1 Course Project - Step 2: Solving the problem under uncertainty

Author: Sebastian Troitzsch
Last updated: 27 January 2021

1 Introduction

As outlined in the project report for step 1, this work considers the combined optimal operation problem of a district-level energy system with air-conditioned buildings as flexible loads (FLs), a district cooling system (DCS) as thermal grid and an electric distribution grid. To study uncertainty in the context of this problem, the scope is extended to consider 1) photovoltaic (PV) generators and 2) the participation in day-ahead and real-time markets. PV generators are modelled as flexible generators which are normally dispatched based on to the incident solar irradiation, but can be curtailed if required. The system operator purchases active / reactive power from the transmission system at the day-ahead and real-time markets. To this end, the day-ahead market price c_t^{DA} is expected to be lower than the real-time market price c_t^{RT}, but the realisation of solar irradiation is only known during clearing of the real-time market. Thus, the solar irradiation enters the optimal operation problem as an uncertain parameter. To address the uncertainty, a stochastic optimization problem is formulated in the following.

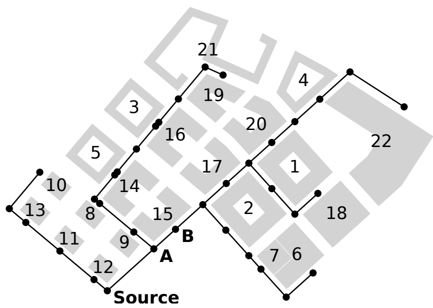

Figure 1: Test case with 22 commercial buildings as FLs and a PV generator at node A. The thermal and electric grid layout is congruent, i.e. thermal and electric grid nodes coincide.

The considered test case is based on a neighbourhood in Singapore and was developed as part of [1]. The test case consists of thermal and electric grids with identical layout, i.e. thermal and electric grid nodes coincide according to fig. 1, and 22 commercial buildings modelled as FLs. In addition, one PV generator is connected at node A in the electric grid. The day-ahead market price c_t^{DA} is derived from one day of the Universal Singapore Energy Price (USEP) [1], whereas the real-time market price c_t^{RT} is assumed to correlate with the

[1] https://www.emcsg.com/MarketData/PriceInformation

day-ahead price as $c_t^{RT} = 2c_t^{DA}$.

1.1 Notation

Let $\mathbb{R}$ be the domain of real numbers. Non-bold letters x, X denote scalars $\mathbb{R}^1$, bold lowercase letters $\boldsymbol{x}$ denote vectors $\mathbb{R}^n$ and bold uppercase letters $\boldsymbol{X}$ denote matrices $\mathbb{R}^{n,m}$. Bold numbers $\mathbf{0}$ and $\mathbf{1}$ denote vectors or matrices of zeros and ones of appropriate sizes. The transpose of a vector or matrix is denoted by $()^\intercal$. Prices are in Singapore Dollar (SGD) which is denoted by S$.

1.2 PV generator model

The PV generator model is formulated simply as:

$$0 \le p_{f,t} \le p_f^{nom} p_t^{irr} \tag{1.1a}$$
$$q_{f,t} = \frac{q_f^{nom}}{p_f^{nom}} p_{f,t} \tag{1.1b}$$

The scalars $p_{f,t}$, $q_{f,t}$ and p_t^{irr} are the active power, reactive power and per-unit solar irradiation of PV generator f at time step t. The scalars p_f^{nom} and q_f^{nom} are the nominal / peak active and reactive power of the generator. Note that eq. (1.1b) assumes a fixed power factor for reactive power dispatch and also assumes p_f^{nom} to be non-zero. To align with the format of the flexible load model, the PV generator model is transformed into a state space form as:

$$\boldsymbol{x}_{f,t+1} = \boldsymbol{A}_f \boldsymbol{x}_{f,t} + \boldsymbol{B}_f^c \boldsymbol{c}_{f,t} + \boldsymbol{B}_f^d \boldsymbol{d}_{f,t} \quad \forall f \in \mathcal{F},\ \forall t \in \mathcal{T}/t^N \tag{1.2a}$$
$$\boldsymbol{y}_{f,t} = \boldsymbol{C}_f \boldsymbol{x}_{f,t} + \boldsymbol{D}_f^c \boldsymbol{c}_{f,t} + \boldsymbol{D}_f^d \boldsymbol{d}_{f,t} \quad \forall f \in \mathcal{F},\ \forall t \in \mathcal{T} \tag{1.2b}$$
$$\boldsymbol{y}_{f,t}^- \le \boldsymbol{y}_{f,t} \le \boldsymbol{y}_{f,t}^+ \quad \forall f \in \mathcal{F},\ \forall t \in \mathcal{T} \tag{1.2c}$$
$$p_{f,t}^{th} = \boldsymbol{M}_f^{p^{th},y} \boldsymbol{y}_{f,t} \quad p_{f,t} = \boldsymbol{M}_f^{p,y} \boldsymbol{y}_{f,t} \quad q_{f,t} = \boldsymbol{M}_f^{q,y} \boldsymbol{y}_{f,t} \quad \forall f \in \mathcal{F},\ \forall t \in \mathcal{T} \tag{1.2d}$$

The entries of the state space model vectors and matrices are defined as:

$$\boldsymbol{x}_{f,t} = [] \in \mathbb{R}^0, \quad \boldsymbol{c}_{f,t} = [p_{f,t}] \in \mathbb{R}^1 \quad \boldsymbol{d}_{f,t} = [] \in \mathbb{R}^0, \tag{1.3a}$$
$$\boldsymbol{y}_{f,t} = [p_{f,t}, q_{f,t}]^\intercal \in \mathbb{R}^2, \quad \boldsymbol{y}_{f,t}^- = [0,0]^\intercal \in \mathbb{R}^2, \quad \boldsymbol{y}_{f,t}^+ = [p_f^{nom} p_t^{irr}, \infty]^\intercal \in \mathbb{R}^2 \tag{1.3b}$$
$$\boldsymbol{A}_f = [] \in \mathbb{R}^{0,0} \quad \boldsymbol{B}_f^c = [] \in \mathbb{R}^{0,0} \quad \boldsymbol{B}_f^d = [] \in \mathbb{R}^{0,0} \tag{1.3c}$$
$$\boldsymbol{C}_f = [] \in \mathbb{R}^{2,0} \quad \boldsymbol{D}_f^c = \begin{bmatrix} 1 \\ \frac{q_f^{nom}}{p_f^{nom}} \end{bmatrix} \in \mathbb{R}^{2,1} \quad \boldsymbol{D}_f^d = [] \in \mathbb{R}^{2,0} \tag{1.3d}$$
$$\boldsymbol{M}_f^{p^{th},y} = [0,0] \in \mathbb{R}^{1,2} \quad \boldsymbol{M}_f^{p,y} = [1,0] \in \mathbb{R}^{1,2} \quad \boldsymbol{M}_f^{q,y} = [0,1] \in \mathbb{R}^{1,2} \tag{1.3e}$$

The vectors $\boldsymbol{y}_{f,t}$, $\boldsymbol{x}_{f,t}$, $\boldsymbol{c}_{f,t}$ and $\boldsymbol{d}_{f,t}$ are the output, state, control and disturbance vectors for FL f at time step t. The matrices $\boldsymbol{A}_f$, $\boldsymbol{C}_f$ are the state and output matrix, and $\boldsymbol{B}_f^c$, $\boldsymbol{D}_f^c$, $\boldsymbol{B}_f^d$, $\boldsymbol{D}_f^d$ are the control and feed-through matrices, on the control and disturbance vectors respectively. The vector $\boldsymbol{x}^0$ is the initial state vector at the initial time step t^0 in $\mathcal{T}$. The symbol t^N denotes the final time step in $\mathcal{T}$. The matrices $\boldsymbol{M}_f^{p^{th},y}, \boldsymbol{M}_f^{p,y}, \boldsymbol{M}_f^{q,y}$ are the mapping matrices for relating the output vector $\boldsymbol{y}_{f,t}$ to the thermal, active and reactive power demand $\boldsymbol{p}_t^{th}, \boldsymbol{p}_t, \boldsymbol{q}_t \in \mathbb{R}^f$.

1.3 Flexible load, thermal grid and electric grid models

Mathematical models for flexible loads, thermal grid and electric grid have been introduced in the project report for step 1 and are omitted here for the sake of brevity.

2 Stochastic optimization problem

The stochastic optimization problem addresses the economic dispatch of FLs and PV generator in the day-ahead and real-time markets, expressed as:

$$\min_{\substack{\boldsymbol{x}_{s,f,t}, \boldsymbol{c}_{s,f,t}, \boldsymbol{y}_{s,f,t}, \\ \boldsymbol{p}_{s,t}^{th}, \boldsymbol{p}_{s,t}, \boldsymbol{q}_{s,t}, p_{s,t}^{th,src}, \\ p_t^{src,DA} \geq 0, p_{s,t}^{src,RT} \geq 0}} \quad \sum_{t \in \mathcal{T}} c_t^{DA} p_t^{src,DA} + \sum_{s \in \mathcal{S}} \sum_{t \in \mathcal{T}} \mathrm{P}(s)\, c_t^{RT} p_{s,t}^{src,RT} \tag{1.4a}$$

$$\text{s.t.}_{\forall s \in \mathcal{S}} \quad \boldsymbol{x}_{s,f,t^0} = \boldsymbol{x}_f^0 \quad \forall f \in \mathcal{F} \tag{1.4b}$$

$$\boldsymbol{x}_{s,f,t+1} = \boldsymbol{A}_f \boldsymbol{x}_{s,f,t} + \boldsymbol{B}_f^c \boldsymbol{c}_{s,f,t} + \boldsymbol{B}_f^d \boldsymbol{d}_{s,f,t} \quad \forall f \in \mathcal{F},\ \forall t \in \mathcal{T}/t^N \tag{1.4c}$$

$$\boldsymbol{y}_{s,f,t} = \boldsymbol{C}_f \boldsymbol{x}_{s,f,t} + \boldsymbol{D}_f^c \boldsymbol{c}_{s,f,t} + \boldsymbol{D}_f^d \boldsymbol{d}_{s,f,t} \quad \forall f \in \mathcal{F},\ \forall t \in \mathcal{T} \tag{1.4d}$$

$$\boldsymbol{y}_{s,f,t}^- \leq \boldsymbol{y}_{s,f,t} \leq \boldsymbol{y}_{s,f,t}^+ \quad \forall f \in \mathcal{F},\ \forall t \in \mathcal{T} \tag{1.4e}$$

$$p_{s,f,t}^{th} = \boldsymbol{M}_f^{p^{th},y} \boldsymbol{y}_{s,f,t} \quad p_{s,f,t} = \boldsymbol{M}_f^{p,y} \boldsymbol{y}_{s,f,t} \quad q_{s,f,t} = \boldsymbol{M}_f^{q,y} \boldsymbol{y}_{s,f,t} \quad \forall f \in \mathcal{F},\ \forall t \in \mathcal{T} \tag{1.4f}$$

$$\boldsymbol{h}^- \leq \boldsymbol{h}^{ref} + \boldsymbol{M}^{h,p^{th}} (\boldsymbol{p}_{s,t}^{th} - \boldsymbol{p}^{th,ref}) \quad \forall t \in \mathcal{T} \tag{1.4g}$$

$$\boldsymbol{v}^{ref} + \boldsymbol{M}^{v,p^{th}} (\boldsymbol{p}_{s,t}^{th} - \boldsymbol{p}^{th,ref}) \leq \boldsymbol{v}^+ \quad \forall t \in \mathcal{T} \tag{1.4h}$$

$$\frac{1}{\eta^{ch}} \left(p_{s,t}^{th,src} - \boldsymbol{1}^\intercal \boldsymbol{p}_{s,t}^{th} \right) = p^{pm,ref} + \boldsymbol{M}^{p^{pm},p^{th}} (\boldsymbol{p}_{s,t}^{th} - \boldsymbol{p}^{th,ref}) \quad \forall t \in \mathcal{T} \tag{1.4i}$$

$$\boldsymbol{u}^- \leq \boldsymbol{u}^{ref} + \boldsymbol{M}^{u,p} (\boldsymbol{p}_{s,t} - \boldsymbol{p}^{ref}) + \boldsymbol{M}^{u,q} (\boldsymbol{q}_{s,t} - \boldsymbol{q}^{ref}) \leq \boldsymbol{u}^+ \quad \forall t \in \mathcal{T} \tag{1.4j}$$

$$|\boldsymbol{s}^{f,ref}| + \boldsymbol{M}^{s^f,p} (\boldsymbol{p}_{s,t} - \boldsymbol{p}^{ref}) + \boldsymbol{M}^{s^f,q} (\boldsymbol{q}_{s,t} - \boldsymbol{q}^{ref}) \leq |\boldsymbol{s}^{f,+}| \quad \forall t \in \mathcal{T} \tag{1.4k}$$

$$|\boldsymbol{s}^{t,ref}| + \boldsymbol{M}^{s^t,p} (\boldsymbol{p}_{s,t} - \boldsymbol{p}^{ref}) + \boldsymbol{M}^{s^t,q} (\boldsymbol{q}_{s,t} - \boldsymbol{q}^{ref}) \leq |\boldsymbol{s}^{t,+}| \quad \forall t \in \mathcal{T} \tag{1.4l}$$

$$\begin{aligned} p_t^{src,DA} + p_{s,t}^{src,RT} - \frac{1}{\eta^{ch}} p_{s,t}^{th,src} - \boldsymbol{1}^\intercal \boldsymbol{p}_{s,t} = p^{ls,ref} &+ \boldsymbol{M}^{p^{ls},p} (\boldsymbol{p}_{s,t} - \boldsymbol{p}^{ref}) \\ &+ \boldsymbol{M}^{p^{ls},q} (\boldsymbol{q}_{s,t} - \boldsymbol{q}^{ref}) \quad \forall t \in \mathcal{T} \end{aligned} \tag{1.4m}$$

The vectors $p_t^{src,DA}$ and $p_{s,t}^{src,RT}$ are introduced to differentiate the amount of active power purchased at the day-ahead and real-time market in time step t and scenario $s \in \mathcal{S}$. The set $\mathcal{S}$ describes the set of in-sample scenarios which are considered for the stochastic optimization. The symbol $\mathrm{P}(s)$ expresses the probability for scenario s to realise. Note that the thermal source power $p_{s,t}^{th,src}$ is considered in the active power balance eq. (1.4m) and the the source reactive power balance is omitted, because the price for reactive power is assumed to be zero. Furthermore, $p_t^{src,DA}$ and $p_{s,t}^{src,RT}$ are constrained to be non-negative to restrict reverse power flow from the distribution grid to the transmission grid. The omission of the PV generator production cost in the objective is intentional and corresponds to zero marginal cost. Lastly, note that the set $\mathcal{F}$ includes all flexible loads and the PV generator.

2.1 Scenario generation

Scenarios for the uncertain solar irradiation p_t^{irr} are obtained from EnergyPlus weather data for Singapore[2], where one year of horizontal irradiation data is divided into 365 individual daily scenarios. To reduce the number of scenarios and consequently the dimension of the stochastic optimization problem, time-series clustering based on k-means is applied according to [2] and utilising the tslearn Python toolbox[3]. The number of clusters can be varied depending on the desired dimension of the stochastic optimization problem. Note that the clustered scenarios represent the mean time series of values within the associated cluster, but do not correspond to any individual input scenario. Therefore, the 365 input scenarios are considered as out-of-sample scenario set in the following.

3 Results

The stochastic optimization problem is solved for different sizes of the in-sample scenario and different settings for the probability parameter $\mathrm{P}(s)$. In the not-probability-weighted solution, the probability is set as $\mathrm{P}(s) = \frac{1}{N^S}$, where N^S is the number of in-sample scenarios. In the probability-weighted solution, the probability is set as $\mathrm{P}(s) = \frac{N_s}{365}$, where N_s is the number of out-sample scenarios which are associated with the cluster of in-sample scenario s.

Table 1: Solution statistics.

Number of in-sample scenarios	**Probability-weighted scenarios**	**Objective expected value** [S$]	**Objective standard deviation** [S$]	**Solution time** [s]
1	-	29679	-	0.98
10	No	40318	12861	8.24
10	Yes	34730	13148	10.46
20	No	40244	8337	19.65
20	Yes	35235	12777	17.67
30	No	38473	13390	25.87
30	Yes	35392	13069	28.67
365	-	35670	10513	433.7

Table 1 presents the solution statistics for different settings of in-sample scenario number and probability weighting. An initial observation is that the non-probability-weighted solution seems to over-emphasise extreme

[2] https://energyplus.net/weather-location/southwest_pacific_wmo_region_5/SGP//SGP_Singapore.486980_IWEC
[3] https://github.com/tslearn-team/tslearn

scenarios and thus leads to high objective values. The probability-weighted solution is able to more closely approximate the objective value of the solution with the full out-of-sample scenario set, which is provided at the bottom of the table for reference. Although the standard deviation of the objective values of the in-sample scenarios remains far from the full out-of-sample scenario set, the gap slightly decreases with a higher number of in-sample scenarios. The solution time clearly increases almost linearly with the number of scenarios considered. However, note that the given solution time includes pre-processing time of the optimization modelling interface CVXPY in addition to the solver time.

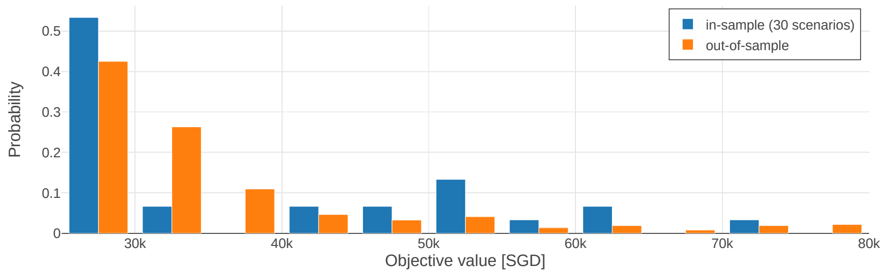

Figure 2: Objective value probability distribution for 30 in-sample scenarios without probability weighting.

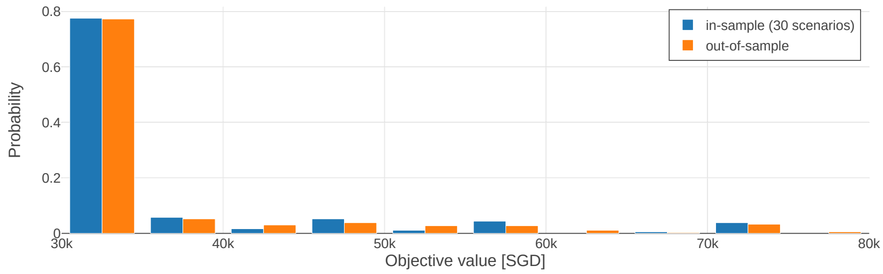

Figure 3: Objective value probability distribution for 30 in-sample scenarios without probability weighting.

Figures 2 and 3 compare the in-sample and out-of-sample objective value probability distribution for 30 in-sample scenarios without and with probability weighting. Note that the out-of-sample solutions are obtained by solving eq. (1.4) with $p_t^{src,DA}$ fixed to the in-sample solution and the out-of-sample scenario set as $\mathcal{S}$. In fig. 2, the in-sample and out-of-sample objective value probability distribution closely resemble each other. In fig. 2, the solution for $p_t^{src,DA}$ yields increased objective values in the out-of-sample solution, pointing to the inadequate weighting of in-sample scenarios.

4 Conclusions

This work presented a stochastic optimization problem formulation for the combined optimal operation problem of a district-level energy system, with participation in day-ahead and real-time markets and uncertainty in the PV generation. An input scenario set of 365 daily solar irradiation time series was reduced using time series clustering. The results demonstrate that the reduced scenario set can adequately represent the uncertainty, but also point to the importance of appropriately weighting clustered scenarios based on the number of time series associated with each cluster in the input scenario set.

Bibliography

[1] S. Troitzsch, B. K. Sreepathi, T. P. Huynh, A. Moine, S. Hanif, J. Fonseca, and T. Hamacher, "Optimal electric-distribution-grid planning considering the demand-side flexibility of thermal building systems for a test case in Singapore," en, *Applied Energy*, vol. 273, 2020, ISSN: 03062619. DOI: 10.1016/j.apenergy.2020.114917.

[2] H. Teichgraeber and A. R. Brandt, "Clustering methods to find representative periods for the optimization of energy systems: An initial framework and comparison," en, *Applied Energy*, vol. 239, pp. 1283–1293, 2019, ISSN: 03062619. DOI: 10.1016/j.apenergy.2019.02.012.