Math 10

Lesson 1–5 Answers

Lesson Questions

Question 1

The key idea for simplifying radicals is to find the largest common factors of the radicand (the number inside the radical sign) that are perfect squares, perfect cubes, perfect quartics, etc. depending on the index.

For $\sqrt{75}$, on the side of the page we write the perfect squares. We note that 25 is a factor of 75.

$$\begin{aligned}\sqrt{75} &= \sqrt{25\cdot 3}\\ &= \sqrt{25}\cdot\sqrt{3}\\ &= \boxed{5\sqrt{3}}\end{aligned}$$

$2^2 = 4$
$3^2 = 9$
$4^2 = 16$
$5^2 = 25$
$6^2 = 36$
$7^2 = 49$

For $\sqrt[4]{288}$, on the side of the page we write the perfect powers of 4. (Note that the numbers become very large very quickly, therefore we usually look for factors of small numbers like 2 or 3 that are being raised to the power of 4.) We note that 16 is a factor of 288 and 81 is not a factor of 288.

$$\begin{aligned}\sqrt[4]{288} &= \sqrt[4]{16\cdot 18}\\ &= \sqrt[4]{16}\cdot\sqrt[4]{18}\\ &= \boxed{2\sqrt[4]{18}}\end{aligned}$$

$2^4 = 16$
$3^4 = 81$
$4^4 = 256$

For $\sqrt[3]{1715}$, on the side of the page we write the perfect cubes. We note that 343 is a factor of 1715.

$$\begin{aligned}\sqrt[3]{1715} &= \sqrt[3]{343\cdot 5}\\ &= \sqrt[3]{343}\cdot\sqrt[3]{5}\\ &= \boxed{7\sqrt[3]{5}}\end{aligned}$$

$2^3 = 8$
$3^3 = 27$
$4^3 = 64$
$5^3 = 125$
$6^3 = 216$
$7^3 = 343$

For $\sqrt{30}$, there are no perfect square factors of 30, therefore it is already in simplest form.

$$\begin{aligned}\sqrt[4]{32} &= \sqrt[4]{16\cdot 2}\\ &= \sqrt[4]{16}\cdot\sqrt[4]{2}\\ &= \boxed{2\sqrt[4]{2}}\end{aligned}$$

$$\begin{aligned}\sqrt[3]{40} &= \sqrt[3]{8\cdot 5}\\ &= \sqrt[3]{8}\cdot\sqrt[3]{5}\\ &= \boxed{2\sqrt[3]{5}}\end{aligned}$$

Question 2

$3\sqrt{7} = \sqrt{7 \cdot 3^2} = \sqrt{7 \cdot 9} = \sqrt{63}$

$3\sqrt[6]{7} = \sqrt[6]{7 \cdot 3^6} = \sqrt[6]{7 \cdot 729} = \sqrt[6]{5103}$

$2\sqrt[5]{8} = \sqrt[5]{8 \cdot 2^5} = \sqrt[5]{8 \cdot 32} = \sqrt[5]{256}$

Nasty question of the day

Given that $\sqrt{2} \doteq 1.4142$ and <u>without using a calculator</u> determine a decimal approximation for each radical.

(a)
$$\begin{aligned}\sqrt{200} &= \sqrt{100 \cdot 2} \\ &= 10\sqrt{2} \quad \leftarrow \sqrt{2} \doteq 1.4142 \\ &= 10 \cdot 1.4142 \\ &= 14.142\end{aligned}$$

(b)
$$\begin{aligned}\sqrt{20000} &= \sqrt{10000 \cdot 2} \\ &= 100\sqrt{2} \\ &= 100 \cdot 1.4142 \\ &= 141.42\end{aligned}$$

(c)
$$\begin{aligned}\sqrt{8} &= \sqrt{4 \cdot 2} \\ &= 2\sqrt{2} \\ &= 2 \cdot 1.4142 \\ &= 2.8284\end{aligned}$$

(d)
$$\begin{aligned}\sqrt{18} &= \sqrt{9 \cdot 2} \\ &= 3\sqrt{2} \\ &= 3 \cdot 1.4142 \\ &= 4.2426\end{aligned}$$

(e)
$$\begin{aligned}\sqrt{32} &= \sqrt{16 \cdot 2} \\ &= 4\sqrt{2} \\ &= 4 \cdot 1.4142 \\ &= 5.6568\end{aligned}$$

(f)
$$\begin{aligned}\sqrt{50} &= \sqrt{25 \cdot 2} \\ &= 5\sqrt{2} \\ &= 5 \cdot 1.4142 \\ &= 7.0710\end{aligned}$$

Assignment

1.

a) $\sqrt{8} = \sqrt{4 \cdot 2} = \sqrt{4}\sqrt{2} = 2\sqrt{2}$

b) $\sqrt{12} = \sqrt{4 \cdot 3} = 2\sqrt{3}$

c) $\sqrt{32} = \sqrt{16 \cdot 2} = 4\sqrt{2}$

d) $\sqrt{50} = \sqrt{25 \cdot 2} = 5\sqrt{2}$

e) $\sqrt{18} = \sqrt{9 \cdot 2} = 3\sqrt{2}$

f) $\sqrt{27} = \sqrt{9 \cdot 3} = 3\sqrt{3}$

g) $\sqrt{48} = \sqrt{16 \cdot 3} = 4\sqrt{3}$

h) $\sqrt{75} = \sqrt{25 \cdot 3} = 5\sqrt{3}$

2.

a) $5\sqrt{2} = \sqrt{5^2 \cdot 2} = \sqrt{50}$

b) $6\sqrt{2} = \sqrt{6^2 \cdot 2} = \sqrt{72}$

e) $5\sqrt{3} = \sqrt{5^2 \cdot 3} = \sqrt{75}$

f) $6\sqrt{3} = \sqrt{6^2 \cdot 3} = \sqrt{108}$

3.

d) $\sqrt{600} = \sqrt{100}\sqrt{6} = 10\sqrt{6}$

e) $\sqrt{54} = \sqrt{9}\sqrt{6} = 3\sqrt{6}$

f) cannot be simplified

g) $\sqrt{28} = \sqrt{4}\sqrt{7} = 2\sqrt{7}$

h) cannot be simplied

i) $\sqrt{112} = \sqrt{16}\sqrt{7} = 4\sqrt{7}$

4.

a) $\sqrt[3]{16} = \sqrt[3]{8 \cdot 2} = \sqrt[3]{8}\sqrt[3]{2} = 2\sqrt[3]{2}$

c) $\sqrt[3]{256} = \sqrt[3]{64}\sqrt[3]{4} = 4\sqrt[3]{4}$

e) $\sqrt[3]{60}$ cannot be simplified

g) $\sqrt[3]{135} = \sqrt[3]{27}\sqrt[3]{5} = 3\sqrt[3]{5}$

i) $\sqrt[3]{500} = \sqrt[3]{125}\sqrt[3]{4} = 5\sqrt[3]{4}$

5.

a) $3\sqrt{2} = \sqrt{3^2 \cdot 2} = \sqrt{18}$

c) $6\sqrt{5} = \sqrt{6^2 \cdot 5} = \sqrt{180}$

e) $7\sqrt{7} = \sqrt{7^2 \cdot 7} = \sqrt{343}$

g) $3\sqrt[3]{3} = \sqrt[3]{3^3 \cdot 3} = \sqrt[3]{81}$

i) $5\sqrt[3]{2} = \sqrt[3]{5^3 \cdot 2} = \sqrt[3]{250}$

6. $\sqrt{252} = \sqrt{36 \cdot 7} = 6\sqrt{7}$ ft

7. $\sqrt[3]{200} = \sqrt[3]{8 \cdot 25} = 2\sqrt[3]{25}$ cm

8. $x = \sqrt{54}$

   $x = \sqrt{9 \cdot 6}$

   $x = 3\sqrt{6}$

   $\text{perimeter} = 4 \cdot 3\sqrt{6} = 12\sqrt{6}$

   $\text{perimeter} = 12\sqrt{6}$ in.

9.

   a) $\sqrt[4]{48} = \sqrt[4]{16 \cdot 3} = 2\sqrt[4]{3}$

   c) $\sqrt[4]{1250} = \sqrt[4]{625 \cdot 2} = 5\sqrt[4]{2}$

10.

   a) $6\sqrt[4]{3} = \sqrt[4]{6^4 \cdot 3} = \sqrt[4]{3888}$

   c) $3\sqrt[5]{4} = \sqrt[5]{3^5 \cdot 4} = \sqrt[5]{972}$

11.

$$
\begin{aligned}
8\sqrt[3]{2} &= 8 \cdot \sqrt[3]{2} \\
&= \cancel{\sqrt[3]{2}} \cdot \sqrt[3]{2} \rightarrow &&= \sqrt[3]{512} \cdot \sqrt[3]{2} \\
&= \sqrt[3]{2 \cdot 2} && \boxed{= \sqrt[3]{1024}} \\
&= \sqrt[3]{4}
\end{aligned}
$$

12.

$$
\begin{aligned}
\sqrt{96} &= \sqrt{4} \cdot \cancel{\sqrt{48}} \rightarrow &&= \sqrt{4} \cdot \sqrt{24} \\
&= 2 \cdot \sqrt{48} && = 2 \cdot \sqrt{24} \\
&= 2 \cdot \sqrt{8} \cdot \sqrt{6} && = 2 \cdot \sqrt{4} \cdot \sqrt{6} \\
&= 2 \cdot \cancel{4} \cdot \sqrt{6} && = 2 \cdot 2 \cdot \sqrt{6} \\
&= 8\sqrt{6} && \boxed{= 4\sqrt{6}}
\end{aligned}
$$

13.

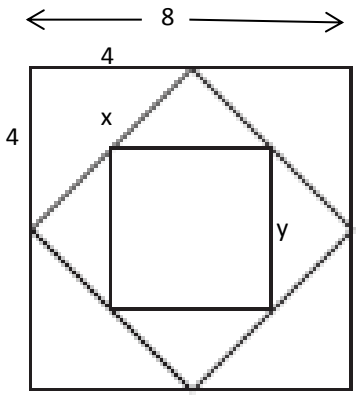
a)

$$
\begin{aligned}
\sqrt{4} &= 2 \\
\sqrt{400} &= 20 \\
\sqrt{40000} &= 200 \\
\sqrt{4000000} &= 2000 \\
\sqrt{400000000} &= 20000
\end{aligned}
$$

c)

$$
\begin{aligned}
\sqrt{8} &= 2\sqrt{2} \\
\sqrt{800} &= 20\sqrt{2} \\
\sqrt{80000} &= 200\sqrt{2} \\
\sqrt{8000000} &= 2000\sqrt{2} \\
\sqrt{800000000} &= 20000\sqrt{2}
\end{aligned}
$$

14.



Find x using Pythagorus

$x^2 = 4^2 + 4^2$

$x = \sqrt{4^2 + 4^2}$

$x = \sqrt{16 + 16}$

$x = \sqrt{32}$

Find y using Pythagorus

$y^2 = \left(x/2\right)^2 + \left(x/2\right)^2$

$y^2 = \left(\sqrt{32}/2\right)^2 + \left(\sqrt{32}/2\right)^2$

$y^2 = 32/4 + 32/4$

$y^2 = 8 + 8$

$y^2 = 16$

$y = \sqrt{16}$

$y = 4$

For the x square

$x = \sqrt{32} = \sqrt{16 \cdot 2}$

$x = 4\sqrt{2}\,\text{cm}$

$\text{Area}_x = x^2$

$\text{Area}_x = \sqrt{32}^2$

$\text{Area}_x = 32\,\text{cm}^2$

For the y square

$y = 4$

$\text{Area}_y = y^2$

$\text{Area}_y = 4^2$

$\text{Area}_y = 16\,\text{cm}^2$