CONTOUREDUCATION

Website: contoureducation.com.au | **Phone:** 1800 888 300
Email: hello@contoureducation.com.au

Year 10 Mathematics
AOS 9 Revision [10.4]
Workbook

Outline:

Functions and Transformations — Pg 2 - 9 ▶ Correspondence ▶ Domain and Range	Gallery of Graphs — Pg 10 - 24 ▶ Square Root Functions ▶ Hyperbola ▶ Circles

Section A: Functions and Transformations

Sub-Section: Correspondence

Functions

$$y = f(x)$$

— ~~many~~ many to one
one to one

- Functions are relations that make one y-value at any given x-value.

Vertical Line Test

Every function only intersects a vertical line once.

Space for Personal Notes

Question 1

For each table of values below, decide whether the relation is a function.

a.

x	y
0	−2
1	0
2	4
3	8

function

one to one

YES

b.

x	y
−3	0
−1	2
1	2
3	0

function

many to one

YES

c.

x	y
−1	5
−1	6
0	7
1	8

relation

one to many ×

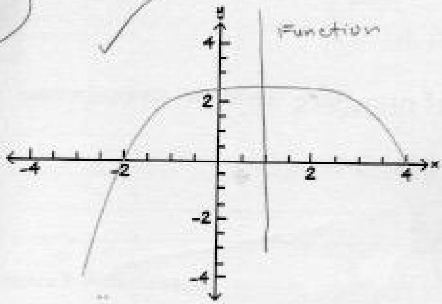
NO.

Question 2

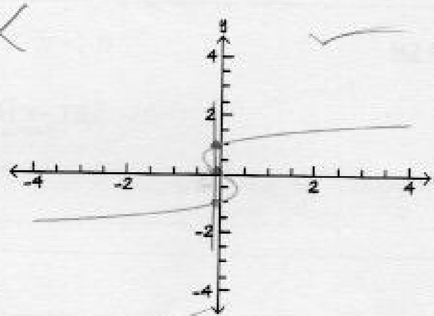
Identify the functions from the following:

W.

Function

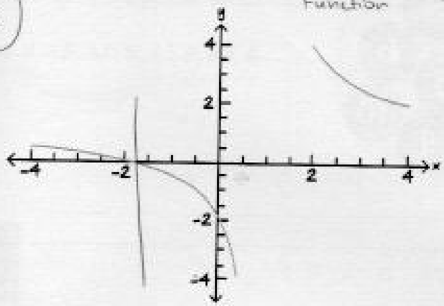


~~X.~~



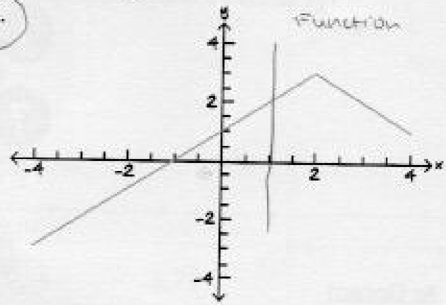
y.

Function



Z.

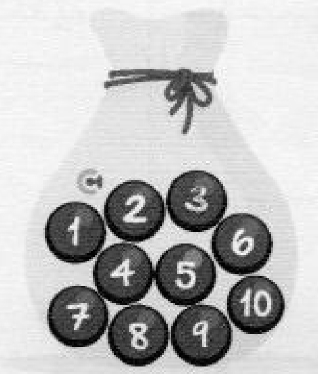
Function



Sub-Section: Domain and Range

A Set

Set → Bunch of numbers



An Element

A number in the set.

Set Operators

- Intersection: "AND".

$$A \cap B = \text{What values are in set } A \text{ AND in set } B.$$

- Union: "OR".

$$A \cup B = \text{What values are in set } A \text{ OR in set } B.$$

- Set difference: "except".

$$A \backslash B = \text{What values are in set } A\text{, except those also in set } B.$$

Definition

Interval Notation

- Parentheses (non-inclusive):

~~element of of~~ x is an element of a (min) to b (max)
* = and b not included

$$\boxed{x} \in (a, b) \Rightarrow a < x < b$$

- Square brackets [inclusive]:

x is an element of a (min) to b (max)

$$x \in [a, b] \Rightarrow a \leq x \leq b$$

* a and b included

Question 3 Walkthrough.

Let $A = \{x: x \text{ is a perfect square less than } 30\}$, and $B = \{x: x \text{ is a positive multiple of 3 less than } 30\}$.

Find $A \cup B, A \cap B, A \setminus B, B \setminus A$.

$A = \{1, 4, 9, 16, 25\}$

$B = \{3, 6, 9, 12, 15, 18, 21, 24, 27\}$

* do not repeat

$A \cup B = \{1, 3, 6, 12, 15, 16, 18, 21, 24, 25, 27\}$

$A \cap B = \{9, \}$

* A values except and 'A∩B' value

$A \setminus B = \{1, 4, 16, 25\} \neq \{9\}$

$B \setminus A = \{3, 6, 12, 15, 18, 21, 24, 27\} \neq \{9\}$

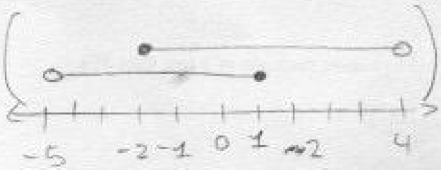
Question 4 Walkthrough.

Find the following sets:

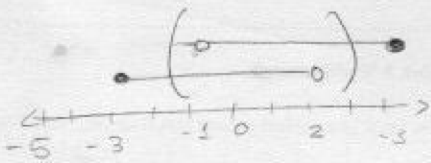
a. $(-5,1] \cup [-2,4)$

Union

$(-5,4)$



b. $[-3,2) \cap (-1,5]$

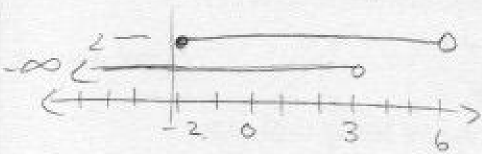


$(-1, 2)$

Question 5

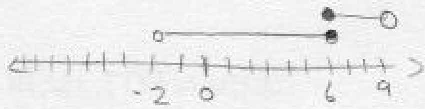
Find the following sets:

a. $(-\infty,3) \setminus [-2,6)$



$(-\infty, -2)$

b. $(-2,6] \cap [6,9)$ one member → { }



~~(-2, 9)~~ ~~[6,6]~~

$\{6\}$

Domain and Range

- Domain: All suitable x-values.
- Range: All suitable y-values.

Question 6 Walkthrough.

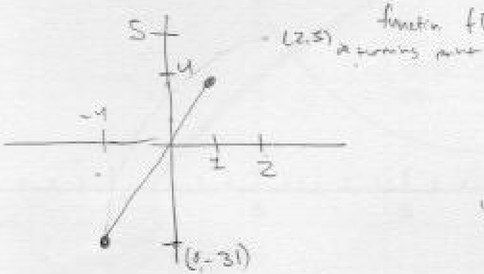
Consider a quadratic function $f: [-4, 1] \to R, f(x) = -(x-2)^2 + 5$, written in functional notation.

What is the domain, range and equation of the function?

function (f) has a domain of -4 to 1

always a domain.

(2, 5)



function $f(x) = -(x-2)^2 + 5$

↳ Quadratic (parabola)

~~cannot calc range by~~ ??

sub domain

you have to graph it.

I thought it was quadratic??? (due to turning point).

$f(-4) = -31$

$-(-4-2)^2 + 5$

$-(-6)^2$

$-36 + 5 = -31$

$[-4, -31)$

$f(1) = 4$

Domain $= [-4, 1]$

Range $= [-31, 4]$

we just subbed it tho ??

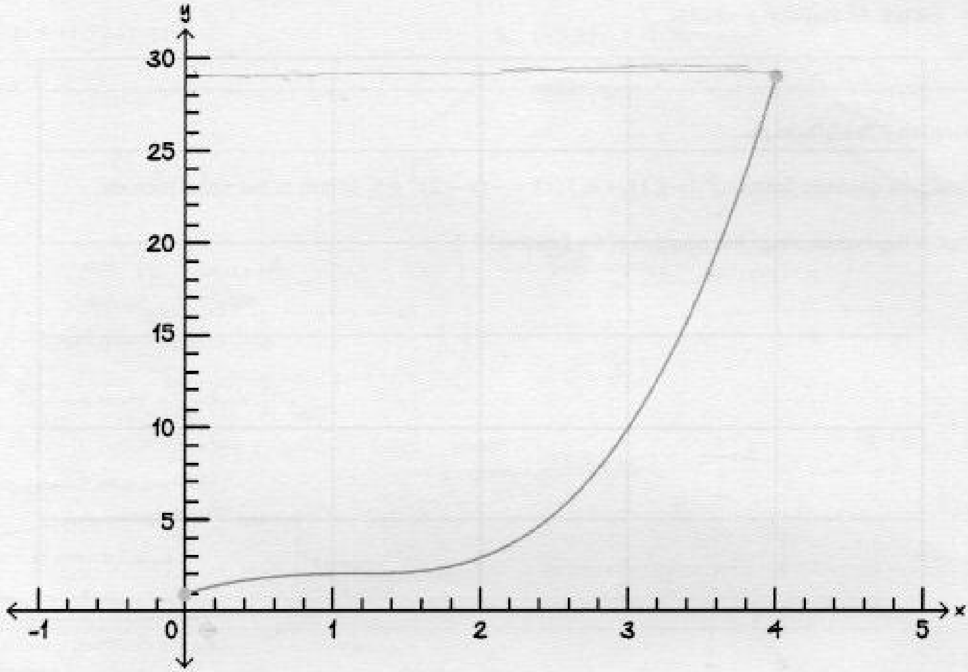
$f(-4) = -31 \quad = (-4, -31)$

$f(1) = 4 \quad (1, 4)$

Range $= [-31, 4]$

Question 7

From the graph, state the domain and range.



Domain = [0, 4]

Range = [1, 29]

will be same

Section B: Gallery of Graphs

Sub-Section: Square Root Functions

Square Root Functions

- General form:

dilation

outside square roots affects y value

$$y = a\sqrt{b(x - h)} + k$$

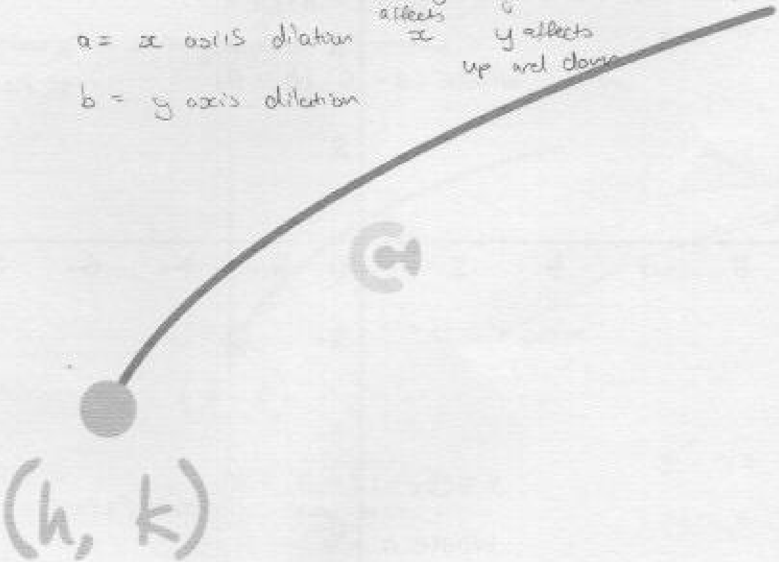
inside square root affect x value

affects x

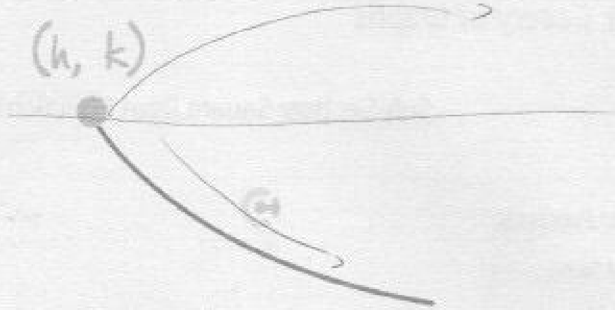
y affects up and down

a = x axis dilation

b = y axis dilation



Where: a = positive ($a > 0$), b = positive ($b \geq 0$)

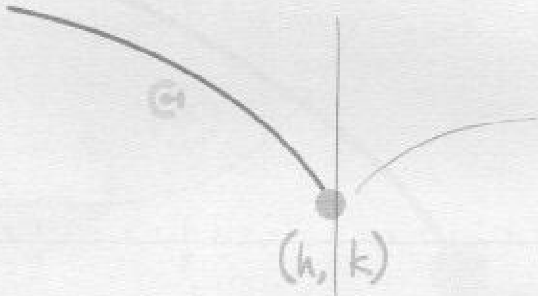


> reflect x axis

$$y = \boxed{-}\sqrt{\left(b(x-h)\right)} + k$$

a is negative = x axis reflection

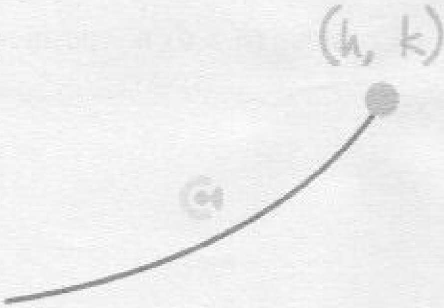
Where: $(a < 0), (b > 0)$



$$y = +\sqrt{-(x-h)} + k$$

a positive y axis reflection

Where: $a > 0, b < 0$



$$y = -\sqrt{-(x-h)} + k$$

Where: $a < 0, b < 0$

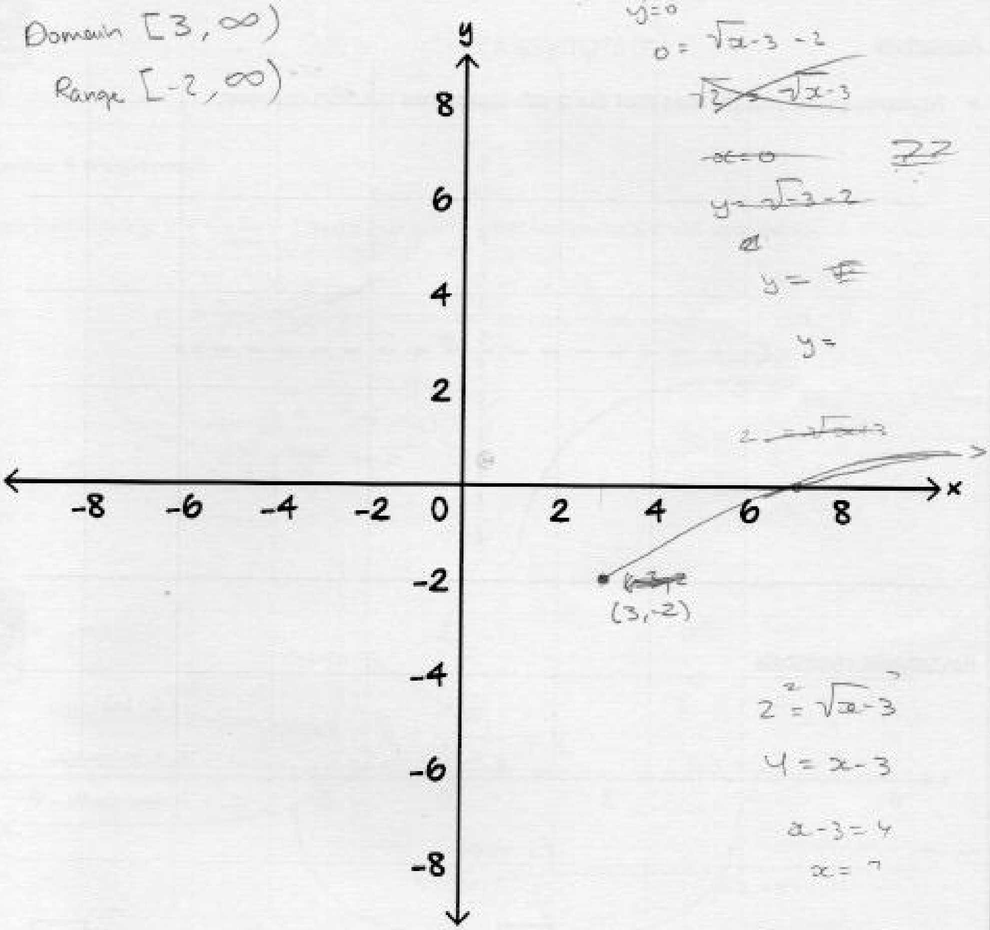
Question 8

$(3, -2)$

Identify the domain and range of $y = \sqrt{x-3} - 2$. Also, sketch the graph on the axes below, label all key points.

Domain $[3, \infty)$

Range $[-2, \infty)$



$y = 0$

$0 = \sqrt{x-3} - 2$

$\cancel{\sqrt{2} = \sqrt{x-3}}$

~~$x = 0$~~

??

~~$y = \sqrt{-3} - 2$~~

$y =$

$y =$

~~$2 = \sqrt{x+3}$~~

$2^2 = \sqrt{x-3}^2$

$4 = x - 3$

$x - 3 = 4$

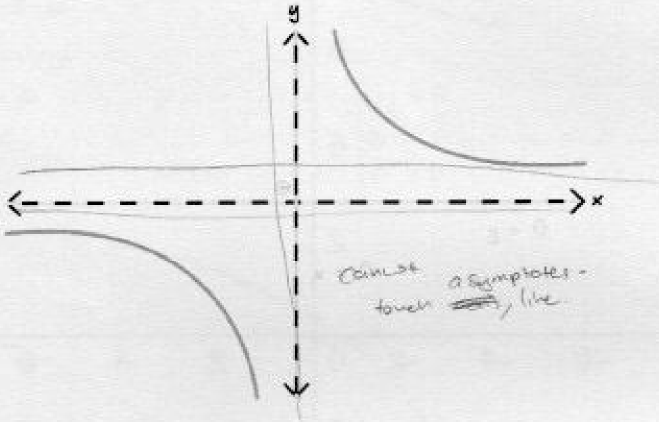
$x = 7$

Space for Personal Notes

Sub-Section: Hyperbola

Asymptote

- Asymptotes are straight lines that the graph approaches but does not meet.

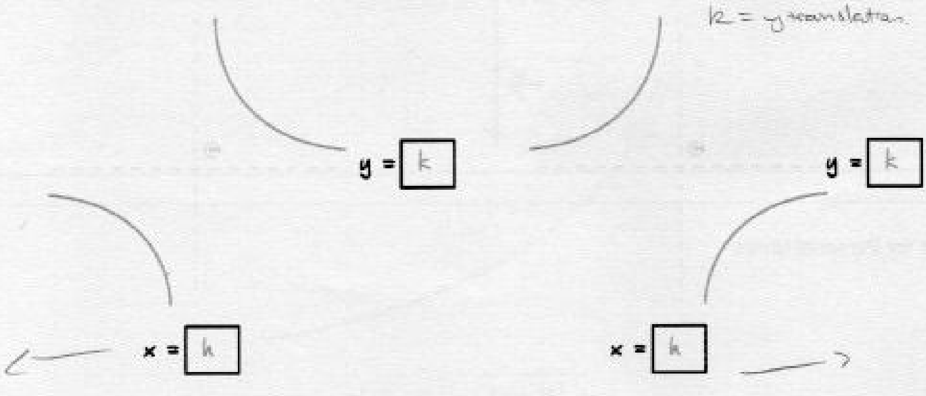


Rectangular Hyperbola

dilation

$$y = \frac{a}{x - h} + k$$

translation (h, k)
h = x translation
k = y translation



$a > 0$
a is positive

$a < 0$
a is negative

Finding Asymptotes

→ translation (h, k) $x = h$ $y = k$

Sub $x = \infty \rightarrow$ (y asymptote)

Sub $y = \infty \rightarrow$ (x asymptote)

Question 9 Walkthrough.

Graph the following: $y = -\frac{1}{\boxed{2x+3}} - 1$ on the axes below. Label the intercept(s) and asymptote(s).

$\rightarrow 2x+3=0$

1. $\boxed{x = -\frac{3}{2}}$

$y = -\frac{1}{2x+3} - 1$

$y = \frac{1}{2(x+\frac{3}{2})} - 1$

2. $\boxed{x = -\frac{3}{2}}$

$y = k$

$y = -1$

$x = 0$

$y = \frac{-1}{3} - 1$

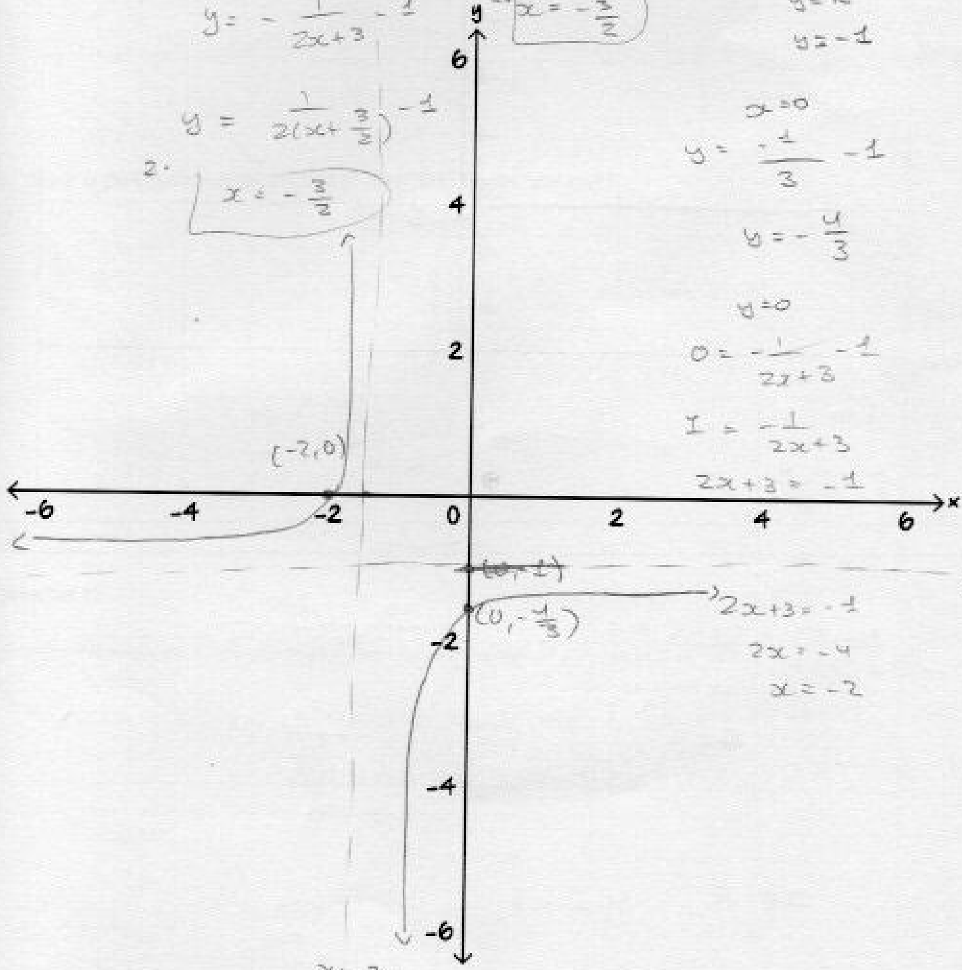
$y = -\frac{4}{3}$

$y = 0$

$0 = -\frac{1}{2x+3} - 1$

$1 = -\frac{1}{2x+3}$

$2x+3 = -1$



$2x+3 = -1$

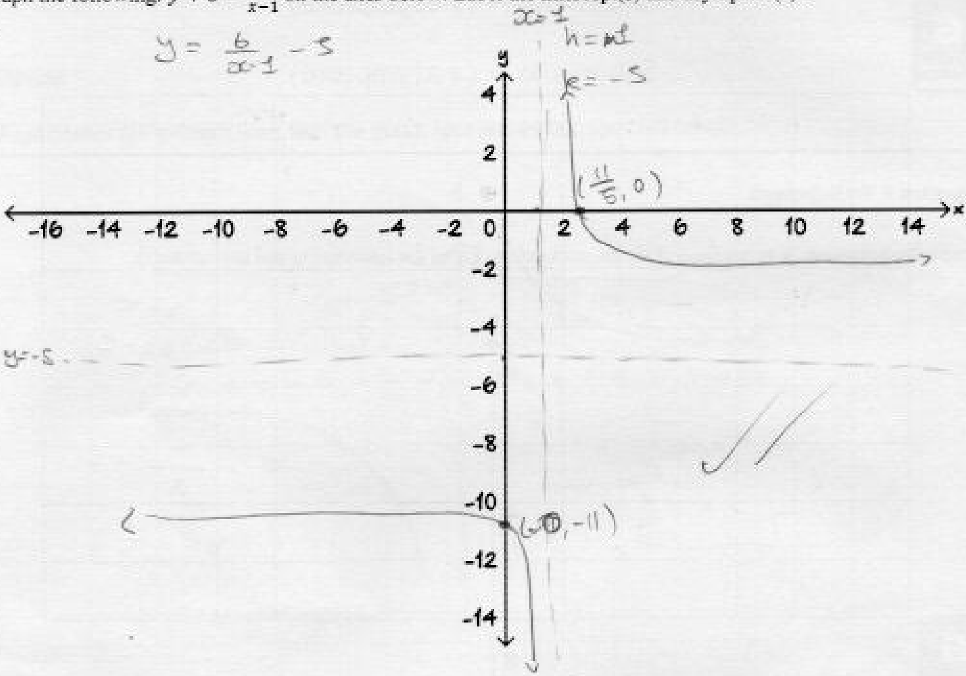
$2x = -4$

$x = -2$

Question 10 a is postie so 1, 3 quadrant.

Graph the following: $y + 5 = \frac{6}{x-1}$ on the axes below. Label the intercept(s) and asymptote(s).

$y = \frac{6}{x-1} - 5$



$y=0$

$0 = \frac{6}{x-1} - 5$

$5 = \frac{6}{x-1}$

$5(x-1) = 5x-5 = 6$

$5x = 11$

$x = \frac{11}{5} = 5\overline{)11}^{\,2}$

$x=0$

$-6-5=-11$

$y = \frac{6}{-1} - 5$

$\cancel{\frac{6}{-1} - \frac{5}{1}}$

$y = -\frac{1}{5}$

$x = \frac{11}{5}, \quad y = -11$

↳ because $x>0$

Question 11 Walkthrough.

A computer animation shows a laser beam following the path of the hyperbola $y = \frac{1}{x}$ for $x > 0$. To create the next scene, the animator applies two transformations: a reflection in the x-axis, followed by a translation of 3 units up.

a. Write the equation of the transformed path of the laser beam.

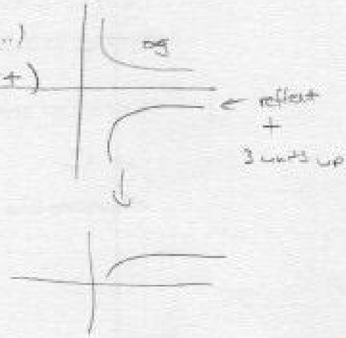
a -> reflection + dilation of x axis

↳ Size of a = dilation (1, 2, 3...)

Sign of a = reflection (−, ~~+~~ +)

k = ↳ translation UP = y = k

$y = -\frac{1}{x} + 3$



$y = -\frac{1}{x} + 3$

b. What is the equation of the horizontal asymptote for the new path?

↳ y asymptope = k

k = 3

Question 12

Describe the sequence of transformations that maps the graph of $y = \sqrt{x}$ to $y = -\sqrt{x} + 3$.

1. reflection in the x axis ($\sqrt{x} \to -\sqrt{x}$)
2. translation ~~up~~ 3 units up ($\sqrt{x} \to -\sqrt{x} + \underline{3}$)

Question 13 Extension.

The path of a drone flying away from an operator is modelled by the function $D(t) = 3\sqrt{t}$, where D is the distance in metres and t is the time in seconds, for $0 \leq t \leq 16$.

$t = [0, 16]$

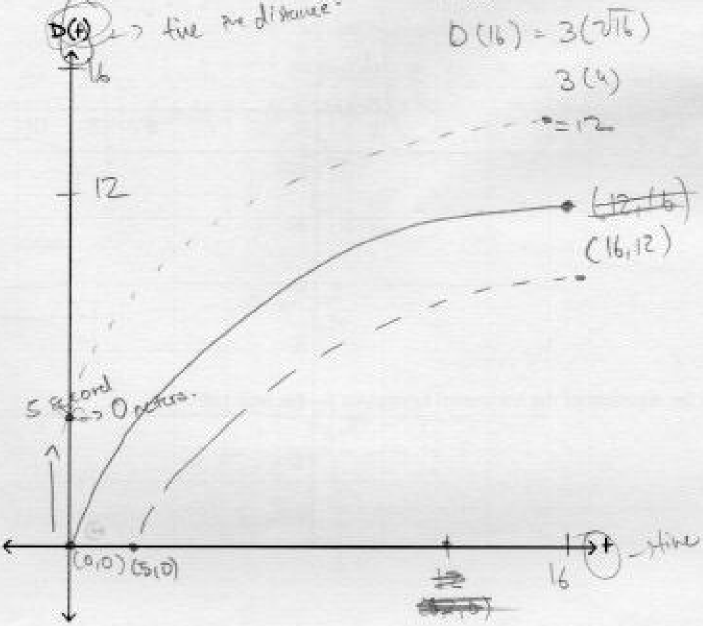
a. Sketch the path of the drone, labelling the start and endpoints.

$D(0) = (0)$

$D(16) = 3(\sqrt{16})$

$3(4)$

$= 12$



b. State the range of the drone's distance from the operator.

Range = $[0, 12]$ → bounce included in $[0, 16]$

↳ Range: $D(t) \in [0, 12]$

c. The drone is reprogrammed to start 5 seconds later but follow the same-shaped path. Describe the transformation required and write the new function for the path. (2 marks)

+5 $\quad$ +5 = t so affects

$y = 3\sqrt{t}\ \cancel{+5}$

↳ ~~translation in 5 units~~

$y = 3\sqrt{t-5}$

because the = positive 5.

Question 14 Extension.

A new skate park design includes a semi-circular bowl and an entry ramp.

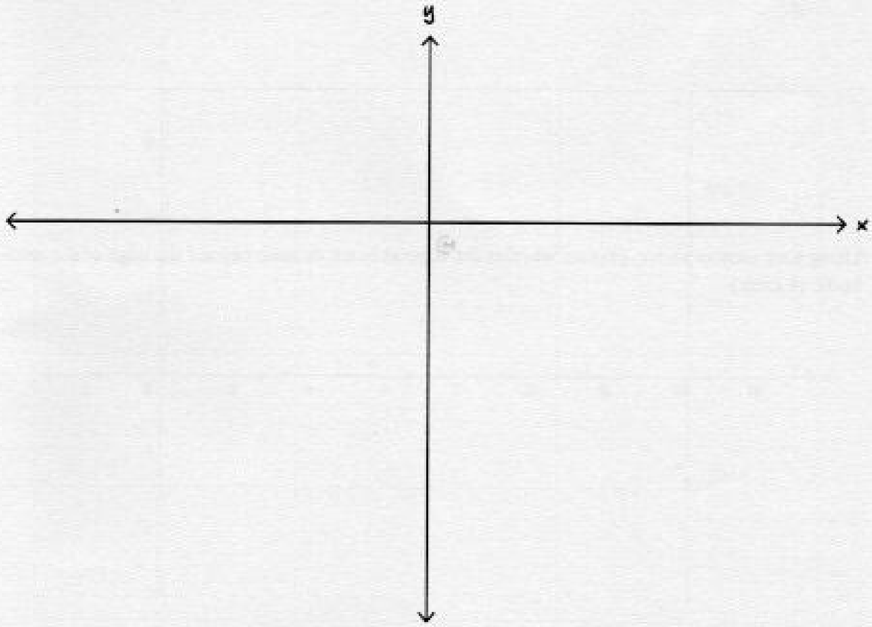
The main bowl is a semicircle below the x-axis. It is centred at the origin and has a diameter of 8 metres.

a. The Bowl

i. Write the equation of the full circle that the bowl is part of. (1 mark)

$y =$

ii. Sketch the semicircular bowl on a set of axes, labelling the coordinates of the intercepts. (3 marks)



iii. State the domain and range for the relation that describes the bowl. (1 mark)

b. The Support Beam

A straight metal support beam runs along the line with the equation $y = x - 4$.

i. Find the coordinates of the points where the support beam intersects the full circular structure of the bowl.

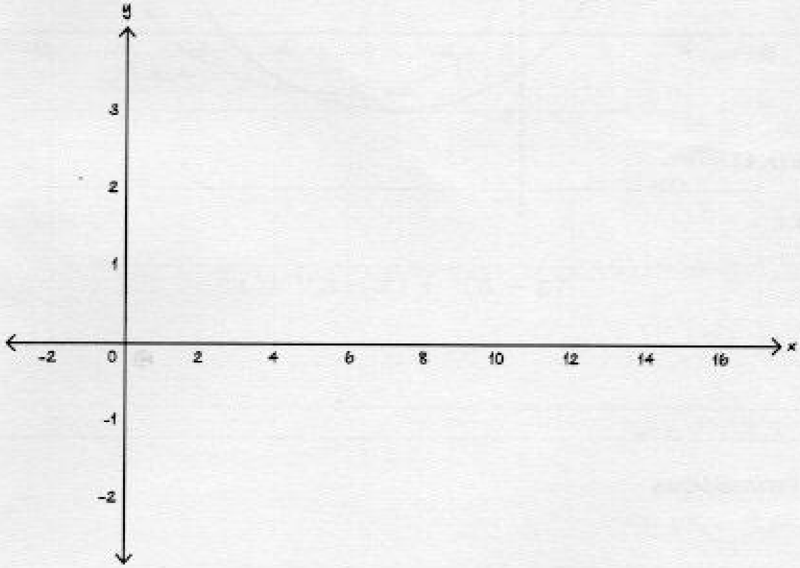
ii. Using your answer above, explain whether the support beam extends beyond the edge of the semi-circular bowl. (1 mark)

c. The Entry Ramp

An entry ramp to the park starts at ground level at the point $(6, 0)$ and curves upwards. Its shape is a transformation of $y = \sqrt{x}$ and it passes through the point $(10, 2)$.

i. Determine the equation of the ramp in the form $y = a\sqrt{x - h} + k$. (2 marks)

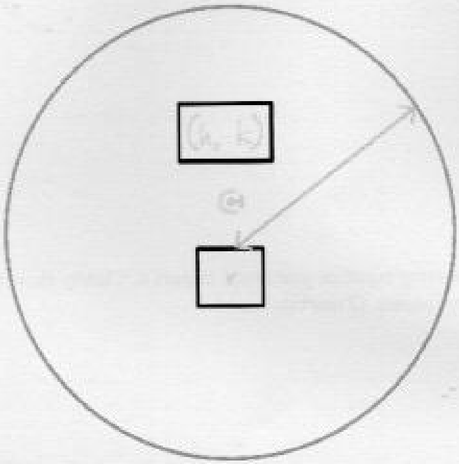
ii. Sketch the graph of the ramp equation you found in **part i.**, Clearly show the starting point and the point through which the curve passes. (2 marks)



iii. Using the equation of the ramp, calculate the height of the ramp when the horizontal distance is 15 metres. (1 mark)

Sub-Section: Circles

Circles



- Centre: (h, k)
- Radius: r

$$(x-h)^2+(y-k)^2=r^2$$

» Where $r > 0$

Space for Personal Notes

Question 15 Walkthrough.

Sketch the graph of $(x+5)^2 + (y-3)^2 = 25$ on the axes below and label all key points.

~~Cent~~

$(x-(-5))^2 = (5,3)$

y-intercept

Centre

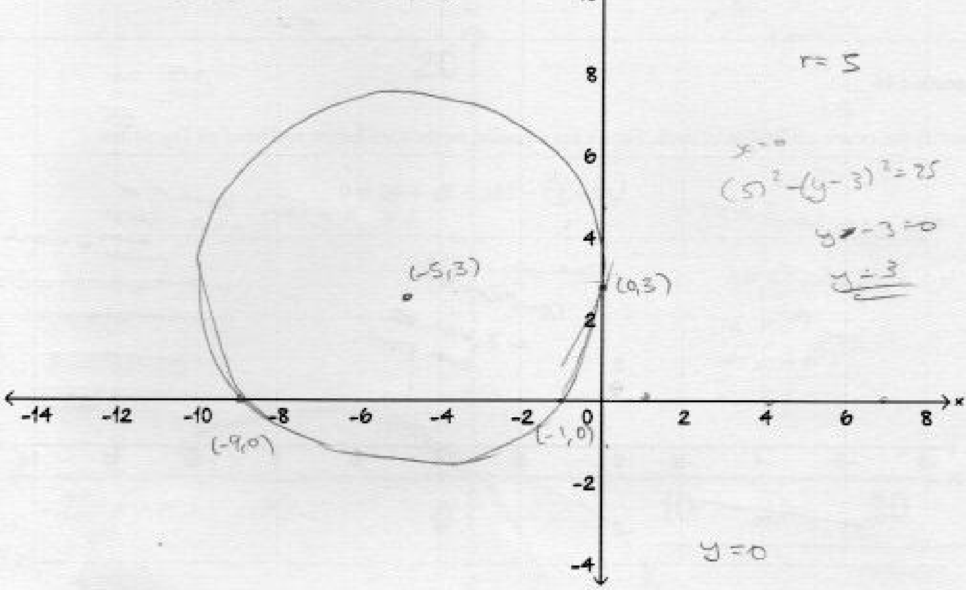
$r = 5$

$x = 0$

$(5)^2 - (y-3)^2 = 25$

$y - 3 = 0$

$y = 3$



$y = 0$

$(x+5)^2 + (0-3)^2 = 25$

$(x+5)^2 + 9 = 25$

$(x+5)^2 = 16$

$x + 5 = \pm 4$

$x = -1, x = -9$

Active Recall: Complete the Square

$$x^2 + bx + c = \left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c$$

Question 16

$49 - 25 = 24$

Identify the centre and radius of each. Sketch the equation on the axes below and label all key points.

$x = 0$

$$x^2 + y^2 - 14x + 8y + 40 = 0$$

No 'xy' term so no cross terms makes it a circle.

have to complete the square each part.

~~$(-7)^2 + (y+4)^2 = 25$~~

$49 + (y+4)^2 = 25$

$(y+4)^2 = 24$

~~$y+4 = \sqrt{24}$~~

$y + 4 = 2\sqrt{6}$

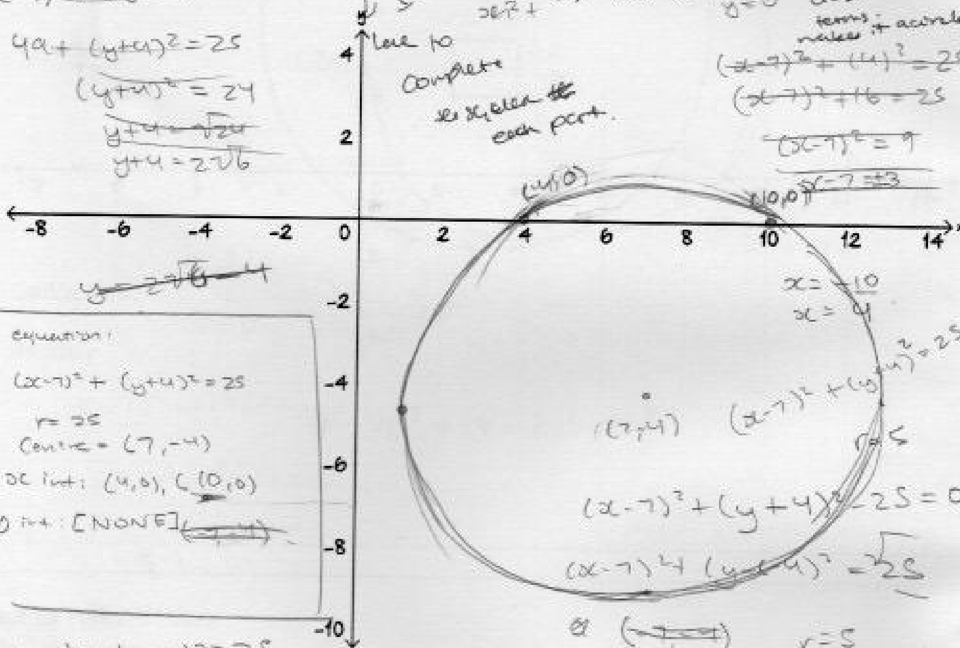
$y = 0$

~~$(x-7)^2 + (4)^2 = 25$~~

~~$(x-7)^2 + 16 = 25$~~

$(x-7)^2 = 9$

$x - 7 = \pm 3$



~~$y = 2\sqrt{6} - 4$~~

$x = 10$

$x = 4$

$(x-7)^2 + (0+4)^2 = 25$

equation:

$(x-7)^2 + (y+4)^2 = 25$

$r = 25$

Centre $= (7, -4)$

x int: $(4,0)$, $(10,0)$

y int: [NONE]

$(x-7)^2 + (y+4)^2 - 25 = 0$

$(x-7)^2 + (y+4)^2 = 25$

$r = 5$

$(7,4)$

$(x-7)^2 + (y+4)^2 = 25$

$(x-7)^2 + (4)^2 = 25$

$\left(\frac{b}{2}\right)^2 = 16$

$x - 7 = 9$ $\quad x = 10$

$x - 7 = \pm 3$ $\quad x = 4$

$-9 - 16 =$

-25

$x^2 - 14x + y^2 + 8y + 40$

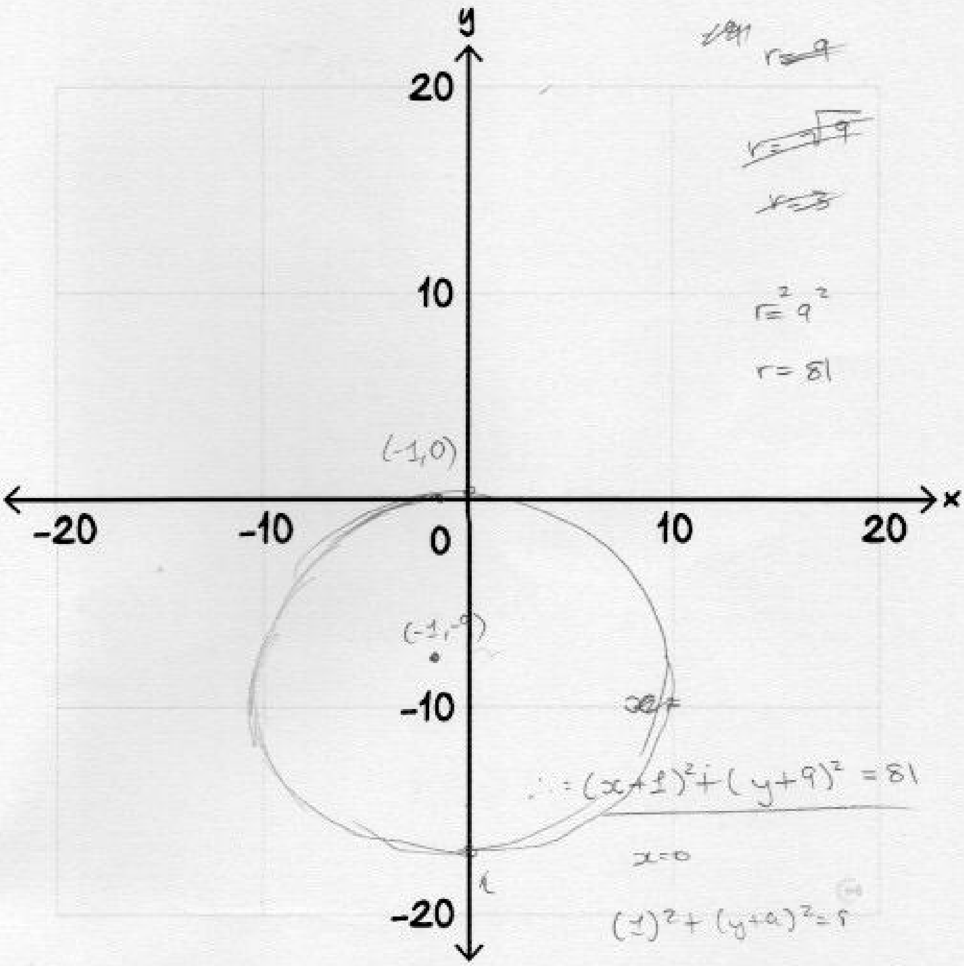
$x^2 - 14x + 49 + y^2 + 8y + 40 - 49 = 0$

$\left(\frac{14}{2}\right)^2 = 49$

$(x-7)^2 + (y+4)^2 + 40 - 49 - 16 = 0$

Question 17 Extension.

A circle centred at $(-1, -9)$ has an x-intercept at $(-1, 0)$. Find the exact radius of the circle. Hence, state the equation of circle and sketch the equation on the axes below and label all key points.



~~r=9~~

~~r=√9~~

~~r=3~~

$r^2 = 9^2$

$r = 81$

$\therefore (x+1)^2 + (y+9)^2 = 81$

$x=0$

$(1)^2 + (y+9)^2 = 81$

$1 + (y+9)^2 = 81$

$(y+9)^2 = \sqrt{80} =$

$y+9 = \pm\sqrt{80}$

$y = \pm\sqrt{80} - 9$

$\sqrt{4 \times 20}$

$\sqrt{16 \times 5}$

$\pm 4\sqrt{5} - 9$

y int $= \pm 4\sqrt{5} - 9$