1 Course Project - Step 3: Solving the problem as a bilevel program

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1 Introduction

As outlined in the project report for step 1, this work considers the combined optimal operation problem of a district-level energy system with air-conditioned buildings as flexible loads (FLs), a district cooling system (DCS) as thermal grid and an electric distribution grid. In this context, the strategic scheduling problem of the FLs can be expressed as a bilevel program with 1) a FL aggregator as leader and 2) a district energy system (DES) operator as follower. To this end, the FL aggregator is responsible for the cost-optimal dispatch of the FLs, i.e. to schedule the thermal, active and reactive power demand, subject to the FLs operation limits. The DES operator ensures the optimal operation of the thermal and electric grids, subject to the grid operation limits, where it schedules additional thermal, active and reactive backup power generation to maintain the operation limits if needed. The flexible load aggregator is charged distribution location marginal prices (DLMPs) for thermal, active and reactive power demand, where DLMPs determined by the DES operator after submission of demand schedules by the FL aggregator. The DES operator purchases active / reactive power from the transmission system at the cleared electricity wholesale market price c_t^{ref} and from thermal, active, reactive backup generators at the prices $\boldsymbol{c}^{p^{th},gen}, \boldsymbol{c}^{p^{th},gen}, \boldsymbol{c}^{p^{th},gen} > c_t^{ref}$. The following assumes that there is exactly one FL aggregator and one DES operator in the district.

The considered test case is based on a neighbourhood in Singapore and was developed as part of [1]. The test case consists of thermal and electric grids with identical layout, i.e. thermal and electric grid nodes coincide, and 22 commercial buildings modelled as FLs. The electricity wholesale market price c_t^{ref} is derived from one day of the Universal Singapore Energy Price (USEP) [1].

1.1 Notation

Let $\mathbb{R}$ be the domain of real numbers. Non-bold letters x, X denote scalars $\mathbb{R}^1$, bold lowercase letters $\boldsymbol{x}$ denote vectors $\mathbb{R}^n$ and bold uppercase letters $\boldsymbol{X}$ denote matrices $\mathbb{R}^{n,m}$. Bold numbers $\boldsymbol{0}$ and $\boldsymbol{1}$ denote vectors or matrices of zeros and ones of appropriate sizes. The transpose of a vector or matrix is denoted by $()^\intercal$ and

[1] https://www.emcsg.com/MarketData/PriceInformation

$[x_e]_{e\in\mathcal{E}}$ denotes the column vector of scalar values x_e for all $e \in \mathcal{E}$. Prices are in Singapore Dollar (SGD) which is denoted by S$.

1.2 Flexible load, thermal grid and electric grid models

Mathematical models for flexible loads, thermal grid and electric grid have been introduced in the project report for step 1 and are omitted here for the sake of brevity.

2 Bilevel program

The strategic scheduling problem[2] of the FLs is expressed as a bilevel program with the upper-level problem in eq. (1.2) denoting the FL operation problem and the lower-level problem in eq. (1.1) describing the DES operation problem. In the following, the lower-level problem is introduced first.

2.1 Lower-level problem

The DES operation problem is expressed as:

$$\min_{\substack{\boldsymbol{p}_t^{th}, \boldsymbol{p}_t, \boldsymbol{q}_t, \\ \boldsymbol{p}_t^{th,gen}, \boldsymbol{p}_t^{gen}, \boldsymbol{q}_t^{gen}, \\ p_t^{th,src}, p_t^{src}, q_t^{src}}} \sum_{t\in\mathcal{T}} -c_t^{ref}\left(p_t^{src} + \frac{1}{\eta^{ch}} p_t^{th,src}\right) \tag{1.1a}$$

$$+ \sum_{t\in\mathcal{T}} \left(\left(\boldsymbol{c}^{p^{th},gen}\right)^{\intercal} \boldsymbol{p}_t^{th,gen} + (\boldsymbol{c}^{p,gen})^{\intercal} \boldsymbol{p}_t^{gen} + (\boldsymbol{c}^{q,gen})^{\intercal} \boldsymbol{q}_t^{gen}\right)$$

$$\text{s.t.} \quad \boldsymbol{p}_t^{th} = \left[p_{f,t}^{th}\right]_{f\in\mathcal{F}} + \boldsymbol{p}_t^{th,gen} \;:\boldsymbol{\lambda}_t^{p^{th}} \quad \boldsymbol{p}_t = \left[p_{f,t}\right]_{f\in\mathcal{F}} + \boldsymbol{p}_t^{gen} \;:\boldsymbol{\lambda}_t^{p} \tag{1.1b}$$

$$\boldsymbol{q}_t = \left[q_{f,t}\right]_{f\in\mathcal{F}} + \boldsymbol{q}_t^{gen} \;:\boldsymbol{\lambda}_t^{q} \quad \forall t\in\mathcal{T}$$

$$\boldsymbol{p}_t^{th,gen} \geq 0 \;:\boldsymbol{\mu}^{p^{th},gen} \quad \boldsymbol{p}_t^{gen} \geq 0 \;:\boldsymbol{\mu}^{p,gen} \quad \boldsymbol{q}_t^{gen} \geq 0 \;:\boldsymbol{\mu}^{q,gen} \tag{1.1c}$$

$$\boldsymbol{h}^{-} \leq \boldsymbol{h}^{ref} + \boldsymbol{M}^{h,p^{th}}(\boldsymbol{p}_t^{th} - \boldsymbol{p}^{th,ref}) \quad :\boldsymbol{\mu}_t^{h^-} \quad \forall t\in\mathcal{T} \tag{1.1d}$$

$$\boldsymbol{v}^{ref} + \boldsymbol{M}^{v,p^{th}}(\boldsymbol{p}_t^{th} - \boldsymbol{p}^{th,ref}) \leq \boldsymbol{v}^{+} \quad :\boldsymbol{\mu}_t^{v^+} \quad \forall t\in\mathcal{T} \tag{1.1e}$$

$$\frac{1}{\eta^{ch}}\left(p_t^{th,src} - \boldsymbol{1}^{\intercal}\boldsymbol{p}_t^{th}\right) = p^{pm,ref} + \boldsymbol{M}^{p^{pm},p^{th}}(\boldsymbol{p}_t^{th} - \boldsymbol{p}^{th,ref}) \quad :\lambda_t^{p^{pm}} \quad \forall t\in\mathcal{T} \tag{1.1f}$$

$$\boldsymbol{u}^{-} \leq \boldsymbol{u}^{ref} + \boldsymbol{M}^{u,p}(\boldsymbol{p}_t - \boldsymbol{p}^{ref}) + \boldsymbol{M}^{u,q}(\boldsymbol{q}_t - \boldsymbol{q}^{ref}) \leq \boldsymbol{u}^{+} \quad :\boldsymbol{\mu}_t^{u^+}, \boldsymbol{\mu}_t^{u^-} \quad \forall t\in\mathcal{T} \tag{1.1g}$$

$$|\boldsymbol{s}^{f,ref}| + \boldsymbol{M}^{s^f,p}(\boldsymbol{p}_t - \boldsymbol{p}^{ref}) + \boldsymbol{M}^{s^f,q}(\boldsymbol{q}_t - \boldsymbol{q}^{ref}) \leq |\boldsymbol{s}^{f,+}| \quad :\boldsymbol{\mu}_t^{s^{f,+}} \quad \forall t\in\mathcal{T} \tag{1.1h}$$

$$|\boldsymbol{s}^{t,ref}| + \boldsymbol{M}^{s^t,p}(\boldsymbol{p}_t - \boldsymbol{p}^{ref}) + \boldsymbol{M}^{s^t,q}(\boldsymbol{q}_t - \boldsymbol{q}^{ref}) \leq |\boldsymbol{s}^{t,+}| \quad :\boldsymbol{\mu}_t^{s^{t,+}} \quad \forall t\in\mathcal{T} \tag{1.1i}$$

$$p_t^{src} - \boldsymbol{1}^{\intercal}\boldsymbol{p}_t = p^{ls,ref} + \boldsymbol{M}^{p^{ls},p}(\boldsymbol{p}_t - \boldsymbol{p}^{ref}) + \boldsymbol{M}^{p^{ls},q}(\boldsymbol{q}_t - \boldsymbol{q}^{ref}) \quad :\lambda_t^{p^{ls}} \quad \forall t\in\mathcal{T} \tag{1.1j}$$

$$q_t^{src} - \boldsymbol{1}^{\intercal}\boldsymbol{q}_t = q^{ls,ref} + \boldsymbol{M}^{q^{ls},p}(\boldsymbol{p}_t - \boldsymbol{p}^{ref}) + \boldsymbol{M}^{q^{ls},q}(\boldsymbol{q}_t - \boldsymbol{q}^{ref}) \quad :\lambda_t^{q^{ls}} \quad \forall t\in\mathcal{T} \tag{1.1k}$$

The vectors $\boldsymbol{p}_t^{th,gen}, \boldsymbol{p}_t^{gen}, \boldsymbol{q}_t^{gen}$ are the thermal, active and reactive power supplied by backup generators, where $\boldsymbol{c}^{p^{th},gen}, \boldsymbol{c}^{p^{th},gen}, \boldsymbol{c}^{p^{th},gen} > c_t^{ref}$ are the price associated with thermal, active and reactive power for

[2] The term strategic scheduling problem is used here rather than strategic offering problem to capture that demand schedules rather price / quantity offers are submitted to the DES operator.

each backup generator. The scalars $p^{th}_{f,t}, p_{f,t}, q_{f,t}$ denote the thermal, active and reactive power demand of FLs, which are parameters in the lower-level problem and are negative by convention. Note that this formulation for convenience assumes that backup generators are co-located which each FL. Notations for other thermal grid and electric grid variables have been introduced in the project report for step 1 and are omitted here for the sake of brevity. There is a slight deviation from the project report for step 1 in the objective eq. (1.1a), where $-c^{ref}_t$ is taken to account for the negative sign in $p^{th,src}_t, p^{src}_t$.

2.2 Upper-level problem

The FL aggregator problem is expressed as:

$$\min_{\substack{\boldsymbol{x}_{f,t},\boldsymbol{c}_{f,t},\boldsymbol{y}_{f,t},p^{th}_{f,t},p_{f,t},q_{f,t},\\ \boldsymbol{p}^{th}_t,\boldsymbol{p}_t,\boldsymbol{q}_t,p^{th,src}_t,p^{src}_t,q^{src}_t,\\ \boldsymbol{\lambda}^{p^{th}}_t,\boldsymbol{\lambda}^{p}_t,\boldsymbol{\lambda}^{q}_t\boldsymbol{\mu}^{h^-}_t\geq\boldsymbol{0},\boldsymbol{\mu}^{v^+}_t\geq\boldsymbol{0},\\ \lambda^{p^{pm}}_t,\boldsymbol{\mu}^{u^-}_t\geq\boldsymbol{0},\boldsymbol{\mu}^{u^+}_t\geq\boldsymbol{0},\boldsymbol{\mu}^{s^{f,+}}_t\geq\boldsymbol{0},\\ \boldsymbol{\mu}^{s^{t,+}}_t\geq\boldsymbol{0},\lambda^{p^{ls}}_t,\lambda^{q^{ls}}_t}} \quad \sum_{t\in\mathcal{T}}\left(-\left(\boldsymbol{\lambda}^{p^{th}}_t\right)^\intercal\left[p^{th}_{f,t}\right]_{f\in\mathcal{F}}-\left(\boldsymbol{\lambda}^{p}_t\right)^\intercal\left[p_{f,t}\right]_{f\in\mathcal{F}}-\left(\boldsymbol{\lambda}^{q}_t\right)^\intercal\left[q_{f,t}\right]_{f\in\mathcal{F}}\right) \tag{1.2a}$$

$$\text{s.t.} \quad (1.1) \tag{1.2b}$$

$$\boldsymbol{x}_{f,t^0} = \boldsymbol{x}^0_f \quad \forall f\in\mathcal{F} \tag{1.2c}$$

$$\boldsymbol{x}_{f,t+1} = \boldsymbol{A}_f\boldsymbol{x}_{f,t} + \boldsymbol{B}^c_f\boldsymbol{c}_{f,t} + \boldsymbol{B}^d_f\boldsymbol{d}_{f,t} \quad \forall t\in\mathcal{T}/t^N,\ \forall f\in\mathcal{F} \tag{1.2d}$$

$$\boldsymbol{y}_{f,t} = \boldsymbol{C}_f\boldsymbol{x}_{f,t} + \boldsymbol{D}^c_f\boldsymbol{c}_{f,t} + \boldsymbol{D}^d_f\boldsymbol{d}_{f,t} \quad \forall t\in\mathcal{T},\ \forall f\in\mathcal{F} \tag{1.2e}$$

$$\boldsymbol{y}^-_{f,t} \leq \boldsymbol{y}_{f,t} \leq \boldsymbol{y}^+_{f,t} \quad \forall t\in\mathcal{T},\ \forall f\in\mathcal{F} \tag{1.2f}$$

$$p^{th}_{f,t} = \boldsymbol{M}^{p^{th},y}_f\boldsymbol{y}_{f,t} \quad p_{f,t} = \boldsymbol{M}^{p,y}_f\boldsymbol{y}_{f,t} \quad q_{f,t} = \boldsymbol{M}^{q,y}_f\boldsymbol{y}_{f,t} \quad \forall t\in\mathcal{T},\ \forall f\in\mathcal{F} \tag{1.2g}$$

The vectors $\boldsymbol{\lambda}^{p^{th}}_t, \boldsymbol{\lambda}^{p}_t, \boldsymbol{\lambda}^{q}_t$ denote the DLMPs for thermal, active and reactive power. Notations for other flexible load variables have been introduced in the project report for step 1 and are omitted here for the sake of brevity.

2.3 Mathematical problem with equilibrium constraints (MPEC)

The bilinear problem is transformed into a mathematical problem with equilibrium constraints (MPEC) as:

$$
\begin{aligned}
\min_{\substack{\boldsymbol{x}_{f,t},\boldsymbol{c}_{f,t},\boldsymbol{y}_{f,t},p^{th}_{f,t},p_{f,t},q_{f,t},\\ \boldsymbol{p}^{th}_t,\boldsymbol{p}_t,\boldsymbol{q}_t,p^{th,src}_t,p^{src}_t,q^{src}_t,\\ \boldsymbol{\lambda}^{p^{th}}_t,\boldsymbol{\lambda}^{p}_t,\boldsymbol{\lambda}^{q}_t\boldsymbol{\mu}^{h^-}_t\geq\boldsymbol{0},\boldsymbol{\mu}^{v^+}_t\geq\boldsymbol{0},\\ \lambda^{p^{pm}}_t,\boldsymbol{\mu}^{u^-}_t\geq\boldsymbol{0},\boldsymbol{\mu}^{u^+}_t\geq\boldsymbol{0},\boldsymbol{\mu}^{s^{f,+}}_t\geq\boldsymbol{0},\\ \boldsymbol{\mu}^{s^{t,+}}_t\geq\boldsymbol{0},\lambda^{p^{ls}}_t,\lambda^{q^{ls}}_t}} \quad & \sum_{t\in\mathcal{T}}\Bigg(-c^{ref}_t\left(p^{src}_t+\frac{1}{\eta^{ch}}p^{th,src}_t\right) && \text{(1.3a)}\\
&+\left(\boldsymbol{c}^{p^{th},gen}\right)^{\intercal}\boldsymbol{p}^{th,gen}_t+(\boldsymbol{c}^{p,gen})^{\intercal}\boldsymbol{p}^{gen}_t+(\boldsymbol{c}^{q,gen})^{\intercal}\boldsymbol{q}^{gen}_t\\
&-\left(\boldsymbol{\mu}^{h^-}_t\right)^{\intercal}\left(\boldsymbol{h}^{-}-\boldsymbol{h}^{ref}+\boldsymbol{M}^{h,p^{th}}\boldsymbol{p}^{th,ref}\right)\\
&-\left(\boldsymbol{\mu}^{v^+}_t\right)^{\intercal}\left(\boldsymbol{v}^{ref}-\boldsymbol{M}^{v,p^{th}}\boldsymbol{p}^{th,ref}-\boldsymbol{v}^{+}\right)\\
&-\lambda^{p^{pm}}_t\left(-p^{pm,ref}+\boldsymbol{M}^{p^{pm},p^{th}}\boldsymbol{p}^{th,ref}\right)\\
&-(\boldsymbol{\mu}^{u^-}_t)^{\intercal}\left(\boldsymbol{u}^{-}-\boldsymbol{u}^{ref}+\boldsymbol{M}^{u,p}\boldsymbol{p}^{ref}+\boldsymbol{M}^{u,q}\boldsymbol{q}^{ref}\right)\\
&-\left(\boldsymbol{\mu}^{u^+}_t\right)^{\intercal}\left(\boldsymbol{u}^{ref}-\boldsymbol{M}^{u,p}\boldsymbol{p}^{ref}-\boldsymbol{M}^{u,q}\boldsymbol{q}^{ref}-\boldsymbol{u}^{+}\right)\\
&-\left(\boldsymbol{\mu}^{s^{f,+}}_t\right)^{\intercal}\left(|\boldsymbol{s}^{f,ref}|-\boldsymbol{M}^{s^f,p}\boldsymbol{p}^{ref}-\boldsymbol{M}^{s^f,q}\boldsymbol{q}^{ref}-|\boldsymbol{s}^{f,+}|\right)\\
&-\left(\boldsymbol{\mu}^{s^{t,+}}_t\right)^{\intercal}\left(|\boldsymbol{s}^{t,ref}|-\boldsymbol{M}^{s^t,p}\boldsymbol{p}^{ref}-\boldsymbol{M}^{s^t,q}\boldsymbol{q}^{ref}-|\boldsymbol{s}^{t,+}|\right)\\
&-\lambda^{p^{ls}}_t\left(-p^{ls,ref}+\boldsymbol{M}^{p^{ls},p}\boldsymbol{p}^{ref}+\boldsymbol{M}^{p^{ls},q}\boldsymbol{q}^{ref}\right)\\
&-\lambda^{q^{ls}}_t\left(-q^{ls,ref}+\boldsymbol{M}^{q^{ls},p}\boldsymbol{p}^{ref}+\boldsymbol{M}^{q^{ls},q}\boldsymbol{q}^{ref}\right)\Bigg)\\
\text{s.t.}\quad & (1.2c), (1.2d), (1.2e), (1.2f), (1.2g), (1.4), (1.1b), (1.1f), (1.1j), (1.1k), (1.5) && \text{(1.3b)}
\end{aligned}
$$

The constraints of the MPEC are composed of the upper-level constraints as well as the KKT conditions of the lower-level problem, which are derived in appendix A. The objective function of the MPEC is equivalent to the upper-level objective in eq. (1.2a), where the multiplication of primal and dual variables leads to nonlinear terms. To this end, the strong duality equality of the lower-level problem in eq. (1.7) in appendix C is utilised to obtain an equivalent linear expression for the MPEC objective. The non-linear complementarity constraints in eq. (1.5) are reformulated with the Fortuny-Amat (Big-M) approach. Therefore, the complementarity condition $0 \leq a \perp b \geq 0$ is replaced with $0 \leq a \leq \psi M$, $0 \leq b \leq (1-\psi)M$, $\psi \in \{0,1\}$, where ψ denotes an auxiliary binary variable and M denotes a large constant. The explicit expressions of the equivalent formulations for the complementarity constraints in eq. (1.5) are omitted here for the sake brevity.

3 Results

The outcome of the strategic scheduling problem in the following is compared with its non-strategic counterpart, i.e. the solution of the upper-level problem in eq. (1.2) without considering the lower-level problem. For the non-strategic problem, the DLMPs are set to be equivalent to the wholesale market price, i.e. $\boldsymbol{\lambda}^{p^{th}}_t = \boldsymbol{1}\frac{c^{ref}_t}{\eta^{ch}}$, $\boldsymbol{\lambda}^{p}_t\boldsymbol{1}c^{ref}_t$, $\boldsymbol{\lambda}^{q}_t = \boldsymbol{0}$. The prices of backup generators are arbitrarily set as $\boldsymbol{c}^{p^{th},gen} = \boldsymbol{1}\max_{t\in\mathcal{T}}(2\frac{c^{ref}_t}{\eta^{ch}})$, $\boldsymbol{c}^{p,gen} = \boldsymbol{1}\max_{t\in\mathcal{T}}(2c^{ref}_t)$, $\boldsymbol{c}^{q,gen} = \boldsymbol{1}\max_{t\in\mathcal{T}}(2c^{ref}_t)$. Furthermore, an arbitrary electric grid branch power limit is introduced to investigate the strategic scheduling in the face of congestion.

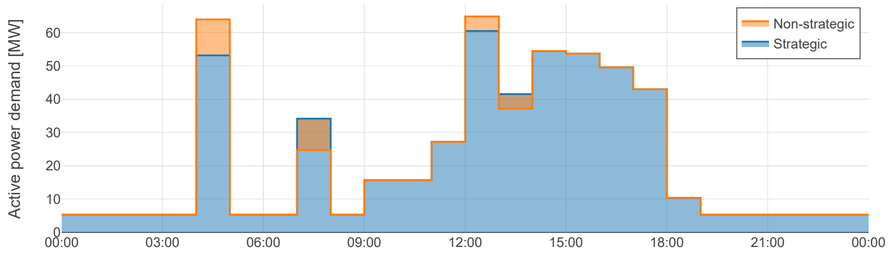

Figure 1: Aggregate FL active power demand schedule ($p_t = \sum_{f \in \mathcal{F}} p_{f,t}$).

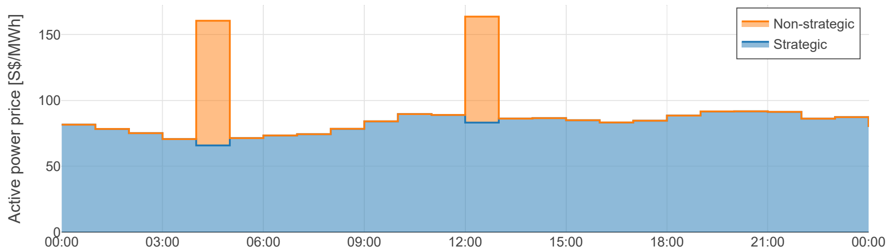

Figure 2: Average active power DLMP schedule ($\lambda_t = \sum_{f \in \mathcal{F}} \frac{\lambda_{f,t}}{n_F}$)

Figures 1 and 2 depict the aggregate FL active power demand and average FL active power DLMP schedules as solutions of the strategic and non-strategic scheduling problems. The non-strategic scheduling leads to peak loads at 4AM and 12PM, which coincide with dips in the wholesale electricity price. Since the load peaks cause congestion in the electric grid, the DES operator dispatches backup generators to maintain the branch power limits. Therefore, the non-strategic scheduling of FLs causes peaks in the active power DLMPs at 4AM and 12PM, related to the marginal operation cost of the back generators. With strategic scheduling, these peaks can be anticipated and avoided by rescheduling FL active power demand as shown in fig. 1.

Table 1: Solution statistics for strategic and non-strategic scheduling problems.

	Non-strategic scheduling	**Strategic scheduling**
FL active power demand [MWh]	529	527
FL active power cost [S$]	56254	43354

As shown in table 1, the active power cost can be reduced by approx. 23 % through strategic scheduling. The total active power demand also reduced slightly, which however does not mean that the cost savings are related to demand reduction. Rather, the demand schedule in the non-strategic case is driven by consuming as much power as possible during low wholesale price periods, thereby accepting additional FL storage losses.

4 Conclusion

This work presented the formulation of the strategic scheduling problem of a FL aggregator as a bilevel program with the DES operator problem as lower-level problem. The strategic scheduling problem was expressed as a tractable MPEC and its ability to strategically avoid high DLMPs due to grid congestion was demonstrated for an exemplary test case.

Appendix A: KKT conditions of the lower-level problem

To formulate the KKT conditions of the lower-level problem, the Lagrangian function is first derived in appendix B, eq. (1.6). Then, the KKT stationarity, primal feasibility and complimentary slackness conditions can be deduced. The KKT stationarity condition $\frac{\delta L}{\delta x} = 0$ yields:

$$\mathbf{0} = \left(\boldsymbol{c}^{p^{th},gen}\right)^\top - \left(\boldsymbol{\lambda}_t^{p^{th}}\right)^\top - \left(\boldsymbol{\mu}^{p^{th},gen}\right)^\top \tag{1.4a}$$

$$\mathbf{0} = (\boldsymbol{c}^{p,gen})^\top - (\boldsymbol{\lambda}_t^{p})^\top - (\boldsymbol{\mu}^{p,gen})^\top \tag{1.4b}$$

$$\mathbf{0} = (\boldsymbol{c}^{q,gen})^\top - (\boldsymbol{\lambda}_t^{q})^\top - (\boldsymbol{\mu}^{q,gen})^\top \tag{1.4c}$$

$$\mathbf{0} = \left(\boldsymbol{\lambda}_t^{p^{th}}\right)^\top - \left(\boldsymbol{\mu}_t^{h^-}\right)^\top \boldsymbol{M}^{h,p^{th}} + \left(\boldsymbol{\mu}_t^{v^+}\right)^\top \boldsymbol{M}^{v,p^{th}} - \lambda_t^{p^{pm}} \left(\frac{1}{\eta^{ch}}\mathbf{1}^\top + \boldsymbol{M}^{p^{pm},p^{th}}\right) \quad \forall t \in \mathcal{T} \tag{1.4d}$$

$$\begin{aligned}\mathbf{0} = (\boldsymbol{\lambda}_t^{p})^\top - (\boldsymbol{\mu}_t^{u^-})^\top \boldsymbol{M}^{u,p} + \left(\boldsymbol{\mu}_t^{u^+}\right)^\top \boldsymbol{M}^{u,p} + \left(\boldsymbol{\mu}_t^{s^{f},+}\right)^\top \boldsymbol{M}^{s^f,p} + \left(\boldsymbol{\mu}_t^{s^{t},+}\right)^\top \boldsymbol{M}^{s^t,p} \\ - \lambda_t^{p^{ls}} \left(\mathbf{1}^\top + \boldsymbol{M}^{p^{ls},p}\right) - \lambda_t^{q^{ls}} \boldsymbol{M}^{q^{ls},p}\end{aligned} \quad \forall t \in \mathcal{T} \tag{1.4e}$$

$$\begin{aligned}\mathbf{0} = (\boldsymbol{\lambda}_t^{q})^\top - (\boldsymbol{\mu}_t^{u^-})^\top \boldsymbol{M}^{u,q} + \left(\boldsymbol{\mu}_t^{u^+}\right)^\top \boldsymbol{M}^{u,q} + \left(\boldsymbol{\mu}_t^{s^{f},+}\right)^\top \boldsymbol{M}^{s^f,q} + \left(\boldsymbol{\mu}_t^{s^{t},+}\right)^\top \boldsymbol{M}^{s^t,q} \\ - \lambda_t^{p^{ls}} \boldsymbol{M}^{p^{ls},q} - \lambda_t^{q^{ls}} \left(\mathbf{1}^\top + \boldsymbol{M}^{q^{ls},q}\right)\end{aligned} \quad \forall t \in \mathcal{T} \tag{1.4f}$$

$$0 = -c_t^{ref} + \lambda_t^{p^{pm}}, \quad 0 = -c_t^{ref} + \lambda_t^{p^{ls}}, \quad 0 = \lambda_t^{q^{ls}} \quad \forall t \in \mathcal{T} \tag{1.4g}$$

The KKT primal feasibility condition $h(x) = 0$ yields that primal equality constraints eqs. (1.1b), (1.1f), (1.1j), (1.1k) and (1.2c) to (1.2e) have to hold. The KKT complementary slackness condition $0 \leq -g(x) \perp \mu \geq 0$ yields:

$$\mathbf{0} \leq \boldsymbol{p}_t^{th,gen} \perp \boldsymbol{\mu}^{p^{th},gen} \geq \mathbf{0} \quad \forall t \in \mathcal{T} \tag{1.5a}$$

$$\mathbf{0} \leq \boldsymbol{p}_t^{gen} \perp \boldsymbol{\mu}^{p,gen} \geq \mathbf{0} \quad \forall t \in \mathcal{T} \tag{1.5b}$$

$$\mathbf{0} \leq \boldsymbol{q}_t^{gen} \perp \boldsymbol{\mu}^{q,gen} \geq \mathbf{0} \quad \forall t \in \mathcal{T} \tag{1.5c}$$

$$\mathbf{0} \leq -\left(\boldsymbol{h}^- - \boldsymbol{h}^{ref} - \boldsymbol{M}^{h,p^{th}}(\boldsymbol{p}_t^{th} - \boldsymbol{p}^{th,ref})\right) \perp \boldsymbol{\mu}_t^{h^-} \geq \mathbf{0} \quad \forall t \in \mathcal{T} \tag{1.5d}$$

$$\mathbf{0} \leq -\left(\boldsymbol{v}^{ref} + \boldsymbol{M}^{v,p^{th}}(\boldsymbol{p}_t^{th} - \boldsymbol{p}^{th,ref}) - \boldsymbol{v}^+\right) \perp \boldsymbol{\mu}_t^{v^+} \geq \mathbf{0} \quad \forall t \in \mathcal{T} \tag{1.5e}$$

$$\mathbf{0} \leq -\left(\boldsymbol{u}^- - \boldsymbol{u}^{ref} - \boldsymbol{M}^{u,p}(\boldsymbol{p}_t - \boldsymbol{p}^{ref}) - \boldsymbol{M}^{u,q}(\boldsymbol{q}_t - \boldsymbol{q}^{ref})\right) \perp \boldsymbol{\mu}_t^{u^-} \geq \mathbf{0} \quad \forall t \in \mathcal{T} \tag{1.5f}$$

$$\mathbf{0} \leq -\left(\boldsymbol{u}^{ref} + \boldsymbol{M}^{u,p}(\boldsymbol{p}_t - \boldsymbol{p}^{ref}) + \boldsymbol{M}^{u,q}(\boldsymbol{q}_t - \boldsymbol{q}^{ref}) - \boldsymbol{u}^+\right) \perp \boldsymbol{\mu}_t^{u^+} \geq \mathbf{0} \quad \forall t \in \mathcal{T} \tag{1.5g}$$

$$\mathbf{0} \leq -\left(|\boldsymbol{s}^{f,ref}| + \boldsymbol{M}^{s^f,p}(\boldsymbol{p}_t - \boldsymbol{p}^{ref}) + \boldsymbol{M}^{s^f,q}(\boldsymbol{q}_t - \boldsymbol{q}^{ref}) - |\boldsymbol{s}^{f,+}|\right) \perp \boldsymbol{\mu}_t^{s^f,+} \geq \mathbf{0} \quad \forall t \in \mathcal{T} \tag{1.5h}$$

$$\mathbf{0} \leq -\left(|\boldsymbol{s}^{t,ref}| + \boldsymbol{M}^{s^t,p}(\boldsymbol{p}_t - \boldsymbol{p}^{ref}) + \boldsymbol{M}^{s^t,q}(\boldsymbol{q}_t - \boldsymbol{q}^{ref}) - |\boldsymbol{s}^{t,+}|\right) \perp \boldsymbol{\mu}_t^{s^t,+} \geq \mathbf{0} \quad \forall t \in \mathcal{T} \tag{1.5i}$$

Appendix B: Lagrangian function of the lower-level problem

$$
\begin{aligned}
L = & \sum_{t\in\mathcal{T}} -c_t^{ref}\left(p_t^{src} + \frac{1}{\eta^{ch}} p_t^{th,src}\right) \\
& + \sum_{t\in\mathcal{T}} \left(\left(\boldsymbol{c}^{p^{th},gen}\right)^\intercal \boldsymbol{p}_t^{th,gen} + (\boldsymbol{c}^{p,gen})^\intercal \boldsymbol{p}_t^{gen} + (\boldsymbol{c}^{q,gen})^\intercal \boldsymbol{q}_t^{gen}\right) \\
& + \sum_{t\in\mathcal{T}} \left(\boldsymbol{\lambda}_t^{p^{th}}\right)^\intercal \left(\boldsymbol{p}_t^{th} - \left[p_{f,t}^{th}\right]_{f\in\mathcal{F}} - \boldsymbol{p}_t^{th,gen}\right) \\
& + \sum_{t\in\mathcal{T}} (\boldsymbol{\lambda}_t^{p})^\intercal \left(\boldsymbol{p}_t - \left[p_{f,t}\right]_{f\in\mathcal{F}} - \boldsymbol{p}_t^{gen}\right) \\
& + \sum_{t\in\mathcal{T}} (\boldsymbol{\lambda}_t^{q})^\intercal \left(\boldsymbol{q}_t - \left[q_{f,t}\right]_{f\in\mathcal{F}} - \boldsymbol{q}_t^{gen}\right) \\
& + \sum_{t\in\mathcal{T}} \left(-\left(\boldsymbol{\mu}^{p^{th},gen}\right)^\intercal \boldsymbol{p}_t^{th,gen} - (\boldsymbol{\mu}^{p,gen})^\intercal \boldsymbol{p}_t^{gen} - (\boldsymbol{\mu}^{q,gen})^\intercal \boldsymbol{q}_t^{gen}\right) \\
& + \sum_{t\in\mathcal{T}} \left(\boldsymbol{\mu}_t^{h^-}\right)^\intercal \left(\boldsymbol{h}^- - \boldsymbol{h}^{ref} - \boldsymbol{M}^{h,p^{th}}(\boldsymbol{p}_t^{th} - \boldsymbol{p}^{th,ref})\right) \\
& + \sum_{t\in\mathcal{T}} \left(\boldsymbol{\mu}_t^{v^+}\right)^\intercal \left(\boldsymbol{v}^{ref} + \boldsymbol{M}^{v,p^{th}}(\boldsymbol{p}_t^{th} - \boldsymbol{p}^{th,ref}) - \boldsymbol{v}^+\right) \\
& + \sum_{t\in\mathcal{T}} \lambda_t^{p^{pm}} \left(\frac{1}{\eta^{ch}}\left(p_t^{th,src} - \boldsymbol{1}^\intercal \boldsymbol{p}_t^{th}\right) - p^{pm,ref} - \boldsymbol{M}^{p^{pm},p^{th}}(\boldsymbol{p}_t^{th} - \boldsymbol{p}^{th,ref})\right) \\
& + \sum_{t\in\mathcal{T}} (\boldsymbol{\mu}_t^{u^-})^\intercal \left(\boldsymbol{u}^- - \boldsymbol{u}^{ref} - \boldsymbol{M}^{u,p}(\boldsymbol{p}_t - \boldsymbol{p}^{ref}) - \boldsymbol{M}^{u,q}(\boldsymbol{q}_t - \boldsymbol{q}^{ref})\right) \\
& + \sum_{t\in\mathcal{T}} \left(\boldsymbol{\mu}_t^{u^+}\right)^\intercal \left(\boldsymbol{u}^{ref} + \boldsymbol{M}^{u,p}(\boldsymbol{p}_t - \boldsymbol{p}^{ref}) + \boldsymbol{M}^{u,q}(\boldsymbol{q}_t - \boldsymbol{q}^{ref}) - \boldsymbol{u}^+\right) \\
& + \sum_{t\in\mathcal{T}} \left(\boldsymbol{\mu}_t^{s^{f,+}}\right)^\intercal \left(|\boldsymbol{s}^{f,ref}| + \boldsymbol{M}^{s^f,p}(\boldsymbol{p}_t - \boldsymbol{p}^{ref}) + \boldsymbol{M}^{s^f,q}(\boldsymbol{q}_t - \boldsymbol{q}^{ref}) - |\boldsymbol{s}^{f,+}|\right) \\
& + \sum_{t\in\mathcal{T}} \left(\boldsymbol{\mu}_t^{s^{t,+}}\right)^\intercal \left(|\boldsymbol{s}^{t,ref}| + \boldsymbol{M}^{s^t,p}(\boldsymbol{p}_t - \boldsymbol{p}^{ref}) + \boldsymbol{M}^{s^t,q}(\boldsymbol{q}_t - \boldsymbol{q}^{ref}) - |\boldsymbol{s}^{t,+}|\right) \\
& + \sum_{t\in\mathcal{T}} \lambda_t^{p^{ls}} \left(p_t^{src} - \boldsymbol{1}^\intercal \boldsymbol{p}_t - p^{ls,ref} - \boldsymbol{M}^{p^{ls},p}(\boldsymbol{p}_t - \boldsymbol{p}^{ref}) - \boldsymbol{M}^{p^{ls},q}(\boldsymbol{q}_t - \boldsymbol{q}^{ref})\right) \\
& + \sum_{t\in\mathcal{T}} \lambda_t^{q^{ls}} \left(q_t^{src} - \boldsymbol{1}^\intercal \boldsymbol{q}_t - q^{ls,ref} - \boldsymbol{M}^{q^{ls},p}(\boldsymbol{p}_t - \boldsymbol{p}^{ref}) - \boldsymbol{M}^{q^{ls},q}(\boldsymbol{q}_t - \boldsymbol{q}^{ref})\right)
\end{aligned}
\tag{1.6}
$$

Appendix C: Strong duality equality of the lower-level problem

$$
\begin{aligned}
&\sum_{t \in \mathcal{T}} -c_t^{ref} \left(p_t^{src} + \frac{1}{\eta^{ch}} p_t^{th,src} \right) + \sum_{t \in \mathcal{T}} \left(\left(\boldsymbol{c}^{p^{th},gen} \right)^\intercal \boldsymbol{p}_t^{th,gen} + (\boldsymbol{c}^{p,gen})^\intercal \boldsymbol{p}_t^{gen} + (\boldsymbol{c}^{q,gen})^\intercal \boldsymbol{q}_t^{gen} \right) \\
&\quad = \sum_{t \in \mathcal{T}} \Bigg(- \left(\boldsymbol{\lambda}_t^{p^{th}} \right)^\intercal \left[p_{f,t}^{th} \right]_{f \in \mathcal{F}} - (\boldsymbol{\lambda}_t^{p})^\intercal \left[p_{f,t} \right]_{f \in \mathcal{F}} - (\boldsymbol{\lambda}_t^{q})^\intercal \left[q_{f,t} \right]_{f \in \mathcal{F}} \\
&\qquad + \left(\boldsymbol{\mu}_t^{h^-} \right)^\intercal \left(\boldsymbol{h}^- - \boldsymbol{h}^{ref} + \boldsymbol{M}^{h,p^{th}} \boldsymbol{p}^{th,ref} \right) \\
&\qquad + \left(\boldsymbol{\mu}_t^{v^+} \right)^\intercal \left(\boldsymbol{v}^{ref} - \boldsymbol{M}^{v,p^{th}} \boldsymbol{p}^{th,ref} - \boldsymbol{v}^+ \right) \\
&\qquad + \lambda_t^{p^{pm}} \left(-p^{pm,ref} + \boldsymbol{M}^{p^{pm},p^{th}} \boldsymbol{p}^{th,ref} \right) \\
&\qquad + (\boldsymbol{\mu}_t^{u^-})^\intercal \left(\boldsymbol{u}^- - \boldsymbol{u}^{ref} + \boldsymbol{M}^{u,p} \boldsymbol{p}^{ref} + \boldsymbol{M}^{u,q} \boldsymbol{q}^{ref} \right) \\
&\qquad + \left(\boldsymbol{\mu}_t^{u^+} \right)^\intercal \left(\boldsymbol{u}^{ref} - \boldsymbol{M}^{u,p} \boldsymbol{p}^{ref} - \boldsymbol{M}^{u,q} \boldsymbol{q}^{ref} - \boldsymbol{u}^+ \right) \\
&\qquad + \left(\boldsymbol{\mu}_t^{s^f,+} \right)^\intercal \left(|\boldsymbol{s}^{f,ref}| - \boldsymbol{M}^{s^f,p} \boldsymbol{p}^{ref} - \boldsymbol{M}^{s^f,q} \boldsymbol{q}^{ref} - |\boldsymbol{s}^{f,+}| \right) \\
&\qquad + \left(\boldsymbol{\mu}_t^{s^t,+} \right)^\intercal \left(|\boldsymbol{s}^{t,ref}| - \boldsymbol{M}^{s^t,p} \boldsymbol{p}^{ref} - \boldsymbol{M}^{s^t,q} \boldsymbol{q}^{ref} - |\boldsymbol{s}^{t,+}| \right) \\
&\qquad + \lambda_t^{p^{ls}} \left(-p^{ls,ref} + \boldsymbol{M}^{p^{ls},p} \boldsymbol{p}^{ref} + \boldsymbol{M}^{p^{ls},q} \boldsymbol{q}^{ref} \right) \\
&\qquad + \lambda_t^{q^{ls}} \left(-q^{ls,ref} + \boldsymbol{M}^{q^{ls},p} \boldsymbol{p}^{ref} + \boldsymbol{M}^{q^{ls},q} \boldsymbol{q}^{ref} \right) \Bigg)
\end{aligned} \tag{1.7a}
$$

Bibliography

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