Course Project - Step 4: Solving the problem in a decomposed manner

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Last updated: 1 March 2021

1 Introduction

As outlined in the project report for step 1, this work considers the combined optimal operation problem of a district-level energy system with air-conditioned buildings as flexible loads (FLs), a district cooling system (DCS) as thermal grid and an electric distribution grid. The combined operation problem of the thermal grid, the electric grid and the DERs can in principle be formulated as a centralized economic dispatch problem. However, the operation of each system is typically managed by dedicated organizations, i.e. thermal grid operator, electric grid operator and DER aggregators, where each entity may prefer to retain control and data of their respective systems. To this end, a decentralized local market organization is proposed for combined thermal and electric distribution grid operation, enabled by the ADMM solution methodology as a market-clearing mechanism to facilitate the energy trade between different entities while retaining organizational boundaries [1].

The considered test case is based on a neighbourhood in Singapore and was developed as part of [2]. The test case consists of thermal and electric grids with identical layout, i.e. thermal and electric grid nodes coincide. and 22 commercial buildings modelled as FLs. The electricity wholesale market price c_t^{ref} is derived from one day of the Universal Singapore Energy Price (USEP) [1].

1.1 Notation

Let $\mathbb{R}$ be the domain of real numbers. Non-bold letters x, X denote scalars $\mathbb{R}^1$, bold lowercase letters $\boldsymbol{x}$ denote vectors $\mathbb{R}^n$ and bold uppercase letters $\boldsymbol{X}$ denote matrices $\mathbb{R}^{n,m}$. Bold numbers $\boldsymbol{0}$ and $\boldsymbol{1}$ denote vectors or matrices of zeros and ones of appropriate sizes. The p-norm of a vector is denoted by $||\boldsymbol{x}||_p$. Prices are in Singapore Dollar (SGD) which is denoted by S$. The transpose of a vector or matrix is denoted by $()^\intercal$ and $[x_e]_{e\in\mathcal{E}}$ denotes the column vector of scalar values x_e for all $e \in \mathcal{E}$.

1.2 Flexible load, thermal grid and electric grid models

Mathematical models for flexible loads, thermal grid and electric grid have been introduced in the project report for step 1 and are omitted here for the sake of brevity.

[1] https://www.emcsg.com/MarketData/PriceInformation

2 ADMM-based market clearing

2.1 Combined optimal operation problem

The combined optimal operation problem addresses the economic dispatch of FLs subject to the operational constraints of the thermal grid, the electric grid and the FLs. The ADMM-based market-clearing algorithm (alg.) is applied as a distributed optimization solution methodology for the combined optimal operation problem. To form the basis for deriving the individual sub-problems of the market clearing alg., first, the combined optimal operation is formulated as follows:

$$\min_{\substack{\boldsymbol{p}_t^{th}, \boldsymbol{p}_t, \boldsymbol{q}_t, \\ p_t^{th,src}, p_t^{src}, q_t^{src}}} \quad \sum_{t \in \mathcal{T}} c_t^{ref} \left(p_t^{src} + \frac{1}{\eta^{ch}} p_t^{th,src} \right) \tag{1a}$$

$$\text{s.t.} \quad \boldsymbol{x}_{f,t^0} = \boldsymbol{x}_f^0 \tag{1b}$$

$$\boldsymbol{x}_{f,t+1} = \boldsymbol{A}_f \boldsymbol{x}_{f,t} + \boldsymbol{B}_f^c \boldsymbol{c}_{f,t} + \boldsymbol{B}_f^d \boldsymbol{d}_{f,t} \quad \forall t \in \mathcal{T} / t^N \tag{1c}$$

$$\boldsymbol{y}_{f,t} = \boldsymbol{C}_f \boldsymbol{x}_{f,t} + \boldsymbol{D}_f^c \boldsymbol{c}_{f,t} + \boldsymbol{D}_f^d \boldsymbol{d}_{f,t} \quad \forall t \in \mathcal{T} \tag{1d}$$

$$\boldsymbol{y}_{f,t}^- \leq \boldsymbol{y}_{f,t} \leq \boldsymbol{y}_{f,t}^+ \quad \forall t \in \mathcal{T} \tag{1e}$$

$$p_{f,t}^{th} = \boldsymbol{M}_f^{p^{th},y} \boldsymbol{y}_{f,t} \quad p_{f,t} = \boldsymbol{M}_f^{p,y} \boldsymbol{y}_{f,t} \quad q_{f,t} = \boldsymbol{M}_f^{q,y} \boldsymbol{y}_{f,t} \quad \forall t \in \mathcal{T} \tag{1f}$$

$$\boldsymbol{p}_t^{th} = \left[p_{f,t}^{th} \right]_{f \in \mathcal{F}} : \boldsymbol{\lambda}_t^{p^{th}} \quad \boldsymbol{p}_t = \left[p_{f,t} \right]_{f \in \mathcal{F}} : \boldsymbol{\lambda}_t^p \quad \boldsymbol{q}_t = \left[q_{f,t} \right]_{f \in \mathcal{F}} : \boldsymbol{\lambda}_t^q \quad \forall t \in \mathcal{T} \tag{1g}$$

$$\boldsymbol{h}^- \leq \boldsymbol{h}^{ref} + \boldsymbol{M}^{h,p^{th}} (\boldsymbol{p}_t^{th} - \boldsymbol{p}^{th,ref}) \quad : \boldsymbol{\mu}_t^{h^-} \quad \forall t \in \mathcal{T} \tag{1h}$$

$$\boldsymbol{v}^{ref} + \boldsymbol{M}^{v,p^{th}} (\boldsymbol{p}_t^{th} - \boldsymbol{p}^{th,ref}) \leq \boldsymbol{v}^+ \quad : \boldsymbol{\mu}_t^{v^+} \quad \forall t \in \mathcal{T} \tag{1i}$$

$$\frac{1}{\eta^{ch}} \left(p_t^{th,src} - \boldsymbol{1}^\intercal \boldsymbol{p}_t^{th} \right) = p^{pm,ref} + \boldsymbol{M}^{p^{pm},p^{th}} (\boldsymbol{p}_t^{th} - \boldsymbol{p}^{th,ref}) \quad : \lambda_t^{p^{pm}} \quad \forall t \in \mathcal{T} \tag{1j}$$

$$\boldsymbol{u}^- \leq \boldsymbol{u}^{ref} + \boldsymbol{M}^{u,p} (\boldsymbol{p}_t - \boldsymbol{p}^{ref}) + \boldsymbol{M}^{u,q} (\boldsymbol{q}_t - \boldsymbol{q}^{ref}) \leq \boldsymbol{u}^+ \quad : \boldsymbol{\mu}_t^{u^+}, \boldsymbol{\mu}_t^{u^-} \quad \forall t \in \mathcal{T} \tag{1k}$$

$$|\boldsymbol{s}^{f,ref}| + \boldsymbol{M}^{s^f,p} (\boldsymbol{p}_t - \boldsymbol{p}^{ref}) + \boldsymbol{M}^{s^f,q} (\boldsymbol{q}_t - \boldsymbol{q}^{ref}) \leq |\boldsymbol{s}^{f,+}| \quad : \boldsymbol{\mu}_t^{s^{f,+}} \quad \forall t \in \mathcal{T} \tag{1l}$$

$$|\boldsymbol{s}^{t,ref}| + \boldsymbol{M}^{s^t,p} (\boldsymbol{p}_t - \boldsymbol{p}^{ref}) + \boldsymbol{M}^{s^t,q} (\boldsymbol{q}_t - \boldsymbol{q}^{ref}) \leq |\boldsymbol{s}^{t,+}| \quad : \boldsymbol{\mu}_t^{s^{t,+}} \quad \forall t \in \mathcal{T} \tag{1m}$$

$$p_t^{src} - \boldsymbol{1}^\intercal \boldsymbol{p}_t = p^{ls,ref} + \boldsymbol{M}^{p^{ls},p} (\boldsymbol{p}_t - \boldsymbol{p}^{ref}) + \boldsymbol{M}^{p^{ls},q} (\boldsymbol{q}_t - \boldsymbol{q}^{ref}) \quad : \lambda_t^{p^{ls}} \quad \forall t \in \mathcal{T} \tag{1n}$$

$$q_t^{src} - \boldsymbol{1}^\intercal \boldsymbol{q}_t = q^{ls,ref} + \boldsymbol{M}^{q^{ls},p} (\boldsymbol{p}_t - \boldsymbol{p}^{ref}) + \boldsymbol{M}^{q^{ls},q} (\boldsymbol{q}_t - \boldsymbol{q}^{ref}) \quad : \lambda_t^{q^{ls}} \quad \forall t \in \mathcal{T} \tag{1o}$$

2.2 Market clearing

For the ADMM-based market clearing, individual sub-problems are formulated for 1) thermal grid operator, 2) electric grid operator and 3) flexible load aggregator. Each sub-problem consists of a subset of the objective and constraints from eq. (1) which are augmented by ADMM Lagrangian terms. In the following, $\rho > 0$ denotes the ADMM penalty factor, which serves as the convergence tuning parameter for the market clearing alg. and remains constant during ADMM iterations.

Thermal grid operator

The optimal operation problem of the thermal grid operator is formulated as:

$$\begin{aligned} \min_{\boldsymbol{p}_t^{th,th}} \; & \sum_{t \in \mathcal{T}} \left(\frac{c_t^{ref}}{\eta^{ch}} \mathbf{1}^\intercal \boldsymbol{p}_t^{th,th} + \left(\boldsymbol{\pi}_t^{th,th}\right)^\intercal \left(\boldsymbol{p}_t^{th,th} - \boldsymbol{p}_t^{th,ex}\right) + \frac{\rho}{2} \left\| \boldsymbol{p}_t^{th,th} - \boldsymbol{p}_t^{th,ex} \right\|_2^2 \right) \\ \text{s.t. } & (1h), (1i), (1j) \end{aligned} \tag{2}$$

The vector $\boldsymbol{p}_t^{th,th}$ is the thermal grid sub-problem variable for the thermal power demand $\boldsymbol{p}_t^{th}$. The vectors $\boldsymbol{\pi}_t^{th,th}$ and $\boldsymbol{p}_t^{th,ex}$ are the nodal thermal power price and the thermal power demand which are obtained in the previous market clearing iteration in alg. 1. Note that $\boldsymbol{\pi}_t^{th,th}$ represents the prices which are perceived by the thermal grid operator during market clearing, but upon convergence of alg. 1, the cleared nodal thermal power price is identical for the thermal grid operator and the flexible load aggregator.

Electric grid operator

The optimal operation problem of the electric grid operator is formulated as:

$$\begin{aligned} \min_{\boldsymbol{p}_t^{el}, \boldsymbol{q}_t^{el}} \; & \sum_{t \in \mathcal{T}} \left(c_t^{ref} \mathbf{1}^\intercal \boldsymbol{p}_t^{el} + \left(\boldsymbol{\pi}_t^{p,el}\right)^\intercal \left(\boldsymbol{p}_t^{el} - \boldsymbol{p}_t^{ex}\right) + \frac{\rho}{2} \left\| \boldsymbol{p}_t^{el} - \boldsymbol{p}_t^{ex} \right\|_2^2 + \left(\boldsymbol{\pi}_t^{q,el}\right)^\intercal \left(\boldsymbol{q}_t^{el} - \boldsymbol{q}_t^{ex}\right) + \frac{\rho}{2} \left\| \boldsymbol{q}_t^{el} - \boldsymbol{q}_t^{ex} \right\|_2^2 \right) \\ \text{s.t. } & (1k), (1l), (1m), (1n), (1o) \end{aligned} \tag{3}$$

The vectors $\boldsymbol{p}_t^{el}$ and $\boldsymbol{q}_t^{el}$ are the electric grid sub-problem variables for the active power demand $\boldsymbol{p}_t$ and reactive power demand $\boldsymbol{q}_t$. The vectors $\boldsymbol{\pi}_t^{p,el}$ and $\boldsymbol{\pi}_t^{q,el}$ are the nodal active power price and nodal reactive power price. Similar to the thermal grid operator above, the vectors $\boldsymbol{\pi}_t^{p,el}$ and $\boldsymbol{\pi}_t^{q,el}$ represent the prices which are perceived by the electric grid operator during market clearing, but upon convergence of alg. 1, the cleared nodal price is identical for the electric grid operator and the flexible load aggregator. The vectors $\boldsymbol{p}_t^{ex}$ and $\boldsymbol{q}_t^{ex}$ are the active and reactive power demand which are obtained in the previous market clearing iteration in alg. 1.

Flexible load aggregator

The optimal operation problem of the flexible load aggregator is formulated as:

$$\begin{aligned} \min_{\boldsymbol{p}_t^{th,fl}, \boldsymbol{p}_t^{fl}, \boldsymbol{q}_t^{fl}} \; & \sum_{t \in \mathcal{T}} \left(\frac{c_t^{ref}}{\eta^{ch}} \mathbf{1}^\intercal \boldsymbol{p}_t^{th,th} + \boldsymbol{\pi}_t^{th,fl\intercal} \left(\boldsymbol{p}_t^{th,fl} - \boldsymbol{p}_t^{th,ex}\right) + \frac{\rho}{2} \left\| \boldsymbol{p}_t^{th,fl} - \boldsymbol{p}_t^{th,ex} \right\|_2^2 \right. \\ & \left. + \left(\boldsymbol{\pi}_t^{p,fl}\right)^\intercal \left(\boldsymbol{p}_t^{fl} - \boldsymbol{p}_t^{ex}\right) + \frac{\rho}{2} \left\| \boldsymbol{p}_t^{fl} - \boldsymbol{p}_t^{ex} \right\|_2^2 + \left(\boldsymbol{\pi}_t^{q,fl}\right)^\intercal \left(\boldsymbol{q}_t^{fl} - \boldsymbol{q}_t^{ex}\right) + \frac{\rho}{2} \left\| \boldsymbol{q}_t^{fl} - \boldsymbol{q}_t^{ex} \right\|_2^2 \right) \\ \text{s.t. } & (1b), (1c), (1d), (1e), (1f) \end{aligned} \tag{4}$$

The vectors $\boldsymbol{p}_t^{th,fl}$, $\boldsymbol{p}_t^{fl}$ and $\boldsymbol{q}_t^{fl}$ are the flexible load aggregator sub-problem variables for the thermal power demand $\boldsymbol{p}_t^{th}$, active power demand $\boldsymbol{p}_t$ and reactive power demand $\boldsymbol{q}_t$. The vectors $\boldsymbol{\pi}_t^{th,fl}$, $\boldsymbol{\pi}_t^{p,fl}$ and $\boldsymbol{\pi}_t^{q,fl}$ are the nodal thermal power price, nodal active power price and nodal reactive power price, as perceived by the flexible load aggregator during market clearing. The vectors $\boldsymbol{p}_t^{th,ex}$, $\boldsymbol{p}_t^{ex}$ and $\boldsymbol{q}_t^{ex}$ are the thermal power demand, active power demand and reactive power demand which are obtained in the previous market clearing iteration in alg. 1.

Market-clearing algorithm

Algorithm 1: Market clearing based on ADMM

begin initialization:

$\boldsymbol{p}_t^{th,ex} = \mathbf{0},\ \boldsymbol{p}_t^{ex} = \mathbf{0},\ \boldsymbol{q}_t^{ex} = \mathbf{0},\ \boldsymbol{\pi}_t^{th,th} = \mathbf{0},\ \boldsymbol{\pi}_t^{p,el} = \mathbf{0},\ \boldsymbol{\pi}_t^{q,el} = \mathbf{0},\ \boldsymbol{\pi}_t^{th,fl} = \mathbf{0},\ \boldsymbol{\pi}_t^{p,fl} = \mathbf{0},\ \boldsymbol{\pi}_t^{q,fl} = \mathbf{0};$

repeat ADMM loop:

begin local optimization:

$\boldsymbol{p}_t^{th,th} = \arg\min$ (2);

$\boldsymbol{p}_t^{el}, \boldsymbol{q}_t^{el} = \arg\min$ (3);

$\boldsymbol{p}_t^{th,fl}, \boldsymbol{p}_t^{fl}, \boldsymbol{q}_t^{fl} = \arg\min$ (4);

begin variable update:

$\boldsymbol{p}_t^{th,ex} = \frac{1}{2}(\boldsymbol{p}_t^{th,th} + \boldsymbol{p}_t^{th,fl});$

$\boldsymbol{p}_t^{ex} = \frac{1}{2}(\boldsymbol{p}_t^{el} + \boldsymbol{p}_t^{fl});$

$\boldsymbol{q}_t^{ex} = \frac{1}{2}(\boldsymbol{q}_t^{el} + \boldsymbol{q}_t^{fl});$

begin price update:

$\boldsymbol{\pi}_t^{th,th} = \boldsymbol{\pi}_t^{th,th} + \rho(\boldsymbol{p}_t^{th,th} - \boldsymbol{p}_t^{th,ex});$

$\boldsymbol{\pi}_t^{p,el} = \boldsymbol{\pi}_t^{p,el} + \rho(\boldsymbol{p}_t^{el} - \boldsymbol{p}_t^{ex});$

$\boldsymbol{\pi}_t^{q,el} = \boldsymbol{\pi}_t^{q,el} + \rho(\boldsymbol{q}_t^{el} - \boldsymbol{q}_t^{ex});$

$\boldsymbol{\pi}_t^{th,fl} = \boldsymbol{\pi}_t^{th,fl} + \rho(\boldsymbol{p}_t^{th,fl} - \boldsymbol{p}_t^{th,ex});$

$\boldsymbol{\pi}_t^{p,fl} = \boldsymbol{\pi}_t^{p,fl} + \rho(\boldsymbol{p}_t^{fl} - \boldsymbol{p}_t^{ex});$

$\boldsymbol{\pi}_t^{q,fl} = \boldsymbol{\pi}_t^{q,fl} + \rho(\boldsymbol{q}_t^{fl} - \boldsymbol{q}_t^{ex});$

until $r(\boldsymbol{a}, \boldsymbol{b}) < \epsilon$
$\forall(\boldsymbol{a}, \boldsymbol{b}) \in \{(\boldsymbol{p}^{th,th}, \boldsymbol{p}^{th,fl}), (\boldsymbol{p}^{el}, \boldsymbol{p}^{fl}), (\boldsymbol{q}^{el}, \boldsymbol{q}^{fl})\};$

Alg. 1 presents the market-clearing alg. based on ADMM which is utilised to coordinate the solution of eqs. (2) to (4) to obtain the market equilibrium. Note that all ADMM iteration variables are initialized to zero at the beginning of the ADMM loop. The scalar r denotes the primal residuals, which are defined as $r(\boldsymbol{a}, \boldsymbol{b}) = \sum_{t\in\mathcal{T}} \left\|\boldsymbol{a}_t - \boldsymbol{b}_t\right\|_1$ for thermal power $(\boldsymbol{p}^{th,th}, \boldsymbol{p}^{th,fl})$, active power $(\boldsymbol{p}^{el}, \boldsymbol{p}^{fl})$ and reactive power $(\boldsymbol{q}^{el}, \boldsymbol{q}^{fl})$. Since the residuals approach zero in the final solution [3], the ADMM loop is terminated once the residuals reach the desired termination threshold given by $\epsilon > 0$.

3 Results

The presented methodology is demonstrated for a test case with an artificial branch volume flow constraint is defined in thermal grid, to demonstrate the ability of the market clearing alg. to achieve a solution in the face of congestion.

Figures 1 and 2 depict the dispatch and price schedules for thermal power which are obtained from centralized operation, thermal grid operator and flexible load aggregator. In this context, the centralized operation solution serves as a theoretical reference and is obtained from the solution of eq. (1). The solutions for thermal grid operator and flexible load aggregator are obtained as a result of the market clearing with alg. 1. To this end, the results in figs. 1 and 2 demonstrate that the solutions for the individual sub-problems through alg. 1 align with the solution of the combined problem in eq. (1). The results for active and reactive power coincide with

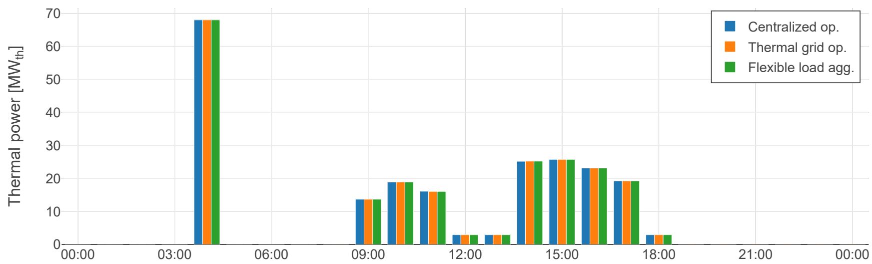

Figure 1: Thermal power dispatch at FL 22.

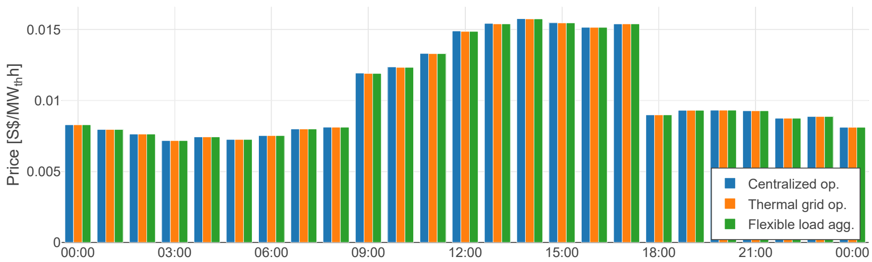

Figure 2: Thermal power price at FL 22.

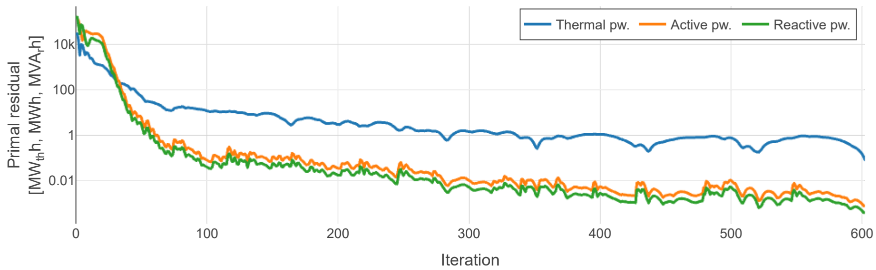

Figure 3: Primal residuals.

the results for thermal power and are omitted for brevity. The mid-day surge in prices is due to congestion, which highlights the ability of alg. 1 to determine a market-clearing solution despite the presence of congestion. Figure 3 depicts the primal residuals, where the alg. 1 converged after approx. 600 iterations for $\epsilon = 10^{-1}$. The ADMM parameters were not tuned for convergence speed according to [3], although this would be required for practical applications.

4 Conclusion

This work presented a local market organization for combined thermal and electric distribution grid operation along with an ADMM solution methodology as a market-clearing mechanism. The proposed market-clearing mechanism was shown to achieve the market equilibrium in the presence of congestion.

Bibliography

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