

The Atomic Theory as Applied To Gases, with Some Experiments on the Viscosity of Air

by
Silas W. Holman

Submitted to the Department of Physics
in partial fulfillment of the requirements for the degree of
BACHELOR OF SCIENCE IN PHYSICS
at the
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
June 1876

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ABSTRACT

The developments of the “kinetic theory” of gases made within the last ten years have enabled it to account satisfactorily for many of the laws of gases. The mathematical deductions of Clausius, Maxwell and others, based upon the hypothesis of a gas composed of molecules acting upon each other at impact like perfectly elastic spheres, have furnished expressions for the laws of its elasticity, viscosity, conductivity for heat, diffusive power and other properties. For some of these laws we have experimental data of value in testing the validity of these deductions and assumptions. Next to the elasticity, perhaps the phenomena of the viscosity of gases are best adapted to investigation.¹

Thesis supervisor: Edward C. Pickering
Title: Professor of Physics

¹Text from Holman (1876): doi:[10.2307/25138434](https://doi.org/10.2307/25138434).

Acknowledgments

Write your acknowledgments here.

Biographical Sketch

Silas Whitcomb Holman was born in Harvard, Massachusetts on January 20, 1856. He received his S.B. degree in Physics from MIT in 1876, and then joined the MIT Department of Physics as an Assistant. He became Instructor in Physics in 1880, Assistant Professor in 1882, Associate Professor in 1885, and Full Professor in 1893. Throughout this period, he struggled with increasingly severe rheumatoid arthritis. At length, he was defeated, becoming Professor Emeritus in 1897 and dying on April 1, 1900.

Holman's light burned brilliantly before his tragic and untimely death. He published extensively in thermal physics, and authored textbooks on precision measurement, fundamental mechanics, and other subjects. He established the original Heat Measurements Laboratory. Holman was a much admired teacher among both his students and his colleagues. The reports of his department and of the Institute itself refer to him frequently in the 1880's and 1890's, in tones that gradually shift from the greatest respect to the deepest sympathy.

Holman was a student of Professor Edward C. Pickering, then head of the Physics department. Holman himself became second in command of Physics, under Professor Charles R. Cross, some years later. Among Holman's students, several went on to distinguish themselves, including: the astronomer George E. Hale ('90) who organized the Yerkes and Mt. Wilson observatories and who designed the 200 inch telescope on Mt. Palomar; Charles G. Abbot ('94), also an astrophysicist and later Secretary of the Smithsonian Institution; and George K. Burgess ('96), later Director of the Bureau of Standards.

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Chapter 1

Introduction

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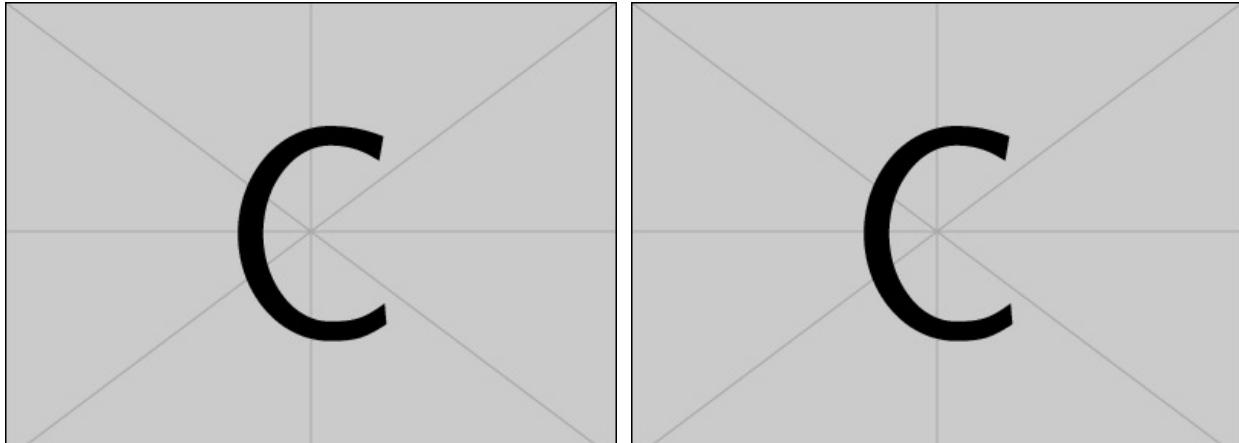
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1.1 A section discussing the first issue: J/ψ

We begin with some ideas from the literature [6,7].

$$\frac{\partial}{\partial t} [\rho(e + |\vec{u}|^2/2)] + \nabla \cdot [\rho(h + |\vec{u}|^2/2) \vec{u}] = -\nabla \cdot \vec{q} + \rho \vec{u} \cdot \vec{g} + \frac{\partial}{\partial x_j} (d_{ji} u_i) \quad (1.1)$$

Nulla malesuada porttitor diam. Donec felis erat, congue non, volutpat at, tincidunt tristique, libero. Vivamus viverra fermentum felis. Donec nonummy pellentesque ante. Phasellus adipiscing semper elit. Proin fermentum massa ac quam. Sed diam turpis, molestie vitae, placerat a, molestie nec, leo. Maecenas lacinia. Nam ipsum ligula, eleifend at, accumsan nec, suscipit a, ipsum. Morbi blandit ligula feugiat magna. Nunc eleifend



(a) First subfigure

(b) Second subfigure

Figure 1.1: A figure with two subfigures: (a) first subfigure; (b) second subfigure.

consequat lorem. Sed lacinia nulla vitae enim. Pellentesque tincidunt purus vel magna. Integer non enim. Praesent euismod nunc eu purus. Donec bibendum quam in tellus. Nullam cursus pulvinar lectus. Donec et mi. Nam vulputate metus eu enim. Vestibulum pellentesque felis eu massa.

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Quisque ullamcorper placerat ipsum. Cras nibh. Morbi vel justo vitae lacinia tincidunt ultrices. Lorem ipsum dolor sit amet, consectetur adipiscing elit. In hac habitasse platea dictumst. Integer tempus convallis augue. Etiam facilisis. Nunc elementum fermentum wisi. Aenean placerat. Ut imperdiet, enim sed gravida sollicitudin, felis odio placerat quam, ac pulvinar elit purus eget enim. Nunc vitae tortor. Proin tempus nibh sit amet nisl. Vivamus quis tortor vitae risus porta vehicula.

1.1.1 Subsection eqn. (1.2)

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Suspendisse vel felis. Ut lorem lorem, interdum eu, tincidunt sit amet, laoreet vitae, arcu. Aenean faucibus pede eu ante. Praesent enim elit, rutrum at, molestie non, nonummy vel, nisl. Ut lectus eros, malesuada sit amet, fermentum eu, sodales cursus, magna. Donec eu purus. Quisque vehicula, urna sed ultricies auctor, pede lorem egestas dui, et convallis elit erat sed nulla. Donec luctus. Curabitur et nunc. Aliquam dolor odio, commodo pretium, ultricies non, pharetra in, velit. Integer arcu est, nonummy in, fermentum faucibus, egestas vel, odio.

A subsubsection

Sed commodo posuere pede. Mauris ut est. Ut quis purus. Sed ac odio. Sed vehicula hendrerit sem. Duis non odio. Morbi ut dui. Sed accumsan risus eget odio. In hac habitasse platea dictumst. Pellentesque non elit. Fusce sed justo eu urna porta tincidunt. Mauris felis odio, sollicitudin sed, volutpat a, ornare ac, erat. Morbi quis dolor. Donec pellentesque, erat ac sagittis semper, nunc dui lobortis purus, quis congue purus metus ultricies tellus. Proin et quam. Class aptent taciti sociosqu ad litora torquent per conubia nostra, per inceptos hymenaeos. Praesent sapien turpis, fermentum vel, eleifend faucibus, vehicula eu, lacus.

$$L(\mathbf{A}) = \begin{pmatrix} \frac{\varphi}{(\varphi_1, \varepsilon_1)} & 0 & \dots & \dots & \dots & 0 \\ \frac{\varphi k_{2,1}}{(\varphi_2, \varepsilon_1)} & \frac{\varphi}{(\varphi_2, \varepsilon_2)} & 0 & \dots & \dots & 0 \\ \frac{\varphi k_{3,1}}{(\varphi_3, \varepsilon_1)} & \frac{\varphi k_{3,2}}{(\varphi_3, \varepsilon_2)} & \frac{\varphi}{(\varphi_3, \varepsilon_3)} & 0 & \dots & 0 \\ \vdots & \vdots & & \ddots & & \vdots \\ \frac{\varphi k_{n-1,1}}{(\varphi_{n-1}, \varepsilon_1)} & \frac{\varphi k_{n-1,2}}{(\varphi_{n-1}, \varepsilon_2)} & \dots & \frac{\varphi k_{n-1,n-2}}{(\varphi_{n-1}, \varepsilon_{n-2})} & \frac{\varphi}{(\varphi_{n-1}, \varepsilon_{n-1})} & 0 \\ \frac{\varphi k_{n,1}}{(\varphi_n, \varepsilon_1)} & \frac{\varphi k_{n,2}}{(\varphi_n, \varepsilon_2)} & \dots & \dots & \frac{\varphi k_{n,n-1}}{(\varphi_n, \varepsilon_{n-1})} & \frac{\varphi}{(\varphi_n, \varepsilon_n)} \end{pmatrix} \quad (1.2)$$

1.2 Description our paradigm

Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Donec odio elit, dictum in, hendrerit sit amet, egestas sed, leo. Praesent feugiat sapien aliquet odio. Integer vitae justo. Aliquam vestibulum fringilla lorem. Sed neque lectus, consectetur at, consectetur sed, eleifend ac, lectus. Nulla facilisi. Pellentesque eget lectus. Proin eu metus. Sed porttitor. In hac habitasse platea dictumst. Suspendisse eu lectus. Ut mi mi, lacinia sit amet, placerat et, mollis vitae, dui. Sed ante tellus, tristique

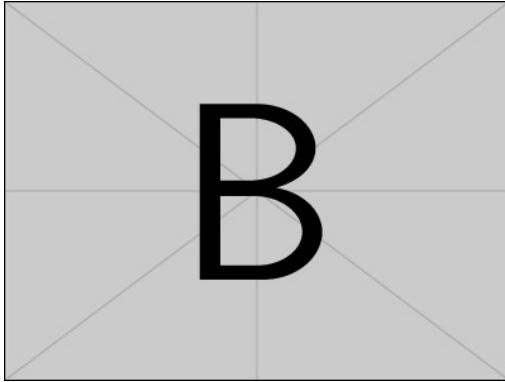


Figure 1.2: Caption text [8].

ut, iaculis eu, malesuada ac, dui. Mauris nibh leo, facilisis non, adipiscing quis, ultrices a,
dui. No dissertation is complete without footnotes.^{1,2,3}

1.2.1 Conversion to a metaheuristic

Sed feugiat. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur
ridiculus mus. Ut pellentesque augue sed urna. Vestibulum diam eros, fringilla et,
consectetuer eu, nonummy id, sapien. Nullam at lectus. In sagittis ultrices mauris.
Curabitur malesuada erat sit amet massa. Fusce blandit. Aliquam erat volutpat. Aliquam
euismod. Aenean vel lectus. Nunc imperdiet justo nec dolor.

Etiam euismod. Fusce facilisis lacinia dui. Suspendisse potenti. In mi erat, cursus id,
nonummy sed, ullamcorper eget, sapien. Praesent pretium, magna in eleifend egestas,
pede pede pretium lorem, quis consectetur tortor sapien facilisis magna. Mauris quis
magna varius nulla scelerisque imperdiet. Aliquam non quam. Aliquam porttitor quam
a lacus. Praesent vel arcu ut tortor cursus volutpat. In vitae pede quis diam bibendum
placerat. Fusce elementum convallis neque. Sed dolor orci, scelerisque ac, dapibus nec,
ultricies ut, mi. Duis nec dui quis leo sagittis commodo. This concept is discussed further
in section 1.4, and Refs. [18,19].

1.3 Other generalizations

1.3.1 The most general case

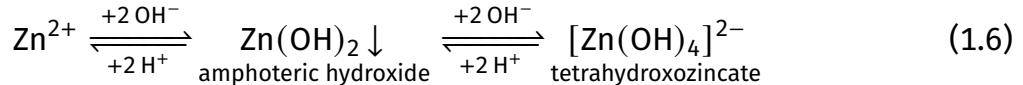
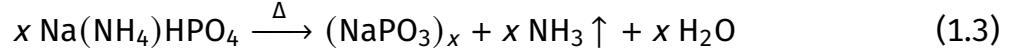
Sed commodo posuere pede. Mauris ut est. Ut quis purus. Sed ac odio. Sed vehicula
henderit sem. Duis non odio. Morbi ut dui. Sed accumsan risus eget odio. In hac
habitasse platea dictumst. Pellentesque non elit. Fusce sed justo eu urna porta tincidunt.
Mauris felis odio, sollicitudin sed, volutpat a, ornare ac, erat. Morbi quis dolor. Donec

¹First footnote. $a_h = F_m$ See section 1.4.

²Another interesting detail.

³And another really important idea to have in mind [12–17].

pellentesque, erat ac sagittis semper, nunc dui lobortis purus, quis congue purus metus ultricies tellus. Proin et quam. Class aptent taciti sociosqu ad litora torquent per conubia nostra, per inceptos hymenaeos. Praesent sapien turpis, fermentum vel, eleifend faucibus, vehicula eu, lacus. And another citation, so that our sources will be unambiguous [20].



These examples of chemical formulæ are copied directly from the documentation of the `mhchem` package, which was used to typeset them.

1.4 Baroclinic generation of vorticity

Substitution of the particle acceleration and application Stokes theorem leads to the *Kelvin-Bjerknes circulation theorem*, for $\rho \neq \text{fn}(p)$:

$$\frac{d\Gamma}{dt} = \frac{d}{dt} \int_C \mathbf{u} \cdot d\mathbf{r} \quad (1.7)$$

$$= \int_C \frac{D\mathbf{u}}{Dt} \cdot d\mathbf{r} + \underbrace{\int_C \mathbf{u} \cdot d\left(\frac{d\mathbf{r}}{dt}\right)}_{=0} \quad (1.8)$$

$$= \iint_S \nabla \times \frac{D\mathbf{u}}{Dt} \cdot d\mathbf{A} \quad (1.9)$$

$$= \iint_S \nabla p \times \nabla \left(\frac{1}{\rho} \right) \cdot d\mathbf{A} \quad (1.10)$$

Baroclinic generation of vorticity accounts for the sea breeze and various other atmospheric currents in which temperature, rather than pressure, creates density gradients. Further, this phenomenon accounts for ocean currents in straits joining more and less saline seas, with surface currents flowing from the fresher to the saltier water and with bottom current going oppositely.

1.5 Summary

Etiam euismod. Fusce facilisis lacinia dui. Suspendisse potenti. In mi erat, cursus id, nonummy sed, ullamcorper eget, sapien. Praesent pretium, magna in eleifend egestas,

Table 1.1: The error function and complementary error function

| x | $\text{erf}(x)$ | $\text{erfc}(x)$ | x | $\text{erf}(x)$ | $\text{erfc}(x)$ |
|------|-----------------|------------------|--------|-----------------|------------------|
| 0.00 | 0.00000 | 1.00000 | 1.10 | 0.88021 | 0.11980 |
| 0.05 | 0.05637 | 0.94363 | 1.20 | 0.91031 | 0.08969 |
| 0.10 | 0.11246 | 0.88754 | 1.30 | 0.93401 | 0.06599 |
| 0.15 | 0.16800 | 0.83200 | 1.40 | 0.95229 | 0.04771 |
| 0.20 | 0.22270 | 0.77730 | 1.50 | 0.96611 | 0.03389 |
| 0.30 | 0.32863 | 0.67137 | 1.60 | 0.97635 | 0.02365 |
| 0.40 | 0.42839 | 0.57161 | 1.70 | 0.98379 | 0.01621 |
| 0.50 | 0.52050 | 0.47950 | 1.80 | 0.98909 | 0.01091 |
| 0.60 | 0.60386 | 0.39614 | 1.8214 | 0.99000 | 0.01000 |
| 0.70 | 0.67780 | 0.32220 | 1.90 | 0.99279 | 0.00721 |
| 0.80 | 0.74210 | 0.25790 | 2.00 | 0.99532 | 0.00468 |
| 0.90 | 0.79691 | 0.20309 | 2.50 | 0.99959 | 0.00041 |
| 1.00 | 0.84270 | 0.15730 | 3.00 | 0.99998 | 0.00002 |

pede pede pretium lorem, quis consectetur tortor sapien facilisis magna. Mauris quis magna varius nulla scelerisque imperdiet. Aliquam non quam. Aliquam porttitor quam a lacus. Praesent vel arcu ut tortor cursus volutpat. In vitae pede quis diam bibendum placerat. Fusce elementum convallis neque. Sed dolor orci, scelerisque ac, dapibus nec, ultricies ut, mi. Duis nec dui quis leo sagittis commodo.

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Nomenclature for Chapter 1

Roman letters

- C material curve
- r material position [m]
- u velocity [m s^{-1}]

Greek letters

- Γ circulation [$\text{m}^2 \text{s}^{-1}$]
- ρ mass density [kg m^{-3}]
- ω vorticity [s^{-1}]

Appendix A

One-term coefficients for heat conduction

A.1 A multipage table of numbers

This example uses the `longtable` package: $\theta = A_1 f_1 \exp(-\lambda_1^2 \text{Fo})$, $\bar{\theta} = D_1 \exp(-\lambda_1^2 \text{Fo})$.

Table A.1: One-term coefficients for one-dimensional heat conduction with a convective boundary condition. Data follow H. D. Baehr and K. Stephan [21].

| Bi | Plate | | | Cylinder | | | Sphere | | |
|------|-------------|--------|--------|-------------|--------|--------|-------------|--------|--------|
| | λ_1 | A_1 | D_1 | λ_1 | A_1 | D_1 | λ_1 | A_1 | D_1 |
| 0.01 | 0.09983 | 1.0017 | 1.0000 | 0.14124 | 1.0025 | 1.0000 | 0.17303 | 1.0030 | 1.0000 |
| 0.02 | 0.14095 | 1.0033 | 1.0000 | 0.19950 | 1.0050 | 1.0000 | 0.24446 | 1.0060 | 1.0000 |
| 0.03 | 0.17234 | 1.0049 | 1.0000 | 0.24403 | 1.0075 | 1.0000 | 0.29910 | 1.0090 | 1.0000 |
| 0.04 | 0.19868 | 1.0066 | 1.0000 | 0.28143 | 1.0099 | 1.0000 | 0.34503 | 1.0120 | 1.0000 |
| 0.05 | 0.22176 | 1.0082 | 0.9999 | 0.31426 | 1.0124 | 0.9999 | 0.38537 | 1.0150 | 1.0000 |
| 0.06 | 0.24253 | 1.0098 | 0.9999 | 0.34383 | 1.0148 | 0.9999 | 0.42173 | 1.0179 | 0.9999 |
| 0.07 | 0.26153 | 1.0114 | 0.9999 | 0.37092 | 1.0173 | 0.9999 | 0.45506 | 1.0209 | 0.9999 |
| 0.08 | 0.27913 | 1.0130 | 0.9999 | 0.39603 | 1.0197 | 0.9999 | 0.48600 | 1.0239 | 0.9999 |
| 0.09 | 0.29557 | 1.0145 | 0.9998 | 0.41954 | 1.0222 | 0.9998 | 0.51497 | 1.0268 | 0.9999 |
| 0.10 | 0.31105 | 1.0161 | 0.9998 | 0.44168 | 1.0246 | 0.9998 | 0.54228 | 1.0298 | 0.9998 |
| 0.15 | 0.37788 | 1.0237 | 0.9995 | 0.53761 | 1.0365 | 0.9995 | 0.66086 | 1.0445 | 0.9996 |
| 0.20 | 0.43284 | 1.0311 | 0.9992 | 0.61697 | 1.0483 | 0.9992 | 0.75931 | 1.0592 | 0.9993 |
| 0.25 | 0.48009 | 1.0382 | 0.9988 | 0.68559 | 1.0598 | 0.9988 | 0.84473 | 1.0737 | 0.9990 |
| 0.30 | 0.52179 | 1.0450 | 0.9983 | 0.74646 | 1.0712 | 0.9983 | 0.92079 | 1.0880 | 0.9985 |
| 0.40 | 0.59324 | 1.0580 | 0.9971 | 0.85158 | 1.0931 | 0.9970 | 1.05279 | 1.1164 | 0.9974 |
| 0.50 | 0.65327 | 1.0701 | 0.9956 | 0.94077 | 1.1143 | 0.9954 | 1.16556 | 1.1441 | 0.9960 |
| 0.60 | 0.70507 | 1.0814 | 0.9940 | 1.01844 | 1.1345 | 0.9936 | 1.26440 | 1.1713 | 0.9944 |
| 0.70 | 0.75056 | 1.0918 | 0.9922 | 1.08725 | 1.1539 | 0.9916 | 1.35252 | 1.1978 | 0.9925 |
| 0.80 | 0.79103 | 1.1016 | 0.9903 | 1.14897 | 1.1724 | 0.9893 | 1.43203 | 1.2236 | 0.9904 |
| 0.90 | 0.82740 | 1.1107 | 0.9882 | 1.20484 | 1.1902 | 0.9869 | 1.50442 | 1.2488 | 0.9880 |
| 1.00 | 0.86033 | 1.1191 | 0.9861 | 1.25578 | 1.2071 | 0.9843 | 1.57080 | 1.2732 | 0.9855 |
| 1.10 | 0.89035 | 1.1270 | 0.9839 | 1.30251 | 1.2232 | 0.9815 | 1.63199 | 1.2970 | 0.9828 |
| 1.20 | 0.91785 | 1.1344 | 0.9817 | 1.34558 | 1.2387 | 0.9787 | 1.68868 | 1.3201 | 0.9800 |
| 1.30 | 0.94316 | 1.1412 | 0.9794 | 1.38543 | 1.2533 | 0.9757 | 1.74140 | 1.3424 | 0.9770 |
| 1.40 | 0.96655 | 1.1477 | 0.9771 | 1.42246 | 1.2673 | 0.9727 | 1.79058 | 1.3640 | 0.9739 |

Table A.1: (continued)

| Bi | Plate | | | Cylinder | | | Sphere | | |
|----------|-------------|--------|--------|-------------|--------|--------|-------------|--------|--------|
| | λ_1 | A_1 | D_1 | λ_1 | A_1 | D_1 | λ_1 | A_1 | D_1 |
| 1.50 | 0.98824 | 1.1537 | 0.9748 | 1.45695 | 1.2807 | 0.9696 | 1.83660 | 1.3850 | 0.9707 |
| 1.60 | 1.00842 | 1.1593 | 0.9726 | 1.48917 | 1.2934 | 0.9665 | 1.87976 | 1.4052 | 0.9674 |
| 1.70 | 1.02725 | 1.1645 | 0.9703 | 1.51936 | 1.3055 | 0.9633 | 1.92035 | 1.4247 | 0.9640 |
| 1.80 | 1.04486 | 1.1695 | 0.9680 | 1.54769 | 1.3170 | 0.9601 | 1.95857 | 1.4436 | 0.9605 |
| 1.90 | 1.06136 | 1.1741 | 0.9658 | 1.57434 | 1.3279 | 0.9569 | 1.99465 | 1.4618 | 0.9570 |
| 2.00 | 1.07687 | 1.1785 | 0.9635 | 1.59945 | 1.3384 | 0.9537 | 2.02876 | 1.4793 | 0.9534 |
| 2.20 | 1.10524 | 1.1864 | 0.9592 | 1.64557 | 1.3578 | 0.9472 | 2.09166 | 1.5125 | 0.9462 |
| 2.40 | 1.13056 | 1.1934 | 0.9549 | 1.68691 | 1.3754 | 0.9408 | 2.14834 | 1.5433 | 0.9389 |
| 2.60 | 1.15330 | 1.1997 | 0.9509 | 1.72418 | 1.3914 | 0.9345 | 2.19967 | 1.5718 | 0.9316 |
| 2.80 | 1.17383 | 1.2052 | 0.9469 | 1.75794 | 1.4059 | 0.9284 | 2.24633 | 1.5982 | 0.9243 |
| 3.00 | 1.19246 | 1.2102 | 0.9431 | 1.78866 | 1.4191 | 0.9224 | 2.28893 | 1.6227 | 0.9171 |
| 3.50 | 1.23227 | 1.2206 | 0.9343 | 1.85449 | 1.4473 | 0.9081 | 2.38064 | 1.6761 | 0.8995 |
| 4.00 | 1.26459 | 1.2287 | 0.9264 | 1.90808 | 1.4698 | 0.8950 | 2.45564 | 1.7202 | 0.8830 |
| 4.50 | 1.29134 | 1.2351 | 0.9193 | 1.95248 | 1.4880 | 0.8830 | 2.51795 | 1.7567 | 0.8675 |
| 5.00 | 1.31384 | 1.2402 | 0.9130 | 1.98981 | 1.5029 | 0.8721 | 2.57043 | 1.7870 | 0.8533 |
| 6.00 | 1.34955 | 1.2479 | 0.9021 | 2.04901 | 1.5253 | 0.8532 | 2.65366 | 1.8338 | 0.8281 |
| 7.00 | 1.37662 | 1.2532 | 0.8932 | 2.09373 | 1.5411 | 0.8375 | 2.71646 | 1.8673 | 0.8069 |
| 8.00 | 1.39782 | 1.2570 | 0.8858 | 2.12864 | 1.5526 | 0.8244 | 2.76536 | 1.8920 | 0.7889 |
| 9.00 | 1.41487 | 1.2598 | 0.8796 | 2.15661 | 1.5611 | 0.8133 | 2.80443 | 1.9106 | 0.7737 |
| 10.00 | 1.42887 | 1.2620 | 0.8743 | 2.17950 | 1.5677 | 0.8039 | 2.83630 | 1.9249 | 0.7607 |
| 12.00 | 1.45050 | 1.2650 | 0.8658 | 2.21468 | 1.5769 | 0.7887 | 2.88509 | 1.9450 | 0.7397 |
| 14.00 | 1.46643 | 1.2669 | 0.8592 | 2.24044 | 1.5828 | 0.7770 | 2.92060 | 1.9581 | 0.7236 |
| 16.00 | 1.47864 | 1.2683 | 0.8541 | 2.26008 | 1.5869 | 0.7678 | 2.94756 | 1.9670 | 0.7109 |
| 18.00 | 1.48830 | 1.2692 | 0.8499 | 2.27556 | 1.5898 | 0.7603 | 2.96871 | 1.9734 | 0.7007 |
| 20.00 | 1.49613 | 1.2699 | 0.8464 | 2.28805 | 1.5919 | 0.7542 | 2.98572 | 1.9781 | 0.6922 |
| 25.00 | 1.51045 | 1.2710 | 0.8400 | 2.31080 | 1.5954 | 0.7427 | 3.01656 | 1.9856 | 0.6766 |
| 30.00 | 1.52017 | 1.2717 | 0.8355 | 2.32614 | 1.5973 | 0.7348 | 3.03724 | 1.9898 | 0.6658 |
| 35.00 | 1.52719 | 1.2721 | 0.8322 | 2.33719 | 1.5985 | 0.7290 | 3.05207 | 1.9924 | 0.6579 |
| 40.00 | 1.53250 | 1.2723 | 0.8296 | 2.34552 | 1.5993 | 0.7246 | 3.06321 | 1.9942 | 0.6519 |
| 50.00 | 1.54001 | 1.2727 | 0.8260 | 2.35724 | 1.6002 | 0.7183 | 3.07884 | 1.9962 | 0.6434 |
| 60.00 | 1.54505 | 1.2728 | 0.8235 | 2.36510 | 1.6007 | 0.7140 | 3.08928 | 1.9974 | 0.6376 |
| 80.00 | 1.55141 | 1.2730 | 0.8204 | 2.37496 | 1.6013 | 0.7085 | 3.10234 | 1.9985 | 0.6303 |
| 100.00 | 1.55525 | 1.2731 | 0.8185 | 2.38090 | 1.6015 | 0.7052 | 3.11019 | 1.9990 | 0.6259 |
| 200.00 | 1.56298 | 1.2732 | 0.8146 | 2.39283 | 1.6019 | 0.6985 | 3.12589 | 1.9998 | 0.6170 |
| ∞ | 1.57080 | 1.2732 | 0.8106 | 2.40483 | 1.6020 | 0.6917 | 3.14159 | 2.0000 | 0.6079 |

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