CONTOUREDUCATION

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Year 10 Mathematics
AOS 7 Revision [10.2]
Workbook

YeWoo

Outline:

Announcements

Link: https://bit.ly/Contour-Class-Announcements

Contour Resources

Core		Mastery	
Workbook + Test, CAT	Homework	Workshop	Bound Reference, Mock CAT, Exams
In Class	At Home	In Class	At Home

Subject Outline for AOS 10 - End of Year Exam Revision

In Class (Workbook + Test, CAT) | At Home (Homework)	In Class (Workshop)
☐ MA10 [10.1] - AOS 6 Revision	MA10 [10.1] - Mock CAT 3
☐ MA10 [10.2] - AOS 7 Revision	MA10 [10.2] - Mock CAT 3
☐ MA10 [10.3] - AOS 8 Revision	MA10 [10.3] - Mock CAT 3
☐ MA10 [10.4] - AOS 9 Revision	MA10 [10.4] - Mock CAT 3

Additional Resources

Mock CAT	Exam
☐ MA10 [10.1] - AOS 5 Revision 3 x Mock CATs ☐ MA10 [10.2] - AOS 6 Revision 3 x Mock CATs ☐ MA10 [10.3] - AOS 7 Revision 3 x Mock CATs	☐ MA10 [10.4] - EOY - Exam

Section A: Congruence and Similarity

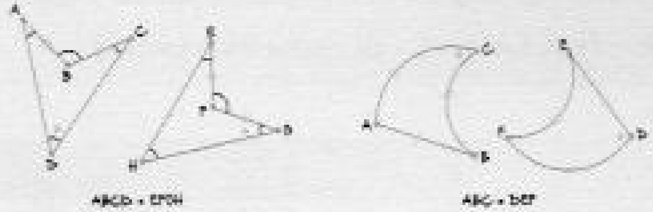
Sub-Section: Congruence

Congruence

Definition

Congruence means "exactly the same".

- If two objects are congruent, we use '≡'. Same side length and angles

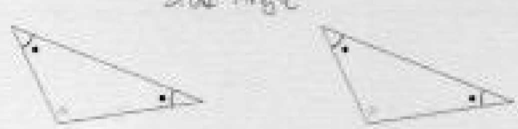


Congruence in Triangles

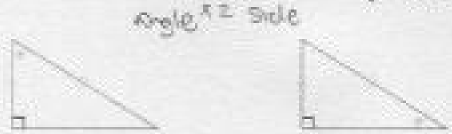
Definition

SSS case ✓
side ×3

SAS case ×2 sides
side×2 Angle



AAS case × Angles
Angle×2 Side

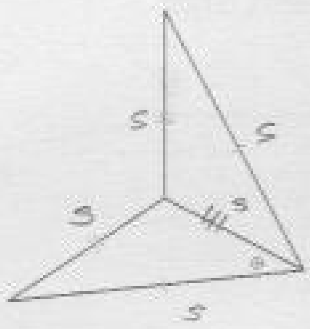


RHS case
Right hand side

Question 1 Walkthrough.

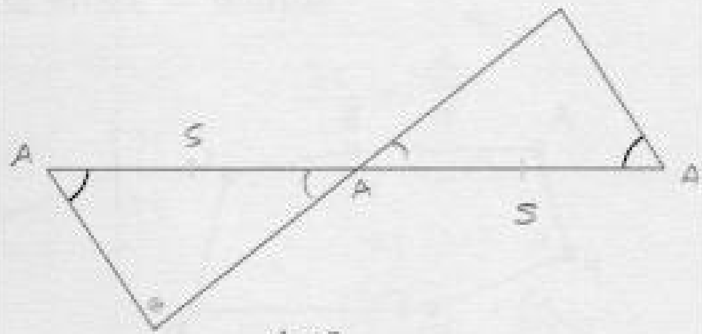
For each pair of triangles, state the postulate or theorem that can be used to conclude that the triangles are congruent.

a.



= SSS

b.



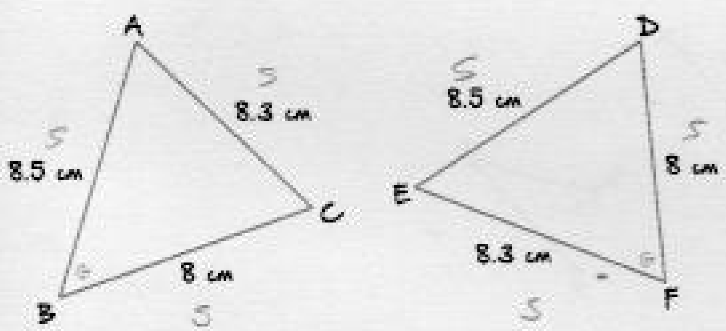
AAS

Question 2

Are the following sets of triangles congruent? Explain your answer.

Example 1.

a.



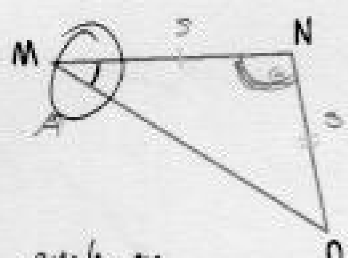
• SSS

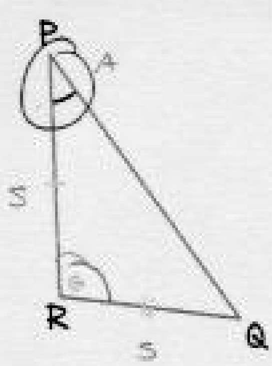
$\triangle ABC \cong \triangle DEF$ SSS

∠AB = ∠DE = 8.5 (S)
∠BC = ∠DF = 8cm(S) = SSS
∠AC = ∠EF = 8.3cm (S)

Example 2

b.





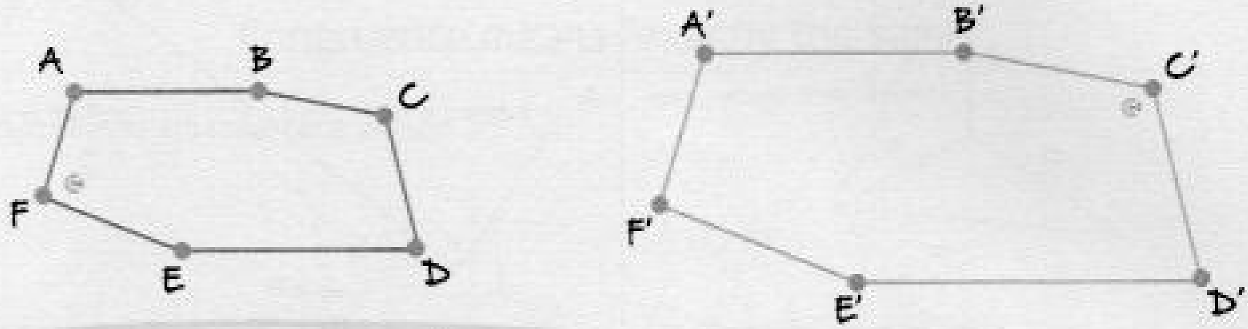
angle on ~~outs~~ outside cannot be, it has to be ~~SAS~~ Not congruent inbetween S, S.

Sub-Section: Similarity

Similarity

same order

Similar = Same shape, different size

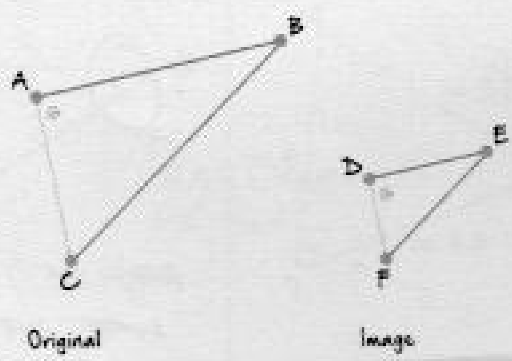


- The symbol '~' is used to describe similarity and to write similarity statements.
- For example,

$$ABCDEF \sim A'B'C'D'E'F'$$

The Scale Factor

- The scale factor between two similar figures is the ratio of corresponding side lengths.



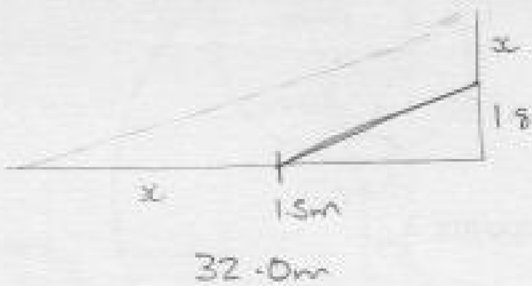
$$\text{Scale factor} = \frac{\text{Image length}}{\text{Original length}}$$

order from triangle does not matter.

$$\text{Scale factor} = \frac{DF}{AC} = \frac{DE}{AB} = \frac{EF}{BC}$$

Question 3

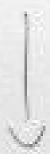
A lighthouse casts a shadow 32.0 m long on level ground. At the same time, a 1.8 m tall park sign casts a shadow **1.5 m** long. What is the height of the lighthouse?



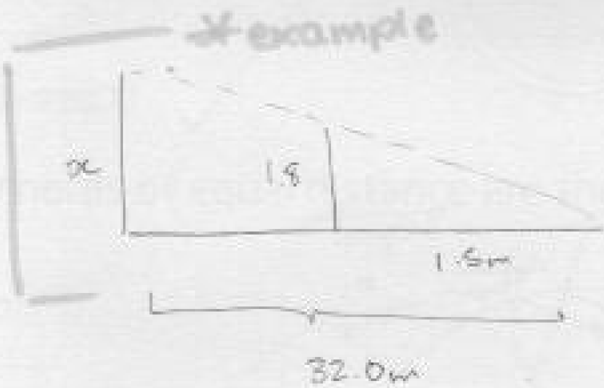
$$\frac{32}{15} = \frac{x}{18}$$

$$\begin{array}{r} 18 \\ \times\ 15 \\ \hline 90 \\ +\ 180 \\ \hline 270 \end{array}$$

$$\frac{966}{270}$$



$$\frac{966}{270} =$$



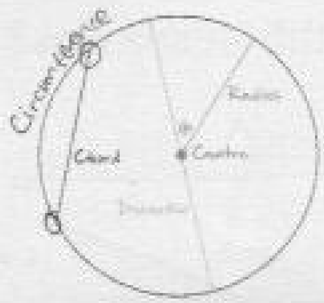
$$\frac{1.8}{x} = \frac{1.5}{3.2} = 38.4m$$

Section B: Chord Theorems

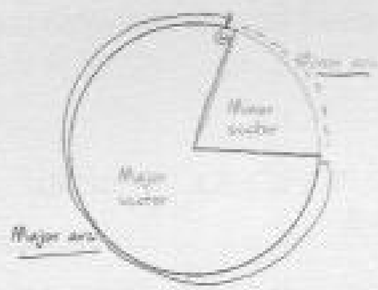
Definition

Key Terminologies in Circles

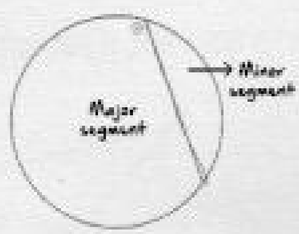
- **Chord:** A straight line that connects any two points located on the circumference.



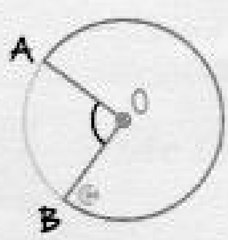
- **Arc:** A smooth curve joining two endpoints.
- **Sector:** A portion of a circle.



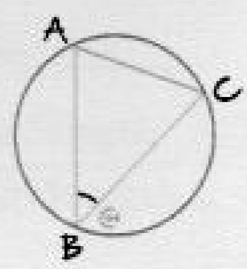
- **Segment:** An interior region of a circle.



- An angle is **subtended** by an arc or chord if the arms of the angle meet the endpoints of the arc or chord.



∠AOB is subtended at the centre by the minor arc AB.



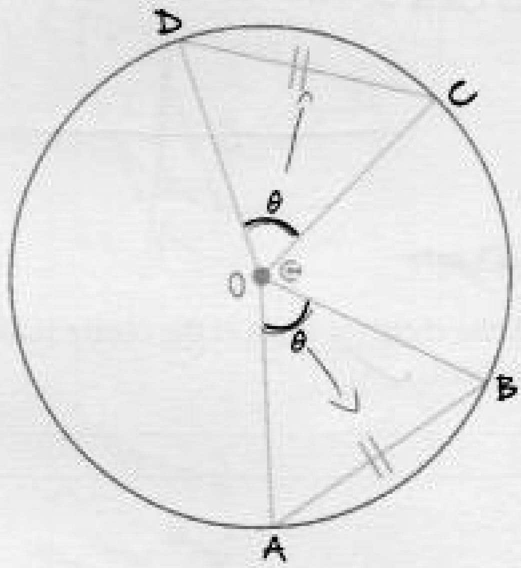
∠ABC is subtended at the circumference by the chord AC.

Chord Theorem 1 all chord theorems

Definition

Chords of equal length subtend equal angles at the centre of the circle.

- Conversely, if chords subtend equal angles at the centre of the circle, then the chords are of equal length.



Chord DC = Chord AB

then

Angle ∠DC = Angle AB

viceversa

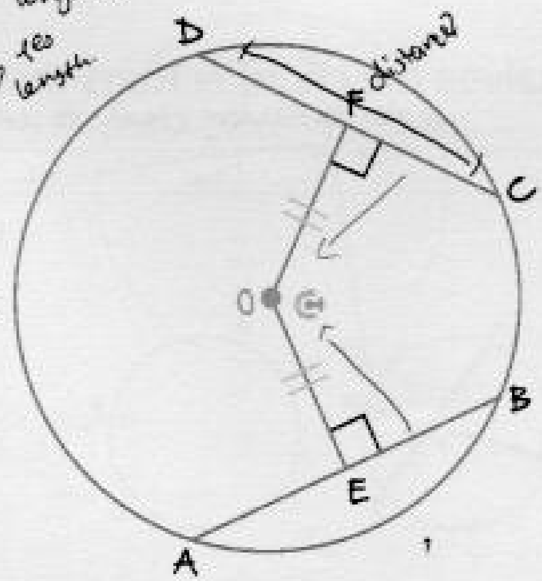
Chord Theorem 2

Definition

2 chords of equal distance are the same distance from the centre.

length?

? too length



length

Chord AB = Chord DC

then

equidistant from centre.

- Conversely, if chords are equidistant from the centre of the circle, then the chords are of equal length.

equal distance

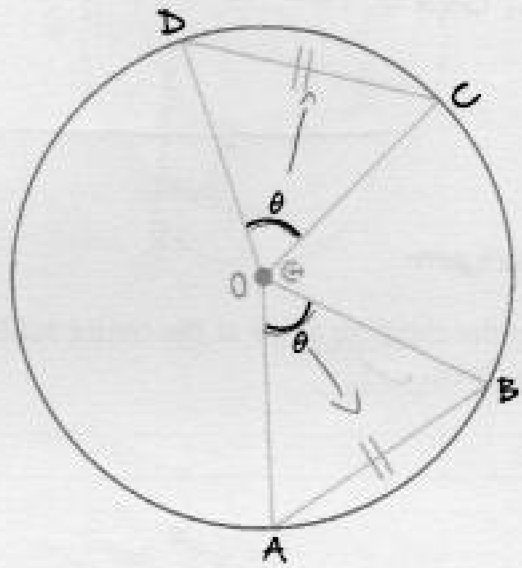
Definition

Chord Theorem 1

all chord theorems

Chords of equal length subtend equal angles at the centre of the circle.

- Conversely, if chords subtend equal angles at the centre of the circle, then the chords are of equal length.



Chord DC = Chord AB
then
Angle ∠DC = Angle AB

viceversa

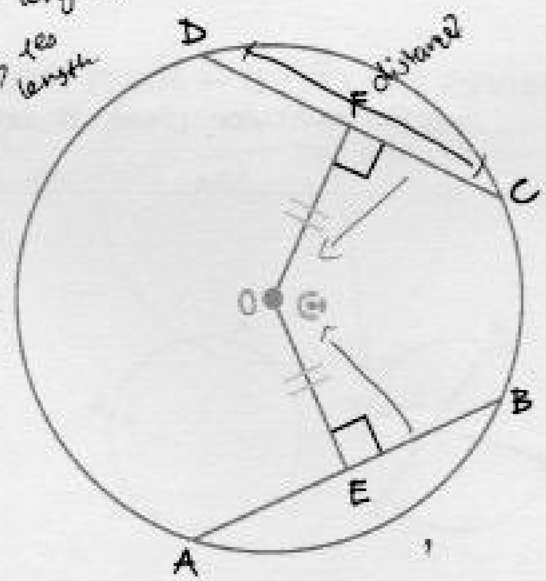
Definition

Chord Theorem 2

2 chords of equal distance are the same distance from the centre.

length?

yes length



length
Chord AB = Chord DC
then
equidistant from
centre.

- Conversely, if chords are equidistant from the centre of the circle, then the chords are of equal length.

equal distance

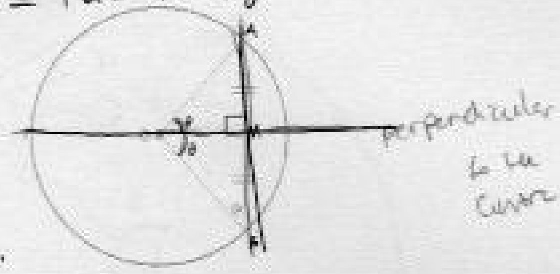
Definition

Chord Theorem 3

- The perpendicular from the centre of the circle to the chord bisects the chord and the angle at the centre subtended by the chord.

Half the chord → Half the angle

perpendicular
centre to chord $= \frac{1}{2}$ of chord $=$ Half the angle.

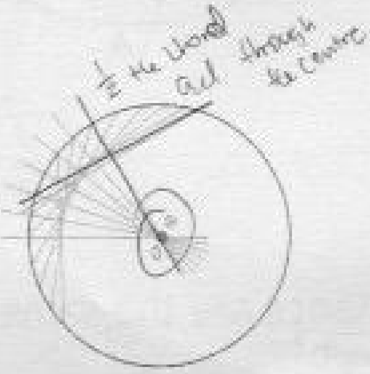


divide into 2 parts.

- Conversely, if a radius bisects the chord (or angle at the centre subtended by the chord), then the radius is perpendicular to the chord.

Definition

Chord Theorem 4



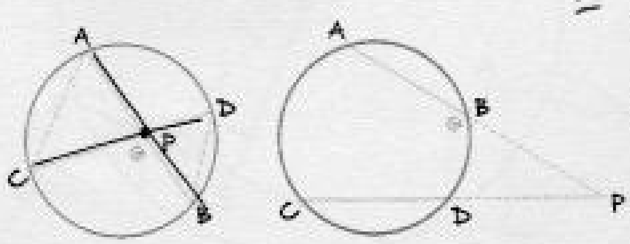
Corrected.

The line that halves the chord → Runs through the centre

= perpendicular chord to centre runs through centre

Definition

Chord Theorem 5



IF similarity

$= \frac{PA}{PD} = \frac{PC}{PB}$

then to

$PA \times PB = PC \times PD$

+ Similarity.

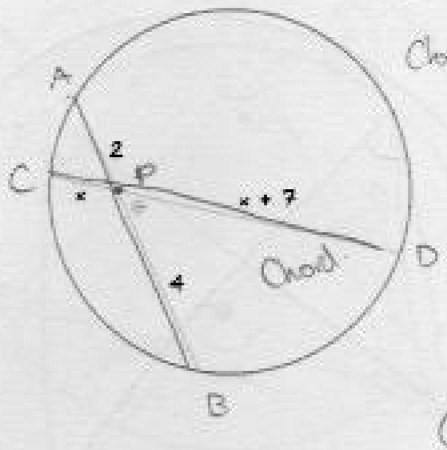
- If AB and CD are two chords that cut at a point P, then,

$$PA \times PB = PC \times PD$$

why?

Question 4 Walkthrough.

Find the value of x in the diagram below.



∠ intersection (P)

Chord: AB, CD

$PA \times PB = PC \times PD$

$2 \times 4 = x \times (x+7)$

① $8 = x^2 + 7x$

$0 = x^2 + 7x + 8$

② Quadratic involved

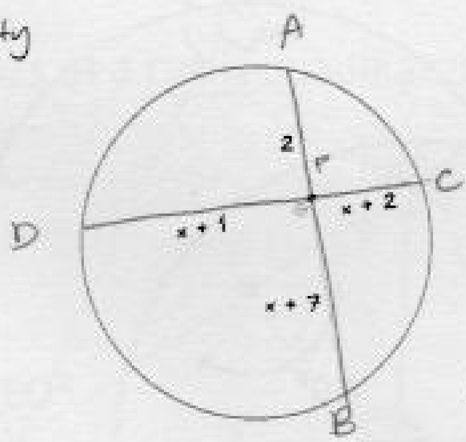
$0 = (x+8)(x-1)$

$x = -8, x = 1$

Question 5 ∀ example

explain with similarity

Find x.



$PA \times PB = PC \times PD$

$2 \times x+7 = (x+2)(x+1)$

$2x + 14 = x^2 + 3x + 2$

$0 = x^2 + 3x - 2x + 2 - 14$

$= x^2 + x - 12$

$(x+4)(x-3)$

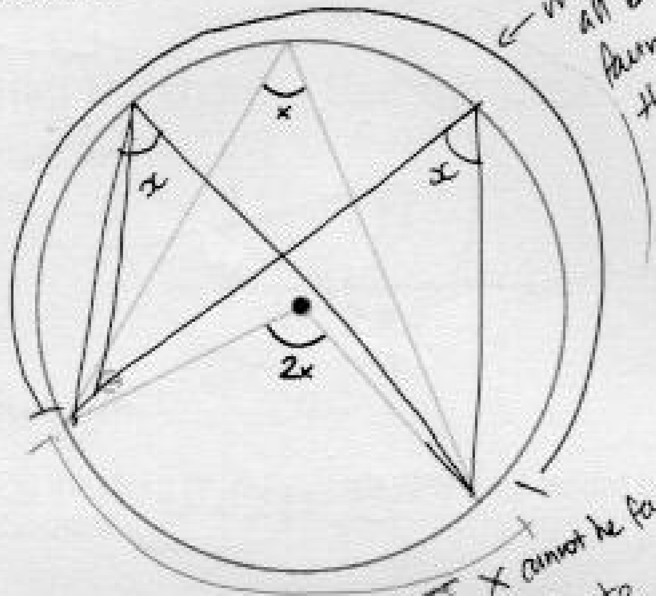
$x = -4, x = 3$

Section C: Angle Theorems

all angle theorems.

need more study on angle theorems

Angle Theorem 1: Angle at the Centre



← must all be facing this arc.

?

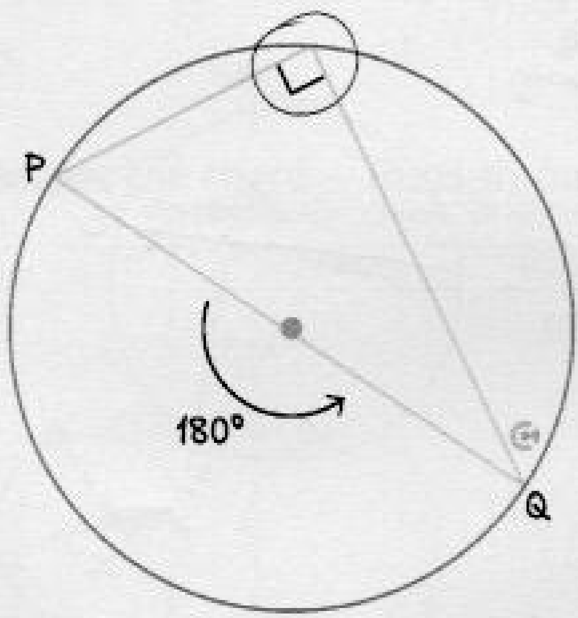
X cannot be facing this arc

- If 2 angles are subtended by the same arc, then,

↳ looking, ~~have~~ the vertex towards to same arc. both angles

$$\text{Angle at the centre} = 2 \times \text{Angle at the circumference}$$

Angle Theorem 2: Angles in a Semi-circle



How many angles can be a angle at circumference?

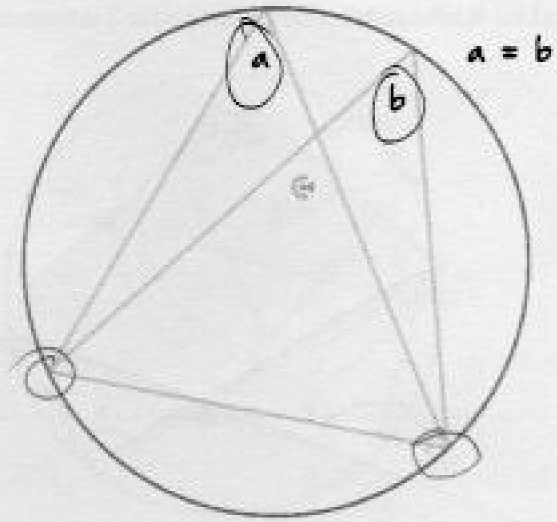
- If an angle is subtended by the same arc and one side is the diameter, then, 180°

$$\text{Angle at the circumference} = 90^\circ$$

only one angle?

Definition

Angle Theorem 3: Angles in the Same Segment



a and b being the circumference angle.

?

- If 2 (or more) circumference angles are:
  - Subtended by the same endpoints. ?
  - In the same segment. Then,

$$a = b$$

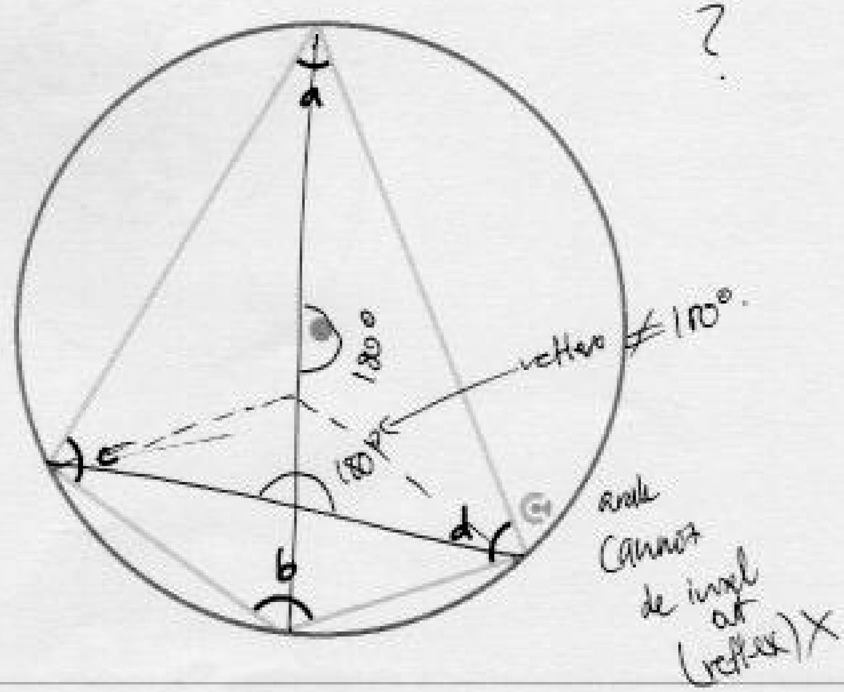
Definition

Angle Theorem 4: Cyclic Quadrilateral

Opposite angles of a cyclic quadrilateral → Add to 180°

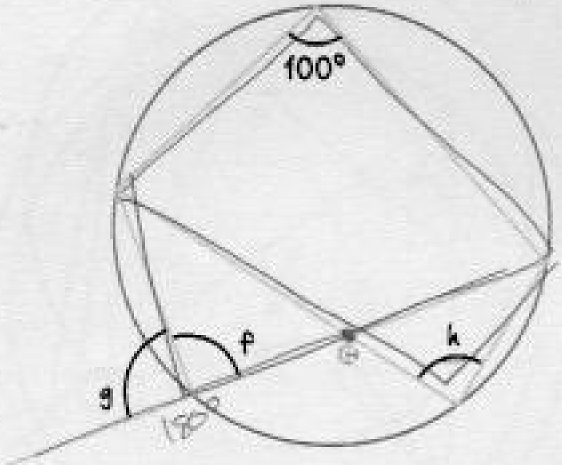
?

$a + b = 180°$
$c + d = 180°$



Question 6

Work out the size of each angle marked with a letter. Give reasons for your answers.



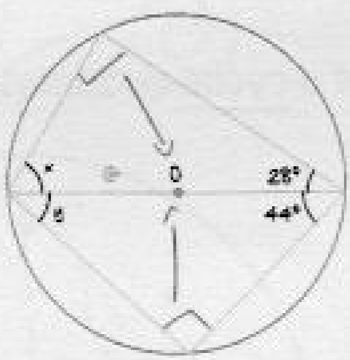
f = 180 − 100 = 80°

k = 80°

g = 180 − 80 = 100°

Question 7

Work out the size of each angle marked with a letter. Give reasons for your answers.



x = ~~49~~ 62°

y = ~~38~~ 46°

corresponding??

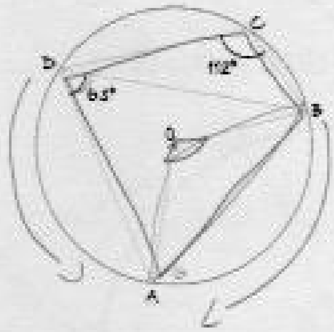
90 + 44 =

y = 180 − 90 − 44 = 46

x = 180 − 90 − 28 = 62

Question 8

A, B, C and D are points on a circle, centre O. Given $\angle ADB = 63°$ and $\angle BCD = 112°$.



a. Calculate the size of the angle AOB.

$\angle AOB = 2 \times 63$
$= 126°$

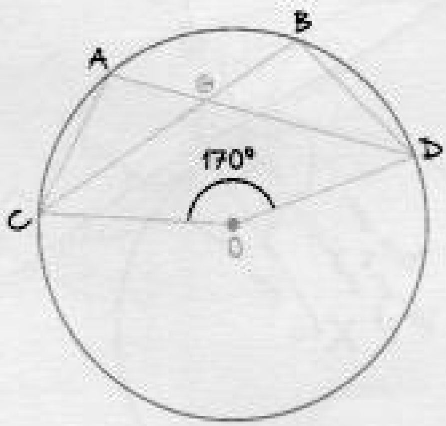
b. Calculate the size of the angle BAD.

$\angle BAD = 180 - 112$
$= 68°$

Question 9 Extension.

Given the circle below $\angle COD = 170°$.

Show with proper reasoning that $\angle CAD = \angle CBD$. Hence, find their values.



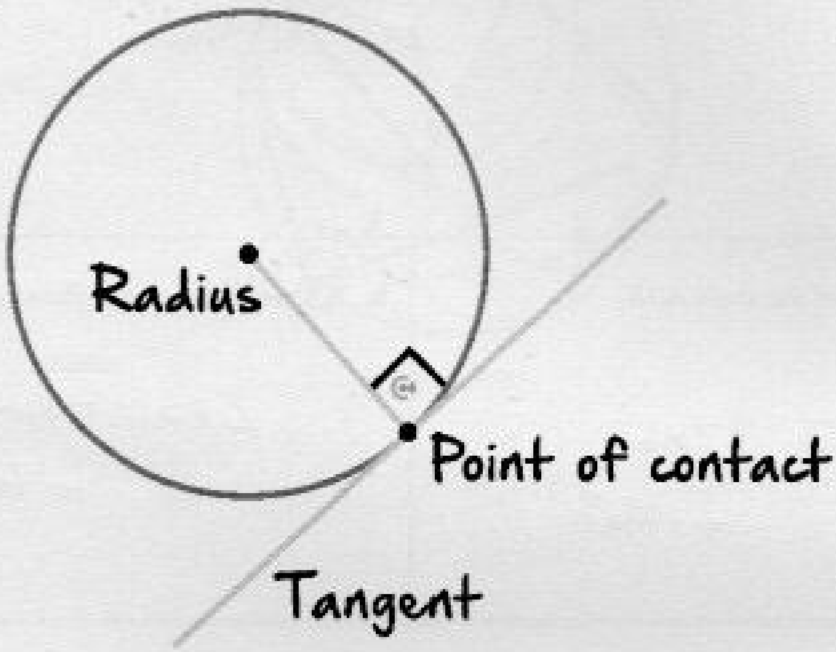
Section D: Tangent Theorems

all angle theorems

Definition

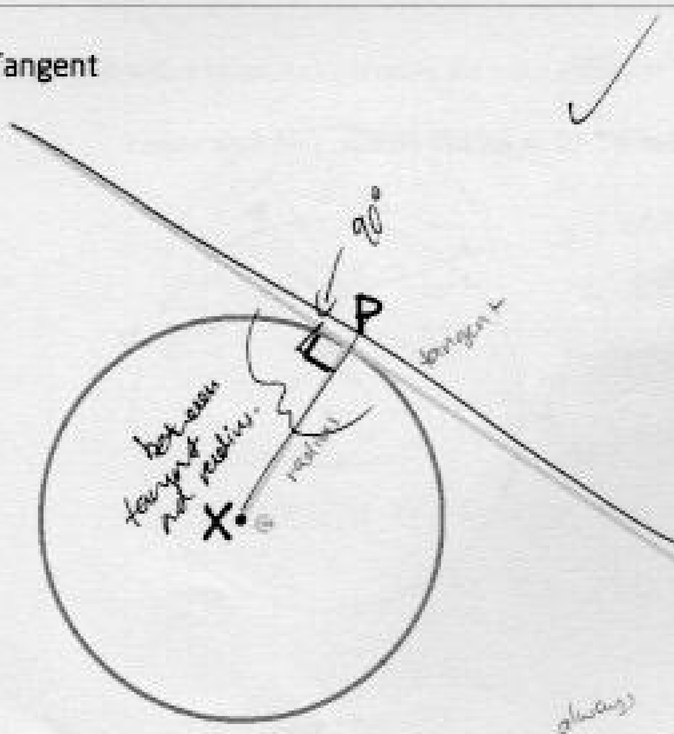
Tangent

- A straight line that only touches the circle at one point.



Definition

Tangent Theorem 1: The Tangent



Angle between tangent and radius always $= 90^\circ$

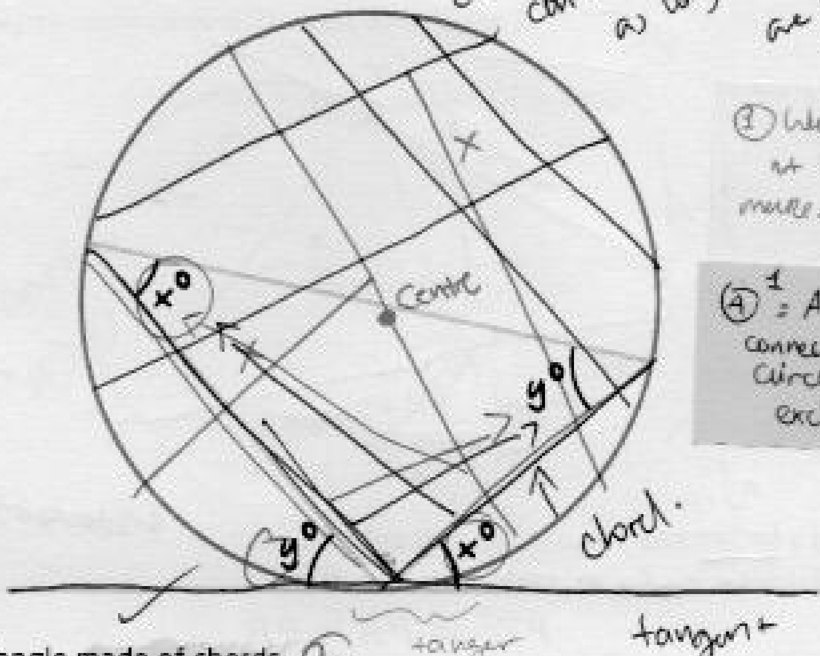
Definition

Tangent Theorem 2: Alternate Segment Theorem

chords can be anywhere as long as 2 points are from circumference.

① When does something become and not become a chord? What exactly makes something a chord?

④ [1]: A chord is a straight line that connects ~~a line~~ 2 points on the circumference. This applies ANYWHERE excluding the ~~cen~~ centre.



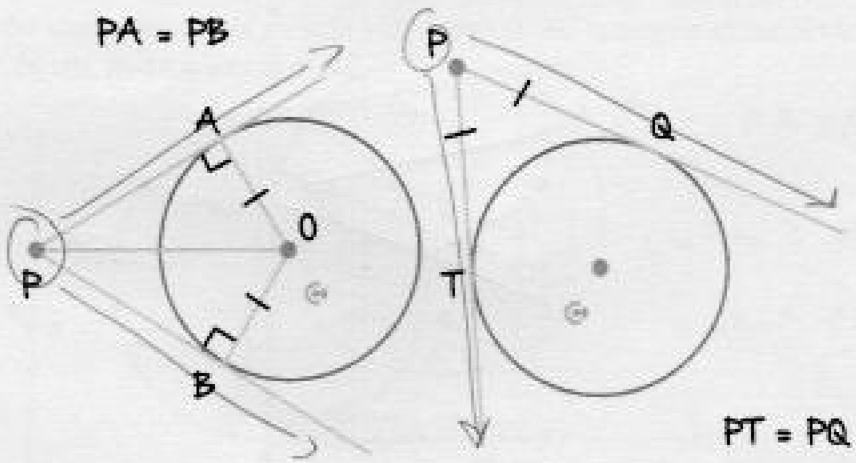
$x = x°$
$y = y°$.

- For an internal triangle made of chords, ②

Angle between a tangent and a chord = Angle on the opposite corner

Definition

Tangent Theorem 3: Two Tangents



- From a single point outside the circle,

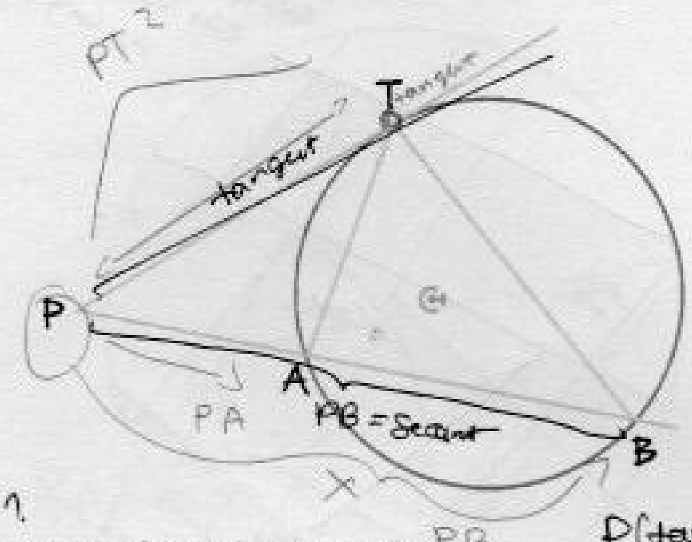
Only two tangents exist.

- And for each tangent,

The distance from the point to the tangent point is equivalent.

Definition

Tangent Theorem 4



Secant PB

= A.

First bit

P(tangent)^2 = PA × PB

First bit

secant

- For a tangent and a secant starting at the same point:

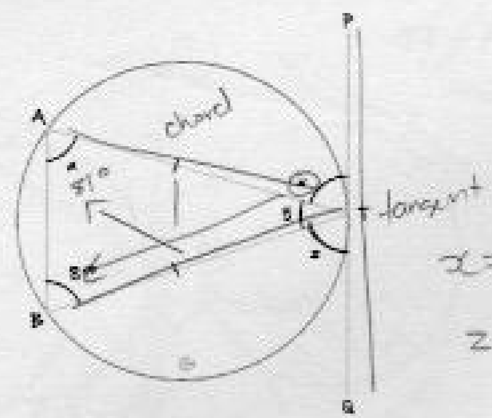
crosses the circle 2 times = PB

$$PT^2 = PA \times PB$$

Question 10 Walkthrough.

ΔATB is isosceles. PQ is a tangent to the circle at T. Work out the size of each angle marked with a letter. Give reasons for your answers.

a = 81°



x = 81°

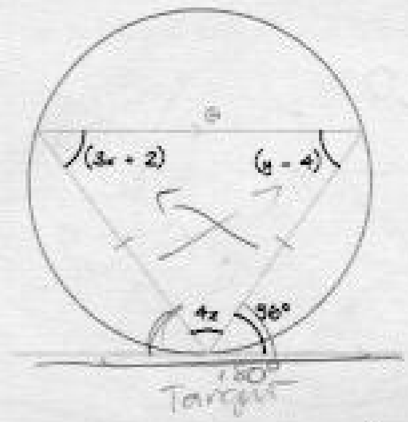
z = 81°

y = 180 − 81 − 81 = 18°

y = 18°

Question 13

Determine the values of the unknowns in the following diagram.



$z = 17$
$y = 60$
$x = 18$

$y - 4 = 56$
$y = 60$
$60 - 4 = 56$

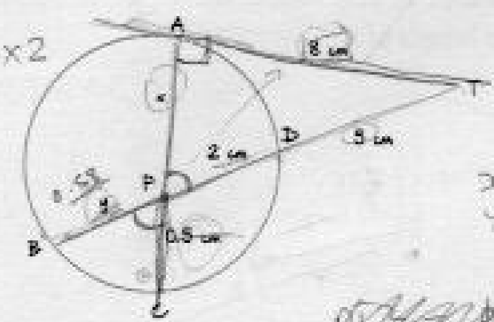
$4z = 68$
$z = \frac{68}{4}$

$56 + 56 =$
112
$180 - 112 = 68$

$3x + 2 = 56$
$3x = 54$
$x = \frac{54}{3}$
$x = 18$

Question 14

Find the unknown lengths in the diagram. Given: $AP = x$ cm, $PC = 0.5$ cm, $PD = 2$ cm, $PB = y$ cm, $TD = 5$ cm, and $AT = 8$ cm.



$x \times 0.5 = 0.5 \times 2$

$8^2 = 0.5 \times (x + 0.5)$

$PT^2 = PA \times PB$

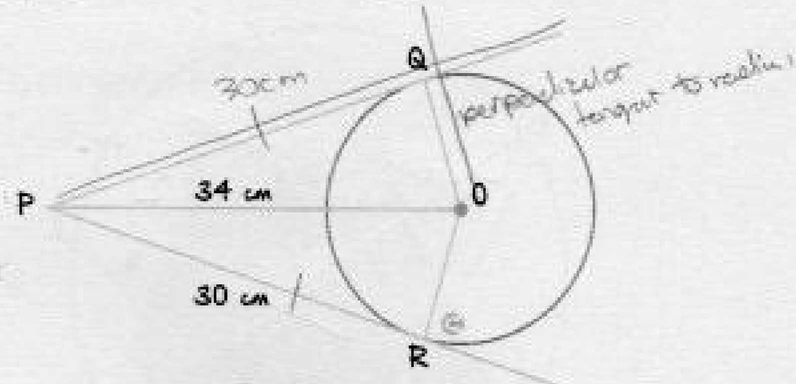
$5 + 2 = 7$

$8^2 = 5 \times (7 + y)$
$64 = 35 + 5y$
$29 = 5y$
$y = \frac{29}{5}$

$y = 0.58$

Question 15

PQ and PR are two tangents to a given circle. We know that $PQ = 30$ cm and $PO = 34$ cm.



Find the length of OQ. $= \sqrt{\ }$

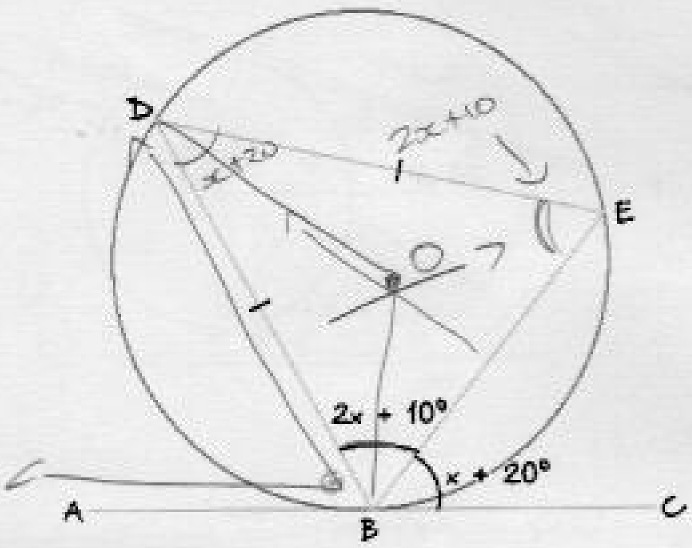
$OQ = \sqrt{34^2 - 30^2}$

$= 16 cm$

$OQ = 16cm$

Question 16

B, D, E lie on a circle. AC is a tangent at B. Given that $BD = DE$, $\angle DBE = \mathbf{2x + 10}^\circ$, $\angle CBE = \mathbf{x + 20}^\circ$.



a. Find the value of x.

$2x+10+2x+10+x+20=180°$

$x=28$

b. Work out the size of $\angle ABD$.

$\angle ABD = 2x+10$

$2x+10$

$2(28)+10 = \underline{\underline{66°}}$

c. Let O be the centre of the circle. Find $\angle BOD$.

~~2x+10~~

$\angle BOD = 2\times 66°$

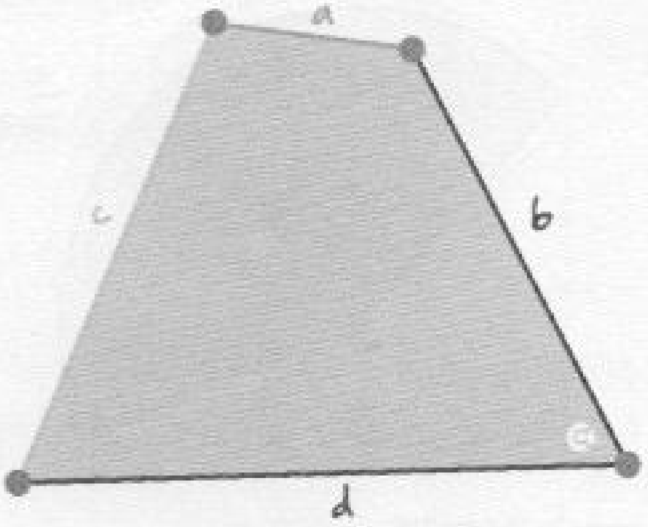
$= 132°$

Section E: Perimeter and Composite Area

Perimeter

Definition

- The perimeter of a shape is the sum of a, b, c, d.

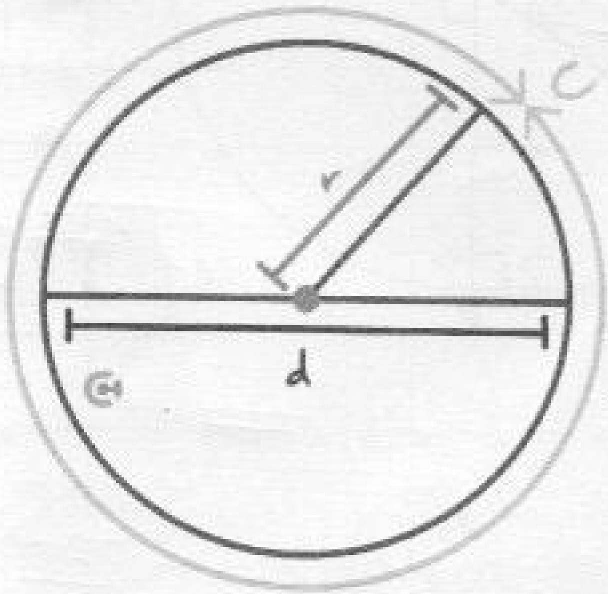


$$P = a + b + c + d$$

Circumference

Definition

- The circumference of circle is given by:



$C = \pi d$ $\quad C = 2\pi r$

- π is an irrational number. Rounded to 8 decimal places, $\pi = 3.14159265\ldots$

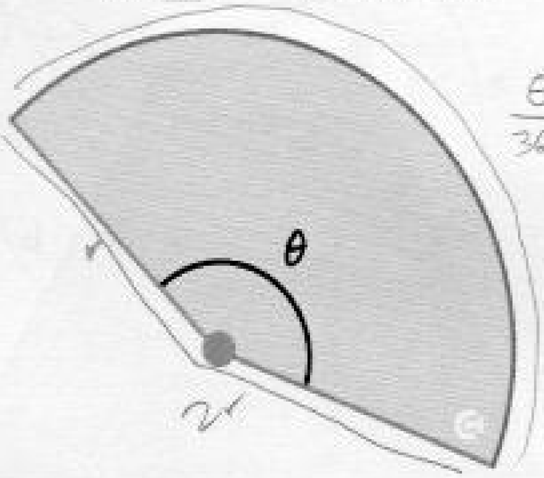
Definition

The Perimeter of a Sector

$$\text{Perimeter of a sector} = \text{Arc length} + \text{Radius} + \text{Radius}$$

$$= \boxed{\frac{\theta}{360}} \times 2\pi r + 2r$$

circumference formula

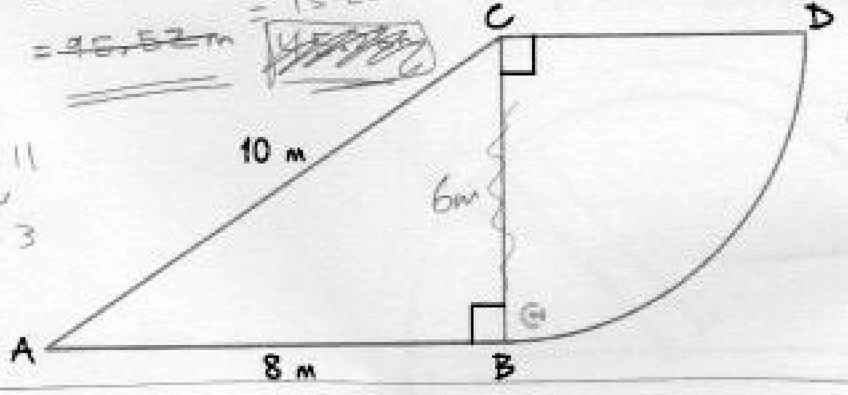


$\frac{\theta}{360}$ a part of a circle

- Where θ is the interior angle in the sector.

Question 17

Calculate the perimeter of the following shape. Given $AC = 10$ m, $AB = 8$ m. Give your answers correct to two decimal places.



= 37.26m + 18 = 15.26+36 = 33.42m

= ~~95.52m~~

18 × 3.14

$\frac{\cancel{3\pi+6}}{}$

9π

9×3.14

$\frac{2\pi 6}{4}$ $\frac{12\pi}{4}$

$\frac{1}{4} \times 2\pi(6) \times \cancel{3(6)}$

~~10² − 8² =~~

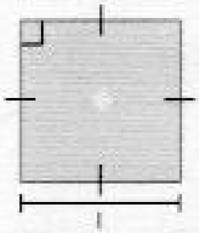
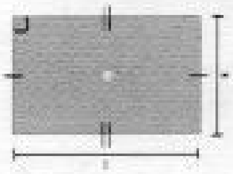
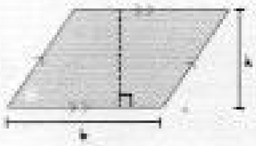
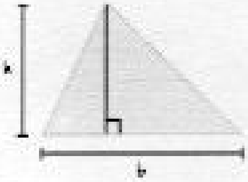
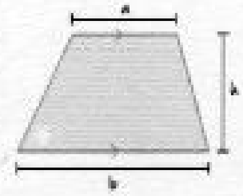
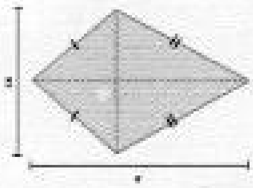
~~100 − 64 = √36~~ = 6

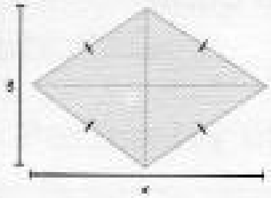
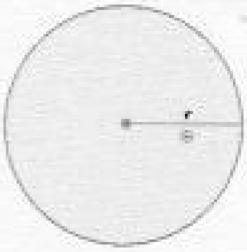
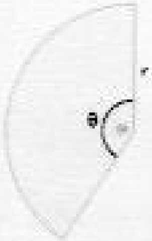
10² − 8² = 100 − 64 = √36 = 6

$\frac{2\pi(6)\times 2(6)}{4}$ = 2×6×6 = $\frac{72\pi}{4}$ $\frac{36\pi}{2}$

Section F: Surface Area of Prism, Cylinder, Pyramid and Cone

Active Recall: Area of Triangle and Quadrilaterals

Shapes	Names	Area Formula
	Square	l^2
	Rectangle	$l \times w$
	Parallelogram	$b \times h$
	Triangle	$\frac{b \times h}{2}$
	Trapezium	$\frac{h(a+b)}{2}$
	Kite	$\frac{1}{2}(x)(y)$

	Rhombus	$\frac{1}{2}(x)(y)$
	Circle	πr^2
	Sector	$\frac{\theta}{360} \times \pi r^2$

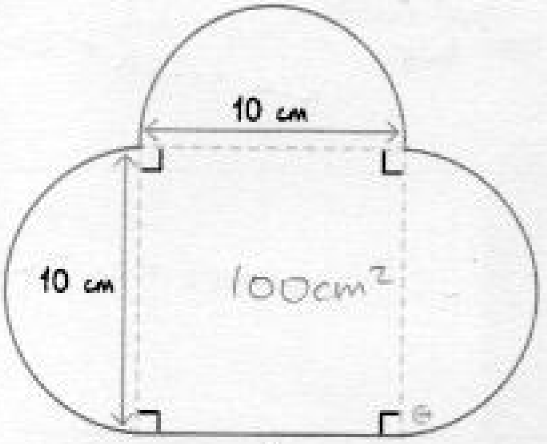
Space for Personal Notes

Rhombus = kite

Question 18

Work out the area of each of these shapes.

a. Correct to one decimal place.



$3\left(\frac{1}{2} \times \pi \cancel{(10)^2}\right) + 100$ $(5)^2$

$3\left(\frac{25\pi}{2}\right) + 100$

$\cancel{\frac{300\pi}{2} = 150\pi}$

~~150 × 3.14 circles = 20.2cm²~~

~~150 × 314 = 1570 + 4500 = 2020~~

~~= 120.2cm²~~

$3\left(\frac{25\pi}{2}\right)$ $25 \times 3 = 75$

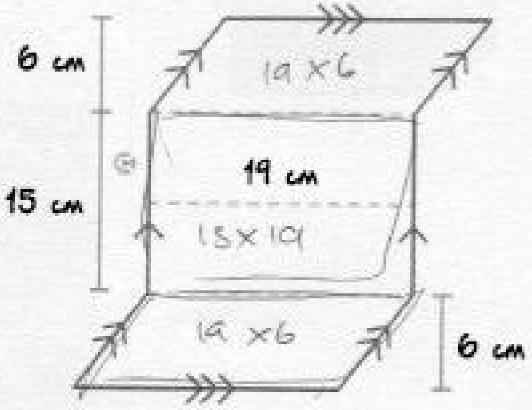
$\frac{75\pi}{2}$

$314 \times 75 = 1570 + 21980 = 23550$

$= 175.30cm^2 + 100$

$275.30cm^2$

b.



$(15 \times 19) = 285cm$

$15 \times 19 = 135 + 150 = 285$

19×6

$19 \times 6 = 114$

$2(114)$

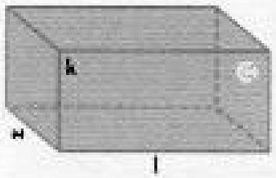
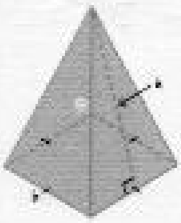
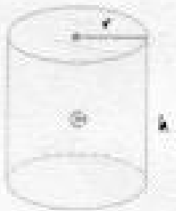
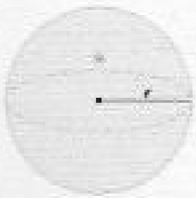
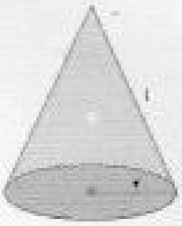
228

$285 + 228$

$= 285 + 228 = 513$

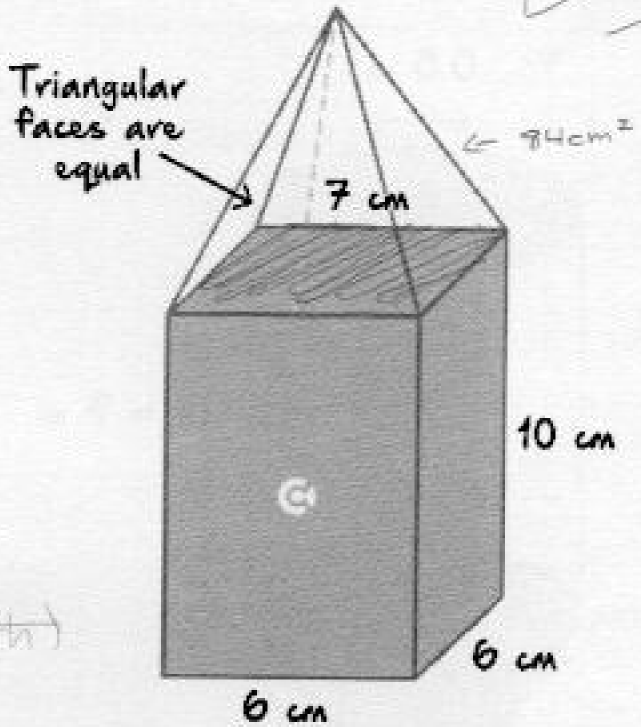
$= 513cm^2$ ✓

Surface Area of Figures

Shapes	Names	Area Formula
	Rectangular Prism	$2lw + 2lh + 2hw$
	Triangular Prism	$bh + al + bl + cl$
	Square-based Pyramid	$b^2 + \cancel{b}\ 2bh$
	Cylinder	$2\pi r^2 + 2\pi rh$
	Sphere	$4\pi r^2$
	Cone	$\pi r(r+l)$

Question 19

Calculate the surface area of the following shape.



$\Delta = \cancel{(6)^2 + 2(6)(h)}$

$2(6)(7)$

$12 \times 7 = 84 \text{ cm}^2$

$4(10 \times 6) = 240 \text{ cm}^2$

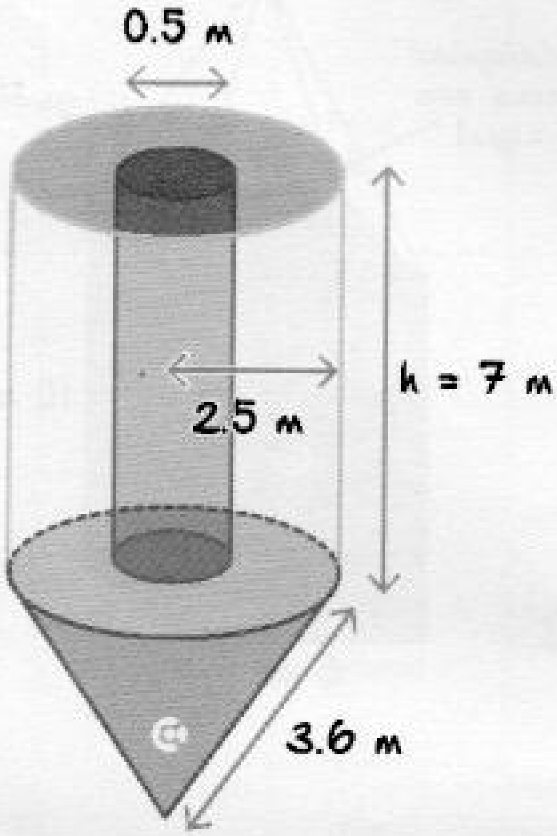
$6^2 = 36 \text{ cm}^2$

$$\begin{array}{r} 36 \\ 84 \\ +\ 240 \\ \hline 360 \end{array}$$

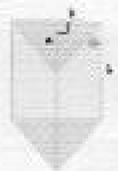
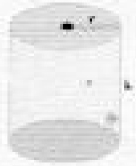
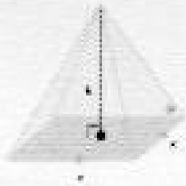
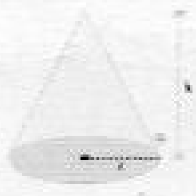
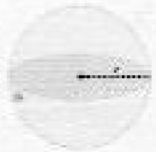
$= 360 \text{ cm}^2$

Question 20 Extension.

Find the total surface area of the given combined solid object having an inner cylindrical hole with a diameter 0.5 m. Give your answer correct to two decimal places.



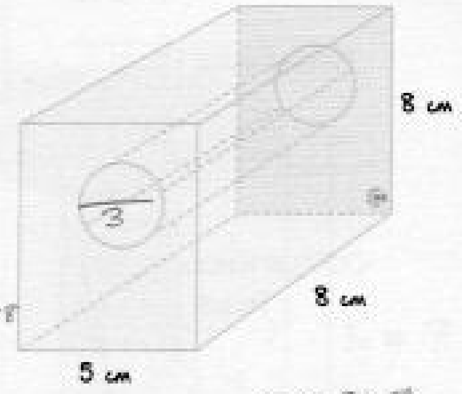
Volume of Figures

Shapes	Name	Volume Formula
	Rectangular Prism	$l \times w \times h$
	Oblique Rectangular Prism	$x^2 \times h$
	Triangular Prism	$\frac{1}{2} \times a \times b \times h$
	Cylinder	$\pi r^2 h$
	Right Square Pyramid	$\frac{1}{3} x^2 h$
	Right Cone	$\frac{1}{3} \pi r^2 h$
	Sphere	$\frac{4}{2} \pi r^3$

Question 21

The diagram shows a steel block that has had a hole drilled in it. The diameter of the hole is 3 cm.

Calculate the volume of this solid, giving your answer correct to two decimal places.



7065.0 4+4
× 8
565200
56.52

$443.12 cm^3$

$= \pi(1.5)^2 \times 8$

$(1.5)^2 = 2.25 \times 3.14 \times 8$

64
× 5
500 2+3

500 −

15
× 15
75
+150
225 2+6

2.25
× 3.14
900
2250
67500
7.0650 2+8 1+8 1+6

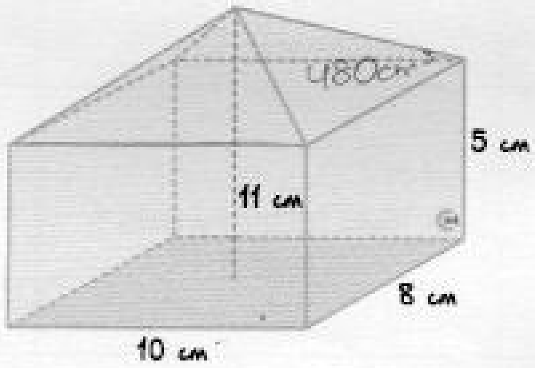
5 × 8 × 8

64 × 5

500 − 56.52
500.00
− 56.52
443.12

Question 22

Shown is a solid that is made of a pyramid and a cuboid.



Calculate the volume of the solid.

$\frac{1}{3}x^2h$

$80 \times 6 = \frac{480 cm^3}{3} = 160 cm^3$

$=$

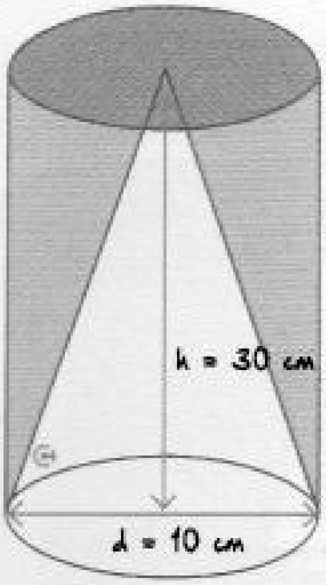
$10 \times 8 \times 5$

$80 \times 5 = 400 +$

$560 cm^3$

Question 23

A solid block of wood is shaped like a cylinder with a height of 30 cm and a diameter of 10 cm. A conical hole, with the same base and height as the cylinder, is drilled out from the centre of the block. Find the volume of the remaining wood, correct to one decimal place.



25 × 30

25
× 30
00
750

1+6

250
3)750
− 6
150

250 2+6
× 314
1000
+ 2500
3300

~~33.00~~ cm^3

~~$\pi(5)^2$~~

$\frac{1}{3}\pi r^2 h$

$\frac{1}{3}\pi(5)^2(30)$

$\frac{750\pi}{3}$

250π

$750\pi - 250\pi$

$= 500\pi\ cm^3$

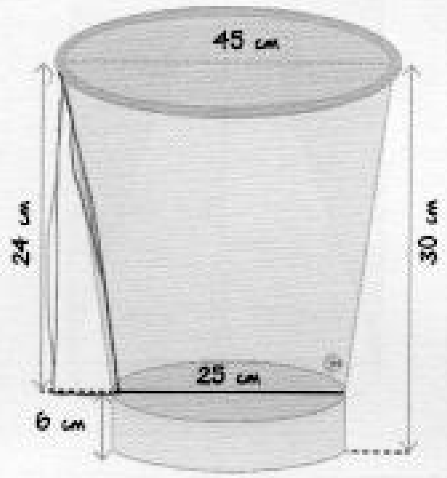
500
× 314
2000
5000
+150000
1570.00

1570 cm^3

Question 24 Extension.

A sheet-metal pail is a frustum of a cone, open at the top.

- Top circular rim diameter = 45 cm
- Base circular diameter = 25 cm
- Vertical depth = 24 cm



Give your answer correct to the nearest integer.

a. Find the total surface area of metal used to make the pail.

$24^2 + 6^2 =$

b. Find the capacity (volume of water it can hold) in cm^3.