - V(c, X; \chi_1)$. I need to prove that, for a given payout threshold X, $h(c) > 0$ for any $c \in [0, X]$. At X, the function is positive as

$$h(X) = (rX + \mu)\left(\frac{1}{\rho + \Phi_2} - \frac{1}{\rho + \Phi_1}\right) = (rX + \mu)\frac{\Phi_1 - \Phi_2}{(\rho + \Phi_1)(\rho + \Phi_2)} > 0,$$

since $\Phi_1 > \Phi_2$ as $\chi_1 > \chi_2$, and $h'(X) = h''(X) = 0$. In addition, the function evolves as

$$[rc + \mu]h'(c) + \frac{\sigma^2}{2}h''(c) - (\rho + \lambda + \Phi_2)h(c) + \lambda h(X) = (\Phi_2 - \Phi_1)V(c, X; \chi_1)$$

and the right hand side is negative. Differentiating, we have $[rc + \mu]h''(c) + \frac{\sigma^2}{2}h'''(c) - (\rho + \lambda + \Phi_2 - r)h'(c) = (\Phi_2 - \Phi_1)V_s'(c, X; \chi_1)$. At X, I get $\frac{\sigma^2}{2}h'''(X) = \Phi_2 - \Phi_1$, meaning that $h'''(X) < 0$. This means that the second derivative is decreasing in a neighbourhood of X, so one has $h''(c) > 0$ in a left neighbourhood of X. In turn, this means that $h'(c)$ is increasing in such neighbourhood of X, then implying that $h'(c) < 0$ in a left neighbourhood of X. Now I need to prove that the function is decreasing for any c smaller that X. Note that, by the ODE above, $h'(c)$ cannot have a negative local minimum. As $h'(X) = 0$ and it is negative and increasing in a left neighbourhood of X, this means that $h'(c)$ should be negative for any $c < X$,

[19] I set $\ell = \phi\frac{\lambda}{\lambda+1}\frac{\mu}{\rho}$, to capture the idea that the liquidation value of the firm is a function of profitability and of supply frictions.

so $h(c)$ is always decreasing. As it is positive at X, it means that it should be always positive, so $h(c) > h(X) > 0$ so it is positive for any $c < X$. ■

Exploiting the results above, I have the following.

Lemma 14 *For any $\chi_1 > \chi_2$, $C_V(\chi_1) < C_V(\chi_2)$.*

Proof. The payout thresholds $C_V(\chi_1)$ and $C_V(\chi_2)$ are the unique solution to the boundary conditions $V(0, C_V(\chi_2); \chi_2) - \ell = 0 = V(0, C_V(\chi_1); \chi_1) - \ell$. Exploiting the result in Lemma 13, I now take, for instance, $X = C_V(\chi_1)$. It then follows that

$$V(0, C_V(\chi_1); \chi_2) - \ell \; > \; 0 \; = \; V(0, C_V(\chi_1); \chi_1) - \ell.$$

As V is decreasing in the payout threshold, this means that $C_V(\chi_1) < C_V(\chi_2)$ to get the equality $\ell - V(0, C_V(\chi_2); \chi_2) = 0$. The claim follows. ■

The result below is a straightforward consequence of Lemma 14.

Corollary 15 *In the presence of secondary market frictions, the target cash level is lower than in the benchmark case with perfectly liquid secondary market, i.e. $C_V < C^*$.*

Note also that all the results in this section can be extended for two parameters $\delta_1 > \delta_2$ and $\eta_1 > \eta_2$. The following result is then straightforward.

Corollary 16 *For any $\delta_1 > \delta_2$ and $\eta_1 > \eta_2$, $C_V(\delta_1) < C_V(\delta_2)$ and $C_V(\eta_1) < C_V(\eta_2)$.*

Finally, I prove the monotonicity of C_V with respect to the severity of primary market frictions, represented by the parameter λ.

Lemma 17 *C_V are monotone decreasing in λ.*

Proof. The monotonicity of C^* stems from Lemma B.10 in HMM (2013). Thus, the result for C_V is a corollary of this Lemma, as the presence of market illiquidity increases the required return from ρ to $\rho + \Phi$, preserving the monotonicity. ■

A.3 Proof of Proposition 4

I derive the results concerning the probability of liquidation $P_l(c, X)$, since the probability of external financing is just $P_f(c, X) = 1 - P_l(c, X)$. Using standard methods (see e.g., Dixit and Pyndick, 1994), the dynamics of $P_l(c, X)$ are given by

$$P_l'(c)(rc + \mu) + \frac{\sigma^2}{2} P_l''(c) - \lambda \; P_l(c) = 0$$

$$\text{s.t. } P_l(0) = 1 \tag{26}$$

$$P_l'(X) = \; 0, \tag{27}$$

where the first boundary condition is given by the definition of P_l, while the second boundary condition is due reflection at the payout threshold.

Now I prove that the probability of liquidation is higher in the presence of secondary market frictions. To do so, I first prove that the probabilities $P_l(c, C^*)$ and $P_l(c, C_V)$ are decreasing and convex in c. To do so, I employ the generic function $P_l(c, X)$.

Lemma 18 *The probability $P_l(c, X)$ is decreasing and convex for any $c \in [0, X]$.*

Proof. To prove the claim, I exploit arguments analogous to Lemma 11. As $P_l'(X) = 0$ and $P_l(X) \geq 0$, by the dynamics of $P_l(c)$ it must be that $P_l''(X) > 0$. Then, there exists a left neighbourhood of X, $[X - \epsilon, X]$ with $\epsilon > 0$, over which $P_l'(c) < 0$ and $P_l''(c) > 0$. Toward a contradiction, suppose that there exists some $c \in [0, X - \epsilon]$ where $P_l'(c) > 0$. Then, there should be a $\bar{C}$ such that $P_l'(\bar{C}) = 0$, while $P_l'(c) < 0$ for $c \in [\bar{C}, X]$. Then, for any $c \in [\bar{C}, X]$ it must be that

$$P_l''(c) = \frac{2}{\sigma^2}\left[\lambda P_l(c) - P_l'(c)(rc + \mu)\right] > \frac{2}{\sigma^2}\,\lambda P_l(X) \;>\; 0\,.$$

Then, $P_l''(c) > 0$ for any $c \in [\bar{C}, X]$ means that $P_l'(c)$ is always increasing on $c \in [\bar{C}, X]$, contradicting $P_l'(\bar{C}) = P_l'(X) = 0$. The claim follows. ■

Now I prove that $P_l(c, C_V) \geq P_l(c, C^*)$.

Lemma 19 *For any $\chi_1 > \chi_2$, $P_l(c, C_V(\chi_1)) \geq P_l(c, C_V(\chi_2))$.*

Proof. By Lemma 14, $C_V(\chi_1) < C_V(\chi_2)$. To ease the notation in the proof, I denote $X_1 = C_V(\chi_1)$ and $X_2 = C_V(\chi_2)$. By Lemma 18, the functions $P_l(c, X_1)$ and $P_l(c, X_2)$ are positive, decreasing and convex over the interval of definition. I define the auxiliary function

$$h(c) = P_l(c, X_1) - P_l(c, X_2).$$

Note that $h(c)$ cannot have neither a positive local maximum ($h(c) > 0$, $h'(c) = 0$, $h''(c) < 0$) nor a negative local minimum ($h(c) < 0$, $h'(c) = 0$, $h''(c) > 0$) on $[0, X_1]$, as otherwise the equation $h''(c)\frac{\sigma^2}{2} + h'(c)[rc + \mu] - \lambda h(c) = 0$ would not hold. In addition, $h(0) = 0$, and $h'(X_1) = -P_l'(c, X_2) > 0$ because of the boundary conditions at zero and at X_1. This means that the function is null at the origin, and increasing at C_V. Toward a contradiction, assume that $h(X_1)$ is negative. This would imply the existence of a negative local minimum, given that the function is null at zero and it is increasing at X_1. This cannot be the case as argued above, contradicting that $h(X_1) < 0$. Therefore, the function must be always positive, and the claim follows. ■

The result below is a straightforward consequence of Lemma 19 and the fact that, in the absence of secondary market frictions, $\delta = 0$ or $\chi = 0$ or $\eta = 0$ (or combinations of these).

Corollary 20 *In the presence of secondary market frictions, the probability of liquidation P_l is larger than in the benchmark case with perfectly liquid secondary market, i.e. $P_l(c, C^*) < P_l(c, C_V)$.*

Again, the results in this section can be extended for two parameters $\eta_1 > \eta_2$ or $\delta_1 > \delta_2$. The following result is then straightforward.

Corollary 21 *For any $\eta_1 > \eta_2$ or $\delta_1 > \delta_2$, $P_l(c, C_V(\eta_1)) \geq P_l(c, C_V(\eta_2))$ and $P_l(c, C_V(\delta_1)) \geq P_l(c, C_V(\delta_2))$.*

A.4 Proof of Proposition 5

I exploit the dynamic programming result in Décamps and Villeneuve (2007) and HMM, establishing that the growth option has a non-positive NPV if and only if $V(c) > V_+(c - I)$ for any $c \geq 0$, where I denote by $V_+(c - I)$ the value of equity after investment. To prove the claim, I rely on the following lemma.

Lemma 22 *$V(c) \geq V_+(c - I)$ for any $c \geq I$ if and only if $I \geq I_V$, where I_V is defined as in Proposition 5.*

Proof. I define $\bar{c} = \max[C_V, I + C_{V+}]$. The inequality $V(c) \geq V_+(c - I)$ for $c > \bar{c}$ means that $c - C_V + V(C_V) \geq c - C_{V+} - I + V_+(C_{V+})$. Using the definition of I_V, the former inequality is equivalent to the inequality $I \geq I_V$, by straightforward calculations.

To prove the sufficient condition, I can just prove that $V(c) \geq V_+(c - I_V)$ for any $c \geq I_V$, and I exploit the inequalities $C_V < C_{V+} + I_V$ and $\mu_+ - \mu - rI_V > 0$ (this is a slight modification of Lemma C.3 in HMM, so I omit the details). For $c \geq C_V$, the following inequality

$$V_+(c - I_V) \leq V_+(C_{V+}) + c - I_V - C_{V+} = c - C_V + V(C_V) = V(c)$$

holds. Note that the first inequality is due to the concavity of V_+, the first equality is given by the definition of I_V, while the second equality is due to the linearity of V above C_V. Then, I now need to proof the result for $c \in [I_V, C_V]$. To this end, I define the auxiliary function $u(c) = V(c) - V_+(c - I_V)$. The function $u(c)$ is positive at C_V as argued above, $u'(C_V) < 0$ and $u''(C_V) > 0$. On the interval of interest it satisfies

$$(\rho+\Phi+\lambda)u(c) = (rc+\mu)u'(c)+\frac{\sigma^2}{2}u''(c)+(\mu+rI_V-\mu_+)V_+'(c-I_V)+\lambda(V(C_V)-C_V-V_+(C_{V+})+C_{V+}+I_V)$$

where the last term on the right hand side is zero by the definition of I_V, while the third term is negative. Then, the function cannot have a positive local maximum here, because otherwise $u(c) > 0$, $u''(c) < 0 = u'(c)$, and the ODE above would not hold. Jointly with the fact that $u(C_V)$ is positive, decreasing and convex means that the function is always decreasing on this interval. Then, $u(c)$ is also always positive, and the claim holds. ■

If instead χ, η or δ are zero, the zero-NPV cost is given by 12, and the result in HMM and Décamps and Villeneuve (2007) obtains.

B Proof of the results in Section 4

The proofs in this section are based on the general case in which $\underline{C} \in [0, C_S]$, according to which shareholders' value satisfies the system of ODEs composed by (17) and (18). Note that the cases in which $\ell > \underline{S}$ (and therefore equity value satisfies 18 for any $c \in [0, C_S]$) or the $S(C_S) < \underline{S}$ (and therefore equity value satisfies 17 for any $c \in [0, C_S]$) are corner cases (as derived separately below).

B.1 Proof of Proposition 6

To prove this claim, I show that the value function is strictly monotone and concave over $0 \le c < C_S$, as follows.

Lemma 23 *$S'(c) > 1$ and $S''(c) < 0$ for any $c \in [0, C_S)$.*

Proof. The first part of this proof follows the same arguments as Lemma 11. Accordingly, I differentiate equation (18) and get the following ODE $(\rho + \lambda + \Phi - r) S'(c) = S''(c)(rc + \mu) + \frac{\sigma^2}{2} S'''(c) + \lambda$. Jointly with the boundaries $S'(C_S) = 1$ and $S''(C_S) = 0$, this ODE implies that $S'''(C_S) > 0$, meaning that there exists a left neighbourhood of C_S such that for any $c \in (C_S - \epsilon, C_S)$, with $\epsilon > 0$, the inequalities $S'(c) > 1$ and $S''(c) < 0$ hold. Toward a contradiction, I assume that $S'(c) < 1$ for some $c \in [0, C_S - \epsilon]$. Then, there should be a point $C_c \in [0, C_S - \epsilon]$ such that $S'(C_c) = 1$ and $S'(Cc) > 1$ over (C_c, C_S), so $S(C_S) - S(c) > C_S - c$ for any $c \in (C_c, C_S)$. The point C_c could belong either in the interval $[0, \underline{C}]$ or in the interval $[\underline{C}, C_S]$. I now discriminate between these two cases. If $\underline{C} < C_c < C_S$, it must be that for any $c \in (C_c, C_S)$

$$S''(c) = \frac{2}{\sigma^2}\left\{(\rho + \lambda + \Phi) S(c) - (rc + \mu)S'(c) + \frac{\alpha\gamma(1-\eta)}{\eta} - \lambda[S(C_S) + c - C_S]\right\}$$

Using the fact that $S(C_S) - S(c) > C_S - c$, jointly with $S(C_S) = \frac{rC_S + \mu - \frac{\alpha\gamma(1-\eta)}{\eta}}{\rho + \Phi}$, it follows that

$$S''(c) < \frac{2}{\sigma^2}\left\{(\rho + \Phi)(S(C_S) + c - C_S) - rc - \mu + \frac{\alpha\gamma(1-\eta)}{\eta}\right\} = \frac{2}{\sigma^2}(c - C_S)(\rho + \Phi - r) < 0.$$

This means that $S'(c)$ is decreasing for any $c \in (C_c, C_S)$, which contradicts $S'(C_c) = S'(C_S) = 1$. So, $S'(c) > 1$ for any $c \in [\underline{C}, C_S)$, so such C_c does not exist on $[\underline{C}, C_S]$.

I now consider the case $0 < C_c < \underline{C}$. Should such point C_c exist, the strict concavity of $S(c)$ over $[\underline{C}, C_S]$ means that there should be a maximum $C_m \in [C_c, \underline{C}]$ for the first derivative over the interval $(C_c, \underline{C})$, such that $S'(C_m) > 1$, $S''(C_m) = 0$ and $S'''(C_m) < 0$. Simply differentiating equation 17, I get that

$$S''(c)[rc + \mu] + S'''(c)\frac{\sigma^2}{2} - S'(c)(\rho + \delta\chi - r) + \lambda(1 - S'(c)) = 0.$$

Then, $S'''(C_m)\frac{\sigma^2}{2} = (\rho + \delta\chi - r)S'(C_m) + \lambda(S'(C_m) - 1) > 0$, which contradicts the existence of such a maximum C_m for $S'(c)$. It follows that C_c cannot exists, and the claim follows. ■

B.2 Proof of Proposition 7

In this proof I exploit the results in Appendix A.1. Differently from that environment, however, the value function in the more general case with $\underline{C} \in [0, C_S]$ is defined over three intervals, namely $[0, \underline{C}]$, $[\underline{C}, C_S]$, and $[C_S, \infty]$.

On the first interval $[0, \underline{C}]$, I define

$$a_d = -\frac{\rho + \lambda + \delta\chi}{2r},\ b = \frac{1}{2},\ z = -\frac{(rc + \mu)^2}{r\sigma^2}.$$

and the solutions of the homogeneous ODE is given by

$$F_d(c) = M(a_2, b, z) \quad \text{and} \quad G_d(c) = z^{1-b} M(1 + a_2 - b, 2 - b, z)$$

where $M(.)$ is the confluent hypergeometric function. The general solution on $[0, \underline{C}]$ is therefore given by

$$w_1 F_d(c) \ + \ w_2 G_d(c) + \Pi_d(c)$$

where

$$\Pi_d(c) \ = \frac{\lambda}{\lambda + \rho + \delta\chi - r} c + \frac{\lambda}{\lambda + \rho + \delta\chi} \left[\frac{(r - \rho - \Phi)C_S + \mu - \frac{\alpha\gamma(1-\eta)}{\eta}}{\rho \ + \ \Phi} + \frac{\mu}{\rho + \lambda + \delta\chi - r} \right]. \tag{28}$$

On the second interval $[\underline{C}, C_S]$, the general solution is

$$w_3 F(c) \ + \ w_4 G(c) + \Pi_u(c)$$

where $F(c)$ and $G(c)$ are defined as in Appendix A.1, while the inhomogeneity $\Pi_u(c)$ (guessing it is linear) is given by

$$\Pi_u(c) \ = \frac{\lambda}{\lambda + \rho + \Phi - r} c + \frac{\lambda}{\lambda + \rho + \Phi} \left[\frac{(r - \rho - \Phi)C_S + \mu}{\rho \ + \ \Phi} + \frac{\mu}{\rho + \lambda + \Phi - r} \right] - \frac{\alpha\gamma(1 - \eta)}{\eta(\rho + \Phi)}$$

Finally, on the third interval $[C_S, \infty]$, the solution is linear and given by

$$S(C_S) + c - C_S.$$

The solution reported in the proposition exploits the following functions

$$L_d(c) = E\left[e^{-(\rho+\delta\chi+\lambda)\tau} 1_{\tau \leq \bar{\tau}_S} \right], \qquad H_d(c) = E\left[e^{-(\rho+\delta\chi+\lambda)\bar{\tau}_S} 1_{\tau \geq \bar{\tau}_S} \right]$$
$$L_u(c) = E\left[e^{-(\rho+\Phi+\lambda)\tau} 1_{\bar{\tau}_S \leq \tau_S} \right], \qquad H_u(c) = E\left[e^{-(\rho+\Phi+\lambda)\tau_S} 1_{\bar{\tau}_S \geq \tau_S} \right]$$

where τ is defined as in (2), while $\tau_S = \inf\{t \geq 0 : C_t = \ C_S\}$ and $\bar{\tau}_S = \inf\{t \geq 0 : C_t = \ \underline{C}\}$ are respectively the first time that the thresholds C_s and $\underline{C}$ are hit. Analogously to Lemma 10, we have the following results.

Lemma 24 *The functions $L_d(c), L_u(c), H_d(c), H_u(c)$ solve*

$$L_d(c) \ = \frac{G_d(\underline{C})F_d(c) - F_d(\underline{C})G_d(c)}{G_d(\underline{C})F_d(0) - F_d(\underline{C})G_d(0)}, \qquad H_d(c) \ = \frac{G_d(c)F_d(0) - F_d(c)G_d(0)}{G_d(\underline{C})F_d(0) - F_d(\underline{C})G_d(0)}$$
$$L_u(c) \ = \frac{G(C_S)F(c) - F(C_S)G(c)}{G(C_S)F(\underline{C}) - F(C_S)G(\underline{C})}, \qquad H_u(c) \ = \frac{G(c)F(0) - F(c)G(0)}{G(C_S)F(\underline{C}) - F(C_S)G(\underline{C})}$$

Proof. The result follows Lemma 10, so the proof is omitted. ■

As mentioned, two corner cases may arise. Namely, if $\ell > \underline{S}$, then the non-trading region collapses, and therefore firm value satisfies (18) for any $c \in [0, C_S]$. In this case, firm value is given by

$$\Pi_u(c) + \left[\ell - \Pi_u(0)\right] \, L_a(c) \, + \, \left[S(C_S) - \Pi_u(C_S)\right] H_a(c)$$

on $c \in [0, C_S]$ where, similarly to the previous cases, the function L_a (respectively H_a) denotes the first time that the cash process hits zero (the target cash level) before hitting the target cash level (zero). As before, $S(c) = S(C_S) - C_S + c$ for any $c > C_S$.

If instead $\underline{S} > S(C_S)$, then the trading region collapses, and firm value satisfies (17) for any $c \in [0, C_S]$. In this case, firm value is given by

$$\Pi_d(c) + \left[\ell - \Pi_d(0)\right] \, L_n(c) \, + \, \left[\frac{rC_S + \mu}{\rho + \delta\chi} - \Pi_d(C_S)\right] H_n(c)$$

on $c \in [0, C_S]$ and the functions L_n and H_n are defined similarly as before. Above $c > C_S$, $S(c) = \frac{rC_S+\mu}{\rho+\delta\chi} - C_S + c$ (so note that the expression for the value of equity at the target cash level is different).

B.3 Proof of Proposition 8

In this section I prove that endogenous participation, and therefore the strategic interaction of the inter-dealer sector, makes the internal liquidity buffer even lower, i.e. $C_S < C_V < C^*$. In addition, I prove that the threshold C_S is decreasing in the severity of inter-dealer market frictions, i.e. in the parameters α and γ.

I start by proving that $C_S < C_V$. To this end, I exploit the following auxiliary result.

Lemma 25 *$S(c)$ is decreasing in X.*

Proof. As before, I consider two thresholds $X_1 < X_2$ satisfying the smooth-pasting and super-contact conditions $S'(c) - 1 = 0 = S''(c)$. I define the auxiliary function $k(c) = S(c, X_1) - S(c, X_2)$. At X_2, $k(X_2) = (X_2 - X_1)\left(1 - \frac{r}{\rho+\Phi}\right) > 0$, and $k'(X_2) = k''(X_2) = 0$. At X_1,

$$\begin{aligned} \frac{rX_1 + \mu - \frac{\alpha\gamma(1-\eta)}{\eta}}{\rho + \Phi} - S(X_1, X_2) >& \frac{rX_1 + \mu - \frac{\alpha\gamma(1-\eta)}{\eta}}{\rho + \Phi} - \left(\frac{rX_2 + \mu - \frac{\alpha\gamma(1-\eta)}{\eta}}{\rho + \Phi} - X_2 + X_1\right) \\ =& (X_2 - X_1)\left(1 - \frac{r}{\rho + \Phi}\right) > 0 \end{aligned}$$

and $k'(X_1) < 0$ and $k''(X_1) > 0$, so the function is positive, decreasing and convex at X_1. Note also that $k'(c) < 0 < k''(c)$ for any $[X_1, X_2]$. I conjecture (and verify later in this proof that this is indeed the case) that $\underline{C}(X_1) < \underline{C}(X_2)$. As $\underline{C}$ is uniquely identified by the level of the value function equal to $\frac{\alpha\gamma}{\chi\delta\eta}$, the strict monotonicity of S in c (see Lemma 23), jointly with $\underline{C}(X_1) < \underline{C}(X_2)$, means that $k(\underline{C}(X_1))$ and $k(\underline{C}(X_2))$ are non-negative.

I now focus on $k'(c)$. On the interval $[\underline{C}(X_2), X_1]$, the dynamics of $k'(c)$ are $(\rho + \Phi + \lambda - r)k'(c) = (rc + \mu)k''(c) + \frac{\sigma^2}{2}k'''(c)$ so $k'(c)$ cannot have neither a negative local minimum nor a positive local maximum on this interval. On the interval $[\underline{C}(X_1), \underline{C}(X_2)]$, $k'(c)$ evolves as

$$(\rho + \lambda + \Phi - r)k'(c) \; = [rc + \mu]k''(c) + \frac{\sigma^2}{2}k'''(c) + \delta\chi(1 - \eta)S'(c, X_2)$$

where the last term is positive, meaning that $k'(c)$ cannot have a negative local minimum. Finally, on $[0, \underline{C}(X_1)]$, the first derivative satisfies $(\rho+\delta\chi+\lambda-r)k'(c) = [rc+\mu]k''(c) + \frac{\sigma^2}{2}k'''(c)$ so it cannot have neither a positive local maximum nor a negative local minimum. The fact that $k'(c)$ is negative and increasing over $[X_1, X_2)$, jointly with the fact that a negative local minimum can never occur, means that $k'(c)$ must be negative and increasing over all the interval of definition. Then, $k(c)$ is decreasing for any $c \in [0, X_2)$. Jointly with the fact that $k(X_2)$ is positive, this means that $k(c)$ is indeed positive for any $c \in [0, X_2)$.

Toward a contradiction, let us now assume that $\underline{C}(X_1) > \underline{C}(X_2)$. Given the monotonicity of S, the inequality $\underline{C}(X_1) > \underline{C}(X_2)$ would mean that $k(c) < 0$ at least on the interval $[\underline{C}(X_1), \underline{C}(X_2)]$. By previous results, $k(X_1)$ is positive, decreasing and convex, and also on the interval $[X_1, X_2)$. Then, if $k(c) < 0$ on the interval $[\underline{C}(X_1), \underline{C}(X_2)]$, there should exist a positive local maximum a for $k(c)$ on the interval $[\underline{C}(X_1), X_1]$. At a, the first derivative $k'(c)$ would go from positive to negative, and k'' would be negative here. Nevertheless, $k''(X_1) > 0$, and k' cannot have neither a negative local minimum on the interval $[\max(\underline{C}(X_2), \underline{C}(X_1)), X_1]$, contradicting the existence of a. In turn, this means that $\underline{C}(X_1) > \underline{C}(X_2)$ cannot hold. ■

Exploiting Lemma 25, I prove the following result.

Lemma 26 *The inequality $C_V > C_S$ holds.*

Proof. To prove the claim, I first show that for a given payout threshold X, $V(c, X) > S(c, X)$ for any $c \in [0, X]$ (Consistently with the positiveness of the functions V and S, I am picking values of X such that $S(0)$ as well as $V(0)$ are non-negative). I define the auxiliary function $k(c) = V(c, X) - S(c, X)$. At X, the function is positive as

$$k(X) = \frac{\alpha\gamma(1-\eta)}{\eta(\rho+\Phi)} > 0.$$

In addition, $k'(X) = k''(X) = 0$ because of smooth-pasting and super-contact at X. On the interval $[\underline{C}, X]$ the function satisfies

$$[rc+\mu]k'(c) + \frac{\sigma^2}{2}k''(c) - (\rho+\lambda+\Phi)k(c) + \lambda k(X) + \frac{\alpha\gamma(1-\eta)}{\eta} = 0$$

Note that the function cannot have a negative local minimum, because the sum of the last two terms on the left hand side is positive. Differentiating the ODE above, it follows that k' satisfies

$$[rc+\mu]k''(c) + \frac{\sigma^2}{2}k'''(c) - (\rho+\lambda+\Phi-r)k'(c) = 0.$$

So, k' cannot have neither a positive local maximum nor a negative local minimum on $[\underline{C}, X]$. In addition, $\frac{\sigma^2}{2}k'''(X) = 0$. On the interval $[0, \underline{C}]$, $k(c)$ satisfies

$$[rc+\mu]k'(c) + \frac{\sigma^2}{2}k''(c) - (\rho+\lambda+\Phi)k(c) + \lambda k(X) = (\Phi-\delta\chi)S(c) = -\delta\chi(1-\eta)S(c)$$

so that $k(c)$ cannot have a negative local minimum over this interval. In addition, $k'(c)$ cannot have a negative local minimum either on this interval, as its dynamics are given by

$$[rc+\mu]k''(c)+\frac{\sigma^2}{2}k'''(c)-(\rho+\Phi+\lambda-r)k'(c)= \ -\delta\chi(1-\eta)S'(c) \tag{29}$$

as $S'(c)\geq 1$ by Lemma 23.

Now, S and S' are continuous because of the value-matching and smooth-pasting conditions at $\underline{C}$. Moreover, for $\underline{C}^-=\underline{C}-\epsilon$ (with $\epsilon>0$ and small enough),

$$\frac{\sigma^2}{2}S''(\underline{C}^-) \ = \ (\rho+\lambda+\delta\chi)\,S(\underline{C}^-)-[r\underline{C}^-+\mu]S'(\underline{C}^-)-\lambda(S(X)-X+\underline{C}^-)$$

while for $\underline{C}^+=\underline{C}+\epsilon$,

$$\frac{\sigma^2}{2}S''(\underline{C}^+) \ = \ (\rho+\lambda+\Phi)\,S(\underline{C}^+)-[r\underline{C}^++\mu]S'(\underline{C}^+)-\lambda(S(X)-X+\underline{C}^+)+\frac{\alpha\gamma(1-\eta)}{\eta}$$

By simply subtracting the two equations above, taking the limit for $\epsilon\to 0$, and keeping in mind the continuity of $S(c)$ and $S'(c)$, I get

$$\lim_{\epsilon\to 0}\frac{\sigma^2}{2}\left(S''(\underline{C}^-)-S''(\underline{C}^+)\right) \ = \ (\delta\chi-\Phi)\,S(\underline{C})-\frac{\alpha\gamma(1-\eta)}{\eta} \ = \ 0$$

that establishes the continuity of the second derivative at $\underline{C}$. On the interval $[\underline{C},X]$ the two functions V and S satisfy the same ODE but for the inhomogeneity term $-\frac{\alpha\gamma(1-\eta)}{\eta}(1+\lambda/(\rho+\Phi))$ in the equation for S. Recall that we are just imposing the two boundaries at the payout threshold, i.e. X is exogenously taken since the boundary at zero is lax at this stage of the proof. Then, $k'(\underline{C}^+)= \ k''(\underline{C}^+)= \ 0$, and this means that $k'(\underline{C}^-)= \ k''(\underline{C}^-)= \ 0$ because of continuity of the first and second derivative of V and S. By equation (29), it follows that $\frac{\sigma^2}{2}k'''(\underline{C}^-) \ = \ -\delta\chi(1-\eta)S'(\underline{C})<0$. In turn, this means that $k'(\underline{C})$ is increasing in a left neighbourhood of $\underline{C}$, and given that $k'(\underline{C})$ is null at $\underline{C}$, it means that $k'(c)$ is negative in such neighbourhood. Jointly with the fact that on $[0,\underline{C}]$ there cannot exist a negative local minimum for k', this means that $k'(c)$ is negative for any $[0,\underline{C}]$. Therefore, $k(c)$ is decreasing for any $c\in[0,\underline{C}]$. As $k(X)$ is positive, $k(c)>0$ for any $c\in[0,X]$. This means that $V(c,X)>S(c,X)$. Now, taking $X=C_S$, it follows that the boundary condition at zero is satisfied for the function V if $C_V>C_S$, using arguments analogous to the proof of Lemma 14. ■

Finally, I prove that C_S is decreasing in α e γ. Note that, when $\alpha=\gamma=0$, it follows that $C_V=C_S$.

Lemma 27 *For any $\alpha_1<\alpha_2$, $C_S(\alpha_1)>C_S(\alpha_2)$.*

Proof. First I prove that, for a given payout threshold X satisfying $S'(X)-1=0=S''(X)$ (but keeping the boundary at zero lax), the inequality $S(c,X;\alpha_1)>S(c,X;\alpha_2)$ holds. To this end, I define the auxiliary function $h(c)= \ S(c,X;\alpha_1)-S(c,X;\alpha_2)$. At X, it holds that

$$h(X) \ = \ \frac{\gamma(1-\eta)}{\eta(\rho+\Phi)}\,(\alpha_2-\alpha_1) \ > \ 0,$$

as well as $h'(X) = 0 = h''(X)$. Over the interval $[\max(\underline{C}(X,\alpha_1), \underline{C}(X,\alpha_2)), X]$, $h(c)$ evolves as $(\rho+\lambda+\Phi)h(c) = [rc+\mu]h'(c) + \frac{\sigma^2}{2}h''(c) - \frac{\gamma(1-\eta)}{\eta}(\alpha_1-\alpha_2) + \lambda h(X)$ and note that the sum of the last two terms is positive, meaning that the function cannot have a negative local minimum. Over the interval $[0, \min(\underline{C}(X,\alpha_1), \underline{C}(X,\alpha_2))]$, $h(c)$ satisfies $(\rho+\lambda+\delta\chi)h(c) = [rc+\mu]h'(c) + \frac{\sigma^2}{2}h''(c) + \lambda h(X)$, so there cannot be a negative local minimum either. Conjecturing (and verifying below in the proof) that $\underline{C}(X,\alpha_1) < \underline{C}(X,\alpha_2)$, it follows that $h(c)$ satisfies

$$(\rho+\lambda+\delta\chi)h(c) = (rc+\mu)h'(c) + \frac{\sigma^2}{2}h''(c) + (\delta\chi-\Phi)\,S(c,X;\alpha_1) - \frac{\alpha_1\gamma(1-\eta)}{\eta} + \lambda h(X)$$

on $[\underline{C}(X,\alpha_1), \underline{C}(X,\alpha_2)]$. Note that $(\delta\chi-\Phi)\,S(c,X;\alpha_1) = \delta\chi(1-\eta)S(c,X,\alpha_1)$, so it means $(\delta\chi-\Phi)\,S(c,X;\alpha_1) - \frac{\alpha_1\gamma(1-\eta)}{\eta} > 0$ on $[\underline{C}(X,\alpha_1), \underline{C}(X,\alpha_2)]$ by the definition of $S(\underline{C})$ and the monotonicity of S, so the function cannot have a negative local minimum because the sum of the last three terms in the equation above is positive. Simply differentiating, note also that $h'(c)$ cannot have a negative local minimum either. Exploiting arguments analogous to Lemma 26, $h'''(\underline{C}(X,\alpha_2))$ is negative and so $h(c)$ is positive for any c if $\underline{C}(X,\alpha_1) < \underline{C}(X,\alpha_2)$. Toward a contradiction, I assume that the inequality $\underline{C}(X,\alpha_1) > \underline{C}(X,\alpha_2)$ holds. If this were the case, $h(c)$ would be negative at least on the interval $[\underline{C}(X,\alpha_2), \underline{C}(X,\alpha_1)]$. However, $h(X) > 0 = h'(X)$. Exploiting arguments analogous to Lemma 26 regarding the dynamics of $S(c,X,\alpha_1)$ and $S(c,X,\alpha_2)$ on $[\max(\underline{C}(X,\alpha_1), \underline{C}(X,\alpha_2)), X]$, it follows that $h'(c)$ cannot be positive here. Then, $h(\underline{C}(X,\alpha_2))$ cannot be negative, contradicting that $\underline{C}(X,\alpha_1) > \underline{C}(X,\alpha_2)$.

Using arguments analogous to the proof of Lemma 14, it then follows that the boundary condition at zero implies that $C_S(\alpha_1) > C_S(\alpha_2)$. Then, the claim follows. ■

Following the same steps as for Lemma 27 but for γ, the result below holds.

Lemma 28 *For any $\gamma_1 < \gamma_2$, $C_S(\gamma_1) > C_S(\gamma_2)$.*

B.4 Proof of Proposition 9

The result is based on the fact that $S(c)$ is always increasing and concave, as shown in Lemma 23. Given such result, $\theta'_S(c) > 0 > \theta''_S(c)$ as well as $\pi'(c) > 0 > \pi''(c)$ on $c > \underline{C}$ follow by simply differentiating the functions $\theta(c)$ and $\pi(c)$ on $c \geq \underline{C}$, obtaining

$$\theta'(c) = \delta\eta\frac{\chi}{\gamma}S'(c) \geq 0, \qquad \theta''(c) = \delta\eta\frac{\chi}{\gamma}S''(c) \leq 0$$
$$\pi'(c) = \frac{\alpha\gamma}{\delta\chi\eta}\frac{S'(c)}{(S(c))^2} \geq 0, \qquad \pi''(c) = \frac{\alpha\gamma}{\delta\chi\eta}\frac{S(c)S''(c) - 2(S'(c))^2}{(S(c))^3} \leq 0$$

Therefore, the claim follows.

C External and internal liquidity: Some evidence

Market liquidity is broadly defined as the ability to exchange securities quickly and at a low cost. Ceteris paribus, a liquid stock is more attractive for investors than an illiquid one, as it can be converted easily into a liquid medium, such as cash. Investors may refrain from buying illiquid stocks, as they fear being locked into undesirable positions.

Over the last few decades, the joint effect of technology and regulation has multiplied trading venues and improved the ease with which investors buy and sell securities. However, stocks listed on the largest U.S. equity markets still suffer from illiquidity, as I document in the following. Specifically, I investigate trading patterns of ordinary stocks exchanged on the NYSE, AMEX, and NASDAQ markets, as recorded in CRSP. I remove preferred stocks, stock rights and warrants, stock funds, and ADRs. I perform the same analysis over two non-consecutive years for robustness, and choose 2006 and 2012 to avoid abnormal patterns related to the 2007-2009 financial crisis. I remove observations with missing volume, where volume is defined as the number of firm shares sold in one day. The sample consists of 1,184,794 daily observations (936,195) in 2006 (in 2012), out of which 704,050 (546,801) represent the NASDAQ-traded stocks.

I start by investigating the daily trading patterns for the year 2006. For each trading day, stocks are ranked by ascending trading volume. An average of 96% of the stocks constitutes less than half of the daily volume. In turn, the least-traded 50% of the stocks only represents 2.3% of the daily volume.[20] The results reported in Table 2 for the year 2012 confirm these figures. Moreover, these trading patterns are reinforced when performing the analysis on NASDAQ stocks only. The share of stocks accounting for half of the daily volume can be as high as 99.4%, whereas the least-traded 50% of the stocks may account for less than 1.2% of the daily volume.

Table 3 investigates the cross-sectional characteristics of listed stocks over a trading year. To this end, I split the sample into quartiles of annual trading volume, where the first quartile contains the least actively-traded stocks. The scale of trading volume differs substantially across quartiles: In the highest quartile, it is more than 200 times higher than in the lowest quartile. The same pattern holds for a normalized measure of volume, defined as the number of firm shares sold in one day normalized by the number of shares outstanding. Normalized trading volume is indeed ten times higher in the highest quartile than in the lowest quartile. The maximum number of days without trading also differs dramatically across the quartiles. In the lowest quartile, it peaks at 204 days (174) in the year 2006 (2012); this number decreases sharply and non-linearly for more liquid stocks. The frequency at which a stock is traded matters in as much as infrequently-traded stocks are subject to higher transaction costs (among others, Easley, Kiefer, O'Hara, and Paperman, 1996, Barclay and Hendershott, 2004). Table 3 indeed shows that bid-ask spreads considerably decrease as trading volume increases, being the highest for the least active stocks. The same holds true for half spreads.[21]

Fact 1. *Trading on the largest U.S. equity markets involves costs, which are larger for less frequently-traded stocks.*

One may wonder whether stock market illiquidity matters for the policies and the performance of corporations. While this question may span the whole set of corporate decisions, I

[20] This evidence recalls Easley, Kiefer, O'Hara, and Paperman (1996), reporting that "on the London Stock Exchange, 50 percent of listed stocks account for only 1.5 percent of trading volume, and over 1000 stocks average less than one trade a day. On the [...] NYSE, it is common for individual stocks not to trade for days or even weeks at a time, while one stock in London never traded in an eleven-year period."

[21] Half spreads measure the percentage trading cost incurred in a one-way trade; they are computed as $HS = 100 * (A_{it} - B_{it})/(2M_{it})$, where A_{it} (B_{it}) is the closing ask (bid) price for security i at time t, and M_{it} is the quote midpoint, as Bessembinder and Kaufman (1997) and Huang and Stoll (1996).

narrow the field of investigation by focusing on a firm's retention and payout decisions. I match CRSP data to Compustat annual files for the period 1980-2012. I pool daily data into monthly aggregates, and then average on the given year. I focus again on NYSE, AMEX, and NASDAQ firms, and exclude financial firms (SIC 6000-6999) and regulated industries (SIC 4900-4999). I delete observations with missing or negative total assets as well as observations with negative sales. The sample consists of 86,968 firm-year observations. For each observation, balance-sheet liquidity is measured by cash and short-term investment deflated by total assets (as Bates, Kahle, and Stulz, 2009). Market liquidity is proxied by trading volume (see Chordia et al., 2001, or Barclay and Hendershott, 2004) and normalized by the number of shares outstanding. Confirming previous contributions, Figure 2 shows that balance-sheet liquidity has significantly increased over time. Parallel to this increase, Figure 2 also illustrates an important increase in stock market trading volume.

A question follows as to whether there is a cross-sectional relation between internal and external liquidity. While abstracting from a comprehensive empirical investigation (see however Nyborg and Wang, 2014, and Gopalan, Kadan, and Pevzner, 2012), I describe some pieces of the data. For both small and large firms, I examine the cash ratios of the top and bottom terciles of the stocks ranked by normalized trading volume for each year. I define *size* as the logarithm of sales and define large (small) firms to be in the top (bottom) tercile of *size*. Figure 1 illustrates that firms with liquid stocks, on average, hold more cash. By examining the top and bottom terciles of the stocks ranked by cash ratios, moreover, I find that cash-rich firms issue securities that are more liquid on average.

FACT 2. *Firms with more (less) liquid stocks are those that keep more (less) cash.*

Balance-sheet and market liquidity appear to be positively related. Such a positive link is supported by the *"corporate savings glut"* displayed by stocks such as Microsoft, Exxon, or Apple, an issue that has frequently become headlines in the financial press.[22] These corporations display immense cash reserves on their balance-sheets and their securities are among the most liquid on the U.S. stock markets.

[22]To name a few, "Pressure mounts for corporate cash piles to be put to work" (*Financial Times*, January 21, 2014), "Investors have a right to ask for cash" (*Financial Times*, March 20, 2013), "Wave of money eludes the real economy" (*Financial Times*, February 8, 2013), or the most cited "The corporate savings glut" (*The Economist*, July 7, 2005).

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Table 1: Benchmark parameters.

Symbol	Description	Value
	Firm parameters	
r	Return on cash	3.0%
ρ	Discount rate	5.0%
μ	Cash flow drift	0.20
σ	Cash flow volatility	0.25
λ	Arrival rate of financing opportunities	2.50
ϕ	Asset tangibility	0.55
	Secondary market parameters	
δ	Intensity of liquidity shocks	1.00
X_h	Holding cost	2.50%
X_d	Cost of non-intermediated trading	2.25%
η	Intermediaries' bargaining power	0.60
γ	Intermediaries' participation cost	0.08
α	Market inefficiencies	0.35

Table 2: Trading patterns I

The table reports the daily trading patterns of active ordinary stocks in CRSP for the years 2006 and 2012. The columns headed by (W) refer to the whole sample (NYSE, AMEX and NASDAQ), while those headed by (N) refer to NASDAQ stocks only. The first pair of columns report the daily trading volume, defined as the number of shares sold in one day. The second pair of columns report the percentage of stocks representing 50% of the daily traded volume. The third pair reports the share of the daily volume accounted by 50% of the stocks (being the least traded when sorted by volume). Finally, the fourth pair reports the percentage of stocks that are not traded in one day.

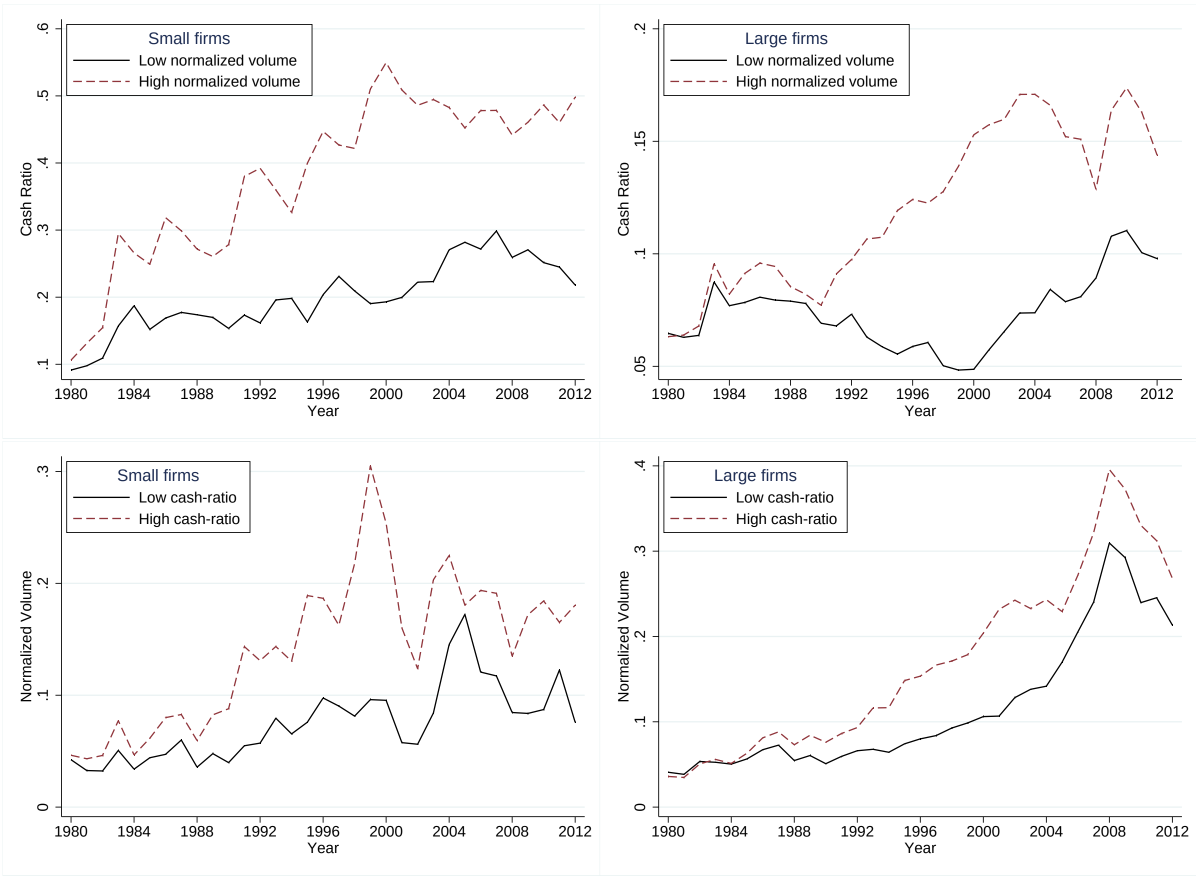
	Daily volume (thousands)		Stock constituting 50% volume		Share of volume by 50% stocks		Share of non-traded stocks	
	(W)	(N)	(W)	(N)	(W)	(N)	(W)	(N)
Year 2006								
Average	3,803,795	1,767,535	96.8%	98.4%	2.30%	1.71%	2.65%	3.26%
Median	3,861,490	1,766,268	96.8%	98.4%	2.29%	1.69%	2.63%	3.25%
Minimum	1,257,030	583,961	94.8%	96.1%	1.71%	1.19%	1.20%	1.29%
Maximum	6,229,979	3,398,085	98.3%	99.4%	4.06%	3.46%	5.76%	6.83%
Std. Dev.	561,591.7	301,552	0.0043	0.0039	0.0032	0.0029	0.0055	0.0069
Year 2012								
Average	4,275,441	1,511,968	96.7%	97.8%	1.99%	1.63%	1.96%	2.57%
Median	4,289,221	1,510,397	96.7%	97.7%	1.97%	1.59%	1.98%	2.62%
Minimum	1,432,418	514,329	94.9%	95.6%	1.29%	0.90%	0.48%	0.22%
Maximum	7,505,599	3,212,558	98.0%	99.0%	3.54%	2.66%	4.41%	5.74%
Std. Dev.	672,877	250,053	0.004	0.0048	0.003	0.0029	0.005	0.0067

Table 3: Trading patterns II

The table reports some cross-sectional characteristics of listed stocks, when sorted into quartiles of annual trading volume. The columns headed by (W) refer to the whole sample (NYSE, AMEX and NASDAQ), while those headed by (N) refer to NASDAQ stocks only. The first and the second pair of columns report respectively the average daily volume and the normalized volume. The third pair of columns report the maximum number of days without trading over one trading year. Finally, the fourth and the fifth pairs of columns report respectively the average bid-ask spread and the half bid-ask spread.

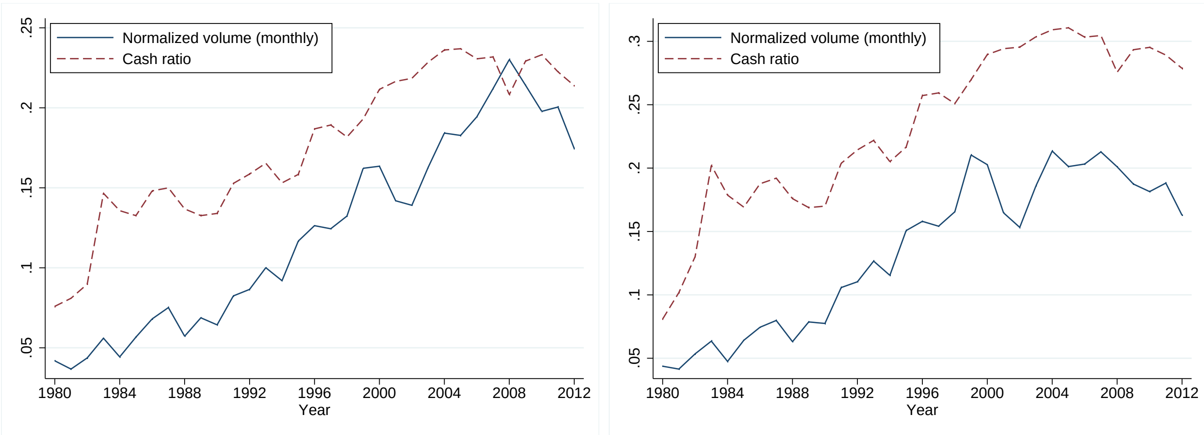
	Mean daily volume		Mean daily volume (normalized)		Maximum number of non-trading days		Average bid-ask spread (%)		Average half bid-ask spread (%)	
	(W)	(N)	(W)	(N)	(W)	(N)	(W)	(N)	(W)	(N)
Year 2006										
All	805,808	630,124	0.77%	0.80%	204	204	0.65%	0.76%	0.33%	0.38%
Lowest	12,578	8,464	0.15%	0.13%	204	204	1.58%	1.76%	0.81%	0.90%
II	100,031	63,494	0.57%	0.46%	36	73	0.61%	0.72%	0.31%	0.36%
III	341,361	220,049	0.98%	0.90%	15	5	0.28%	0.36%	0.14%	0.18%
Highest	2,768,543	2,228,325	1.39%	1.70%	1	1	0.14%	0.19%	0.07%	0.10%
Year 2012										
All	114,115	691,014	0.76%	0.68%	174	174	0.91%	1.25%	0.47%	0.65%
Lowest	15,755	9,687	0.16%	0.13%	174	174	2.70%	3.41%	1.40%	1.78%
II	113,703	59,828	0.50%	0.36%	26	32	0.62%	0.99%	0.32%	0.51%
III	424,667	225,214	0.88%	0.76%	1	26	0.22%	0.40%	0.11%	0.20%
Highest	4,009,909	2,468,484	1.51%	1.49%	1	1	0.11%	0.20%	0.05%	0.10%

Figure 1: Balance-sheet liquidity and market liquidity.



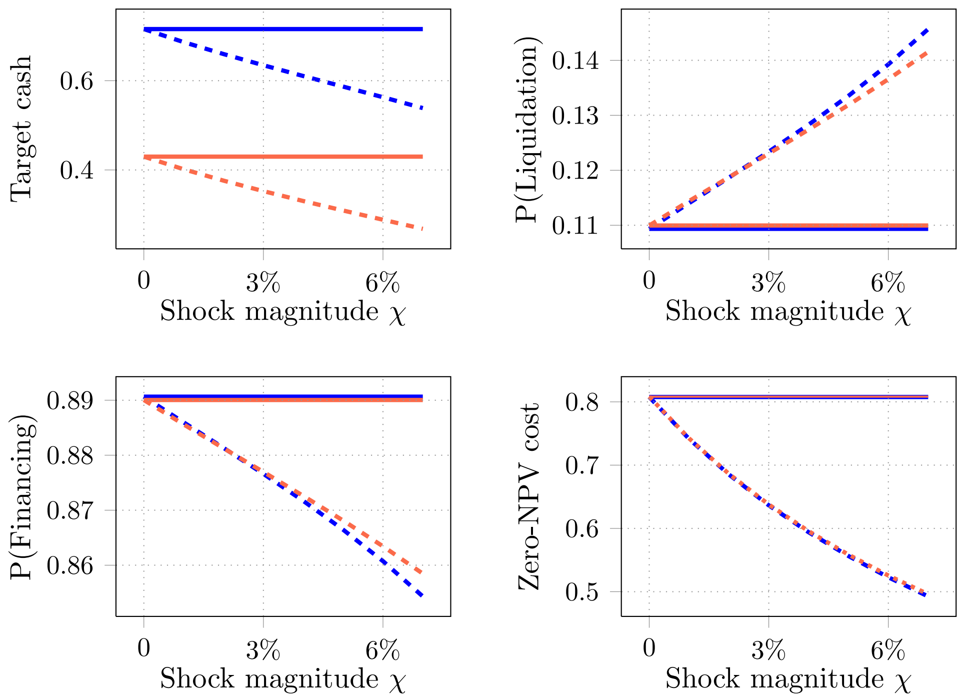
The top panel plots the average cash ratio for firms ranked by the market liquidity of their stock, as measured by normalized trading volume. The bottom panel plots the average normalized trading volume for firms ranked by cash ratios. The left (respectively, right) panel reports the patterns of small (large) firms.

Figure 2: Balance-sheet liquidity and market liquidity (II).



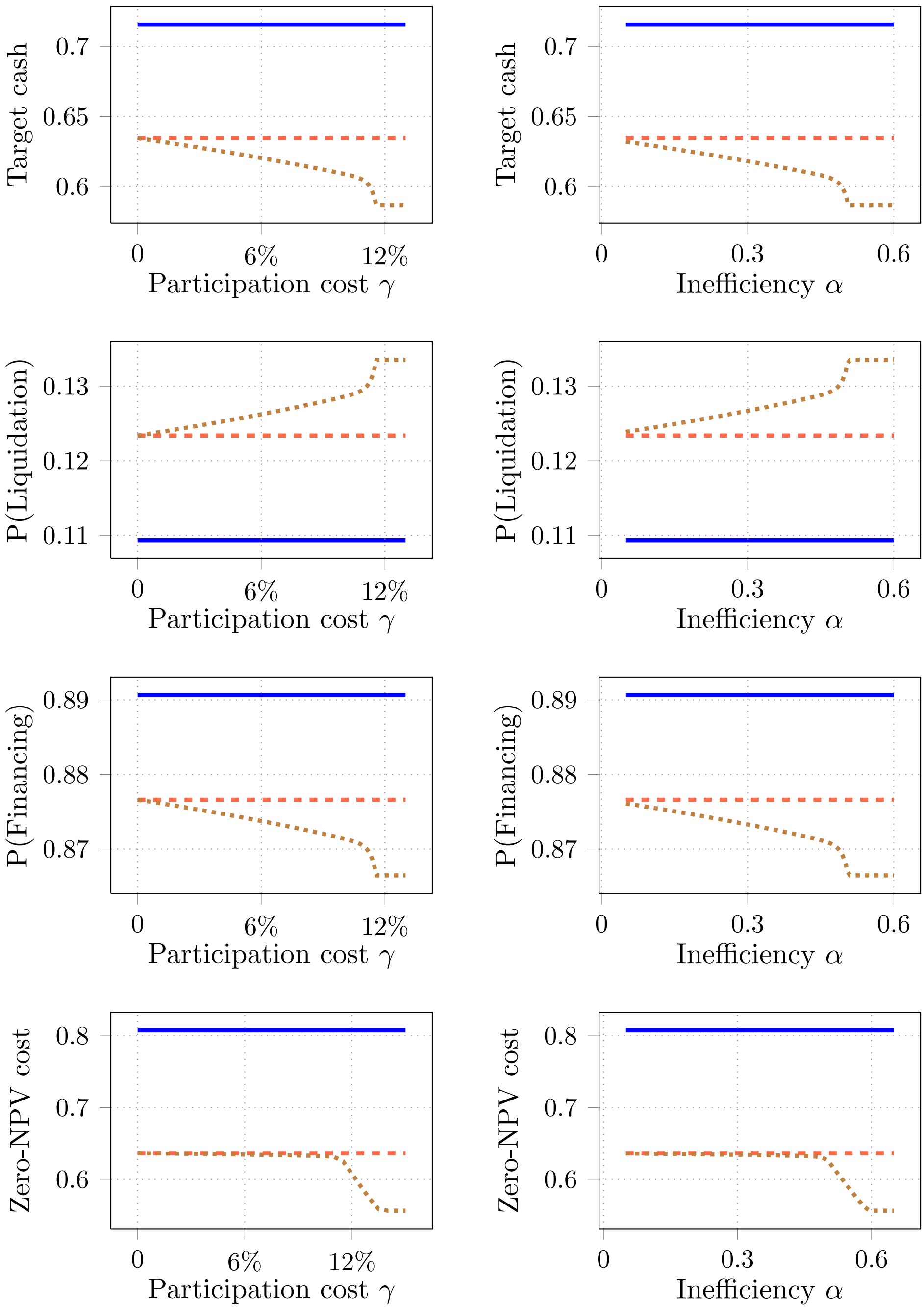
The graph plots the average cash ratio and the average monthly trading volume normalized by the number of outstanding shares over the period 1980-2012, for (a) NYSE, AMEX, and NASDAQ firms (top panel), (b) NASDAQ firms only (bottom panel).

Figure 3: Market illiquidity and corporate policies.



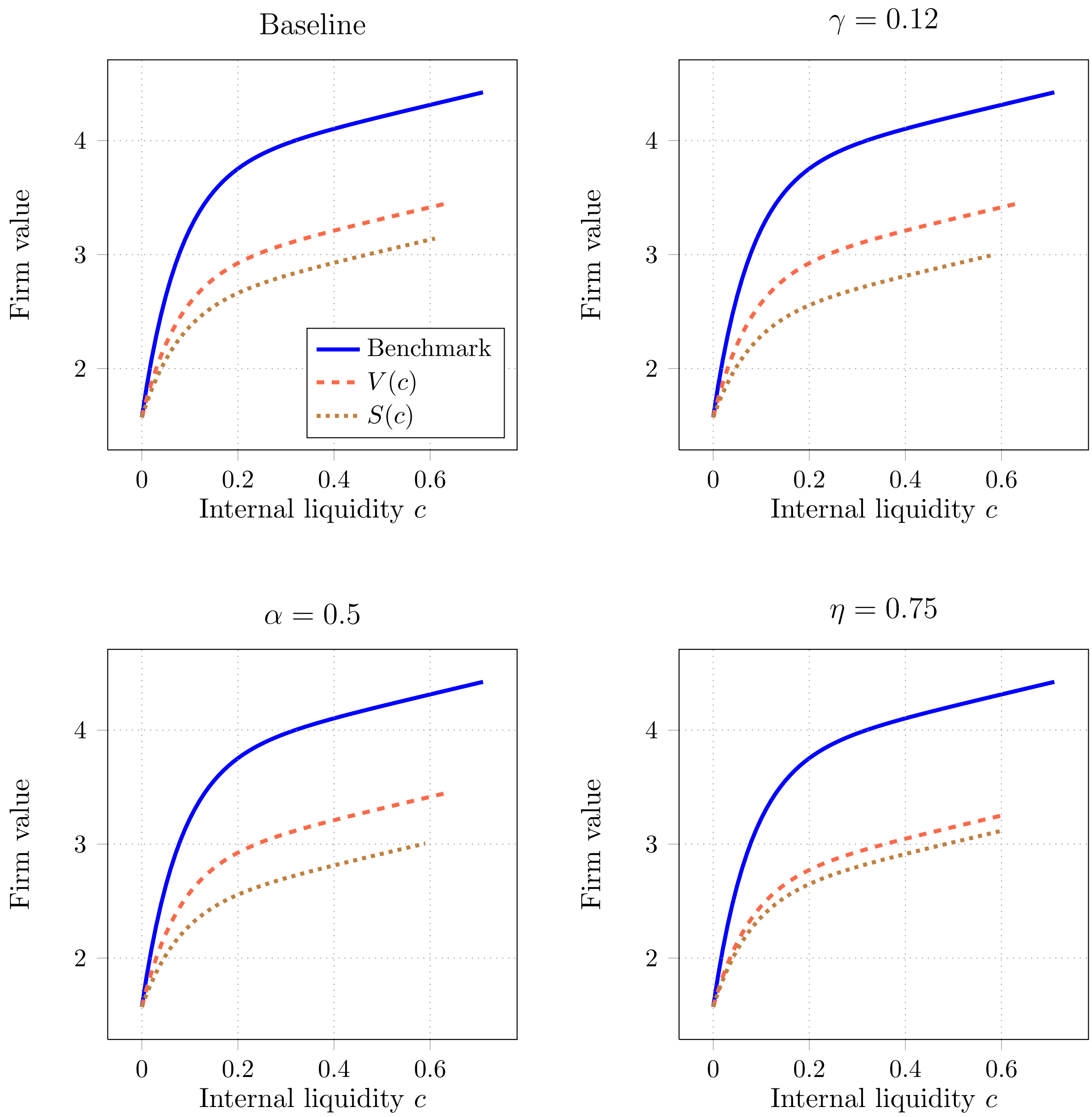
The figure shows the target level of cash holdings, the probability of liquidation, the probability of external financing, and the zero-NPV cost, as a function of the cost of investors' outside options χ (being positively related to the bid-ask spread). The blue lines refer to a firm with no access to bank credit and whose stocks are perfectly (solid line) or imperfectly (dashed line) liquid. The red lines refer to a firm having access to bank credit and whose stocks are perfectly (solid line) or imperfectly (dashed line) liquid.

Figure 4: Endogenous liquidity provision and corporate policies.



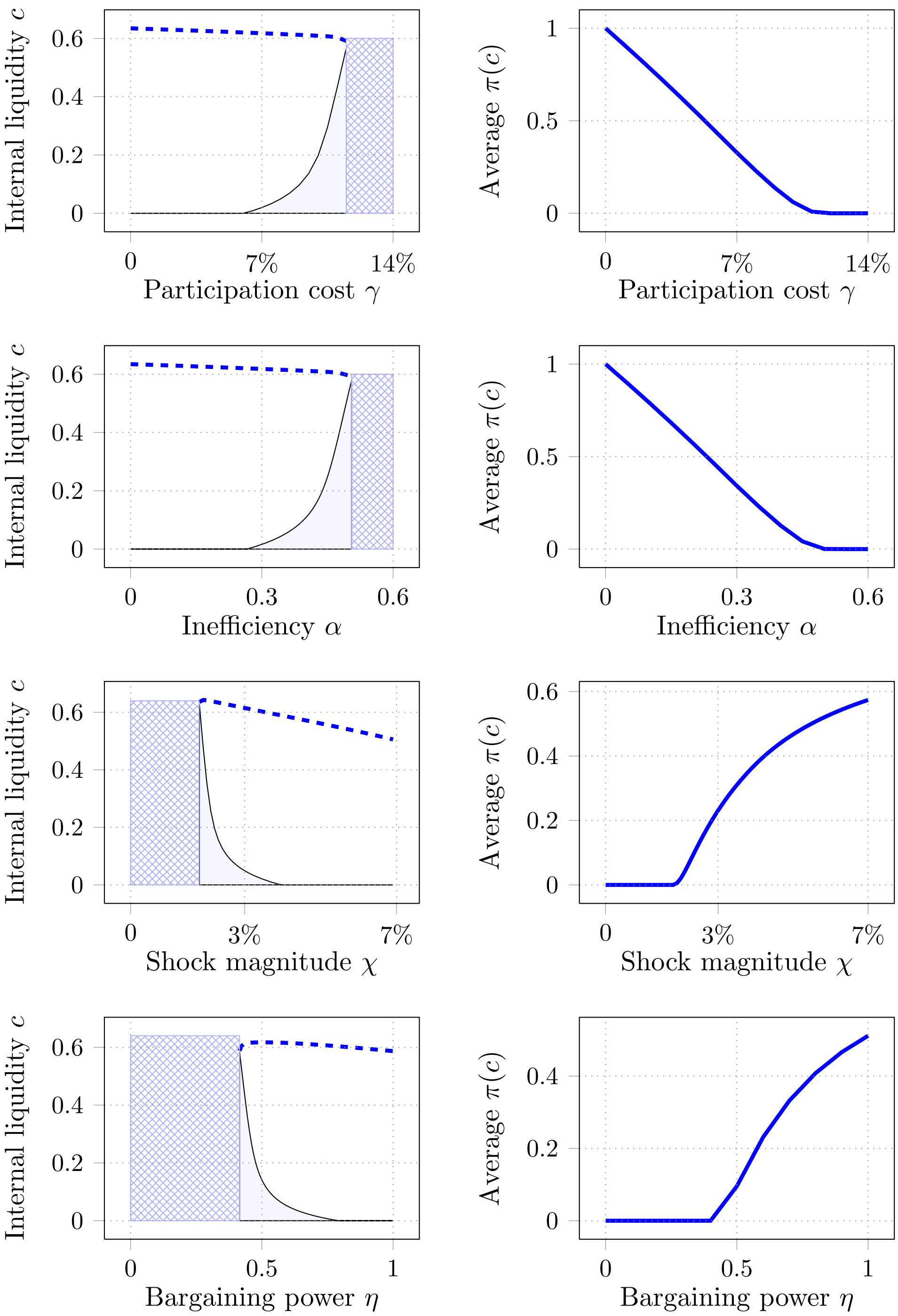
The figure shows the target level of cash holdings, the probability of liquidation, the probability of external financing, and the zero-NPV cost, as a function of the participation cost faced by the intermediaries γ and market inefficiencies α. The solid blue line depicts the benchmark with perfect market liquidity, the dashed red line depicts the environment with exogenous liquidity provision, and the dotted brown line depicts the environment with endogenous liquidity provision.

Figure 5: Endogenous liquidity provision and firm value.



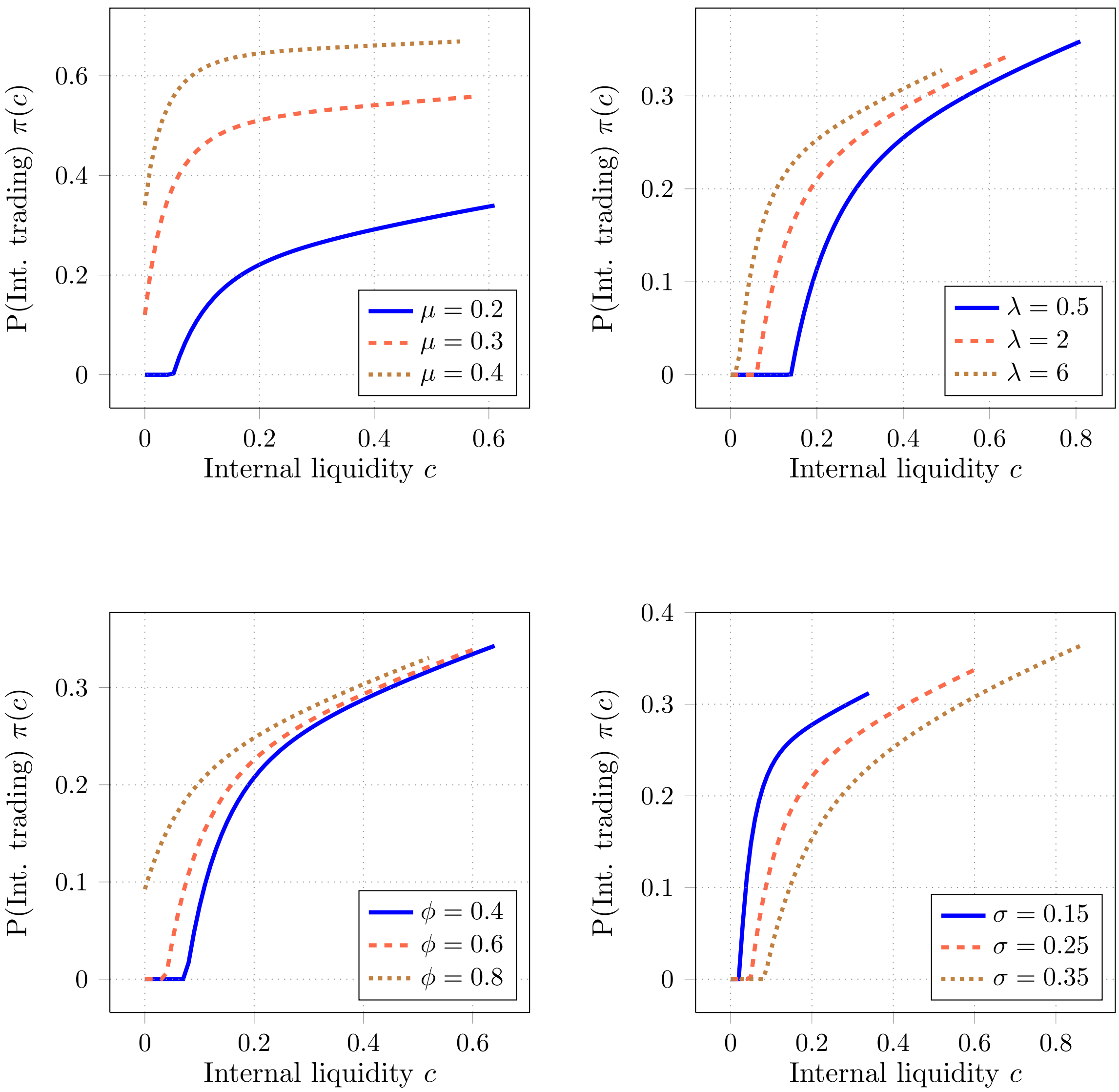
The figure shows firm value as a function of corporate cash holdings c under the baseline parametrization and when increasing: γ up to 0.12, α up to 0.5, and η up to 0.75. The solid blue line depicts the benchmark with perfect market liquidity, the dashed red line depicts the environment with exogenous liquidity provision, and the dotted brown line depicts the environment with endogenous liquidity provision.

Figure 6: Probability of intermediated trading.



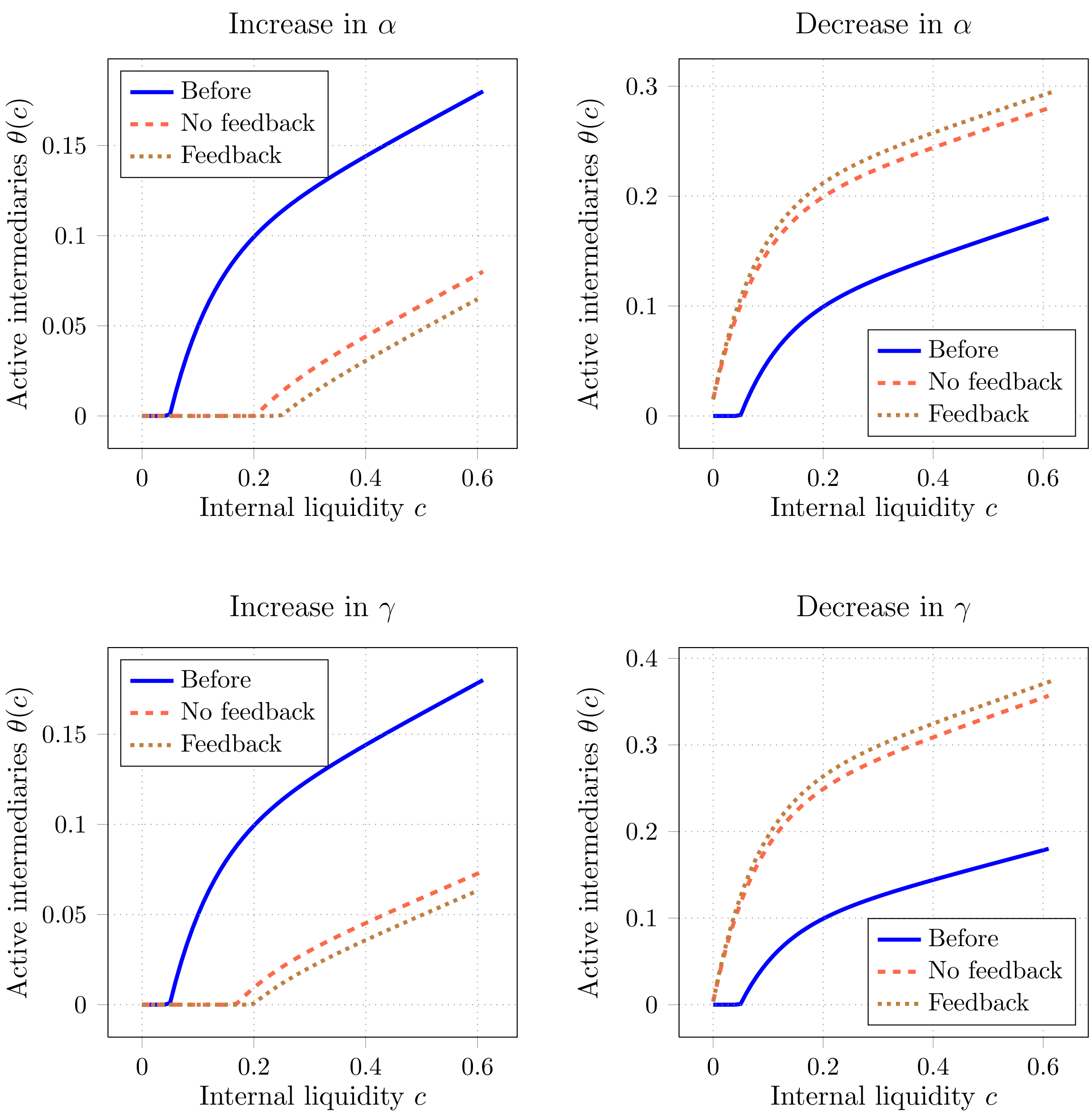
The left-hand side panel shows the parametric regions where intermediated trading is available (or not) to investors. In these plots, the solid black line represents the trading threshold $\underline{C}$ and the dashed blue line represents the target cash level C_S. The white area marks the region where investors have a positive probability to find intermediaries, while the light-shaded area denotes the region where intermediaries stay away from the market (temporary dry-up). The dark-shaded area marks the parametric region for which intermediaries stay away from the market for the stock for any c. The right-hand side panel plots the investors' average probability of intermediated trading $\pi(c)$.

Figure 7: Liquidity provision and firm characteristics.



The figure shows the probability of intermediated trading $\pi(c)$ as a function of cash holdings c and for different values of the profitability of cash flows μ, of the arrival rate of financing opportunities λ, of the tangibility of assets ϕ, and of the volatility of cash flows σ.

Figure 8: Amplification effects – Participation shocks.



The figure shows the measure of active intermediaries $\theta(c)$ before (solid blue line) and after (dashed red line and brown dotted line) a 25% variation in the magnitude of market inefficiencies α (top panel) and the participation cost γ (bottom panel). In particular, the dashed red line plots $\theta(c)$ if firm value was insensitive to the change in α or γ, whereas the dotted brown line plots $\theta(c)$ by taking into account that firm value internalizes this change.