Problem 3

Use truth tables to show:

Law I

$$\neg(p \land q) \equiv \neg p \lor \neg q$$

$$\begin{aligned}
\neg(\mathit{true} \land \mathit{true}) &\equiv \neg\mathit{true} \lor \neg\mathit{true} \\
\neg\mathit{true} &\equiv \mathit{false} \lor \mathit{false} \\
\mathit{false} &\equiv \mathit{false}
\end{aligned}$$

$$\begin{aligned}
\neg(\mathit{true} \land \mathit{false}) &\equiv \neg\mathit{true} \lor \neg\mathit{false} \\
\neg\mathit{false} &\equiv \mathit{false} \lor \mathit{true} \\
\mathit{true} &\equiv \mathit{true}
\end{aligned}$$

$$\begin{aligned}
\neg(\mathit{false} \land \mathit{true}) &\equiv \neg\mathit{false} \lor \neg\mathit{true} \\
\neg\mathit{false} &\equiv \mathit{true} \lor \mathit{false} \\
\mathit{true} &\equiv \mathit{true}
\end{aligned}$$

$$\begin{aligned}
\neg(\mathit{false} \land \mathit{false}) &\equiv \neg\mathit{false} \lor \neg\mathit{false} \\
\neg\mathit{false} &\equiv \mathit{true} \lor \mathit{true} \\
\mathit{true} &\equiv \mathit{true}
\end{aligned}$$

Law II

$$\neg(p \lor q) \equiv \neg p \land \neg q$$

$$\begin{aligned}
\neg(\mathit{true} \lor \mathit{true}) &\equiv \neg\mathit{true} \land \neg\mathit{true} \\
\neg\mathit{true} &\equiv \mathit{false} \land \mathit{false} \\
\mathit{false} &\equiv \mathit{false}
\end{aligned}$$

$$\begin{aligned}
\neg(\mathit{true} \lor \mathit{false}) &\equiv \neg\mathit{true} \land \neg\mathit{false} \\
\neg\mathit{true} &\equiv \mathit{false} \land \mathit{true} \\
\mathit{false} &\equiv \mathit{false}
\end{aligned}$$

$$\begin{aligned}
\neg(\mathit{false} \lor \mathit{true}) &\equiv \neg\mathit{false} \land \neg\mathit{true} \\
\neg\mathit{true} &\equiv \mathit{true} \land \mathit{false} \\
\mathit{false} &\equiv \mathit{false}
\end{aligned}$$

$$\begin{aligned}
\neg(\mathit{false} \lor \mathit{false}) &\equiv \neg\mathit{false} \land \neg\mathit{false} \\
\neg\mathit{false} &\equiv \mathit{true} \land \mathit{true} \\
\mathit{true} &\equiv \mathit{true}
\end{aligned}$$

Problem 6

Show $(p \implies q) \equiv \neg p \vee q$:

$$\begin{aligned}
(\mathit{true} \implies \mathit{true}) &\equiv \neg \mathit{true} \vee \mathit{true} \\
\mathit{true} &\equiv \mathit{false} \vee \mathit{true} \\
\mathit{true} &\equiv \mathit{true}
\end{aligned}$$

$$\begin{aligned}
(\mathit{true} \implies \mathit{false}) &\equiv \neg \mathit{true} \vee \mathit{false} \\
\mathit{false} &\equiv \mathit{false} \vee \mathit{false} \\
\mathit{false} &\equiv \mathit{false}
\end{aligned}$$

$$\begin{aligned}
(\mathit{false} \implies \mathit{true}) &\equiv \neg \mathit{false} \vee \mathit{true} \\
\mathit{true} &\equiv \mathit{true} \vee \mathit{true} \\
\mathit{true} &\equiv \mathit{true}
\end{aligned}$$

$$\begin{aligned}
(\mathit{false} \implies \mathit{false}) &\equiv \neg \mathit{false} \vee \mathit{false} \\
\mathit{true} &\equiv \mathit{true} \vee \mathit{false} \\
\mathit{true} &\equiv \mathit{true}
\end{aligned}$$

Problem 22

Symbolize each of the following phrases:

a) Each x has property P: $\forall x, P(x)$.

b) Every x has property P: $\forall x, P(x)$.

c) Some x has property P: $\exists x \ st \ P(x)$.

d) All x have property P: $\forall x, P(x)$.

e) At least one x has property P: $\exists x \ st \ P(x)$.

Problem 23

Negate each statement in the previous exercise:

a) $\exists x \ st \ \neg P(x)$.

b) $\exists x \ st \ \neg P(x)$.

c) $\forall x, \neg P(x)$.

d) $\exists x \ st \ \neg P(x)$.

e) $\forall x, \neg P(x)$.

Additional Problem

Define: $p \implies q$, as well as $p \not\implies q$.
$p \implies q$ means that, given p, q is true: if p, then q. This means that either p is false, which means that you can prove anything given it; or both p and q are true.
$p \not\implies q$ means that we cannot get q from p: p must be true, since if it were false, we could prove anything; and additionally, q must be false, because otherwise we could prove it to be true.

$$\begin{aligned}
true &\implies true \equiv true \\
true &\implies false \equiv false \\
false &\implies true \equiv true \\
false &\implies false \equiv true
\end{aligned}$$

$$\begin{aligned}
true &\not\implies true \equiv false \\
true &\not\implies false \equiv true \\
false &\not\implies true \equiv false \\
false &\not\implies false \equiv false
\end{aligned}$$