

# Math 10

## Lesson 1–8 Answers

### Lesson Questions

#### Question 1

Simplify the following expressions. Write all answers as positive exponents.

$$x^3 \cdot x^{-2}$$

$$= x^{3+(-2)}$$

$$\boxed{= x^1}$$

$$(x^2 y^{-3})^4$$

$$= x^{2 \cdot 4} y^{-3 \cdot 4}$$

$$= x^8 y^{-12}$$

$$\boxed{= \frac{x^8}{y^{12}}}$$

$$y^4 \cdot y^2$$

$$= y^{4+2}$$

$$\boxed{= y^6}$$

$$y^4 + y^2$$

Since the terms are being added the exponent rules do not apply. Therefore, the expression is already in simplest form.

$$(x^2)^3$$

$$= x^{2 \cdot 3}$$

$$\boxed{= x^6}$$

$$\left(\frac{z^7}{z^3}\right)^2$$

$$= (z^{7-3})^2 \text{ or } = \frac{z^{7 \cdot 2}}{z^{3 \cdot 2}}$$

$$= (z^4)^2 = \frac{z^{14}}{z^6}$$

$$= z^{4 \cdot 2} = z^{14-6}$$

$$\boxed{= z^8}$$

$$\boxed{= z^8}$$

$$m^2 \cdot m^{-5}$$

$$= m^{2+(-5)}$$

$$= m^{-3}$$

$$\boxed{= \frac{1}{m^3}}$$

$$(x^{-3} x^{-5})^{-2}$$

$$= x^{-3 \cdot -2} x^{-5 \cdot -2} \text{ or } = (x^{-3+(-5)})^{-2}$$

$$= x^6 x^{10}$$

$$= x^{6+10}$$

$$\boxed{= x^{16}}$$

$$= (x^{-8})^{-2}$$

$$= x^{-8 \cdot -2}$$

$$\boxed{= x^{16}}$$

$$\frac{a^3 b^4 c^{-2}}{a^2 b^{-3} c^{-4}}$$

$$= a^{3-2} b^{4-(-3)} c^{-2-(-4)}$$

$$\boxed{= ab^7 c^2}$$

$$\left[\left(-\frac{3}{2}\right)^{-4}\right]^{-2}$$

$$= \left(-\frac{3}{2}\right)^{-4 \cdot -2} \text{ or } = \left[\left(-\frac{2}{3}\right)^4\right]^2$$

$$= \left(-\frac{3}{2}\right)^8 = \left[\frac{(-2)^4}{3^4}\right]^2$$

$$= \left(-\frac{2}{3}\right)^8 = \left[\frac{2^4}{3^4}\right]^2$$

$$= -^8 \frac{2^8}{3^8} = \frac{2^{4 \cdot 2}}{3^{4 \cdot 2}}$$

$$\boxed{= \frac{2^8}{3^8}}$$

$$\boxed{= \frac{2^8}{3^8}}$$

$$\begin{aligned}
& \left[ \left( -\frac{3}{2} \right)^{-4} \right]^2 \cdot \left[ \left( -\frac{3}{2} \right)^2 \right]^3 \\
&= \left( -\frac{3}{2} \right)^{-4 \cdot 2} \cdot \left( -\frac{3}{2} \right)^{2 \cdot 3} \\
&= \left( -\frac{3}{2} \right)^{-8} \cdot \left( -\frac{3}{2} \right)^6 \\
&= \left( -\frac{3}{2} \right)^{-2} \\
&= \left( -\frac{2}{3} \right)^2 \\
&= \frac{2^2}{3^2} \\
&= \frac{4}{9}
\end{aligned}$$

$$\begin{aligned}
& \frac{(1.4^3)(1.4^4)}{1.4^{-2}} \\
&= 1.4^{3+4-(-2)} \\
&= 1.4^9
\end{aligned}$$

## Question 2

Simplify the following expressions. Write all answers as positive exponents.

$$\begin{aligned}
& \left( \frac{7^{\frac{2}{3}}}{7^{\frac{1}{3}} \cdot 7^{\frac{5}{3}}} \right)^6 \\
&= \left( 7^{\frac{2}{3} - \frac{1}{3} - \frac{5}{3}} \right)^6 \\
&= \left( 7^{-\frac{4}{3}} \right)^6 \\
&= 7^{-\frac{4}{3} \cdot 6} \\
&= 7^{-8} \\
&= \frac{1}{7^8}
\end{aligned}$$

$$\begin{aligned}
& (8a^3b^6)^{\frac{1}{3}} \\
&= 8^{\frac{1}{3}} a^{3 \cdot \frac{1}{3}} b^{6 \cdot \frac{1}{3}} \\
&= 2ab^2
\end{aligned}$$

$$\begin{aligned}
& \left( x^{\frac{3}{2}} y^2 \right) \left( x^{\frac{1}{2}} y^{-1} \right) \\
&= x^{\frac{3}{2} + \frac{1}{2}} y^{2+(-1)} \\
&= x^2 y
\end{aligned}$$

$$\begin{aligned}
& \left( \frac{100a}{25a^5b^{\frac{1}{2}}} \right)^{\frac{1}{2}} \\
&= \left( \frac{100}{25} \frac{1}{a^{5-1}b^{\frac{1}{2}}} \right)^{\frac{1}{2}} \\
&= \left( 4 \frac{1}{a^4b^{\frac{1}{2}}} \right)^{\frac{1}{2}} \\
&= 4^{\frac{1}{2}} \frac{1}{a^{4 \cdot \frac{1}{2}} b^{\frac{1}{2} \cdot \frac{1}{2}}} \\
&= \frac{2}{a^2b^{\frac{1}{4}}}
\end{aligned}$$

$$\begin{aligned}
& \frac{4a^{-2}b^{\frac{2}{3}}}{2a^2b^{\frac{1}{3}}} \\
&= \frac{4}{2} a^{-2-2} b^{\frac{2}{3} - \frac{1}{3}} \\
&= 2a^{-4}b^{\frac{1}{3}} \\
&= \frac{2b^{\frac{1}{3}}}{a^4}
\end{aligned}$$

### Question 3

$$V = \frac{4}{3}\pi r^3$$

$$425 = \frac{4}{3}\pi r^3$$

$$\frac{3 \cdot 425}{4} = \pi r^3$$

$$318.75 = \pi r^3$$

$$\frac{318.75}{\pi} = r^3$$

$$101.4613 = r^3$$

$$\sqrt[3]{101.4613} = r$$

$$r = 4.664$$

### Assignment

1. a)  $x^7$       b)  $a^{-3}$   
c)  $b^2$       d)  $m^{-1}$
2. a)  $0.5^5$       b)  $0.5^{-1}$   
c)  $0.5^{-1}$       d)  $0.5^5$
3. a)  $x^2$       b)  $x^{-3}$   
c)  $n^1$       d)  $a^{-4}$
4. a)  $n^6$       b)  $z^{-6}$   
c)  $n^{12}$       d)  $c^{-4}$
5. a)  $\left(\frac{3}{5}\right)^{12}$       b)  $\left(\frac{3}{5}\right)^{-12}$   
c)  $\left(\frac{3}{5}\right)^{12}$       d)  $\left(\frac{3}{5}\right)^{12}$
6. a)  $\frac{a^2}{b^2}$       b)  $\frac{n^6}{m^3}$   
c)  $\frac{c^{-8}}{d^{-8}}$  or  $\frac{d^8}{c^8}$       d)  $\frac{4b^2}{25c^2}$   
e)  $a^2b^2$       f)  $n^6m^3$   
g)  $c^{-12}d^{-8}$       h)  $x^3y^{-3}$





$$\begin{aligned}
 11. \quad V &= \frac{4\pi r^3}{3} \\
 r &= \sqrt[3]{\frac{3V}{4\pi}} \\
 r &= \sqrt[3]{\frac{3(375)}{4\pi}} \\
 r &= 4.4735...
 \end{aligned}$$

$$\begin{aligned}
 A &= 4\pi r^2 \\
 A &= 4\pi(4.4735...) ^2 \\
 A &= 251.5 \text{ ft}^2
 \end{aligned}$$

$$\begin{aligned}
 12. \text{ a) } & \frac{(a^2b^{-1})^{-2}}{(a^{-3}b)^3} \\
 &= \frac{a^{-4}b^2}{a^{-9}b^3} \text{ power rule} \\
 &= \frac{a^5}{b} \text{ quotient rule}
 \end{aligned}$$

$$\begin{aligned}
 \text{b) } & \left( \frac{(c^{-3}d)^{-1}}{c^2d} \right)^{-2} \\
 &= \frac{(c^{-3}d)^2}{c^{-4}d^{-2}} \text{ power rule} \\
 &= \frac{c^{-6}d^2}{c^{-4}d^{-2}} \text{ power rule} \\
 &= \frac{d^4}{c^2} \text{ power rule}
 \end{aligned}$$

$$\begin{aligned}
 13. \text{ a) } & (a^3b^2)(a^2b^3) \\
 &= a^5b^5 \\
 &= (-2)^5(1)^5 \\
 &= -32
 \end{aligned}$$

$$\begin{aligned}
 \text{b) } & (a^{-1}b^{-2})(a^{-2}b^{-3}) \\
 &= a^{-3}b^{-5} \\
 &= \frac{1}{a^3b^5} \\
 &= \frac{1}{(-2)^3(1)^5} \\
 &= -\frac{1}{8}
 \end{aligned}$$

$$\begin{aligned}
 \text{c) } & \frac{a^{-4}b^5}{ab^3} \\
 &= \frac{b^2}{a^5} \\
 &= \frac{(1)^2}{(-2)^5} \\
 &= -\frac{1}{32}
 \end{aligned}$$

$$\begin{aligned}
 \text{d) } & \left( \frac{a^{-7}b^7}{a^{-9}b^{10}} \right)^{-5} \\
 &= \frac{a^{35}b^{-35}}{a^{45}b^{-50}} \\
 &= \frac{b^{15}}{a^{10}} \\
 &= \frac{(1)^{15}}{(-2)^{10}} \\
 &= \frac{1}{1024}
 \end{aligned}$$

$$\begin{aligned}
 14. \text{ a) } & m^{\frac{2}{3}} \cdot m^{\frac{4}{3}} \\
 &= m^{\frac{2}{3} + \frac{4}{3}} \\
 &= m^2
 \end{aligned}$$

$$\begin{aligned}
 \text{b) } & x^{\frac{3}{2}} \div x^{-\frac{1}{4}} \\
 &= x^{\frac{3}{2} - (-\frac{1}{4})} \\
 &= x^{\frac{7}{4}}
 \end{aligned}$$

$$\begin{aligned}
 \text{c) } & \frac{-9a^{-4}b^{\frac{3}{4}}}{3a^2b^{\frac{1}{4}}} \\
 &= \frac{-3b^{\frac{3}{4} - \frac{1}{4}}}{a^{2+4}} \\
 &= \frac{-3b^{\frac{1}{2}}}{a^6}
 \end{aligned}$$

$$\begin{aligned}
 \text{d) } & \left( \frac{-64c^6}{a^9b^{-\frac{1}{2}}} \right)^{\frac{1}{3}} \\
 &= \frac{(-64)^{\frac{1}{3}} c^{\frac{6}{3}}}{a^{\frac{1}{3} \cdot 9} b^{\frac{1}{3} \cdot (-\frac{1}{2})}} \\
 &= \frac{-4c^2}{a^3 b^{-\frac{1}{6}}} \\
 &= -\frac{4b^{\frac{1}{6}}c^2}{a^3}
 \end{aligned}$$



$$\begin{aligned}
 15. \text{ a) } (x^2 y^{-3}) (x^{\frac{1}{2}} y^{-1}) &= x^2 \cdot x^{\frac{1}{2}} \cdot y^{-3} \cdot y^{-1} \\
 &= \cancel{x^1} \cdot \cancel{y^3} \rightarrow = x^{2+\frac{1}{2}} \cdot y^{-3+(-1)} \\
 &= xy^3 \qquad = x^{2+\frac{1}{2}} \cdot y^{-4} \\
 & \qquad \qquad = \frac{x^{\frac{5}{2}}}{y^4}
 \end{aligned}
 \qquad
 \begin{aligned}
 \text{b) } \left( \frac{-5a^2}{b^{\frac{1}{2}}} \right)^{-2} &= \frac{\cancel{10}a^{-4}}{b^{-1}} \rightarrow = \frac{(-5)^{-2} a^{-4}}{b^{-1}} \\
 &= \frac{10b}{a^4} \qquad = \frac{b}{(-5)^2 a^4} \\
 & \qquad \qquad = \frac{b}{25a^4}
 \end{aligned}$$

$$\begin{aligned}
 16. \text{ a) } \frac{(m^{-3} n^2)^{-4}}{(m^2 n^{-3})^2} &= (m^{-5} n^5)^{-6} \\
 &= m^{30} n^{-30} \rightarrow = m^{30} n^{-30} \\
 &= (mn)^{30} \qquad = \frac{m^{30}}{n^{30}} = \left( \frac{m}{n} \right)^{30}
 \end{aligned}
 \qquad
 \begin{aligned}
 \text{b) } \left( r^{\frac{1}{2}} s^{-\frac{3}{2}} \right)^{\frac{1}{2}} \left( r^{-\frac{1}{4}} s^{\frac{1}{2}} \right)^{-1} &= \cancel{r^1} \cdot \cancel{s^{-1}} \cdot \cancel{r^{\frac{5}{4}}} \cdot \cancel{s^{-\frac{1}{2}}} \rightarrow = r^{\frac{1}{4}} \cdot s^{-\frac{3}{4}} \cdot r^{\frac{1}{4}} \cdot s^{-\frac{1}{2}} \\
 &= r^{1-\frac{5}{4}} \cdot s^{-1-\frac{1}{2}} \qquad = r^{\frac{1}{4}+\frac{1}{4}} \cdot s^{-\frac{3}{4}-\frac{1}{2}} \\
 &= r^{-\frac{1}{4}} \cdot s^{-\frac{3}{2}} \qquad = r^{\frac{1}{2}} \cdot s^{-\frac{5}{4}} \\
 &= \frac{1}{r^{\frac{1}{4}} \cdot s^{\frac{3}{2}}} \qquad = \frac{r^{\frac{1}{2}}}{s^{\frac{5}{4}}}
 \end{aligned}$$

$$\begin{aligned}
 17. \text{ a) } \left( \frac{a^{-3} b}{c^3} \right)^{-4} \cdot \left( \frac{c^5}{a^4 b^{-3}} \right)^{-1} &= \left( \frac{a^{12} b^{-4}}{c^{-12}} \right) \cdot \left( \frac{c^{-5}}{a^{-4} b^3} \right) \\
 &= \frac{a^{12+4} c^{-5+12}}{b^{3+4}} \\
 &= \frac{a^{16} c^7}{b^7}
 \end{aligned}
 \qquad
 \begin{aligned}
 \text{b) } \frac{(2a^{-1} b^4 c^{-3})^{-2}}{(4a^2 b c^{-4})^2} &= \frac{(2^{-2} a^2 b^{-8} c^6)}{(4^2 a^4 b^2 c^{-8})} \\
 &= \frac{c^{6+8}}{4^2 2^2 a^{4-2} b^{2+8}} \\
 &= \frac{c^{14}}{64 a^2 b^{10}}
 \end{aligned}$$

$$18. \text{ a) } \left( x^{\frac{1}{2}} y^{\frac{2}{3}} \right)^2$$

$$= xy^{\frac{4}{3}}$$

$$= a^{-2} \left( a^{\frac{2}{3}} \right)^{\frac{4}{3}}$$

$$= a^{-2} a^{\frac{8}{9}}$$

$$= a^{\frac{-10}{9}}$$

$$= \frac{1}{a^{\frac{10}{9}}}$$

$$\text{b) } \left( x^{\frac{3}{4}} \div y^{-\frac{1}{2}} \right)^3$$

$$= \left( x^{\frac{3}{4}} \cdot y^{\frac{1}{2}} \right)^3$$

$$= x^{\frac{9}{4}} y^{\frac{3}{2}}$$

$$= \left( a^{-2} \right)^{\frac{9}{4}} \left( a^{\frac{2}{3}} \right)^{\frac{3}{2}}$$

$$= a^{-\frac{9}{2}} a^1$$

$$= a^{-\frac{7}{2}}$$

$$= \frac{1}{a^{\frac{7}{2}}}$$

19. For example:

$$\text{a) } x^1 \cdot x^{\frac{1}{2}}, x^{\frac{3}{4}} \cdot x^{\frac{3}{4}}, x^2 \cdot x^{-\frac{1}{2}}$$

$$\text{b) } x^2 \div x^{\frac{1}{2}}, x^{\frac{5}{2}} \div x^1, x^{-1} \div x^{-\frac{5}{2}}$$

$$\text{c) } \left( x^{\frac{1}{2}} \right)^3, \left( x^6 \right)^{\frac{1}{4}}, \left( x^3 \right)^{\frac{1}{2}}$$