Math 10

Lesson 1–4 Answers

Lesson Questions

Question 1

When we calculate the radical, radicals that are rational numbers result in a rational number while radicals that are irrational result in an irrational number.

$\sqrt{0.24} = 0.489897...$irrational

$\sqrt[5]{-32} = -2$ rational

$\frac{5}{7}$ rational

$\sqrt{64} = 8$ rational

$\sqrt{\frac{16}{49}} = \frac{\sqrt{16}}{\sqrt{49}} = \frac{4}{7}$ rational

$\sqrt{\frac{1}{5}} = \frac{1}{\sqrt{5}} = 0.4473235...$ irrational

$\sqrt{0.25} = 0.5$ rational

$\sqrt[4]{12} = 1.122462...$ irrational

$0.6^2 = 0.36$ rational

$\sqrt{3} = 1.17320508...$ irrational

Question 2

Compute the following numbers and classify them as natural, whole, integer, rational, and/or irrational:

$\sqrt{16} = 4$ rational, integer, whole and natural

$\sqrt[3]{30} = 3.1072325...$ irrational

$\sqrt[4]{\frac{16}{81}} = \frac{\sqrt[4]{16}}{\sqrt[4]{81}} = \frac{2}{3}$ rational

Question 3

Which numbers below belong to each set: natural, whole, integer, rational, and/or irrational?

$\frac{3}{5}$ rational

$0.21\overline{7}$ rational

-6 rational, integer

41275 rational, integer, whole, natural

$3\sqrt{2}$ irrational

6π irrational

$-2\frac{1}{4}$ rational

$\sqrt[3]{8} = 2$ rational, integer, whole, natural

$\sqrt{121} = 11$ rational, integer, whole, natural

6.121121... rational

Question 4

Classify each of the following numbers as rational or irrational. Provide an explanation.

Number	Rational or irrational	Explanation
0	rational	whole number
π	irrational	non-repeating number
$\sqrt{36}=6$	rational	whole number
$-4.2558...$	irrational	non-repeating number
-4.2558	rational	terminating decimal
$\frac{99}{13}$	rational	division of two integers
$\sqrt{500}=22.36067...$	irrational	non-repeating number
$6.\overline{3}$	rational	repeating number
$\sqrt[3]{343}=7$	rational	whole number

Question 5

Which of the following numbers are irrational. Provide an explanation.

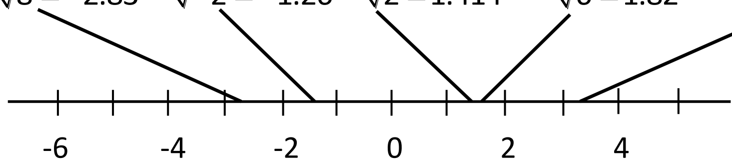
Number	Irrational (yes or no)	Explanation
$\sqrt{3}=1.73205...$	yes	non-terminating decimal
$\sqrt{36+64}=\sqrt{100}=10$	no	integer
$\sqrt{24}=4.898979...$	yes	non-terminating decimal
$2\sqrt{36}=12$	no	integer
$\sqrt{2+\sqrt{4}}=2$	no	integer
$\sqrt{36}+\sqrt{64}=14$	no	integer
$\sqrt{2\frac{1}{4}}=1.5$	no	terminating decimal
$\sqrt{434}=20.83266...$	yes	non-terminating decimal
$2+\sqrt{36}=8$	no	integer

Question 6

Use a number line to order the following numbers from least to greatest

$\sqrt{2} = 1.414$ $\quad \sqrt[3]{-2} = -1.26$ $\quad \sqrt[3]{6} = 1.82$ $\quad \sqrt{11} = 3.32$ $\quad -\sqrt{8} = -2.83$

$-\sqrt{8} = -2.83$ $\quad \sqrt[3]{-2} = -1.26$ $\quad \sqrt{2} = 1.414$ $\quad \sqrt[3]{6} = 1.82$ $\quad \sqrt{11} = 3.32$



Question 7

$\sqrt{-4}$ is not a real number. It is impossible to find a root value that, when multiplied by itself, results in a negative number (i.e. 2·2 = 4 and –2 · –2= 4).

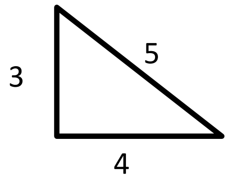
(However, the idea of the square root of a negative number eventually led to a whole new branch of mathematics called Complex Numbers.)

Nasty question of the day

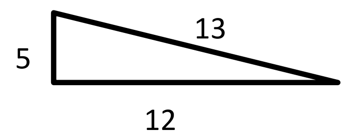
For a right angle triangle, the lengths of the sides must obey Pythagorus' equation

$c^2 = a^2 + b^2$

(a) All sides have rational number lengths.
There are an infinite number of triangles like this (Google whole number right triangles):

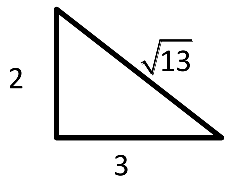


$$\begin{aligned} c^2 &= a^2 + b^2 \\ 5^2 &= 4^2 + 3^2 \\ 25 &= 16 + 9 \\ 25 &= 25 \end{aligned}$$



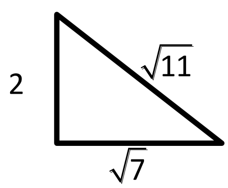
$$\begin{aligned} c^2 &= a^2 + b^2 \\ 13^2 &= 12^2 + 5^2 \\ 169 &= 144 + 25 \\ 169 &= 169 \end{aligned}$$

(b) Exactly 2 sides have rational number lengths.
There are an infinite number of triangles like this:



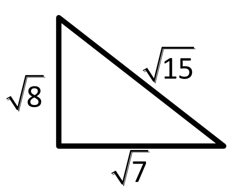
$$\begin{aligned} c^2 &= a^2 + b^2 \\ c^2 &= 2^2 + 3^2 \\ c^2 &= 4 + 9 \\ c^2 &= 13 \\ c &= \sqrt{13} \end{aligned}$$

(c) Exactly 1 side has a rational number length.
There are an infinite number of triangles like this:



$$\begin{aligned} c^2 &= a^2 + b^2 \\ \left(\sqrt{11}\right)^2 &= 2^2 + \left(\sqrt{7}\right)^2 \\ 11 &= 4 + 7 \\ 11 &= 11 \end{aligned}$$

(d) No sides have rational number lengths.
There are an infinite number of triangles like this:



$$\begin{aligned} c^2 &= a^2 + b^2 \\ \left(\sqrt{15}\right)^2 &= \left(\sqrt{8}\right)^2 + \left(\sqrt{7}\right)^2 \\ 15 &= 8 + 7 \\ 15 &= 15 \end{aligned}$$

Assignment

1. a) The square root of 8 is between the root of 4 (2) and the root of 9 (3). Since 8 is close to 9, we try 2.9 and 2.8 and find that 2.8 is the best answer

   b) The cube root of 9 is between the cube root of 8 (2) and the cube root of 27 (3). Since 9 is very close to 8 we try 2.1 and 2.2 and find that 2.1 is the best answer

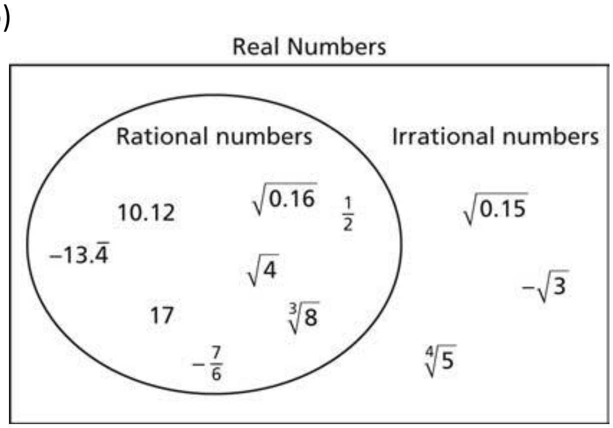
   c) 1.8

   d) 3.6

2.
a) The calculator returns an error message; the square of a real number will always be positive.
b) Any non-zero <u>even</u> index
c) i) Any odd index
  ii) Any even index

3. a) As written the number 12.247 448 71 is rational since it terminates.
   b) The root of 150 is irrational since it results in a non-terminating and non-repeating number.

4. a), b)



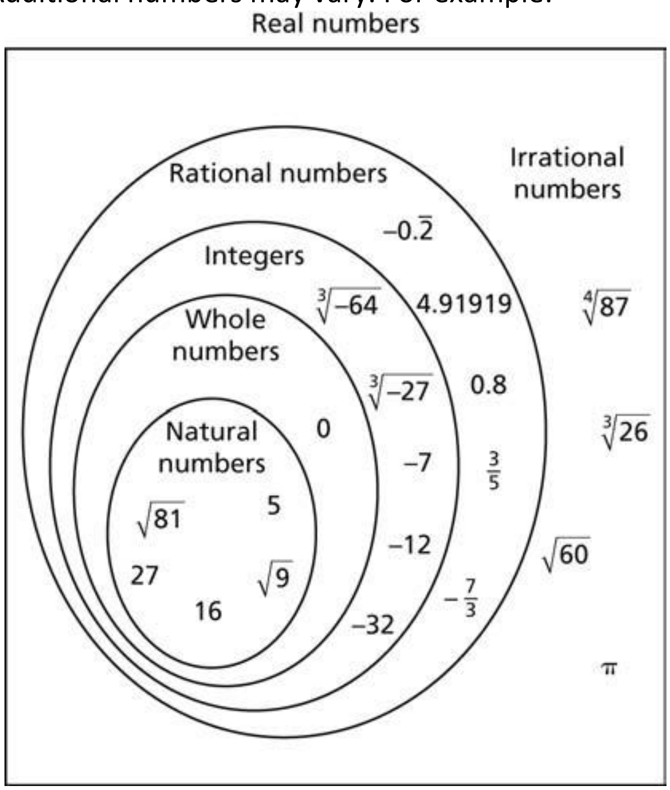
5. $\sqrt[3]{8}=2$ $\sqrt[3]{64}=4$ $\sqrt[3]{30}=3.10723...$ $\sqrt[3]{300}=6.6943295...$
   The cubes roots of the numbers in parts c and d will be irrational.

6. $\sqrt[3]{98},\sqrt{40},\sqrt[3]{300},\sqrt[3]{500},\sqrt{75},\sqrt{98}$

7. a) i) True – Natural numbers are a subset of Integers
   ii) True – Integers are a subset of Rational numbers.
   iii) False – The set of Whole numbers includes 0. 0 is not a Natural number.
   iv) False – Other irrational numbers are numbers like π and ζ and e.
   v) True – Natural numbers are a subset of rational numbers
   b) iii) 0 iv) π

8. Answers will vary. For example:
   a) any fraction or decimal 0.75
   b) 0
   c) any non-repeating, non-terminating number like $\sqrt{7}$

9. Additional numbers may vary. For example:



10. a) $\sqrt{40} = 6.3245...$ Irrational number
    b) $\sqrt{81} = 9$ Rational number

11. a) Yes. Any number that is not a perfect square like $\sqrt{40}$.
    b) No. If the original number is irrational then the square root will be "doubly" irrational.
    For example $\sqrt{\sqrt{7}} = 1.62657...$