Practical Business Python

Taking care of business, one python script at a time

Mon 18 February 2019

Monte Carlo Simulation with Python

Posted by Chris Moffitt in articles

Introduction

There are many sophisticated models people can build for solving a forecasting problem. However, they frequently stick to simple Excel models based on average historical values, intuition and some high level domain-specific heuristics. This approach may be precise enough for the problem at hand but there are alternatives that can add more information to the prediction with a reasonable amount of additional effort.

One approach that can produce a better understanding of the range of potential outcomes and help avoid the “flaw of averages” is a Monte Carlo simulation. The rest of this article will describe how to use python with pandas and numpy to build a Monte Carlo simulation to predict the range of potential values for a sales compensation budget. This approach is meant to be simple enough that it can be used for other problems you might encounter but also powerful enough to provide insights that a basic “gut-feel” model can not provide on its own.

Problem Background

For this example, we will try to predict how much money we should budget for sales commissions for the next year. This problem is useful for modeling because we have a defined formula for calculating commissions and we likely have some experience with prior years' commissions payments.

This problem is also important from a business perspective. Sales commissions can be a large selling expense and it is important to plan appropriately for this expense. In addition, the use of a Monte Carlo simulation is a relatively simple improvement that can be made to augment what is normally an unsophisticated estimation process.

In this example, the sample sales commission would look like this for a 5 person sales force:

	A	B	C	D	E	F
1	Sales Rep	Sales Target	Actual Sales	Percent To Plan	Commission Rate	Commission Amount
2	1	$100,000	$88,000	88.0%	2.0%	$1,760
3	2	$200,000	$202,000	101.0%	4.0%	$8,080
4	3	$75,000	$90,000	120.0%	4.0%	$3,600
5	4	$400,000	$360,000	90.0%	0.0%	$0
6	5	$500,000	$350,000	70.0%	0.0%	$0
7						
8	Total	$1,275,000	$1,090,000			$13,440

In this example, the commission is a result of this formula:

Commission Amount = Actual Sales * Commission Rate

The commission rate is based on this Percent To Plan table:

Rate Schedule	
0 – 90%	2.00%
91-99%	3.00%
>= 100	4.00%

Before we build a model and run the simulation, let's look at a simple approach for predicting next year's commission expense.

Naïve Approach to the Problem

Imagine your task as Amy or Andy analyst is to tell finance how much to budget for sales commissions for next year. One approach might be to assume everyone makes 100% of their target and earns the 4% commission rate. Plugging these values into Excel yields this:

11	*Base Case*					
12	**Sales Rep**	**Sales Target**	**Actual Sales**	**Percent To Plan**	**Commission Rate**	**Commission Amount**
13	1	$100,000	$100,000	100.0%	4.0%	$4,000
14	2	$200,000	$200,000	100.0%	4.0%	$8,000
15	3	$75,000	$75,000	100.0%	4.0%	$3,000
16	4	$400,000	$400,000	100.0%	4.0%	$16,000
17	5	$500,000	$500,000	100.0%	4.0%	$20,000
18						
19	Total	$1,275,000	$1,275,000			$51,000

Imagine you present this to finance, and they say, "We never have everyone get the same commission rate. We need a more accurate model."

For round two, you might try a couple of ranges:

21	*Scenario #1*					
22	**Sales Rep**	**Sales Target**	**Actual Sales**	**Percent To Plan**	**Commission Rate**	**Commission Amount**
23	1	$100,000	$95,000	95.0%	3.0%	$2,850
24	2	$200,000	$204,000	102.0%	4.0%	$8,160
25	3	$75,000	$60,000	80.0%	2.0%	$1,200
26	4	$400,000	$480,000	120.0%	4.0%	$19,200
27	5	$500,000	$400,000	80.0%	2.0%	$8,000
28						
29	Total	$1,275,000	$1,239,000			$39,410

Or another one:

31	*Scenario #2*					
32	**Sales Rep**	**Sales Target**	**Actual Sales**	**Percent To Plan**	**Commission Rate**	**Commission Amount**
33	1	$100,000	$105,000	105.0%	4.0%	$4,200
34	2	$200,000	$140,000	70.0%	2.0%	$2,800
35	3	$75,000	$74,250	99.0%	3.0%	$2,228
36	4	$400,000	$352,000	88.0%	2.0%	$7,040
37	5	$500,000	$550,000	110.0%	4.0%	$22,000
38						
39	Total	$1,275,000	$1,221,250			$38,268

Now, you have a little bit more information and go back to finance. This time finance says, "this range is useful but what is your confidence in this range? Also, we need you to do this for a sales force of 500 people and model several different rates to determine the amount to budget." Hmmm… Now, what do you do?

This simple approach illustrates the basic iterative method for a Monte Carlo simulation. You iterate through this process many times in order to determine a range of potential commission values for the year. Doing this manually by hand is challenging. Fortunately, python makes this approach much simpler.

Monte Carlo

Now that we have covered the problem at a high level, we can discuss how Monte Carlo analysis might be a useful tool for predicting commissions expenses for the next year. At its simplest level, a Monte Carlo analysis (or simulation) involves running many scenarios with different random inputs and summarizing the distribution of the results.

Using the commissions analysis, we can continue the manual process we started above but run the program 100's or even 1000's of times and we will get a distribution of potential commission amounts. This distribution can inform the likelihood that the expense will be within a certain window. At the end of the day, this is a prediction so we will likely never predict it exactly. We can develop a more informed idea about the potential risk of under or over budgeting.

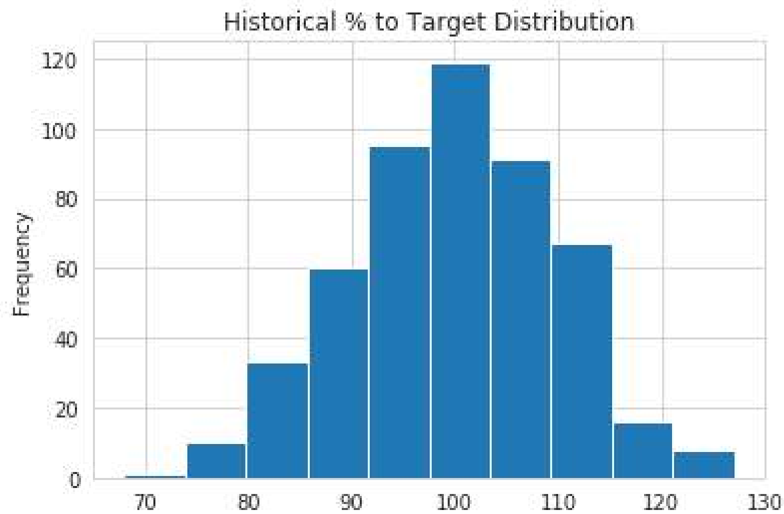
There are two components to running a Monte Carlo simulation:

- the equation to evaluate
- the random variables for the input

We have already described the equation above. Now we need to think about how to populate the random variables.

One simple approach would be to take a random number between 0% and 200% (representing our intuition about commissions rates). However, because we pay commissions every year, we understand our problem in a little more detail and can use that prior knowledge to build a more accurate model.

Because we have paid out commissions for several years, we can look at a typical historical distribution of percent to target:



This distribution looks like a normal distribution with a mean of 100% and standard deviation of 10%. This insight is useful because we can model our input variable distribution so that it is similar to our real world experience.

If you are interested in additional details for estimating the type of distribution, I found this article interesting.

Building a Python Model

We can use pandas to construct a model that replicates the Excel spreadsheet calculation. There are other python approaches to building Monte Carlo models but I find that this pandas method is conceptually easier to comprehend if you are coming from an Excel background. It also has the added benefit of generating pandas dataframes that can be inspected and reviewed for reasonableness.

First complete our imports and set our plotting style:

```
import pandas as pd
import numpy as np
import seaborn as sns

sns.set_style('whitegrid')
```

For this model, we will use a random number generation from numpy. The handy aspect of numpy is that there are several random number generators that can create random samples based on a predefined distribution.

As described above, we know that our historical percent to target performance is centered around a a mean of 100% and standard deviation of 10%. Let's define those variables as well as the number of sales reps and simulations we are modeling:

```
avg = 1
std_dev = .1
num_reps = 500
num_simulations = 1000
```

Now we can use numpy to generate a list of percentages that will replicate our historical normal distribution:

```
pct_to_target = np.random.normal(avg, std_dev, num_reps).round(2)
```

For this example, I have chosen to round it to 2 decimal places in order to make it very easy to see the boundaries.

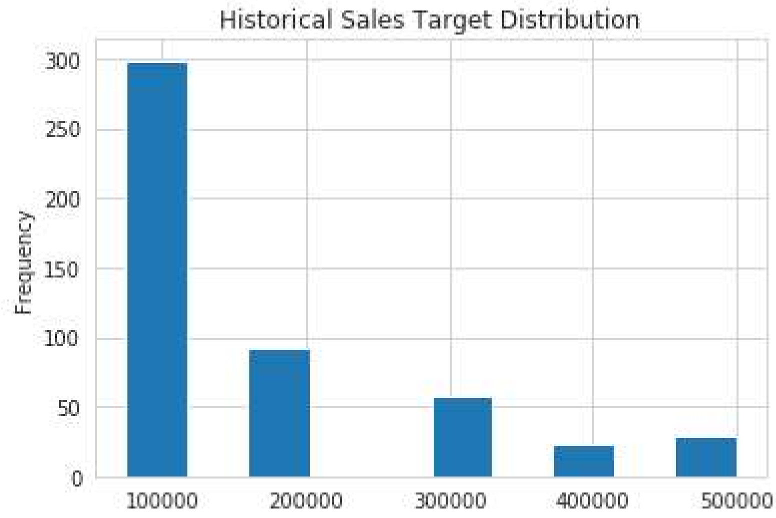
Here is what the first 10 items look like:

```
array([0.92, 0.98, 1.1 , 0.93, 0.92, 0.99, 1.14, 1.28, 0.91, 1.  ])
```

This is a good quick check to make sure the ranges are within expectations.

Since we are trying to make an improvement on our simple approach, we are going to stick with a normal distribution for the percent to target. By using numpy though, we can adjust and use other distribution for future models if we must. However, I do warn that you should not use other models without truly understanding them and how they apply to your situation.

There is one other value that we need to simulate and that is the actual sales target. In order to illustrate a different distribution, we are going to assume that our sales target distribution looks something like this:



This is definitely not a normal distribution. This distribution shows us that sales targets are set into 1 of 6 buckets and the frequency gets lower as the amount increases. This distribution could be indicative of a very simple target setting process where individuals are bucketed into certain groups and given targets consistently based on their tenure, territory size or sales pipeline.

For the sake of this example, we will use a uniform distribution but assign lower probability rates for some of the values.

Here is how we can build this using `numpy.random.choice`

```
sales_target_values = [75_000, 100_000, 200_000, 300_000, 400_000, 500_000]
sales_target_prob = [.3, .3, .2, .1, .05, .05]
sales_target = np.random.choice(sales_target_values, num_reps, p=sales_target_prob)
```

Admittedly this is a somewhat contrived example but I wanted to show how different distributions could be incorporated into our model.

Now that we know how to create our two input distributions, let's build up a pandas dataframe:

```
df = pd.DataFrame(index=range(num_reps), data={'Pct_To_Target': pct_to_target,
                                               'Sales_Target': sales_target})

df['Sales'] = df['Pct_To_Target'] * df['Sales_Target']
```

Here is what our new dataframe looks like:

	Pct_To_Target	Sales_Target	Sales
0	0.92	100000	92000.0
1	0.98	75000	73500.0
2	1.10	500000	550000.0
3	0.93	200000	186000.0
4	0.92	300000	276000.0

You might notice that I did a little trick to calculate the actual sales amount. For this problem, the actual sales amount may change greatly over the years but the performance distribution remains remarkably consistent. Therefore, I'm using the random distributions to generate my inputs and backing into the actual sales.

The final piece of code we need to create is a way to map our `Pct_To_Target` to the commission rate. Here is the function:

```
def calc_commission_rate(x):
    """ Return the commission rate based on the table:
    0-90% = 2%
    91-99% = 3%
    >= 100 = 4%
    """
    if x <= .90:
        return .02
    if x <= .99:
        return .03
    else:
        return .04
```

The added benefit of using python instead of Excel is that we can create much more complex logic that is easier to understand than if we tried to build a complex nested if statement in Excel.

Now we create our commission rate and multiply it times sales:

```
df['Commission_Rate'] = df['Pct_To_Target'].apply(calc_commission_rate)
df['Commission_Amount'] = df['Commission_Rate'] * df['Sales']
```

Which yields this result, which looks very much like an Excel model we might build:

	Pct_To_Target	Sales_Target	Sales	Commission_Rate	Commission_Amount
0	97.0	100000	97000.0	.03	2910.0
1	92.0	400000	368000.0	.03	11040.0
2	97.0	200000	194000.0	.03	5820.0
3	103.0	200000	206000.0	.04	8240.0
4	87.0	75000	65250.0	.02	1305.0

There you have it!

We have replicated a model that is similar to what we would have done in Excel but we used some more sophisticated distributions than just throwing a bunch of random number inputs into the problem.

If we sum up the values (only the top 5 are shown above) in the `Commission_Amount` column, we can see that this simulation shows that we would pay $2,923,100.

Let’s Loop

The real “magic” of the Monte Carlo simulation is that if we run a simulation many times, we start to develop a picture of the likely distribution of results. In Excel, you would need VBA or another plugin to run multiple iterations. In python, we can use a `for` loop to run as many simulations as we’d like.

In addition to running each simulation, we save the results we care about in a list that we will turn into a dataframe for further analysis of the distribution of results.

Here is the full for loop code:

```
# Define a list to keep all the results from each simulation that we want to analyze
all_stats = []

# Loop through many simulations
for i in range(num_simulations):

    # Choose random inputs for the sales targets and percent to target
    sales_target = np.random.choice(sales_target_values, num_reps, p=sales_target_pro
    pct_to_target = np.random.normal(avg, std_dev, num_reps).round(2)

    # Build the dataframe based on the inputs and number of reps
    df = pd.DataFrame(index=range(num_reps), data={'Pct_To_Target': pct_to_target,
                                                   'Sales_Target': sales_target})

    # Back into the sales number using the percent to target rate
    df['Sales'] = df['Pct_To_Target'] * df['Sales_Target']

    # Determine the commissions rate and calculate it
    df['Commission_Rate'] = df['Pct_To_Target'].apply(calc_commission_rate)
    df['Commission_Amount'] = df['Commission_Rate'] * df['Sales']

    # We want to track sales,commission amounts and sales targets over all the simula
    all_stats.append([df['Sales'].sum().round(0),
                      df['Commission_Amount'].sum().round(0),
                      df['Sales_Target'].sum().round(0)])
```

While this may seem a little intimidating at first, we are only including 7 python statements inside this loop that we can run as many times as we want. On my standard laptop, I can run 1000 simulations in 2.75s so there is no reason I can't do this many more times if need be.

At some point, there are diminishing returns. The results of 1 Million simulations are not necessarily any more useful than 10,000. My advice is to try different amounts and see how the output changes.

In order to analyze the results of the simulation, I will build a dataframe from `all_stats` :

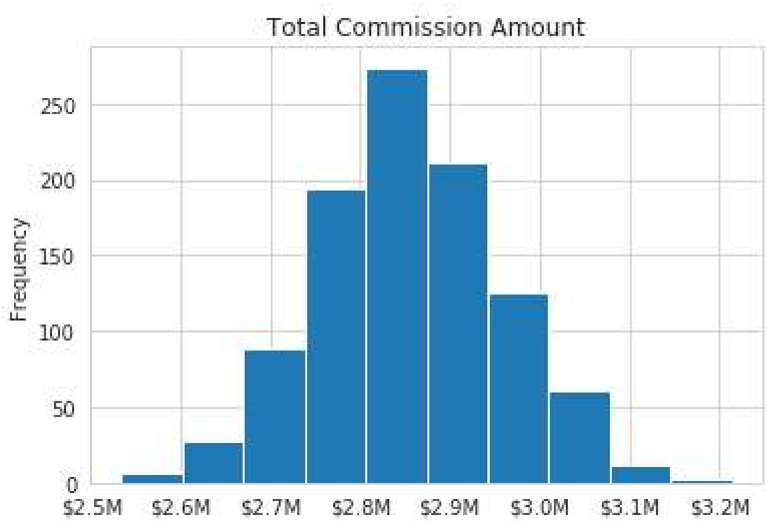
```
results_df = pd.DataFrame.from_records(all_stats, columns=['Sales',
                                                           'Commission_Amount',
                                                           'Sales_Target'])
```

Now, it is easy to see what the range of results look like:

```
results_df.describe().style.format('{:,}')
```

	Sales	Commission_Amount	Sales_Target
count	1,000.0	1,000.0	1,000.0
mean	83,617,936.0	2,854,916.1	83,619,700.0
std	2,727,222.9	103,003.9	2,702,621.8
min	74,974,750.0	2,533,810.0	75,275,000.0
25%	81,918,375.0	2,786,088.0	81,900,000.0
50%	83,432,500	2,852,165.0	83,525,000.0
75%	85,318,440.0	2,924,053.0	85,400,000.0
max	92,742,500.0	3,214,385.0	91,925,000.0

Graphically, it looks like this:



So, what does this chart and the output of describe tell us? We can see that the average commissions expense is $2.85M and the standard deviation is $103K. We can also see that the commissions payment can be as low as $2.5M or as high as $3.2M.

Based on these results, how comfortable are you that the expense for commissions will be less than $3M? Or, if someone says, “Let’s only budget $2.7M” would you feel comfortable that your expenses would be below that amount? Probably not.

Therein lies one of the benefits of the Monte Carlo simulation. You develop a better understanding of the distribution of likely outcomes and can use that knowledge plus your business acumen to make an informed estimate.

The other value of this model is that you can model many different assumptions and see what happens. Here are some simple changes you can make to see how the results change:

- Increase top commission rate to 5%
- Decrease the number of sales people
- Change the expected standard deviation to a higher amount
- Modify the distribution of targets

Now that the model is created, making these changes is as simple as a few variable tweaks and re-running your code. You can view the notebook associated with this post on github.

Another observation about Monte Carlo simulations is that they are relatively easy to explain to the end user of the prediction. The person receiving this estimate may not have a deep mathematical background but can intuitively understand what this simulation is doing and how to assess the likelihood of the range of potential results.

Finally, I think the approach shown here with python is easier to understand and replicate than some of the Excel solutions you may encounter. Because python is a programming language, there is a linear flow to the calculations which you can follow.

Conclusion

A Monte Carlo simulation is a useful tool for predicting future results by calculating a formula multiple times with different random inputs. This is a process you can execute in Excel but it is not simple to do without some VBA or potentially expensive third party plugins. Using numpy and pandas to build a model and generate multiple potential results and analyze them is relatively straightforward. The other added benefit is that analysts can run many scenarios by changing the inputs and can move on to much more sophisticated models in the future if the needs arise. Finally, the results can be shared with non-technical users and facilitate discussions around the uncertainty of the final results.

I hope this example is useful to you and gives you ideas that you can apply to your own problems. Please feel free to leave a comment if you find this article helpful for developing your own estimation models.

Updates

- 19-March-2019: Based on comments from reddit, I have made another implementation which is faster.

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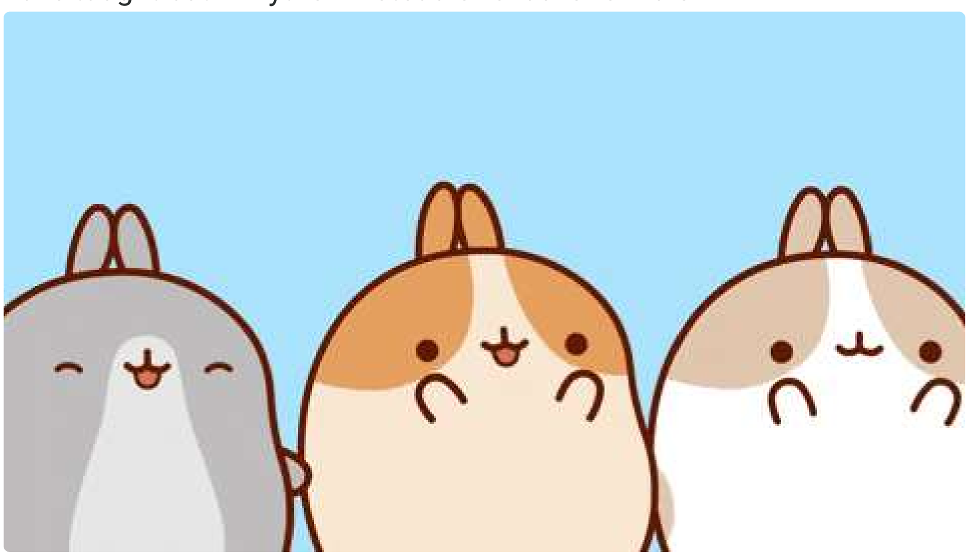
Ali A.E. Ahmed
4 years ago

This is just gold documentation for MCS in Python, just what my university professor would have taught but in Python instead of excel or simul8!

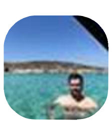


1 0 Reply

Chris Moffitt Mod → Ali A.E. Ahmed
4 years ago

Thank you. I appreciate the kind words.

1 0 Reply



Afonso O. Lenzi
5 years ago

best python approachs, thanks Chris

1 0 Reply

C
Cong Tran
2 years ago

Thank you for the example.
May i ask why you need a loop? can we just add the input arrays together to get the output array? it would save lots of computing resources and it wouldn't matter as they're all random anyway.

0 0 Reply

Yuvraj Singh
3 years ago

sales_target_prob = [.3, .3, .2, .1, .05, .05]
How are these probabilities calculated?

0 0 Reply

Chris Moffitt Mod → Yuvraj Singh
3 years ago

They are probabilities I chose based on real-world experience. In other words, you would need to choose appropriate values that all add up to 1 in order to do your own simulation. If you don't, you could assume that all values are equally likely.

0 0 Reply

Yuvraj Singh → Chris Moffitt
3 years ago

Thanks!

1 0 Reply

G
Gourab
3 years ago

Very helpful article. It would be great if you can please share the code for the histogram.

0 0 Reply

P
Padmanabhan Sairam
3 years ago

sales_target_values = [75_000, 100_000, 200_000, 300_000, 400_000, 500_000]
sales_target_prob = [.3, .3, .2, .1, .05, .05]

How did you determine the **sales_target_prob** ?
Is it randomly chosen ?
And why did you chose to keep the probability values low ?

please help !

0 0 Reply

P

Padmanabhan Sairam ➦ Padmanabhan Sairam

3 years ago

OH SORRY I GOT IT xD

And by the way this article was really helpful
Thanks a lot !!!!

0 0 Reply

Yuvraj Singh ➦ Padmanabhan Sairam

3 years ago

could you please explain it once

0 0 Reply

Sid

4 years ago

very well articulated...Thanks a ton for this article.

0 0 Reply

V

Venali Sonone

4 years ago

quite explanatory ! thanks !

0 0 Reply

C

Chris H.

5 years ago

Hello, thanks for the great post. I am trying it by my own and I am not really sure that the output are gaussian distributions, not always. Can you suggest something to implement that can check if the output dataset is a gaussian?

0 0 Reply

Chris Moffitt Mod ➦ Chris H.

5 years ago

Good question. I really liked this article for some good examples.

0 0 Reply

T

Thakib Kayodé Adéchinan Salami

5 years ago

Awesome. Thanks!

0 0 Reply

A

Alfredo Rossi

5 years ago edited

Awesome article, everything is clear and well explained.Can you provide us the finished code or the full folder so that I can try it, i am having problems building it by myself.
Thank you for your help.
Regards,
Alfredo

0 0 Reply

Chris Moffitt Mod → Alfredo Rossi
5 years ago

Sure. It's all in a jupyter notebook here.

0 0 Reply

A

Alfredo Rossi → Chris Moffitt
5 years ago

I run it, and now everything is working, i'd love to know how to build graphics and tables, maybe transform them into respectively a png and a .csv file.
Thank you very much!!

0 0 Reply

Reece Althoff
5 years ago

Great write up Chris. It inspired me a bit (along with my 4 yr old) to do a monte carlo simulation on the game of Trouble. That was a fun side project.

(Hope its ok to add a link to my not as well written post. It references back to this article)
https://www.redgamut.xyz/mo...

0 0 Reply

Chris Moffitt Mod → Reece Althoff
5 years ago

Reece - Awesome write up. Thanks for sharing. I like seeing some more examples of simulating real world issues - even if it is a game.

By any chance, have you heard of SimPy before? I recently heard about it and think it could be an interesting tool to do some real world simulations but I'm not quite sure I understand how to use it yet!

0 0 Reply

Reece Althoff → Chris Moffitt
5 years ago

Thanks! I have not heard much about SimPy, I will add it to my list of things to look into. Thanks again!

0 0 Reply

Pietro B.
5 years ago edited

thanks for the great article! love your blog, btw.

0 0 Reply

Deniell
5 years ago

Hi thanks for the great post!
I have one question:

Do we have to assume that Actual Sales & Percent_to_target are independent variables?

0 0 Reply

Chris Moffitt Mod → Deniell
5 years ago

No. I don't think we do. In my case, they are independent but there is no reason you have to assume that. Does that help?

0 0 Reply

Deniell → Chris Moffitt
5 years ago

Sorry I meant Sales Target & Percent_to_Target, not Actual_Sales & Percent_to_Target.

For example:
I think it is very possible that the higher the Sales_Target, the lower the Percent_to_Target. (If the Sales Target is very high, the sales person may not be able to meet the target.)

The process in the article doesn't seem to take that into account. Does that make sense?

0 0 Reply

Chris Moffitt Mod → Deniell
5 years ago

Ah. Yes. I understand. You are correct, that it would be useful to understand the impact of target size on the potential target achievement. Unfortunately I am not sure the best way to do that kind of model. I think you would have to do some sort of more sophisticated model. Maybe using PyMC3?

0 0 Reply

A

Amir

5 years ago

This blog in super super cool! Thanks for sharing your knowlege.

0 0 Reply

M

Michael Biermann

5 years ago

I've just listened to the talk python to me episode where you where hosted. Awesome stuff, i absolutely love what you are doing! I hope you can keep it up.

0 0 Reply

Chris Moffitt Mod → Michael Biermann

5 years ago

Thank you! I am glad you enjoyed it and thank you for the kind words.

1 0 Reply

J

Jide

5 years ago

doesn't get clearer than this!..

0 0 Reply

Tom Fawcett

5 years ago

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