interferometric interpolation

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basic equations

The Nyquist equations for Fourier transform interferometry are

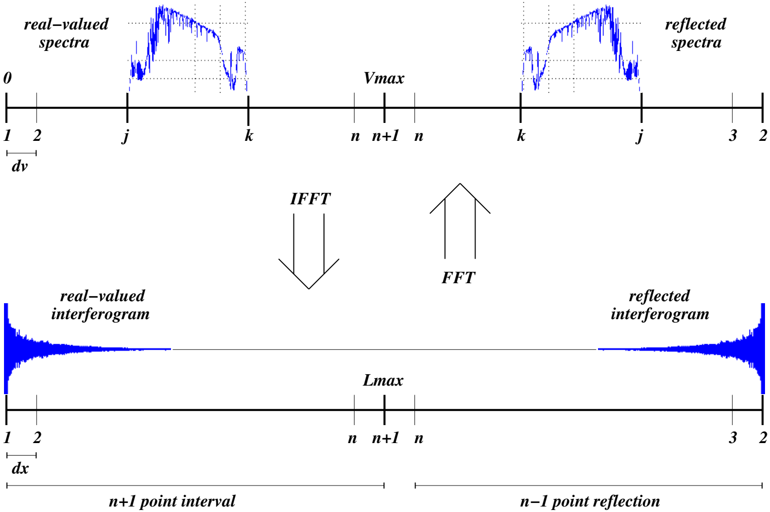
$$V = Ndv = 1/2dx$$

$$L = Ndx = 1/2dv$$

where

- L is maximum OPD
- V is maximum frequency
- dx is the distance step
- dv is the frequency step
- the transform size is $2N$

example

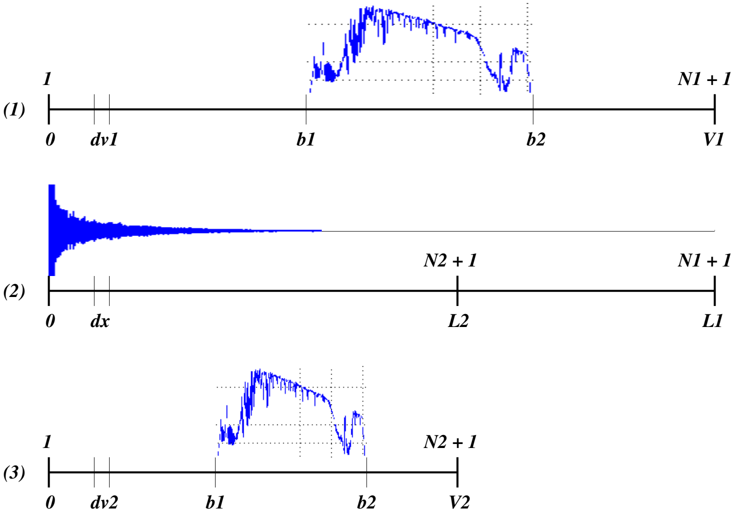


n+1 point real–valued cosine transform with a 2n point FFT

discussion

- the previous slide shows the special case of a symmetrical real-valued interferogram being transformed to symmetrical real-valued spectra. Note that the transform is invertible.
- the $2n$ point real-valued FFT is equivalent to an $n+1$ point cosine transform.
- applications of the $n+1$ point representation include
  - calculating expected observed radiances
  - applying apodization and response functions
  - interpolation between instrument grids
- for a typical interpolation application we start with two frequency steps, dv_1 and dv_2, and want to move between them

interpolation



basic equations

The Nyquist equations for (1) and (2) are

$$V_1 = N_1 dv_1 = 1/2dx$$

$$L_1 = N_1 dx = 1/2dv_1$$

and for (2) and (3) are

$$V_2 = N_2 dv_2 = 1/2dx$$

$$L_2 = N_2 dx = 1/2dv_2$$

If we are given dv_1 and dv_2 then we need dx, N_1, or N_2 to fill out the values above. N_1 and N_2 must be integers.

constraints

Note that dx is the same for both pairs of equations, so $V_1 = V_2$. Then $N_1 dv_1 = N_2 dv_2$, and we have

$$N_2/N_1 = dv_1/dv_2,$$

a constraint on the transform sizes that can only be satisfied if dv_1/dv_2 is rational. In addition, the transforms must include the band of interest, so we require

$$V_1 = N_1 dv_1 \geq b_2$$

Suppose dv_1/dv_2 is rational. Let m_1 and m_2 be the smallest integers such that $m_1/m_2 = dv_1/dv_2$.

constraints

Let k be the smallest integer such that $m_2 \cdot 2^k \cdot dv_1 \geq b_2$, and let $N_1 = m_2 \cdot 2^k$ and $N_2 = m_1 \cdot 2^k$.

Then the constraints above are satisfied, and in addition if m_1 and m_2 are not too large then N_1 and N_2 will have mostly small prime factors, making the FFT calculation more efficient.

If dv_1/dv_2 is not rational or m_1 or m_2 are very large we may want to proceed differently, for example by finding a value for dv_1 close to the desired dv but with a more tractible rational representation. In that case we might want to do a conventional interpolation from dv to dv_1 as a preliminary step.