CONTOUREDUCATION

Website: contoureducation.com.au | **Phone:** 1800 888 300
Email: hello@contoureducation.com.au

Year 10 Mathematics
AOS 6 Revision [10.1]
Workbook YeWoo

Outline:

Two-Step Experiments and Venn Diagrams Pg 3 - 11
- Probability Fundamentals
- Multi-Stage Experiments
- Venn Diagram and Combining Events
- Apply the Addition Rule to Find Unknown Probabilities
- Two-Way Table

Conditional Probability and Independent Events Pg 12 - 17
- Conditional Probability
- Independent Events

Measure of Centre and Spread, Standard Deviation Pg 18 - 27
- Understanding Data
- Measure of Spread
- Standard Deviation
- z (standardised) score

Bivariate Data & Scatter Plots Pg 28 - 31
- Bivariate Data
- Line of Best Fit by Eye

Announcements

Link: https://bit.ly/Contour-Class-Announcements

Contour Resources

Core		Mastery	
Workbook + Test, CAT	Homework	Workbook	Bound Reference, Mock CAT, Exams
In Class	At Home	In Class	At Home

Subject Outline for AOS 10 - End of Year Exam Revision

In Class (Workbook + Test, CAT) | At Home (Homework)	In Class (Workshop)
☐ MA10 [10.1] - AOS 6 Revision	MA10 [10.1] - Mock CAT 3
☐ MA10 [10.2] - AOS 7 Revision	MA10 [10.2] - Mock CAT 3
☐ MA10 [10.3] - AOS 8 Revision	MA10 [10.3] - Mock CAT 3
☐ MA10 [10.4] - AOS 9 Revision	MA10 [10.4] - Mock CAT 3

Additional Resources

Mock CAT	Exam
☐ MA10 [10.1] - AOS 5 Revision 3 x Mock CATs	
☐ MA10 [10.2] - AOS 6 Revision 3 x Mock CATs	☐ MA10 [10.4] - EOY - Exam
☐ MA10 [10.3] - AOS 7 Revision 3 x Mock CATs	

Section A: Two-Step Experiments and Venn Diagrams

Sub-Section: Probability Fundamentals

Calculating Probabilities for Equally Likely Outcomes

- When there is a number of equally likely outcomes, the probability of a "successful" outcome can be calculated as:

$$\Pr(\text{success}) = \frac{\text{Number of successful outcomes}}{\text{Number of total outcomes}}$$

Question 1

Write all the sample spaces for the following experiments.

a. A random letter from the word "MAT" and a random digit from the set {2,4}.

	M	A	T
2	(M,2)	(A,2)	(T,A)
4			

b. Tossing a coin and rolling a standard six-sided die.

	1	2	3	4	5	6
H						
T						

Question 2 Additional.

A basket has 6 oranges and 4 apples. You select one fruit at random.

a. Consider an orange as *O* and an apple as *A*. What is the sample space?

$\{O, O, O, O, O, O, A, A, A, A\}$

or

$\{O, A\}$

b. What is the probability of selecting an orange?

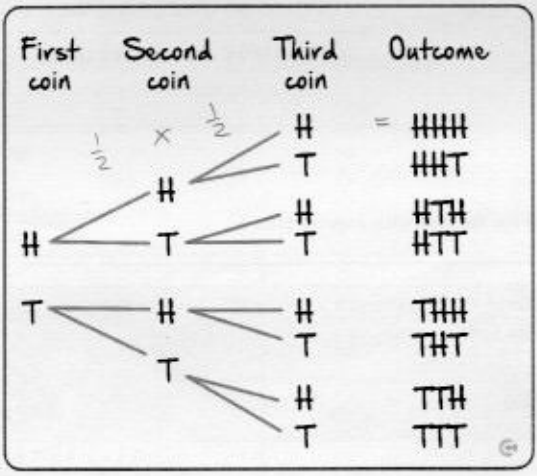
$\frac{6}{10} = \frac{3}{5}$

$\Pr(O) = \frac{3}{5}$

Sub-Section: Multi-Stage Experiments

Tree Diagram

- Useful for multiple sequence events.
- To calculate the probability of a sequence, we **multiply the probabilities along the relevant branches.**
- For instance, the following tree diagram shows the outcomes of three successive coin tosses.



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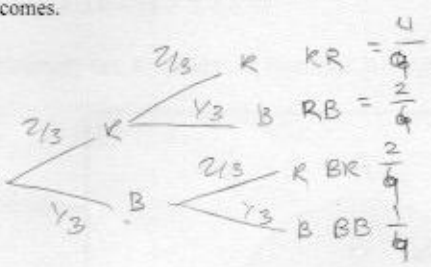
Question 3 Walkthrough.

A bag contains 2 red (R) marbles and 1 blue (B) marble. A marble is selected, its colour is noted, and then it's replaced.

This is done twice.

a. Draw a tree diagram showing all possible outcomes.



b. Find the probability of the event (R, B).

$Pr(R, B) = \frac{2}{9}$

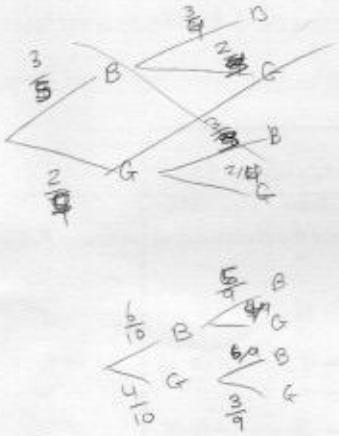
c. Find the probability of the event (R, R).

$Pr(R, R) = \frac{4}{9}$

Question 4

A class has 10 students: 6 boys and 4 girls. A student is randomly selected to be the teacher's assistant, and then a different student is randomly selected to be the class representative. ~~replaced?~~ Not replaced.

a. Draw a tree diagram for the situation, showing the outcomes of selecting a boy or a girl.



b. Find the probability that a girl is selected first, and then a boy.

$\frac{2}{5} \times \frac{3}{9} = \frac{6}{20}$

$Pr(G,B) = \frac{3}{10}$

$\frac{4}{10} \times \frac{6}{9} = \frac{24}{90}$

c. Find the probability that two girls are selected.

$\frac{2}{5} \times \frac{2}{9} = \frac{4}{20} = \frac{1}{5}$

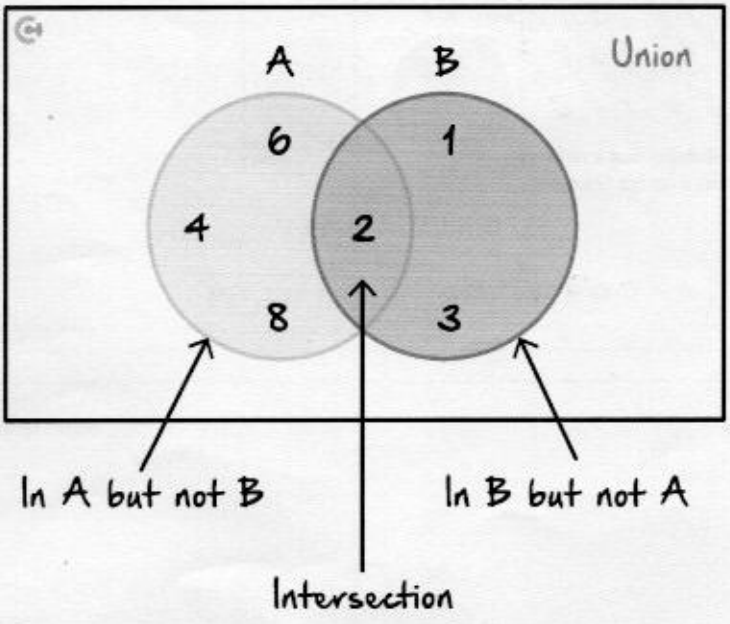
P $\frac{4}{10} \times \frac{3}{9} = \frac{12}{90}$

$Pr(G,G) = \frac{12}{90}$

Sub-Section: Venn Diagram and Combining Events

List of Set Notation and Terms

- A **set** is a collection or group of elements that can include numbers, letters or other objects.
  - E.g., $A = \{2, 4, 6, 8\}, B = \{1, 2, 3\}$
- Intersection ($A \cap B$): Elements that belong to both A and B.
  - E.g., $A \cap B = \{2\}$
- Union ($A \cup B$): Elements that belong to either event A or B.
  - E.g., $A \cup B = \{1, 2, 3, 4, 6, 8\}$
- Null/empty set: A set with no elements, it is symbolised by { } or Ø.



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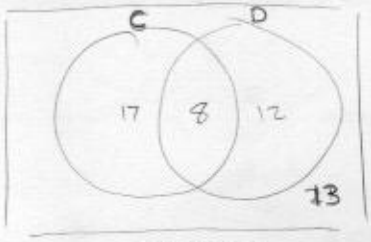
Question 5

In a survey of 50 households, it was found that 25 have a cat, 20 have a dog, and 8 have both a cat and a dog.

a. Draw a Venn diagram to represent the above information.



25-8 = 17
20-8 = 12
17+12+8+13 = 50
50-29 = 21
21
2n+8 = 87
50-37 = 13
the 'neither' circle
C∩D+C∩D'
17+12+8 = 37
50-37 = 13

b. Find the probability that a randomly selected household has a cat but not a dog.

$Pr(C \cap D') = \frac{17}{50}$

c. Find the probability that a randomly selected household has neither a cat nor a dog.

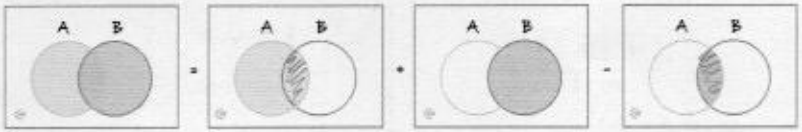
$Pr(C \cap D)' \frac{13}{50}$

Sub-Section: Apply the Addition Rule to Find Unknown Probabilities

Addition Rule

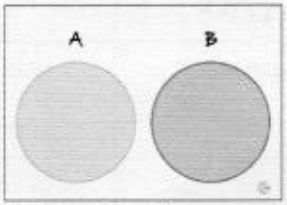
- The **addition rule** for two events, A and B, is:

$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$$



without $-\Pr(A \cap B)$ it will duplicate.

If A and B are mutually exclusive, then: So minus it by itself once.



if

$$\Pr(A \cap B) = 0$$

then

$$\Pr(A \cup B) = \Pr(A) + \Pr(B)$$

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Question 6

Two events, A and B, are such that $\Pr(A) = 0.4$, $\Pr(B) = 0.7$ and $\Pr(A \cap B) = 0.3$.

Find: total = 1.00

a. $\Pr(A \cup B)$

0.4+0.7+0.3
= 0.11+0.3 = 0.14
0.14-0.3
0.11
0.4-0.3 = 0.1
0.7-0.3 = 0.4
0.1+0.4 = 0.5
0.5+0.3 = 0.8
Pr(A∪B) = 0.8

b. $\Pr(A' \cap B')$

1 - 0.8 ·
$Pr(A' \cap B') = 0.2$

Question 7

Two events, X and Y, are such that $\Pr(Y) = 0.6$ and $\Pr(X \cup Y) = 0.9$. If X and Y are mutually exclusive, find the value of $\Pr(X)$.

Pr(X) + 0.6 = 0.9
Pr(X) = 0.3

0.9 - 0.6 = 0.3
Pr(X) = 0.3

Sub-Section: Two-Way Table

	B	B'	
A	$\Pr(A \cap B)$	$\Pr(A \cap B')$	$\Pr(A)$
A'	$\Pr(A' \cap B)$	$\Pr(A' \cap B')$	$\Pr(A')$
	$\Pr(B)$	$\Pr(B')$	1

Question 8

A school tracked student attendance for a week. The results are shown in a two-way table with some missing values.

	Was Absent	Was not Absent	
Grade 9	15	135	150
Grade 10	30	45	75
	45	180	225

a. Complete the two-way table with the missing values.

b. Find the probability that a randomly selected student was from Grade 10.

$\Pr(\text{Grade 10}) \quad \frac{75}{225} = \frac{1}{3}$

c. Find the probability that a randomly selected student was from Grade 9 and was absent.

$\frac{15}{225} = \frac{1}{15}$

$\Pr(\text{Year 9}) = \frac{15}{225} = \frac{1}{15}$

Section B: Conditional Probability and Independent Events

Sub-Section: Conditional Probability

Conditional Probability

- Involves calculating the probability of an event given that another event has occurred.
- We simply divide by the probability of the given condition, as it has already occurred.

$$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$

"Likehood of A given that B has happened."

The Multiplication Rule

$$\Pr(A \cap B) = \Pr(A) \times \Pr(B|A)$$

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Question 9 Walkthrough.

A group of 50 people were surveyed about their preference for pizza and burgers. The results are summarised in the table below.

	Prefers Pizza	Does not prefer Pizza	Total
Prefers Burgers	20	10	30
Does not prefer Burgers	15	5	20
Total	35	15	50

A person is selected at random from the group.

a. Find the probability that the person prefers burgers given that they prefer pizza.

$Pr(B|P) = \frac{Pr(B \cap P)}{Pr(P)}$

$= \frac{20}{35} = \frac{4}{7}$

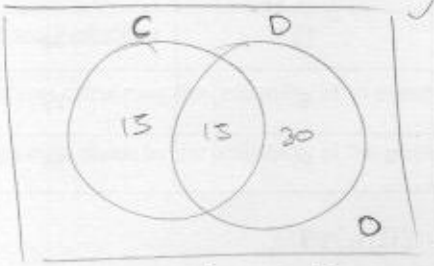
b. Find the probability that the person does not prefer pizza, given that they do not prefer burgers.

$Pr(P'|$

Question 10

Of 60 students in a club, 30 are in the chess club, 45 are in the debate club, and 15 are in both clubs.

a. Draw a Venn diagram to represent the information.



$30 - 15 = 15$ $30 + 30$
$45 - 15 = 30$

b. Find the probability that a randomly selected student is in the debate club but not the chess club.

$\frac{3}{60} = \frac{1}{2}$

$Pr(D \cap C') = \frac{1}{2}$

c. Find the probability that a randomly selected student is in the chess club, given that they are not in the debate club.

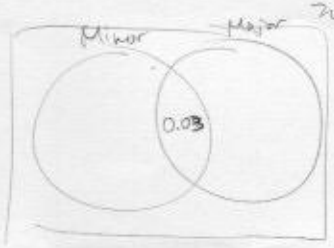
$Pr(C|D') = \frac{Pr(C \cap D')}{Pr(D')}$

$\frac{30}{45} = \frac{6}{9} = \frac{2}{3}$

$Pr(C|D') = \frac{2}{3}$

Question 11 Extension.

A company finds that 20% of the products they manufacture have a minor defect, and 5% have a major defect. The probability of a product having a minor defect and a major defect is 0.03. Given a product has a minor defect, what is the probability it also has a major defect?



$20\% \times 5\% = 0.03$

$20\% \times 5\% = 0.03\%$

$(Minor \cap Major) = 0.03$

$Minor$

Sub-Section: Independent Events

Independent Events

- Two events A and B are independent when the occurrence of one **does not affect** the occurrence of the other.
- When two events are independent:

$$\Pr(A \cap B) = \Pr(A) \times \Pr(B)$$

- For multi-stage experiments with the selection made **with replacement**, successive events are independent.
- For multi-stage experiments with the selection made **without replacement**, successive events are not independent.

Combining Addition Rule and Independent Event Rule

- Given:

$$\Pr(A \cap B) = \Pr(A) \times \Pr(B)$$

$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$$

- Becomes:

$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A) \times \Pr(B)$$

Space for Personal Notes

Question 12 Walkthrough.

Determine if the events A and B are independent.

$$\Pr(A) = \frac{3}{4}, \Pr(B) = \frac{1}{2}, \Pr(A \cap B) = \frac{3}{8}$$

$Pr(A) \times Pr(B) = \frac{3}{4} \times \frac{1}{2}$

$= \frac{3}{8}$ $= Pr(A \cap B)$

$\therefore$ Independent ✓

Question 13

If events A and B are independent, find the missing probability.

a. $\Pr(A) = \frac{3}{5}, \Pr(B) = \frac{1}{2}$, find $\Pr(B|A)$.

$3 \times 1 = 3$

$Pr(A \cap B) = \frac{3}{5} \times \frac{1}{2} = \frac{3}{10}$

$Pr(B) = \frac{1}{2}$

$Pr(B|A) = \frac{Pr(A \cap B)}{Pr(A)}$ ∴

b. $\Pr(A) = \frac{4}{5}, \Pr(B) = \frac{7}{10}$, find $\Pr(A \cap B)$.

$Pr(A) \times P(B) = Pr(A \cap B)$

$\frac{4}{5} \times \frac{7}{10} = \frac{28}{50} =$ ✓

Question 14 Extension.

Determine if the events A and B are independent.

$$\Pr(A) = \frac{3}{7}, \Pr(B) = \frac{1}{2}, \Pr(A|B) = \frac{7}{20}$$

Section C: Measure of Centre and Spread, Standard Deviation

Sub-Section: Understanding Data

Mean

$$\text{Mean} = \frac{\text{Add everything together}}{\text{Number of things}}$$

$$\bar{x} = \frac{\Sigma x}{n}$$

Finding the Median

- **Odd Number of Data Values**

  Median is the middle value.

  - Example:

    $2, 4, 5, 8, 9$

    - The median is the middle value.

- **Even Number of Data Values**

  - When there is an even number of data values, the median is the average of the two middle values.
  - Example:

    $3, 5, 7, 11, 13, 15$

    - The middle values are 7 and 11.
    - The median is the average of these two values.

Mode

The number that occurs the most often.

Question 15

A survey was conducted on the number of times 50 students used a vending machine in a month. Find the missing frequencies if the **mean** number of uses is 2.8.

Number of Uses (x)	1	2	3	4	5
Number of Students (f)	f_1	10	15	f_4	5

$= 2.8$

$= 50$

$50 =$

$2.8 =$

$2 \text{ row} = 20 + 45 + 25 + f_2 + 4(f_4)$

$3 \times 15 =$

$\frac{20 + 45 + 25 + f_2 + 4(f_4)}{50} = 2.8$

$65 + 25 =$

Sub-Section: Measure of Spread

Five-Figure Summary

Minimum (min): The smallest value.

Lower quartile (Q_1): The number above 25% of the ordered data.

Median (Q_2): The middle value above 50% of the ordered data.

Upper quartile (Q_3): The number above 75% of the ordered data.

Maximum (max): The largest value.

Measure of Spread

- **Range:** The difference between the maximum and minimum data values.

$$\text{Range} = \max - \min$$

- **Interquartile range (IQR):** The difference between the upper quartile and lower quartile data values.
- For example,

2, 2, [2], 6, 13, | 14, 14, [15], 17, 19

$Q_1 = 2$ $Q_3 = 15$

- **Interquartile range (IQR)** is the range of the middle 50% of a data set.

$$\text{IQR} = Q_3 - Q_1$$

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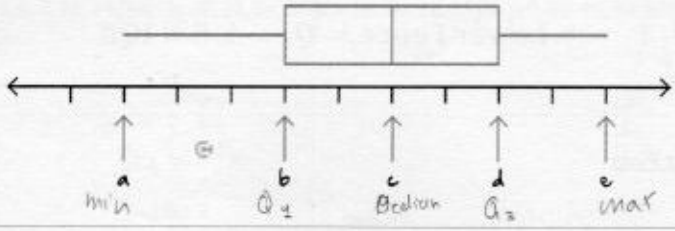
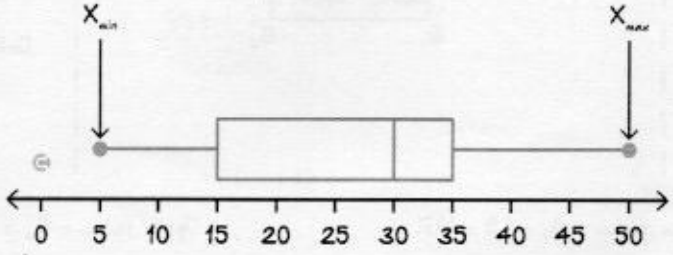
Box Plot

- A boxplot requires the five-number summary - minimum, Q_1, median, Q_3, and the maximum.



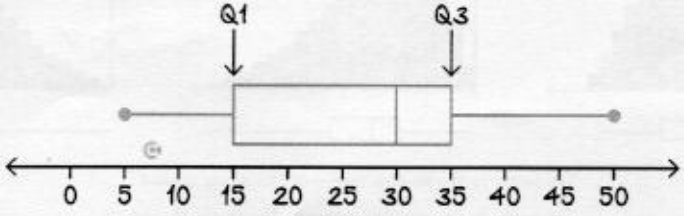
The Range



$$\text{Range} = \text{highest value}(X_{max}) - \text{lowest value}(X_{min})$$

IQR (Inter-Quartile Range)

$$\text{IQR} = Q_3 - Q_1$$



- Gives:

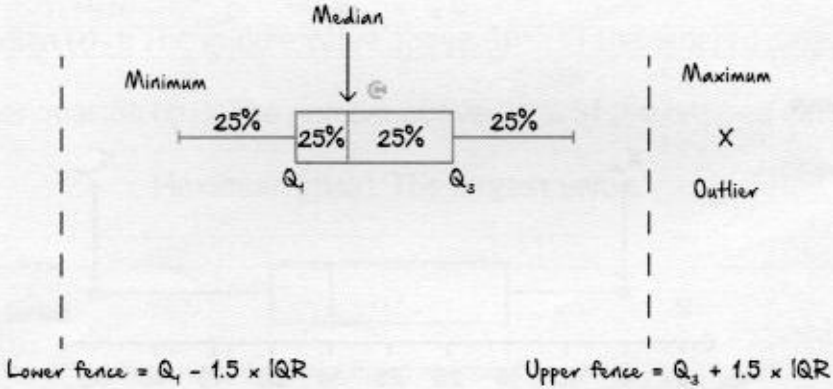
The middle 50% of data.

The Fences

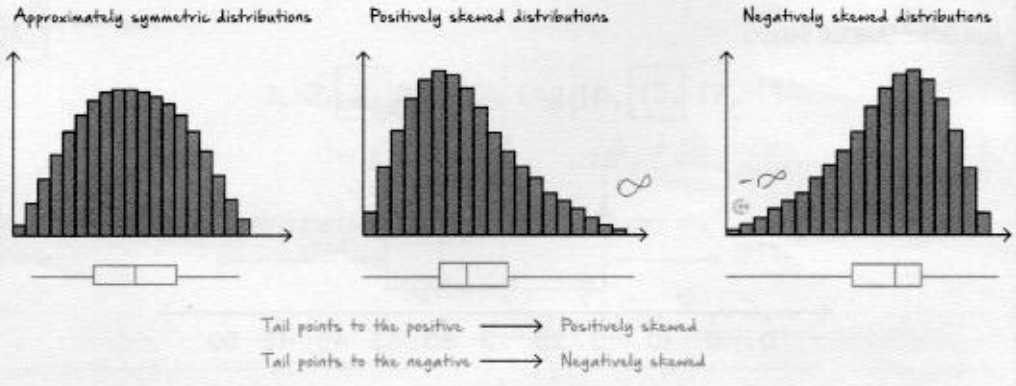
$$\text{Upper Fence} = Q_3 + 1.5 \times \text{IQR}$$

$$\text{Lower Fence} = Q_1 - 1.5 \times \text{IQR}$$

Box-and-Whisker Plots



Distribution of Data



TIP: We can find the outlier by using the box plot on CAS calculator!

Question 16 Tech Active.

Find the outlier(s), if any, in the given dataset.

1, 5, 8, 12, 14, 16, 18, 20, 22, 24, 25, 26, 28, 30, 32, 35, 38, 40, 42, 45, 50, 100

outlier

71

Min = 1

Q_1 = 16

Median = 25.5

Q_3 = 38

Max = 100

Upper fence = 71

Lower fence

$38 + 1.5 \times 22 = 71$

Sub-Section: Standard Deviation

Standard Deviation

- Standard deviation is given by:

$$s = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n-1}}$$

- s represents the sample standard deviation.
- Σ represents 'the sum of'.
- x represents an observation.
- $\bar{x}$ represents the mean.
- n represents the number of observations.

Question 17 Tech Active. Menu 4,1,1

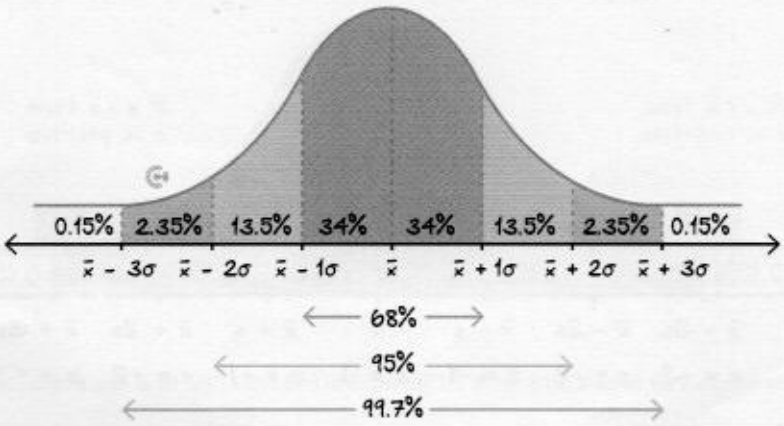
The heights (in cm) of 6 plants are:

12,15,11,14,13,17

Calculate the standard deviation correct to two decimal places.

Sub-Section: z (Standardised) Score

Standardised Values (68%-95%-99.7% Rule)



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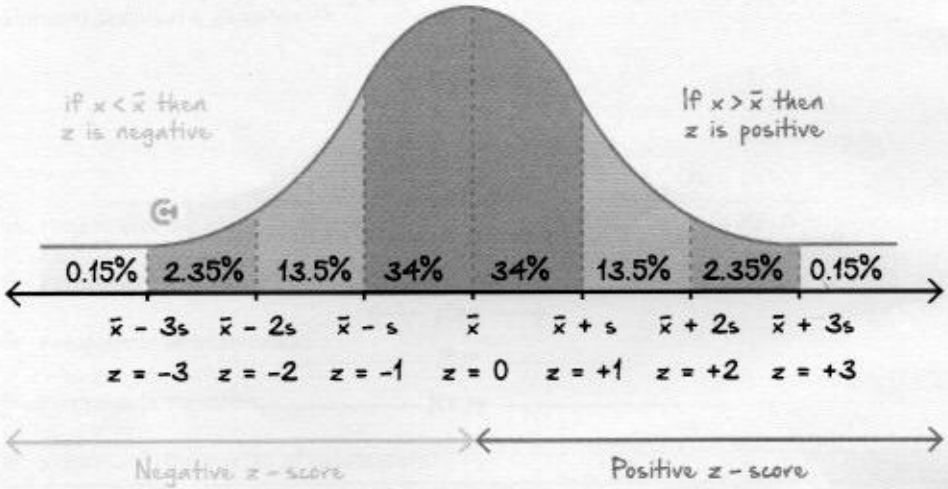
Z-score

- To decide whether a data point is acceptable, we look at its Z-score.

Z-score → Number of standard deviations from the mean



- Formula to calculate standard Z-scores:

$$Z = \frac{x - \bar{x}}{s}$$

  - Where:
    - x = Score
    - $\bar{x}$ = Mean
    - s = Standard deviation

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Question 18 Walkthrough. Tech Active.

The average weight of newborn babies is 3.5 kg with a standard deviation of 0.5 kg. What is the **z-score** for a baby who weighs 2.9 kg.

Question 19

The average score on a quiz is 80 with a standard deviation of 10. What is the **z-score** for a student who scored 95?

~~80 − 80~~ $\frac{95-80}{10} = \frac{15}{10} = 1.5$ ✓

Question 20 Extension.

The scores on a science test have a standard deviation of 8. A student who scored 70 on the test has a **z-score** of −1.5. What is the mean score of the test?

Section D: Bivariate Data and Scatter Plots

Sub-Section: Bivariate Data

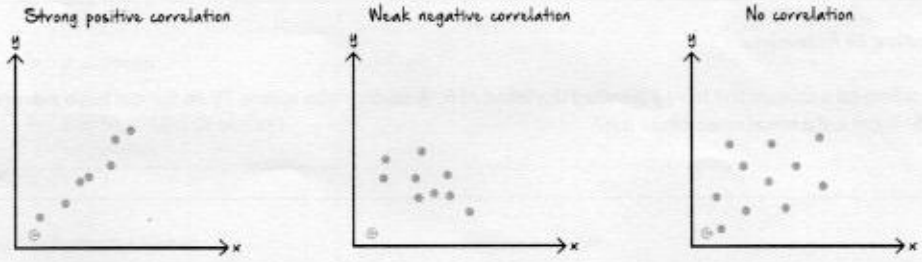
Bivariate Data

- Bivariate data includes data for two variables:
  - The independent variable (the variable being changed) is on the x-axis.
  - The dependent variable (the variable being measured) is on the y-axis.
- Examples of relationships:
  - Height of person and Weight of person.
  - Temperature and Life of Milk.
  - Years of Education and Income.

response and explanatory variable

Scatter Plot and Types of Correlation

- A scatter plot is a dotted graph on a number plane corresponding to two variables from the bivariate data.
- Types of correlations:



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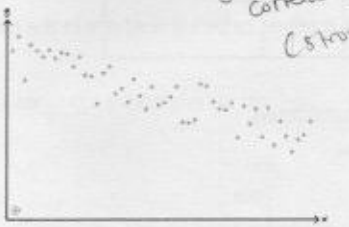
Question 21

For each of the following scatter plots, determine whether the bivariate data is positively correlated, negatively correlated, or has no correlation.

a.

negative correlation (strong)



b.

(moderate)

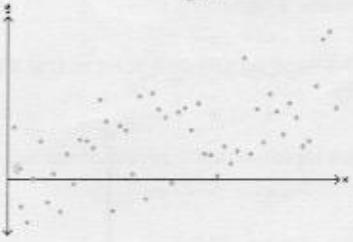
positive correlation

c.

no correlation

d.

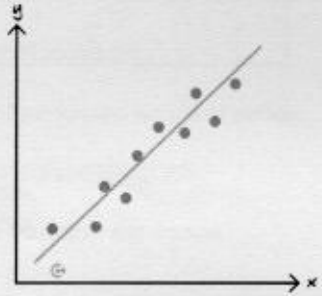
positive (weak) correlation

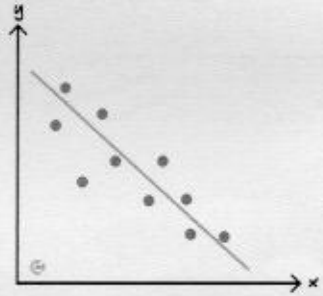


Sub-Section: Line of Best Fit by Eye

Line of Best Fit

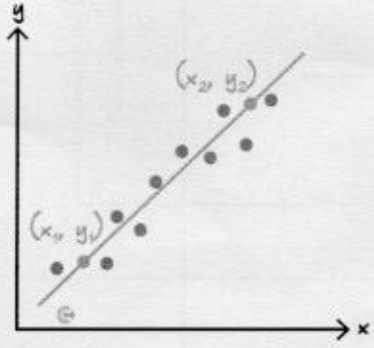
- A line of best fit is positioned in a way that:
  - The line is as close to all points as possible, which means it needs to be somewhere in the middle of the points.
  - The distance of each point from the trend line must be considered.





The Equation of the Line

- For $y = mx + c$,
- $m = \frac{y_2 - y_1}{x_2 - x_1}$ and substitute a point to find the value of c.

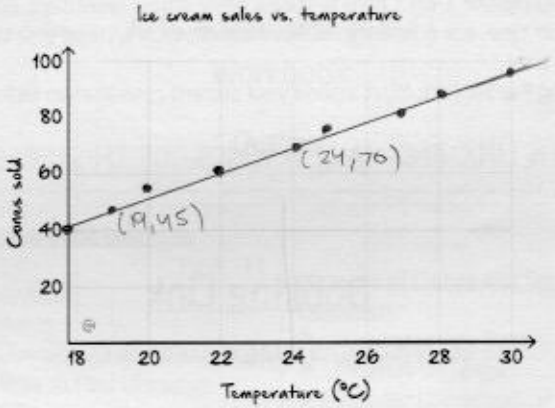


Question 25 Tech Active.

The table below shows the daily temperature in degrees Celsius and the number of ice cream cones sold at a small shop over seven days.

Temperature (°C)	18	20	22	25	27	28	30
Cones sold	40	55	60	75	80	85	95

a. Construct a scatter plot for the data.



60 - 110 = C

b. Draw a line of best fit on the scatter plot and find the equation of the line of best fit.

$\frac{60-40}{22-18} = \frac{20}{4} = 5$ $y = 5x + C$

$60 = 5(22) + C$

$60 = 110 + C$

c. Use your equation to estimate the number of cones sold, to the nearest integer, when the temperature is:

i. 24 °C

(22,60) (18,40) $y = 5x - 50$

$x = 24$ $y = 5(24) - 50$ 70 cones

$y = 120 - 50$, $y = 70$

ii. 19 °C

$y = 5(19) - 50$

$y = 95 - 50$ $y = 45$ 45 cones.