CONTOUREDUCATION

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Year 10 Mathematics
AOS 8 Revision [10.3]
Workbook

Yewbo

Outline:

Announcements

Link: https://bit.ly/Contour-Class-Announcements

Contour Resources

Core		Mastery	
Workbook + Test, CAT	Homework	Workshop	Bound Reference, Mock CAT, Exams
In Class	At Home	In Class	At Home

Subject Outline for AOS 10 - End of Year Exam Revision

In Class (Workbook + Test, CAT) | At Home (Homework)	In Class (Workshop)
☐ MA10 [10.1] - AOS 6 Revision	MA10 [10.1] - Mock CAT 3
☐ MA10 [10.2] - AOS 7 Revision	MA10 [10.2] - Mock CAT 3
☐ MA10 [10.3] - AOS 8 Revision	MA10 [10.3] - Mock CAT 3
☐ MA10 [10.4] - AOS 9 Revision	MA10 [10.4] - Mock CAT 3

Additional Resources

Mock CAT	Exam
☐ MA10 [10.1] - AOS 5 Revision 3 x Mock CATs ☐ MA10 [10.2] - AOS 6 Revision 3 x Mock CATs ☐ MA10 [10.3] - AOS 7 Revision 3 x Mock CATs	☐ MA10 [10.4] - EOY - Exam

Section A: Polynomials

Polynomials

- Functions of the form:

leading term

degree of!

$$f(x) = a_n x^{n} + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots + a_1 x + a_o$$

- Where all powers are positive integers (whole numbers).

Question 1

Expand and simplify:

$$(5y-2)(2y^2+y-3)$$

$10y^3+5y^2-15y-4y^2-2y+6$

$10y^3+5y^2-4y^2-15y-2y+6$

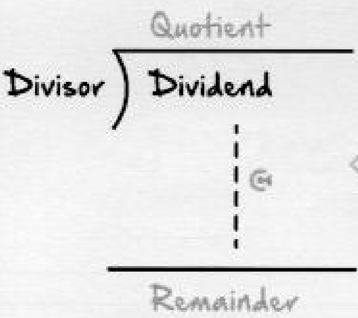
$= 10y^3+y^2-17y+6$

Sub-Section: Long Division

Expressing Long Division

Definition

- So, to summarise:



$$\begin{array}{r|l} & 14 \\ \hline 7 & 100 \\ & -7 \\ \hline & 30 \\ & -28 \\ \hline & 2 \end{array}$$

$$100 = (14 \times 7) + 2$$

$$\frac{100}{7} = 14 + \frac{2}{7}$$

$$\frac{\text{Dividend}}{\text{Divisor}} = \text{Quotient} + \frac{\text{Remainder}}{\text{Divisor}}$$

Question 2 Walkthrough.

Apply long division to find the quotient and the remainder in the division of $p(x)$ by $g(x)$ as given below:

$$p(x) = 2x^4 - 3x^2 + 8x - 1, \qquad g(x) = x^2 + 1$$

$-3x^2 - 2x^2$

$2x^2 - 5$

$x^2 + 1 \,\big)\, 2x^4 - 3x^2 + 8x - 1$

$2x^4 + 2x^2$

$-5x^2 + 8x - 1$

$-5x^2 + 0x - 5$

$8x + 4$

Question 3

Apply long division to find the quotient $q(x)$ and the remainder $r(x)$ in the division of $p(x)$ by $g(x)$ as given below:

$$p(x) = 4x^3 - 3x^2 - 10x + 17, g(x) = x^2 + x - 2$$

$$
\begin{array}{r|l}
 & 4x + 7 \\
x^2 + x - 2 & 4x^3 - 3x^2 - 10x + 17 \\
 & 4x^3 + 4x^2 - 8x \\
 & -7x^2 - 2x + 17 \\
 & -7x^2 + x - 2 \\
 & \cancel{6x + 19} \\
 & 5x + 7
\end{array}
$$

$-3x^2 - -4x^2 = x^2$

$-10x - -8x = -2x$

$-2x - x$

$17 - -2$

$-7x^2 - -x^2$

$= -6x^2$

$-3x^2 - 4x^2$

$= \cancel{4x + 7} + \frac{6x + 19}{x^2 + \cancel{x} - 2}$

$4x - 1 + \frac{5x + 3}{x^2 + x - 2}$

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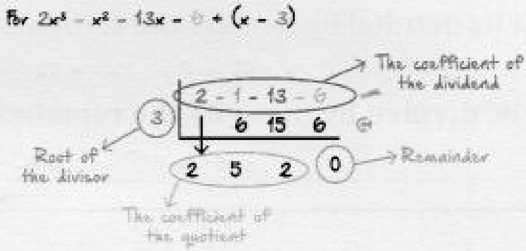
Sub-Section: Short Division/Synthetic Division

Short Division/Synthetic Division

Definition

- For $2x^3 - x^2 - 13x - 6 \div (x - 3)$



Question 4 Walkthrough.

Use short division to simplify the following:

$$(2x^3 - 3x^2 - 8x + 12) \div (x - 3)$$

$= 2x^2 + 3x + 1 + \frac{15}{x-3}$

$3 \,|\,$ 2   -3   -8   12
6   9   3
2   3   1   15 → remainder

Question 5

Factor!

Use short division to simplify the following:

-4+5

$$(x^3 - 4x^2 - 7x + 10) \div (x - 5)$$

$x^2 + x - 2$

$5 \,|\,$ 1   -4   -7   10
5   5   -10
1   1   -2   0

Sub-Section: Remainder Theorem

Remainder Theorem

- Simply sub in the x-value which makes the $f(x)$ 0 into the dividend (TOP)!

When $f(x)$ is divided by $(x-a)$, the remainder is $f(a)$.

$x-a=0, \quad x=a$

When $f(x)$ is divided by $(ax-b)$, the remainder is $f\left(\frac{b}{a}\right)$.

$ax-b=0 \quad ax=b$

$x=\frac{b}{a}$

Question 6 Walkthrough.

The polynomials $p(x)=3x^3-2x^2+ax+4$ and $q(x)=x^3+5x^2-7x+1$ are divided by $x-2$ if the remainder in each case is the same, find the value of a.

$x=2$

$p(2)=3(2)^3-2(2)^2+a(2)+4=(2)^3+5(2)^2-7(2)+1$

$3(8)-2(4)+2a+4=8+5(4)-14+1$

$24-8+2a+4=8+20-14+1$

$24-8+4=16+4=20 \qquad 8+20-13=28-13=15$

$20+2a=15-20$

$2a=-5$

$a=-\frac{5}{2}=-2.5$

Question 7

Find the remainder when $f(x)=-3x^3+4x^2-5x+12$ is divided by $2x-3$.

$f(\frac{3}{2})=-3(\frac{3}{2})^3+4(\frac{3}{2})-5(\frac{3}{2})+12 \qquad 2x-3=0$

$x=\frac{3}{2}$

$=\frac{27}{4}$

Question 8 Extension.

What will be the remainder when $x^{101} - 1$ is divided by $(x - 1)$?

Question 9 Extension.

The polynomials $ax^3 + 3x^2 - 13$ and $2x^3 - 5x + a$ leave the same remainder when divided by $x - 2$. Find the value of a.

Sub-Section: Factor Theorem

Factor Theorem

Factor theorem is used when you check if $(x-a)$ is a factor of $f(x)$ don't use long division.

- For every x-intercept (root) $x = a$, there is a corresponding factor $(x - a)$.

$$\textbf{If } f(a) = 0, \textbf{ then } (x - a) \textbf{ is a factor of } f(x).$$

Definition

Question 10 Walkthrough. Example.

If $(x - a)$ is a factor of $-5x^4 + 3x^3 + 7ax^2$, what must be the value of a, $\boxed{\text{where } a \neq 0}$?

important

$a = 2$

$x = a$

$-5(a)^4 + 3(a)^3 + 7a(a)^2 = 0$

$-5a^4 + 3a^3 + 7a^3 = 0$

$-5a^4 + 10a^3 = 0$

$-5a^3(a-2) = 0$

$-5a^3 = 0$; $a - 2 = 0$

$\cancel{a=0}$ $a = 2$

Question 11

Determine if $x - 2$ is a factor of $P(x) = x^3 + 4x^2 - 5x - 6$.

$x = 2$

$P(2) = (2)^3 + 4(2)^2 - 5(2) - 6 = 0$

$8 + 16 - 10 - 6 = 0$

$24 - 16 \neq 0$

$8 \neq 0$

NO

not a factor

Question 12

If $2x + 5$ is a factor of $2x^2 - k$, then find the value of k.

always simplify!

$2x + 5 = 0$

$x = -\frac{5}{2}$

$2(-\frac{5}{2})^2 - k = 0$

$2\left(\frac{25}{4}\right)$

$\frac{2}{1} \times \frac{25}{4} = \frac{50}{4} - k = 0$

$k = \frac{50}{4} = \frac{25}{2}$

$= 12.5$

Sub-Section: Solving Polynomial Equations

Null Factor Law

$$a \times b \times c = 0$$

$(x-a)(x-b)(x-c)$

- Then:

$x = a, \quad x = b, \quad x = c$

$$a = 0 \text{ or } b = 0 \text{ or } c = 0$$

Question 13

Find the roots of the following polynomials:

$$\left(2x+\frac{3}{5}\right)^3 (x-6)^2(7-x) = 0$$

$\left(2x + \frac{3}{5}\right) = 0$

$2x = -\frac{3}{5}$

$x = \cancel{-\frac{3}{5} \times \frac{2}{1} = -\frac{6}{5}}$ $\quad -\frac{3}{5} \times \frac{1}{2} = -\frac{3}{10}$

$x - 6 = 0$

$x = 6$

$x = -\frac{6}{5}, \ x = 6, \ x = 7$

$-x + 7 = 0$

$x = \frac{-7}{-1}$

$x = 7$

Question 14 Extension.

Given $p(x)$ is a cubic polynomial. $p(x) = 0$, has solutions at $x = 2, x = 4$ and $x = 6$.

a. Write down one possible equation of the polynomial.

~~$(x-2)(x-4)(x-6)$~~

$(x-2)(x-4)(x-6)$

b. If the polynomial passes through $(3, 6)$, find the exact equation for the polynomial.

$x=3 \quad y=6$ * solve for k

$6=k(x-2)(x-4)(x-6)$

$k(3-2)(3-4)(3-6)$

$6=(1)(-1)(-3)$

$k=2$

Question 15 Extension.

Let:

$$Q(x) = k(x+2)^2(x-1)(x-a)$$

where $k \neq 0$. Suppose the remainder when $Q(x)$ is divided by $(x-4)$ is 0 and $Q(0) = 48$. Find a and k. Hence, state the x-intercepts of $Q(x)$.

?

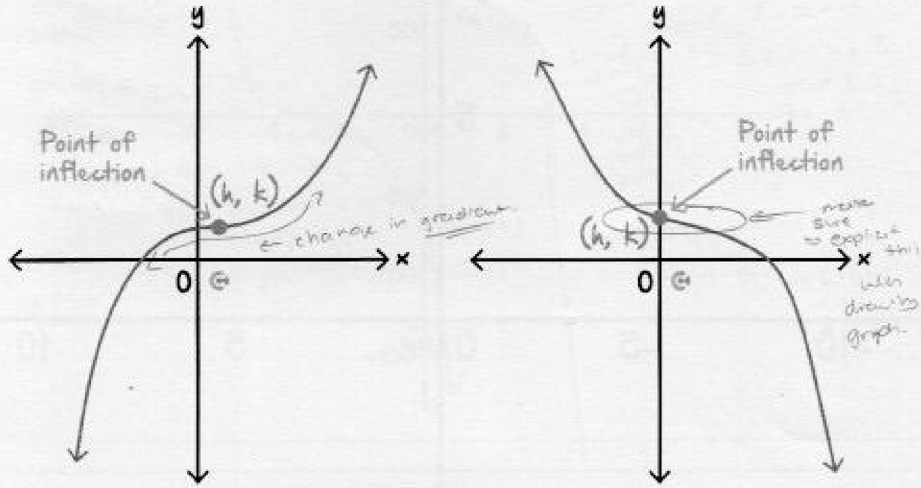
Section B: Sketching Polynomial Graphs

Sub-Section: Sketching Cubic Graphs in POI Form

Definition

Graphs of $a(x-h)^n + k$, where n is an Odd Positive Integer

- All graphs look like a "cubic".



Where (h, k) is the point of inflexion.

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Question 16 Walkthrough.

Sketch the graph of $y = (x + 1)^3 + 8$ on the axes below. Label the intercepts and turning point.

$y = a(x+h)+k$

$h = -1,\ k = 8$

$(-1, 8)$

$x = 0 \quad y = (1)^3 + 8$

$y = (0, 9$

$y = 0$

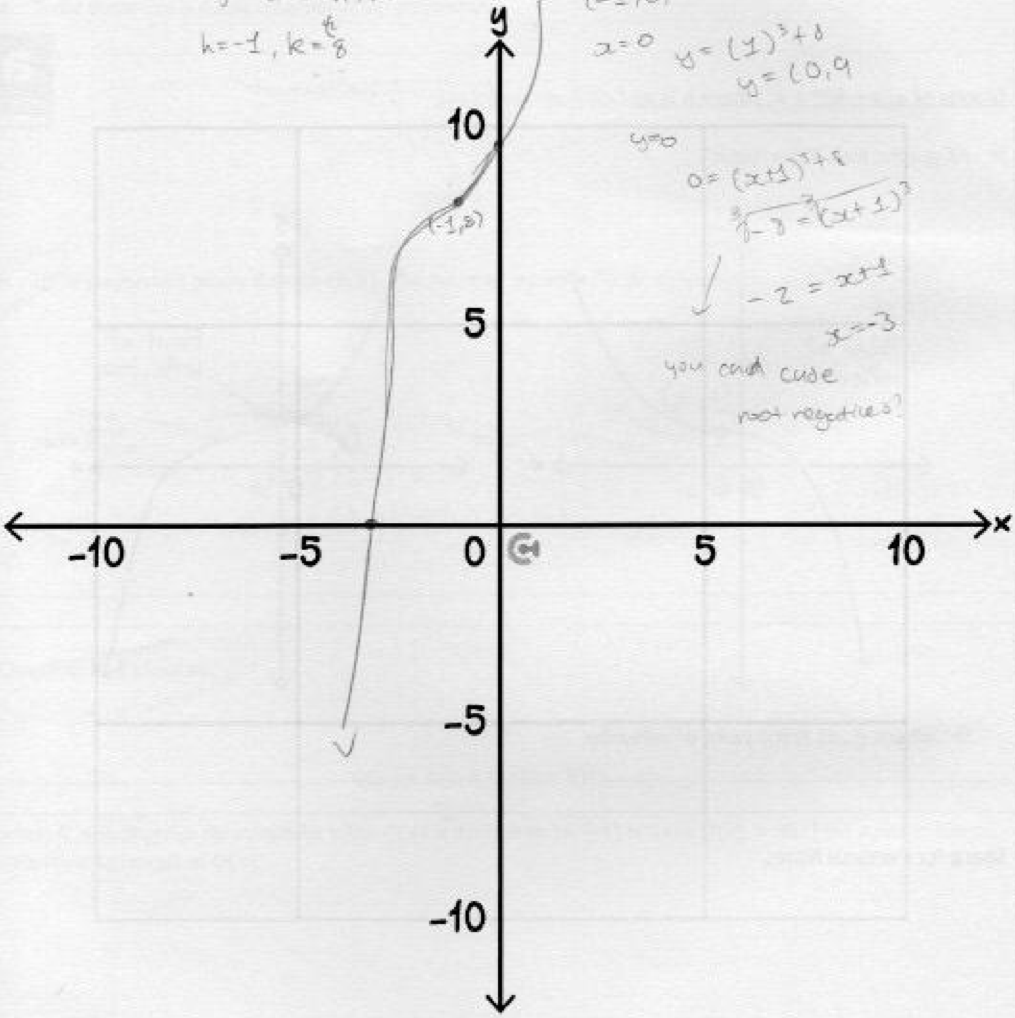
$0 = (x+1)^3 + 8$

$\sqrt[3]{-8} = \sqrt[3]{(x+1)^3}$

$-2 = x + 1$

$x = -3$

you cant cube not negatives?



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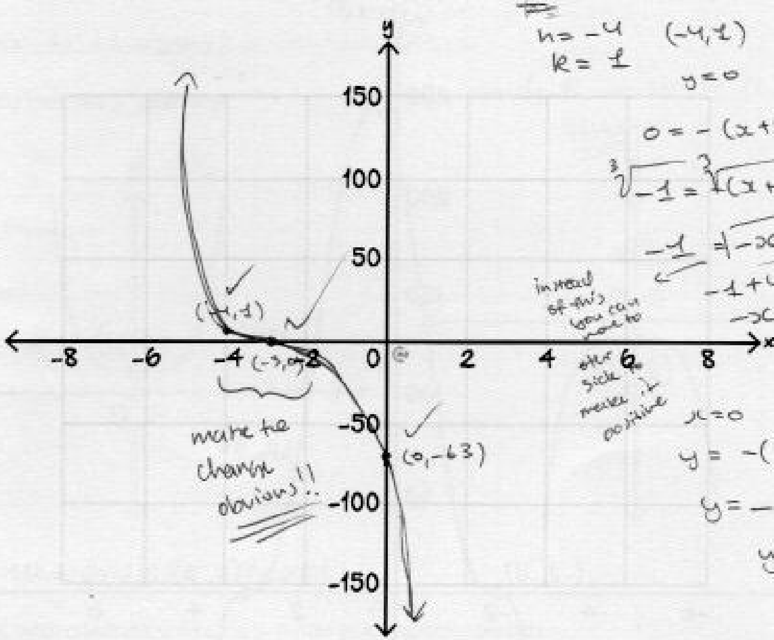
Question 17

Sketch the graph of $y = -(x+4)^3 + 1$ on the axes below. Label the intercepts and turning point.



$h = -4$ $(-4, 1)$
$k = 1$

$y = 0$

$0 = -(x+4)^3 + 1$

$\sqrt[3]{-1} = \sqrt[3]{(x+4)^3}$

$\boxed{-1 = \sqrt[3]{-x-4}}$

$-1 + 4 = 3$

$-x = 3$

$x = -3$

$x = 0$

$y = -(4)^3 + 1$

$y = -64 + 1$

$y = -63$

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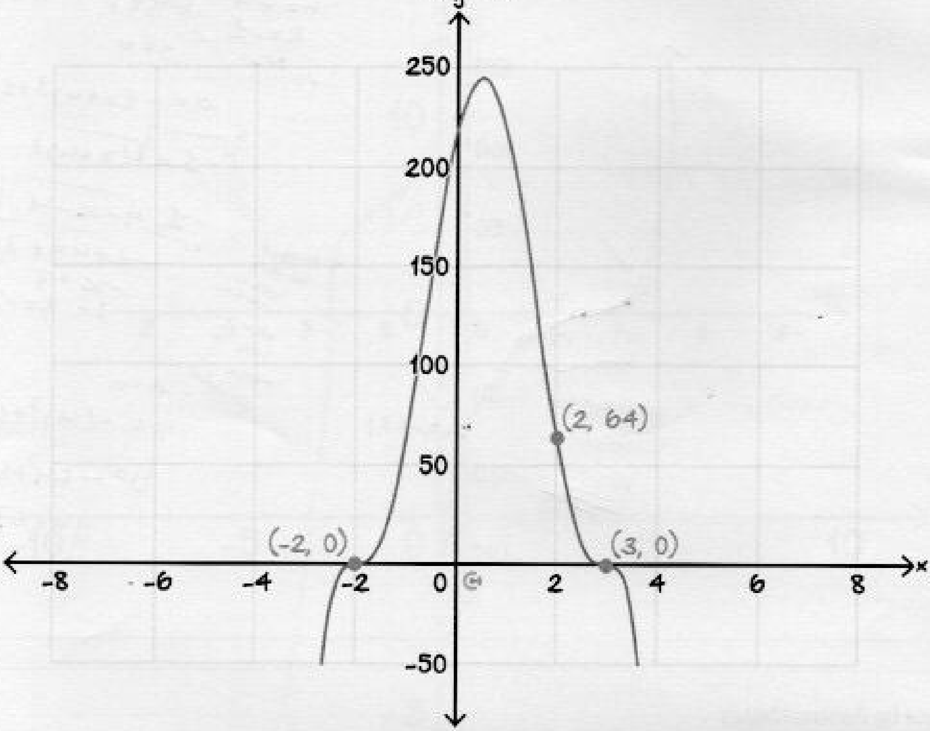
Question 18 Extension.

Consider the function of the form $f(x) = a(x - b)^3(x - c)^3$, where $b > c$, depicted on the graph below.

intercepts



Find the values of $a, b,$ and c.

a = dilation, subbing and solving only, to do that find b and c first, these are the x intercepts. (ok)

$= f(x) = a(x-3)^3(x+2)^3$

$64 = a(2-3)^3(2+2)^3$

b > c meaning b is (3,0) and c is (-2,0)

$a = -1$. y sub.

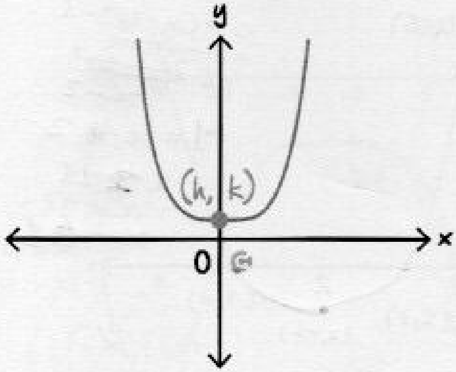
$b = 3,\ c = -2,\ a = -1.$

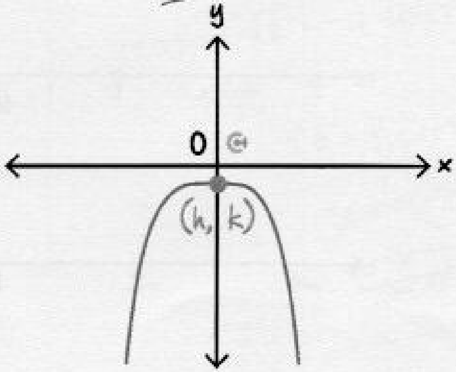
Sub-Section: Sketching Quartic Graphs in Turning Point Form

Graphs of $a(x-h)^n + k$, where n is an Even Positive Integer

- All graphs look like a "quadratic".

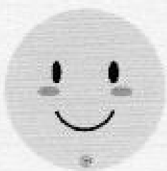
4, 6, 8 → larger is flater





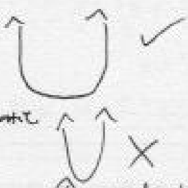
- The point (h, k) gives us the turning point.
- Similarly, with quadratics, when $a > 0$, the graph opens upwards.
- When $a < 0$, the graph opens downwards.

← turning point has to be flat



positive

negative



← quadratic

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Question 19 Walkthrough.

Sketch the graph of $y = (x-2)^4 - 1$ on the axes below. Label the vertex and the point of intercepts.

$h = 2, \; k = -1$

$x = 0$

$y = (-2)^4 - 1$

$y = 16 - 1 = 15$

$y = 0$

$0 = (x-2)^4 - 1$

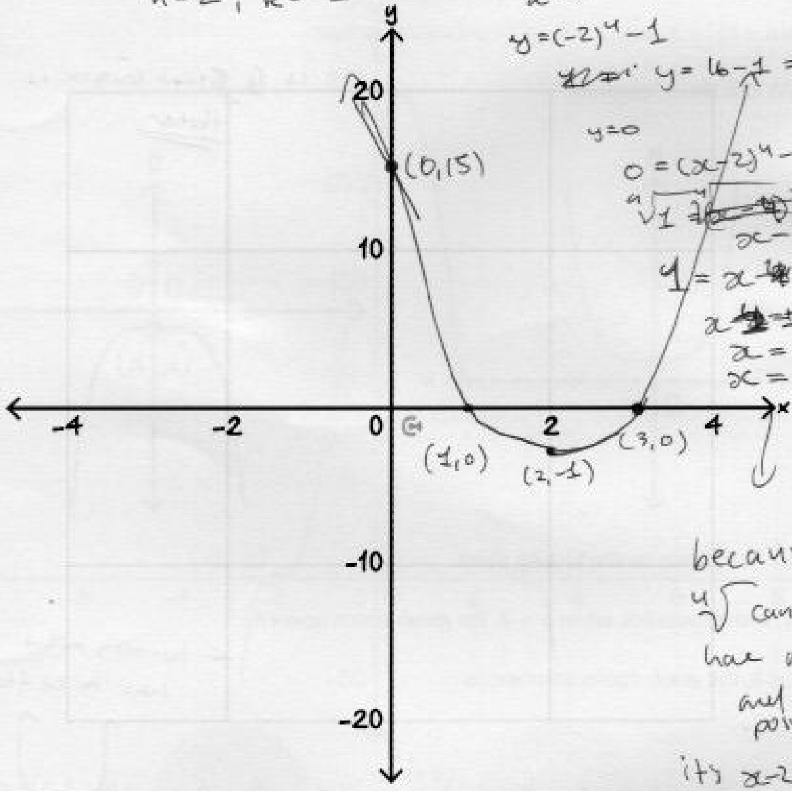
$\sqrt[4]{1} = x - 2$

$1 = x - 2$

$x - 2 = \pm 1$

$x = 3$

$x = 1$



because $\sqrt[4]{\;}$ can have negative and positive

its $x - 2 = \pm 1$

$x = 3, \; x = 1$

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Question 20

Sketch the graph of $y = -\frac{1}{4}(x + 2)^4 - 1$ on the axes below. Label the vertex and the point of intercepts.

$h = -2,\ k = -1$

$y = 0 \quad 0 = -\frac{1}{4}(x+2)^4 - 1$

$1 = -\frac{1}{4}(x+2)^4$

$\sqrt[4]{1} = -\frac{1}{4}\sqrt[4]{(x+2)^4}$

$1\pm = -\frac{1}{4}(x+2)$

~~intercept?~~

no x intercept

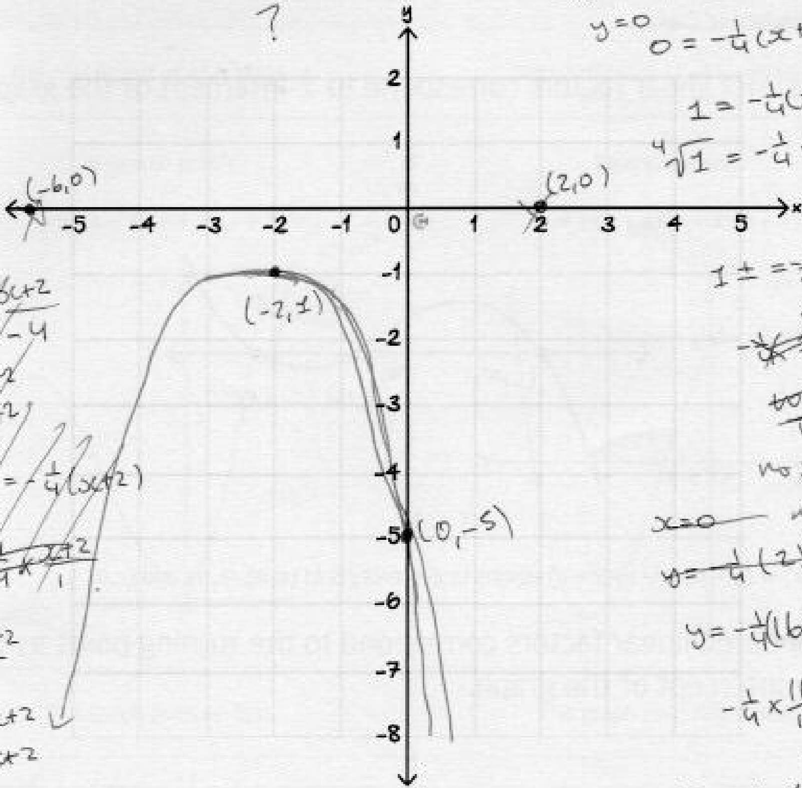
~~x=0~~

$\cancel{y = \frac{1}{4}(2)^4 - 1}$

$y = -\frac{1}{4}(16) - 1$

$-\frac{1}{4} \times \frac{16}{1} = \frac{16}{-4} = -4$

$y = -4 - 1 = y = -5$



$\cancel{\pm 1 = \frac{x+2}{-4}}$

$\cancel{4 = x+2}$

$\cancel{-4 = x+2}$

$\cancel{\pm 1 = -\frac{1}{4}(x+2)}$

$\pm 1 = \frac{x+2}{-4}$

$-4 = x+2$

$4 = x+2$

$x = -6$

$x = 2$

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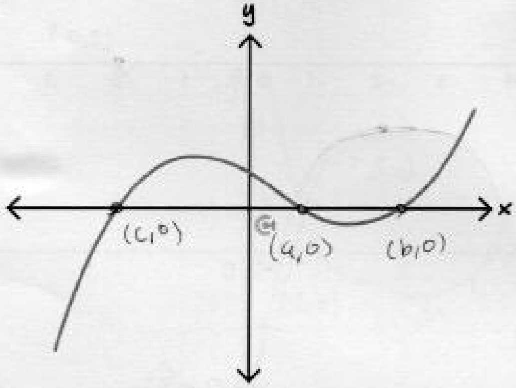
Sub-Section: Graph of Cubics

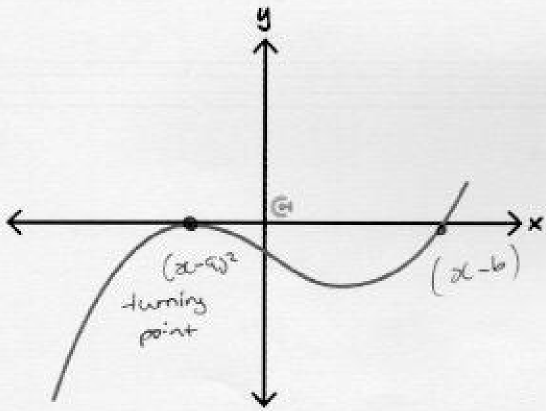
Graphs of Factorised Cubics

- All distinct linear factors correspond to x-intercept of the graph.



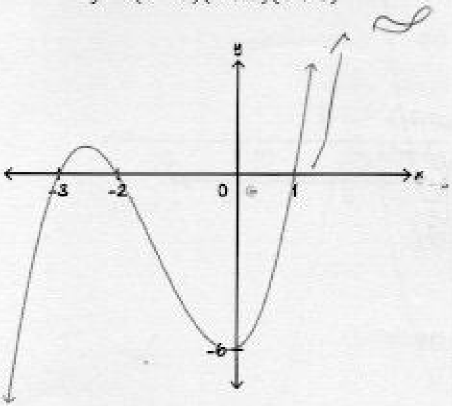
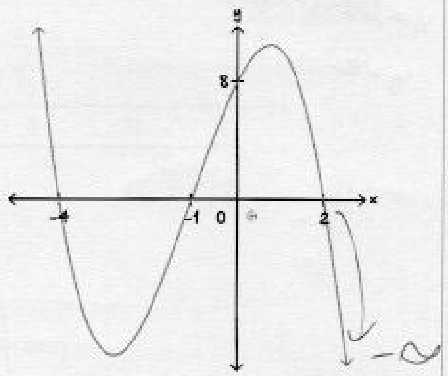
- E.g., $f(x) = (x-a)(x-b)(x-c)$ results in x-intercepts at $(a,0)$, $(b,0)$, and $(c,0)$.

- All repeated linear factors correspond to the turning point as well as the x-intercept of the graph.



- E.g., $f(x) = (x-a)^2(x-b)$ will have an x-intercept $(a,0)$ which is also a turning point.

Positive vs Negative Coefficient of x^3

Positive Cubic	**Negative Cubic**
The coefficient of x^3 is positive.	The coefficient of x^3 is negative.
Example graph: $y = (x-1)(x+2)(x+3)$ 	Example graph: $y = -(x+4)(x-2)(x+1)$ 
The graph goes up first.	The graph goes down first.

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Question 21 Walkthrough.

Sketch the graph of $y = (x - 2)(3x - 1)(4x + 1)$ on the axes below. Label the intercepts.

$x = 2$

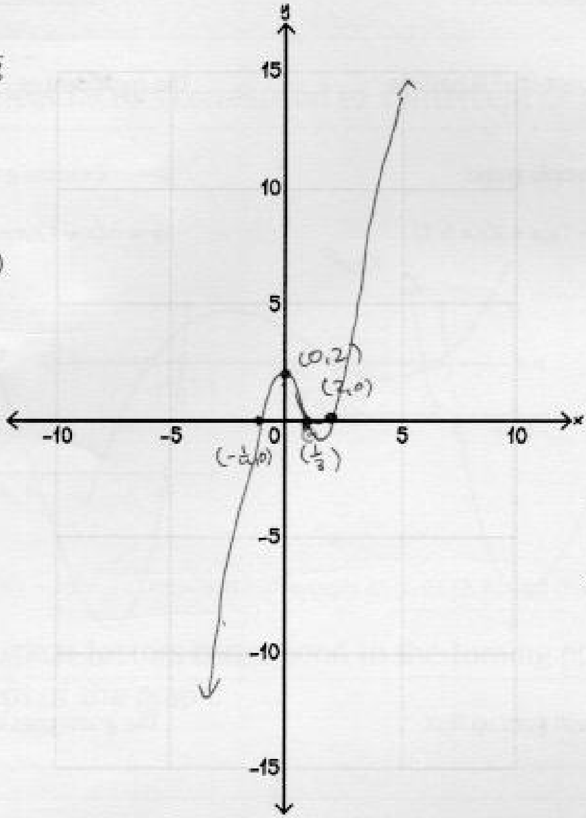
$x = \frac{1}{3}$

$x = -\frac{1}{4}$

$x = 0$

$y = (-2)(-1)(1)$

$y = 2$



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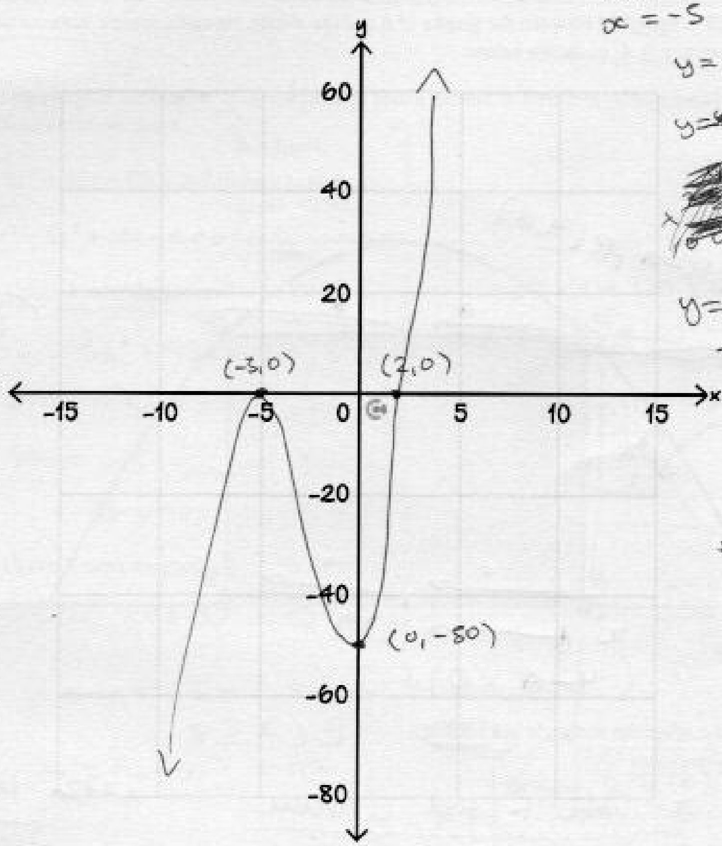
Question 22

Sketch the graph of $y = (x-2)(x+5)^2$ on the axes below. Label the intercepts.

$x = 2$
$x = -5$



$y = (-2)(5)^2$

$y = \cancel{4}(25)$

$y = \cancel{100}$
-50

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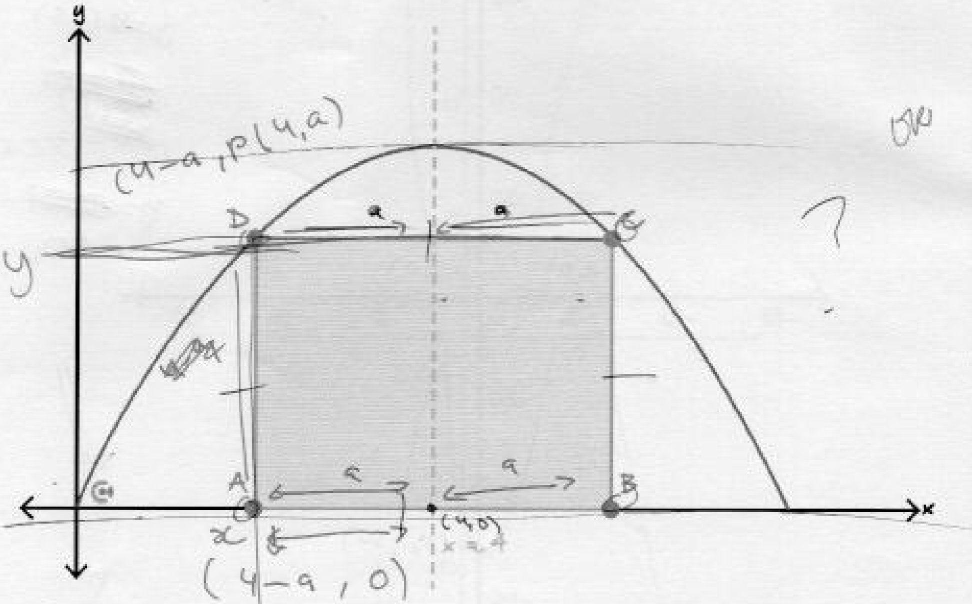
Question 23 Extension.

Quadratic domain restriction

Consider the parabola $p(x) = x(8 - x)$, where $0 \le x \le 8$.

$0 \le x \le 8$ $x \ge 0$ $x \le 8$

A rectangle $ABCD$ is inscribed between the graphs of p and the x-axis. Its vertices are a distance of a units from the axis of symmetry, $x = 4$, as shown below.



Find the value of a when the rectangle is a square.

$0 \le x \le 8$

x value to find y value

$W = L$, $x = y$

$p(4-a) = (4-a)(8-(4-a))$

$(4-a)(4+a)$

DOPS

$16 - a^2$

$w = \cancel{16 - a^2}$

$2a = 16 - a^2$

$a^2 + 2a - 16 = 0$

$a = \dfrac{-2 \pm \sqrt{4 - 4(1)(-16)}}{2(1)}$

$-1 \pm \sqrt{17}$.

$a = -1 + \sqrt{17}$

Sub-Section: Application of Polynomials

Question 24

Two polynomial curves give the heights of two structures above ground as functions of horizontal distance x (in metres) from a fixed reference point.

quadratic

Curve A: $y = -5x^2 + 45x + 50, x \geq 0$ (height in metres).

~~quat~~ cubic.

Curve B: $y = x^3 - 6x^2 + 11x - 6, x \geq 0$ (height in metres).

$0 =$

-10×1

a. Where does Curve A meet the ground? ? $y=0$

$-5x^2 + 45x + 50 = 0$

$-5(x^2 - 9x - 10) = 0$

$0 = -5x^2 + 45x + 50, x \geq 0$

-10×1

~~$-50 = -5x^2 + 45x$~~

$(x-10)(x+1)$

~~$-50 = -5x(x-9)$~~

$\boxed{x = 10}$

~~$x = 0, x = 9$~~

~~$x = -1$~~

$36 - 44$

b. Where does Curve B meet the ground?

$x^3 - 6x^2 + 11x - 6 = 0$

$\sqrt{36-44}$

$0 = x^3 - 6x^2 + 11x - 6, x \geq 0$

$6 = x^3 - 6x^2 + 11x$

$x = 1$

$\frac{\pm b \sqrt{(b)^2 - 4ac}}{2a}$

$6 = x(x^2 - 6x + 11)$

$\pm(-6)\sqrt{36 - 4(1)(11)}$

$a = 1$

$(1)^3 - 6(1)^2 + 11(1) - 6$ $b = -6$

$-5 + 5 = 0$ $c = 11$

$\pm(-6)\sqrt{36-44}$

$1 - 6 + 11 - 6$

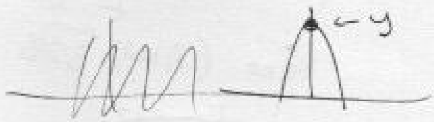
$x = 1, x = 2, x = 3$ 2

c. Find the distance x, at which Curve A is the tallest in decimal form.

$x = 0$

$y = -5(0) + 45(0) + 50$

$y = 50$



$x =$

d. For which value(s) of x, are the two curves at the same height, and what is the common height in metres?

$-5x^2+45x+80 = x^3-6x^2+11x-6$

$-5x^2+6x^2+45x+80 = x^3+11x-6$

$x^2+45x-11x+80 = x^3-6$

$x^2+34x+80+6 = x^3$

$x^2+34x+56 = x^3$

$-x^3+x^2+34x+56 = 0$

$y = x^3-x^2-34-56$

$34 \times 5 = 170$ $2+15$

$120 - 56 = 114$

$34 \times 8 = 272$ $3+24$

$64 \times 8 = 512$ $3+48$ 51

e. Hence or otherwise, sketch the graph of the polynomial $y = x^3 - x^2 - 34x - 56$ from $x = -5$ to $x = 8$ on the axes below, clearly labelling all axes intercepts and endpoints. (3 marks)

$[-5, 8]$

$x = -5$

$y=(-5)^3-(-5)^2-34(-5)-56$

$y=-125-25+170-56$

$-150+144$

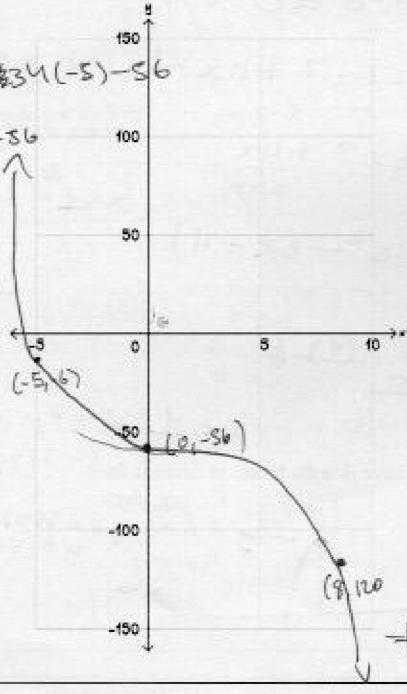
$-150+144$

$150 - 144 = 006$ $=-6$

$(-5,-6)$

$x=0$

$y=-56$



$x=8$

$(8)^3-(8)^2-34(8)-56$

$48-64-272-56$

$48-64$

$64 - 48 = 16$ $-16-272$

$512-64-272-56$

$272 + 56 = 328$

$(8,120)$

~~$-16 = 328$~~