Central limit Theorem Assignment

1) $\mu = 10$, $n = 100$, $\sigma_{Pop} = 4$ $\sigma_{sample\ distribution} = \frac{\sigma_{Pop}}{\sqrt{n}} \Rightarrow \frac{4}{\sqrt{100}} = 0.4$

$$Z\ score = \frac{x - \mu}{\sigma} = \frac{9 - 10}{0.4}$$

$$\Rightarrow \frac{-1}{0.4} = -2.5$$

Z Score of $(-2.5) = 0.0062$ or 0.62%.

2) Let x be the weight of Students

$x \sim N(\mu, \sigma^2)$

$x \sim N(50, 15^2)$

$n = 10$

Prob(that all 10 Students will Reach Safety 8th floor?

$$\Rightarrow P\left(\sum_{x}^{10} x \leq 550\right)$$

$$\Rightarrow \bar{x} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$$

$$\Rightarrow \Sigma x_i \sim N(n\mu, n\sigma^2)$$

$$\Rightarrow P(\Sigma x \leq 550) = P\left(Z \leq \frac{550 - 10 \times 50}{\sqrt{10 \times 15^2}}\right)$$

$$\Rightarrow = P\left(Z \leq \frac{50}{15 \times \sqrt{10}}\right)$$

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Subject :

$\Rightarrow (P\ Z \leq 1.0541)$

$= 0.8531$ or 85.31%.

Prob (all students can safely reach 8th floor) $= 1 - 0.8531$
$= 0.15\%$.

3)

$x = 250/100 = 2.5 \quad \mu = 2.4 \quad \sigma = 2$

$$\text{ZScore} = \frac{x - \mu}{\sigma} = \frac{2.5 - 2.4}{\frac{2.0}{\sqrt{100}}}$$

$$= \frac{0.1}{2.0} \times 10$$

$$= \frac{1}{2} = 0.5$$

ZScore of $0.5 = 0.6915$

Prob(100 Passengers will be able to Purchase all 250 tickets) $= 1 - 0.6915$

$= 0.3085$

Date

Subject :

4) $x = 98$, $n = 35$, $\sigma_{POP} = 16$, $\mu_{POP} = 96$

$$Z\text{Score} = \frac{x - \mu}{\sigma} = \frac{98 - 96}{\frac{16}{\sqrt{35}}}$$

$$\Rightarrow 2 \times \frac{\sqrt{35}}{16}$$

$$= \frac{5.91}{8} = 0.73$$

ZScore of $0.73 = 0.7673$

Prob (average IQ Score > 98 pt) $= 1 - 0.7673$

$= 0.2327$

5) (a) $\mu = 6$ ~~SD =~~ $\sigma = 1$

$$Z = \frac{x - \mu}{\sigma} = \frac{6.2 - 6}{1} = 0.2$$

ZScore of $0.2 = 0.5793$

Prob(head breadth < 6.2 inch) = 57.93%.

Subject :

b) $\mu = 6 \quad n = 100, \sigma_{POP} = 1$

$\sigma_{Sample\ distribution} = \frac{\sigma_{POP}}{\sqrt{n}} = \frac{1}{\sqrt{100}} = 0.1$

$Z\ Score = \frac{x - \mu}{\sigma} = \frac{6.2 - 6}{0.1} = \frac{0.2}{0.1} = 2$

Z Score of 2 = 0.9772

Prob(Mean < 6.2 inch for 100 men) = 97.72 %.

b) Prob(head breadth < 6.2 inch) = 57.93

This will turn out a wrong for men who fall in;

$\epsilon \Rightarrow (1 - 57.93) = 42.07$

By this we can say that 42.07 % men won't ~~fit~~ in helmet whose breadth < 6.2 inches.

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7) $x = 260$, $\mu = 266$, $\sigma = 15$, $n = 25$

$$Z\text{Score} = \frac{x-\mu}{\sigma} = \frac{260-266}{\frac{15}{\sqrt{25}}}$$

$$= \frac{-6}{3} = -2.66$$

ZScore of $-2.66 = 0.0039$ or 0.39%.

8) $(0.39 < .1)$ - This is a sequel of the above Problem. I am not able to figure out the solution for this.

9)(a) $x = 190$, $\mu = 172$, $\sigma = 29$

$$Z = \frac{x-\mu}{\sigma} = \frac{190-172}{29} = \frac{18}{29} = 0.620\%.$$

ZScore of $0.62 = 0.7324$

$Prob(Z > 0.62) = 1 - 0.7324 = 0.2676$ or 26.76%

(b) $Z = \frac{x-\mu}{\sigma} \Rightarrow \frac{190-172}{29}$

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→) $\frac{190 - 172}{\frac{29}{\sqrt{25}}}$

⇒) $\frac{18}{29} \times 5 = \frac{90}{29} = 3.10$

2 Sided of $3.10 = 0.9990$

$Prob(Z > 3.10) = 1 - 0.9990$

$= 0.0010$ or 0.1%.

(c) $Z = \frac{x - \mu}{\sigma}$ = ~~$4750 - 25 \times 225 \times$~~

⇒) $\frac{4750 - 25 \times 172}{\frac{29}{\sqrt{25}} \times 25}$

⇒) $\left(\frac{4750 - 4300}{29 \times 5}\right)$

⇒) $\frac{450 \times 5}{29}$

⇒) $\frac{2250}{29} = 3.103$

2-Sided of $3.103 = 0.9990$

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$P(x < 3.10) = 1 - 0.9990 = 0.01$ or 0.1%.

10) $x_1 = 3.59$

$$z_1 = \frac{x_1 - \mu}{\sigma} = \frac{3.5 - 4}{\frac{1.5}{\sqrt{50}}}$$

$$= \frac{0.5}{1.5} \times \sqrt{50}$$

$$= \frac{7.0710}{3}$$

$$= 2.357$$

z Score of $2.357 = 0.9906$

$x_2 = 3.89$

$$z_2 = \frac{x_2 - \mu}{\sigma}$$

$$= \frac{3.8 - 4}{\frac{1.5}{\sqrt{50}}}$$

$$= \frac{0.2}{1.5} \times \sqrt{50}$$

$$= 0.942$$

z Score of $0.942 = 0.8264$

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Subject :

Prob (Z1) - P(Z>2)

= 0.9906 - 0.8264

= 0.1642 or 16.42 %.

11) $Z\text{Score} = \frac{x-\mu}{\sigma} = \frac{27 - 23.1}{\frac{3.1}{\sqrt{8}}}$

$= \frac{3.9}{1.265}$ 23.081

ZScore of 3.081 = 0.9990

Prob(8 students >27) = 1 - 0.9990

= 0.0010 or 0.1%.

12) $x_1 = 20$

$Z_1 = \frac{x_1 - \mu}{\sigma}$

$= \frac{20 - 21.50}{\frac{2.22}{\sqrt{8}}}$

$= \frac{-1.50}{0.7848}$

$= -1.9110$

$x_2 = 23$

$Z_2 = \frac{x_2 - \mu}{\sigma}$

$= \frac{23 - 21.50}{\frac{2.22}{\sqrt{8}}}$

$= \frac{1.50}{0.7848}$

$= 1.9110$

Z Score for -1.9110 = 0.0281

Z Score of 1.9110 = 0.9719

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Subject :

Prob (avg of 8 checks between $20 & $23) :

$= 0.9719 - 0.0281$

$\Rightarrow 0.9438$ or 94.38%.

13) (a) $\mu = 75, \sigma = 5$

$$\text{ZScore} = \frac{x-\mu}{\sigma} = \frac{83-75}{5} = \frac{8}{5} = 1.6$$

ZScore of $1.6 = 0.9452$

Prob (Student has a grade of atleast 83) $= 1 - 0.9452$

$= 0.0548$

(b) $$\text{ZScore} = \frac{x-\mu}{\sigma} = \frac{83-75}{\frac{5}{\sqrt{5}}} = \frac{8}{\sqrt{5}}$$

$= 3.57771$

ZScore of $3.57771 = 0.9999$

Prob (avg grade of 5 Students atleast 83) :

$= 1 - 0.9999$

$= 0.0001$

Subject :

14) $\mu = 28.3$, $\sigma = 2.3$, $n = 10$, $x = 27$

$$Z\text{Score} = \frac{x - \mu}{\sigma}$$

$$= \frac{27 - 28.3}{\frac{2.3}{\sqrt{10}}}$$

$$= -1.79$$

ZScore of $1.79 = 0.0367$

Prob(Avg age of 10 Players < 27 years) $= 0.3671$.

The End