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Year 10 Mathematics
AOS 8 Revision [10.3]
Mock CAT 1

50 Marks. 60 Minutes Writing.

Results:

Short Answer Questions	22 / 34
Extended Response Questions	~~7~~ 9 / 16

29/50 = FAIL!

Section A: Short Answer Questions (34 Marks)

Question 1 (1 mark)

State the x-intercepts of the graph of the function $y = (x-2)(x-3)$.

$y=0$

$x=2$, $x=3$

$(2,0)$, $(3,0)$

Question 2 (1 mark)

A

How many different solutions does $(x-3)(x+5)(x-2)(x+5) = 0$ have?

your final answer should answer the question clearly.

3 solutions → be clear about your answer

$x = 3, 2, -5$

Question 3 (1 mark)

Show that $(x-2)$ is a factor of $P(x) = x^3 - 7x + 6$.

$x=2$

$P(2) = (2)^3 - 7(2) + 6 = 0$

$8 - 14 + 6 = 0$

$8 - 8 = 0$

$\therefore (x-2)$ is a factor of $P(x)$

Question 4 (1 mark)

Solve the equation $(3x-1)(x+5)(x-2) = 0$.

$3x-1=0$ $\quad x+5=0$ $\quad x-2=0$

$x=\frac{1}{3}$ $\quad x=-5$ $\quad x=2$

$\therefore x = \frac{1}{3}, -5, 2$

Question 5 (3 marks) ① - correct intercepts ② - shape ⊖1 - label intercepts.

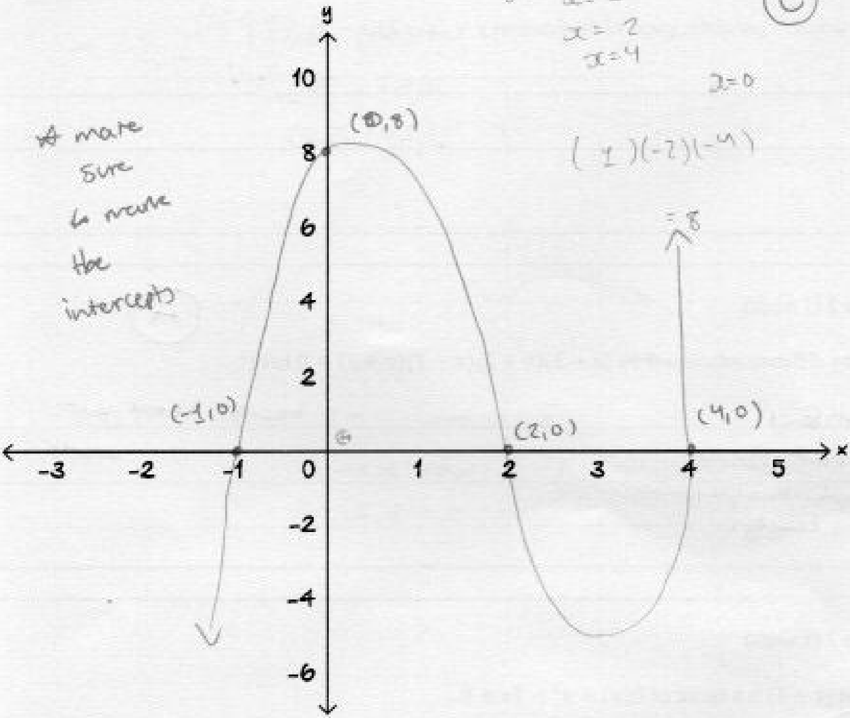
Sketch the graph of $y = (x+1)(x-2)(x-4)$, clearly labelling all axis intercepts.

C

$x=-1$
$x=2$
$x=4$

$x=0$

$(1)(-2)(-4)$
$=8$



Question 6 (2 marks) -2 C

Use the remainder theorem to find the remainder when $P(x) = 2x^3 - 4x^2 + x - 5$ is divided by $(x+1)$.

$2x^2 - 2x + 3$
$x+1 \overline{)\,2x^3 - 4x^2 + x - 5}$
$-\;2x^3 - 2x^2$
$-2x^2 + x$
$-2x^2 - 2x$
$3x - 5$
$3x + 3$
-8

$-4x^2 - (-2x^2)$
$-2x^2 - (-2x^2) = 0$
$x - (-2x) = 3x$
$-5 - (3)$
$-5 - 3$
$= \dfrac{-8}{x+1}$

Question 7 (3 marks)

-3

D

Let $P(x) = x^3 + ax^2 + bx + 12$.

It is known that $P(x)$ is exactly divisible by $(x - 3)$ and $(x + 2)$. Using the Factor Theorem, determine the values of a and b.

$x = 3$
$x = -2$

$P(x) = 0$

$P(3) = (3)^3 + a(3)^2 + b(3) + 12$

$27 + 9a + 3b + 12 = 0$

$39 + 9a + 3b = 0$

$39 + 9a = -3b$

$-3b = 39 + 9a$

$b = \frac{39 + 9a}{-3}$ $b = \frac{\cancel{3}(13 + 3a)}{-3}$

$\frac{39 + 9a + 3b}{3}$

$= 13 + 3a + b$

$x = 2$

$P(2) = (2)^3 + a(2) + b(2) + 12$

$8 + 2a + 2b + 12 = 20 + 2a + 2b$

$27 + 9a + 3(-13 + 3a) + 12 = 0$

$39 + 9a - 39 + 9a = 0$

$9a + 9a = 0$

$18a = 0$

$a =$

$b = -13 + 3a$

Question 8 (3 marks)

3

A cubic polynomial has x-intercepts at $x = -2, 1, 3$ and a y-intercept at $y = 12$. Write a possible equation for this polynomial.

$y = a(x+2)(x-1)(x-3)$

$12 = a(x+2)(x-1)(x-3)$

$12 = a(2)(-1)(-3)$ $(2)(3) = 6$

$12 = 6a$

$2 = a$

$a = 2$

$y = 2(x+2)(x-1)(x-3)$

Question 9 (3 marks) 3

$x^2 \times x = 3$ $x \times x = x^4$

Let $A(x) = 7x^3 - 2x + 9$ and $B(x) = x(x-4)^2(x+2)$.

a. State the degree of x for $A(x)$ and for $B(x)$. (2 marks)

A = 3rd degree , B = 4th degree

b. Using your answers from **part a.**, give the degree of the product $A(x)B(x)$. (1 mark) 1

7th degree

$(7x^3 - 2x + 9)($

$x^2 - 8x + 16$

$(x^3 - 8x^2 + 16x)(x+2) = \boxed{x^4} - 8x^3 + 16x^2 + 2x^3 - 16x$

$x^3 \times x^4 = x^7$

Question 10 (3 marks) x intercept

-3

y intercept

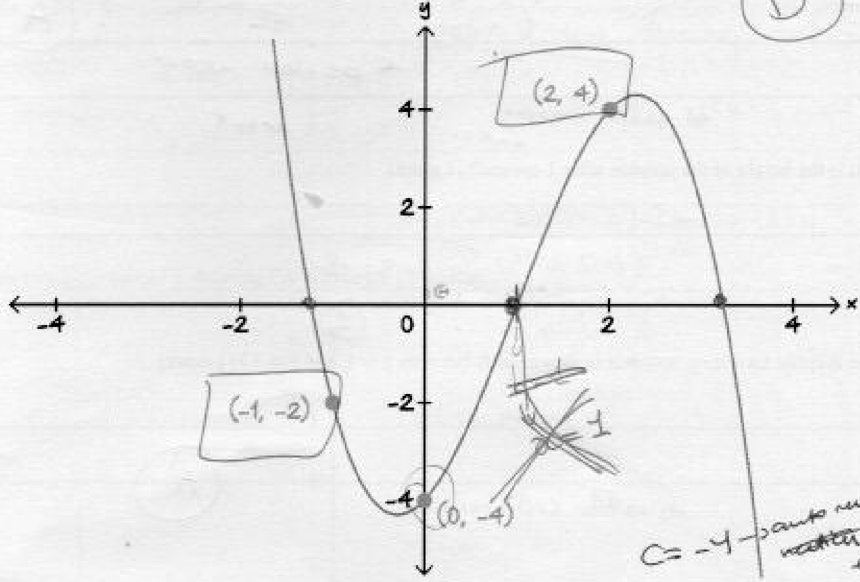
The cubic $P(x) = -x^3 + ax^2 + bx + c$ is shown below.

leading term x intercept

The curve passes through the points $(-1, -2)$, $(0, -4)$ and $(2, 4)$.

~~C~~

D



$c = -4$ — automatically in equation

Find the values of a, b and c. y intercept = −4

b = 1

$-4 = -x^2 + ax^2 + bx + c$ $0 = -x^3 + ax^2 + bx + 4$

$-4 = -x^3 + ax^2 + x + c$

then sub in the coordinates and solve for a and b.

$P(1) = -(1)^3 + a(1)^2 + b(1) - 4$

$P(2) = -(2)^3 + a(2)^2 + b(2) - 4$

$-(-1 \times -1 \times -1) = -(-1) = 1$

$p(1) = 1 + a - b + c = -2$

$1 + a - b + 4 = -2$

$1 + a - b = -6$

$a - b = -7$

Question 9 (3 marks)

3

$x^2 \times x = 3 \times x = x^4$

Let $A(x) = 7x^3 - 2x + 9$ and $B(x) = x(x-4)^2(x+2)$.

a. State the degree of x for $A(x)$ and for $B(x)$. (2 marks)

A = 3rd degree , B = 4th degree

b. Using your answers from **part a.**, give the degree of the product $A(x)B(x)$. (1 mark)

1

7th degree

$(7x^3 - 2x + 9)($

$x^2 - 8x + 16$

$(x^3 - 8x^2 + 16x)(x+2) = \boxed{x^4} - 8x^3 + 16x^2 + 2x^3 - 16x$

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The cubic $P(x) = -x^3 + ax^2 + bx + c$ is shown below.

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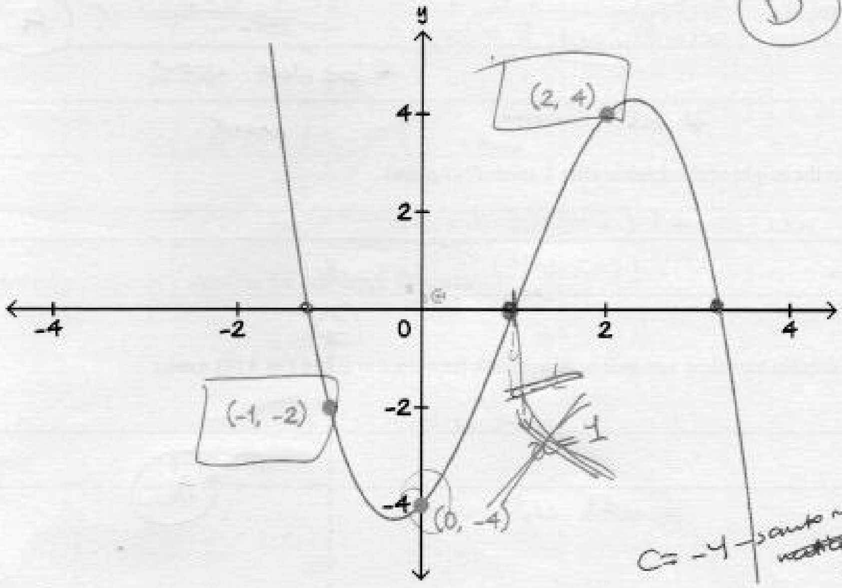
The curve passes through the points $(-1, -2)$, $(0, -4)$ and $(2, 4)$.



C = -4 → auto matically in equation

Find the values of a, b and c. y intercept = -4

b=1

-4 = $-x^3 + ax^2 + bx + c$ 0 = $-x^3 + ax^2 + bx + 4$

then sub in the (coordinates) and solve for a and b.

-4 = $-x^3 + ax^2 + x + c$

$P(1) = -(1)^3 + a(1)^2 + b(1) - 4$

$P(2) = -(2)^3 + a(2)^2 + b(2) - 4$

$-(-1 \times -1 \times -1) = -(-1) = 1$

$P(1) = 1 + a - b + c = -2$

$1 + a - b + 4 = -2$

$1 + a - b = -6$

$a - b = -7$

Question 11 (3 marks)

3

The path of a dolphin leaping out of the water is modelled by the polynomial $h(t) = -t(t-3)(t-5)$, where h is the height in metres above water and t is the time in seconds.

a. At what times is the dolphin at water level? (1 mark) $y=0$

$t=3,\ t=5$? A

1 3 seconds and 5 seconds

* be clear about answer

* show working out

b. What is the height of the dolphin after 1 second? (1 mark)

$h(1) = -1(1-3)(1-5)$

$h = -1(-2)(-4) = -1\times 8 = -8$

$h = -8m$ 1

c. Is the dolphin travelling upwards or downwards between $t = 0$ and $t = 1$? (1 mark)

downwards. 1

A

* add explanation.

Question 12 (5 marks) 5

The vertical displacement of a roller-coaster car above ground (in metres) is modelled by the function $y = -2(x-1)(x+3)(x-5)$, where x is the horizontal distance (in metres) from the station.

a. Find the axis intercepts of the graph. (2 marks) 2

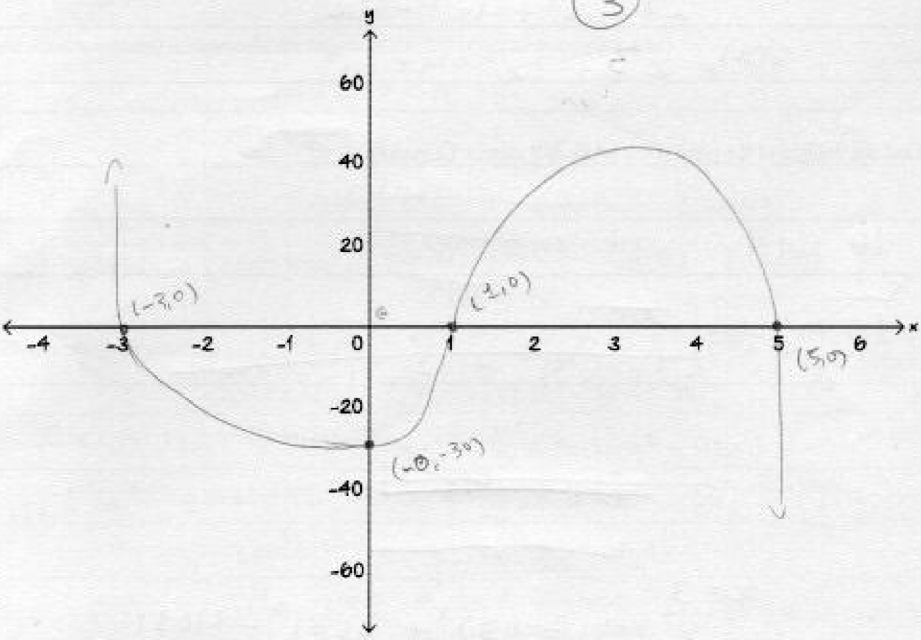
y=0 x=0: $x = 1, -3, 5$

x int: $(1,0)$, $(-3,0)$, $(5,0)$ $(-1)(3)(-5)$

$= 15 \times -2 = -30$

y int: $(0,-30)$

b. Sketch a rough graph of y showing the intercepts. (3 marks) 3



Question 13 (5 marks) -5 "in terms of x"

A rectangular garden bed has a length that is 4 metres longer than its width. Let the width be x metres.

L is +4 longer than W and W = x

a. Write an expression for the area, $A(x)$, of the garden bed. (1 mark)

LxW → $A(x) = x(x+4)$ ← L, ← W

W = x $A(x) = L(x-4)$ ← W, ↑ L answer = $x(x+4)$ ← W; me = $L(x-4)$ ← LX; W = x, into $L = x+4$

b. The garden bed is to be converted into a pond with a uniform depth of $(x - 1)$ metres. Write an expression for the volume, $V(x)$, of the pond. (**Hint:** $V = A(x) \times \text{depth}$.) (2 marks)

~~$L(x-4)(x-1)$~~ correct thinking wrong equation

$(x^2+4x)(x-1)$

$x^3 - x^2 + 4x^2 - 4x$

$V(x) = x^3 + 3x^2 - 4x \text{ m}^3$

c. Find the volume of the pond if the width is 3 metres. (2 marks)

~~$A(3)\ L(3-4)(3-1)$~~

16 1+4 ~~$L(-1)(2)$~~

x^3 ~~48~~ $-2L =$

~~$x=4$~~

~~$V(4) = (4)^3 + 3(4)^2 - 4(4)$~~

~~$= 64 + 48 - 16$~~

$x = 3$

$V(3) = (3)^3 + 3(3)^2 - 4(3)$

$27 + 27 - 12$

$54 - 12 = 42 \text{ m}^3$

Section B: Extended Response Questions (16 Marks)

Question 14 (9 marks)

The cross-section of a hill is modelled by the polynomial $H(x) = -\frac{1}{4}x(x-8)(x-12)$, where H is the height in metres and x is the horizontal distance in metres from a starting point.

a. Find the height of the hill when $x = 4$ metres. (2 marks)

2

$H(4) = -\frac{1}{4}(4)(4-8)(4-12)$

$H = \underbrace{-1(-4)(-8)}$

$H = -32$ metres

b. At what horizontal distances is the height of the hill equal to zero? (2 marks)

-2

C

$0 = -\frac{1}{4}x(x-8)(x-12)$

$x = 8,\ x = 12,\ x = 0$

8 metres and 12 metres and 0 meters

~~refer~~ it in coordinates = $(0,0),\ (8,0),\ (12,0)$

*do it in lowest → highest.

c. Expand the polynomial $H(x)$ into the form $ax^3 + bx^2 + cx + d$. (2 marks)

2

$-\frac{1}{4}x(x-8)(x-12)$ $\quad -\frac{1}{4}(x^2-20x+96)$

$\frac{1}{4}x(x-8)(x-12)$ $\quad -\frac{1}{4}(x^3-20x^2+96x)$

$\frac{1}{4}x(x^2-20x+96)$ $\quad -\frac{1}{4}x^3+5x^2-24x$

$\frac{x^3-20x^2+96x}{-4}$ $\quad -\frac{1}{4}x^3+5x^2-24x$

$4\overline{)96}$ = 24, -8, 16

D

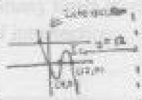
state a, b, c values

$a = -\frac{1}{4},\ b = 5,\ c = -24,\ d = 0.$

d. Another path on the hill is modelled by a straight line with equation $y = 12$. How [many times does this path] meet the cross-section of the hill? (No calculation required, just state the number o[f times.])

Once

guessed + no provided ...

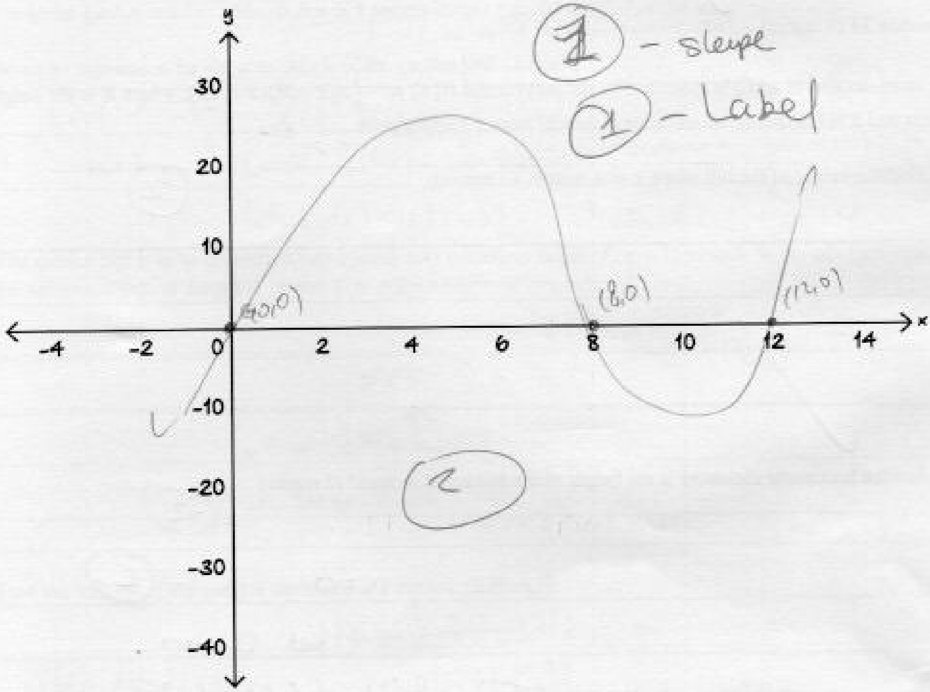


Because H(x) of the ... hump between (8,0) and ... local maximum is < y= ... and only 1 intersection, since we can't calculate turning point of the ...

turning point

y=12 → H(x), how do we know the local maximum is < y=12?

e. Sketch the graph of the polynomial $H(x)$. (2 marks)



1 - slope

1 - Label

2

$-\frac{1}{4}x(x-8)(x-12)$

x int = $y=0$

$0 = \quad x=8, \ x=12$

$-\frac{1}{4}x=0$

$x=0$

$x=0$

$y=(-8)(-12)=96$

Question 15 (7 marks) −7 D

A pop-up bakery sells limited-edition pastry boxes. Because of a sliding discount, the price per box decreases as more boxes are sold.

- Number of boxes sold: x
- Price per box: $p(x) = 24 - x$ dollars (valid while $0 \le x \le 24$). → Domain.
- Total cost: $C(x) = -x^3 + 8x^2 + 4x + 12$ dollars.

a. Write an expression for the revenue $R(x)$ in terms of x. (**Hint**: Revenue = Units sold × price per unit) (1 mark)

$R(x) = x \times p(x)$ ← Rewrite in $24 - x$ −1

↳ remember always in the form of a polynomial (with leading term.)

$= x \times 24 - x = 24x - x^2$ $R(x) = 24x - x^2$

b. Write an expression for the profit $P(x)$ in terms of x. (**Hint:** Profit = Revenue − Cost) (1 mark)

↳ $p(x) \neq P(x)$

$P(x) = R(x) - C(x)$ −1

$P(x) = 24x^2 - (-x^3 + 8x^2 + 4x + 12)$

$P(x) = x^3 - 9x^2 + 20x$

c. Factor the profit $P(x)$ completely into linear factors. (2 marks)

↳ $(x-a)(x-b)(x-c)$

~~$24 - x = x \times 24 - x$~~

↳ linear fo...

$\boxed{24 \quad -1}$

~~$x + x^3 + 8x^2 + 4$~~

~~$x - (-x^3 + 8x^2 + 4x + 12)$~~

$\frac{x + x^3 - 8x^2 - 4x - 12}{x^3 - 8x^2 - 3x - 12}$ $(x-1)(x-2)(x-6)$

Linear factors are factors with the highest degree. Remember Linear = $(x-a)(x-b)(x-c)$

There's no specific way to efficiently get linear factors in cubic so always start with $(x-1)$

d. Find the break-even point(s) for the bakery (solve $P(x) = 0$). (1 mark) −1

$P(x) = 0$

$P(x) = 0$ $(x-1)(x-2)(x-6)$

When a company's total revenue equals its total cost so there's no lost or gain.

$x = 1$, $x = 2$, $x = 6$

↳ consider the domain

$(0 \le x \le 24)$

condition where $R(x) = C(x)$

e. For each break-even point, calculate the corresponding revenue and cost. (At break-even, revenue equals cost.) (2 marks)

$R(x) \stackrel{?}{=} C(x)$

$R(x) = 24x - x^2$, $C(x) = -x^3 + 8x^2 + 4x + 12$

$x = 1$: $R = 24(1) - (1)^2 = 23$, $C = -1 + 8 + 4 + 12 = 23$

$x = 2$: $R = 48 - 4 = 44$, $C = -8 + 32 + 8 + 12 = 44$

$x = 6$: $R = 144 - 36 = 108$, $C = -216 + 288 + 24 + 12 = 108$

$R = C$: $\underbrace{(23, 23), (44, 44), (108, 108)}_{\text{are not coordinates } (R, C)}$ dollars

sub the x intercept values where ~~to cost~~ no cost, no gain (break even point)