Website: contoureducation.com.au | Phone: 1800 888 300
Email: hello@contoureducation.com.au

Year 10 Mathematics
AOS 8 Revision [10.3]
Mock CAT 2

50 Marks. 60 Minutes Writing.

Results:

Short Answer Questions	24 / 34
Extended Response Questions	2 / 16

65%

Section A: Short Answer Questions (34 Marks)

Question 1 (1 mark)

Expand and simplify: $(x + 4)(x - 3)$.

$x^2 + x - 12$

Question 2 (1 mark)

Find the degree of $p(x) = x(3x^5 - 2x + k)$, where k is an integer.

$3x^6 \rightarrow$ 6 th degree

$=$ ~~M~~ $p(x)$ is 6th degree

Question 3 (1 mark)

State the x-intercepts of $y = -x(x - 4)^2$.

~~0~~ $= \; -x = 0$ $\qquad x = 4$

$x = 0$

$(0,0)$ ~~$(x,4)$~~ ~~$(x,4)$~~ ~~$(0,4)$~~ $(0,4)$

Question 4 (1 mark)

What is the remainder when $x^7 + 5$ is divided by $(x - 1)$?

$x = 1$

$(1) + 5 = 6$

remainder $= 6$

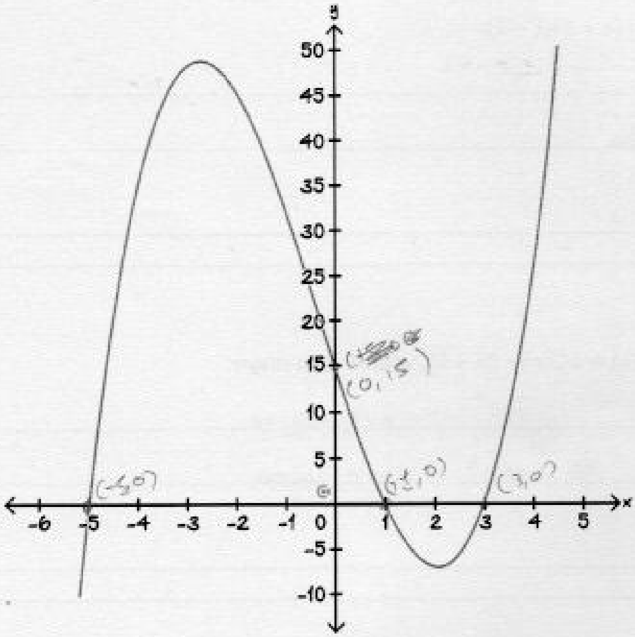
x^6

$x - 1 \,\big)\, \cancel{x^7 + 8x^6 + 5}$

$\cancel{x^7 - x^6}$

Question 5 (2 marks)

Find the x-intercepts and the y-intercept of the following cubic graph given below.



when ($x=0$) y int: $(0,15)$

where ($y=0$) x int: $(-5,0)$ $(1,0)$ $(3,0)$

Question 6 (3 marks)

Solve the following polynomial equations:

a. $x(x-5)(x+2)=0$. (1 mark)

$x=0$, $x=5$, $x=-2$

~~(0,0)~~ (5,0) (-2,0)

(0,0)

b. $x^4-10x^2+9=0$. (2 marks) D $a^2=x^4$ (-1)

$\cancel{x^2(x^2-10x)+9=0}$ $x^2(x^2-10)+9=0$

$\cancel{x^2(x^2-10)+9=-9}$ $x^2(x^2-10)=-9$

$x^4-10x^2=-9$ $x^2(x-\sqrt{10})(x+\sqrt{10})=-9$

$x^2(x^2-10)$

$a^2+10a+9=0$

$(a-1)(a-9)$ $a=1, 9$

$\boxed{\therefore\ x=\pm1, \pm3}$ $x^2=\pm1, x^2=\pm3$

Question 7 (3 marks)

Given $(x-1)$ is a factor of $P(x)=x^3+2x^2-13x+10$, fully factorise $P(x)$.

$x=1$ $2x^2-(-x^2)$

$\cancel{(1)^3+2(1)^2-13(1)+10}$

$\cancel{1+2-13+10}=$

$$
\begin{array}{r}
x^2+3x-10 \\
x-1\,\overline{)\,x^3+2x^2-13x+10} \\
-\ x^3-x^2 \\
\hline
3x^2-13x \\
-\ 3x^2-3x \\
\hline
-10x+10 \\
-10x+10 \\
\hline
0
\end{array}
$$

$-13x($

$P(x)=\dfrac{x^3+2x^2-13x+10}{x-1}=x^2+3x-10$

↓

$=\cancel{(x-5)(x+2)}$ 5×2 -5×2

$(x-1)(x+5)(x-2)$

Question 8 (3 marks)

Given $P(x) = 5x^3 - 2x^2 - 23x + 14$.

a. Verify that $x = 2$ is a root of the equation $P(x) = 0$. (1 mark)

$5(2)^3 - 2(2)^2 - 23(2)+14$

$5\times 8 - 2\times 4 - 23\times 2+14$

$40 - 8 - 46+14 =$

$32-46=-10$

$-14+14=0$

$\therefore$ ~~not a root~~, $4\neq 0$

$\therefore$ yes a root $x=2$

b. Determine the remainder when $P(x)$ is divided by $(x + 3)$. (1 mark)

$x=-3$

$P(-3) = 5(-3)^3 - 2(-3)^2 - 23(-3)+14$

$5\times -27 - 2\times 9 \quad -23\times -3+14$

$-135 - 18 + 69 + 14 =$

c. It is known that $P(x) \div (x - a) = Q(x) + \frac{25}{x-a}$, where $Q(x)$ is a quadratic polynomial. Find $P(a)$. (1 mark)

~~$P(x) \div (x-a) = ax^2+bx+c + \frac{25}{x-a}$~~ $\quad P(a)\neq 0$

$P(a) = 25$

remainder just number.

Question 9 (3 marks)

$y=a(x-h)+k$

$(-2,-1)$

Sketch the graph of $y = (x + 2)^4 - 1$ on the axes below, labelling all axes intercepts and turning points.

$y=0$

$0=(x+2)^4-1$

$\pm 1 = x+2$

$x+2=\pm 1$

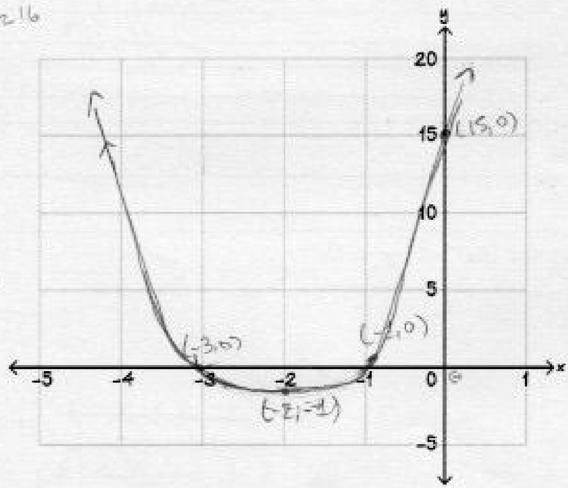
$x=-3$

$x=-1$

$x=0$

$y=(2)^4-1$

$y=16-1=15$



Question 10 (2 marks)

Use long division to find the quotient and remainder when $2x^3 - 5x^2 + 8x - 5$ is divided by $(x - 3)$.

$2x^2 + x + 11$

$x - 3 \overline{)2x^3 - 5x^2 + 8x - 5}$

$-\ 2x^3 - 6x^2$

$x^2 + 8x$

$x^2 - 3x$

$11x - 5$

$11x - 33$

28

$-5x^2 - (-6x^2$

$8x +$

$-5 - -(33$

$-5 + 33 =$

28

$\boxed{(x-3)(2x^2+x+11) + 28}$

quotient $= 2x^2 + x + 11$

remainder $= 28$

Question 11 (2 marks)

Find the value of k if $(x - 2)$ is a factor of $P(x) = x^3 + kx^2 - 3x - 6$.

$x = 2$

$(2)^3 + k(2)^2 - 3(2) - 6$

$8 + 4k - 6 - 6 = 0$

$8 + 6 - 6$

$8 - 12 = -4$

$4k - 4 = 0$

$4k = 4$

$k = 1$

$\boxed{\therefore k = 1}$

Question 12 (2 marks)

Expand and simplify $(x - 5)(x^2 - 2x + 3)$.

$x^3 - 2x^2 + 3x - 5x^2 + 10x - 15$

$x^3 - 2x^2 - 5x^2 + 3x + 10x$

$\therefore = x^3 - 7x^2 + 13x - 15$

Question 13 (5 marks)

The weekly profit, P, in thousands of dollars, of a company is modelled by $P(x) = x^3 - 8x^2 + 16x$, where x is the number of units produced, in hundreds.

B

-3

6-9

a. Factorise the profit function $P(x)$. (2 marks)

$x(x^2 - 8x + 16)$

$x(x-4)^2$

b. For what production levels does the company break even (make zero profit)? (2 marks)

$x(x-4)^2 = 0$

$x=0$ $x=4$ $4 \times 100 = 400$

~~(0,0) (4,0)~~

so 0 units and 400 units

But on graph it would just be $(x=4)$ since $x=100$.

c. What is the profit if 200 units are produced? (1 mark)

remember $x=100$ so every

$200(200-4)^2 = y$

$(196)^2$

$\frac{200}{100} = \boxed{x=2}$ on graph.

$2(2-4)^2 = y$

$2(-2)^2$

$2(2-4)^2$

$2(-2)^2$

3+1

5+8=13

1+9

~~8×100~~

~~\$800~~

12

40

400

18000

40000

200000

52

452

18452

58452

\$258452

196 × 196

6486 (?)

03140

+19600

129226

129226 × 200 =

∴ = \$258,452

Question 14 (5 marks)

-3

A reservoir's net water balance $W(x)$ (in hundreds of litres) after x hours of alternating pump cycles is modelled by $W(x) = x^3 - 5x^2 - 2x + 24$.

~~A~~ = 0

a. Using the Factor Theorem and trial-and-error, find a time x at which the net balance is zero. (2 marks)

$\cancel{x^3 - 5x^2 - 2x + 24 = 0}$ D , 24 is the constant.

$\cancel{x(x^2 - 5x - 2) + 24 = 0}$ ↳ multiplied by other constants:

$\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24$

$\cancel{W(3) = (3)^3 - 5(3)^2 - 2(3) + 24}$

$\cancel{27 - 45 - 6 + 24}$

$= 0.$ $x - 3 = 0$, $x = 3$ Breakeven at $\underline{\underline{x = 3}}$.

b. Once you have this value, factorise $W(x)$ completely to determine all times when the reservoir is in balance ($W(x) = 0$). (3 marks)

D

$x - 3 \div x^3 - 5x^2 - 2x + 24$

$-5 + 3 = -2$

$-2 + (-6) = -8$

$$\begin{array}{c|cccc} 3 & 1 & -5 & -2 & 24 \\ & & 3 & -6 & -24 \\ \hline & 1 & -2 & -8 & 0 \end{array}$$

$\cancel{x^2 - 2x - 8}$

$\cancel{x}(x-4)(x+2)\cancel{x}$

$\cancel{(x+4)(x-2)(x-3)}$

$(x-4)(x+2)(x-3)$

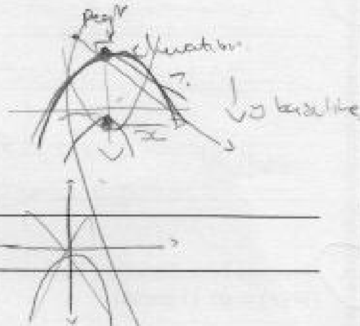
$x = -2, 3, 4$

Section B: Extended Response Questions (16 Marks)

Question 15 (9 marks)

Two trail elevation profiles give the vertical position (relative to a baseline; negative values indicate dips below the baseline) as a function of horizontal distance x metres from the trailhead.

- Trail A: $y = -2(x-4)(x-1), x \geq 0$ (elevation in metres).
- Trail B: $y = (x-1)(x-2)(x-4), x \geq 0$ (elevation in metres).



a. Where does Trail A meet the baseline (ground)? (1 mark)

$-2(x-4)(x-1)$

$x=4, \quad x=1$

$\therefore (4,0), (1,0)$

b. Where does Trail B meet the baseline? (1 mark)

$x=1, x=2, x=4 = 0$

$\therefore (1,0), (2,0), (4,0)$

c. Find the horizontal distance x at which Trail A reaches its maximum elevation (give your answer in decimal form). (2 marks)

$x \geq 0$, when $x=0$ $\quad -2(x-4)(x-1)$

$y = -2(-4)(-1) = \quad -2(x^2-5x+4)$

$-2(-5)(4) = y$

How? when $x=0$, y is negative

but $x \geq 0$? $-2(0-0+4)$

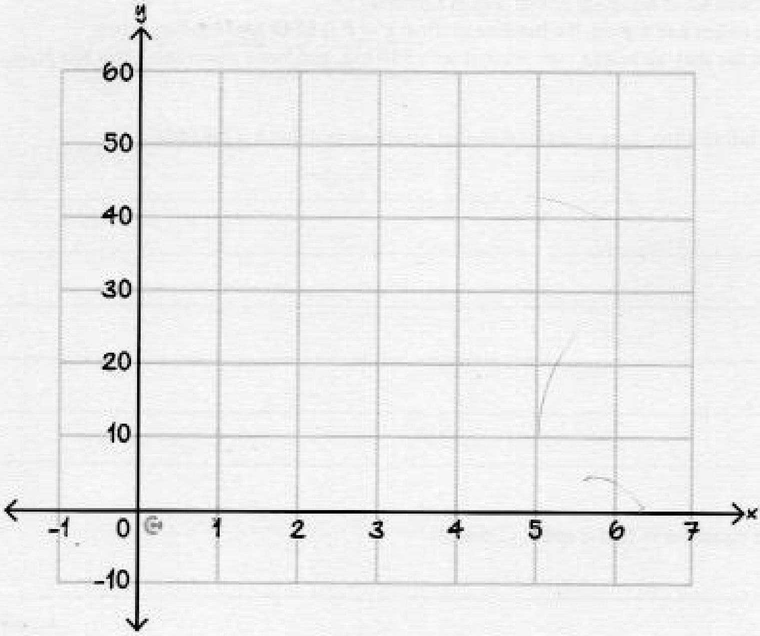
$-2x^2+10x-8$ $\quad$ -8

d. For which value(s) of x are the two trails at the same elevation, and what is that common elevation (in metres)? (2 marks)

e. Hence or otherwise, sketch the graph of the polynomial:

$$y = x^3 - 5x^2 + 4x$$

In the interval $-1 \le x \le 6$. Clearly label all axis intercepts and endpoints. (3 marks)



Question 16 (7 marks)

A robotics start-up models its net monthly profit $P(x)$ (in thousands of dollars) by a monic cubic:

$$P(x) = x^3 + ax^2 + bx + 24$$

Where x is the number of hundreds of kits sold in a month.
Because of pre-orders and a grant, the baseline profit at $x = 0$ is \$24k (the constant term).
It is known that the start-up breaks even when it sells 300 kits, and that a pilot run of 100 kits produced a profit of \$10k.

a. Using this information, form two simultaneous equations in a and b. (3 marks)

b. Solve your equations to find a and b. (2 marks)

c. Write down the complete polynomial $P(x)$. (1 mark)

d. Hence, factor $P(x)$ fully and state the other two linear factors (besides $x - 3$). (1 mark)