Physics 20 - Lesson 28
Simple Harmonic Motion – Energy & Dynamics

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1) /2 A simple harmonic oscillator experiences a restoring force toward the equilibrium point which depends on the distance from the equilibrium point. The further from the equilibrium point the greater the restoring force and therefore the greater the acceleration.

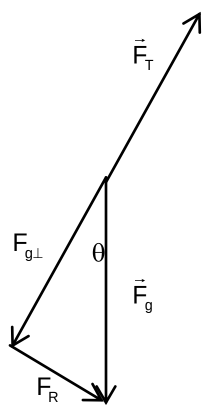
2) /3

(a) acceleration is at a maximum at the points of greatest displacement from the equilibrium position

(b) the velocity is at a maximum as the oscillator passes through the equilibrium position

(c) the restoring force is at a maximum at the points of greatest displacement from the equilibrium position

3) /5



$$\vec{F}_R = F_g \sin\theta\,[\text{right}]$$
$$\vec{F}_R = mg \sin\theta\,[\text{right}]$$
$$\vec{F}_R = (0.3000\,kg)(9.81\,^{m}/_{s^2})\sin 12.0^\circ\,[\text{right}]$$
$$\boxed{\vec{F}_R = 0.612\,N\,[\text{right}]}$$

4) /3

$$F_R = F_g \sin\theta$$
$$F_R = mg \sin\theta$$
$$\theta = \sin^{-1}\frac{F_R}{mg}$$
$$\theta = \sin^{-1}\frac{4.00N}{(0.50000kg)(9.81\,^{m}/_{s^2})}$$
$$\boxed{\theta = 54.6^\circ}$$

5)

/7

$$E_T = E_k + E_p$$
$$E_T = \tfrac{1}{2}mv^2 + mgh$$
$$E_T = \tfrac{1}{2}(1.35kg)(2.40m/s)^2 + 1.35kg(9.81 \tfrac{m}{s^2})(0.85m)$$
$$E_T = 15.14\,J$$

$$T = 2\pi\sqrt{\frac{l}{g}}$$
$$T = 2\pi\sqrt{\frac{3.20\,m}{9.81N/m}}$$
$$\boxed{T = 3.59\,s}$$

$$E_{k_{\max}} = E_T$$
$$\tfrac{1}{2}mv_{\max}^2 = E_T$$
$$v_{\max} = \sqrt{\frac{2E_T}{m}}$$
$$v_{\max} = \sqrt{\frac{2(15.14J)}{1.35kg}}$$
$$\boxed{v_{\max} = 4.74\,m/s}$$

6)

/7

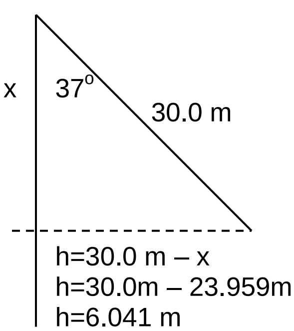
$$E_{k_{\max}} = E_{P_{\max}}$$
$$\tfrac{1}{2}mv_{\max}^2 = mgh_{\max}$$
$$\tfrac{1}{2}mv_{\max}^2 = mgh_{\max}$$
$$h_{\max} = \frac{\tfrac{1}{2}v_{\max}^2}{g}$$
$$h_{\max} = \frac{\tfrac{1}{2}(6.26 \tfrac{m}{s})^2}{9.81 \tfrac{m}{s^2}}$$
$$\boxed{h_{\max} = 2.00\,m}$$

$$T = 2\pi\sqrt{\frac{l}{g}}$$
$$T = 2\pi\sqrt{\frac{2.75\,m}{9.81N/m}}$$
$$\boxed{T = 3.33\,s}$$

7)

/8

x=30.0cos37°
x=23.959m



h=30.0 m – x
h=30.0m – 23.959m
h=6.041 m

a. from rest

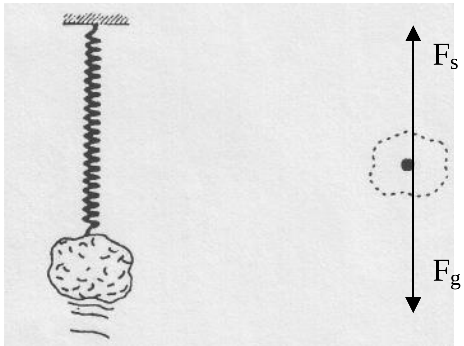
$$E_k = E_p$$
$$\frac{\cancel{m}v^2}{2} = \cancel{m}gh$$
$$v = \sqrt{2gh} = \sqrt{2(9.81 \tfrac{m}{s^2})(6.041m)}$$
$$\boxed{v = 10.9\,m/s}$$

b. starts at 4.00 m/s

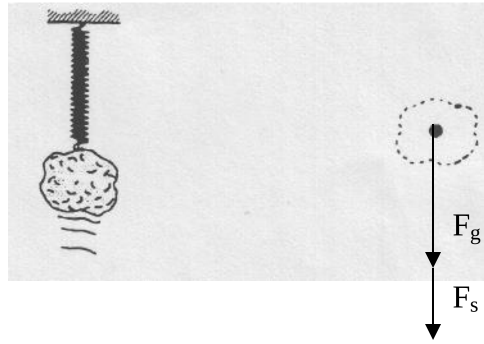
$$E_k = E_p + E_{ki}$$
$$\frac{\cancel{m}v^2}{2} = \cancel{m}gh + \frac{\cancel{m}v_i^2}{2}$$
$$v = \sqrt{2gh + v_i^2}$$
$$v = \sqrt{2(9.81 \tfrac{m}{s^2})(6.041m) + (4.00m/s)^2}$$
$$\boxed{v = 11.6\,m/s}$$

8) 2 marks each for a total of 8 marks.

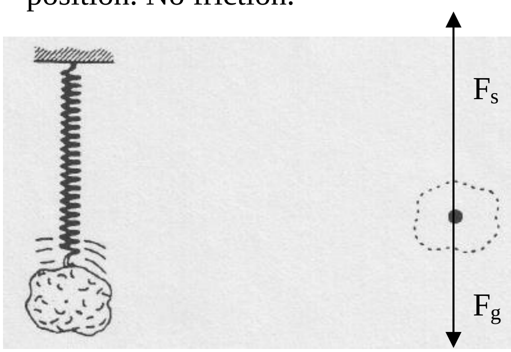
a. Suspended from a spring. Pulled downward slightly and released. No friction.



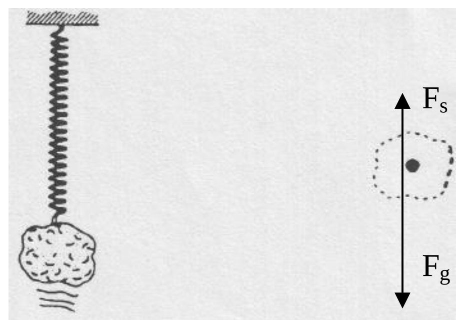
b. Suspended from a spring. Instantaneously at rest at the top of its travel.



c. Suspended from a spring. Moving downward through the equilibrium position. No friction.



d. Suspended from a spring. Moving upward through the equilibrium position. No friction.



9) /6

first calculate k

$$T = 2\pi\sqrt{\frac{m}{k}} \rightarrow T^2 = \frac{4\pi^2 m}{k}$$

$$\therefore k = \frac{4\pi^2 m}{T^2}$$

$$k = \frac{4\pi^2(3.08kg)}{(0.323s)^2}$$

$$k = 1165.48\,{}^{N}\!/\!{}_{m}$$

second calculate a

$$\vec{F}_{NET} = \vec{F}_s$$

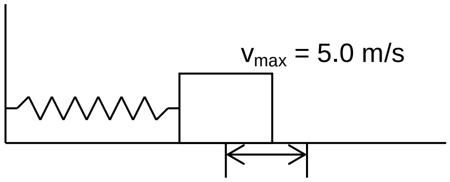
$$m\vec{a} = -kx$$

$$\vec{a} = \frac{-kx}{m}$$

$$\vec{a} = \frac{-(1165.48\,{}^{N}\!/\!{}_{m})(2.85m)}{(3.08kg)}$$

$$\boxed{\vec{a} = 1.08\times10^3\,{}^{m}\!/\!{}_{s^2}\,[\text{left}]}$$

10) /5



$$E_{k_{max}} = E_{p_{max}}$$
$$\tfrac{1}{2}mv_{max}^2 = \tfrac{1}{2}kx_{max}^2$$
$$k = \frac{mv_{max}^2}{x_{max}^2}$$
$$k = \frac{(4.0kg)(5.0m/s)^2}{(0.110m)^2}$$
$$\boxed{k = 8.3\times10^3\ N/m}$$

11) /6

first find m

$$E_{k_{max}} = E_{p_{max}}$$
$$\tfrac{1}{2}mv_{max}^2 = \tfrac{1}{2}kx_{max}^2$$
$$m = \frac{kx^2}{v^2}$$
$$m = \frac{18N/m(0.29m)^2}{(0.35m/s)^2}$$
$$m = 12.4\,kg$$

Second, calculate the speed when x = 0.20 m

$$E_{p\max} = E_k + E_p$$
$$\tfrac{1}{2}kx_{max}^2 = \tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2$$
$$mv^2 = kx_{max}^2 - kx^2$$
$$v = \sqrt{\frac{k(x_{max}^2 - x^2)}{m}}$$
$$v = \sqrt{\frac{18\,{}^{N}\!/\!{}_{m}((0.29m)^2 - (0.20m^2))}{12.4kg}}$$
$$\boxed{v = 0.25\,m/s}$$

12) /9

$$E_T = E_k + E_p$$
$$E_T = \tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2$$
$$E_T = \tfrac{1}{2}(0.750kg)(2.50m/s)^2 + \tfrac{1}{2}(995N/m)(0.145m)^2$$
$$E_T = 12.8J$$

$$E_{k_{max}} = E_T$$
$$\tfrac{1}{2}mv_{max}^2 = E_T$$
$$v_{max} = \sqrt{\frac{2E_T}{m}}$$
$$v_{max} = \sqrt{\frac{2(12.8J)}{0.750kg}}$$
$$\boxed{v_{max} = 5.84\,m/s}$$

$$E_{p_{max}} = E_T$$
$$\tfrac{1}{2}kx_{max}^2 = E_T$$
$$x_{max} = \sqrt{\frac{2E_T}{k}}$$
$$x_{max} = \sqrt{\frac{2(12.8J)}{995N/m}}$$
$$\boxed{x_{max} = 0.160\,m}$$

13) /9

a) calculate acceleration

$$\vec{F}_{NET} = \vec{F}_s$$
$$m\vec{a} = -kx$$
$$\vec{a} = \frac{-kx}{m}$$
$$\vec{a} = \frac{-(40.0\,{}^{N}\!/\!_{m})(+0.300m)}{(2.50kg)}$$
$$\boxed{\vec{a} = -4.80\,{}^{m}\!/\!_{s^2}}$$

b) maximum speed

$$E_{k_{\max}} = E_{p_{\max}}$$
$$\tfrac{1}{2}mv_{\max}^2 = \tfrac{1}{2}kx_{\max}^2$$
$$v_{\max} = \sqrt{\frac{kx_{\max}^2}{m}}$$
$$v_{\max} = \sqrt{\frac{40.0\,{}^{N}\!/\!_{m}(0.300m)^2}{2.50kg}}$$
$$\boxed{v_{\max} = 3.20\,{}^{m}\!/\!_{s}}$$

c) period

$$T = 2\pi\sqrt{\frac{m}{k}}$$
$$T = 2\pi\sqrt{\frac{2.50kg}{40.0\,{}^{N}\!/\!_{m}}}$$
$$\boxed{T = 1.57s}$$

14) /6

$$E_{k_{\max}} = E_{p_{\max}}$$
$$\tfrac{1}{2}mv_{\max}^2 = \tfrac{1}{2}kx_{\max}^2$$
$$k = \frac{mv_{\max}^2}{x_{\max}^2}$$
$$k = \frac{(0.200kg)(0.803\,{}^{m}\!/\!_{s})^2}{(0.120m)^2}$$
$$k = 8.96\,{}^{N}\!/\!_{m}$$

$$\vec{F}_{NET} = \vec{F}_s$$
$$m\vec{a} = -k\vec{x}$$
$$\vec{x} = -\frac{m\vec{a}}{k}$$
$$\vec{x} = -\frac{(0.200kg)(3.58\,{}^{m}\!/\!_{s^2}[\text{west}])}{8.96\,{}^{N}\!/\!_{m}}$$
$$\boxed{\vec{x} = 0.0799m[\text{east}]}$$

15) /6

First, find m

$$E_{k_{\max}} = E_{p_{\max}}$$
$$\tfrac{1}{2}mv_{\max}^2 = \tfrac{1}{2}kx_{\max}^2$$
$$m = \frac{kx^2}{v^2}$$
$$m = \frac{18\,{}^{N}\!/\!_{m}(0.29m)^2}{(0.35\,{}^{m}\!/\!_{s})^2}$$
$$m = 12.4\,kg$$

Second, calculate the speed when x = 0.20 m

$$E_{p\max} = E_k + E_p$$
$$\tfrac{1}{2}kx_{\max}^2 = \tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2$$
$$mv^2 = kx_{\max}^2 - kx^2$$
$$v = \sqrt{\frac{k(x_{\max}^2 - x^2)}{m}}$$
$$v = \sqrt{\frac{18\,{}^{N}\!/\!_{m}(0.29m^2 - 0.20m^2)}{12.4kg}}$$
$$\boxed{v = 0.25\,{}^{m}\!/\!_{s}}$$