Problem 3

Use truth tables to show:

Law I

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$\begin{aligned}\neg(\text{true} \wedge \text{true}) &\equiv \neg\text{true} \vee \neg\text{true}\\ \neg\text{true} &\equiv \text{false} \vee \text{false}\\ \text{false} &\equiv \text{false}\end{aligned}$$

$$\begin{aligned}\neg(\text{true} \wedge \text{false}) &\equiv \neg\text{true} \vee \neg\text{false}\\ \neg\text{false} &\equiv \text{false} \vee \text{true}\\ \text{true} &\equiv \text{true}\end{aligned}$$

$$\begin{aligned}\neg(\text{false} \wedge \text{true}) &\equiv \neg\text{false} \vee \neg\text{true}\\ \neg\text{false} &\equiv \text{true} \vee \text{false}\\ \text{true} &\equiv \text{true}\end{aligned}$$

$$\begin{aligned}\neg(\text{false} \wedge \text{false}) &\equiv \neg\text{false} \vee \neg\text{false}\\ \neg\text{false} &\equiv \text{true} \vee \text{true}\\ \text{true} &\equiv \text{true}\end{aligned}$$

Law II

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

$$\begin{aligned}\neg(\text{true} \vee \text{true}) &\equiv \neg\text{true} \wedge \neg\text{true}\\ \neg\text{true} &\equiv \text{false} \wedge \text{false}\\ \text{false} &\equiv \text{false}\end{aligned}$$

$$\begin{aligned}\neg(\text{true} \vee \text{false}) &\equiv \neg\text{true} \wedge \neg\text{false}\\ \neg\text{true} &\equiv \text{false} \wedge \text{true}\\ \text{false} &\equiv \text{false}\end{aligned}$$

$$\begin{aligned}\neg(\text{false} \vee \text{true}) &\equiv \neg\text{false} \wedge \neg\text{true}\\ \neg\text{true} &\equiv \text{true} \wedge \text{false}\\ \text{false} &\equiv \text{false}\end{aligned}$$

$$\begin{aligned}\neg(\text{false} \vee \text{false}) &\equiv \neg\text{false} \wedge \neg\text{false}\\ \neg\text{false} &\equiv \text{true} \wedge \text{true}\\ \text{true} &\equiv \text{true}\end{aligned}$$

Problem 6

Show $(p \implies q) \equiv \neg p \vee q$:

$$
\begin{aligned}
(\text{true} \implies \text{true}) &\equiv \neg\text{true} \vee \text{true} \\
\text{true} &\equiv \text{false} \vee \text{true} \\
\text{true} &\equiv \text{true} \\
(\text{true} \implies \text{false}) &\equiv \neg\text{true} \vee \text{false} \\
\text{false} &\equiv \text{false} \vee \text{false} \\
\text{false} &\equiv \text{false} \\
(\text{false} \implies \text{true}) &\equiv \neg\text{false} \vee \text{true} \\
\text{true} &\equiv \text{true} \vee \text{true} \\
\text{true} &\equiv \text{true} \\
(\text{false} \implies \text{false}) &\equiv \neg\text{false} \vee \text{false} \\
\text{true} &\equiv \text{true} \vee \text{false} \\
\text{true} &\equiv \text{true}
\end{aligned}
$$

Problem 22

Symbolize each of the following phrases:

a) Each x has property P: $\forall x, P(x)$.

b) Every x has property P: $\forall x, P(x)$.

c) Some x has property P: $\exists x \text{ st } P(x)$.

d) All x have property P: $\forall x, P(x)$.

e) At least one x has property P: $\exists x \text{ st } P(x)$.

Problem 23

Negate each statement in the previous exercise:

a) $\exists x \text{ st } \neg P(x)$.

b) $\exists x \text{ st } \neg P(x)$.

c) $\forall x, \neg P(x)$.

d) $\exists x \text{ st } \neg P(x)$.

e) $\forall x, \neg P(x)$.

Additional Problem

Define: $p \Longrightarrow q$, as well as $p \not\Longrightarrow q$.
$p \Longrightarrow q$ means that, given p, q is true: if p, then q. This means that either p is false, which means that you can prove anything given it; or both p and q are true.
$p \not\Longrightarrow q$ means that we cannot get q from p: p must be true, since if it were false, we could prove anything; and additionally, q must be false, because otherwise we could prove it to be true.

$$\begin{aligned}
true &\Longrightarrow true \equiv true \\
true &\Longrightarrow false \equiv false \\
false &\Longrightarrow true \equiv true \\
false &\Longrightarrow false \equiv true
\end{aligned}$$

$$\begin{aligned}
true &\not\Longrightarrow true \equiv false \\
true &\not\Longrightarrow false \equiv true \\
false &\not\Longrightarrow true \equiv false \\
false &\not\Longrightarrow false \equiv false
\end{aligned}$$