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Email: hello@contoureducation.com.au

Year 10 Mathematics

AOS 9 Revision [10.4]

Mock CAT 1

50 Marks. 60 Minutes Writing.

Results:

Short Answer Questions	28 / 36
Extended Response Questions	0 / 14

26/50

Question 15 (4 marks)

A circular spray pattern is $(x-4)^2+(y+1)^2=16$ (metres).

a. State the centre and radius. (1 mark)

centre = (4, -1)

radius = 4

b. Is (8, 1) on the boundary? Justify. (1 mark)

$(8-4)^2+(1+1)^2=16$

$16+4 \neq 16$ no it is not.

$20 \neq 16$

c. The sprinkler is moved 2 m left and 3 m up. Write the new equation. (2 marks)

x y

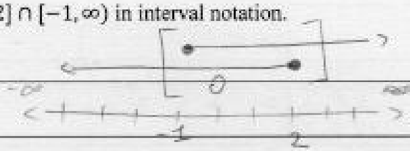
+4 − 2 = 2

$(x-2)^2+(y-2)^2=16$; 1 + 3 = 2

Section A: Short Answer Questions (36 Marks)

Question 1 (1 mark)

Write the intersection $(-\infty, 2] \cap [-1, \infty)$ in interval notation.



$[-1, 2]$

Question 2 (1 mark)

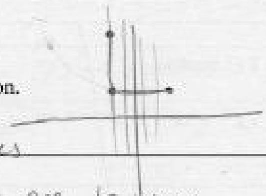
Factorise completely: $9p^2 - 25$.

$\sqrt{9p^2 - 25}$

$(3p-5)(3p+5)$

Question 3 (1 mark)

Does the relation $\{(-1, 2), (2, 2), (-1, 5)\}$, define a function? Give a reason.



no, because it has 2 y values for one x value makeing it one to many.

Question 4 (1 mark)

Expand: $(x + 3)(x - 5)$.

$x^2 - 2x - 15$

Question 5 (1 mark)

For $y = (x-1)^2 - 3$, find the y-intercept.

$x = 0$

$y = (-1)^2 - 3$

$y = 1 - 3$

$y = -2$

$y = (0, -2)$

Question 6 (2 marks)

2

C

Use long division to find the quotient and remainder when $2x^4 - x^3 + 5x - 7$ is divided by $x^2 + 1$.

$2x^2 - 2$

$x^2 + 1 \enclose{longdiv}{2x^4 + 0x^3 + 0x^2}$

$-\ 2x^4 + 2x^2$

$-2x^2 + 0$

$-\ -2x^2 - 2$

-2

$0 - 2x^2$

quotient $= 2x^2 - 2$

remainder $= -2$

Question 7 (2 marks)

2

C

Solve $x^3 - x^2 - 9x + 9 = 0$.

$x = 1$

$1 - 1 - 9 + 9 =$

$x = 1$

$1 - 1 = 0$

$-9 + 9 = 0$

Question 8 (2 marks)

Sketch $y = -(x-1)^2(x+3)$. Label all intercepts.

$x = 1, \quad x = -3$

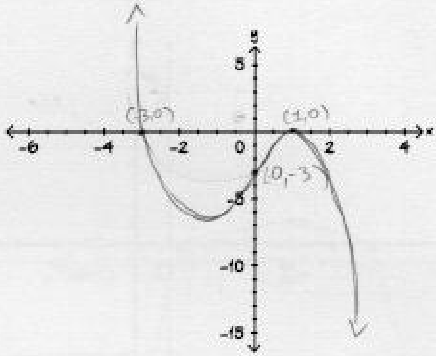
$x = 0$

$y = -(-1)^2(3)$

$y = -(1)(3)$

$y = -(3)$

$y = -3$



Question 9 (2 marks)

Find the centre and radius of $x^2 + y^2 + 4x - 10y + 13 = 0$.

$\left(\frac{4}{2}\right)^2 = 4$

$\left(\frac{10}{2}\right)^2 = 25$

$x^2 + 4x + y^2 - 10y + 13$

$(x^2 + 4x + 4)$

$\downarrow$

$(x+2)^2 + y^2 - 10y + 25$

$\downarrow$

$(x-5)^2$

$(x+2)^2 + (x-5)^2 + 13 - 25 - 4 = 0$

$(x+2)^2 + (x-5)^2 - 16 = 0$

$(x+2)^2 + (x-5)^2 = 16$

$-25 - 4 = -29$

$13 - 29$

$29 - 13 = 16$

$\boxed{\text{centre} = (-2, 5) \quad \text{radius} = 4 \text{ units}}$

Question 10 (2 marks)

Let $f(x) = 3x^3 - 2x^2 + x + 5$. Using the Remainder Theorem, find the remainder on division by $x - 2$. Hence, decide if $x - 2$ is a factor.

$x = 2$

$3(2)^3 - 2(2)^2 + (2) + 5$

$3(8) - 2(4) + 2 + 5$

$24 - 8 + 2 + 5 = 0$

$16 + 7 \neq 0$

$23 \neq 0$ ← remember to finalise.

$\therefore x - 2$ is not a factor

~~Not a f~~

Question 11 (2 marks)

Sketch $y = -(x+2)^4 + 1$. Label the turning point and any intercepts.

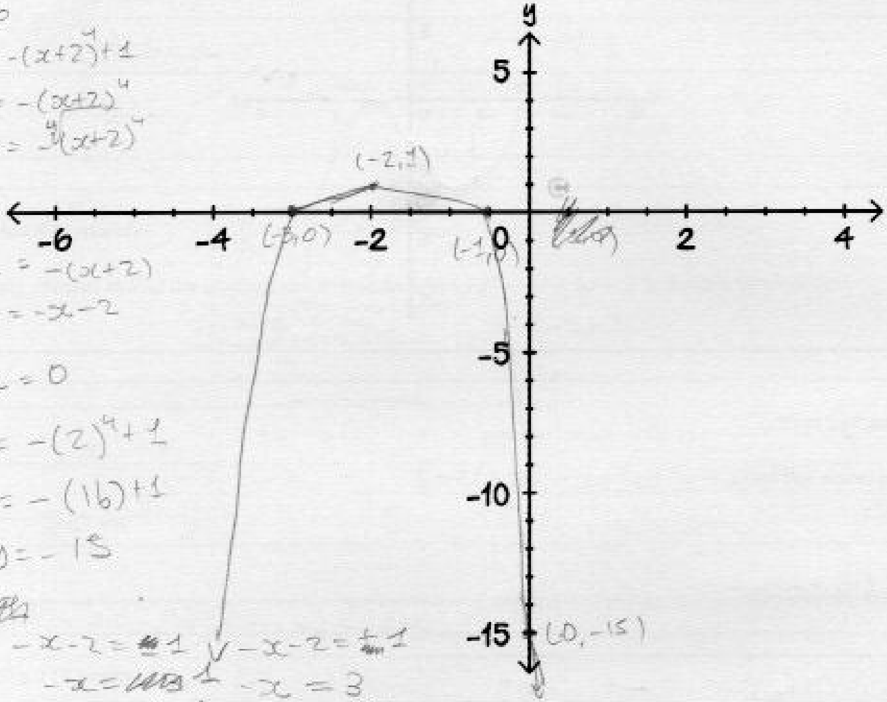
$(-2,1)$

$y=0$

$0 = -(x+2)^4 + 1$

$-1 = -(x+2)^4$

$\sqrt[4]{1} = \sqrt[4]{(x+2)^4}$



$\pm 1 = -(x+2)$

$\pm 1 = -x-2$

$x = 0$

$y = -(2)^4 + 1$

$y = -(16)+1$

$y = -15$

$-x-2 = 1 \quad \lor \quad -x-2 = -1$

$-x = 1 \quad -x = 3$

$x = -1 \quad x = -3$

Question 12 (2 marks)

Find k so that $(x+1)$ is a factor of $x^3 + kx^2 - x - 6$. Hence, factor the polynomial completely.

$x = -1$

$(-1)^3 + k(-1)^2 - (-1) - 6$ $\quad -1+(-5)$

$-1 + k + 1 - 6 = 0$ $\quad -1+6=5$

$-1+1-6+k=0$

$-1-5=-6+k=0$

$\boxed{k=6}$

$x^3 + 6x^2 - x - 6$

$-1 \,|\, 1 \quad 6 \quad -1 \quad -6$

$\quad 1 \quad 5 \quad -6 \quad 0$

$x^2 + 5x - 6$

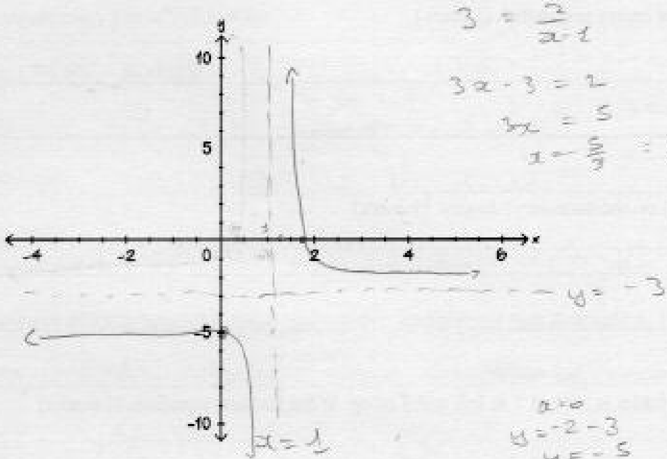
$\boxed{= (x-1)(x+6)(x+1)}$ ← don't skip steps!

Question 13 (2 marks)

Sketch the graph $y = \frac{2}{x-1} - 3$. Label the asymptotes.

$h = 1$
$k = -3$
$y = 0$
$3 = \frac{2}{x-1}$
$3x - 3 = 2$
$3x = 5$
$x = \frac{5}{3}$
$x = 0$
$y = \frac{2}{-1} - 3$
$y = -2 - 3$
$y = -5$

$1, -3$
x int $y = 0$
$3 = \frac{2}{x-1}$
$3x - 3 = 2$
$3x = 5$
$x = \frac{5}{3}$: $(\frac{5}{3}, 0)$



$x = 0$
$y = -2 - 3$
$y = -5$

Question 14 (4 marks)

From the platform edge at $x = 1$ m to $x = 10$ m along the deck, the slide height (m) is $h(x) = -\sqrt{x-1} + 5$.

a. State the domain and range over this section. (2 marks)

Domain $[1, 10]$
Range $= [2, 5]$

$h(1) = -\sqrt{1-1} + 5$
$= -\sqrt{0} + 5$
$h(1) = 5$
$h(10) = -\sqrt{10-1} + 5$
$= -\sqrt{9} + 5$
$= -3 + 5$
$= 2$

b. How far from the edge does the slide first reach 3 m high? (2 marks)

$h(3) = -\sqrt{3-1} + 5$
$h(3) = -\sqrt{2} + 5$
$-\sqrt{2} + 5$ m

a
c

Question 16 (3 marks)

A stage light is aimed so that its intensity curve is $y = -(2x + 1)^4 + 3$.

C

a. Describe the transformations from $y = x^4$. (2 marks)

-1

- Dilation in the ~~x~~ y axis by 2 units
- Translation in the x axis to the left ~~down~~ by 1 unit
- Translation in the y axis up by 3 units
- Reflection in the ~~x~~ x axis.

b. Identify the turning point and whether the curve opens up or down; justify from your description. (1 mark)

turning=(-1, 3), ~~Opened~~ opens downwards because of the reflection in the x axis

+1

C

Question 17 (4 marks)

A tunnel cross-section is $x^2 + y^2 = 16$ (metres), ground is $y = 0$. A truck travels along the centreline.

$\cancel{(x-a)^2+(x}$ $(x-h)^2+(y-k)^2 = r^2$

a. Determine the maximum truck width that fits at the height $y = 3$ m. (2 marks)

$r = \sqrt{16}$

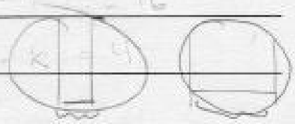
$r = 4\text{ m}$

$\cancel{(x^2-h)+(y-k)^2 = r^2}$

$\sqrt{x^2+9} = 16$

$x + 3 = 4$

$x = 1\text{ m}$



oh okay!

b. If the truck is 3 m wide, what is the maximum height it can have at the centreline? Give an exact value. (2 marks)

$3 + y^2 = 16$

$y^2 = 13$

$y = \sqrt{13}\text{ m}$

-4

C

Section B: Extended Response Questions (14 Marks)

Question 18 (14 marks) -14 8

The city is finalising Compass Court, a plaza drawn on the Cartesian plane (units in metres).

The outer rim of a round seating area is a circle whose diameter has endpoints $A(1, -1)$ and $B(7, 5)$.

a. Find the equation of the circle. (3 marks)

$[-1, 5]$

Domain $= [1, 7]$ $\quad 7 - 1 = 6 \quad r = \frac{6}{2} = 3$

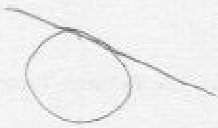
$(x-h)^2 + (y-k)^2 = r^2 \quad r^2 = 9$

$(x-4)^2 + (y-2)^2 = 9 \quad 1 + 3 = 4 \quad x = 4$

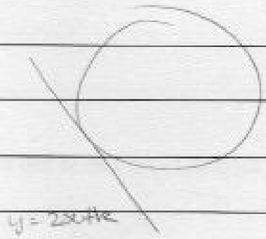
centre $(4, 2)$ correct but use midpoints $\quad -1 + 3 = 2$

$\underbrace{\left(\frac{x+x}{2}\right)\left(\frac{y+y}{2}\right)}_{\text{midpoint}} \quad (4, 2)$

A family of straight paths is planned with equations $L_k: y = 2x + k$.



b. Determine the value(s) of k for which L_k is tangent to the circle. (3 marks)

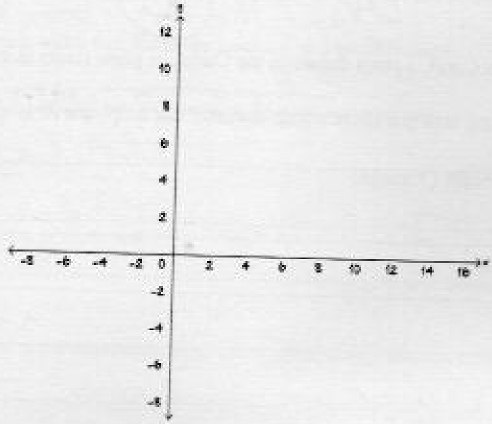


$(x-4)^2 + (y-2)^2 = 9$

$(x-4)^2 + (2x+k-2)^2 = 9$

c. Hence, determine the equation of the tangent line(s). (1 mark)

d. **Tech-Active.** Sketch the graph of a circle and a line L_k for the k found in **part b.** Label the point of intersections, correct to two decimal places. (3 marks)



A raised planter's front edge is modelled by the cubic:

$$G(x) = x^3 - 3x^2 - 4x + 12$$

e.

i. Use the Factor Theorem to show that $(x-3)$ is a factor of $G(x)$. (1 mark)

ii. Perform polynomial division to factorise $G(x)$ completely, and use the Null Factor Law to find all x-intercepts. (3 marks)