DOES TICK SIZE INFLUENCE PRICE DISCOVERY? EVIDENCE FROM THE TORONTO STOCK EXCHANGE

MARIE-CLAUDE BEAULIEU*
SHAFIQ K. EBRAHIM
IEUAN G. MORGAN

We investigate the price discovery role of an exchange-traded fund and the futures contract for the same market index. We find that the fund predicts the index in the subperiod after but not in the subperiod before a substantial decrease in the minimum tick size. The futures predict the index in both subperiods. The results are consistent with the view that the factors leading to successful price discovery do not depend on zero investment, as in futures markets, but do depend on a small tick size. © 2003 Wiley Periodicals, Inc. Jrl Fut Mark 23:49–66, 2003

We thank Phillips, Hager, and North for their financial support, and Jean Bédard from the SIT at Université Laval for his help in organizing the data. M.-C. Beaulieu also thanks Queen's University School of Business Research Visiting Program, SSHRCC, Fonds FCAR, and Institut de Finance mathématique de Montréal (IFM2) for funding this research. I.G. Morgan acknowledges financial support from SSHRCC. We are grateful to Jean-Marie Gagnon, Zengxiang Wang, and members of the Queen's workshop in finance for suggestions. The usual disclaimer applies.
*Correspondence author, Département de Finance et Assurance et CRÉFA, Université Laval, Québec, Canada G1K 7P4, e-mail: Marie-Claude.Beaulieu@fas.ulaval.ca

Received May 2000; Accepted February 2002

- *Marie-Claude Beaulieu is with the Département de Finance et Assurance et CRÉFA, Université Laval, Québec, Canada.*
- *Shafiq K. Ebrahim is with the Financial Engines, Palo Alto, California.*
- *Ieuan G. Morgan is with the School of Business, Queen's University, Kingston, Ontario, Canada.*

The Journal of Futures Markets, Vol. 23, No. 1, 49–66 (2003)

Published online in Wiley InterScience (www.interscience.wiley.com). DOI: 10.1002/fut.10053

INTRODUCTION

Previous work has tended to show that futures prices incorporate new information faster than market indexes. Does the introduction of an exchange-traded index-mimicking security provide a useful alternative instrument with price discovery properties and, if so, what can lead-lag relationships with this security reveal about sufficient or necessary conditions for successful price discovery? These are the issues motivating this article. We will refer to the exchange-traded fund as the spot asset. Specifically, we examine the lead-lag relationships between a spot asset, TSE 35 Index Participation Units also known as TIPS,[1] and the associated futures contract. First, we want to determine whether the spot asset has a price discovery role similar to that of the futures contract. When a futures contract and an exchange-traded fund are defined in relation to the same market index, three pairs of time series are available for examination of the price discovery role through lead-lag relationships. If there is an empirically detectable lead-lag relationship in a given pair, can this be attributed to differences in microstructure or to some other comparative advantage? Second, given that there was a tick size decrease in the particular spot asset in the time period we study, we can assess the effect of this change on the lead-lag relationship between the spot market for the asset, the futures market, and the index itself.

In previous work on the lead-lag relationship, Stoll and Whaley (1990) used ordinary least-squares analysis of residuals from an ARMA model fitted to 5-minute interval data for the S&P 500 futures and index. Kawaller, Koch, and Koch (1987) used three-stage least squares in examining the lead-lag relationship of the S&P 500 futures and index with data for 1-minute intervals markets. Both of these articles concluded that lags were long, the futures index leading the index by anywhere from 10 to as much as 45 minutes. Chan, Chan, and Karolyi (1991) inferred that 5-minute interval data volatility in the index transmits to the futures and vice versa, so that both markets have a role in price discovery. De Jong and Nijman (1997) showed that the S&P 500 futures led the index by at least 10 minutes, but the index led the futures by at most 2 minutes. With later data, Ebrahim and Morgan (2000) used an index mimicking security

[1]TIPS trading started in March 1990 on the Toronto Stock Exchange (TSE). Each TIPS unit represented an interest in a trust that held baskets of the stocks in the TSE 35 index and reproduces the portfolio of the TSE 35 index. Each unit was traded as an individual security with a cash redemption value of one-tenth of the index value. TIPS later became known as TIPS 35, and they were finally delisted when investors exchanged their holdings for shares in the exchange traded fund created for the more diversified S&P TSE 60 index.

(SPDR) for the spot market and found that the S&P 500 futures led the index by 2 minutes and the SPDR by 1 minute. In turn, the SPDR led the index by 1 minute. Their empirical analysis suggests that the price discovery role is no longer restricted to the futures market.

Different relative tick sizes in the futures and the spot markets may well be important for our work. Accordingly, some of the previous research on microstructure and tick size is relevant here. Ronen and Weaver (2001) surveyed previous work on the theoretical and empirical implications of tick size reductions. In this literature, at least two opposing views have emerged about the importance of minimum tick size rules in financial markets. In one view, exchanges have good reasons for imposing such rules and, in principle, an optimum tick size exists. Harris (1991) noted the pervasive tendency of markets in financial and other assets to generate trades from a relatively coarse distribution of discrete prices. He argued the benefits from not using a finer gradation are a lower cost of negotiation between buyers and sellers and a greater incentive for more market-makers to participate and provide liquidity to the market by doing so. This implies the optimal tick size is not arbitrarily small. Empirical work (Chan & Hwang, 2000; Goldstein & Kavajecz, 2000) has focused on measures of trading costs and benefits such as liquidity. Among others, Ahn, Coe and Choe (1998) showed that the 1996 reduction in tick size for all stocks selling for at least $5 on the Toronto Stock Exchange (TSE) reduced cross-sectional averages of costs. Some of the other empirical work to date (e.g., Ahn et al., 1998; Huson et al., Mehrotra, 1997) has examined the effect of tick size changes by comparing stocks on an exchange that reduced tick size with the same stocks listed in a different country. The second country provides the benchmark. This approach is also cross-sectional.

In the second view, tick size rules may not matter, because it is possible to circumvent the tick size rules imposed by the exchange. Harris (1994) and Angel (1997) noted that firms are free to split their stocks, up or down, to achieve what they consider to be an optimum trading range for price. For example, by refusing to split (to a lower price per share) firms can allow their stock price to climb eventually to a range for which the bid ask spread is not constrained by the tick size. Similarly, in some markets, investors may be able—by breaking a trade into two transactions—to negotiate a price that is effectively between the bid and ask set by dealers. As a result, if the tick size rules are binding for the bid ask spread, the negotiated price circumvents the rules. In these circumstances the effective spread is smaller than the quoted spread.

The focus of this article is on time series instead of cross-sections. This allows us to follow the behavior of the three time series over a given period of time and to attribute our results to the behavior of these series rather than to an average of assets. In that context, there are two main empirical questions. First, does the spot asset lead or lag the index? In other words, does the spot asset play a role in price discovery in a way similar to the futures contract for the same underlying portfolio? The second one concerns the role of tick size. What are the relevant conditions for successful price discovery? Does the price of the spot asset adequately incorporate available information if the tick size is large? Our results show important differences between the spot asset and futures in both respects. In the spot asset the tick size is almost always binding for the bid ask spread; in the futures it is not. The futures lead the index. After, but not before, the reduction in tick size, the spot asset also leads the index. Given the more competitive trading costs for the futures, it may be difficult to evaluate how much the smaller relative tick size and the zero investment property of futures contracts contribute to the comparative advantage of the futures market over the spot market. Nonetheless, at a minimum, our results imply two things about successful price discovery in the two instruments: a zero investment property is not a necessary condition, and a small tick size is a sufficient condition.

The remainder of the article is organized as follows. The next section discusses the 1991 spot tick size reduction. Then we describe the data and present summary statistics for the time series. Then we present estimates for the effective bid ask spread for the futures and the spot. The test time series section presents the models used to test the hypotheses for differences across the subperiods before and after the tick size reduction. Then we show the analysis of lead-lag relationships, and we then conclude.

THE 1991 TICK SIZE REDUCTION

On September 27, 1991 the Toronto Stock Exchange (TSE) introduced decimal trading for the TSE 35 Index Participation Units. The TSE thereby reduced the spot asset tick size from 12.5 cents (a *relative* tick size of about 0.60% of the prevailing price) to 5 cents (0.25%). Because the bid ask spread is a major component of the cost of trading, if the tick size sets a lower bound on the bid ask spread, the reduction in tick size should have reduced the effective transaction cost for traders of the spot asset. The values of the index itself and the prices of futures and the spot asset should reflect changes in the unobservable equilibrium value of the same underlying portfolio.[2] Given the survival of both the futures and the spot asset, these cannot be perfect mutual substitutes, and must complement each other in some way, perhaps by improving hedging opportunities. Nonetheless, a preference for one instrument should result in its price responding to new information more rapidly than the price of the other.

There are important differences between the futures and the spot asset. The relative tick size in the futures is much smaller, and the futures also have the potential advantage for the trader to exploit the zero investment property of the futures contracts. As a result of one or the other or both of these differences, the futures contract trades more frequently. The most important difference between the index and the two traded instruments is that the index does not reflect all relevant information at a given instant. It reflects the most recently observed prices of the component stocks instead of their unobservable up-to-date equilibrium values. Not all stocks trade every few minutes of each day, and the resulting nonsynchronous trading is known to introduce positive autocorrelation in the time series of returns or price changes (Fisher, 1966; Lo & MacKinlay, 1990). This characteristic of stock market indices is a source of noise in studies that want to assess whether the futures market leads the index. Nonsynchronous trading applies neither to the spot asset nor to the futures, both being individually traded entities.

Tick Size and Bid Ask Spread

It has been well recognized that short-term price changes of traded stocks or futures contracts are prone to negative first-order autocorrelation induced by bid ask bounce. Roll (1984) measured the *implied* bid ask spread from the first-order autocovariance of returns. His model assumes that bid and ask prices straddle the unobservable equilibrium price. In the absence of new information, the observed price follows a two-state Markov process with equal probabilities of no change from the existing state (a price at the bid or one at the ask) and a reversal to the other state. With trade and quote information, it is also possible to measure the *effective* bid ask spread (Blume & Goldstein, 1992) as twice the absolute difference between the transaction price and the midpoint of the extant bid and ask prices. If the tick size is binding, its effect might be detectable in changes in the measures of the effective bid

[2]We detected a significant increase in daily volume of TIPS following the tick reduction but we do not report the results since there was a similar increase in futures volume. Inspection of Table II below shows increases in the average number of transactions per day in each series, since the sample contains 39 days before and only 24 days after the change.

ask spread at the time of the tick size reduction in the spot asset. We address this question with analyses of time series data for the months straddling the 1991 reduction in the spot tick size.

Tick Size and the Prediction of the Index

Kawaller, Koch, and Koch (1987) show that price movements in the S&P 500 futures lead movements in the S&P 500 index, and argue that this is a consequence of nonsynchronous trading in the index. Because the futures and the spot are both traded instruments, either of them might be useful as predictors for the index value. For the TSE 35 index, if the tick size is unimportant or if it is binding before and after the tick reduction, there should be no difference between subperiods in the ability of the lagged value of the spot price change to predict the current change in the market index. If tick size matters, however, and it is binding before but not after the reduction, we might expect improved predictive power following the reduction if the spot price then responds more rapidly to the arrival of new information. In this way, our work can shed light on the question of the relevance of tick size.

DATA DESCRIPTION

Data Sources

The data for this article come from the Toronto Stock Exchange. The raw data consist of trade-by-trade transaction prices and bid and ask quotes for the spot and futures and 15-s calculations of the index values from August 1, 1991 to October 31, 1991. This sample period includes the date of the tick size reduction, September 27. Stock market trading ends at 4:00 p.m., futures trading continues until 4:15 p.m. We discard the last 15 minutes of trading for the futures.

Futures contracts on the TSE 35 index expire on the third Thursday of every month. Contract months are the 2 nearest months and the next 2 calendar quarterly months from the March, June, September, and December cycle. Futures contracts trade from 9:15 a.m. to 4:15 p.m. Eastern Time.

Describing the Time Series of Price Changes

We begin with a summary of time series of 15-minute intervals of spot, futures, and the index. These series consist of price changes for the values closest to the midpoint of each interval. In these preliminary analyses, we discard any 15-minute interval that did not contain a trade and calculate price changes from the remaining intervals within the day. This procedure excludes overnight price changes, so the earliest possible price change is for the second 15-minute interval of each day. The procedure also ensures that no futures price change mixes prices from two different contracts at the time a contract expires. As a result, futures price changes are always for the nearby contract.

Figures 1–3 show the raw data for changes in price, not rates of change. The futures price changes in Figure 1 exemplify a discrete distribution, with many changes of one and two ticks of 10 cents. There is no obvious change in this pattern at the date of the spot tick size reduction, shown as 910827 in Figure 1. In contrast, the spot price changes in Figure 2 show a sharp break at the time of the tick size reduction. In Figure 2, in the earlier subperiod all nonzero price changes correspond to either one or two ticks of 12.5 cents. In the later subperiod all nonzero price changes correspond to either one or two ticks of 5 cents. Because the spot asset cash redemption value is one-tenth of the index value, 5 cents for the spot corresponds to 50 cents for the futures contract. A comparison of Figures 1 and 2 reveals that price changes often exceed one tick in the futures but rarely do so, even after the tick size reduction, in the spot. In fact, there was only one interval with a change of two ticks (25 cents) before and only seven intervals with a change of two ticks (10 cents) after the reduction. The index changes in Figure 3 show no trace of the discrete patterns so readily discernible in Figures 1 and 2 but Figure 3 resembles Figure 1 more closely than Figure 2.

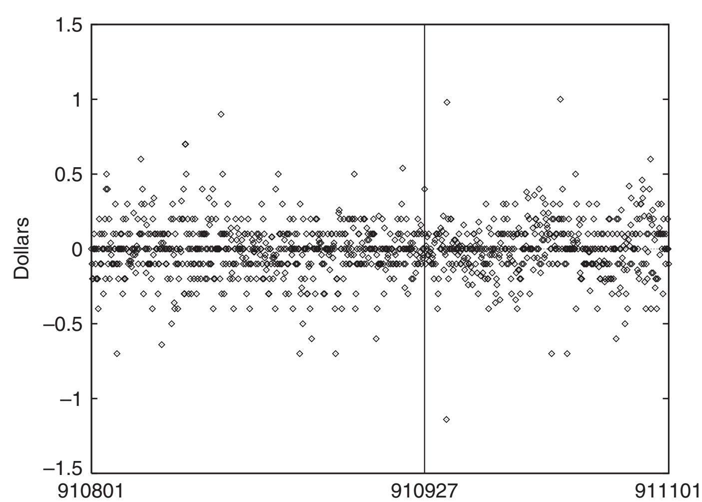

FIGURE 1
Trade-to-trade futures price changes.

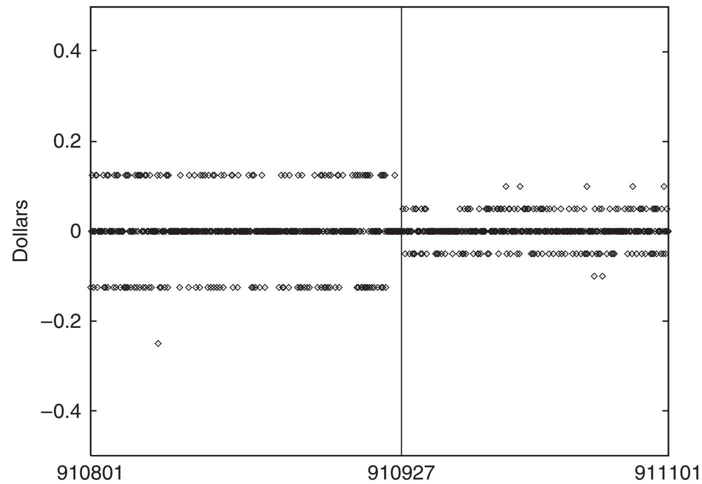

FIGURE 2
Trade-to-trade spot price changes.

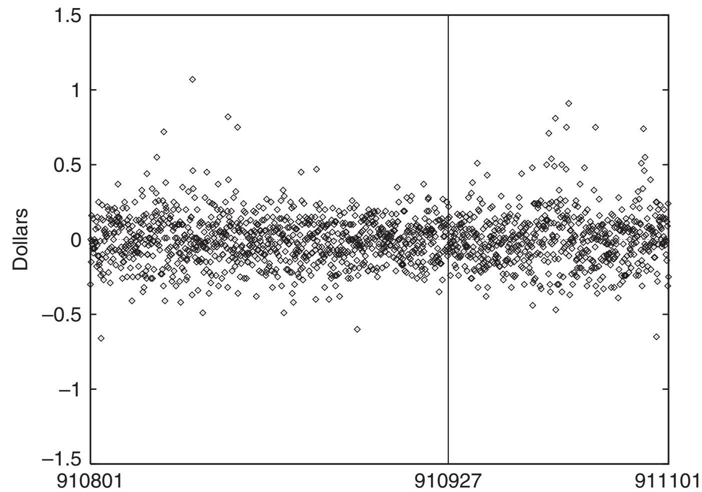

FIGURE 3
Index value changes.

Table I shows descriptive statistics for the 15-minute interval series of changes in log price for the futures, spot, and index. For these statistics the log price changes are multiplied by 100. Of particular interest are the features of the autocorrelation function. The futures show no evidence of significant first-order autocorrelation or any evidence of higher order autocorrelation in the first 48 lags. In contrast, both the spot and the index have significantly negative first-order autocorrelation. Negative autocorrelation in the spot is consistent with bid ask bounce in a traded security. Negative autocorrelation in the index is consistent with the Stoll and Whaley (1990) argument that poor diversification allows

TABLE I

Futures, Spot, and Index: Descriptive Statistics for 15-Min Interval Log Price Changes

Sample Period	*Statistic*	*Futures*	*Spot*	*Index*
1991.08.01–1991.10.31	Mean	−0.0029	−0.0018	−0.0010
	σ	0.097	0.307	0.091
	ρ_1	0.024	−0.338	−0.129
		[0.030]	[0.036]	[0.025]
	$Q(48)$	53.07	150.03	84.37
		(0.28)	(0.00)	(0.00)
	$Q^2(48)$	90.99	687.61	104.36
		(0.00)	(0.00)	(0.00)
	T	1110	780	1576
1991.08.01–1991.09.26	Mean	−0.0061	−0.0077	−0.0029
	σ	0.094	0.386	0.086
	ρ_1	0.046	−0.351	−0.109
		[0.039]	[0.049]	[0.032]
	$Q(48)$	58.05	99.17	51.14
		(0.15)	(0.00)	(0.35)
	$Q^2(48)$	77.95	71.03	79.03
		(0.00)	(0.02)	(0.00)
	T	640	419	975
1991.09.27–1991.10.31	Mean	0.0014	0.0050	0.0022
	σ	0.100	0.175	0.099
	ρ_1	−0.005	−0.269	−0.156
		[0.046]	[0.052]	[0.041]
	$Q(48)$	53.23	109.20	81.54
		(0.28)	(0.00)	(0.00)
	$Q^2(48)$	62.28	57.39	86.50
		(0.08)	(0.17)	(0.00)
	T	470	361	601

Note. σ is the sample standard deviation; ρ_1 is the first-order autocorrelation coefficient, with standard errors in brackets; $Q(48)$ is the Ljung-Box (1978) portmanteau test for the first 48 lags of the autocorrelation function (corresponding to almost 2 days of trading); p-values for the $\chi^2(48)$ distribution are in parentheses; $Q^2(48)$ is the corresponding statistic for the squared data; T is the sample size. All log price changes are scaled by multiplication by 100.

the bid ask bounce in the more actively traded individual stocks to carry over to the whole when there is periodic nontrading of the less actively traded stocks.

The portmanteau statistics for the squared deviations from the mean reveal evidence of heteroscedasticity in some but not all of the series. In particular, the portmanteau test statistic for the spot rejects homoscedasticity in the overall sample but does not reject homoscedasticity at the 0.01 level in either subperiod. This is indicative of a sharp change in the properties of the series at the time of the tick change but with each subperiod behaving as a more or less homoscedastic series. Figure 2 makes it clear that the change was sharp. We test the hypotheses about homoscedasticity within each subperiod but heteroscedasticity across the two subperiods in a later subsection. Because heteroscedasticity across subperiods in the spot may be partly due to a reduction in first-order autocovariance at the time of the tick size change, we first show a summary of the evidence about the effective bid ask spread.

EFFECTIVE BID ASK SPREAD ESTIMATES

In measuring the effective bid ask spread from tick-by-tick trade and quote data, we use almost all the raw data and compute twice the absolute difference between the transaction price and the midpoint of the extant bid and ask prices (Blume & Goldstein, 1992). For those cases in which the bid and ask quotes were equal (as happened occasionally in the spot data but never for the futures) we substitute the previous pair of valid quotes with a positive spread. We start with the first quotes at the open of trading and recording, for each transaction during the day, the quote midpoint for the most recent valid simultaneous bid and ask quotes preceding the transaction.

Table II shows the estimates of the average effective and quoted spreads for the subperiods before and after the spot tick size reduction for the spot and the futures only. For the futures there is some evidence of a reduction in the effective spread in the second subperiod. The

TABLE II
Effective and Quoted Bid Ask Spreads

	Futures		*Spot*	
Sample Period	*Effective Spread*	*Quoted Spread*	*Effective Spread*	*Quoted Spread*
1991.08.01–1991.09.26	0.199	0.198	0.092	0.157
	(0.0121)	(0.0142)	(0.0018)	(0.0015)
	2766	1287	992	1730
1991.09.27–1991.10.31	0.149	0.170	0.039	0.063
	(0.0041)	(0.0106)	(0.0008)	(0.0010)
	2313	984	1001	1708
t-test	3.90	1.60	27.16	52.07

Note. Each group of three values consists of the mean, standard error (in parentheses), and number of observations. For the effective spread, the number of observations is the number of trades. The *t*-test is for the difference between the means of the first and second subperiods in each case. The spot price here corresponds to one-tenth of the index value or roughly one-tenth of the futures price.

apparent reduction is approximately 25% for the effective spread and 14% for the quoted spread. The t-test shows only the former to be significant. For the spot, however, there is a very sharp drop in both the effective and quoted spreads after the tick size reduction. The effective spread drops by 58% and the quoted spread falls by 60%.

TESTS OF TIME SERIES PROPERTIES ACROSS SUBPERIODS

The Model for Trade-to-Trade Returns

The purpose of this model is to test the significance of the differences, apparent in Table I for returns, in properties of the time series of 15-minute interval rates-of-return across the two subperiods. We use a GARCH model (Bollerslev, 1986; Engle, 1982) with different unconditional variances and different MA(1) terms in the two subperiods. This model allows us to test two null hypotheses: that there is no difference in unconditional variance, and that there is no difference in the first-order autocorrelation property between the two subperiods.

Let d_{jt}, $j = 1, 2$, represent indicator variables for the subperiods before and after the spot asset tick size reduction, let r_t represent the rate of change in price from $t - 1$ to t, and let ε_t represent a normally distributed error term with zero mean and conditional variance h_t. These indicator variables allow both the unconditional variance and the MA(1) coefficient to differ across subperiods. The conditional variance specification is based on Baillie and Bollerslev (1989). The system is

$$\begin{gathered} r_t = \alpha + \varepsilon_t + \psi_1 d_{1t-1}\varepsilon_{t-1} + \psi_2 d_{2t-1}\varepsilon_{t-1} \\ \mathrm{var}_{t-1}(\varepsilon_t) = h_t \\ h_t = c + a\varepsilon_{t-1}^2 + bh_{t-1} + \phi[d_{1t} - (a + b)d_{1t-1}]. \end{gathered} \tag{1}$$

The model specifies the unconditional variance to be

$$h = \phi = c/[1 - (a + b)] \tag{2}$$

in the first subperiod and

$$h = c/[1 - (a + b)] \tag{3}$$

in the second subperiod. A coefficient ϕ with value significantly different from zero would reject the hypothesis of equal unconditional variances across subperiods.

Estimates and Tests of the Models for Trade-to-Trade Returns

One of the main features of Table III is the highly significant positive value of ϕ for the spot, in contrast to the futures and the index, for which the estimates are numerically small. The unconditional variance for the spot seems to be much larger in the first subperiod, but there is no apparent corresponding change for the futures or the index. After adjustments for the scaling of the data for the numerical work, the estimated unconditional variance for the spot is 0.1344 before and 0.0262 after the tick size reduction to 5 cents. The estimated standard deviations are 0.367 and 0.162, respectively and they correspond well with the sample values of 0.386 and 0.175 in Table I. This correspondence, together with the fact that the estimated GARCH parameter for the lagged conditional variance hit its lower bound of zero supports the contention that, for the

TABLE III

Futures, Spot, and Index: Estimates of the Model for Log Price Changes

$$r_t = \alpha + \varepsilon_t + \psi_1 d_{1t-1}\varepsilon_{t-1} + \psi_2 d_{2t-1}\varepsilon_{t-1}$$

$$\mathrm{var}_{t-1}(\varepsilon_t) = h_t$$

$$h_t = c + a\varepsilon_{t-1}^2 + bh_{t-1} + \phi[d_{1t} - (a+b)d_{1t-1}]$$

Parameter	*Futures*	*Spot*	*Index*
α	−0.058	−0.004	−0.027
	(0.040)	(0.088)	(0.025)
ψ_1	0.040	−0.432	−0.146
	(0.074)	(0.072)	(0.059)
ψ_2	0.044	−0.260	−0.177
	(0.068)	(0.070)	(0.075)
c	0.520	2.621	0.175
	(0.166)	(0.606)	(0.091)
a	0.128	0.083	0.126
	(0.044)	(0.126)	(0.054)
b	0.356	0.0	0.636
	(0.308)		(0.217)
ϕ	−0.0014	0.097	0.0023
	(0.0043)	(0.032)	(0.0042)

Note. Robust standard errors (Bollerslev and Wooldridge, 1992) are in parentheses. The subscripts on the MA(1) parameters and indicator variables refer to the first and second subperiods (before and after the reduction in the spot tick size). The dependent variable was scaled by multiplication by 1000; the variable d_{1t} in the conditional variance function was scaled by multiplication by 100.

TABLE IV

Futures, Spot, and Index: Tests of Model Restrictions

	Futures	*Spot*	*Index*
$\psi_1 = \psi_2$	0.002	6.29	0.31
	(0.964)	(0.012)	(0.530)
$\phi = 0$	2.41	154.57	6.58
	(0.120)	(0.000)	(0.010)
$a = b = 0$	47.14	3.83	56.93
	(0.000)	(0.050)	(0.000)

Note. *p*-values in parentheses are from $\chi^2(1)$ for the likelihood ratio tests of restrictions $\psi_1 = \psi_2$ and $\phi = 0$; and from $\chi^2(2)$ for the restriction $a = b = 0$, except for the spot, for which the *p*-value is from $\chi^2(1)$.

spot, the time series is essentially homoscedastic within subperiods. Table IV provides a test of homoscedasticity within subperiods by also suppressing the parameter a and the hypothesis is not rejected at the 0.01 level. A second feature of Table III is a possible difference between the values of the MA(1) coefficients in the two subperiods for the spot. This coefficient appears to be more strongly negative in the first subperiod, when the tick size was 12.5 cents. Table IV tests the restriction of equal MA(1) coefficients, $\psi_1 = \psi_2$, in the model. The restriction is not rejected at the 0.01 level but would be rejected at the 0.02 level. Combining the results for the tests of the restrictions $\phi = 0$ and $\psi_1 = \psi_2$ establishes that the first-order autocovariance is greater before than after the tick size reduction, as Table I suggests.

LEAD-LAG RELATIONSHIPS

To assess the price discovery role of the spot asset, we compare lead-lag relationships between the spot asset, the futures contract, and the index in each subperiod. As Table I indicates, many 15-minute intervals contain no trades for either the spot asset or the futures contract. In dealing with irregularly spaced data, researchers frequently impute zero returns for these intervals. However, this approach may bias correlation estimates toward zero. Instead, De Jong and Nijman (1997) propose a covariance estimator that takes into account these missing observations and assumes that the price process is independent of the trading pattern. De Jong and Donders (1998) apply the estimator to infer that the futures lead both the Amsterdam European Options Exchange AEX stock index and the options on the index. De Jong, Mahieu, and Schotman (1998) apply it to the foreign exchange market. This estimator is described below.

Econometric Methodology

Let p_t and q_t be the logarithms of prices of two assets observed at time t. We assume that price levels are nonstationary and become stationary after differencing. The differenced values are

$$\Delta p_t = p_t - p_{t-1}, \quad \Delta q_t = q_t - q_{t-1}.$$

The crosscovariance function is

$$\gamma(k) = \operatorname{cov}(\Delta p_t, \Delta q_t).$$

These price changes are not always observable because of occasional nontrading. We can index the observed values of p_t by i and the observed values of q_t by j. Then, the log price change over a time period spanning a number of intervals can be written as the sum of unobserved single-interval changes,

$$p_{t_{i+1}} - p_{t_i} = \sum_{t=t_i+1}^{t_{i+1}} \Delta p_t \tag{4}$$

where t_i is the clock time of the ith observation. The crossproduct of log price changes in two markets is

$$y_{ij} = (p_{t_{i+1}} - p_{t_i})(q_{t_{j+1}} - q_{t_j}) = \sum_{t=t_i+1}^{t_{i+1}} \Delta p_t \cdot \sum_{s=t_j+1}^{t_{j+1}} \Delta q_t. \tag{5}$$

The expectation of this linear combination of crossproducts is a linear combination of crosscovariances $\gamma(k)$,

$$E(y_{ij}) = E\left(\sum_{t=t_i+1}^{t_{i+1}} \Delta p_t \cdot \sum_{s=t_j+1}^{t_{j+1}} \Delta q_t \right) = \sum_{t=t_i+1}^{t_{i+1}} \sum_{s=t_j+1}^{t_{j+1}} \gamma(t-s). \tag{6}$$

De Jong and Nijman (1997) show that the number of times that $\gamma(k)$ appears in Equation (6) is given by

$$x_{ij}(k) = \max(0, \min(t_{i+1}, t_{j+1} + k) - \max(t_i, t_j + k)). \tag{7}$$

Assuming that all crosscovariances of order $k > K$ are equal to zero, $E(y_{ij})$ can be rewritten as a linear combination of crosscovariances $\gamma(k)$

with the x_{ij} values as coefficients,

$$E(y_{ij}) = \sum_{k=-K}^{k=K} x_{ij}(k)\gamma(k). \tag{8}$$

It is possible to think of Equation (8) as a regression equation with the unknown crosscovariances $\gamma(k)$ as parameters and the x_{ij} values as explanatory variables. The unknown crosscovariances can be estimated consistently by ordinary least squares. We use only adjacently observed prices to construct the values for y_{ij} and x_{ij}, because nonadjacent observations are linearly related to differences of adjacent observations, and do not add more information. In addition, crossproducts for which $|t_{i+1} - t_j| \geq K$ and $|t_i - t_{j+1}| \geq K$ are omitted from the regression as all explanatory variables would be zero otherwise. We set K equal to 5 in this study, and we select observed prices nearest to the midpoint of each 15-minute interval. We choose midpoints instead of using the last transaction in each interval to avoid a possible bias toward finding that a relatively active market appears to lead a less active market merely because the latest transaction is more likely to be closer to the end point in the more active market. Autocovariances are estimated by imposing the constraint $\gamma(n) = \gamma(-n)$ in the regression equation.

We form estimates of crosscorrelations of log price changes using variances and crosscovariances obtained from the above procedure. For example, denoting the estimated variances of p_t and q_t by $\hat{\gamma}_0^p$ and $\hat{\gamma}_0^q$, the crosscorrelation, $\hat{\rho}_k$, is given by

$$\hat{\rho}_k = \frac{\hat{\gamma}_k}{\sqrt{(\hat{\gamma}_0^p \cdot \hat{\gamma}_0^q)}}.$$

Discussion of the Results

Table V presents the crosscorrelations between the index, the futures, and the spot asset, as computed with the De Jong and Nijman procedure for the subperiods before and after the spot tick size reduction. In the first subperiod, the futures contract leads the index by 15 minutes. On the other hand, the spot asset does not play a price discovery role for the index. Indeed, none of the crosscorrelations between the index and the spot is significant at the 5% level. Similarly, there does not appear to be a discernible relationship between the spot and the futures in the first subperiod: of 11 coefficients only one is significant at the 5% level and it is negative. In the second subperiod, the futures leads the index by 30 minutes. The spot asset does have a price discovery role for the index in the second subperiod—the crosscorrelations between the current index

TABLE V
Crosscorrelations of Futures, Spot, and Index Log Price Changes

	1991.08.01–1991.09.26			*1991.09.27–1991.10.31*		
Lag	*Index & Futures*	*Index & Spot*	*Futures & Spot*	*Index & Futures*	*Index & Spot*	*Futures & Spot*
−5	0.1226	0.0472	0.2342	0.0750	0.1359	0.5487
	(1.47)	(0.65)	(1.05)	(0.62)	(1.54)	(2.04)
−4	0.0263	0.0769	0.2545	0.0986	0.0194	−0.4066
	(0.36)	(0.95)	(1.08)	(1.03)	(0.28)	(−1.53)
−3	0.0065	0.0283	0.1916	−0.0733	−0.0336	0.2340
	(0.09)	(0.37)	(0.98)	(−0.70)	(−0.33)	(0.96)
−2	−0.0202	−0.0370	−0.3767	0.2493	0.0821	0.1644
	(−0.20)	(−0.50)	(−2.24)	(2.16)	(0.82)	(0.80)
−1	−0.0828	0.0726	0.1394	0.0105	−0.0045	−0.0006
	(−0.80)	(0.90)	(0.86)	(0.10)	(−0.06)	(−0.00)
0	0.6684	0.1463	0.2964	0.7369	0.1658	0.9778
	(4.56)	(1.74)	(1.43)	(3.70)	(2.07)	(3.07)
1	0.2705	0.0916	−0.0145	0.2216	0.2014	−0.2697
	(2.71)	(1.20)	(−0.07)	(2.05)	(2.40)	(−1.28)
2	−0.0793	−0.0970	0.1095	0.2166	−0.0255	0.1801
	(−0.90)	(−1.20)	(0.50)	(2.02)	(−0.30)	(0.85)
3	0.1095	0.0384	−0.0405	0.1728	0.1813	0.1990
	(1.18)	(0.42)	(−0.22)	(1.67)	(2.37)	(1.03)
4	0.0820	−0.0130	−0.1609	0.0240	−0.0811	−0.0209
	(1.19)	(−0.20)	(−1.13)	(0.23)	(−1.21)	(−0.11)
5	0.0211	0.1307	−0.0275	0.1642	0.0470	0.3206
	(0.26)	(1.75)	(−0.15)	(1.41)	(0.64)	(1.49)

Note. The table shows the crosscorrelations between current log price changes of the first instrument and lagged changes of the second. Heteroscedasticity-consistent *t*-statistics are in parentheses.

changes and the current, first lagged, and third lagged spot changes are positive and significant. In addition, there is a large contemporaneous crosscorrelation between the spot asset and the futures contract in the second subperiod, reflecting the fact that both instruments now share similar price dynamics. These results indicate that the tick size of an instrument linked to an underlying portfolio has an important effect on its price discovery role for that portfolio. It is only after the tick size reduction that the spot asset price is able adequately to incorporate information about the underlying portfolio in a manner akin to that of the futures.

SUMMARY AND CONCLUSION

We have examined the price discovery role of an exchange-traded fund and the futures contract for the same market index. The direct question was to determine whether or not the fund, as the spot asset, could play a role similar to that of the futures. The results show that the spot asset led the index only in the subperiod following a sharp reduction in tick size. In contrast, the futures led the index in both subperiods.

The underlying question in this work is concerned with the factors leading to successful price discovery. If the success of the futures depends on zero investment or at least the greater flexibility in margin rules, encouraging market timers with limited capital to trade in futures rather than the spot market, we would not expect the results to differ across subperiods. Instead, our results imply two things about the factors important for price discovery in the two instruments: a zero investment property is not a necessary condition, and a small tick size is a sufficient condition.

BIBLIOGRAPHY

Ahn, H.-J., Cao, C. Q., & Choe, H. (1998). Decimalization and competition among stock markets: Evidence from the Toronto Stock Exchange cross-listed securities. Journal of Financial Markets 1, 51–87.

Angel, J. (1997). Tick size, share prices, and stock splits. Journal of Finance, 52, 655–682.

Baillie, R. T., & Bollerslev, T. (1989). The message in daily exchange rates: A conditional variance tale. Journal of Business and Economic Statistics, 7, 297–305.

Blume M., & Goldstein, M. (1992). Displayed and effective spreads. Rodney L. White Center for Financial Research Working Paper 27–92, The Wharton School.

Bollerslev, T. (1986). Generalized autoregressive conditional heteroscedasticity. Journal of Econometrics, 31, 307–327.

Bollerslev, T., & Wooldridge, J. M. (1992). Quasi-maximum likelihood estimation and inference in dynamic models with time-varying covariances. Econometric Reviews, 11, 143–178.

Chan, K. C., & Hwang, C.-Y. (2000). The impact of tick size on market quality: An empirical investigation of the stock exchange of Hong Kong tick size and market quality. Working paper, Hong Kong University of Science and Technology.

Chan, K., Chan, K. C., & Karolyi, G. A. (1991). Intraday volatility in the stock index and stock index futures markets. The Review of Financial Studies, 4(4), 657–684.

De Jong, F., & Donders, M. W. M. (1998). Intraday lead-lag relationships between the futures, options and stock market. European Financial Review, 1, 337–359.

De Jong, F., & Nijman, T. (1997). High frequency analysis of lead-lag relationships between financial markets. Journal of Empirical Finance, 4(2–3), 259–277.

De Jong, F., Mahieu, R., & Schotman, P. (1998). Price discovery in the foreign exchange market: An empirical analysis of the yen/dmark rate. Journal of International Money and Finance, 17, 5–27.

Ebrahim, S. K., & Morgan, I. G. (2000). High frequency relationships between the S&P 500 index, futures and depository receipts (manuscript). Queen's University.

Engle, R. (1982). Autoregressive conditional heteroscedasticity with estimates of variance of United Kingdom inflation. Econometrica, 50, 987–1007.

Fisher, L. (1966). Some new stock market indexes. Journal of Business, 39, 191–225.

Goldstein, M., & Kavajecz, K. (2000). Eights, sixteenths and market depth: Changes in tick size and liquidity provision on the New York Stock Exchange. Journal of Financial Economics, 56(1), 125–149.

Harris, L. (1991). Stock price clustering and discreteness. Review of Financial Studies, 4, 389–415.

Harris, L. (1994). Minimum price variations, discrete bid-ask spreads and quotation sizes. Review of Financial Studies, 7, 149–178.

Huson, M., Kim, Y., & Mehrotra, V. (1997). Decimal quotes, market quality, and competition for order flow: Evidence from the Toronto Stock Exchange. Unpublished manuscript, University of Alberta.

Kawaller, I. G., Koch, P. D., & Koch, T. W. (1987). The temporal relationship between S&P 500 futures and the S&P 500 index. Journal of Finance, 42, 1309–1329.

Ljung, G. M., & Box, G. E. P. (1978). On a measure of lack of fit in time series models. Biometrika, 65, 297–303.

Lo, A., & MacKinlay, A. C. (1990). An econometric analysis of nonsynchronous trading. Journal of Econometrics, 45, 181–212.

Ronen, T., & Weaver, D. G. (2001). "Teenies" anyone? Journal of Financial Markets, 4(3), 231–260.

Roll, R. (1984). A simple implicit measure of the effective bid-ask spread in an efficient market. Journal of Finance, 39, 1127–1139.

Stoll, H., & Whaley, R. (1990). The dynamics of stock index futures returns. Journal of Financial and Quantitative Analysis, 25, 441–468.