Year 10 Mathematics
AOS 8 Revision [10.3]
Mock CAT 1 - Version B

Name: _____ **Date:** _____

Instructions:

- Answer all questions in the spaces provided.
- Show all working clearly.
- Calculators are permitted.
- Total marks: 50
- Time allowed: 60 minutes

Section A: Short Answer Questions (34 Marks)

Question 1 [1 mark]

State the x-intercepts of the graph of the function $y = (x + 3)(x - 1)$.

Question 2 [1 mark]

How many different solutions does $(x - 1)(x + 4)(x - 1)(x + 2) = 0$ have?

Question 3 [1 mark]

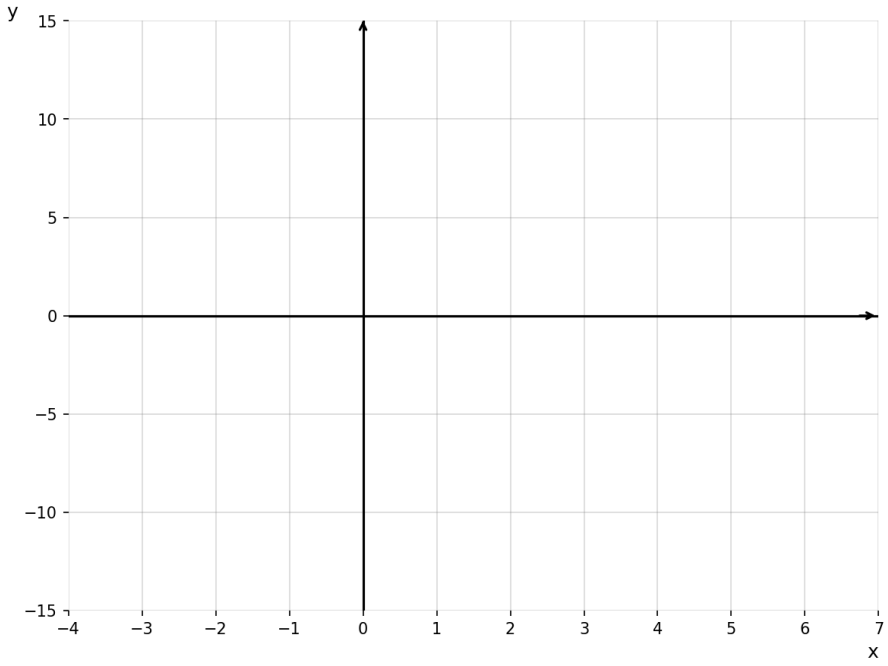
Show that $(x + 3)$ is a factor of $P(x) = x^3 + 2x^2 - 7x + 12$.

Question 4 [1 mark]

Solve the equation $(2x + 3)(x - 4)(x + 1) = 0$.

Question 5 [3 marks]

Sketch the graph of $y = (x-1)(x+2)(x-5)$, clearly labelling all axis intercepts.



Question 6 [2 marks]

Use the remainder theorem to find the remainder when $P(x) = 3x^3 - 5x^2 + 2x - 7$ is divided by $(x-2)$.

Question 7 [3 marks]

Let $P(x) = x^3 + ax^2 + bx - 6$.

It is known that $P(x)$ is exactly divisible by $(x-1)$ and $(x+3)$. Using the Factor Theorem, determine the values of a and b.

Question 8 [3 marks]

A cubic polynomial has x-intercepts at $x = -3, 2, 4$ and a y-intercept at $y = -24$. Write a possible equation for this polynomial.

Question 9 [3 marks]

Let $A(x) = 5x^4 - 3x + 7$ and $B(x) = x^2(x + 1)^3$.

a. State the degree of x for $A(x)$ and for $B(x)$. (2 marks)

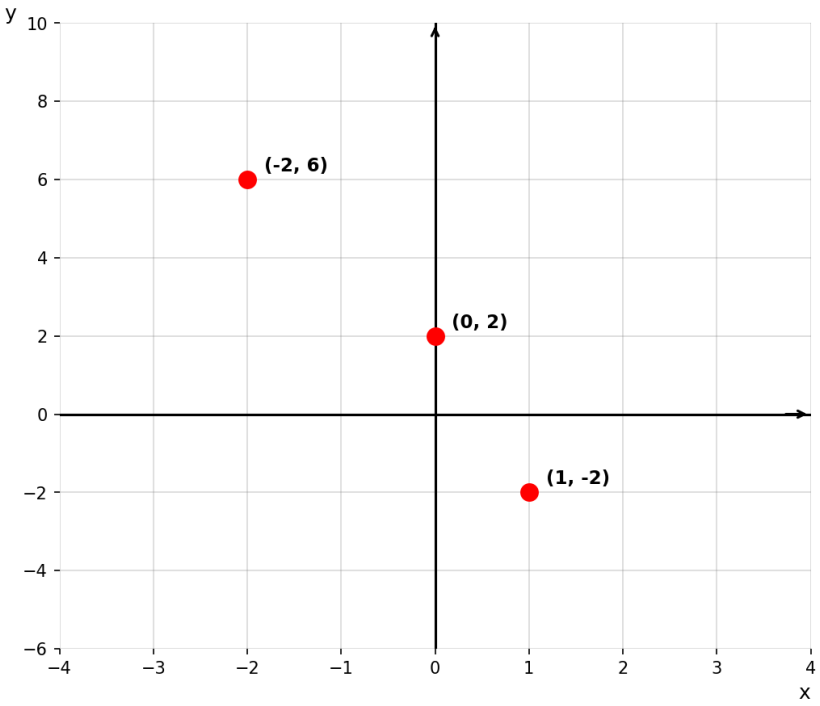
b. Using your answers from **part a.**, give the degree of the product $A(x)B(x)$. (1 mark)

Question 10 [3 marks]

The cubic $P(x) = x^3 + ax^2 + bx + c$ is shown below.

The curve passes through the points $(-2, 6)$, $(0, 2)$ and $(1, -2)$.

Find the values of a, b and c.


y
10
8
6
4
2
0
−2
−4
−6
−4
−3
−2
−1
0
1
2
3
4
x
(-2, 6)
(0, 2)
(1, -2)

Question 11 [3 marks]

The path of a ball thrown into the air is modelled by the polynomial $h(t) = -t(t-2)(t-6)$, where h is the height in metres above ground and t is the time in seconds.

a. At what times is the ball at ground level? (1 mark)

b. What is the height of the ball after 3 seconds? (1 mark)

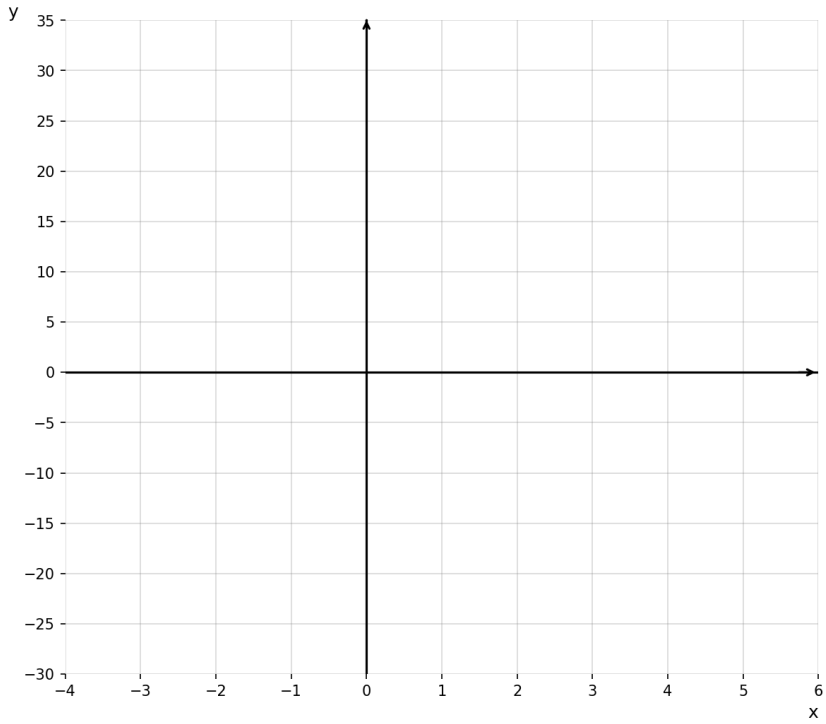
c. Is the ball travelling upwards or downwards between $t = 3$ and $t = 4$? (1 mark)

Question 12 [5 marks]

The vertical displacement of a cable car above ground (in metres) is modelled by the function $y = 3(x+2)(x-1)(x-4)$, where x is the horizontal distance (in metres) from the starting point.

a. Find the axis intercepts of the graph. (2 marks)

b. Sketch a rough graph of y showing the intercepts. (3 marks)


y
35
30
25
20
15
10
5
0
−5
−10
−15
−20
−25
−30
−4
−3
−2
−1
0
1
2
3
4
5
6
x

Question 13 [5 marks]

A rectangular swimming pool has a length that is 6 metres longer than its width. Let the width be x metres.

a. Write an expression for the area, $A(x)$, of the swimming pool. (1 mark)

b. The pool is to be filled to a uniform depth of $(x - 2)$ metres. Write an expression for the volume, $V(x)$, of water in the pool. (Hint: $V = A(x) \times \text{depth}$) (2 marks)

c. Find the volume of water if the width is 5 metres. (2 marks)

Section B: Extended Response Questions (16 Marks)

Question 14 [9 marks]

The cross-section of a valley is modelled by the polynomial $H(x) = \frac{1}{3}x(x-6)(x-10)$, where H is the height in metres (with negative values below ground level) and x is the horizontal distance in metres from a starting point.

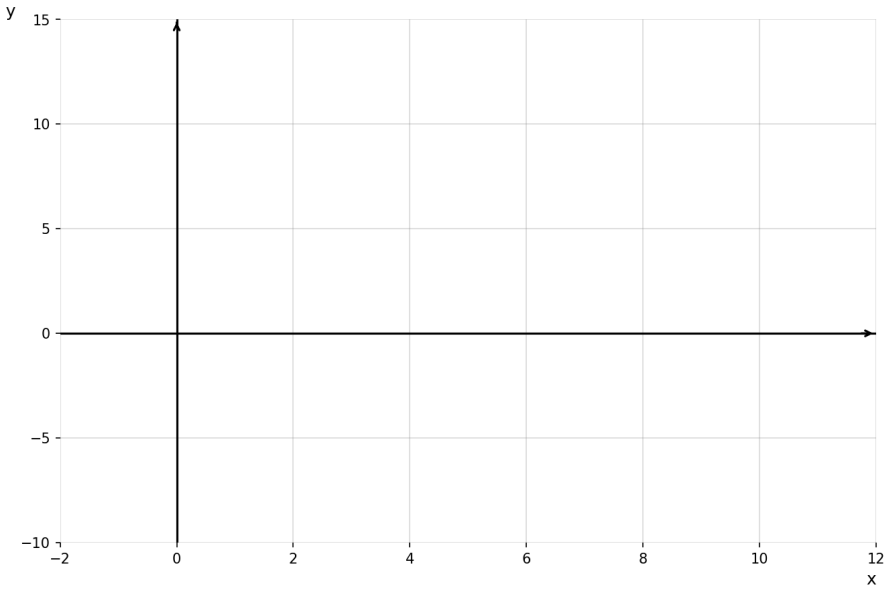
a. Find the height when $x = 3$ metres. (2 marks)

b. At what horizontal distances is the valley at ground level? (2 marks)

c. Expand the polynomial $H(x)$ into the form $ax^3 + bx^2 + cx + d$. (2 marks)

d. A straight road is modelled by the line $y = -3x + 18$. How many points does this road meet the cross-section of the valley? (No calculation required, just state the number of intersection points.) (1 mark)

e. Sketch the graph of the polynomial $H(x)$. (2 marks)



Question 15 [7 marks]

A food truck sells gourmet sandwiches. Due to bulk discounts, the price per sandwich decreases as more sandwiches are sold.

- Number of sandwiches sold: x
- Price per sandwich: $p(x) = 18 - x$ dollars (valid while $0 \leq x \leq 18$)
- Total cost: $C(x) = -x^3 + 6x^2 + 3x + 10$ dollars

a. Write an expression for the revenue $R(x)$ in terms of x. (Hint: Revenue = Units sold $\times$ price per unit) (1 mark)

b. Write an expression for the profit $P(x)$ in terms of x. (Hint: Profit = Revenue − Cost) (1 mark)

c. Factor the profit $P(x)$ completely into linear factors. (2 marks)

d. Find the break-even point(s) for the food truck (solve $P(x) = 0$). (1 mark)

e. For each break-even point, calculate the corresponding revenue and cost. (At break-even, revenue equals cost.) (2 marks)

— END OF TEST —