

# Algorithmic Complexity

## Big- $\Theta$ Notation

We compute the big- $\Theta$  of an algorithm by counting the number of iterations the algorithm *always* takes with an input of  $n$ . For instance, the loop in the pseudo code below will always iterate  $N$  times for a list size of  $N$ . The runtime can be described as  $\Theta(N)$ .

```
for each item in list:  
    print item
```

## Asymptotic Notation

Asymptotic Notation is used to describe the running time of an algorithm – how much time an algorithm takes with a given input,  $n$ . There are three different notations: big O, big Theta ( $\Theta$ ), and big Omega ( $\Omega$ ). big- $\Theta$  is used when the running time is the same for all cases, big-O for the worst case running time, and big- $\Omega$  for the best case running time.

## Adding Runtimes

When an algorithm consists of many parts, we describe its runtime based on the slowest part of the program.

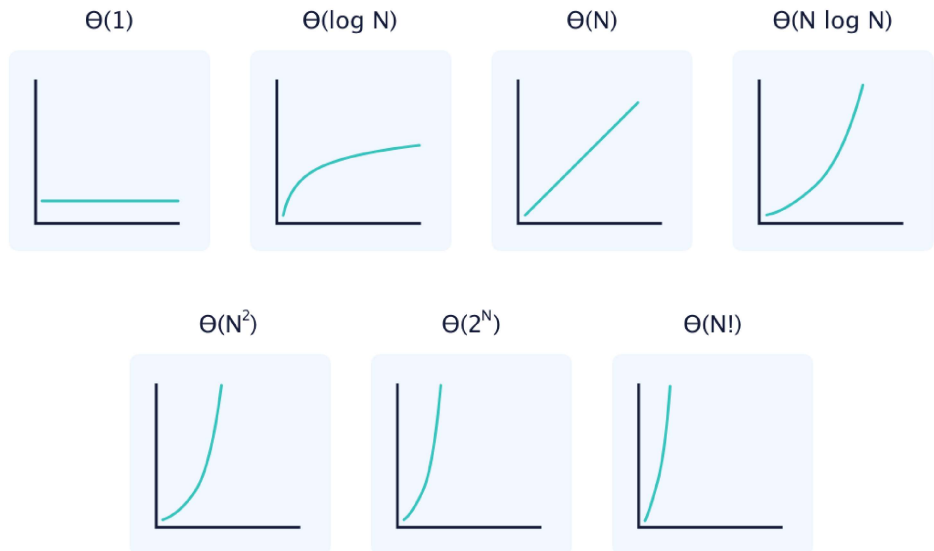
An algorithm with three parts has running times of  $\Theta(2N) + \Theta(\log N) + \Theta(1)$ . We only care about the slowest part, so we would quantify the runtime to be  $\Theta(N)$ . We would also drop the coefficient of 2 since when  $N$  gets really large, the multiplier 2 will have a small effect.

## Algorithmic Common Runtimes

The common algorithmic runtimes from fastest to slowest are:

- constant:  $\Theta(1)$
- logarithmic:  $\Theta(\log N)$
- linear:  $\Theta(N)$
- polynomial:  $\Theta(N^2)$
- exponential:  $\Theta(2^N)$
- factorial:  $\Theta(N!)$

## Common Runtimes



## Big-O Notation

The Big-O notation describes the worst-case running time of a program. We compute the Big-O of an algorithm by counting how many iterations an algorithm will take in the worst-case scenario with an input of  $N$ . We typically consult the Big-O because we must always plan for the worst case. For example,  $O(\log n)$  describes the Big-O of a binary search algorithm.

## Big- $\Omega$ Notation

Big- $\Omega$  (Omega) describes the best running time of a program. We compute the big- $\Omega$  by counting how many iterations an algorithm will take in the best-case scenario based on an input of  $N$ . For example, a Bubble Sort algorithm has a running time of  $\Omega(N)$  because in the best case scenario the list is already sorted, and the bubble sort will terminate after the first iteration.

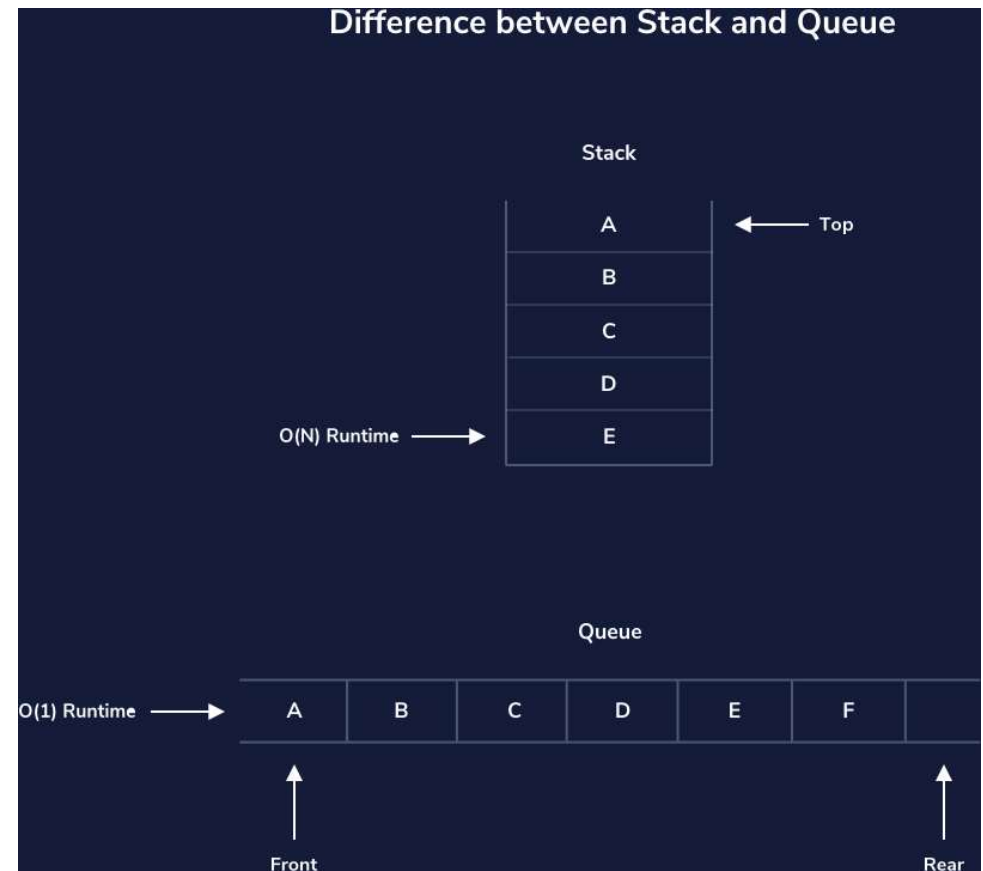
## Analyzing Runtime

The speed of an algorithm can be analyzed by using a while loop. The loop can be used to count the number of iterations it takes a function to complete.

```
def half(N) :  
    count = 0  
    while N > 1:  
        N = N//2  
        count += 1  
    return count
```

## Queue Versus Stack

A Queue data structure is based on First In First Out order. It takes  $O(1)$  runtime to retrieve the first item in a Queue. A Stack data structure is based on First In Last Out order. Therefore, it takes  $O(N)$  runtime to retrieve the first value in a Stack because it is all the way at the bottom.



## Max Value Search in List

The big-O runtime for locating the maximum value in a list of size N is  $O(N)$ . This is because the entire list of N members has to be traversed.

```
# O(N) runtime
def find_max(linked_list):
    current = linked_list.get_head_node()
    maximum = current.get_value()
    while current.get_next_node():
        current = current.get_next_node()
        val = current.get_value()
        if val > maximum:
            maximum = val
    return maximum
```

## Bubble Sort with Linked List

Bubble Sort is the simplest sorting algorithm for a list. For every element in the list, it compares it with its subsequent neighbor and swaps them if they are in descending order. Each pass of the swap takes  $O(N)$ . Since there are N elements in the list, it would take  $N*N$  swaps. The Big O runtime would be  $O(N^2)$ .