AIRS Deconvolution
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1 AIRS spectral response functions

The AIRS spectral response functions model channel response as a function of frequency and associate channels with nominal center frequencies [1]. Each AIRS channel i has an associated spectral response function or SRF $\sigma_i(v)$ such that the channel radiance $c_i = \int \sigma_i(v) r(v)\, dv$. The center or peak of this function is the channel frequency.

Suppose we have n channels and a frequency grid v of k points spanning the domains of the functions σ_i. The grid step size for our applications is often 0.0025 cm^{-1}, the kcarta resolution [2]. Let S_k be an $n \times k$ array such that $s_{i,j} = \sigma_i(v_j)/w_i$, where $w_i = \sum_j \sigma_i(v_j)$, that is where row i is $\sigma_i(v)$ tabulated at the grid v and normalized so the row sum is 1. If the channel centers are in increasing order S_k is banded, and if they are not too close, the rows are linearly independent. S_k is a linear transform whose domain is radiance at the grid v and whose range is channel radiances. If r is radiance at the grid v, then $c = S_k r$ gives a good approximation of the channel radiances $c_i = \int \sigma_i(v) r(v)\, dv$.

In practice this is how we convolve kcarta or other simulated radiances to get AIRS channel radiances, or transmittances for our fast transmittance models[3]. We construct S_k either explicitly or implicitly from AIRS SRF tabulations. The matrix S_k in the former case is large but managable with a banded or sparse representation.

Suppose we have S_k and channel radiances c and want to find r, that is, to deconvolve c. Consider the linear system $S_k x = c$. Since $n < k$ for the kcarta grid mentioned above, this is underdetermined, with infinitely many solutions. We could add constraints, take a pseudo-inverse, consider a new matrix S_b with columns tabulated at some coarser grid, or some combination of the above. We begin with two cases, S_a with SRFs tabulated at the AIRS channel grid, and S_b with SRFs at an intermediate grid, typically 0.1 cm^{-1}, the approximate resolution of the original measurements. We then consider the effect of choice of basis set for the intermdiate representation.

2 Deconvolution to the AIRS channel grid

Let $v_a = v_1, v_2, \ldots, v_n$ be channel center frequencies associated with a set of SRFs. Similar to S_k, let S_a be an $n \times n$ array where row i is $\sigma_i(v)$ tabulated at the v_a grid, with rows normalized to 1. If the channel centers are in increasing order S_a is banded, and if they are not too close the rows are linearly independent. S_a is a linear transform whose domain is radiance at the grid v_a and whose range is channel radiances. If r is radiance at the grid v_a, then $c = S_a r$ is still an approximation of $\int \sigma_i(v) r(v)\, dv$, though not as good an approximation as for the high-resolution case.

Consider the linear system $S_a x = c$, similar to the case $S_k x = c$ above, where we are given S_a and channel signals c and want to find radiances x. Since S_a is an $n \times n$ matrix and may have linearly independent rows, we might hope to solve for r. However in practice there are problems. If we take v_a as the standard AIRS L1b channel set, we find S_a is poorly conditioned, without a usable inverse. This is due to the L1b channel spacing, which because of module overlaps is quite variable, with the closest channels only 0.0036 cm^{-1} apart.

If we use the the linear function $g(v) = 4 \cdot 10^{-4} \cdot v - 0.04$, where v is frequency, as a lower bound on the acceptable channel spacing, we drop about 64 out of the 2378 L1b channels and the condition number of S_a is much improved, to around 30. With the partly synthetic L1c channel set, $g(v)$ drops only 4 channels and $\mathrm{cond}(S_a)$ is about 250, still low enough for a useful inverse.

- show figures for channel trimming
- compare this approach with more recent results
- the main point of this section is the channel trimming and discussion of condition number—maybe merge with the following section into a single discussion where the intermediate grid size varies from n to k.

3 Deconvolution to the SRF tabulation grid

We now consider deconvolution to an intermediate grid. Typically this will be at 0.1 cm^{-1}, the resolution of the tabulated SRFs. Let $v_b = v_1, v_2, \ldots, v_m$ be channel center frequencies associated with a set of SRFs. Similar to S_k, let S_b be an $n \times m$ array where row i is $\sigma_i(v)$ tabulated at the v_m grid, with rows normalized to 1. If the channel centers are in increasing order S_m is banded, and if they are not too close the rows are linearly independent. S_m is a linear transform whose domain is radiance at the grid v_m and whose range is channel radiances. If r is radiance at the grid v_b, then $c = S_b r$ is a good approximation of $\int \sigma_i(v) r(v)\, dv$.

Consider the linear system $S_b x = c$, similar to the case $S_k x = c$ above, where we are given S_b and channel signals c and want to find radiances x. Since $n < m < k$, as with S_k, the system will be underdetermined, but more manageable because for the default resolutions m is approximately 40 times less than k, so finding the pseudo-inverse of S_b becomes tractible.

- this works well in practice
- show representative results from cris_test4
- there is some redundant detail in the descriptions of $S_{a,b,k}$ that will go away if the sections are merged

4 A sinc basis for the intermediate representation

5 Applications