Subject: Distributions Assignment Date

2) (a) $\mu = 38000$, $\sigma = 10{,}000$ $x = 50{,}000$

$$Z = \frac{x-\mu}{\sigma} \Rightarrow \frac{50{,}000 - 38000}{10{,}000} = \frac{12000}{10{,}000} = 1.2$$

Z Score for 1.2 = 0.8849

$P(x > 50{,}000) = P(Z > 1.2)$

$\Rightarrow 1 - P(Z > 1.2)$

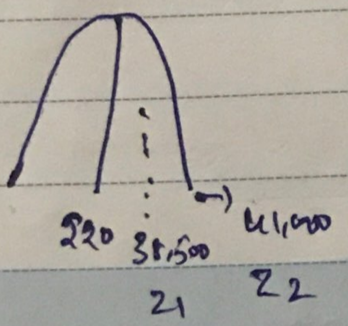
$\Rightarrow 1 - 0.8849 = 0.11$ or 11 %.

11 % of the firms have sales over 50,000

~~No of sales~~ No of firms making sales > 50,000 = 0.11 × 2000

= 220 firms

(b)



$P(38500 < x < 41000)$

$$Z_1 = \frac{38500 - 38000}{10{,}000} = 0.05$$

$$Z_2 = \frac{41000 - 38000}{10{,}000} = 0.3$$

Experience certainty.

TATA

Subject :

Date

$P(Z<0.3) = 0.6179$

$P(Z<0.05) = 0.5199$

$\therefore P(0.05<Z<0.3) = 0.6179 - 0.5199 = 0.098$ or 10%. ~~approx~~

(c) $x_1 = 30,000$

$x_2 = 50,000$

$$z_1 = \frac{30,000 - 38,000}{10,000} = -0.8$$

$$z_2 = \frac{50,000 - 38,000}{10,000} = 1.2$$

$\therefore P(z_1 < Z < z_2)$

$\Rightarrow P(-0.8 < Z < 1.2)$

$= \cancel{P(Z<} \; P(Z<1.2) - P(Z<-0.8)$

$\Rightarrow P(Z<1.2) - [1 - P(Z<0.8)]$

$= \cancel{0.88493 - 1 + 0.77}$

$= 0.88493 - 1 + 0.78814$

$= 0.673$ or 67.3%.

No of firms sales between Rs 30,000 and Rs 50,000 =

$0.673 \times 2000 = 1346$ firms

Subject :　　　　　　　　　　　　　Date

3) Probability of Right answer = 1/4
Probability of wrong answer = 3/4

Probability of 5 wrong answers = Probability of 15 right answers

$P(x=15) = {}^nC_r\, p^r q^{n-r}$

$= {}^{20}C_{15} \left(\frac{1}{4}\right)^{15} \left(\frac{3}{4}\right)^{5}$

$= 15504 * (9.313 \times 10^{-10}) * 0.2373$

$= 3.43 \times 10^{-6} = 0.00000342$

4) $\lambda = 4$ Photons/sec

$PMF = P(x=0) = \frac{e^{-\lambda}\lambda^x}{x!} = \frac{e^{-4}4^0}{0!}$

$\Rightarrow \frac{e^{-4} \times 1}{1} = e^{-4}$

$= 0.0183$ or 1.83%.

a) $\lambda = 23$ = No of calls coming Per week
$P(x=0)$ Prob of no of calls coming in Per minute

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$$\frac{e^{-3} \times 3^0}{0!} = e^{-3} = \cancel{0.0498}\ 0.0498 \text{ or } 4.98\%.$$

(b) In 1 minutes, 3 calls arrive on an average
In 2 minutes, 6 calls arrive on an average
i.e. $\lambda = 6$ $\quad i = 2$

$$P(x \geq 2) = \frac{e^{-\lambda} \lambda^{x_i}}{x_i!}$$

$$\Rightarrow \sum_{i=2} \frac{e^{-6}\, 6^{x_i}}{x_i!}$$

$$\Rightarrow \cancel{1 - P[P(} \quad 1 - [P(x=0) + P(x=1)]$$

$$\Rightarrow \left[1 - \frac{e^{-6} 6^0}{0!} + \frac{e^{-6} 6^1}{1!}\right]$$

$$\Rightarrow 1 - [e^{-6} + 6e^{-6}]$$

$$= 1 - 7e^{-6}$$

$$= 0.983 \text{ or } 9.83\%.$$

~~Production line has 20% defective rate~~

Subject :

6) Average number of inspections for the first defective part

$= \frac{1}{P} = 5$

$P = 0.2$ (defective) $q = 0.8$ (good)

Probability of obtaining 1st defective part after 3 good parts

$P(x=4)$ 1 2 3 4

G G G D

$\Rightarrow 0.8 \times 0.8 \times 0.8 \times 0.2$

$= \cancel{0.12}\ 0.1024$

$= 10.24\%$

7) Success $= 0.3 = P$, failure $= 0.7 = q$, $n = 5$ students

$r = 2$ students

$P(x) = {}^nC_r\, p^r q^{n-r} = {}^5C_2 (0.3)^2 (0.7)^3$

$P(x \leq r) = \sum_{i}^{r} {}^nC_i\, p^i q^{n-i} = \frac{5!}{(5-2)!\,2!} (0.3)^2 (0.7)^3$

$= \frac{5 \times \overset{2}{\cancel{4}} \times \cancel{3 \times 2 \times 1}}{\cancel{3 \times 2 \times 1} \times \cancel{2}} = (0.09)(0.343)$

$= 0.3087$

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$P(x=0) = {}^nC_r\, p^r q^{n-r}$

$= {}^5C_0\, p^r q^{5-0}$

$= 1 \times (0.3)^0 (0.7)^5$

$= (0.7)^5$

$= 0.1681$

$P(x=1) = {}^nC_r\, p^r q^{n-r}$

$\Rightarrow {}^5C_1 (0.3)^1 (0.7)^{(5-1)}$

$= \frac{5!}{4! \times 1!} \times (0.3) \times (0.7)^4$

$= 0.3601$

Prob(almost 2 are accepted) $= 0.1681 + 0.3601 + 0.3087$

$= 0.8369$

8) $\mu = 70$ ~~SD~~ $\sigma = \sqrt{\text{variance}} = \sqrt{200} = 14.14$

$n = 10$ ~~Prob~~

$Z = \frac{x - \mu}{\sigma} = \frac{800 - (70 \times 10)}{\frac{14.14 \times 10}{\sqrt{10}}} = \frac{800 - 700}{14.14 \times \sqrt{10}}$

$\Rightarrow \frac{100}{44.71} = 2.23$

Subject: Date

Z score for 2.23 = 0.9871 or 98.71

Prob(12 Adults) - $Z = \frac{x - \mu}{\sigma}$

$= \cancel{(800 - 70 \times 12)}$ $\frac{800 - (70 \times 12)}{\frac{14.14}{\sqrt{12}} \times 12}$

$= \frac{800 - 840}{14.14 \times \sqrt{12}} = \frac{-40}{48.9} \quad \frac{-40}{48.98} = -0.816$

or

≈ -0.82

Z score for -0.82 = 0.2061 or 20.61%

9) Let us assume 'P' be the Probability that the student answers correctly by choosing correct option.

$n = 50$

$P = 1/2$

Probability that Student will Pass

$= P(x \geq 20)$ → x is the number of questions Student answered correctly.

$= 1 - P(x < 19)$

~~Since n is la~~ Since n is large we ~~can~~ estimate x to normal distribution.

$x \sim Bin(50, 1/2)$ is approximate to

$x \sim N(nP, nP(1-P))$

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$\Rightarrow x \sim N(50 \times 1/2, 20 \times 1/2 \times 1/2)$

$\Rightarrow x \sim N(25, 12.5)$

$\Rightarrow P(x \geq 20)$

$= 1 - P(x < 19)$

$\Rightarrow 1 - P\left(z < \frac{19 - 25}{\sqrt{12.5}}\right)$

$= 1 - P\left(z < \frac{-6}{\sqrt{12.5}}\right)$

$\Rightarrow \cancel{1 - P(z < -\ldots)} \Rightarrow 1 - P(z < -1.697)$

$\Rightarrow 1 - [1 - P(z < 1.697)]$

$= 1 - (1 - 0.95543)$

$= 1 - 0.045 = 0.96$

As Per our question we have 4 option, then $P = 1/4$

$x \sim N(50 \times 1/4, 50 \times 1/4 \times 3/4)$

$x \sim N(12.5, 18.75)$

$\Rightarrow 1 - P(x < 19)$

$= 1 - P\left(z < \frac{19 - 12.5}{\sqrt{\ldots}}\right)$

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$\Rightarrow 1 - P(z < 1.5)$

$\Rightarrow 1 - 0.93319$

$\Rightarrow 0.067 = 6.7\%$ → Probability of Student Passing exam = 6.7%

10) Failure = 30%

Success = 70%

Prob of Randomly Select 6 bulbs, exactly 2 are faulty

$n = 6$

$r = 2$

$p = \frac{30}{100} = 3/10$

$q = \frac{70}{100} = 7/10$

$$P(x=2) = {}^nC_r \, p^r \, q^{n-r}$$

$$= {}^6C_2 \left(\frac{3}{10}\right)^2 \left(\frac{7}{10}\right)^{6-2}$$

$$= \frac{6!}{4! \times 2!} = (0.3)^2 (0.7)^4$$

$$= \frac{\cancel{6}^{3} \times 5 \times \cancel{4 \times 3 \times 2 \times 1}}{\cancel{4 \times 3 \times 2 \times 1} \, \cancel{2} \times 1} = (0.09) \times (0.2401)$$

$$= 0.3241$$

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Subject :

11) $\lambda = 6$ errors/hr $= \frac{6}{60}$ Per min $= 0.1$ error/min

$x = 2$ errors

$P(x=x) = \frac{e^{-\lambda}\lambda^{r}}{r!}$

$P(x=2) = \frac{e^{0.1}(0.1)^2}{2!}$

$= 0.00452$

$= 0.45\%$

12(a) Probability of (<1 site) exceeds the recommended level of dioxin

$P(x<1) = P(x=0) = {}^{n}C_{r}\, p^{r} q^{n-r}$

$= {}^{20}C_{0}\,(0.05)^0 (0.95)^{20}$

$= 0.3585$

b) Probability that ≤ 1 site exceed

$P(x=0) + P(x=1)$

$P(x=0) = 0.3585$

$P(x=1) = {}^{n}C_{r}\, p^{r} q^{n-r}$

$= {}^{20}C_{1}\,(0.05)^1 (0.95)^{20-1} \Rightarrow 0.3774$

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Subject :

$P(x \leq 1) = 0.3585 + 0.3774$
$= 0.7359$

(c) $P(x \leq 2)$. Probability (max of 2) sites exceed

$P(x \leq 2) = ?$

$P(x=0) = 0.3584$
$P(x=1) = 0.3774$
$P(x=2) = {}^{n}C_{r}\, p^r q^{n-r}$

$= {}^{20}C_{2}\, \cancel{0.05}(0.05)^2 (0.95)^{18}$

$= 0.1887$

$P(x \leq 2) = 0.3584 + 0.3774 + 0.1887$
$= 0.9246$

13 (a) $P = 0.05 \quad n = 5, \ x = 2$
$P(x=2) = {}^{5}C_{2}\, 0.05^2 \times 0.95^3$

$= 0.0214 \simeq 2\%$

(b) ~~Rfexatel~~ ~~Rfexa~~

Subject : Date

(b) P(exactly twice in 2 years)

$= 2_{C_2} \times 0.05^2 \times 0.95^0$

$= 0.0025 \simeq 0.25\%$.

(c) $P(x \geq 1) \Rightarrow$ Probability of atleast once in 4 years

$\Rightarrow 1 - P(x=0)$

$= 1 - ({}^4C_0\ 0.05^0\ 0.95^4)$

$= 1 - (0.8145)$

$= 0.1855$

14) (a) $P = 0.2 \quad n = 15$

$P(x=2) = 15_{C_2}\ 0.2^2 \times 0.8^{13}$

$\Rightarrow 0.2309 \simeq 23\%$.

(b) $P(x \geq 1)$

$= 1 - P(x < 1)$

$= 1 - P(x=0) = 1 - 15_{C_0} \times 0.2^0 \times 0.8^{15}$

$= 0.9648 \approx 97\%$.

The End