Year 10 Mathematics
AOS 8 Revision [10.3] Mock CAT 2
Version B

Instructions: Answer all questions. Show all working.
Total Marks: 50
Time: 60 minutes writing

Section A: Short Answer Questions (34 Marks)

Question 1   [1 mark]

Expand and simplify: $(x + 7)(x - 2)$.

Question 2   [1 mark]

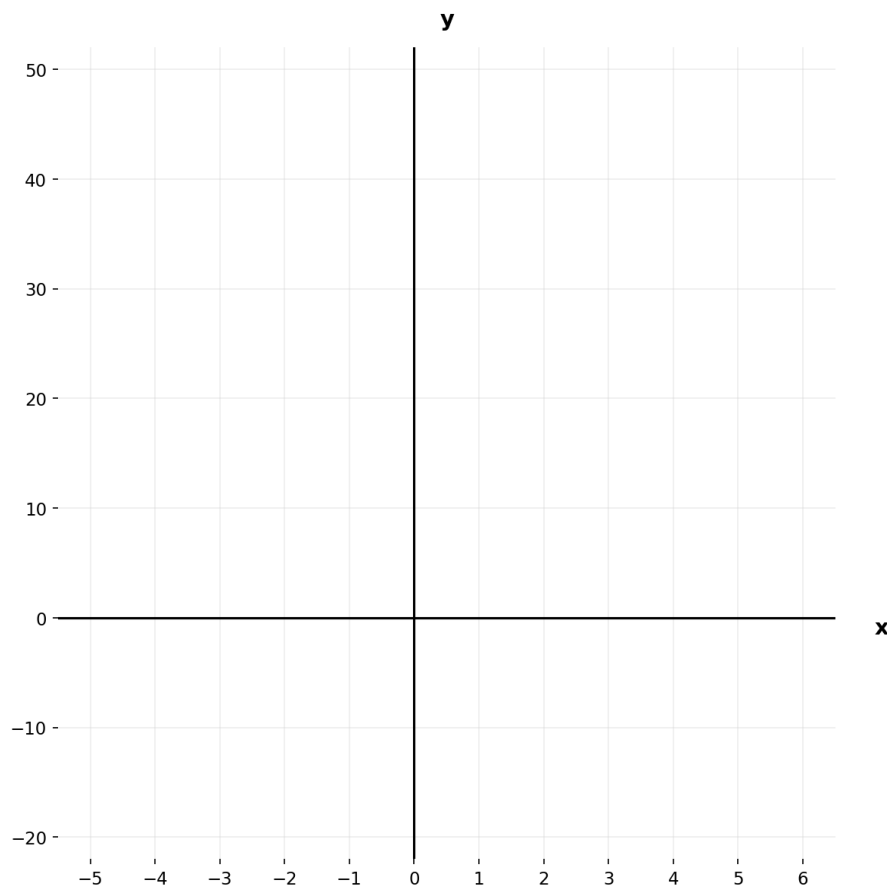Find the degree of $p(x) = x^2(2x^4 - 5x + 3)$, where the constant term is an integer.

Question 3   [1 mark]

State the x-intercepts of $y = x(x + 3)^2$.

Question 4   [1 mark]

What is the remainder when $x^5 - 8$ is divided by $(x + 2)$?

Question 5   [2 marks]

Sketch the graph of $y = (x+3)(x-1)(x-4)$ on the axes below, clearly labelling all x-intercepts and the y-intercept.

Question 6   [3 marks]

Solve the following polynomial equations:

a. $x(x + 4)(x - 7) = 0$ (1 mark)

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b. $x^4 - 13x^2 + 36 = 0$ (2 marks)

Question 7   [3 marks]

Given $(x + 2)$ is a factor of $P(x) = x^3 - 2x^2 - 13x - 10$, fully factorise $P(x)$.

Question 8   [3 marks]

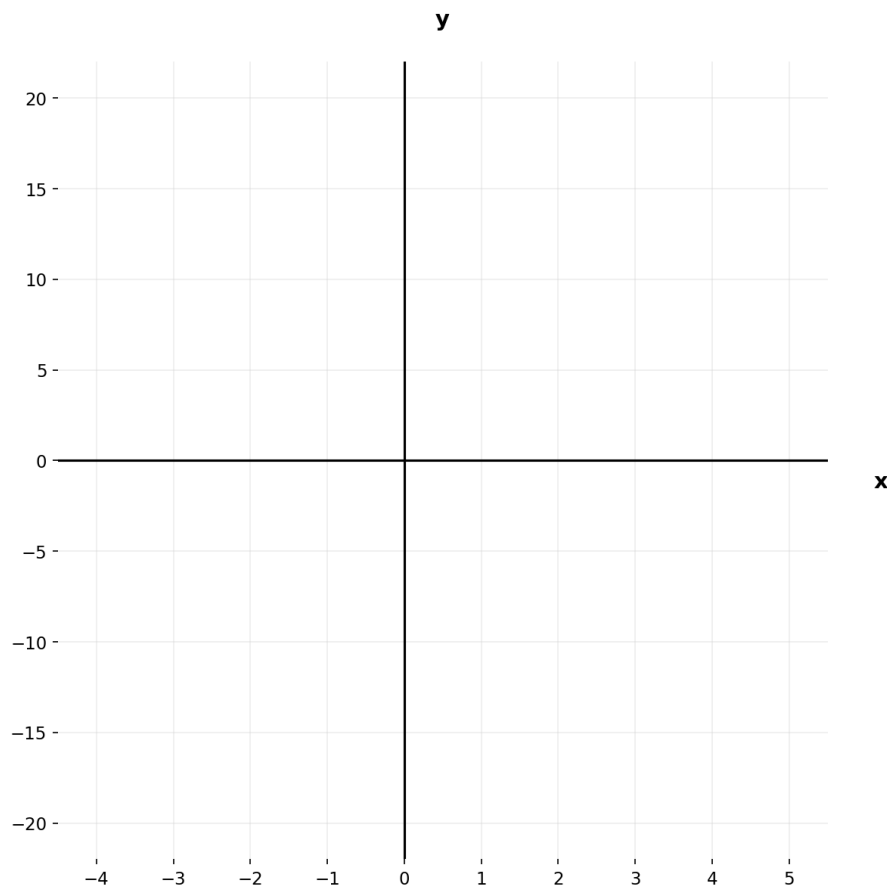Given $P(x) = 3x^3 + 5x^2 - 26x + 8$:

a. Verify that $x = 2$ is a root of the equation $P(x) = 0$.   (1 mark)

b. Determine the remainder when $P(x)$ is divided by $(x - 1)$.   (1 mark)

c. It is known that $P(x) \div (x - a) = Q(x) + \frac{18}{x-a}$, where $Q(x)$ is a quadratic polynomial. Find $P(a)$.   (1 mark)

Question 9   **[3 marks]**

Sketch the graph of $y = (x-1)^4 - 16$ on the axes below, labelling all axes intercepts and turning points.

Question 10   **[2 marks]**

Use long division to find the quotient and remainder when $3x^3 + 7x^2 - 4x + 1$ is divided by $(x + 2)$.

Question 11   [**2 marks**]

Find the value of k if $(x + 3)$ is a factor of $P(x) = x^3 + kx^2 + 7x + 12$.

Question 12   [**2 marks**]

Expand and simplify $(x + 3)(x^2 - 4x + 5)$.

Question 13   [**5 marks**]

The monthly revenue, R, in thousands of dollars, of a startup company is modelled by $R(x) = x^3 - 12x^2 + 27x$, where x is the number of units sold, in hundreds.

a. Factorise the revenue function $R(x)$.   (2 marks)

b. For what sales levels does the company earn zero revenue?   (2 marks)

c. What is the revenue if 500 units are sold?   (1 mark)

Question 14 [5 marks]

A population model $N(x)$ (in hundreds) after x months of environmental changes is modelled by $N(x) = x^3 - 3x^2 - 10x + 24$.

a. Using the Factor Theorem and trial-and-error, find a time x at which the population returns to its initial level (where $N(x) = 24$). (2 marks)

b. Once you have this value, factorise $N(x) - 24$ completely to determine all times when the population equals 24. (3 marks)
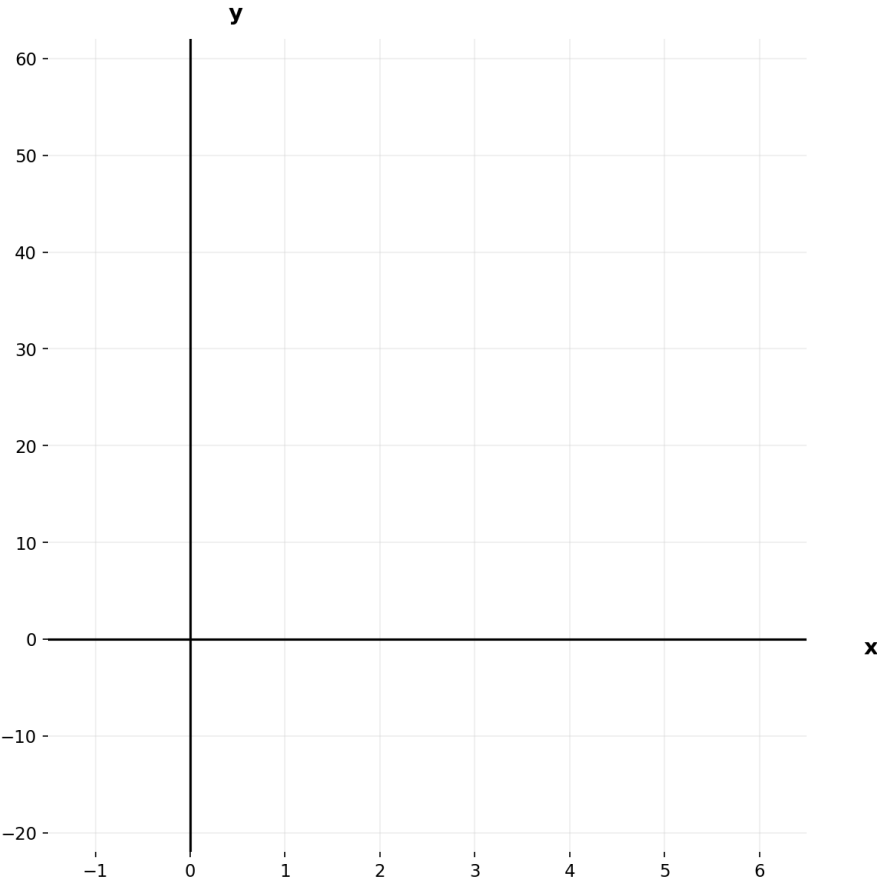
Section B: Extended Response Questions (16 Marks)

Question 15   [9 marks]

Two trail elevation profiles give the vertical position (relative to a baseline) as a function of horizontal distance x metres from the trailhead.

- Trail A: $y = -3(x - 3)(x - 1), x \geq 0$ (elevation in metres)

- Trail B: $y = (x - 1)(x - 2)(x - 4), x \geq 0$ (elevation in metres)

a. Where does Trail A meet the baseline (ground)?   (1 mark)

b. Where does Trail B meet the baseline?   (1 mark)

c. Find the horizontal distance $\bar{x}$ at which Trail A reaches its maximum elevation (give your answer in decimal form).   (2 marks)

d. For which value(s) of x are the two trails at the same elevation, and what is that common elevation (in metres)?   (2 marks)

e. Hence or otherwise, sketch the graph of the polynomial: $y = x^3 - 7x^2 + 14x - 8$ in the interval $-1 \leq x \leq 5$. Clearly label all axis intercepts and endpoints.   (3 marks)

Question 16   [7 marks]

An investment fund models its net monthly return $R(x)$ (in thousands of dollars) by a monic cubic:

$$R(x) = x^3 + ax^2 + bx - 6$$

Where x is the number of hundreds of clients in a month.

It is known that the fund breaks even when it has 200 clients, and that with 100 clients the fund produces a loss of \$8k.

a. Using this information, form two simultaneous equations in a and b.   (3 marks)

b. Solve your equations to find a and b.   (2 marks)

c. Write down the complete polynomial $R(x)$.   (1 mark)

d. Hence, factor $R(x)$ fully and state all three linear factors.   (1 mark)

END OF TEST